Seventh Heaven - Teachers' Notes

Adapted from http://illuminations.nctm.org/BrainTeasers.aspx?id=4686)

Curriculum Links:

This problem is linked to Level 4-5 of the Number and Algebra strand of the Maths curriculum. It requires a student to be able to work multiplicatively and generalise patterns from small numbers to larger numbers knowing what effect multiplication has when working with exponents.

Background:

The problem requires students to work with patterns created by multiplication and have an understanding of exponents.

The solution is as follows:

1. Students may start with finding the whole number for the 2 initial expressions.
   777^2 = 777 x 777 = 603,729. The ones digit is 9.
   777^3 = 777 x 777 x 777 = 469,097,433. The ones digit is 3.
   They will find that the numbers get large very quickly and therefore their calculator will not be able to solve 777^7.

2. This is where they will have to back up and work with smaller numbers to derive the pattern required.
   7^2 = 49 is the same as the ones digit of 777^2 and
   7^3 = 343 is the same as the ones digit of 777^3.

3. A student then may work out all the exponents up to 7^7 and make the assertion that the ones digit will be the same is the same as for 777^7.

Note that 7^7 = 823,453, so the ones digit of 7^7 and 777^7 is 3.

In fact, the ones digit of a product is only influenced by the ones digits of the factors. The ones digit for the seventh power of any number with ones digit 7 will be 3.

Suggestions:

Following this investigation, students may be interested in pursuing exploration in one of these areas:

On the student page there are suggestions for investigating the cultural relevance of the number seven (seven deadly sins, seventh heaven, seven wonders of the world, lucky number seven etc).

There is also the prompt to consider the application of being able to predict digits of large numbers.

Related learning experiences:

- Numeracy Book 8 Applying Remainders, p.32
  http://www.nzmaths.co.nz/sites/default/files/Numeracy/2008numPDFs/NumBk8.pdf
- Level 6 Number Unit on Exponents
  http://www.nzmaths.co.nz/resource/exponent-power
- Figure It Out, Number Book 6 Powerful Thought p.4
  http://www.nzmaths.co.nz/resource/powerful-thought