Accelerating **Learning in Mathematics**

DRAFT

Learning to: use part–whole thinking to solve problems

**Target group:
students in years 1–4**

Focusing on:
- part–whole thinking
- understanding place value up to 100
- recalling the basic facts required for early additive strategies

Beliefs underpinning effective teaching of mathematics:

- Every student's identity, language, and culture is respected and valued.
- Every student has the right to access effective mathematics education.
- Every student can become a successful learner of mathematics.

Ten principles of effective teaching of mathematics:

1. An ethic of care
2. Arranging for learning
3. Building on students' thinking
4. Worthwhile mathematical tasks
5. Making connections
6. Assessment for learning
7. Mathematical communication
8. Mathematical language
9. Tools and representations
10. Teacher knowledge

See *Effective Pedagogy in Mathematics* by G. Anthony and M. Walshaw, Educational Practices Series 19, International Bureau of Education, available at: www.ibe.unesco.org

TEACHER OBSERVATION OVER A RANGE OF ACTIVITIES

The student may be able to confidently count on or back to solve addition and subtraction problems but cannot use part–whole strategies.

POSSIBLE BARRIERS TO THE STUDENT'S PROGRESS

1	Limited part–whole thinking
2	Limited knowledge of groupings with ten and with the number of tens in decades
3	Limited recall of addition and subtraction basic facts

EXPECTATIONS FOR NUMBER

AFTER 1 YEAR AT SCHOOL	AFTER 2 YEARS AT SCHOOL	AFTER 3 YEARS AT SCHOOL	BY THE END OF YEAR 4	BY THE END OF YEAR 5	BY THE END OF YEAR 6	BY THE END OF YEAR 7	BY THE END OF YEAR 8
COUNTING FROM ONE	ADVANCED COUNTING	EARLY PART–WHOLE THINKING	EARLY ADDITIVE	EARLY ADVANCED ADDITIVE	ADVANCED ADDITIVE – EARLY MULTIPLICATIVE	EARLY ADVANCED MULTIPLICATIVE	ADVANCED MULTIPLICATIVE – EARLY PROPORTIONAL
NZC EARLY LEVEL 1 / NUMERACY STAGE 2 OR 3	NZC LEVEL 1 / NUMERACY STAGE 4	NZC EARLY LEVEL 2 / NUMERACY EARLY STAGE 5	NZC LEVEL 2 / NUMERACY STAGE 5	NZC EARLY LEVEL 3 / NUMERACY EARLY STAGE 6	NZC LEVEL 3 / NUMERACY STAGE 6	NZC EARLY LEVEL 4 / NUMERACY EARLY STAGE 7	NZC LEVEL 4 / NUMERACY STAGE 7

Aligned to *The New Zealand Curriculum* and *Mathematics Standards for years 1–8*

BARRIER BEING ADDRESSED	1	LIMITED PART–WHOLE THINKING

DIAGNOSTIC QUESTIONS

1. a. What number comes after 4?
   b. What number comes before 8?

2. a. What are two numbers that can be added to make 5?
   b. Ask the student to find other pairs of numbers that can be added to make 5.

3. a. What number comes after 9?
   b. What number comes before 13?
   c. What number comes after 24?

4. a. What are two numbers that can be added to make 10?
   b. Ask the student to find other pairs of numbers that can be added to make 10.

WHAT TO NOTICE IN THE STUDENT'S RESPONSE

Does the student know forward and backward number sequences?

Can the student recall basic facts for making 5?

If necessary, provide blue and yellow counters and ask the student to use counters of both colours to make a set of five. Use their response to write an equation. For example, if they use four yellow counters and one blue counter, write out 4 + 1 = 5.

Can the student recall basic facts for making 10?

If necessary, provide blue and yellow counters and a tens frame. Ask the student to use counters of both colours to fill the tens frame. Use their result to write an equation. For example, if they use eight yellow counters and two blue counters, write out 8 + 2 = 10.

DELIBERATE ACTS OF TEACHING

Developing part–whole thinking allows students to be flexible in their use of numbers. This enables students to use different strategies to solve problems, depending on the numbers involved.

Making Five
Ask the student to count out five double-sided counters and to place them in a container.

Shake the container and then scatter the counters. Show the student that they can use the colours of the counters to make an addition equation. For example, if there are two red counters and three blue counters, write out the equation 2 + 3 = 5, using a red pen to write the 2 and a blue pen to write the 3.

Ask the student to put the counters back into the container and to make their own equations by repeating the process you have modelled.

Making Ten
Repeat the process above using ten counters. Ask the student to place the counters on a blank tens frame and to record each result as an equation.

This activity can be repeated using different totals or counters of three colours. For example, if eight counters of three colours are scattered, the result could be 3 + 3 + 2 = 8. Show the student that this is the same as 3 + (3 + 2) = 3 + 5 = 8.

Extend the activity by using a full tens frame and then scattering up to ten counters for the student to add to the counters in the frame. For example, using eight additional counters, the student may get three red and five blue counters and can use them to show that 10 + (5 + 3) = 18.

MATERIALS/LINKS

Double-sided counters. These counters can be bought or can be made by sticking two different-coloured counters together.

Tens frames
(Material master 4-6)

Digital Learning Object: Number Partner
(Number, level 1)
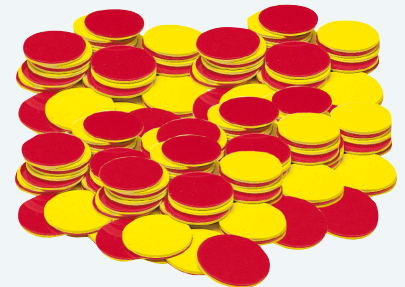
WHAT TO DO NEXT IF THE STUDENT IS STUCK
Work with small sets up to and including five. Build patterns using interlocking cubes blocks and relate each pattern to an equation. Talk about numbers that make 5. Try to help the student see that inside any number greater than 1, there are parts that can be combined to create the number.

INITIATING HOME-BASED ACTIVITIES
Parents can support the student to recognise pairs of numbers that make up a larger number by using activities from www.nzmaths.co.nz. See Stages 0–3: Number Facts with 10 in the "Activities to support your child" section on www.nzmaths.co.nz

NEXT TEACHING STEPS BACK IN THE CLASSROOM
Concentrate on helping the student to identify pairs of numbers within numbers when solving problems. Show the student how part–whole strategies can be used to solve addition problems such as 14 + 8 = (14 + 6) + 2 = 20 + 2 = 22.

BARRIER BEING ADDRESSED	2	LIMITED KNOWLEDGE OF GROUPINGS WITH TEN AND WITH THE NUMBER OF TENS IN DECADES

DIAGNOSTIC QUESTIONS

Write the numbers 14, 40, and 26 on separate cards. Provide the student with a set of bundles and sticks (iceblock sticks bundled in groups of ten and single sticks).

1. Ask the student to count how many sticks there are in a bundle.
2. Show the student the card with 14 written on it.
   a. Ask the student to read out the number.
   b. Ask the student to use the iceblock sticks to show you the number.
3. Show the student the card with 40 written on it.
   a. Ask the student to read out the number.
   b. Ask the student to use the iceblock sticks to show you the number.
4. Show the student the card with 26 written on it.
   a. Ask the student to read out the number.
   b. Ask the student to use the iceblock sticks to show you the number.

WHAT TO NOTICE IN THE STUDENT'S RESPONSE

Can the student correctly read the numbers? (Listen carefully to ensure that they distinguish clearly between fourteen and forty.)

Does the student try to count out the numbers using single sticks?

Does the student identify that the digit in the tens place is the number of groups of ten?

DELIBERATE ACTS OF TEACHING

Students who struggle with sequencing sometimes perform better when working with a vertical number strip. The pattern of the "teen numbers" all beginning with 1 is visually clearer when the numbers are presented vertically.

Constructing Teens

Work with the student to relate each number on the vertical number strip to models made using a place value chart and iceblock sticks. Place a transparent yellow counter over the 1 on the vertical number strip and position one stick in the ones column of the place value chart. Move the counter up to 2 and ask the student to add another stick to the ones column of the place value chart. Continue to move the counter up the number strip until you get to 10.

Explain that when the ones column has ten sticks in it, the sticks need to be bundled together and moved to the tens column. Use a rubber band to bundle the set. Ask the student to place the bundle in the tens column. Draw attention to the labels on the columns. Ask the student how many tens there are and how many ones there are. Have them practise reading 10 as "1 ten and no ones".

Move on to the teen numbers. Ask the student to refer to each number as a number of tens and a number of ones. For example, for 13, the student should say "1 ten and 3 ones".

Constructing Decades

Repeat the process above, using the decade column of a hundreds chart instead of the number strip. Refer to the "-ty" numbers as numbers of tens with no ones, for example, 40 is 4 tens and no ones.

Extend to two-digit numbers, for example, 34 is 3 tens and 4 ones. Keep emphasising the connection between the way the number is written, its model, and its meaning.

MATERIALS/LINKS

Sticks and bundles

Simple place value chart: Create two columns, labelled "tens" and "ones", on an A3 piece of card (portrait orientation). Laminate the card.

A vertical number strip from 1 to 20, with 1 at the base and 20 at the top

A hundreds chart

WHAT TO DO NEXT IF THE STUDENT IS STUCK
Work on one-to-one counting and on recognising the numbers from 1 to 20.

INITIATING HOME-BASED ACTIVITIES
Provide students with iceblock sticks and bundles to take home for extra practice. Give the student a list of two-digit numbers and ask them to make them at home with their parents. For additional activities, use Tens in Tens and "Ten And" Facts on www.nzmaths.co.nz

NEXT TEACHING STEPS BACK IN THE CLASSROOM
Use place value equipment when developing strategies for addition and subtraction. Provide opportunities for targeted work on teen numbers and the "-ty" numbers.

BARRIER BEING ADDRESSED	3	LIMITED RECALL OF ADDITION AND SUBTRACTION BASIC FACTS

DIAGNOSTIC QUESTIONS

1. Tell the student that you are filling boxes of eggs. The boxes are similar to tens frames. Show the student an empty tens frame and ask the following questions:

   a. If I have already put seven eggs in the box, how many eggs do I need to fill the box?

   b. If I give you a full box and you take out four eggs, how many eggs are left in the box?

2. Tell the student that you have a machine that doubles numbers. If you put in a 2, out comes a 4!

   What will the machine produce if you put in the number:

   a. 3?

   b. 5?

   c. 8?

   d. 10?

WHAT TO NOTICE IN THE STUDENT'S RESPONSE

Does the student have fast, fluent recall of their basic facts for:

- addition and subtraction facts to 10?
- doubling to 20?

Observe whether the student uses another method to solve each problem, for example, counting on their fingers.

DELIBERATE ACTS OF TEACHING

Identifying patterns and relationships is a key component of mathematical thinking. When learning a new basic facts set, the emphasis should be on making connections between different facts.

+/– Flashcards

Have the student explore the relationships between the three numbers on each card by writing out families of addition and subtraction equations. For example, for 3, 9, 6, the student could write and read aloud $3 + 6 = 9$, $6 + 3 = 9$, $9 – 6 = 3$, and $9 – 3 = 6$. Cover up one of the numbers and ask the student to work out what the hidden number is. Have them practise with a small number of cards until they can work out the answers without reverting to counting, gradually adding cards as the student recalls each fact more easily.

Memory Doubles

Give the student practice at naming the doubles facts to 20, starting with doubles to 10. Play the Memory Doubles game (www.nzmaths.co.nz) with the student.

MATERIALS/LINKS

+/– Flashcards
(Material master 4-29)

Number Boggle
(Material master 4-35)

Memory Doubles
www.nzmaths.co.nz

WHAT TO DO NEXT IF THE STUDENT IS STUCK

Help the student to develop part–whole thinking by using counters and coloured pens. Ask the student to make a group of eight using counters, starting with one blue counter and seven red counters. Write the equation using blue, red, and black pens. Ask the student to make eight using two blue counters, and then three blue counters, continuing until there are eight blue counters and no red counters. Give the student another number to work with and ask them to write as many equations as they can, using the counters as a guide.

Give the student practice with doubles to 10, encouraging them to use their fingers as a visual aid.

INITIATING HOME-BASED ACTIVITIES

Number Boggle

Print out the Number Boggle game (www.nzmaths.co.nz). Ask the student to use it to find at least five equations and to write them out in full.

Number Facts Activities for Advanced Counting Children (www.nzmaths.co.nz) provides suggestions of activities that can be played at home.

NEXT TEACHING STEPS BACK IN THE CLASSROOM

Continue working on the transition from advanced counting to early additive, using the learning experiences described in *Book 5: Teaching Addition, Subtraction, and Place Value.*