Notes for parents (1).

The purpose of the activity is to help your student to:

- Apply Pythagoras' Theorem in practical situations.
- Apply trigonometric ratios (sine, cosine, tangent) in practical situations.

Here is what to do:

Read through the problem together. The first question is about angle of elevation. This is an application of the trigonometric ratio, tangent. For more information about these important ratios read Points to Note below.

The angle can be worked out in this way:

tangent=opposite/adjacent

In this case tangent θ=18/28 Note that 18/28 is another way to express 18 divided by 28.

To find the angle, θ, we need to apply the inverse tangent (tan-1) function. Inverse tangent is like asking the calculator, "Which angle has a tangent of …?"

18 divided by 28 is 0.643, and the inverse tangent of 0.643 is 32.74°.

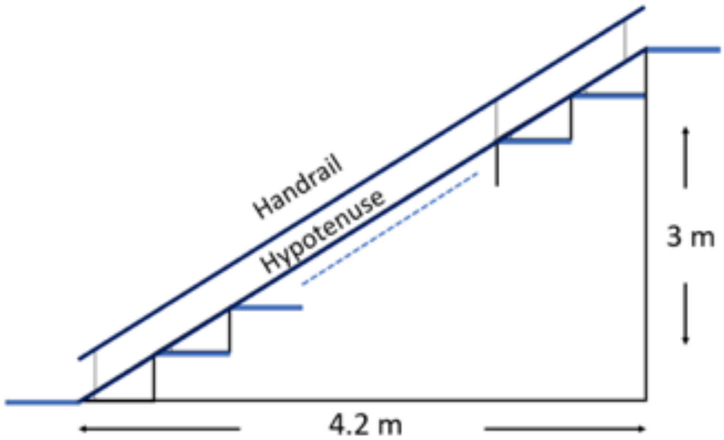
The second question is about working out how many steps are needed to form a staircase that rises 3 metres, a common height between floors. Your student needs to stay as close as possible to the given rise over run ratio of 18:28. Calculating 300 ÷ 18 = 16.666… which means $16\frac{2}{3}$. One way to deal with the remainder is to have the last step with a rise of $\frac{2}{3}$ x 18 = 12. That is not normal practice as steps should be at the same height to make walking easier.

Another option is to find a tidy step-rise that divides into 300cm exactly and is close to 18cm. Two possibilities are 300 ÷ 15 = 20 and 300 ÷ 20 = 15. An architect would probably go for 20 steps with a rise of 15cm rather than a steeper stairwell with steps of 20cm rise. However, that depends on the space available in the house.

Twenty steps with a rise of 15cm each would require a staircase that is 20 x 28 = 560cm or 5.6 metres. That is a very long staircase that occupies a lot of ground floor space. Fifteen steps would require 15 x 28 = 420cm or 4.2 metres. That is considerably shorter than 5.6m.

Your student may come up with other answers that can be justified and are close enough to the 18:28 ratio. The handrail is the length of the long side (hypotenuse) of the triangle.

Notes for parents (2).



To find the length of the hypotenuse your student needs to use apply Pythagoras' Theorem. In the example above $h^2 = 32 + 4.22$, where 32 means "three multiplied by itself" and h represents the length of the hypotenuse. Here is the chain of working:

$h^2 = 32 + 4.22$

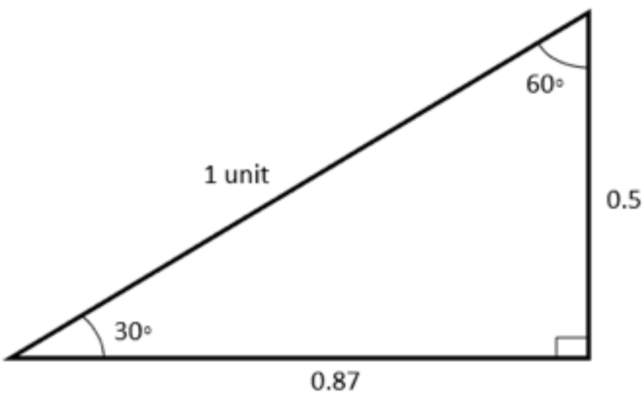
$h^2 = 9 + 17.64$

$h = \sqrt{(26.64)}$ (Finding the square root undoes squaring)

$h = 5.16$ metres

Points to note

Trigonometry is a branch of geometry that deals with measurement of triangles. It was developed by Greek mathematicians from the third century BC on. Right-angled triangles have particular significance in many fields such as construction and navigation. Consider this special right-angled triangle that has a hypotenuse (longest side) of 1 unit.



There are three interesting side to side ratios in relation to the angle of 30°. These ratios are:

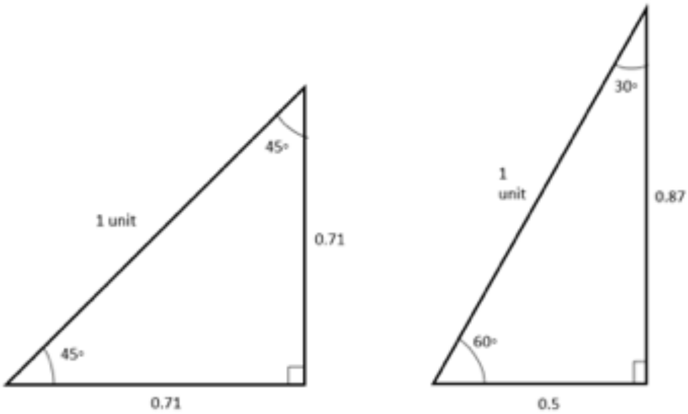
Notes for parents (3).

Sine is the ratio of opposite/hypotenuse (opposite ÷ hypotenuse) which for
Sine 30° = 0.5/1=0.5.

Cosine is the ratio of adjacent/hypotenuse (adjacent ÷ hypotenuse) which for
Cosine 30° = 0.87/1=0.87.

Tangent is the ratio of adjacent/opposite (opposite ÷ adjacent) which for
Tangent 30° = 0.5/0.87=0.58.

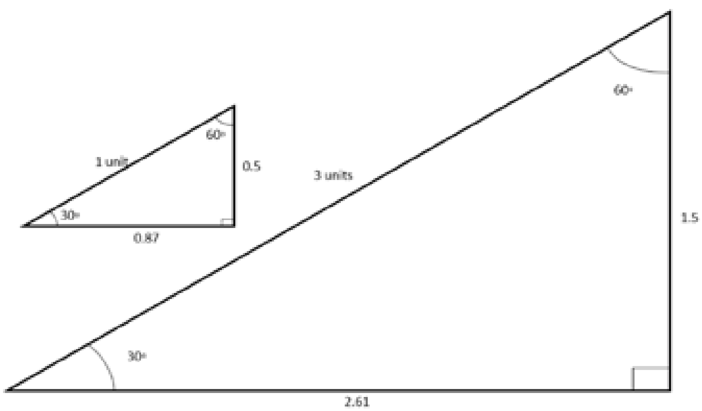
Imagine that the angle in the bottom left increases but the hypotenuse stays at length of one unit. What happens to the lengths of the opposite and adjacent sides?



As the angle increases the length of the opposite side increases but the adjacent side decreases. So sine gets larger and cosine gets smaller. Tangent also increases. The sine, cosine and tangent values are unique to the angle they apply to in this special right-angled triangle with an hypotenuse of one.

So how is that useful?

The ancient greeks realised that side lengths stay in proportion as the same triangle is enlarged or shrunk. The angles stay the same. Look at this enlargement of the 30° and 60° right-angled triangle. Each side length in the enlarged triangle is three times the length of the matching side in the small triangle.



Notes for parents (4). Activity next page.

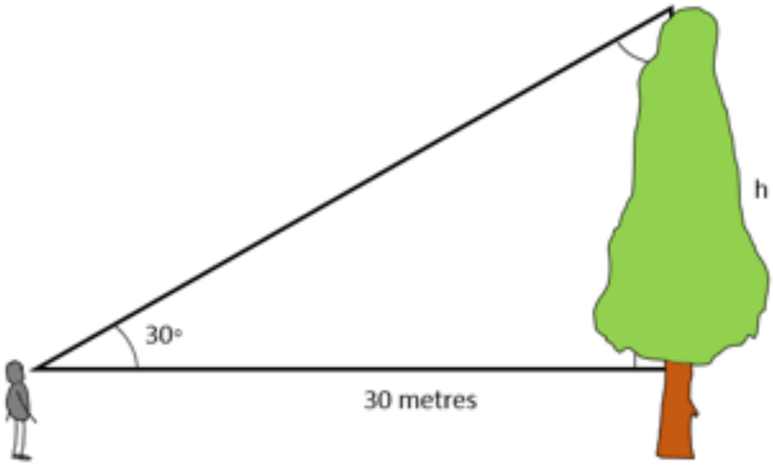
Though the triangles are different sizes the ratios remain constant. Let's calculate the ratios for the larger triangle.

Sine is the ratio of opposite/hypotenuse (opposite ÷ hypotenuse) which for Sine 30° = 1.5/3=0.5.

Cosine is the ratio of adjacent/hypotenuse (adjacent ÷ hypotenuse) which for Cosine 30° = 2.61/3=0.87.

Tangent is the ratio of adjacent/opposite (opposite ÷ adjacent) which for Tangent 30° = 1.5/2.61=0.58.

So for bigger and smaller right-angled triangles with the same angles, the sine, cosine, and tangent ratios can be trusted to stay constant. In the real world you can't always measure lengths and angles because they are hard to access. Suppose you want to know the height of a tree before you fell it. You already know three measurements for the triangle. It is at 90° to the ground and the angle of elevation is 30° when you are 30 metres aways from it.



The height of the tree (h) is the opposite side of the triangle and you know the adjacent side. Tangent is the ratio of opposite to adjacent sides. You can get the tangent of 30° from your calculator.

Tangent 30° = opposite/adjacent

0.58 = h/30

0.58 x 30 = h

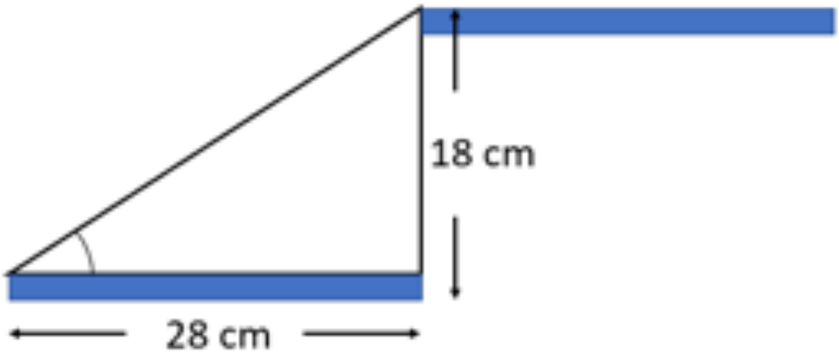
17.4 = h

So the height of the tree is 17.4 metres plus the eye height of the person. The inknown in a real life triangle can be a side or an angle. It is all very useful. Ask someone who designs trusses for roofs of houses.

For thousands of years people have built staircases to connect the floors in buildings.

A rule of thumb for the steepness of a staircase is shown below.

For every 28 centimetre landing step (run) there is a rise of 18 centimetres.



What is the angle of ascent? That is the marked angle.

Suppose you need to build a staircase that rises 3 metres between the ground floor and the first floor of your house.

How many rises would you need to stay close to the rule of thumb?

What would be the horizontal distance (run) of the whole stairway?

How long would the handrail be?