# *Accelerating* Learning in Mathematics



## Learning to: use multiplication and division to solve problems

**Target group:**
**students in years 5–8**

**Focusing on:**

- applying advanced additive strategies to multiplication and division problems
- flexibly using knowledge of basic facts.

## Beliefs underpinning effective teaching of mathematics:

- Every student's identity, language, and culture is respected and valued.
- Every student has the right to access effective mathematics education.
- Every student can become a successful learner of mathematics.

## Ten principles of effective teaching of mathematics:

1. An ethic of care
2. Arranging for learning
3. Building on students' thinking
4. Worthwhile mathematical tasks
5. Making connections
6. Assessment for learning
7. Mathematical communication
8. Mathematical language
9. Tools and representations
10. Teacher knowledge

## TEACHER OBSERVATION OVER A RANGE OF ACTIVITIES

The student may be able to solve addition and subtraction problems with numbers of three or more digits, using different strategies that depend on the numbers involved. The student may not have strategies beyond repeated addition for solving multiplication and division problems with two-digit numbers. The student may have fast and fluent recall of most of the multiplication basic facts but cannot use this knowledge to derive new facts or flexibly solve problems. For example, a student who knows the 3 and 10 times tables does not use this knowledge to solve 13 x 6. The student may not understand the inverse nature of operations, for example, if 3 x 8 = 24 then 24 ÷ 3 = 8. The student is likely to have difficulty solving problems involving fractions.

## POSSIBLE BARRIERS TO THE STUDENT'S PROGRESS

| | |
|---|---|
| 1 | Limited understanding of arrays |
| 2 | Lack of confidence |
| 3 | Limited understanding of the inverse nature of multiplication and division |

See *Effective Pedagogy in Mathematics* by G. Anthony and M. Walshaw, Educational Practices Series 19, International Bureau of Education, available at: www.ibe.unesco.org

## EXPECTATIONS FOR NUMBER

| AFTER 1 YEAR AT SCHOOL | AFTER 2 YEARS AT SCHOOL | AFTER 3 YEARS AT SCHOOL | BY THE END OF YEAR 4 | BY THE END OF YEAR 5 | BY THE END OF YEAR 6 | BY THE END OF YEAR 7 | BY THE END OF YEAR 8 |
|---|---|---|---|---|---|---|---|
| COUNTING FROM ONE | ADVANCED COUNTING | EARLY PART–WHOLE THINKING | EARLY ADDITIVE | EARLY ADVANCED ADDITIVE | ADVANCED ADDITIVE – EARLY MULTIPLICATIVE | EARLY ADVANCED MULTIPLICATIVE | ADVANCED MULTIPLICATIVE – EARLY PROPORTIONAL |
| NZC EARLY LEVEL 1 / NUMERACY STAGE 2 OR 3 | NZC LEVEL 1 / NUMERACY STAGE 4 | NZC EARLY LEVEL 2 / NUMERACY EARLY STAGE 5 | NZC LEVEL 2 / NUMERACY STAGE 5 | NZC EARLY LEVEL 3 / NUMERACY EARLY STAGE 6 | NZC LEVEL 3 / NUMERACY STAGE 6 | NZC EARLY LEVEL 4 / NUMERACY EARLY STAGE 7 | NZC LEVEL 4 / NUMERACY STAGE 7 |

Aligned to *The New Zealand Curriculum* and *Mathematics Standards for years 1–8*

## BARRIER BEING ADDRESSED 1 LIMITED UNDERSTANDING OF ARRAYS

### DIAGNOSTIC QUESTIONS

If each lolly kebab has 15 lollies on it, how many lollies will Jemma need to make 15 kebabs?
Answer: 225

### WHAT TO NOTICE IN THE STUDENT'S RESPONSE

Does the student use repeated addition, for example, 15 + 15 + 15 + …?

Can they suggest an alternative strategy when asked?

Does the student overgeneralise the use of place value partitioning in addition by combining 10 x 10 = 100 and 5 x 5 = 25 to get 100 + 25 = 125?

### DELIBERATE ACTS OF TEACHING

Arrays can help students move beyond repeated addition by providing a visual representation of multiplication concepts.

Show the student an array. Turn it sideways and ask the student whether the array has changed.

Show the student a simple array, for example, a 3 x 4 array. Give the student 10 seconds to look at it, then hide it. Ask the student to use counters to make the array and to count how many counters there are.

**Area Array Models for Multiplication**
Draw a chalk rectangle on the carpet, small enough to be filled by 12–20 memo cube squares. Ask the student to use the squares to find the area of the rectangle. Explore the multiplicative relationship between length, width, and area.

Increase the area of the rectangle to 50–80 squares, making it impractical to fill it with paper squares. Refer the student back to the length, width, and area relationship of the first rectangle. Use chalk lines to identify different arrays within the rectangle. For example, using a 13 x 6 rectangle, mark out the 10 x 6 section of the array and the 3 x 6 section. Tell the student to record the model as a diagram, including known facts.

This activity can be adapted using Digital Learning Objects, sheets of small stickers, or sticky notes on a table top. Encourage students to use the basic facts that they are familiar with and discourage the use of counting strategies.

### MATERIALS/LINKS

20 memo cube squares

### WHAT TO DO NEXT IF THE STUDENT IS STUCK

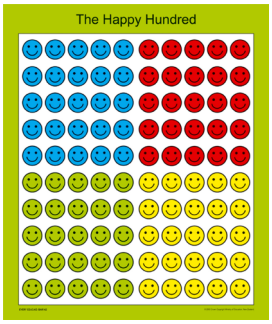
Check the student's knowledge of basic multiplication facts. Revisit smaller arrays. Increase the size of the rectangle gradually until its area can no longer be counted quickly. Note that skip-counting does not indicate the understanding of multiplication as an operation.

### INITIATING HOME-BASED ACTIVITIES

Parents can help the student to recognise and solve array problems within their home and community environments, such as working out the number of cars in a car park, buttons on a remote control, windows in large buildings, or bricks in a wall. The student should locate five array problems each night, record them in their homework book, and show how they used multiplication to solve each problem.

### NEXT TEACHING STEPS BACK IN THE CLASSROOM

Explore square units used in geometry and measurement. Use games such as Multiplication Roundabout (Material master 6-6b) to provide the students with practice in multiplying two-digit numbers by a one-digit number.



**BARRIER BEING ADDRESSED** **2** LACK OF CONFIDENCE

## DIAGNOSTIC QUESTIONS

1. Create a continuum on a piece of paper, with a negative face at one end, a positive face at the other, and a neutral face in the centre.
   Ask the student to move a counter along the continuum to show how they feel when they:
   a. are doing a basic facts test
   b. have to use basic facts to solve a problem
   c. have to learn a new set of basic facts.
2. Ask the student how they feel when they have to recall a basic fact quickly.
3. Ask the student what techniques are useful for learning new facts.
4. Ask the student which basic fact sets they have learned this year.

## WHAT TO NOTICE IN THE STUDENT'S RESPONSE

Does the student indicate any negative feelings towards learning or using basic facts?

Is the student familiar with techniques that they can use to learn basic facts?

## DELIBERATE ACTS OF TEACHING

Needing to memorise a large amount of information can be daunting. Seeing a task broken down into more manageable chunks can help to restore the student's confidence.

**Building up basic facts**
With the student, create a poster showing the sets of basic facts that they need to master. Make sure that the sets are manageable, for example, the poster might include: all the square numbers in the times tables; the 3 times table; doubles to 30; halves involving odd numbers.

Let the student decide which basic fact set on the poster they would like to work on. The set can be as small or as large as the student feels comfortable with. Allow the student to establish a reasonable time target for learning the set.

Make a flow chart to illustrate the following process of learning a set of basic facts:

1. Learn a new group of facts – understand what the facts have in common and find patterns within the group.
2. Practise the new group of facts – move from working out the answer to having the answer come to mind automatically.
3. Use the new facts to solve problems – solve simple problems using basic facts.
4. Quiz me! – attempt a basic fact quiz.

After the student has mastered each set, highlight it on the poster.

### MATERIALS/LINKS

Array game
(Number and Algebra, level 3, www.nzmaths.co.nz)

Digital Learning Object:
Addition and Subtraction Basic Facts

Digital Learning Object:
Multiplication and Division Basic Facts

### WHAT TO DO NEXT IF THE STUDENT IS STUCK

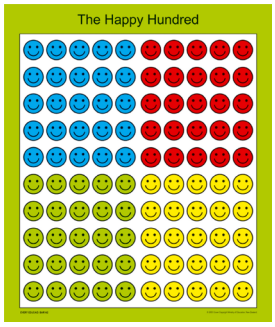
Concentrate on exploring patterns within the set. Break the set into two parts and work on one half at a time. Celebrate any achievements.

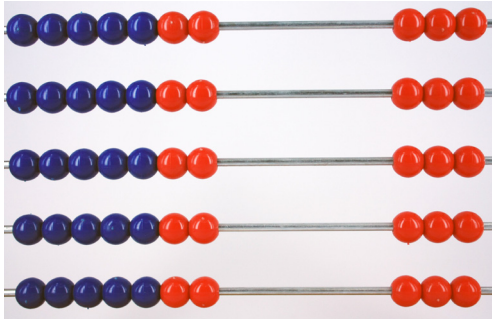
### INITIATING HOME-BASED ACTIVITIES

Direct parents to the Number Facts Activities for Advanced Additive Children page on www.nzmaths.co.nz

### NEXT TEACHING STEPS BACK IN THE CLASSROOM

Ensure that the problems that the student is working on align well with the basic facts set they are trying to master.

**BARRIER BEING ADDRESSED** **3** LIMITED UNDERSTANDING OF THE INVERSE NATURE OF MULTIPLICATION AND DIVISION

## DIAGNOSTIC QUESTIONS

Keri is making packets of mini chocolate fish for a gala. She puts five mini chocolate fish in each packet. The mini chocolate fish come in bags of 90.

Keri wants to know how many packets of five fish can be made from one bag.

a. Write down the division equation that needs to be solved.

Note: If the student cannot record the problem as 90 ÷ 5, record this for them and ask what it means.

b. Solve the equation.
Answer: 18

## WHAT TO NOTICE IN THE STUDENT'S RESPONSE

Does the student attempt to skip-count in fives to 90?

Does the student make connections between multiplication and division facts?

For example, "90 ÷ 10 = 9, and there will be twice as many if you divide by 5" or "9 x 10 = 90 so 18 x 5 = 90".

Does the student correctly explain 90 ÷ 5 as "the number of times 5 goes into 90" or "how many fives there are in 90"?

## DELIBERATE ACTS OF TEACHING

Flexible use of strategies when problem solving relies on the ability to fluently recall basic facts.

**Back and Forth: Family of Facts**
Explore relationships within a family of facts by modelling the relationship between 3 x 4, 4 x 3, 12 ÷ 4, and 12 ÷ 3. Students need to recognise that these are different arrangements of the same set.

Print out the x/÷ flash cards. Take one card and a set of interlocking cubes. Arrange the cubes in an array that models the set. For example, if using the numbers 24, 6, and 4, arrange 24 blocks in four rows of six. Ask the student to show you four groups of six, six groups of four, 24 divided into groups of four, and 24 shared among six people. Record each fact in the family. With practice, students will be able to state the family of facts for any three numbers.

Reinforce that division equations can be found using known multiplication equations.

### MATERIALS/LINKS

x/÷ flashcards
(Material master 4-37)

### WHAT TO DO NEXT IF THE STUDENT IS STUCK

Continue to work with physical models of x/÷ relationships. Use alternative models, such as arrays of stickers or clip art images. When working with paper models, students can use felt pens to circle each factor.

Use the Digital Learning Object: Multiplication and Division Basic Facts.

### INITIATING HOME-BASED ACTIVITIES

Give the students a set of flash cards to practise with at home. Parents can place a card in front of the student, cover one of the numbers, and ask the student to use the other two numbers to work out the covered number. The student can identify one x equation and one ÷ equation for each card.

### NEXT TEACHING STEPS BACK IN THE CLASSROOM

Make a set of flash cards available for independent or pair-practice use.

The Digital Learning Object: Multiplication and Division Basic Facts can be used to provide additional practice.