Task notes | Savings plan

Y9–10

Notes for parents (1).

The purpose of the activity is to help your student to:

- Predict further members of linear patterns.
- Express rules for linear growth using algebra.
- Manipulate linear equations to solve problems.

Here is what to do:

Read the problem together identifying the important information. The most important information is the condition that satisfies the problem; Georgia has twice as much saved as Jack. Your student may use a table to organise how much money Jack and Georgia have saved at the end of each week.

Weeks	1	2	3	4	5	6
Jack's savings	\$110	\$120	\$130	\$140	\$150	\$160

Weeks	1	2	3	4	5	6
Georgia's savings	\$125	\$150	\$175	\$200	\$225	\$250

Interrupt the table completion and ask your student to predict what will happen as the pattern continues.

In what week will Georgia save twice as much as Jack?

You student might note that the gap between savings amounts is increasing week by week. By how much is the gap increasing? (\$15 per week)

Since the problem involves a multiplicative comparison then the gap amount is not particularly useful.

Your student might then complete the table to find the week at which Georgia's savings are twice those of Jack.

Weeks	1	2	...	18	19	20
Jack's savings	\$110	\$120	...	\$280	\$290	\$300

Weeks	1	2	...	18	19	20
Georgia's savings	\$125	\$150	...	\$550	\$575	\$600

Notes for parents (2).

Since $600 is twice $300 the problem has been solved. You might discuss how the problem can be solved more efficiently. That is where algebra comes in.

If t represents the number of weeks, then each person's saving plan can be written like this:

$j = 100 + 10t$ (Jacks savings are $100 plus the number of weeks multiplied by $10)

$g = 100 + 25t$ (Georgia's savings are $100 plus the number of weeks multiplied by $25)

We need the situation where $g = 2j$, that is, Georgia's savings are twice that of Jack's savings. The two savings equations can now be put together.

$$2(100 + 10t) = 100 + 25t$$

(find the value of t that makes the equation true)

(expand the brackets by multiplying)

$$200 + 20t = 100 + 25t$$

(subtract 100 from both sides)

$$100 + 20t = 25t$$

(subtract 20t from both sides)

$$100 = 5t$$

(divide both sides by 5)

$$20 = t$$

Algebra is a lot more efficient than using numbers which is why it was invented.

The final question is designed to show students that number methods can be very laborious. Can Georgia's savings ever be three times as much as Jack's savings? Even setting up a spreadsheet is cumbersome.

	A	B	C	D
1	Time (weeks)	Jack's savings	Georgia's savings	Three times Jack's savings
2	0	100	100	300
3	1	110	125	330
4	2	120	150	360
5	3	130	175	390
6	4	140	200	420
7	5	150	225	450
8	6	160	250	480
9	7	170	275	510
10	8	180	300	540
11	9	190	325	570
12	10	200	350	600

Notes for parents (3). Activity next page.

No matter how far down the spreadsheet is extended it is impossible to find a week where three times Jack's savings equals Georgia's savings. Trying to find a solution using algebra shows that there is no value of t that works.

$$3(100 + 10t) = 100 + 25t$$

(expand the brackets by multiplying)

$$300 + 30t = 100 + 25t$$

(subtract 25t from both sides)

$$300 + 5t = 100$$

(subtract 200 from both sides)

$$5t = -200$$

(divide both sides by -5)

$$t = -40$$

The solution can only be found going backwards in time by 40 weeks and the savings amounts would be negative (-$300 for Jack and -$900 for Georgia). That makes no sense in the context. The ratio of Georgia's saving divided by Jack's savings converges on three as t increases but never reaches it.

Points to note:

Algebra is a powerful system for problem solving. However, it has some foundational ideas that students often get confused about. The equation to solve the first problem was: $2(100 + 10t) = 100 + 25t$

Let's look at the symbols in the equation and what they mean. The equals sign represents a state of balance between the two quantities on both sides. That would be easier if the quantities were numbers like $4 + 5 = 3 + 6$ but t is a quantity that is not known. In fact, t could be any number, so it represents a variable.

Equality is maintained when the same operation is performed on both sides of the equations. For example, consider this step:

$$300 + 30t = 100 + 25t$$

(subtract 25t from both sides)

$$300 + 5t = 100$$

Subtracting a number like 25 from both sides is easier to comprehend than subtracting 25 lots of a number, t, that we do not know the value of. Treating t as a quantity without knowing its value is called "accepting lack of closure."

There also subtle conventions in algebra that can cause confusion as well. The first equation, $2(100 + 10t) = 100 + 25t$ has brackets and a missing multiplication sign. $2(100 + 10t)$ means $100 + 10t$ multiplied by two and that multiplication must follow the distributive rule to get $2(100 + 10t) = 200 + 20t$.

Activity | **Savings plan**

Y9–10

Both Jack and Georgia have $100 already in their savings accounts.

They have an overseas holiday in 6 months time.

Jack plans to save $10 per week.
Georgia plans to save $25 per week.

After how many weeks does Georgia have twice as much saved as Jack?

Try to solve the problem using algebra. You need to use t to represent time in number of weeks. Use j and g to represent the amount, in dollars, that Jack and Georgia have in their accounts.

Saving at those rates how many weeks will it take Georgia to have three times as much saved as Jack?