MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Book 4
Teaching Number Knowledge

Numeracy Professional Development Projects

NUMERACY AND THE CURRICULUM

Numeracy arises out of effective mathematics teaching. All the strands in the mathematics and statistics learning area are important in the pathway to numeracy. Number and Algebra is central to this pathway, although the relative emphasis on this strand changes with the stages of schooling:

- in the first four years of schooling, the main emphasis should be on the Number and Algebra strand;
- in the middle and upper primary years of schooling, the emphasis is spread across all strands of the learning area;
- towards the end of compulsory schooling, number sense becomes a tool for use across the other strands.

The Venn diagrams in the curriculum statement represent this change in balance across the levels. At all stages, students should:

- create models and predict outcomes, conjecture, justify and verify, and seek patterns and generalisations
- estimate with reasonableness and calculate with precision
- understand when results are precise and when they must be interpreted with uncertainty.

From *The New Zealand Curriculum,* page 26

Effective numeracy programmes provide students with a range of tools and skills necessary for everyday life and future endeavours.

> By studying mathematics and statistics, students develop the ability to think creatively, critically, strategically, and logically. They learn to structure and to organise, to carry out procedures flexibly and accurately, to process and communicate information, and to enjoy intellectual challenge.
>
> *The New Zealand Curriculum,* page 26

Although the groundwork is laid in mathematics, other curriculum areas also provide opportunities for numeracy learning. In addition, the home, early childhood settings, and the community assist in the development of numeracy.

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Note: Teachers may copy these notes for educational purposes.

This book is also available on the New Zealand Maths website,
at www.nzmaths.co.nz/Numeracy/2008numPDFs/pdfs.aspx

Teaching Number Knowledge

Knowledge Activities

The activities in this book, if modified appropriately to meet the needs of students, can be used as whole-class warm-ups at the beginning of a lesson. Many of the activities can also be used with individuals and groups of students to build and maintain key knowledge.

Some items of knowledge are critical enablers in helping students make strategy stage transitions. These key items of knowledge are listed in the planning formats provided in *Book 3: Getting Started* under the heading of “Key Knowledge Required”.

It is also important that students are given opportunities to enhance their knowledge while they are developing strategies. Suitable knowledge outcomes for each stage and operational domain are included in the planning formats under the heading of “Knowledge Being Developed”.

The following key is used in each of the teaching numeracy books. Shading indicates which stage or stages the given activity is most appropriate for. Many activities, given suggested modifications, are suitable for a range of stages. Note that CA, “Counting All”, refers to all three counting from one stages.

Strategy Stage

	Strategy Stage
E →	Emergent
CA →	Counting All (One-to-one Counting, Counting from One on Materials or by Imaging)
AC →	Advanced Counting
EA →	Early Additive Part-Whole
AA →	Advanced Additive–Early Multiplicative Part-Whole
AM →	Advanced Multiplicative–Early Proportional Part-Whole
AP →	Advanced Proportional Part-Whole

The table of contents below details the main sections in this book. These sections reflect the key knowledge domains as described on pages 18–22 of *Book One: The Number Framework.*

Number Identification

Number Mat and Lily Pads

E

CA

I am learning to identify single-digit, then two-digit numbers.

Equipment: A large piece of cloth or PVC with the numbers 0 to 9 arranged randomly over it, or an A4 number mat for the students to use in pairs (Material Master 4–13). Large numeral cards showing 0 to 9 (Material Master 4–3).

Activity

Ask a student to jump on a number that you have chosen for them. The students seated around the outside of the mat can write the answer on the floor in front of them with their fingers, and you can quickly see from this which students can identify the number correctly. The students can play "Twister" by having three or four numbers called out consecutively. The students must use their hands, feet, or head to touch each new number while leaving the other numbers touched. Two or three students can be on the mat at the same time and can be given different sets of numbers to touch.

5	8	9	3
2	7	5	0
0	1	4	6
7	9	2	8

Activity

Extend the previous activity to the Number Sequence and Order part of the Number Framework by asking the students to touch the numbers before or after a given number or set of numbers.

Activity

Use numeral cards lined up in order to create lily pads. The students act as frogs and jump on specific numbers, sequences of numbers, or the number just after or just before a given number. Make up different sets of numeral cards to reinforce recognition of larger numbers, for example, 40, 41, 42, 43, ... 50.

Tens Frames

CA

AC

EA

I am learning to identify single-digit numbers and their matching patterns.

Equipment: Tens frames (Material Master 4–6). Numeral cards (Material Master 4–1).

Activity

This activity warrants constant repetition to help establish images of all numbers 10 or less. Begin by linking the dot patterns on tens frames to finger patterns. For example, show a "7" card and ask the students to show five fingers on one hand and two fingers on the other hand. Over time, the students will develop the ability to recognise instantly the number of dots without counting.

Get the students to link these patterns with the numbers to 10. Show a numeral card and tell the students to show the finger pattern or find the matching tens frame. The students should develop the ability to image the finger pattern without counting, by hiding fingers behind their backs.

The students can be seated in pairs, one behind the other. Show a tens frame to one member of each pair. That student draws the pattern as dots with gentle taps on their partner's back. The partner then writes the matching number in the air with their finger then mimics the dot pattern in the air as well.

Extension Activity

When the students are familiar with the tens frames, ask them to identify the number of spaces on the cards as well as the number of dots. In each case, "dots" plus "spaces" equals 10. For example, seven dots plus three spaces makes 10. Record these results using numeral cards or by writing equations like 7 + 3 = 10 on the board or modelling book.

"Teen" and "Ty" Numbers

I am learning to identify "teen" and "ty" numbers.

CA
AC
EA

Equipment: Student copies and class copy of hundreds boards (Material Master 4–4). Tens frames (Material Master 4–6). Slavonic abacus (preferably with a backing sheet to aid in imaging numbers). Transparent counters.

Background

"Teen" numbers can be very troublesome for some students because they do not realise that "teen" means 10, and the usual rule of saying tens before ones is broken. For example, sixteen means literally "six and ten", which really should be "ten and six". Also 11 and 12 break the "teen" pattern by *not* being "oneteen" and "twoteen". Further confusion can occur for the students who fail to realise words ending in "ty" are tens. In particular, many students confuse "sixty" with "sixteen". Persistence in teaching "teen" and "ty" is needed in overcoming these problems as understanding these numbers is essential by the time part-whole thinking emerges.

Activity – "Teen" Numbers

Seat the students in pairs. One partner shows 10 fingers. The other partner shows any number of fingers from one to nine, say six. The "ones" person says "six" and the other partner says "10", and together they say "is six*teen*".

As a class, record teen numbers as equations on the board or modelling book, and get the students to read them out loud. For example 10 + 4 = 14 is on the board or modelling book. The students say "Ten and four is the same as (equals) 14."

Activity – "Teen" Numbers

In pairs, one student points to a number between 10 and 20 on the hundreds board and the other student reads the number. Then together they show that many fingers. Repeat with the roles reversed.

Activity – "Teen" Numbers

On the Slavonic abacus, push across 10 beads on the first row, and then push across extra beads for any amount from 11 to 19, to allow the students to practise recalling the "teen" number facts.

Activity – "Ty" Words

Screen a Slavonic abacus from the students' view while you move complete rows of 10. Turn the abacus around and ask the students to tell you how many beads they can see. Link the number of tens to the structure of the word, for example, "eight tens is eighty", and its numeral on the hundreds board, for example, 80.

Activity –"Teen" and "Ty" Bingo

Every student has a hundreds board and eight transparent counters. Each student places transparent counters on any eight "teen" and "ty" numbers of their choice. You show a succession of "teen" and "ty" numbers on the Slavonic abacus. If any student has a counter on the matching number, they remove that counter. The first player to remove all their counters wins.

Pipe Cleaner Numbers

E
CA
AC

I am learning to identify single-digit numbers, then two-digit numbers.

Equipment: Two pipe cleaners for each student.

Activity

Say a number. (The number size will depend on the ability of the students.) The students are to make that number with their pipe cleaner/s. For two-digit numbers, ensure that the students are placing their digits in the correct order. For example, thirty-one is 31 not 13.

Number Fans

E
CA
AC
EA
AA
AM

I am learning to identify whole numbers and decimals, and to count forwards and backwards.

Equipment: A number fan for each student (Material Masters 4–10 and 4–18), or commercially made number fans.

Activity – Whole Numbers

The students use the fans to show numbers. As the students' knowledge develops, bigger whole numbers may be used. For example: 8, 4, 7 ..., then 24, 48 ..., then 134, 178 ..., then 2 345, 8 034 ..., then 45 702, 803 856 ....

Normally no digit can be repeated in a number because the fans have no repeated digits. However, some commercial fans do have repeated digits.

Activity – Decimal Numbers

Repeat the previous activity. Material Master 4–18 contains a decimal point, or a loose card with a dot printed on it may be used as the decimal point. With decimals, be careful not to say "six point seventy-five" as this may confuse decimals fractions with other uses of the decimal point in money and measurement such as six dollars seventy-five cents and 6.75 metres. Instead say "six point seven, five".

Activity – Number Sequence and Order

Extend the use of number fans to the Number Sequence and Order part of the Number Framework by using problems like:

Show the number that comes after or before	6, 17, 19 ...	456 ...	
Show the number 10 after or before	46 ...	783 ...	41 895 ...
Show the number 100 after or before	357 ...	92 863 ...	
Show the number two greater than	30, 45, 247 ...	64 723 ...	3.5 ...
Show the number in between	23 and 25 ...	456 and 458 ...	
Show the decimal one-tenth more or less than	2.3, 5.7, 8.03, 43.092 ...		
Show the decimal one-hundredth more or less than	1, 5.62, 3.8 ...		
Show the decimal for	$\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{5}$, $\frac{7}{8}$ ...		
Show the decimal for	40%, 85%, 123%, 12.5% ...		

Place Value Houses

AC EA AA AM AP

I am learning to identify multi-digit numbers up to trillions.

Equipment: Place-value houses (Material Master 4–11). Numeral cards (Material Master 4–1).

Activity

Explain to the students how each place-value house is broken into hundreds, tens, and ones. Help the students to read the numbers in their house positions. In particular, assist the students to read numbers like 34 009 083 080, where the zeros must be noticed but are not read out loud. Notice the first house needs no name. (It is called "The Trend Setter House" in Material Master 4–11 because it starts the pattern of column names within every house.)

Billions			Millions			Thousands					
H	T	O	H	T	O	H	T	O	H	T	O

Give the students a number and get them to add the place-value houses then read aloud the number. Once the students' knowledge is secure ask them to read numbers like 34 908 345 002 without houses.

Extension Activity

Go on to reading numbers in the quadrillions and quintillions houses, and/or include the decimal place values tenths, hundredths, thousandths, ...

Number Hangman

CA AC EA AA AM

I am learning to identify ones, tens, hundreds, and thousands.

Equipment: Whiteboard and pen.

Activity

Place dashes on the whiteboard to indicate how many digits are in the number. The students can ask questions about specific places, like, "Is there a five in the tens place?"

They may also ask digit related questions, like, "Does the number have the digit eight anywhere?", "Is the tens digit odd?", or, "Is the hundreds digit greater than five?"

Th	H	T	O
___	___	___	7

Each time you answer "No" to their question, add a piece to the hangman. If they guess the correct digit, place that digit above the appropriate dash in the correct column.

Encourage the students to be systematic by using lists of digits and eliminating as they receive answers.

Extension Activity

Repeat the activity above with decimal numbers.

Fraction Pieces

CA
AC
EA
AA

I am learning to identify unit fractions from regions.

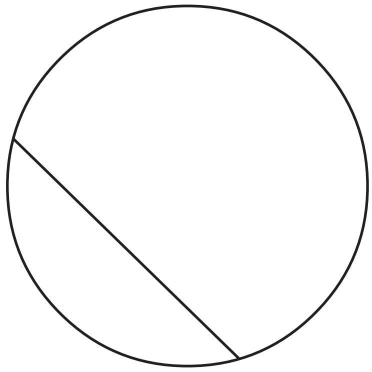
Equipment: Fraction pieces (Material Master 4–19), or commercially made fraction sets.

Background

How to name fractions is not obvious for many students. It is important that the students know that most fraction words end in a "th". For such words, the link to the symbol is clear. For example, one-eighth links to $\frac{1}{8}$ (the denominator is eight). Unfortunately the fractions that are most commonly encountered do not obey this rule; halves, thirds, and quarters are not clearly linked to their symbols $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ by the "th" clue at the end of the word. Linking the words to their symbols needs constant repetition throughout all fraction activities.

Activity

Get the students to sort similar fraction pieces and then create whole circles. Record the words for the unit fractions and their symbols on the board or modelling book. Ask the students to choose the matching piece and describe what the whole (1) looks like. Extend this to writing non-unit fractions like $\frac{3}{4}$, $\frac{2}{3}$, and $\frac{3}{5}$ for the students to model.



Activity

Draw a circle on the board or modelling book with two unequal pieces. Discuss why these pieces are not halves. Extend this to three and four equal and non-equal divisions and ask the students to identify the thirds and quarters (fourths).

Creating Fractions

AC
EA
AA
AM
AP

I am learning to create and name unit fractions for regions.

Equipment: Sheets of paper. Geoboards available commercially or as part of BSM equipment. Rubbers bands. Geoboard recording sheets (Material Master 4–20).

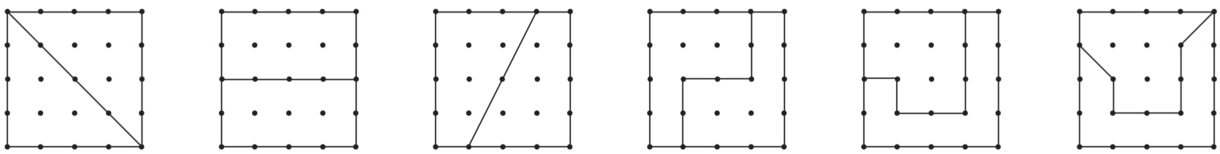
Activity

Get the students to fold a piece of paper a number of times and name the unit fractions created after each fold: halves, quarters, and eighths.

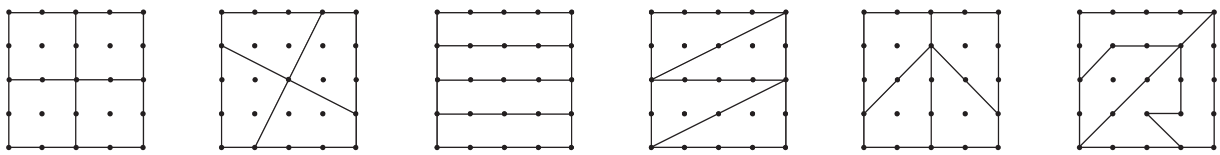
Activity

Get the students to create halves, quarters, and eighths on a geoboard in as many different ways as they can.

For example, all of these geoboards show halves:



All of these geoboards show quarters:



Once they have created the fractions on geoboards, the students can record them on geoboard sheets.

Extension Activity

Draw different-sized geoboards using dots on squared paper and show fractions on them. For example, draw an 8 × 3 dotted geoboard and show sixths on it.

More Geoboard Fractions

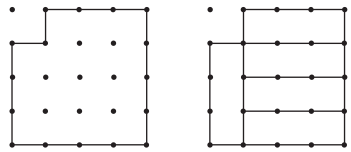
I am learning to create then name harder unit fractions on geoboards.

EA AA AM AP

Equipment: Sheets of paper. Geoboards available commercially or as part of BSM equipment. Rubbers bands. Geoboard recording sheets (Material Master 4–20).

Background

Unlike the previous activity, creating unit fractions for thirds, fifths, sixths, sevenths ... on geoboards requires the students to use multiplicative thinking to define what one whole means. For example, with fifths, only 15, 10, and five squares on the geoboard may be used to represent a whole. The diagram shows a whole with an area of 15 squares that has been divided into fifths.



Activity

Get the students to create a variety of unit fractions for thirds, fifths, sixths, and sevenths. Record the answers on geoboard sheets.

Non-unit Fractions

I am learning to identify, create, and name fractions with more than 1 on the top (the numerator is not 1).

EA AA AM AP

Equipment: Fraction pieces (Material Master 4–19) or commercial fraction kits. Ten sets of digit cards labelled 1, 2, 3, 4, 5, 6, or dice numbered 1 to 6. Fraction Bingo Boards (Material Master 4–22). Transparent counters.

Activity

Play a game of "Close to One". Players take two digit cards or roll the dice twice. They use the numbers to form a fraction, For example, if the "3" and "4" cards are picked up, the student might make $\frac{3}{4}$ or $\frac{4}{3}$. The student then models the fraction they chose with fraction pieces. With each turn, the player with the fraction closest to one (whole) wins.

The concept of "closest to" is about the size of the piece that must be added or taken away from the fraction to form one.

Activity

Play "Fraction Bingo". Form fractions on the overhead projector using the fraction pieces. The students check to see if they have the modelled fraction on their board and, if so, cover the fraction with a counter.

Packets of Lollies

EA AA AM AP

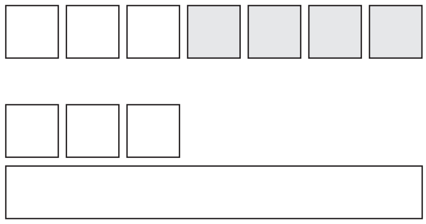
I am learning to create the whole from a fraction part I am given.

Equipment: Unilink cubes in two or more colours. Stacks of five, six, seven, and eight unilink cubes wrapped in paper (two colours of cubes making each stack).

Background

When creating fractions, students generally take a whole and subdivide it to create fractions. This might be called *whole* to *part* thinking. It is equally important that the students are required to form the *whole* from a given *part*.

Activity



Show seven "lollies" assembled and wrapped up in paper, for example, four are one colour, and three are another. Slide the cubes out of the paper to leave the wrapper intact. Place three cubes on top of the wrapper. Ask what fraction of the packet the three cubes represent (three-sevenths). Show the other four cubes against the packet, and ask what fraction they represent. This will reinforce the idea of fractions adding to one. For example, $\frac{3}{7} + \frac{4}{7} = \frac{7}{7} = 1$.

Tell a story about the lolly manufacturer deciding to put in another lolly that makes the packet bigger. Show the wrapped packet of eight lollies and put a cube against it. Ask what fraction of the packet a lolly represents now (one-eighth). Discuss why its fraction name was one-seventh and now is one-eighth, even though the size of each lolly has not altered. Open the packet up and name the fractions of each colour. Repeat for the packets of five and six cubes.

Give the students cubes and get them to make a packet to show $\frac{4}{9}$ (four of one colour and five of another).

Repeat for $\frac{2}{9}$, $\frac{4}{11}$, $\frac{3}{4}$, $\frac{9}{10}$, $\frac{2}{3}$, $\frac{9}{16}$ ...

Record various problems involving fractions and one whole as word stories and equations on the board or modelling book. For example, two-ninths and seven-ninths equals nine-ninths equals one whole, *and* $\frac{2}{9} + \frac{7}{9} = \frac{9}{9} = 1$.

Extension Activity

Make a lolly packet with six of one colour and five of another colour. Hide the five lollies and show the six lollies to the students. Tell them this is six-elevenths of the packet. Record $\frac{6}{11}$ on the board or modelling book. Ask what fraction you are hiding. Repeat for other fractions.

Record 6 elevenths + 5 elevenths = $\square$ elevenths, and the matching equation $\frac{6}{11} + \frac{5}{11} = \frac{\square}{11}$ on the board or modelling book.

Repeat for: $\frac{2}{9} + \frac{7}{9} = \frac{\square}{9}$, $\frac{5}{12} + \frac{\square}{12} = \frac{12}{12}$, $\frac{\square}{13} + \frac{4}{13} = \frac{13}{13}$ ...

Reading Decimal Fractions

AA AM AP

I am learning to read decimal fractions.

Background

Students' early contact with a decimal point is often in the context of money. Using money is not reliant on fraction knowledge. Students can come to think of the decimal point as separating two whole numbers, the dollars and the cents. So students with no experience of decimal fractions can often solve money problems like $1.50 + $2.50 = $4.00. This demonstrates that the use of the decimal point in money is not the same as its use in measurements like 4.567 metres.

Equipment: Large decimal fraction mat (Material Master 4–21). Scissors.

Activity

Check that the students can name the value of each digit in numbers up to 999 999 999 999.

Provide copies of the decimal fraction mat enlarged to A3 size. Get the students to cut out a large piece (one-tenth) and place it on an uncut mat. Discuss how big the piece is compared to the whole mat (one-tenth the size). Get the students to cut out the next largest piece and place it on an uncut mat. Discuss the size of this piece compared to the whole (one-hundredth). Continue this process to progressively smaller pieces (one-thousandth and one ten-thousandth). Get the students to try to cut a ten-thousandth into 10 equal pieces. Discuss the size of each of these tiny pieces compared to the original mat (one hundred-thousandth). Discuss how continued cutting would produce millionths to hundred-billionths. Emphasise that, with each division by 10, the pieces decrease in relative size very rapidly.

Get the students to draw a large table with headings Ones, Tenths, Hundredths, Thousandths, and Ten-thousandths. Use a whiteboard or chalk on the carpet.

Ones	Tenths	Hundredths	Thousandths	Ten-thousandths

Get the students to cut out the matching decimal mat model and place the pieces in the correct column of the table.

More Reading of Decimal Fractions

I am learning to read decimal fractions.

AA
AM
AP

Equipment: None.

Activity

Draw a table with 15 columns on the board or modelling book. Write "Ones" in the middle column, then, going to the left, "Tens", "Hundreds" up to "Ten-millions". To the right of the Ones column discuss why the next heading is "Tenths", noting the "mirroring" of the column names is about the Ones column, *not* the decimal point. Discuss how Hundreds map onto Hundredths, Thousands map onto Thousandths, and so on, up to Ten-million mapping onto Ten-millionths. Enter a number like 456.897603 in the table and get the students to read it as 456 plus eight tenths plus nine hundredths plus seven thousandths ... down to three millionths.

Repeat for other numbers.

Repeat without the table: The students read numbers like 2 009.45609124 by imaging the column place values.

Linking Money and Decimal Fractions

I am learning to turn decimal numbers into money.

AA
AM
AP

Background

In ***Reading Decimal Fractions*** (page 8), the point was made that students contact with a decimal point is often in money where the point really separated two whole numbers: the dollars and the cents. The reverse situation now occurs. Calculators, programmed to work in decimal fractions, do not normally show decimal answers as dollars and cents. The students need to understand how to do this as part of having effective number sense.

Equipment: Calculators. Play money coins (optional).

Activity

Give the students calculators. Pose the problem "Jane has $6 to share among five people. How much does each person get?" Write $6 ÷ 5 on the board or modelling book. The students carry out the operation 6 ÷ 5 = on the calculator, and get 1.2. Ask the students what 1.2 means in money. Discuss, with the aid of 10-cent coins if needed, why one-tenth of a dollar is 10 cents. So two-tenths of a dollar is 20 cents, and so 1.2 = $1.20.

Record $6 ÷ 5 = $1.20 on the board or modelling book.

Repeat for: $7 ÷ 5, $6 ÷ 4, $4 ÷ 5, $14 ÷ 4, $13 ÷ 2 ...

Activity

"Sarah pays $5 for 4 kilograms of apples. How much is this per kilogram?" Get the students to work out 5 ÷ 4 on a calculator. The answer of 1.25 looks familiar in money. Discuss why one whole and two-tenths and five-hundredths is one dollar 25 cents.

Repeat for: $7 ÷ 4, $13 ÷ 4, $3 ÷ 4, $14 ÷ 8, $20 ÷ 16.

Extension Activity

The students use calculators to solve problems like $23 ÷ 16; $13.89 ÷ 11; $345.78 ÷ 17; $14,567.67 ÷ 32; $290 ÷ 64 ...

The students then round the answers to the nearest one cent, or to the nearest five cents. Drawing decimal number lines may help the students visualise the rounding.

Measurement and Zeroes

AP

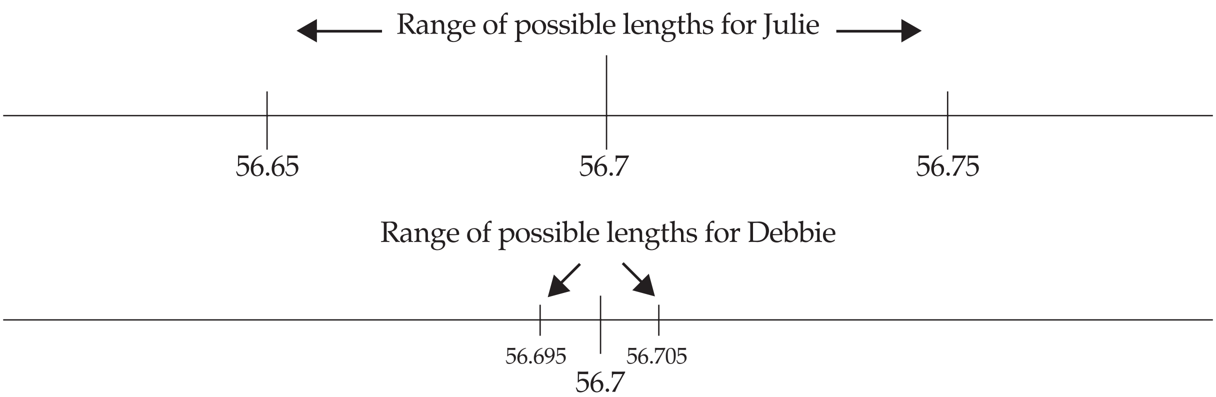
I am learning that for measurements involving decimal numbers zeros *are* significant.

Equipment: None.

Activity

Tell the students this story: "Julie has a long tape measure with divisions every tenth of a metre. She measures the length of a field and writes down 56.7 metres to the nearest tenth of a metre. Debbie measures the same field, and she says the length is 56.70 metres. How many divisions to the metre does Debbie's tape measure have?"

The students will be tempted to think Julie and Debbie have the same answer. But the 0 shows Debbie was measuring to the nearest one-hundredth of a metre (1 cm). Draw these number lines on the board or modelling book and discuss why the 0 implies a greater degree of accuracy in the measurement. 56.7 metres represents any length between 56.65 metres and 56.75 metres. And 56.70 metres represents any length from 56.695 metres and 56.705 metres.



Repeat: Ask the students to draw number lines showing the range of possible values implied by these measurements: 45.6 millilitres, 13.567 kilograms, 1.234 seconds, 14.78 kilometres, 2.3 tonnes, 2.30 tonnes, 2.300 tonnes, 14.0 litres, 14.00 litres.

Number Sequence and Order

Counting

E	CA	AC	EA	AA

I am learning to say the forwards and backwards number word sequences and to skip-count by twos, fives, and tens.

Equipment: Hundreds board with flip capability. Slavonic abacus. Flip strip (Material Master 4–2), or robust commercial version.

Activity

Get the students to clap as they count in ones. Flip over the numbers on the hundreds board as they are said. Practise the number sequences forwards and backwards.

Ask the students to identify individual numbers on the hundreds board from the sequence they have just counted.

Repeat with the Slavonic abacus. Repeat with flip strips.

Activity

Use body "percussion" to skip-count in twos. For example, the students touch their knees and silently think "One", then clap and say "Two", then touch their knees and think "Three", then clap and say "Four" ...

Repeat skip-counting by twos by flipping over every second number on the hundreds board. Similarly skip-count by fives and tens.

Repeat skip-counting by twos by moving pairs of beads on the Slavonic abacus. Relate the counting sequence to sets of objects and ask the students to give the total number of beads. Similarly skip-count by fives and tens.

Extension Activity

Repeat the activity above with skip-counting by threes and fours.

Ask the students to predict whether a given number will be in the pattern of multiples as shown on the hundreds board. For example, "Will 335 be in our fives pattern? Why do you think so?", or "What is the twentieth number in our tens pattern?"

Skip-counting on the Number Line

CA	AC	EA	AA	AM	AP

I am learning to skip-count by twos, fives, and tens.

Equipment: Large number lines (Material Master 4–8), or decimal number lines (Material Master 4–30). Pegs.

Activity

Put pegs on the number line to show the multiples of two, five, and 10 being learned by skip-counting. Say the sequence as you point to the pegs. Ask the students to predict what other numbers would be pegged if the skip-counting continued.

Pose problems like, "When skip-counting by twos (or fives or tens), will the number 27 have a peg put on it? How do you know?"

Gradually remove the pegs until the skip-counting sequence is known. Mask parts of the number line with a strip of card. Point to the position of each multiple and ask the students say it. Repeat the activity by skip-counting backwards. Include skip-counting from new starting points, for example, skip-counting by fives starting at three gives 3, 8, 13, 18 ...

Extension Activity

Develop the idea of common multiples by pegging two sets of skip-counting numbers on the same number line, for example, two, four, six ..., and three, six, nine ... Use different-coloured pegs for each sequence. (Some numbers will have two pegs on them.) Ask the students to predict what other numbers occur in both sequences, for example, 6, 12, 18 ...

Extension Activity

Use the same method to skip-count on the decimal number line. The students need to skip-count in tenths and hundredths. Counting in multiples of two-tenths, three-tenths, five-hundredths ...

Repeat the skip-counting sequences starting in different places. For example, 0.3, 0.34, 0.38, 0.42 ...

Beep

CA
AC
EA
AA
AM

I am reinforcing the forwards and backwards number word sequences and learning to recognise multiples of two, five, and 10.

Equipment: Hundreds board with flip capability.

Activity

The students stand in a circle. Decide on a multiple of two or five that will be the "beep" numbers. Select a student to start counting from one. It is important that all the students count aloud. For example, for counting in fives: "1, 2, 3, 4, beep, 6, 7, 8, 9, beep, 11 ..." When a student says "beep", they sit down. The game continues until only one student is left standing. This activity can be used to reinforce the forwards and backwards counting sequences. Use a hundreds board to assist the students to visualise the patterns. Flip over the spoken numbers but leave the "beep" numbers unflipped.

Extension Activity

Have two multiples going at the same time. For example, threes (say "beep") and fives (say "buzz"). If the number is a multiple of both three and five, then the person says "buzz-beep". So the sequence goes "1, 2, beep, 4, buzz, beep, 7, 8, ... 11, beep, 13, 14, buzz-beep ... "

Begin the counting sequences at different starting numbers. For example, "3, 7, 11 ..." or "100, 97, 94, 91 ..." These patterns will help the students to recognise algebraic relationships.

Extension Activity

Repeat "Beep" for multiples of three, four, and six. Develop increasingly complex sequences by using counts like hundreds, twenties and fifties, in fractions (on multiples of four quarters), like, one-quarter, two-quarters, three-quarters, beep, five-quarters ..., in decimals (on multiples of 0.25), like 0.05, 0.1, 0.15, 0.2, beep ...

Card Ordering

CA
AC
EA
AA
AM

I am learning to order whole numbers, fractions, and decimals.

Equipment: A pack of cards with the picture cards removed (but not the aces).

Activity

The object of the game is to play the cards in order and be the student to play the card that has 10 on it. Each ace is worth one.

Place the aces face up to begin four stacks. Shuffle the cards. Deal each student five cards. A student with a two card begins by placing it on top of an ace. Students take turns putting one card on a stack of their choice. They must add to the stacks in sequence from 1 to 10. After each student has had their turn, they pick up a new card from the pack.

If a student cannot go, then they keep picking up cards from the pack until they can go. The student who plays the 10 collects the stack. They receive a point and put that stack of cards to one side. The students continue to play their cards until there are no cards left and four stacks of 1 to 10 have been completed.

Extension Activity

Repeat the game with different packs of cards using different patterns of numbers. For example, 10 to 20, 50 to 60, 0.1 to 1.0 (in tenths), $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1 to $2\frac{1}{2}$ (in quarters), 2.31 to 3.3 (in hundredths), ...

Arrow Cards

I am learning to identify and order whole numbers and decimals.

AC EA AA AM

Equipment: Whole-number arrow cards (Material Master 4–14). Decimal arrow cards (Material Master 7–2).

Activity

Give the students sets of arrow cards. Choose the number of whole number or decimal places that is appropriate to the students' stages. Ask them to make numbers by overlapping the arrow cards. Note the importance of lining up the points of the cards. For example, for "67" the students combine the 60 card and the 7 card. Record 60 + 7 = 67 on the board or modelling book. With decimal cards, an example might be, "two point five, eight". The students combine the 2, 0.5, and 0.08 cards and record 2 + 0.5 + 0.08 = 2.58.

Repeat with numbers that have zeros as place-holders. For example, 304, 470, 4 080, 4.06, 0.309.

Activity

Play a game of "Arrow Card Order". Spread the whole number or decimal arrow cards on the floor, numbers face down. You may decide to use all of the cards or limit the size of the numbers by removing some cards, for example, thousands or thousandths. The students take turns to pick up a card of each size to form a number. They make the number and display it for the other players to see. The players decide who has the largest number, and that player gets a point. They then remove the highest place-value card (for whole numbers) or lowest place-value card (for decimals), for example, the thousands card, or the thousandths card. Players then compare their numbers to see who has the largest number, and that player gets a point. Players then remove the highest or lowest place-value cards again and compare their new numbers.

Repeat the game with various numbers. Vary the game so the target is the lowest number.

Lucky Dip

I am learning to say the number that is 1, 10, 100, 0.1, 0.01 before and after a given number.

CA AC EA AA AM

Equipment: Container of numeral cards (the range of numbers varies according to the student's stages). Slavonic abacus or plastic container, and cubes. Hundreds board or large number line (Material Master 4–8).

Activity

Ask a student to take a card from the container and show the rest of the class. Ask the students, "What is the number before this?" (the number one less than that shown on the card). Immediately following this, ask the students, "What is the number after this?" (the number one more than that shown on the card). Continue with various other cards.

Activity

Use either the abacus or a container of cubes. Move a number of beads across or put a number of cubes in the container. Ask the students how many beads/cubes are there. Move another bead across/take one off or add a cube to the container or take one out. Ask the students how many beads/cubes are there then. Link this to the number before or after on a hundreds board or number line.

Extension Activity

With decimals, more can appear to be less and vice versa. For example, 0.03 more than 0.67 is 0.7, which appears to be less for the students who are whole-number thinkers. Material models of decimals, like deci-pipes or place-value blocks, may be needed to explain why this occurs.

Examples: Ask the students to say the number 10 more/less, 100 more/less, 0.1 more/less, 0.05 more/less ...

Using Calculators

CA
AC
EA
AA
AM

I am learning to skip-count with whole numbers and decimals.

Equipment: Calculators. Hundreds board with flip capability. Number flip strip (Material Masters 4–12 and 4–31).

Activity

The constant function on the calculator can be used to develop counting patterns. Ask the students to key in the sequence 5 + = = = = ... It will produce a display of increasing multiples of five. Challenge your students to work out the sequence. Note that with some calculators, like Casio, the + key must be pressed twice to activate the constant function.

Use the hundreds board to record the skip-counting sequence. For example, flip over every fifth number. This can also be done by recording the sequence on the blank side of a strip and sliding it into a number flip strip.

Activity

Seat the students in pairs and get one of the pair to put in the first few terms of a sequence, using + (a number) = = =. The student hands the calculator to their partner to push = = = ... The partner tries to work out what number is being repeatedly added.

Tell the students to key in + number but not to press =.

For example, + 4. Instruct them to hold their finger over the equals button, and, without looking, press equals until they think a target number has been reached in the window. For example, aim for 24. This is good practice for skip-counting sequences and multiplication facts.

This can be extended to sequences of two-digit numbers and decimals.
For example, + 23 = = = ..., + 99 = = = ..., + 0.3 = = = ..., + 1.6 = = = ...

Repeat for subtraction. For example: 46 – 5 = = = ... produces the sequence 41, 36, 31, ... on most calculators.

Extension Activity

The students investigate calculator inputs like 4 + 5 = = = ... In this example, most calculators produce the sequence 9, 14, 19, 24, ...

Examples: 0.9 + 0.3 = = = ..., 2.45 + 0.02 = = = ..., 48 – 4 = = = ..., 8.4 – 0.5 = = = ...,
7.5 – 0.25 = = = ..., 2.602 – 0.002 = = = ...

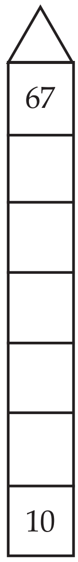
Rocket – Where Will It Fit?

I am learning to order whole numbers and decimals.

AC EA AA AM

Equipment: A piece of scrap paper. Standard 1–6 dice or dodecahedral 0–9 dice.

Activity



Each student needs to draw a "rocket" playing board like the one shown. The number of floors on the rocket can be increased where larger whole numbers or decimals are involved. The aim of the game is to fill every floor of the rocket with numbers in order. If a player cannot place a number they have thrown, they miss that turn. Players take turns to roll a dice twice. From the numbers thrown, the students decide which two-digit number they will use. For example, if five and three is thrown, the student could use 53 or 35.

The students then record the number on a level of the rocket where they think it best fits between 10 and 67. Once a number is written it cannot be moved.

Extension Activity

Repeat: The students can throw the dice three times to make a three-digit whole number and place that number between 110 and 667. Use other variations like using the three throws to make decimals and placing them between 1.1 and 6.67.

Number Line Flips

I am learning to order two-digit numbers.

CA AC EA AA

Equipment: Whole number line flips (Material Master 4–12) or decimal number line flips (Material Master 4–31).

Activity

Give the students an empty number line that shows only the numbers at the start and end. The other numbers are shown on the reverse side as a reference. Have the students place a peg, or paper clip, on the number line and ask their partner to guess which number the peg is on. If the students have difficulty to start with, their partner can help by pointing to and saying reference numbers close to the pegged number. For example, "I am pointing to 15. What number is the peg on?"

Extension Activity

Get the students to develop their own number line flips for other sequences, such as the whole numbers between 40 and 60, the multiples of three between 0 and 30, the decimals, in tenths, between 4.7 and 7.2, the fractions in fifths between 0 and 2.6.

Squeeze – Guess My Number

I am learning to order two-digit numbers.

CA AC EA AA AM

Equipment: A hundreds board with flip capability or a whole number line 0 to 100 (Material Master 4–8). Two pegs. A decimal number line (Material Master 4–31).

Activity

A peg is put at each end of a number line, for example, on 0 and 100. A student chooses a number between the pegs and writes it on a piece of paper. The rest of the students ask "less than" or "greater than" questions to find the mystery number. With each question, a peg is moved to eliminate numbers. For example, if "Is it greater than 25?" is answered by "Yes", then the zero peg is shifted up to 25 to eliminate all the numbers 25 and under.

If "Is it less than 75?" is answered by "Yes", then the top peg is shifted down to 75 to eliminate all the numbers 75 and over.

This continues until the mystery number is finally found by squeezing in from above and below. Note that this game can be played on various number lines, including whole-number lines starting at other than zero, and decimal or fraction number lines.

Alternatively, use a flip hundreds board, showing all the numbers in black and flipping the numbers over to red as they are eliminated.

When the students are confident, remove the number line and do the activity by visualising. List the numbers on the whiteboard:

"I'm thinking of a number between 0 and 5."		Write	0	5
"Is it greater than 2.5?"	"Yes"	Write	2.5	5
"Is it less than 3.6?"	"Yes"	Write	2.5	3.6
"Is it greater than 3.0?"	"No"	Write	2.5	<u>3.0</u>

The game continues until the number is found. Note that the question, "Is it greater than 3.0?" did not eliminate 3.0 itself as the answer was "No." The underlining of 3.0 shows the mystery number *could* be 3.0.

Where the mystery number is a three-place decimal, like 0.456, draw a very long number line on the board board or modelling book showing 0 to 1 at either end. Progressively subdivide the number line until the decimal is found.

Hundreds Boards and Thousands Book

CA
AC
EA
AA
AM

I am learning to count forwards and backwards by ones, tens, and hundreds.

Equipment: Hundreds board. Thousands book (Material Master 4–7). Snake pieces (Material Master 4–36).

Activity

Take a pencil for a walk on a page of the thousands book. The students say each number as the pencil points to it. In the case of pages that have many numbers missing, the students will need to image the numbers.

Discuss what moving one square to the right or left does to the size of the number (increases/decreases by one). Ask what moving one square down or up does (increases/decreases by 10), and what moving through to the next/previous page does (increases/decreases by 100).

Activity

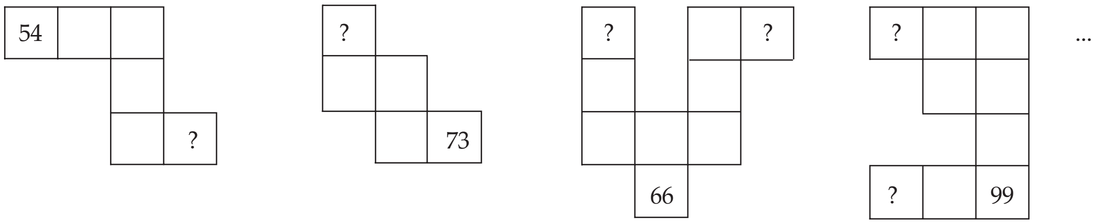
Pose written problems using these symbols:

$\leftarrow$ left,	$\rightarrow$ right,	$\uparrow$ up,	$\downarrow$ down,
$\Longrightarrow$ forwards one page (add 100),		$\Longleftarrow$ back one page (subtract 100)	

Examples: 25 $\downarrow\downarrow\downarrow$ $\leftarrow$ (start at 25, go down three squares to 55 and to the left one square to arrive at 54), 487 $\Longleftarrow\Longleftarrow$ $\uparrow$(start at 487, go back two pages to 287, and up one square to 277), 67 $\downarrow\downarrow\downarrow$ $\leftarrow$ $\uparrow\uparrow$ $\leftarrow\leftarrow$ $\downarrow$ ...

Activity

Use snake pieces, or draw them on the board or modelling book:



The students work out the "mystery numbers". Ask the students "How did you work out what the mystery number was? What different ways could we have used?"

Extension Activity

Repeat the snake pieces activity but now show pages from the thousands book where very few reference numbers are visible.

Figure It Out, Levels 2–3, Number, *page 1 (Activities One and Two), shows examples of how students can look for patterns in missing numbers and how jigsaws can be made with parts of the hundreds board. Similar activities can be applied to larger numbers using the thousands book.*

Bead Strings

I am learning to order whole numbers and decimals.

CA
AC
EA
AA
AM

Equipment: Bead strings with 100 beads in groups of five. Supermarket bag tags.

Activity

Show the students a bead string. Use bag tags to label the beginning and end of the string as 0 and 100. Ask the students to use grouping strategies to locate the multiples of 10, beginning with 50 (half way), 10, and 90. Tag these numbers on the string.

Now record other numbers in the range 0–100 on tags, and ask the students to find efficient ways to locate the numbers. Encourage grouping strategies. For example, 75 is found by identifying the position half way between 50 and 100.

As an independent activity, give the students bead strings and a set of tags with numbers already on them. The students place each tag in its correct position on the string. Partners check each others' strings.

Repeat with several bead strings joined together to form a line extending into other multiples of 100. For example, five bead strings allow numbers from 0 to 500.

Activity

To extend this idea to fractions, and decimals, use tags to label the ends of the string as zero and one. Give the students tags with fractions to place on the string. Be sure that the denominators of the fractions are factors of 100. For example: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{3}{5}$, $\frac{9}{10}$, $\frac{1}{10}$ ...

Join several bead strings together to create fraction lines that extend over one. For example, five bead strings allow fractional numbers from zero to five. Label the whole numbers with tags and ask the students to locate fractions like $\frac{9}{4}$, $\frac{13}{10}$, $\frac{24}{5}$ ...

Extension Activity

Extend to fractions and decimals simultaneously. For example, on a 0 to 3 line (three bead strings joined), tag the whole numbers 0, 1, 2, 3. Ask the students to locate these numbers:
1.5, $\frac{4}{5}$, 2.25, $\frac{19}{10}$, 0.99, $\frac{11}{4}$ ...

Who is the Richest?

AC
EA
AA

I am learning to order large whole numbers up to 1 000 000.

Equipment: Play money. (Material Master 4–9).

Background

In any number, *including* decimals, the students need to realise that the most significant figure is on the far left of the number. For example, 100 000 is more that 99 999 because *any* six-digit number is more than *any* five-digit number. When comparing numbers, the students need to check column by column from the left.

Activity

Write this table of bank balances on the board or modelling book. If necessary, the students model these amounts with play money. Ask who has the most money in the bank. The students need to realise that they start with the left-hand place value of each amount.

Name	Balance
Emiliana	$5,007
Phil	$40,801
Bronwyn	$33,104
Toni	$33,025
Leslie	$4,111

For example, a person with *any* 10 000s in their amount will have more than someone whose largest place value is in the 1 000s.

Repeat with varying amounts of money.

Activity

Move onto doing the same kinds of problems without money. Focus attention on the significance of starting from the left-hand digit when comparing numbers.

Order numbers within these lists. The third list is particularly taxing because of the presence of numerous nines.

List 1	List 2	List 3
100 090	7 090	990 090
345 789	105 740	999 000
99 003	456 003	909 099
56 891	60 840	990 999
45 677	99 470	900 999

Who Has More Cake?

AA
AM
AP

I am learning to order unit fractions.

Background

The key idea in unit fractions follows the general rule that the larger the denominator, the smaller the fraction. This is because the larger the denominator becomes, the more equal parts the whole (one) is divided into. Therefore the pieces must be smaller as the denominator increases.

Equipment: Fraction pieces (Material Master 4–19) or commercial ones.

Activity

"Robyn wants $\frac{1}{5}$ of a cake. Dale wants $\frac{1}{8}$ of a cake. Who will eat more cake?" Give out the fraction pieces and let the students model fifths and eighths. Discuss which fraction is bigger.

Record "One-eighth is less than one-fifth," and "One-fifth is greater than one-eighth" on the board or modelling book, and underneath record "$\frac{1}{5} > \frac{1}{8}$".

Repeat this for $\frac{1}{4}$ and $\frac{1}{3}$, $\frac{1}{6}$ and $\frac{1}{8}$, $\frac{1}{5}$ and $\frac{1}{3}$, $\frac{1}{8}$ and $\frac{1}{4}$ ...

Repeat without materials. Discuss which is larger in the following pairs and why this is true: $\frac{1}{10}$ and $\frac{1}{3}$, $\frac{1}{12}$ and $\frac{1}{8}$, $\frac{1}{15}$ and $\frac{1}{23}$, $\frac{1}{50}$ and $\frac{1}{27}$, $\frac{1}{67}$ and $\frac{1}{68}$, $\frac{1}{59}$ and $\frac{1}{95}$ ...

Little Halves and Big Quarters

I am learning how a half can be smaller than a quarter.

AM
AP

Equipment: None.

Activity

Ask the students whether $\frac{1}{2} > \frac{1}{4}$ or $\frac{1}{2} < \frac{1}{4}$. The expected answer is $\frac{1}{2} > \frac{1}{4}$. Now pose this problem: "Annabelle earns \$100 a week, and Maureen earns \$400 a week." Discuss why, in this case, one-quarter of Maureen's weekly pay would be bigger than half of Annabelle's weekly pay! Record on the board or modelling book, "Half of a small amount can be smaller than a quarter of a larger amount".

Discuss why, when we are making comparisons between fractions, we need to be careful about what the respective "wholes" are.

Extension Activity

Ask the students to work out in this table whether or not half of Annabelle's pay is the same as a quarter of Maureen's pay.

Annabelle's pay	\$8	\$12	\$10
Maureen's pay	\$16	\$24	\$20

Discuss the general rule: Half of Annabelle's pay is the same as a quarter of Maureen's pay provided Maureen earns *exactly* twice as much as Annabelle.

Pose similar problems like: "One half of Peter's pay is the same as one-third of Nicola's pay." "When can this statement be true?" "Come up with some numbers that work." "Try to find the general rule for this statement." (They are equal provided Peter earns exactly two-thirds of Nicola's pay.)

Super Liquorice

I am learning to locate decimal numbers on a number line.

AA
AM

Equipment: Rolls of adding machine tape. Metre rulers. Paper clips. Small dot stickers.

Activity

Get the students to cut lengths of the tape 1 metre long, using metre rulers. Ask them to find the one-half, one-quarter, and three-quarters marks on their metre strip by folding or measuring. Ask for their ideas about how to locate these points exactly. Highlight the fact that there are 100 centimetres in 1 metre, and so 50 out of 100 ($\frac{50}{100}$) is another name for one half. Also $\frac{25}{100} = \frac{1}{4}$, $\frac{75}{100} = \frac{3}{4}$.

Get the students to locate the following fractions on their 1-metre strip:
$\frac{2}{4}$, $\frac{4}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{5}{5}$ ... Discuss how they found each fraction by either folding or measuring.

Discuss why $\frac{4}{4}$, $\frac{5}{5}$, $\frac{12}{12}$ ... are *all* names for 1.

Extension Activity

Ask the students to imagine where the following fractions will be: $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, $\frac{7}{8}$, $\frac{8}{8}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{3}$.

The students then check to find each fraction using a new paper strip and a metre ruler.

Finding eighths by measurement requires the use of millimetres. This requires the students to understand the idea that 10 thousandths of a metre (10 mm) is the same as one-hundredth of a metre (1 cm).

Continue this idea by getting the students to make a 5-metre long strip of tape, and mark the whole number divisions 0, 1, 2, 3, 4, 5 on it. Give each pair of students a set of decimals to put on their number line.

Suitable decimals are: 2.5, 1.75, 4.99, 1.46, 3.5, 2.793, 3.333, 0.079 ...

Who Wins?

AA
AM
AP

I am learning to order decimal numbers up to four decimal places.

Background

It is strongly suggested that the students continue to compare the size of decimal numbers by using the strategy for whole numbers, namely, start at the left-hand end of each number and look column by column. For example, some students think 0.75 is more than 0.8 because 75 is more than 8. Rather, the students should realise that eight-tenths exceeds seven-tenths and therefore 0.8 is more that 0.75. The 5 is *not* needed in making this decision.

Equipment: None.

Activity

Write the table of long-jump distances on the board or modelling book. Ask who wins. Continue listing the distances from largest to smallest.

Name	Distance
Smith	8.06 m
McKay	7.69 m
Hohepa	8.19 m
Tonu'u	7.9 m
Chan	8.7 m

Repeat ordering numbers with the lists in the table. The third list is particularly taxing as it requires the students to realise that, to order a set of very similar numbers, they must search systematically starting at the left-hand end of each number.

List 1	List 2	List 3
145.678	0.09	5.6
145.09	0.081	5.0678
145.005	0.5	5.341
145.9	0.592	5.09
145.2	0.57	5.3481

Who Gets More?

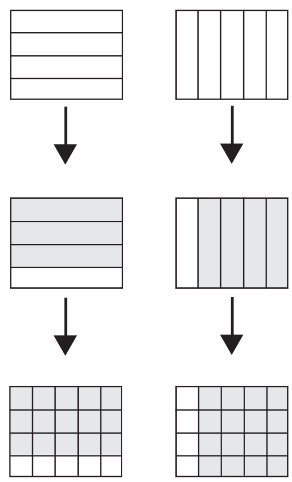
AM
AP

I am learning to compare the size of any fractional numbers.

Equipment: Large squared paper (Material Master 4–27). Ordinary squared paper.

Activity

Set a comparison problem: "Twins receive identical cakes for their birthday. Jose cuts her cake into fifths and Michael cuts his into quarters. Jose eats $\frac{4}{5}$ of her cake and Michael eats $\frac{3}{4}$. Who eats more cake?"



Discuss why a 5 by 4 cake allows fifths and quarters to be drawn easily. The students draw two 5 by 4 grids on the large blank squared paper. The students shade the fractions. Discuss why adding vertical lines and horizontal lines respectively is a good idea; it creates twentieths on both cakes. Discuss why $\frac{4}{5} > \frac{3}{4}$ follows from the pictures. (The pictures show $\frac{16}{20} > \frac{15}{20}$ so $\frac{4}{5} > \frac{3}{4}$.)

Compare this pair of fractions on the large squared paper: $\frac{2}{3}$ and $\frac{3}{4}$.

Repeat with these pairs of fractions on the ordinary squared paper: Compare $\frac{5}{6}$ and $\frac{4}{5}$, $\frac{7}{10}$ and $\frac{2}{3}$, $\frac{5}{8}$ and $\frac{3}{5}$ ...

Repeat with these pairs of fractions without any drawings: $\frac{2}{3}$ and $\frac{13}{20}$, $\frac{7}{11}$ and $\frac{3}{5}$, $\frac{69}{100}$ and $\frac{2}{3}$ ...

Extension Activity

Order these sets of three fractions from smallest to largest each time: $\frac{3}{5}$, $\frac{2}{3}$, and $\frac{1}{2}$; $\frac{7}{10}$, $\frac{2}{3}$, and $\frac{1}{2}$ ...

Equivalent Fractions, Decimals, and Percentages

I am learning the equivalents of fractions, decimal fractions, and percentages.

AP

Equipment: Deci-mats (Material Master 7–3). Source of examples (Material Master 4–28).

Activity

Show 10% as $\frac{10}{100}$ on a deci-mat. Discuss why it equals 0.1.

The students do examples from Material Master 4–28.

Discuss the answers in class. In particular discuss why a number like 0.05 means $\frac{5}{100}$ and equals 5%. Extend this to problems like 15% = $\frac{15}{100}$ = 0.15. The students continue with examples from Material Master 4–28.

Difficult Fractions to Percentages

I am learning to convert any fractional number to a rounded percentage.

AP

Background

Problems involving comparing fractions can be solved by first converting all the fractions to percentages (hundredths).

Equipment: Calculators.

Activity

"In Henry's orchard, 344 apples out of 3 467 were spoiled by hail. In Maria's orchard, 4 567 out of 43 890 apples were spoiled. Whose orchard was worse affected?"

Discuss why the fractions of affected apples are $\frac{344}{3467}$ and $\frac{4567}{43890}$. Turning both these into percentages (hundredths in effect) allows us to compare them directly. Now the issue is how to turn any fraction into a percentage.

A clever method is to recreate a division problem from which fractions might have been derived and then use the calculator:
$\frac{344}{3467} = 344 \div 3467 \approx 0.0992212$ and $\frac{4567}{43890} = 4567 \div 43890 \approx 0.1040556$.

The ability to round to three decimal places is now needed. Here the answers are 0.099 = 9.9% (1 dp) and 0.104 = 10.4% (1 dp). So Henry's orchard suffers fractionally less damage.

Examples: Compare these situations by working out 1 dp percentages.

"The ACE Sweet Company makes a profit of $456,789 on sales of $4,366,888. The Triangle Lolly Company makes a profit of $45,677 on sales of $267,900. Which company is more profitable?"

"A survey of 1 461 days (4 years) shows it rains on 678 days in Circleton. A survey of 1 095 days (3 years) shows it rains on 431 days in Linetown. Which town is more rainy?"

Grouping/Place Value

Fabulous Fives

CA	AC	EA	AA

I am learning to group with fives.

Background

The students need to instantly recognise patterns for numbers from 1 to 10 in a variety of contexts such as on fingers and on the Slavonic abacus.

Equipment: A Slavonic abacus. Fly Flip cards (Material Master 4–5). Murtles 5 and ... game (Material Master 5–9).

Activity

Practise making finger patterns for the numbers 1 to 10. The focus is on grouping not counting. Begin with finger patterns the students will know, like one, two, five, and 10. Find other patterns from these numbers. For example: "Show me five fingers ... Now, show me six. What did you do to make six (add a finger)? What would seven look like? What about four?"

"Show me 10 fingers ... How do you know it is 10?" (five and five) "Change it into nine." "How did you do that?" (took off a finger) "What would five look like?"

Repeat this activity with the students hiding their fingers behind their backs and imagining the finger patterns.

Activity – Fly Flip Cards

Show the students a fly flip card with the five flies and a number on the front, for example, "8". Ask how many flies there will be on the back. Ask the students to show eight on their fingers. (They need to have five fingers showing on one hand.) Ask how many fingers they need to show on their other hand to make eight (three). Turn the card over to reveal the other three flies that make eight.

Repeat for the fly flip cards from six to 10.

Activity

Show an eight-fly card. Ask, "How many more flies would make this a 10-fly card?" (two) Repeat for the fly flip cards from 6 to 10.

Activity – Slavonic Abacus

Push over a small number of beads, for example, six. Ask "How many beads did I just push over?" and "How did you know it was six without counting the beads?" Aim for responses like "I know that one more than five is six." Get the students to show you six fingers to reinforce the connection. Try this for other "five and ..." groupings. For example, "Eight is five and what?"

Alternatively, start by pushing across five beads and ask "How many more to make eight?" Follow up by asking "How did you know?" Ask the students to show you the same fact with their fingers.

Push across a small number of beads and ask how many more are needed to make 10. This is reinforcing the key knowledge of pairs of numbers that add up to 10. Connect this with the finger patterns and matching fly flip card.

Activity – Murtles 5

Murtles 5 and ... (Material Master 5–9) is an excellent game for students to practise groupings with five.

Slavonic Abacus

I am learning to identify a number of objects grouped in tens and ones.

CA

AC

EA

Background

Many students solve problems with tens and ones by first counting in tens. They need to learn not to count but to recognise how many tens there are and convert this into a "ty" word. For example, if there are eight tens instantly the students should know there are 80 objects.

Equipment: A Slavonic abacus, preferably one with a backing sheet.

Activity

Out of sight of the students create a "ty" number on the abacus. Turn the abacus around and show the students the beads briefly. Do not allow time to count by tens. Encourage the students to recognise the numbers of rows from their ability to recognise ones. For example, if the number of beads is 60, they need to see instantly that there are six rows of 10. They do this by spotting five rows of a pair of colours and one other row with the other pair of colours. And they know $5 + 1 = 6$. If the students have to count 10, 20, 30, 40, 50, 60, this possibly indicates that the students do not realise six tens are "sixty". Help them to focus on place-value structure by asking, "How many rows of 10 were there?" If necessary show the beads again, and say, "There are six tens".

Activity

Repeat, but this time include single beads, for example, show 68 briefly. Expect the students to recognise this quickly, without *any* counting, as a collection of six rows of 10 and eight single beads and say "68".

Tens and Ones

I am learning to show two-digit numbers on place-value material.

CA

AC

EA

Equipment: Various types of discrete (individual) materials such as sticks (and pipe cleaners), beans (and film canisters), unilink cubes, hundreds board. Place It game (Material Master 4–32).

Activity

The students amass loose objects into groups of 10. For example, "At Jack's bean shop, they put beans into containers of 10 to make counting easier." or, "Mr Manuka sells sticks to the three little pigs in bundles of 10." The students write the answers down as two-digit numbers.

Reverse the process: Write a small two-digit number, like 37, on the board or modelling book and get the students to collect that number of sticks or beans or cubes. "How many bundles/containers/stacks of 10 can you make?"

Do some "teen" numbers. The problem with the "teen" numbers is that the 10 (teen) is said after the ones, unlike all other two-digit numbers, but this cannot be avoided. It needs constant repetition until the students know the teen numbers reliably.

Activity

The students choose two-digit numbers from the hundreds board, and model them with sticks/beans/cubes. The students play Place It to consolidate grouping in tens and ones.

Close to 100

AC
EA
AA
AM

I am learning to apply my place-value knowledge.

Equipment: Paper. A dice. Any place-value equipment.

Activity

Each player rules up a column for "tens" and a column for "ones". The aim of the game is to get a total as close to 100 as possible. The student tosses a dice and decides whether the number will be put in the ones or the tens place. For example, if a four is thrown, it could either be 40 or four. The dice is rolled a total of seven times. All seven numbers must be used. The total of all the columned numbers may exceed 100, but the students will need to decide which player has got closer to 100.

Tens	Ones

Extension Activity

Use larger numbers and decimals for the target numbers. Vary the number of throws and what the thrown number can represent, such as:

Closest to 1 000: 10 throws of hundreds, tens, or ones.

Closest to 10: 10 throws of ones, tenths, or hundredths.

Closest to 1: 10 throws of tenths, hundredths, or thousandths.

Nudge

AC
EA
AA
AM

I am learning how a number containing nines "rolls" over to leave zeros when 1, 10, 100 ... is added to the number, and how zeros "roll" back to nines with subtraction by 1, 10 ,100 ...

Equipment: Sets of large numeral cards ordered 0 to 9 (Material Master 4–3) or commercially made number flip charts.

Activity

The students use the numeral cards to recreate counting sequences in a way that's similar to the action of a car odometer. They can wear hats marked with the place values involved, for example, ones, tens, hundreds, thousands ...

9	9	9	9

Have a student as the ones counter, counting in ones. Stop them at nine. Ask, "What will happen when one is added?" Discuss how adding one rolls nine over to 10 and that another counting place (tens) is needed.

Count in ones from 95 until 99 rolls over to 100.

Start with 93 and add 10 to it. Discuss how the nine rolls over. Repeat by adding 10 to 94, 99, 90 ...

Add 1, 10, then 100 to 99. Add 1, 10, 100 to 899. Add 1, 10, 100 to 998.

Activity

Roll 1 000 back 1, 10, 100. Roll 3 000 back 1, 10, 100. Roll 309 back 1, 10, 100.

Extension Activity

Increase the size of the numbers to show the students that roll over/back can be applied to all whole numbers and decimals. For example, Add 1, 10, 100, 1 000, 10 000 to 99 999. Add 1, 10, 100 to 99 989. Add 1, 10, 100, 1 000, 10 000 to 109 990. Add 1, 0.1, 0.01, 0.001 to 99.999. Roll 309 000 back 1, 10, 100, 1 000, 10 000. Roll 309.000 back 10, 1, 0.1, 0.01, 0.001.”

Extension Activity

Complete each of these problems and check with a calculator.

395 + $\square$ = 405	36 099 – $\square$ = 34 100
99 962 + $\square$ = 100 062	$\square$ – 99 999 = 1 000
$\square$ + 9 900 = 10 000	100 000 – $\square$ = 90 000 ...

Estimating

I am learning to estimate the number in a collection of single objects.

AC
EA
AA
AM

Equipment: Cubes, iceblock sticks, buttons, or other similar materials. Overhead projector.

Activity

Show the students collections of objects on an overhead projector. Mask the collections initially with a book then uncover them briefly. Encourage strategies like looking for instantly recognisable patterns and using these as a sample with which to estimate the whole collection. You might also develop sampling by using a frame, like a cut out plastic lid, to isolate part of the collection. For example, 10 objects are inside the frame, that is about one-sixth of the collection, so there are about 60 objects in total in the collection.

Activity

The students at the Advanced Additive thinking stage and onwards should apply multiplicative strategies. Fill clear plastic jars with beads, buttons, or similar objects and get the students to estimate the total number. Encourage strategies like taking a cross section and multiplying that by how many times the cross section will fit into the collection.

Traffic Lights

I am learning to use logical reasoning to solve problems with place value.

AC
EA
AA
AM

Equipment: Whiteboard and pen, or overhead projector and coloured transparent counters.

Activity

Draw a grid on the whiteboard or overhead transparency:

Thousands	Hundreds	Tens	Ones

You choose or a student chooses a mystery four-digit number. Each digit can be used only once. The number is then marked using the following code.

•	or a red counter:	Wrong digit, wrong place.
√	or an orange counter:	Correct digit, wrong place.
◯	or a green counter:	Correct digit, correct place.

Write this on the board or modelling book or use red, orange, and green transparent counters on the OHT instead of the code markings to cover the digits provided.

For example, suppose the correct number is 3 567 and the first student guess is 9 637, the table on a whiteboard would look like this:

Thousands	Hundreds	Tens	Ones
9 •	6 √	3 √	7

The next guess might be 6 327. The 7 is definitely correct, the 6 and the 3 must be placed in new columns and the 2 is a possibility as it has not been used in the first try. Guesses and deductions continue until the correct answer is entered in the table.

The students can also play this game in pairs or small groups.

Zap

CA AC EA AA AM

I am learning the meaning of digit in the thousands, hundreds, tens, ones, tenths and hundredths columns.

Equipment: Calculators. Arrow cards (Material Masters 4–14, and 7–2).

Activity

The students work in pairs. One student begins by putting a number into the calculator, for example, 67. Then an instruction is given. "Zap out the 6 in 67". Here 67 – 60 "zaps" the 6. The students "zap" by using numbers and the +, – , ×, or ÷ buttons. The students should also demonstrate the process that the numbers go through with arrow cards. For example, 67 on arrow cards becomes 7 by removing the card that shows 60.

Repeat for: 45 to 75, 56 to 50, 40 to 100, 66 to 23 ...

Activity

Increase the difficulty of the examples to match the abilities of the students by increasing the whole number size, using decimal cards, and getting the students to zap two or more digits simultaneously.

Examples: 5 678 to 5 168, 4 444 to 4 114, 5 678 to 5 600, 678 to 621, 1.32 to 1.29, 3.427 to 3.024 ...

Extension Activity

Use many different operation buttons to convert a number to another given number. For example, when 20 is in the display, it could be converted to 40 by:
+ 20, ÷ 0.5, or + 100%, or × 2!, × 2, – $^{-}$20 ...

Examples: 15 to 30, 0.5 to 2, 100 to 10, 1 000 to 1, 0.56 to 560 ...

Tens in Hundreds and More

EA
AA
AM
AP

I am learning how to convert tens into hundreds, hundreds into thousands, tens into thousands and vice versa.

Background

A core idea of place value is that 10 in *any* column is worth one in the next column to its left, 100 of *any* column is worth one in the column two places to its left, and vice versa. This is also true for decimals.

Equipment: Play money up to $10,000 (Material Master 4–9).

Activity

"Mrs Hau collects $10 from each of her students for a visit to the zoo. She has a total of $180. How many students are going to the zoo?"

Give the students $180 as a $100 note and eight $10 notes and have them break the $100 note down into 10 $10 notes. Discuss the answer. Record $180 = 18 × $10 on the board or modelling book.

Repeat for $230, $140, $200, $160 ... Encourage the students to get the answer before modelling the problem with the money if they can.

Repeat by selecting numbers that are too large to model on materials. For example $980. Discuss the pattern that has emerged. Record $980 = 98 × $10 on the board or modelling book.

Repeat for $860, $900, $340, $660 ...

Activity

Reverse the process. Record 19 × $10 = on the board or modelling book and discuss what it equals. Model with $10 notes if needed. Record 19 × $10 = $190.

Repeat for 13 × $10, 15 × $10, 78 × $10, 34 × $10, 77 × $10 ...

Repeat for 273 × $10, 615 × $10, 798 × $10, 304 × $10, 730 × $10 ...

Activity

"Mrs Wallace's class is going on a trip that costs $100 per student. Mrs Wallace collects $1,600. How many $100 notes does Mrs Wallace have?" The students model $1,600 with play money and swap $1,000 for 10 $100 notes. Discuss the pattern that emerged from the material. Record $1,600 = 16 × $100 on the board or modelling book.

Repeat for $2,000; $1,300; $2,100; $1,100 ... Encourage the students to get the answer before modelling if they can.

Repeat, without money, for $6,000; $7,800; $2,400; $6,600; $5,670 ...

Activity

Record 130 × $10 = on the board or modelling book and discuss what it equals.

Record 130 × $10 = $1300 on the board or modelling book.

Repeat for 120 × $10; 490 × $10; 720 × $10; 840 × $10; 470 × $10 ...

Reverse: Discuss how many $10 notes make $1,000.

Find the number of $10 notes in: $4,000; $7,000; $2,000; $13,300; $2,600; $4,560; $2,380 ...

Extension Activity

How many $1,000 notes to make $1,000,000; $3,500,000; $4,567,000 ... ?

How many $10,000 notes to make $670,000; $560,000 ...?

How many $100,000 notes to make $2.5 million; $5 million ...?

Swedish Rounding

EA
AA
AM
AP

I am learning to round supermarket bills to the nearest 5 cents.

Background

In many supermarkets, the total bill is rounded to the nearest multiple of 5 cents when customers pay cash rather than by credit card or EFTPOS. This is the *Swedish Rounding* system.

Equipment: A number line based on fives divisions *not* tens (Material Master 4–26).

Activity

"Mary's bill at the supermarket is $102.88. How much does she pay in cash?" Discuss why Swedish rounding is needed. (There are no 1- , 2- or 5-cent coins any more.) Get the students to fill in a number line with 102.80 at one end and 102.90 at the other. Discuss why these are the choices. Fill in the other boxes with 102.81, 102.82, 102.83, 102.84, 102.85, 102.86, 102.87, 102.88, and 102.89. Mark 102.88 with an arrow. Discuss why 102.88 is closer to 102.90. So $102.88 is rounded to $102.90.

Repeat rounding with the following amounts. Encourage the students to imagine the answer so that eventually they can do such problems without using the number lines on their sheet: $345.09, $8.53, $5.61, $75.15, $0.36, $149.99, $65.08, $99.99 ...

Sensible Rounding

AM
AP

I am learning to round answers sensibly by thinking about the context of the original problem.

Background

A problem like 44 ÷ 7 produces a calculator answer of 6.285714286. For *any* practical purposes, this number must be rounded. Yet there is no *one* rule for doing this. This is because the context always suggests the method of rounding. The students need to display very good number sense and understanding of real problems to round calculator answers sensibly.

Equipment: Calculators. Number lines (Material Masters 4–24 and 4–25).

Activity

Set a variety of division problems with the same calculator answer yet different rounded answers. For example:

"The supermarket sells seven large tins of peaches for $44. One costs $6.285714286." Discuss why two decimal place rounding is sensible in this scenario. (Supermarkets charge to the nearest cent.) On the class number line, discuss why the lower number is 6.28 and the higher is 6.29, and add them to the ends of the number line. Now discuss which end 6.285714286 is nearer to; 6.28 or 6.29? So conclude $6.285714286 ≈ $6.29 (to two decimal places).

Give the students a copy of their own empty number lines and work through the following problems. Discuss the answers carefully.

"Jane has 44 litres of milk to share among seven families. How much does she measure out for each family?" (6.285714286 litres ≈ 6.286 litres to the nearest millilitre or 6.285714286 litres ≈ 6.29 litres to the nearest hundredth of a litre. More accuracy than this is impractical.)

"The market gardener sends 44 tonnes of potatoes to seven supermarkets. How much does he send to each?" (Here sensibly round to the nearest kilogram. So 6.285714286 tonnes ≈ 6.286 tonnes = 6286 kilograms.)

"The service station sells seven large pizzas for $44. What does a pizza cost?" (Assume the service station charges to the nearest 10 cents. So $6.285714286 ≈ $6.30.)

"John shares 44 hard lollies among seven children. How many does each child get?"

(You cannot cut up hard lollies, so 6.285714286 lollies ≈ 6 lollies. Discuss how many would be left over and who would get the extra lollies.)

"Joel has 44 pizzas to share among seven people. How many pizzas does each person receive?" (Here 6.285714286 is very inappropriate as an answer. Everyone receives six pizzas. There are two whole pizzas left over so probably these should be cut into sevenths and everyone gets two-sevenths, which equals 0.2857142. So $44 \div 7 = 6\frac{2}{7}$ is a sensible answer.)

For each of these division problems, create word problems that are solved by the division yet the rounding rules change with the context. 2 225 ÷ 17   4 567 ÷ 29   7 888 ÷ 11 ...

Locating Decimal Fractions

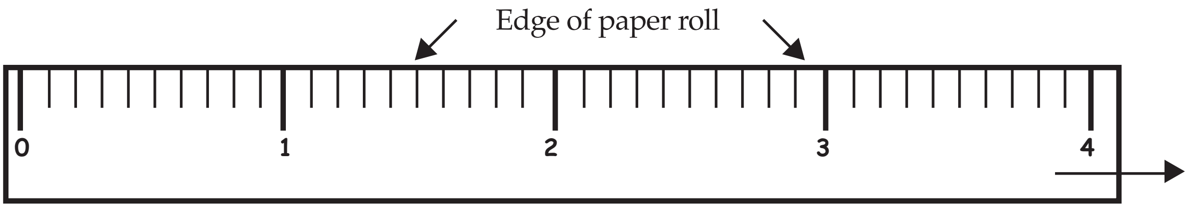
AM
AP

I am learning to locate decimal fractions on a large number line.

Equipment: Decimal fraction number cards (Material Master 4–23).

A roll of adding machine paper, or a number line painted on a paved area, that is at least 15 metres long. Pegs.

Along an edge of the paper roll, mark each metre and label it 0, 1, 2, 3, ... Also mark every tenth of a metre (10 cm). Do *not* label the tenths. The tape should be at least 15 metres long.



Note. You might use a standard tape measure, but you will need to cover all numbers except metres and all marks except tenths of metres with paper whitener.

Activity

Using cards from Material Master 4–23, students working in pairs mark lengths representing one decimal place numbers with pegs, for example, 12.5, then two decimal numbers, for example 13.53, then three decimal place numbers, for example 1.666.

Digits on the Move

AM
AP

I am showing that I understand that adding zeros when multiplying by 10, 100, 1 000 ... is not a reliable method.

Background

A beguilingly simple rule for multiplying by 10, 100, 1 000 ... is to add the correct number of zeros. Yet it is conceptually wrong, and it leads to wrong answers when decimal numbers are involved.

Equipment: Calculators.

Activity

"Janet solved this problem by adding a zero: There are 10 sacks of sugar each with 23 kg of sugar in them. What is the total weight?" Discuss why $10 \times 23 = 230$.

"Janet now measures the weight to greater accuracy and finds each bag more precisely weighs 23.1 kg."

Discuss the answer to 10 × 23.1. At this stage common *wrong* answers are 23.10 and 23.01. Both show the "add zeros" rule failing badly. Allow the students to work out 10 × 23.1 on a calculator. It gives 231. Discuss who is correct; them or the calculator? Revisit problems like 10 × \$23. Discuss the fact that the 20 \$10 notes give two \$100 notes and the 30 \$1 notes give three \$10 notes.

Hundreds	Tens	Units
	2	3
2	3	

Record the table on the board or modelling book and discuss the fact that multiplying by 10 moved the *digits* one place to the left. (In the table, a zero is not needed, but it has to be added when there is no table to differentiate the units.)

Repeat with a chart that has tenths in it. Discuss why 10 × 23.1 is two hundreds, three tens, and one unit (10 tenths). So, again, the digits move.

Hundreds	Tens	Units	Tenths
	2	3	1
2	3	1	

Repeat with similar problems.

Repeat with multiplying by 100, which moves digits two places to the left.

Repeat with multiplying by 1 000, which moves digits three places to the left.

Activity – Division

Problem: "Simon pays \$240 for 10 DVDs. What does one DVD cost?" Model \$240 on money. Discuss why dividing 240 by 10 moves the digits one place to the right.

Problem: "2 345.8 kg of a chemical is packed into 10 containers. What is the weight in each container?" Discuss what a calculator would show for 2 345.8 ÷ 10. Let the students check this on a calculator. Copy the table onto the board or modelling book. Discuss why the digits are moving one place to the right even for the decimal part. Record this in the table. So 2 345.8 ÷ 10 = 234.58.

Thousands	Hundreds	Tens	Units	Tenths	Hundredths
2	3	4	5	8	
	2	3	4	5	8

Repeat for 65.8 ÷ 10, 5.18 ÷ 10, 455 ÷ 10, 1 260 ÷ 100, 63 ÷ 100, 6.8 ÷ 100, 2 345 ÷ 1 000, 200.9 ÷ 1 000 ...

The Same but Different

AM

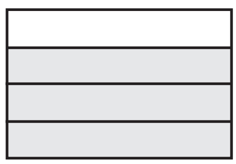
AP

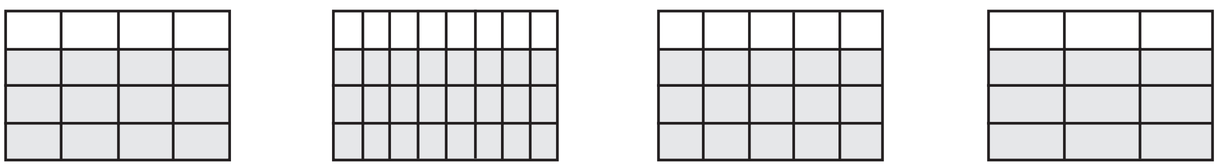
I am learning that every fractional number has an infinite number of different names.

Equipment: Plain paper. Ruler.

Activity

The students draw a rectangle 6 cm by 4 cm and shade $\frac{3}{4}$ as shown in the diagram. The students make six copies. Then they subdivide the various copies "vertically" to produce a range of equivalent fractions. Here are four examples:





These examples show $\frac{3}{4} = \frac{12}{16} = \frac{24}{32} = \frac{15}{20} = \frac{9}{12} = ...$

Collect and write all the different correct answers from the students on the board or modelling book. Discuss ordering them. Use this to spot any missing ones, and get the students to draw them. For example, if $\frac{18}{24}$ is missing, the students need to draw this by splitting the original rectangle into six equal pieces vertically.

Find the missing numbers equal to $\frac{3}{4}$ by looking for patterns:

$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} = \frac{18}{24} = \frac{21}{\square} = \frac{24}{\square} = ... = \frac{\square}{48}$ ...

Find the missing numbers for fractions equal to: $\frac{3}{4} = \frac{\square}{80}$, $\frac{3}{4} = \frac{300}{\square}$ ...

Make drawings to find equivalent fractions to $\frac{2}{3}$.

Find the missing numbers: $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{\square} = \frac{10}{\square} = \frac{\square}{18} = ... = \frac{\square}{30}$ ... $= \frac{200}{\square}$

Basic Facts

Up to Ten

CA
AC
EA

I am learning to recall instantly my addition facts that have the answer 10 or less.

Background

Instant recall of addition facts with answers up to 10, and their associated subtraction facts, is very important as preliminary knowledge ideally *before* early part-whole thinking emerges. Those students using early part-whole thinking can deduce the basic addition facts with answers over 10. For example, a student may work out 8 + 6 = 14 by adding two on to eight and taking two from six. This requires instantly knowing the basic facts 8 + 2 = 10 and 6 – 2 = 4.

Equipment: Tens frames (Material Master 4–6). Counters.

Activity

For 4 + 3, give the students a pre-printed frame with four dots on it. Get the students to add three counters, first by completing a five then putting the other two counters in the blank column. Expect the students to recognise this as totalling to seven *without* counting.

Repeat for a variety of addition facts. Encourage the students to visualise the answer before they put the counters on the tens frame.

For subtraction, use blank tens frames. For example, to work out 8 – 4 get the students to place eight counters on a blank tens frame (as five and three *not* four and four), then encourage them to visualise which four counters will be removed and have them check the answer by removing them.

Repeat for a variety of subtraction facts.

Double Trouble

AC
EA

I am learning to recall instantly my single-digit doubles facts and to work out other facts.

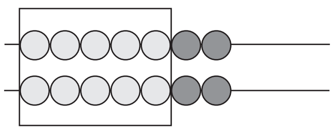
Equipment: A Slavonic abacus. Tens frames (Material Master 4–6). Fly Flip cards (Material Master 4–5). Double Somersaults Plus or Minus One game (Material Master 4–33).

Activity

Show two rows of three beads on the Slavonic abacus. Ask the students to show you three and three on their fingers. Many of them will know that this is six fingers altogether. Ask them what they will need to do in order to change their fingers to four and three (add one finger). Ask them what 4 + 3 must be. Record the equation on the board or modelling book. Ask the students to show you how they changed 3 + 3 into 4 + 3 on the abacus.

Provide other examples like, 2 + 2 = 4 so 2 + 3 = $\square$, 5 + 5 = 10 so 4 + 5 = $\square$, 4 + 4 = 8 so 5 + 3 = $\square$. Ask the students to connect each fact with the other and model it on their fingers and on the abacus.

Activity



Repeat with pairs of numbers from 6 to 10. For example, 7 + 7.

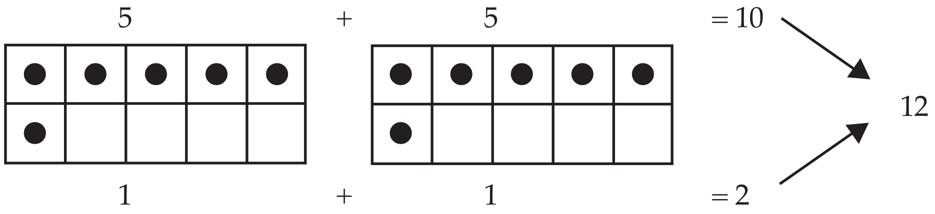
Ask the students to look at the beads and see if they can help work out the answer without counting each bead. Look for answers based on 10 + 4 = 14. Note that this requires knowledge of the teen numbers. Ask the students what they would do to change 7 + 7 into 8 + 7 (add one), and what the answer would be.

Pose similar problems. For example: 6 + 6 = 12 so 7 + 6 = $\square$, 9 + 9 = 18 so 8 + 9 = $\square$, 8 + 8 = 16 so 7 + 8 = $\square$.

Repeat these doubles with fingers, tens frames, and Fly Flip cards. For example, two students act like mirrors facing each other and show seven fingers. They match the full hands (10) and the two twos (four). Similarly two Fly Flip cards showing seven can be put down. Two fives will be visible and two twos will be behind their backs.

Activity – Another Way of Doubling

Another representation is to place two of the same tens frame cards side by side. Talk about the fives structure and point to the two rows of five. "How many dots is that? How many dots altogether?" For example, 6 + 6 = 12.



Activity

The students could play Double Somersaults Plus or Minus One to consolidate their doubles and related facts.

Using the Slavonic Abacus to Reinforce Five Grouping

I am learning some of the basic addition facts that have the answer 10 or less.

CA
AC
EA
AA

Equipment: Slavonic abacus.

Activity

Push over a small number of beads, for example, six, and ask "How many beads did I just push over?" and "How did you know it was six without counting the beads?" Aim for responses like "I know that one more than five is six."

Alternatively, start by pushing across five beads and ask "How many more to make nine?" Follow up by asking "How did you know?"

Activity

Push across a small number of beads and ask how many more are needed to make 10. This is reinforcing the key knowledge of pairs of numbers that add up to 10.

Number Boggle

I am learning my basic addition facts

CA
AC
EA
AA

Equipment: Number Boggle Card (Material Master 4–35).

Activity

Draw this grid on the board or modelling book and give the students two or three minutes to write as many addition and subtraction equations as they can based around these digits. The numbers in the equation must be connected vertically, horizontally, or diagonally.

15	9	6	7
3	12	8	5
7	4	13	2
5	1	11	10

For example, with this grid these are acceptable: 15 – 12 = 3, 5 + 7 = 12 (but *not* 7 + 5 = 12), 15 – 3 – 7 = 5.

Equations involve two or more numbers, and one or more operations, but no number can be used twice.

The students compare their equations in groups. They gain points for each equation only they have recorded. One point is given for each number used in the equation, for example, 15 – 3 – 7 = 4 + 1 earns five points! The player with the most points wins.

Tens Frames Again

CA
AC
EA
AA

I am learning to recall instantly pairs of numbers that add to 10 and their related subtraction facts.

Equipment: Tens frame cards (Material Master 4–6).

Activity

Check that the students can identify the number of dots on any tens frame card instantly without counting. Then focus their attention on the empty spaces so they realise that the dots plus the spaces equal 10. Practise all the combinations that make 10.

For subtraction, take a card with 10 dots on it and ask the students to imagine removing some dots. For example, for 10 minus four the students image four dots removed and see that there are six dots left. Check this by covering four dots. This leaves six dots exposed.

Practice all 12 combinations of every basic fact. For example the basic fact 3 + 7 = 10 is linked to all these problems:

$3 + 7 = \square$	$\square + 7 = 10$	$3 + \square = 10$	$7 + 3 = \square$
$\square + 3 = 10$	$7 + \square = 10$	$10 - 7 = \square$	$\square - 7 = 3$
$10 - \square = 3$	$10 - 3 = \square$	$\square - 3 = 7$	$10 - \square = 7$

Patterns to 10

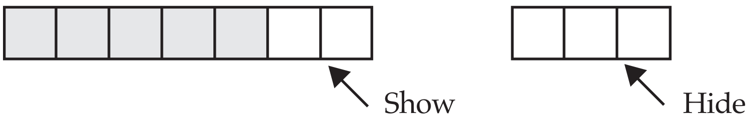
CA
AC
EA

I am learning to recall instantly pairs of numbers that add to 10 and their related subtraction facts.

Equipment: Stacks of 10 cubes made up of five cubes of each colour.

Activity

Group the students in pairs or small groups each with a stack of 10 cubes. Without the others seeing, a student takes some cubes from the stack. They show the other students the remaining cubes and ask the students to say how many cubes were taken.



The problem is made more difficult if cubes are removed from both ends of the stack.

Extension Activity

Take two stacks of 10 cubes, giving 20. Play the same game removing cubes from both stacks, handing the remaining cubes to a partner to work out how many were removed.

Number Mats and Number Fans

AC
EA
AA
AM

I am using my basic facts for addition, subtraction, multiplication, and division.

Equipment: A large piece of cloth or PVC with the numbers 0 to 9 arranged randomly across it or an A4 number mat for the students to use in pairs (Material Master 4–13). Number fans (Material Master 4–10). (Number fans can be altered to include the operations signs +, –, ÷, ×, and a decimal point.)

Activity

By putting their hands and feet on the number mat or picking out leaves of the fan, get the students to solve operational problems like:

"Show me two numbers that add to 10, 20, 50, 100, 1 000 ..."

"Show me two numbers with a difference of 4, 16, 29, 56 ..."

"Show me two or three numbers that multiply to give an answer of 12, 24, 72, 216 ..."

"Show me two numbers that when one number is divided by the other give the answer: 1, 7, 25, 499, 0.5, 3.25 ..."

Bridges

AC | EA | AA | AM

> I am learning to apply my basic facts to adding and subtracting from two-digit numbers.

Equipment: Strings of 100 beads in colour groups of five with the decades 0, 10, 20 ... marked with supermarket tags. Dice marked 5, 6, 7, 8, 9, 10. Pegs. Dominoes. Bridges game (Material Master 4–34).

Activity

Give each small group of students a bead string, a dice, and a different coloured peg for each player. Players take turns to roll the dice and work out where their peg will go when the number of beads is jumped. For example, a player who has their peg at bead 18 and throws a seven must predict that jumping seven beads will get their peg to 25, then check this by moving their peg.

Focus the students on bridging tens. For example, for 18 + 7, firstly 18 + 2 = 20. This leaves five of the seven. So 20 + 5 = 25. If the player incorrectly predicts which bead they will land on, they lose that turn. The player who gets over 100 first wins.

Play the game backwards as well, starting with the pegs in front of 100 and taking the dice number off. This involves going back through tens. For example, to find 83 – 9, firstly do 83 – 3 = 80 leaving six to take off. Then 80 – 6 = 74.

Activity

The students should play the game Bridges to consolidate up through 10 and back through 10 strategies.

Bowl a Fact

AC | EA | AA | AM | AP

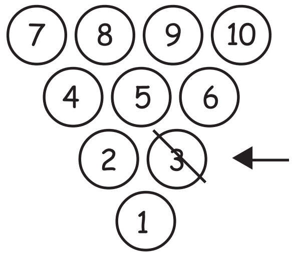
> I am learning to apply my knowledge of basic addition, subtraction, multiplication, and division facts.

Equipment: Whiteboard and whiteboard marker. Three dice.

Activity

Each player draws the 10-pin bowling triangle with the numbers 1 to 10 inside each circle. They take turns to roll three dice and record the numbers that come up. The students use the three numbers once only and combine them with any of the four operations, and brackets if needed, to make up a number sentence that "bowls out" numbers in the triangle.

For example, if 6, 4, and 2 are thrown, a number sentence might be $6 \times 2 \div 4 = 12 \div 4 = 3$, so the ball with 3 in it is "bowled out". Similarly, 8 might be bowled out with 6 + 4 – 2. The students try to "bowl out" as many numbers as possible with each turn, but need to wait for their next turn to have another throw when they are stuck. For the students at the early strategy stages, the focus should be on addition and subtraction, but the activity can be extended for more able students.



$6 \times 2 \div 4 = 12 \div 4 = 3$

Extension Activity – Advanced Multiplicative and Beyond

3 factorial or 3! means $1 \times 2 \times 3$. The recurring decimal $0.\dot{3}$ means 0.333333333 ... and equals $\frac{1}{3}$.

Change the numbers in the circles to 11 to 20. Include the four operations +, –, ×, ÷ and decimals, recurring decimals, and factorials.

For example, a student might roll 3, 3, and 6. They can work out:

$3! \div 3 \times 6 = 12$ $3! + 3! + 6 = 18$ $3 \div 0.\dot{3} + 6 = 15$ $\frac{3!}{0.\dot{3}} - 6 = 12$

In and Out

AC | EA | AA | AM | AP

I am learning to apply my basic multiplication and division facts.

Equipment: A whiteboard and a whiteboard marker. A calculator (on an overhead projector if possible).

Activity

Rule a grid on the whiteboard as shown.

Key a constant function into the calculator unseen. Press the 0 key so that the students cannot get any clue about the rule. For early additive and earlier students, use the addition and subtraction constants. For example, key in + 5 = then 0. Ask the students to provide you with *in* numbers. Record the "In" and corresponding "Out" numbers in a table as shown. The students work out what the calculator is doing to every number. (Provided no clear buttons are pressed the calculator will carry out the function on any number in the window when equals is pressed. This can be checked by entering 0 and pressing equals.)

In	Out
3	8
10	15
1	6
6	11
14	19

Extension Activity

Repeat using the constant multiplier and constant divisor functions on the calculator. Include the use of decimal numbers, for example, ÷ 1.5 =.

Repeat for a combination of two operations. For example, tell the students you have input a number then pressed ÷ 2 then + 4 to get 5 "out". Record the "out" numbers in an In/Out table and ask the students to find the "in" numbers. Two calculators can be used to model this, the first programmed ÷ 2 =, and the second programmed + 4 =. Students can put a number into the first calculator, press =, and take this number as the input into the second calculator. For example, $2 \div 2 = 1$, $1 + 4 = 5$.

Multiplication Madness

EA | AA | AM

I am learning to apply my basic multiplication facts.

Equipment: Game boards (Material Master 4–16). Three dice. Counters.

Activity

Divide the class into groups of three. Give each group a game board, three dice, and counters. Players take turns to roll the three dice. They check to see if the product of the three numbers is on the game board. If it is, they place a counter of their colour on that number. The player who is the first to get three counters in a row is the winner.

Loopy

EA AA AM

I am learning to apply my basic addition, subtraction, multiplication, and division facts.

Equipment: Sets of loopy cards (Material Master 4–17).

Activity

Share the loopy cards out among all the students in the group. The person who has "Start 1.2" on their loopy card begins and says the number at the top of the card and reads out loud the instruction on that card, "Add 0.8." The person with the answer 2.0 (1.2 + 0.8) on one of their cards reads out "2.0. Divide by 4". This keeps going until the card with "60. Finish" comes up.

Addition Flash Cards

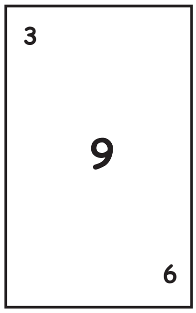
EA AA AM AP

I am learning to apply instantly each of my single-digit addition facts to all the other 11 variations of a basic fact.

Equipment: Flash cards (Material Master 4–29).

Activity

Show the students a flash card. For example, the card shows 3, 9, and 6. Practice all 12 combinations of every basic fact. Ask problems like: $6 + 3$, $9 - 6$, $9 - 3$, $6 + \square = 9$.
The complete list of 12 questions needed for each card is like this:



$3 + 6 = \square$	$\square + 6 = 9$	$3 + \square = 9$
$6 + 3 = \square$	$\square + 3 = 9$	$6 + \square = 9$
$9 - 6 = \square$	$\square - 6 = 3$	$9 - \square = 3$
$9 - 3 = \square$	$\square - 3 = 6$	$9 - \square = 6$

Dividing? Think About Multiplying First

EA AA AM AP

I am learning to divide by using my instant recall of basic multiplication facts where the remainder is sometimes but not always zero.

Background

Division involving basic multiplication facts with zero remainders requires reversibility in the students' thinking. Normally this occurs by the time early part-whole thinking emerges. Non-zero remainders are more complex. For example, $45 \div 6$ requires the student to instantly recall the answers to the six times table, realise that $7 \times 6 = 42$ is needed because only this leads to a remainder less than six, and work out that $45 - 42$ equals three mentally.

Equipment: Counters or similar objects.

Activity

Set a problem involving a multiplication fact that the students know by instant recall: "Five children have 15 sweets altogether. How many does each have?" Let the students model 15 objects in pairs or small groups. Ask how many groups have to be created (5). Without touching the 15 objects, ask the students to imagine how many each child gets and to discuss why they think this is so. Check by sharing out the objects. Record $15 \div 5 = 3$ on the board or modelling book. Link this problem to the multiplication fact $5 \times 3 = 15$ (*not* $3 \times 5 = 15$).

Repeat for other multiplication facts that the students know by instant recall.

Activity – Advanced Additive Onwards

Set a problem like: "Seventeen lollies are shared among three children. How many will each child get?" Suppose an answer like two comes up, record $5 \times 2 = 10$ on the board or modelling book and ask, "How many will be left over?" "Is there enough for everyone to have another lolly?" (Yes.) "How many does everyone now have?" (Three.) "Now is there enough for everyone to have another lolly?" (No.) "Why not?"

Now record $17 \div 5 = 3$ with 2 left over on the board or modelling book.

Repeat with similar examples.

Multiplication Flash Cards

EA
AA
AM
AP

I am learning to apply instantly each of my single-digit multiplication facts to all the other 11 variations of a basic fact.

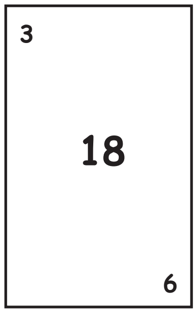
Equipment: Flash cards. (Material Master 4–37).

Activity

Show the students a flash card, for example, the card shows 3, 18, and 6. Practice all 12 combinations of every basic fact. Ask problems like:

6×3, $18 \div 6$, ... The complete list of 12 questions needed for each card is like this:

$3 \times 6 = \square$	$\square \times 6 = 18$	$3 \times \square = 18$
$6 \times 3 = \square$	$\square \times 3 = 18$	$6 \times \square = 18$
$18 \div 6 = \square$	$\square \div 6 = 3$	$18 \div \square = 3$
$18 \div 3 = \square$	$\square \div 3 = 6$	$18 \div \square = 6$



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