MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Book 2
The Diagnostic Interview

Numeracy Professional Development Projects

THE NUMERACY DEVELOPMENT PROJECTS

Teachers are key figures in changing the way in which mathematics and statistics is taught and learned in schools. Their subject matter and pedagogical knowledge are critical factors in the teaching of mathematics and statistics for understanding. The effective teacher of mathematics and statistics has a thorough and deep understanding of the subject matter to be taught, how students are likely to learn it, and the difficulties and misunderstandings they are likely to encounter.

The focus of the Numeracy Development Projects is to improve student performance in mathematics through improving the professional capability of teachers. To date, almost every teacher of year 1 to 6 children and the majority of teachers of year 7 and 8 children have had the opportunity to participate.

A key feature of the projects is their dynamic and evolutionary approach to implementation. This ensures that the projects can be informed by developing understanding about mathematics learning and effective professional development and that flexibility in approach and sector involvement is maximised.

The projects continue to build on the findings and experience associated with the numeracy professional development projects that operated in 2002–2007. These projects made an important contribution to what we know about:

- children's leaming and thinking strategies in early mathematics;
- effective identification of, and response to, children's learning needs;
- the characteristics of professional development programmes that change teaching practice; and
- effective facilitation.

Such findings continue to inform the modification and further development of the projects. National co-ordinators and facilitators from each region provide ongoing feedback about aspects of the projects.

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Numeracy Project Assessment (NumPA)

Introduction

The Numeracy Project Assessment (NumPA) is a diagnostic tool that is designed to give quality information about the knowledge and mental strategies of the students you work with. This information is aligned to The Number Framework (see *Book 1: The Number Framework*).

NumPA takes the form of an individual interview with students. This is necessary for two reasons:

(1) Uncovering students' mental strategies involves finding out how they solve number problems. Pencil and paper assessment tasks show whether or not a student is getting correct answers. However, it is often difficult to find out the strategies that the student uses with pencil and paper assessment.

(2) The interview process is invaluable for your own professional development.

To decide which form of NumPA to use, you will need to use the strategy window questions discussed on pages 4 and 5 or the flowchart on page 7. Use your prior knowledge of the student to choose which question to begin with.

To develop the confidence of the student, it is best to start with a question he or she will find easy.

Use the sequence of questions on pages 4 and 5 to classify the student's responses. From that, use the table on page 6 or the flowchart on page 7 to choose which of the assessment forms, A, B, or C, to use.

The interview consists of two main parts; strategy questions and knowledge questions.

When you first use the assessment, it is important to follow the script provided. This script directs you to skip questions where it is clear that they will be too difficult for the student. This saves time and potential anxiety.

With the **strategy questions**, your primary focus is on how the student solved the problem. It is suggested that you ask the student, "How did you work that out?" for each problem. Sometimes this will not provide adequate information about the strategies the student used, and you will need to ask other more directed questions. It is wise practice where the answer is correct to reassure the student by saying, "Yes, that's right" before asking them about the strategy they used.

With the **knowledge questions**, you are looking for fluent responses. Students who take some time to answer a knowledge question are usually strategising, which means that they do not automatically know the item of knowledge concerned and are working out a solution.

As you become more familiar with the items and how to evaluate students' responses, you will become much quicker at administering the NumPA. You will get better at assigning stages for each area of strategy and knowledge from the assessment using the *least possible number of questions*.

Materials Needed for NumPA

This section provides instructions for assembling the NumPA. Copymasters are provided for all of the materials needed. Since the assessment materials are used frequently, it is advisable to laminate the card items. For each form of the test use the colour of card suggested. Keep all the materials in a see-through plastic bag.

Hardware	Cardware
• Twelve counters of one colour • Ten counters of another colour • Two masking cards (A5 size)	**Strategy Windows:** None **NumPA Form A:** Blue Make up the sets of cards and strips, from pages 14 and 15. **NumPA Form B:** Yellow Make up the sets of cards from pages 24 to 25. **NumPA Form C:** Green Make up the sets of cards from page 39.
Test Booklets: Spiral Bind	**Test Scripts: Spiral Bind**
Strategy Windows: White Photocopy pages 8 and 9. **NumPA Form A:** Blue Photocopy page 16. **NumPA Form B:** Yellow Photocopy pages 26–30. **NumPA Form C:** Green Photocopy pages 39–47.	**Strategy Windows:** White Photocopy pages 4–7. **NumPA Form A:** Blue Photocopy pages 10–13. **NumPA Form B:** Yellow Photocopy pages 17–23. **NumPA Form C:** Green Photocopy pages 31–38.

Individual Assessment Sheets

On pages 48 to 53, you will find individual assessment sheets for each form of NumPA. These sheets make up a three-page profile where you can enter the initial and follow-up NumPA data for a student. Such an individual assessment sheet is a significant document in a student's record of achievement. Note that page 54 provides answers for the Form C questions.

Before beginning the interview, write the student's personal information in the top section of the appropriate assessment sheet. You need to make a judgement about the student's developmental stage after asking the appropriate questions within each section of the assessment.

The strategy windows questions provided in the first box under the student identification details will enable you to determine the student's strategy stage for addition and subtraction. The interview questions are given in the left-hand column of each box. In time, you will learn to interview students directly from the assessment sheets without referring to the detail included in the test scripts.

The panels for entering each judgement are in the columns to the right of the questions being asked. There is a space within each stage to enter either an initial interview result (clear box) or a follow-up result (shaded box). The entry should include the date in abbreviated form, for

example, 4/5. Any comments that you would like to make, including details on how the student solved the strategy windows questions, can be written in the comments space underneath. It is often helpful to write the student's responses for later reference. Where possible, record the student's responses as equations or empty number lines. In deciding which cell to make the entry in, you should use the following criteria:

Strategy Questions: Enter the highest stage the student demonstrates within each operational domain.

Knowledge Questions: Enter the highest stage at which the student answers every item correctly with fluency and certainty.

From these individual assessment sheets, you can transfer the results onto a Class Grouping Sheet for NumPA for your class (see *Book 3: Getting Started*).

The stages that a student achieves within the Knowledge sections of the interview are often different from the stages that they achieve in the Strategy sections. Knowledge can be learned independently of any meaningful ability to apply it. Similarly, students can invent advanced strategies without sufficient knowledge to apply them to a broad range of problems and numbers.

Since the original numeracy development projects in 2001–2002, the assessment tool has been revised annually. In 2003, the operational strategy windows were introduced to refer teachers to each form of NumPA. This allowed the interviews to cover a wide range in student understanding.

In 2004, minor changes were made to separate the place value and basic facts knowledge domains. This followed weak performances in this area and a recommendation from the Te Poutama Tau project evaluation and the Numeracy Reference Group.

The changes for 2005 included extending the addition, subtraction, multiplication and division strategy domains to include fraction and decimal items. The place value knowledge questions were extended to include items related to students' ability with tenths. These items were also the result of recommendations from the project evaluations, 2003.

The 2006 changes included refining the early stages for place value and basic facts to put more emphasis on five as a sub-base and a simplifying of a place value task with large whole numbers. These were the result of feedback from facilitators and a Rasch analysis carried out on the knowledge items.

Interactive review of the project materials is a strength of the numeracy development projects, and will continue.

Operational Strategy Windows
Addition and Subtraction

The answers to these tasks determine which form of NumPA to use. Keep a note of the student's response to each question.

Task (1): Count eight objects.

Actions: Provide the student with access to a pile of counters of the same colour.

Say: **Please get 8 counters for me.**

Decision: If the student did not count eight items, rate him/her as stage 0 on operational strategies. Proceed to form A. Otherwise proceed to task (2).

Task (2): Work out 4 + 3 on materials.

Actions: Place four counters in the student's hand. Place three counters in the student's other hand. Ask the student to close their hands. (Open later if necessary).

Say: **Please hold out your hands for me. Here are 4 counters. Here are another 3 counters. How many counters have you got altogether?**

Decision: If the student was unable to solve 4 + 3 correctly, rate them at stage 1. Proceed to form A. If the student solved 4 + 3 by counting the materials, rate her/him at stage 2. Proceed to form A. Otherwise proceed to task (3).

Task (3): Find 8 + 5 = □.

Actions: Place 8 counters of one colour under a card and 5 counters of another colour under another card. Reveal the collections to the student then cover them, one at a time. Show the problem card to the student.

Say: **There are 8 counters under this card and 5 counters under this card. How many counters are there altogether?**

Decision: If the student solved task (2) by imaging but did not solve task (3) by counting on, rate her/him at stage 3. Proceed to form A. Otherwise proceed to task (4). If the student solved task (3) by counting on, still give her/him task (4) as this may provoke part-whole thinking.

Task (4): Find 9 + 8 = □.

Actions: Place 9 counters of one colour under a card and 8 counters of another colour under another card. Reveal the collections to the student then cover them, one at a time. Show the problem card to the student.

Say: **There are 9 counters under this card and 8 counters under this card. How many counters are there altogether?**

Decision: If the student solved both tasks (3) and (4) by counting on, rate her/him at stage 4. Proceed to form B. For students who used any part-whole strategy for either or both tasks (3) and (4), continue to task (5).

Task (5): Find 37 – 9.

Actions: Show the card with the lolly problem on it to the student.

Say: **You have 37 lollies, and you eat 9 of them. How many lollies have you got left?**

Decision: If the student used any part-whole strategies on tasks (3) and (4) but counted back to solve task (5), rate her/him at stage 5. Proceed to form B. If the student used a part-whole strategy on task (5), proceed to task (6).

Task (6): Find 53 – 26.

Actions: Show the card with the bus problem written on it.

Say: **There are 53 people on the bus. 26 people get off. How many people are left on the bus?**

Decision: If the student imagined using a standard written method, read the notes at the end of task (8). If the student failed to solve this problem correctly, rate her/him at stage 5 and proceed to form B.

Task (7): Find 394 + 79.

Actions: Show the card with the stamp problem written on it.

Say: **Sandra has 394 stamps. She gets another 79 stamps from her brother. How many stamps does she have then?**

Decision: If the student gets both of tasks (6) and (7) correct using part-whole strategies, proceed to task 8. Otherwise rate the student at stage 5 and proceed to form B.

Task (8): Find 5.3 – 2.89 metres.

Actions: Show the card with the sewing problem written on it.

Say: **Marija has a 5.3 metre length of fabric. She uses 2.89 metres of it to make a tracksuit. How much fabric has she got left?**

Decision: Regardless of the student's answer, proceed to task 9.

Task (9): Find $2 - (\frac{3}{4} + \frac{7}{8})$ pizza.

Actions: Show the card with the pizza problem written on it.

Say: **Harry and Sally buy two pizzas. Harry eats $\frac{3}{4}$ of a pizza while his friend Sally eats $\frac{7}{8}$ of a pizza. How much pizza is left over?**

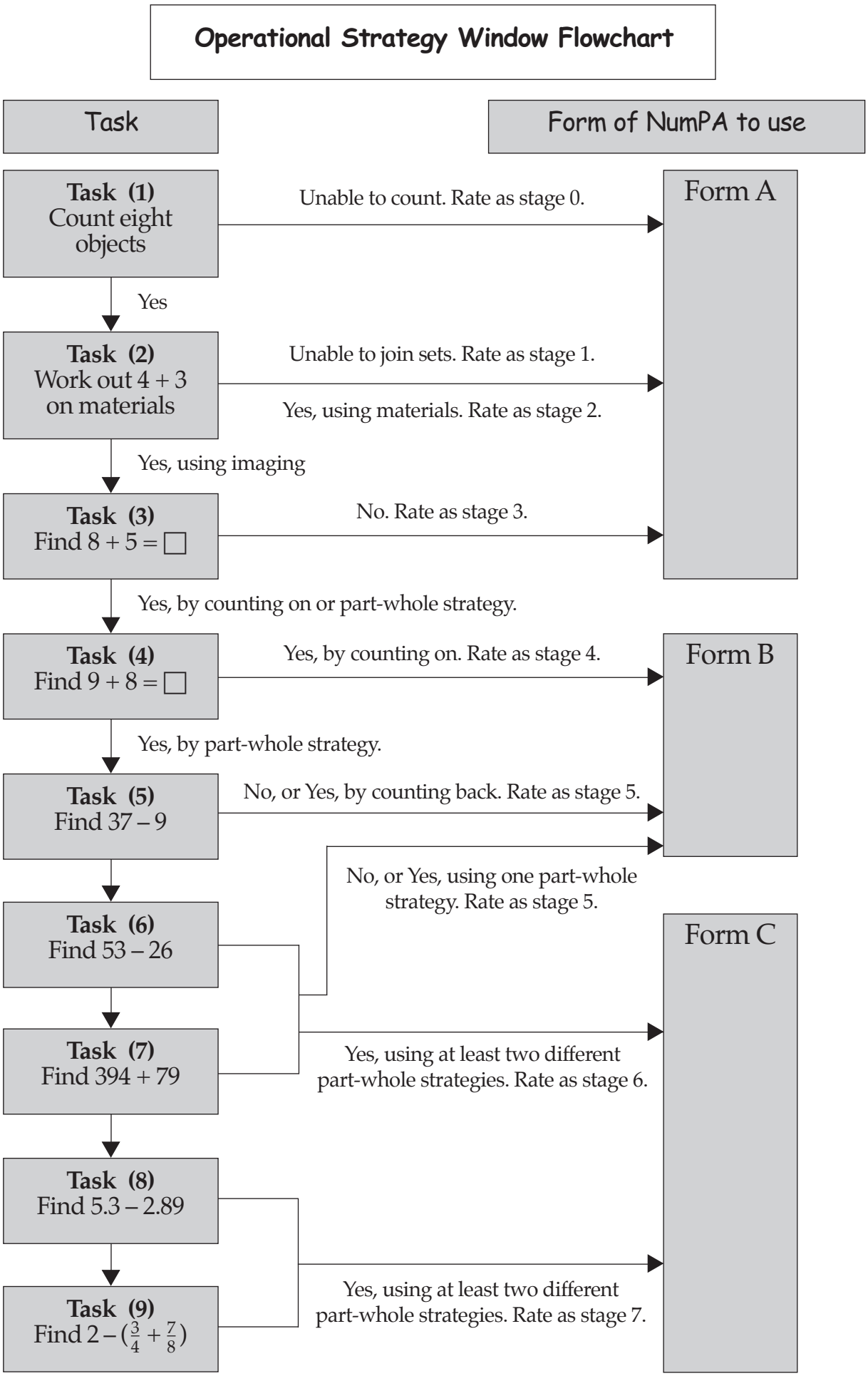
Decision: If the student gets both Tasks (8) and (9) correct using part-whole strategies, rate her/him at stage 7 for addition and subtraction. Otherwise rate the student at stage 6. Proceed to form C.

Note about Standard Written Forms

If the student's method on any problem was to perform standard written form in their head, then no conclusions should be drawn about their strategy stage. Further questioning is needed to establish their strategy stage. For example:

- "Can you do that in a different way?"
- "Explain how the written method works." Look for evidence of part-whole reasoning; for example, for 53 – 26, the student clearly understands that 53 is the same as four tens and 13 ones.

Stage & Behavioural Indicator	Use NumPA form:
0 Emergent The student has no reliable strategy for counting an unstructured collection of items.	**A**
1 One-to-one Counting The student has a reliable strategy for counting an unstructured collection of items.	**A**
2 Counting from One on Materials The student's most advanced strategy is counting from one on materials to solve addition problems.	**A**
3 Counting from One by Imaging The student's most advanced strategy is counting from one without the use of materials to solve addition problems.	**A**
4 Advanced Counting The student's most advanced strategy is counting on or counting back to solve addition or subtraction tasks.	**B**
5 Early Additive Part-Whole The student uses any part-whole strategy to solve addition or subtraction problems mentally by reasoning the answer from basic facts and/or place value knowledge.	**B**
6 Advanced Additive–Early Multiplicative Part-Whole The student is able to use at least *two different* mental strategies to solve addition or subtraction problems with multi-digit numbers.	**C**
7 Advanced Multiplicative–Early Proportional Part-Whole The student is able to use at least *two different* mental strategies to solve addition or subtraction problems with decimals and fractions with related denominators.	**C**


Operational Strategy Window Flowchart
Task
Form of NumPA to use
Task (1)
Count eight objects
Unable to count. Rate as stage 0.
Form A
Yes
Task (2)
Work out 4 + 3 on materials
Unable to join sets. Rate as stage 1.
Yes, using materials. Rate as stage 2.
Yes, using imaging
Task (3)
Find 8 + 5 = □
No. Rate as stage 3.
Yes, by counting on or part-whole strategy.
Task (4)
Find 9 + 8 = □
Yes, by counting on. Rate as stage 4.
Form B
Yes, by part-whole strategy.
Task (5)
Find 37 – 9
No, or Yes, by counting back. Rate as stage 5.
No, or Yes, using one part-whole strategy. Rate as stage 5.
Task (6)
Find 53 – 26
Form C
Task (7)
Find 394 + 79
Yes, using at least two different part-whole strategies. Rate as stage 6.
Task (8)
Find 5.3 – 2.89
Yes, using at least two different part-whole strategies. Rate as stage 7.
Task (9)
Find 2 – (3/4 + 7/8)

Photocopiable Material

Tasks (3) to (8)

Task (3)

$$8 + 5 = \square$$

Task (4)

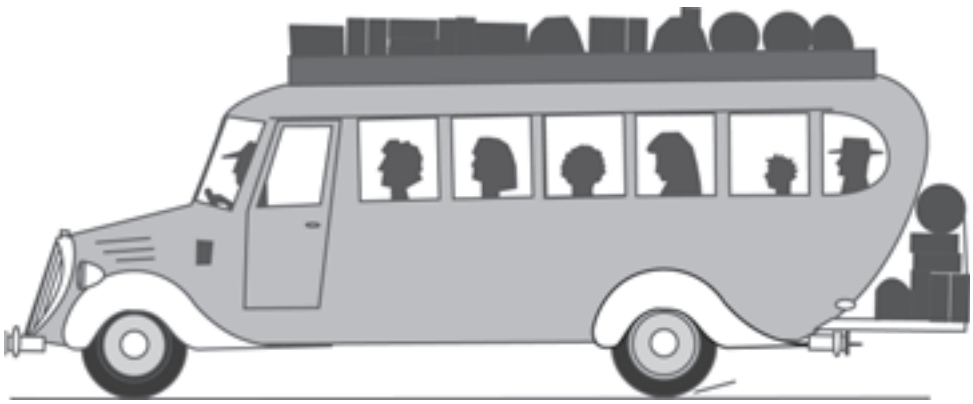
$$9 + 8 = \square$$

Task (5)

You have 37 lollies, and you eat 9 of them. How many lollies have you got left?

Task (6)

There are 53 people on the bus. 26 people get off. How many people are left on the bus?



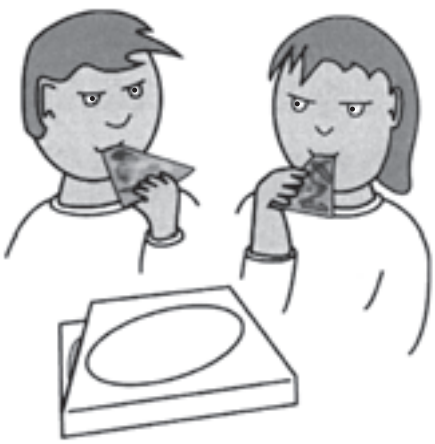
Task (7)

Sandra has 394 stamps. She gets another 79 stamps from her brother. How many stamps does she have then?

Task (8)

Marija has a 5.3 metre length of fabric. She uses 2.89 metres of it to make a tracksuit.
How much fabric has she got left?

Task (9)



Harry and Sally buy two pizzas. Harry eats $\frac{3}{4}$ of a pizza while his friend Sally eats $\frac{7}{8}$ of a pizza. How much pizza is left over?

White

Numeracy Project Assessment (NumPA) Form A

Transfer the notes from the Strategy Windows tasks (pages 4 and 5) to the addition and subtraction stage boxes on the Individual Assessment Sheet.

Knowledge Questions

Forwards Number Word Sequence (FNWS)

Things the interviewer says are in bold. Comments for the interviewer appear in plain type.

(1) **Start counting from 1. I will tell you when to stop.** Stop at 32.

What's the next number after...? If the student does not understand the meaning of the question, say: **The next number after 2 is 3. So if I say 2, you say 3.**

What is the next number after ...? (2) **5** (3) **9**

For questions (4) to (7), listen carefully for confusion between "teen" and "ty". If the student has this confusion, they are assessed at no higher than stage 2.

What's the next number after ...?

(4) **13** (5) **19** (6) **12** (7) **15**

If the student confuses "teen" and "ty" in questions (4) to (7), it is still worthwhile to ask questions (8) to (11) to see if the confusion is only with the "teen" numbers.

What's the next number after ...?

(8) **29** (9) **46** (10) **69** (11) **80** (12) **139** (13) **899**

Stage & Behavioural Indicator
0 Emergent FNWS The student cannot produce the FNWS from 1 to 10.
1 Initial FNWS up to 10 The student can produce the FNWS from 1 to 10 but cannot produce the number just after a given number in the range 1 to 10.
2 FNWS up to 10 The student can produce the number just after a given number in the range 1 to 10 without dropping back.
3 FNWS up to 20 The student can produce the number just after a given number in the range 1 to 20 without dropping back.
4 FNWS up to 100 The student can produce the number just after a given number in the range 1 to 100 without dropping back.
5 FNWS up to 1 000 The student can produce the number just after a given number in the range 1 to 1 000 without dropping back.

For the forwards and backwards number word sequences, *dropping back* means that the student says or mentally counts several numbers up to the given number. He/she then says the number before or after that number. For example, the student finds the number after five by saying one, two, three, four, five, six. A student finds the number before 14 by saying 10, 11, 12, 13, 14, then says 13.

Backwards Number Word Sequence (BNWS)

(14) Count backwards from 10. I will tell you when to stop. Stop at 0 or 1.

(15) Count backwards from 24. I will tell you when to stop. Stop at 11.

What number comes before ...?
If the student does not understand the meaning of the question, say: The number that comes before 2 is 1. So if I say 2, you say 1.

What number comes before ...?

(16) 3 (17) 9 (18) 5 (19) 8

For questions (20) to (23), listen carefully for confusion between "teen and "ty". If the student has this confusion, they are assessed at no higher than stage 2.

What number comes before ...?

(20) 16 (21) 20 (22) 17 (23) 11 (24) 13

If the student confuses "teen" and "ty" in questions (20) to (24), it is still worthwhile to ask questions (25) to (26) to see if the confusion is only with the teen numbers.

What number comes before ...?

(25) 31 (26) 47 (27) 70 (28) 236 (29) 600

Stage & Behavioural Indicator	
0	**Emergent BNWS** The student cannot produce the BNWS from 10 to 0.
1	**Initial BNWS back from 10** The student can produce the BNWS from 10 to 0 but cannot produce the number just before a given number in the range 0 to 10.
2	**BNWS back from 10** The student can produce the number just before a given number in the range 0 to 10 without dropping back.
3	**BNWS back from 20** The student can produce the BNWS from 20 to 0, and the number just before a given number in the range 0 to 20 without dropping back.
4	**BNWS back from 100** The student can produce the BNWS from 100 to 0, and the number just before a given number in the range 0 to 100 without dropping back.
5	**BNWS back from 1 000** The student can produce the BNWS from 1 000 to 0, and the number just before a given number in the range 0 to 1 000 without dropping back.

Numeral Identification

What is this number? Show cards with the number written on them.

(30) 3 (31) 9 (32) 5 (33) 1 (34) 8

(35) 6 (36) 0 (37) 4 (38) 2 (39) 7 (40) 10

For questions (41) to (45), listen carefully for confusion between "teen" and "ty". If the student has this confusion, they are assessed at stage 1.

What is this number?

(41) 13 (42) 19 (43) 11 (44) 16 (45) 12

If the student confuses "teen" and "ty" in questions (41) to (45), it is still worthwhile to ask questions (46) to (49) to see if the confusion is only with the "teen" numbers.

What is this number?

(46) 66 (47) 43 (48) 80 (49) 38 (50) 137 (51) 702

Stage & Behavioural Indicator
0 Emergent Numeral Identification The student cannot identify most of the numerals in the range 0 to 10.
1 Numerals to 10 The student can identify the numerals in the range 0 to 10.
2 Numerals to 20 The student can identify the numerals in the range 0 to 20.
3 Numerals to 100 The student can identify one- and two-digit numbers.
4 Numerals to 1 000 The student can identify two- and three-digit numbers.

Place Value

(52) Place a four-strip horizontally on the table. Now place a ten-strip beneath the four-strip. Say: Here are four dots. Here are ten more dots. How many dots are there now? Continue adding ten-strips to show 24, 34, 44, 54, 64, 74, asking the student to name the total number of dots each time.

If the student cannot count the total number of dots for four and ten, rate him/her at stage 0–1. If the student counts all the dots to find totals of 14 and 24, rate him/her at stage 2. If the student counts in fives and ones to find totals of 14 and 24, rate him/her at stage 3. For students who add ten each time to their previous answer when a new strip is added, 14, 24, 34, 44, ..., tentatively rate them at stage 4. You may wish to further assess their place value understanding using questions 27 and 28 of Form B.

Stage & Behavioural Indicator	
0–1	**Emergent** The student cannot count the number of objects in combined collections.
2	**One as a Unit** The student finds the total number of objects in collections by counting all of the objects by ones. He/she does not use ten as a counting unit.
3	**Five as a Counting Unit** The student uses five as a counting unit, for example, 5, 10, 11, 12, 13, 14.
4	**Ten as a Counting Unit** The student uses ten as a counting unit, for example, 10, 20, 30, 40, 41, 42, 43, 44.

Basic Facts

For questions (53) to (59), show the equation from the test booklet and read it out aloud. Instant recall of the answers is required rather than counting methods.

Tell me the answer to ...

(53) 2 + 3 (54) 5 + 4 (55) 6 and what makes 10?

(56) 6 + 6 (57) 9 + 9 (58) 10 + 4 (59) 7 + 10

Stage & Behavioural Indicator	
0–1	**Emergent** The student is unable to recall instantly facts to five, for example, 2 + 3.
2	**Addition Facts to Five** The student instantly recalls facts to five, for example, 2 + 3.
3	**Addition Facts to Ten** The student instantly recalls facts to ten, for example, 5 + 4, 6 + □ = 10.
4	**Addition Facts with Tens and Doubles** The student recalls the doubles to 20, and teen facts, for example, 14 = 10 + 4.

Based on: Wright, R. J., Martland, J., & Stafford, A. (2000). *Early Numeracy: Assessment for Teaching and Intervention*. London: Paul Chapman Publications/Sage. Acknowledgement is made that some of the ideas for questions were originally sourced from the New South Wales's *Count Me In Too Professional Development Package* (1999). Department of Education and Training Publishers. New South Wales, Australia.

Photocopiable Material

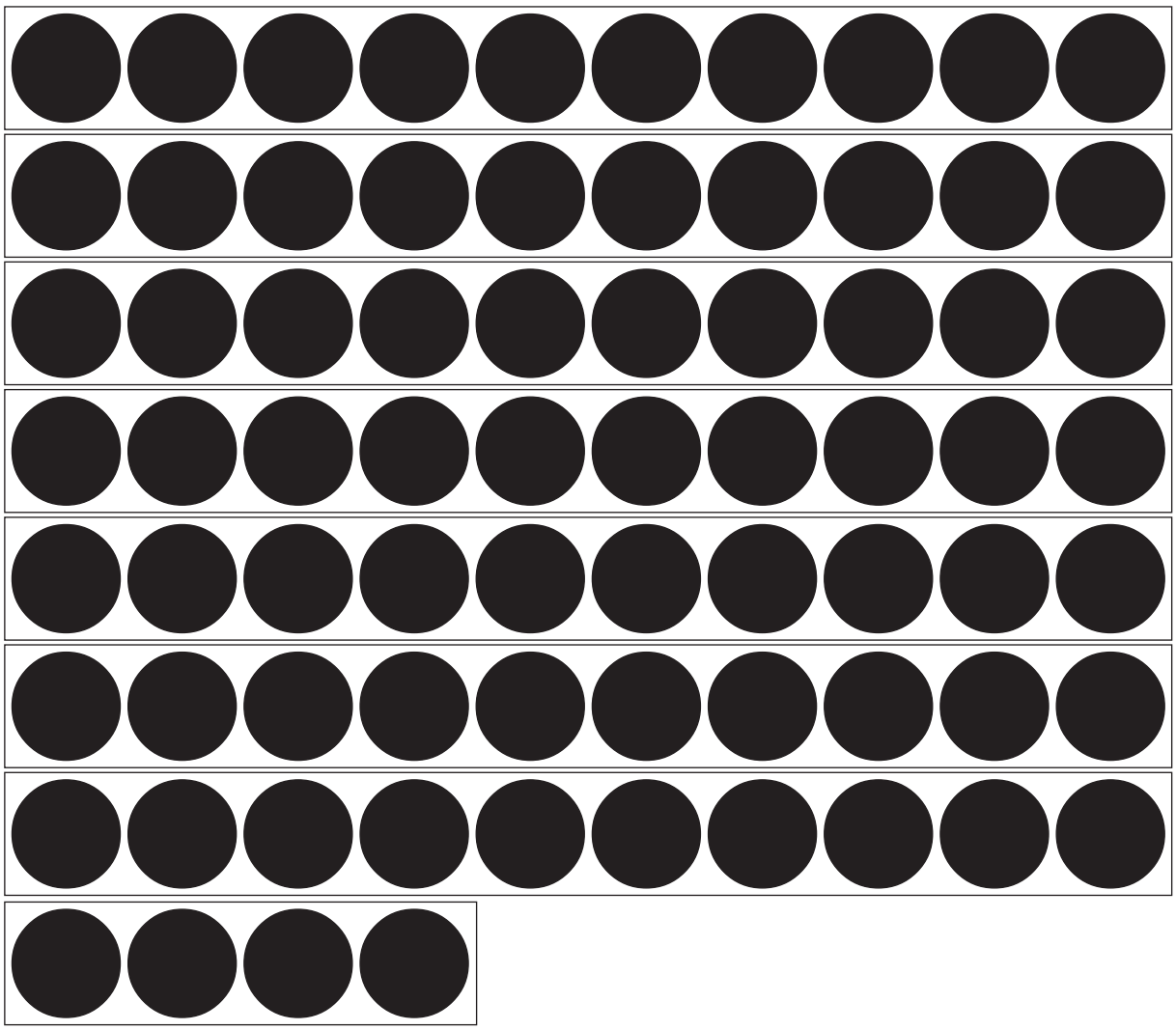
Tasks (30) to (51)

0	1	2
3	4	5
6	7	8
9	10	11
12	13	16
19	38	43

Blue

66	80	137
702		

Question 52



Question 53 2 + 3	Question 54 5 + 4
Question 55 6 + ☐ = 10	Question 56 6 + 6
Question 57 9 + 9	Question 58 10 + 4
Question 59 7 + 10	

Numeracy Project Assessment (NumPA) Form B

Transfer the notes from the Strategy Windows tasks (pages 4 and 5) to the addition and subtraction stage boxes on the Individual Assessment Sheet.

Operational Strategy Questions

Multiplication and Division

Things the interviewer says are in bold. Comments for the interviewer appear in plain type.

(1) **Here is a forest of trees. There are 5 trees in each row, and there are 8 rows.** Use horizontal and vertical sweeps with the index finger. Mask all but one horizontal and one vertical edge of the array. **How many trees are there in the forest altogether?** If the student is unable to give an answer, uncover the rest of the sheet. **If I planted 15 more trees, how many rows of 5 would I have then altogether?**

If the students solves question (1) using one-to-one counting and/or skip-counting, omit questions (2) and (3). Rate him/her at stages 2–3 or 4, as appropriate. Stop the multiplication questions and proceed to the questions on proportions and ratios.

For questions (2) and (3), screen the answer then uncover it if the student responds correctly. If the student gives no response or an incorrect one, go to the proportions and ratios section.

(2) **What is 3 × 20?**
If 3 × 20 = 60, what does 3 × 18 equal?
Does the student derive 3 × 18 by 60 – 6 = 54?

(3) **What is 5 × 8? If 5 × 8 = 40, what does 5 × 16 equal ?**
Does the student derive 5 × 16 = 80 by doubling 40?

Rate the student according to the most advanced strategy he/she uses on questions (2) and (3). Note that the student can know how to derive multiplication facts from other known facts, that is, he/she could be at stage 6 but use counting on or back.

Stage & Behavioural Indicator	
2–3	**Counting from One** The student solves multiplication problems by counting all of the objects.
4	**Advanced Counting** The student solves multiplication problems by skip-counting, where he/she has a known sequence or by using a combination of skip-counting and counting in ones, for example, 5, 10, 15, 20.
5	**Early Additive Part-Whole** The student solves multiplication problems by forming the factors where they have a known multiplication fact or by using repeated addition, for example, for 5 × 8: 5 + 5 = 10, 10 + 10 + 10 + 10 = 40
6	**Advanced Additive–Early Multiplicative Part-Whole** The student solves multiplication problems by deriving from known multiplication facts, for example, 3 × 20 = 60 so 3 × 18 = 60 – (3 × 2) = 54.

Proportions and Ratios

(4) Show the student the fraction circle sheet. **Which of these cakes has been cut into thirds?** If the student responds incorrectly, point to the thirds. **Here are 12 jelly beans to spread out evenly on top of the cake. You eat one-third of the cake. How many jelly beans do you get?** If the student cannot answer the question, allow them to manipulate the beans or counters to solve it. If the student needs to manipulate the materials to solve question (4), rate them at stage 1 or 2–4, as appropriate, and proceed to the knowledge questions.

(5) **What is $\frac{3}{4}$ of 28?** Does the student use a part-whole strategy based on addition and/or multiplication?

Stage & Behavioural Indicator	
1	**Unequal Sharing** The student is unable to find a fraction of a number by sharing the objects into equal subsets.
2–4	**Equal Sharing** The student finds a fraction of a number by sharing the objects into equal subsets, physically or by imaging.
5	**Early Additive Part–Whole** The student finds a unit fraction of a number mentally, using trial and improvement with addition facts, for example, $\frac{1}{3}$ of 12 as $4 + 4 + 4 = 12$.
6	**Advanced Additive–Early Multiplicative Part–Whole** The student finds a fraction of a number mentally, using a combination of addition facts and multiplication, for example, $\frac{3}{4}$ of 28 as: $\frac{1}{4}$ of $20 = 5$ so $\frac{1}{4}$ of $24 = 6$ so $\frac{1}{4}$ of $28 = 7$, $3 \times 7 = 21$; or $\frac{1}{2}$ of 28 is 14, $\frac{1}{2}$ of 14 is 7, $14 + 7 = 21$.

Knowledge Questions

Forwards Number Word Sequence (FNWS)

Ask question (6) only if the student is at the advanced counting stage. For other students proceed to question (7).

(6) **Start counting from 10. I will tell you when to stop.** Stop at 32. If the student has problems counting up through the teens, rate him/her at stage 2 and proceed to the BNWS questions.

For each number I show you, read the number then tell me the number that comes just after it, the number that is one more. For example, if I show you 4, you say 5. Show the FNWS cards. Stop at the point at which the student encounters difficulty and proceed to the BNWS questions.

(7) **12** (8) **17** (9) **29** (10) **99** (11) **209**

(12) **999** (13) **3 049** (14) **989 999**

For the forwards and backwards number word sequences, *dropping back* means that the student says or mentally counts several numbers up to the given number. He/she then says the number before or after that number. For example, the student finds the number after 25 by saying 21, 22, 23, 24, 25, 26. A student finds the number before 14 by saying 10, 11, 12, 13, 14, then says 13.

Rate the student at the highest stage in which they get all relevant questions (7) to (14) correct.

Stage & Behavioural Indicator
2 FNWS up to 10 The student can read and give the number just after a given number in the range 1 to 10 *without dropping back*.
3 FNWS up to 20 The student can read and produce the number just after a given number in the range 1 to 20 *without dropping back*.
4 FNWS up to 100 The student can read and produce the number just after a given number in the range 1 to 100 *without dropping back*.
5 FNWS up to 1 000 The student can read and produce the number just after a given number in the range 1 to 1 000.
6 FNWS up to 1 000 000 The student can read and produce the number just after a given number in the range 1 to 1 000 000.

Backwards Number Word Sequence (BNWS)

Ask question (15) only if the student is at the advanced counting stage. For other students, proceed to question (16).

(15) Start counting backwards from 23. I will tell you when to stop. Stop at 10. If the student has problems counting back through the teens, rate him/her at stage 2 and proceed to the fractional numbers questions.

For each number I show you, read the number then tell me the number that comes just before it, that is, the number that is one less. For example, if I show you 4, you say 3. Show the BNWS cards. Stop at the point the student encounters difficulty and proceed to the fractional number questions.

(16) 13 (17) 19 (18) 30 (19) 100 (20) 680
(21) 900 (22) 2 400 (23) 603 000

Rate the student at the highest stage in which they get all relevant questions (16) to (23) correct.

Stage & Behavioural Indicator
2 BNWS back from 10 The student can read and give the number just before a given number in the range 1 to 10 *without dropping back.*
3 BNWS back from 20 The student can read and produce the number just before a given number in the range 1 to 20 *without dropping back.*
4 BNWS back from 100 The student can read and produce the number just before a given number in the range 1 to 100 *without dropping back.*
5 BNWS back from 1 000 The student can read and produce the number just before a given number in the range 1 to 1 000.
6 BNWS back from 1 000 000 The student can read and produce the number just before a given number in the range 1 to 1 000 000.

Fractional Numbers

(24) **Here are some fractions. Say each fraction as I show it.** Give the student the symbol cards for $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{2}$, $\frac{1}{6}$.

(25) Give the student the unit fraction cards from question (24). **Put these fractions in order from smallest over here,** indicating left, **to largest over here,** indicating right. If correct ask, **Why do you think one-quarter is less than one-third?** Does the student explain the effect of increasing the bottom number (denominator) as decreasing the value of the fraction?

(26) Show the student the test booklet page with $\frac{6}{8}$, $1\frac{2}{6}$, $1\frac{1}{3}$, 1, $\frac{2}{14}$ on it. Point to the fraction $\frac{8}{6}$. **Which of these numbers are the same as $\frac{8}{6}$?** If correct, check that the answer is not a guess by asking **Explain how you know this.**

If the student orders unit fractions but cannot recognise that $\frac{8}{6}$ is equivalent to $1\frac{2}{6}$ or $1\frac{1}{3}$, rate him/her at stage 5.

Rate the student at the highest stage in which he/she gets all relevant questions (24) to (26) correct.

Stage & Behavioural Indicator	
2–3	**Unit Fractions Not Recognised** The student cannot identify symbols for unit fractions.
4	**Unit Fractions Recognised** The student can read unit fraction symbols, for example, the student can read $\frac{1}{3}$ as one-third, $\frac{1}{4}$ as one-quarter.
5	**Ordered Unit Fractions** The student can compare unit fractions, for example, $\frac{1}{3} > \frac{1}{4}$
6	**Co-ordinated Numerators and Denominators** The student describes the size of fractions with reference to both the numerator and denominator, for example, $\frac{8}{6}$ is one whole and two-sixths or one whole and one-third.

Place Value

For the following questions, students should be rated by their fluent recall. Prolonged use of strategising suggests the student does not know the answer.

For each question (27) to (33), show the equation and read it aloud, or use the cards provided [Question (33)].

The student must correctly answer all of questions ...

(27), without counting, to be rated at stage 4, otherwise rate them at stage 3

(28) and (29) to be rated at stage 5

(30) and (31) to be rated at stage 6

(32) and (33) to be rated at stage 7.

Where the student shows knowledge gaps, rate him/her at the previous stage, and move to the Basic Facts section.

Tell me the answer to ...

(27) A toy costs $80. How many $10 notes do you need to pay for it?

(28) A radio costs $230. How many $10 notes do you need to pay for it?

(29) What number is the arrow pointing to? How do you know?

Both 6.8 and 6 and 8 tenths are acceptable answers.

(30) You have $26,700 in $100 notes. How many notes do you have?

(31) What number is three tenths more than 4.8? How do you know?

(32) How many tenths are in all of this number? 4.67 Circle 4.67 with index finger. While 46 tenths is the expected answer, 46.7 tenths is also acceptable.

(33) Put these decimals (0.39, 0.478, 0.8) in order from smallest over here, indicating left, to largest over here, indicating right.

Stage & Behavioural Indicator	
4	**Ten as a Counting Unit** The student uses ten as a counting unit, for example, 10, 20, 30, 40, 50, 60, to find the number of tens in 60.
5	**Tens in numbers to 1 000, Tenth as a Counting Unit** The student knows how many tens are in whole numbers to 1 000 and recognises tenths among whole numbers.
6	**Hundreds in Whole Numbers, Connected Tenths and Ones** The student knows how many hundreds are in any whole number to 100 000 and recognises that ten tenths make one.
7	**Tenths in Decimals/Ordered Decimals** The student knows how many tenths are in numbers with two decimal places, for example, 7.56 has 75 or 75.6 tenths, and orders decimals to three places, for example, 0.539, 0.6, 0.72.

Basic Facts

For the following questions, students should be rated by their fluent recall. Prolonged use of strategising suggests the student does not know the answer, and must work it out. For each question (34) to (48), show the equation in the test booklet and read it aloud. Cease the interview at the line of questions at which the student has knowledge gaps and rate them using the indicators below.

What is the answer to ...

(34)	2 + 3	(35)	5 + 4	(36)	6 and what makes 10?		
(37)	6 + 6	(38)	9 + 9	(39)	10 + 4	(40)	7 + 10
(41)	8 + 6	(42)	6 + 9	(43)	8 × 5	(44)	5 × 7
(45)	17 – 9	(46)	15 – 6	(47)	6 × 7	(48)	8 × 4

Stage & Behavioural Indicator	
2	**Addition Facts to Five** The student instantly recalls facts to five, for example, 2 + 3.
3	**Addition Facts to Ten** The student instantly recalls facts to ten, for example, 5 + 4, 6 + □ = 10.
4	**Addition Facts with Tens and Doubles** The student recalls the doubles to 20, and teen facts, for example, 14 = 10 + 4.
5	**Addition Facts** The student recalls the basic addition facts, and the multiplication facts for 2, 5, and 10.
6	**Subtraction and Multiplication Facts** The student recalls the basic subtraction and multiplication facts.

Photocopiable Material

Questions (7)-(14)

12	17	29	99
209	999	3 049	989 999

Questions (16)-(23)

13	19	30	100
680	900	2 400	603 000

Yellow

Questions (24) and (25)

$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{6}$

Question 33

0.8	0.39	0.478

Question 1

Here is a forest of trees. There are 5 trees in each row, and there are 8 rows. How many trees are there in the forest altogether?

If I planted 15 more trees, how many rows of 5 would I have then altogether?

Question 2

$3 \times 20 = 60$

$3 \times 18 =$ ☐

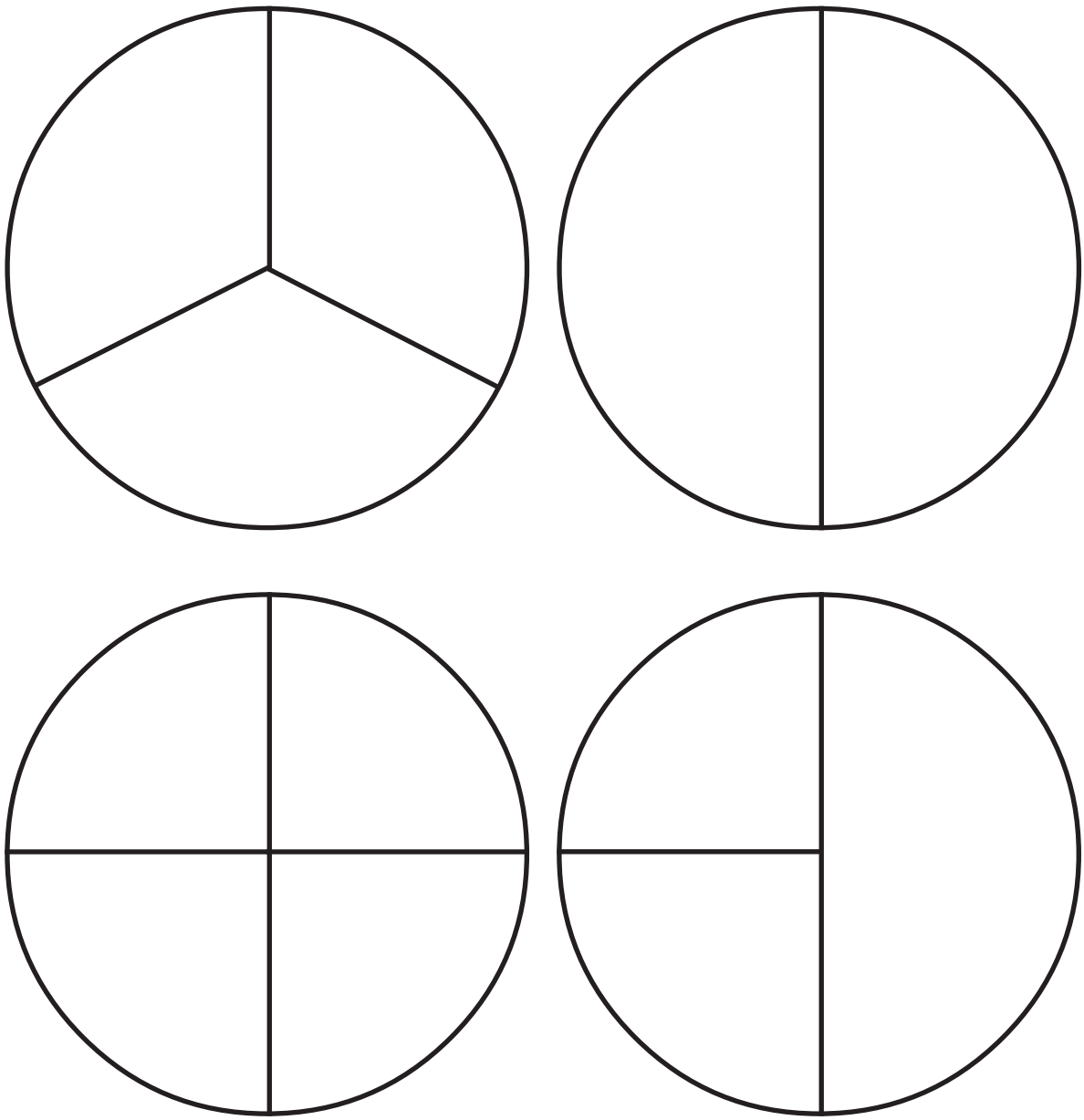
Question 3

$5 \times 8 = 40$

$5 \times 16 =$ ☐

Question 4

Which of these cakes has been cut into thirds?



Question 5

What is $\frac{3}{4}$ of 28?

Yellow

Question 26

Which of these numbers are the same as $\frac{8}{6}$?

$\frac{6}{8}$ $1\frac{2}{6}$ $1\frac{1}{3}$ 1 $\frac{2}{14}$

Question 27

A toy costs $80. How many $10 notes do you need to pay for it?

Question 28

A radio costs $230. How many $10 notes do you need to pay for it?

Question 29

What number is the arrow pointing to? How do you know?

Question 30

You have $26,700 in $100 notes. How many notes do you have?

Question 31

What number is three tenths more than 4.8? How do you know?

Yellow

Question 32

How many tenths are in all of this number? 4.67

Question 34 **2 + 3**	Question 35 **5 + 4**
Question 36 **6 + ☐ = 10**	Question 37 **6 + 6**
Question 38 **9 + 9**	Question 39 **10 + 4**
Question 40 **7 + 10**	Question 41 **8 + 6**
Question 42 **6 + 9**	Question 43 **8 × 5**
Question 44 **5 × 7**	Question 45 **17 – 9**
Question 46 **15 – 6**	Question 47 **6 × 7**
Question 48 **8 × 4**	

Yellow

Numeracy Project Assessment (NumPA) Form C

Transfer the notes from the Strategy Windows tasks (pages 4 and 5) to the addition and subtraction stage boxes on the Individual Assessment Sheet.

Operational Strategy Questions

Multiplication and Division

Things the interviewer says are in bold. Comments for the interviewer appear in plain type.

(1) **Here is a forest of trees. There are 5 trees in each row, and there are 8 rows.**
Use horizontal and vertical sweeps with index finger. Mask all but one horizontal and one vertical edge of the array. **How many trees are there in the forest altogether?** If the student is unable to give an answer, uncover the rest of the sheet.

If I planted 15 more trees, how many rows of 5 would I have then altogether?

For questions (2) and (3), screen the answer then uncover it if the student responds correctly. If the student gives no response or an incorrect response, rate him/her at no higher than stage 5 on multiplication and division and proceed to the proportions and ratios section.

(2) **What is 3 × 20? If 3 × 20 = 60, what does 3 × 18 equal?**
Does the student derive 3 × 18 by 60 – 6 = 54?

(3) **What is 5 × 8? If 5 × 8 = 40, what does 5 × 16 equal?**
Does the student derive 5 × 16 = 80 by doubling 40?

If the student does not derive the answers to questions (2) and (3), rate him/her at either stage 4 or 5 and proceed to the questions on proportions and ratios.

(4) **There are 24 muffins in each basket. How many muffins are there altogether?**
Does the student use a part-whole strategy such as
place value partitioning, for example, 6 × 20 = 120, 6 × 4 = 24, 120 + 24 = 144;
tidy numbers, for example, 6 × 25 = 150, 150 – 6 = 144;
or proportional reasoning, for example, 6 × 24 = 12 × 12 = 144 (doubling and halving)?

(5) **At the car factory, they need 4 wheels to make each car. How many cars could they make with 72 wheels?**
Does the student use a part-whole strategy such as standard place value partitioning, for example, 40 ÷ 4 = 10, 72 – 40 = 32, 32 ÷ 4 = 8, 10 + 8 = 18;
tidy numbers, for example, 80 ÷ 4 = 20 so 72 ÷ 4 = 20 – (8 ÷ 4) = 18;
reversing, for example, 10 × 4 = 40, 8 × 4 = 32 so 18 × 4 = 72 (multiplying to solve a division problem);
proportional reasoning and reversing, for example, 9 × 8 = 72 so 18 × 4 = 72 (doubling and halving) so 72 ÷ 4 = 18 (reversing)?

If the student solves any of questions (4) or (5) successfully using at least two different advanced strategies, rate him/her at stage 7 for multiplication and division and proceed to questions (6) and (7). Otherwise rate the student at stage 6.

(6) **Ivan has 2.4 kilograms of mince. Each pattie takes 0.15 kilograms of mince. How many patties can Ivan make?**
Does the student use a mental part-whole strategy such as
doubling, 2 × 0.15 = 0.3 so 4 × 0.15 = 0.6 so 16 × 0.15 = 2.4;
reversing with rounding and place value, 10 × 0.15 = 1.5 so 20 × 0.15 = 3.0 so 16 × 0.15 = 2.4?

(7) Each day on the life raft, 22 litres of water are shared equally among the 8 survivors. How much water, in litres, does each person get each day?
Does the student use mental part-whole strategies such as:
standard place value, $16 \div 8 = 2$, $6 \div 8 = 0.75$, so $22 \div 8 = 2.75$;
converting equivalent fractions to decimals, $22 \div 8 = 2\frac{6}{8} = 2\frac{3}{4} = 2.75$?

If the student solves both questions (6) and (7) using two different advanced strategies, rate him/her at stage 8 for multiplication and division. Otherwise rate the student at stage 7.

Where the student images a written algorithm, no assumption can be made about their stage. Question the student about their understanding of the processes involved in the algorithm and what other strategies they could use to solve the given problem.

Stage & Behavioural Indicator
4 Advanced Counting The student solves multiplication problems by skip-counting where he/she has a known sequence or by using a combination of skip-counting and counting in ones, for example, 5, 10, 15, 20.
5 Early Additive Part-Whole The student solves multiplication problems by forming the factors where they have a known multiplication fact or using repeated addition, for example, for 6×4: $4 + 4 = 8$, $8 + 4 = 12$, $12 + 12 = 24$.
6 Advanced Additive–Early Multiplicative Part-Whole The student solves multiplication problems by deriving from known multiplication facts, for example, $3 \times 20 = 60$ so $3 \times 18 = 60 - (3 \times 2) = 54$.
7 Advanced Multiplicative–Early Proportional Part-Whole The student is able to use at least *two differ ent* advanced mental strategies to solve multiplication and division problems with whole numbers.
8 Advanced Proportional Part-Whole The student is able to use at least *two differ ent* advanced mental strategies to solve multiplication and division problems with decimals and fractions with related denominators.

Proportions and Ratios

(8) Show the student the fraction circle sheet in the test booklet (page 43). **Which of these cakes has been cut into thirds?** If the student responds incorrectly, identify (point to) the thirds. **Here are 12 jelly beans to spread out evenly on top of the cake. You eat one-third of the cake. How many jelly beans do you get?** If the student cannot answer the question, allow them to manipulate the beans or counters to find it.

For students who need to equally share the beans/counters, either with materials or by imaging the movement of the materials, rate them at stages 2–4 for proportions and ratios and proceed to the knowledge questions (page 36).

(9) **What is $\frac{3}{4}$ of 28?** Does the student use a part-whole strategy based on addition and/or multiplication? If the student uses part-whole strategies based on addition and/or halving, continue on to question (10) as this item may lead them to using division.

(10) **12 is $\frac{2}{3}$ of a number. What is the number?**

For students who are unsuccessful at question (10), rate them at either stage 5 or 6, whatever is appropriate from their response to question (9), and proceed to the knowledge questions. Answering question (10) using multiplication and division places the students at stage 7 at least. If the student uses part-whole strategies successfully for both questions (11) and (12), rate them at stage 8.

(11) **It takes 10 balls of wool to make 15 beanies. How many balls of wool does it take to make 6 beanies?** Show the question in the test booklet (page 44). Does the student use a part-whole strategy based on equivalent fractions such as finding relationships between different units, for example, 10 → 15 so 1→ 1.5 so 4 → 6 (unit fractions), or $6 \times 2\frac{1}{2} = 15$ so $\square \times 2\frac{1}{2} = 10$; or finding relationships within the same units, for example, 10 → 15 so 20→ 30 so 4 → 6?

(12) **There are 21 boys and 14 girls in Ana's class. What percentage of Ana's class are boys?** Show the question in the test booklet (page 44). Does the student use a part-whole strategy based on equivalent fractions such as finding relationships between different units, for example, $21 \div 7 = 3$, $14 \div 7 = 2$ (common factor); 21 → 35 so 3 → 5 so 60 → 100, so $\frac{3}{5} = 60\%$ are boys; or finding relationships within the same units, for example, $35 \times 3 = 105$ so $21 \times 3 = 63$ and adjust down to 60%.

Stage & Behavioural Indicator	
2–4	**Equal Sharing** The student finds a fraction of a number by sharing the objects into equal subsets physically or by imaging.
5	**Early Additive Part-Whole** The student finds a unit fraction of a number mentally using trial and improvement with addition facts, for example, $\frac{1}{3}$ of 12 as $4 + 4 + 4 = 12$.
6	**Advanced Additive–Early Multiplicative Part-Whole** The student finds a fraction of a number mentally using a combination of addition facts and multiplication, for example, $\frac{3}{4}$ of 28 as: $\frac{1}{4}$ of $20 = 5$ so $\frac{1}{4}$ of $24 = 6$ so $\frac{1}{4}$ of $28 = 7$, $3 \times 7 = 21$; or $\frac{1}{2}$ of 28 is 14, $\frac{1}{2}$ of 14 is 7, $14 + 7 = 21$.
7	**Advanced Multiplicative–Early Proportional Part-Whole** The student finds a fraction of a number using division and multiplication, for example, $\frac{2}{3} \times \square = 12$ so $\frac{1}{3} \times \square = 6$ so $\square = 6 \times 3 = 18$, or $1\frac{1}{2} \times 12 = \square$ so $\square = 18$.
8	**Advanced Proportional Part-Whole** The student uses at least two different strategies to solve problems that involve equivalence with and between fractions, ratios, and proportions, for example, 75% of 36 as $\frac{3}{4}$ of 36; or $12 \rightarrow 8$ as $\square \rightarrow 18$: $12 \rightarrow 8$ so $3 \rightarrow 2$ (dividing by four) so $27 \rightarrow 18$ (multiplying by nine).

Knowledge Questions

Forwards and Backwards Number Word Sequence

Show the number sequence cards. Stop at the point the student encounters difficulty and proceed to the fractions questions. For each number I show you, tell me the number that comes just after it, the number that is one more. Also tell me the number that comes just before it, the number that is one less.

(13) 2 400 (14) 3 049 (15) 603 000 (16) 989 999

Stage & Behavioural Indicator
5 FNWS and BNWS within 1 000 The student can produce the number before and after a given number in the range 1 to 1 000.
6 FNWS and BNWS within 1 000 000 The student can produce the number before and after a given number in the range 1 to 1 000 000.

Fractional Numbers

(17) Here are some fractions. Say each fraction as I show it. Show the student the symbol cards for $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{2}$, $\frac{1}{6}$ one at a time. Lay the cards on the table as they are shown.

If the student is unable to recognise the fractions, rate him/her at stage 2–3.

(18) Referring to the fraction cards from question (17) … Put these fractions in order from smallest over here, indicating left, to largest over here, indicating right. If correct, ask, Why do you think one-quarter is less than one-third? Does the student explain the effect of increasing the bottom number (denominator) as decreasing the value of the fraction?

If the student can recognise the unit fractions but cannot order them, rate him/her at stage 4.

(19) Show the student the test booklet page (page 45) with $\frac{6}{8}$, $1\frac{2}{6}$, $1\frac{1}{3}$, 1, and $\frac{2}{14}$ on it. Point to the fraction $\frac{8}{6}$. Which of these numbers are the same as $\frac{8}{6}$? If the student is correct, check that the answer is not a guess by asking Explain how you know this.

If the student orders unit fractions but cannot recognise that $\frac{8}{6}$ is equivalent to $1\frac{2}{6}$ or $1\frac{1}{3}$, rate him/her at stage 5.

(20) Give the student the set of mixed fractions, $\frac{2}{5}$, $\frac{7}{16}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{6}{9}$, $\frac{3}{4}$. Here are some fractions. Put them in order from smallest over here, indicating left, to largest over here, indicating right. Does the student recognise the equivalence of two-thirds and six-ninths? If so, rate them at stage 7 at least, if not rate them at stage 6. Rate a student who successfully orders all the fractions in question (20) at stage 8.

Stage & Behavioural Indicator	
2-3	**Unit Fractions Not Recognised** The student cannot identify symbols for unit fractions.
4	**Unit Fractions Recognised** The student can read unit fraction symbols, for example, the student can read $\frac{1}{3}$ as one-third, $\frac{1}{4}$ as one-quarter.
5	**Ordered Unit Fractions** The student can compare unit fractions, for example, $\frac{1}{3} > \frac{1}{4}$.
6	**Co-ordinated Numerators and Denominators** The student describes the size of fractions with reference to both the numerator and denominator, for example, $\frac{8}{6}$ is one whole and two-sixths.
7	**Equivalent Fractions** The student names equivalent fractions from a set of fractions with different denominators, for example, $\frac{2}{3} = \frac{8}{12}$, $\frac{3}{4} = \frac{6}{8}$.
8	**Ordered Fractions** The student orders fractions with unlike denominators and numerators, for example, $\frac{2}{5} < \frac{7}{16}$.

Place Value

For the following questions, students should be rated by their fluent recall. Prolonged use of strategising suggests the student does not know the answer, and must work it out.

The student must correctly answer all of questions...

(21) and (22) to be rated at stage 5, otherwise rate them at stage 4.

(23) and (24) to be rated at stage 6,

(25) and (26) to be rated at stage 7,

(27) to (30) to be rated at stage 8.

Where the student shows knowledge gaps, rate him/her at the previous stage.

(21) A radio costs $230. How many $10 notes do you need to pay for it?

(22) What number is the arrow pointing to? How do you know?

Both 6.8 and 6 and 8 tenths are acceptable answers.

(23) You have $26,700 in $100 notes. How many notes do you have?

(24) What number is three tenths less than 2? How do you know?

(25) How many tenths are in all of this number? 4.67 Circle 4.67 with index finger. While 46 tenths is the expected answer, 46.7 tenths is also acceptable.

(26) Put these decimals (0.39, 0.478, 0.8) in order from smallest over here, indicating left, to largest over here, indicating right.

(27) How many hundredths are in all of this number? 2.097 Circle 2.097 with index finger. While 209 hundredths is the expected answer, 209.7 hundredths is acceptable.

(28) Round 7.649 to the nearest tenth.

(29) Give three numbers that are between 7.59 and 7.6. If you had time, how many numbers could you find?

(30) Name 137.5% as a decimal.

Stage & Behavioural Indicator
5 Tens in numbers to 1 000, Tenth as a Counting Unit The student knows how many tens are in numbers to 1 000, and recognises tenths among whole numbers.
6 Hundreds in Whole Numbers, Connected Tenths and Ones The student knows how many hundreds are in any whole number to 100 000, and recognises that ten tenths make one.
7 Tenths in Decimals/Ordered Decimals The student knows how many tenths are in numbers with two decimal places, for example, 7.56 has 75 or 75.6 tenths, and orders decimals to three places, for example, 0.539, 0.6, 0.72.
8 Decimal Conversions The student knows how many hundredths are in decimals, and rounds numbers to the nearest tenth, for example, 7.649 → 7.6 to the nearest tenth, not 7.7. The student can identify decimals between others and name a percentage as a decimal and vice versa, for example, 137.5% as 1.375.

Basic Facts

For the following questions, students should be rated by their fluent recall. Prolonged use of strategising suggests the student does not know the answer, and must work it out. For each question (31) to (43), show the equation in the test booklet (page 48) and read it aloud. Cease the interview at the line of questions at which the student has knowledge gaps and rate them using the indicators below.

What is the answer to ...

(31) 8 + 6 (32) 6 + 9 (33) 8 × 5 (34) 5 × 7

(35) 17 – 9 (36) 15 – 6 (37) 6 × 7 (38) 8 × 4

(39) 56 ÷ 7 (40) 63 ÷ 9

For questions (41) to (43) explain the meaning of the terms, factor, common factor, and least common multiple, if necessary.

(41) Name all the factors of 81.

(42) What is the highest common factor of 72 and 81?

(43) What is the least common multiple of 8 and 12?

Stage & Behavioural Indicator	
4	**Addition Facts with Tens and Doubles** The student recalls the doubles to 20, and "teen" facts, for example, 14 = 10 + 4.
5	**Addition Facts** The student recalls the basic addition facts, and the multiplication facts for 2, 5, and 10.
6	**Subtraction and Multiplication Facts** The student recalls the basic subtraction and multiplication facts.
7	**Division Facts** The student recalls the basic division facts and names all the factors of numbers to 100.
8	**Common Factors and Multiples** The student names all the common factors of two numbers to 100, and the least common multiple of numbers to 10.

Photocopiable Material

Questions (13)–(16)

2 400	3 049	603 000	989 999

Questions (17) and (18)

$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{6}$

Question (20)

$\frac{2}{3}$	$\frac{3}{4}$	$\frac{2}{5}$	$\frac{6}{9}$	$\frac{7}{16}$	$\frac{1}{2}$

Question (26)

0.8	0.39	0.478

Green

Question 1

Here is a forest of trees. There are 5 trees in each row, and there are 8 rows. How many trees are there in the forest altogether?

If I planted 15 more trees, how many rows of 5 would I have then altogether?

Question 2

$3 \times 20 = 60$

$3 \times 18 =$ ☐

Question 3

$5 \times 8 = 40$

$5 \times 16 =$ ☐

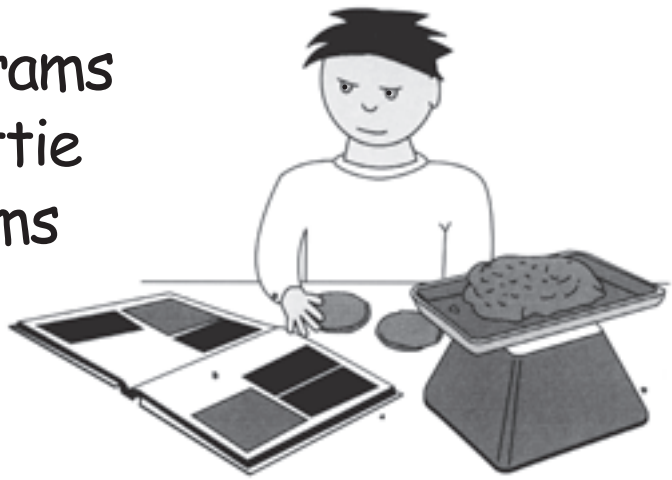
Question 4

There are 24 muffins in each basket. How many muffins are there altogether?

Question 5

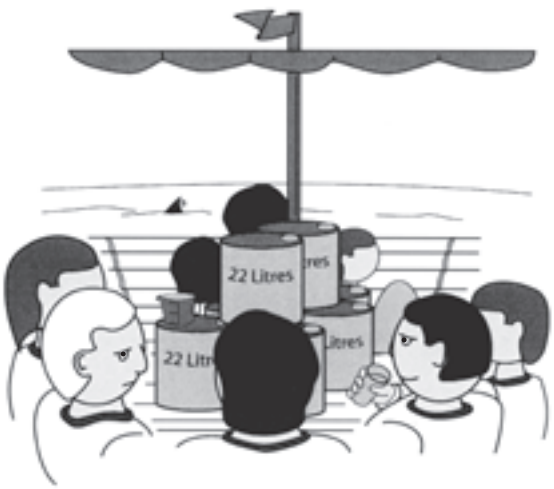
At the car factory, they need 4 wheels to make each car.
How many cars could they make with 72 wheels?

Question 6



Ivan has 2.4 kilograms of mince. Each pattie takes 0.15 kilograms of mince.
How many patties can Ivan make?

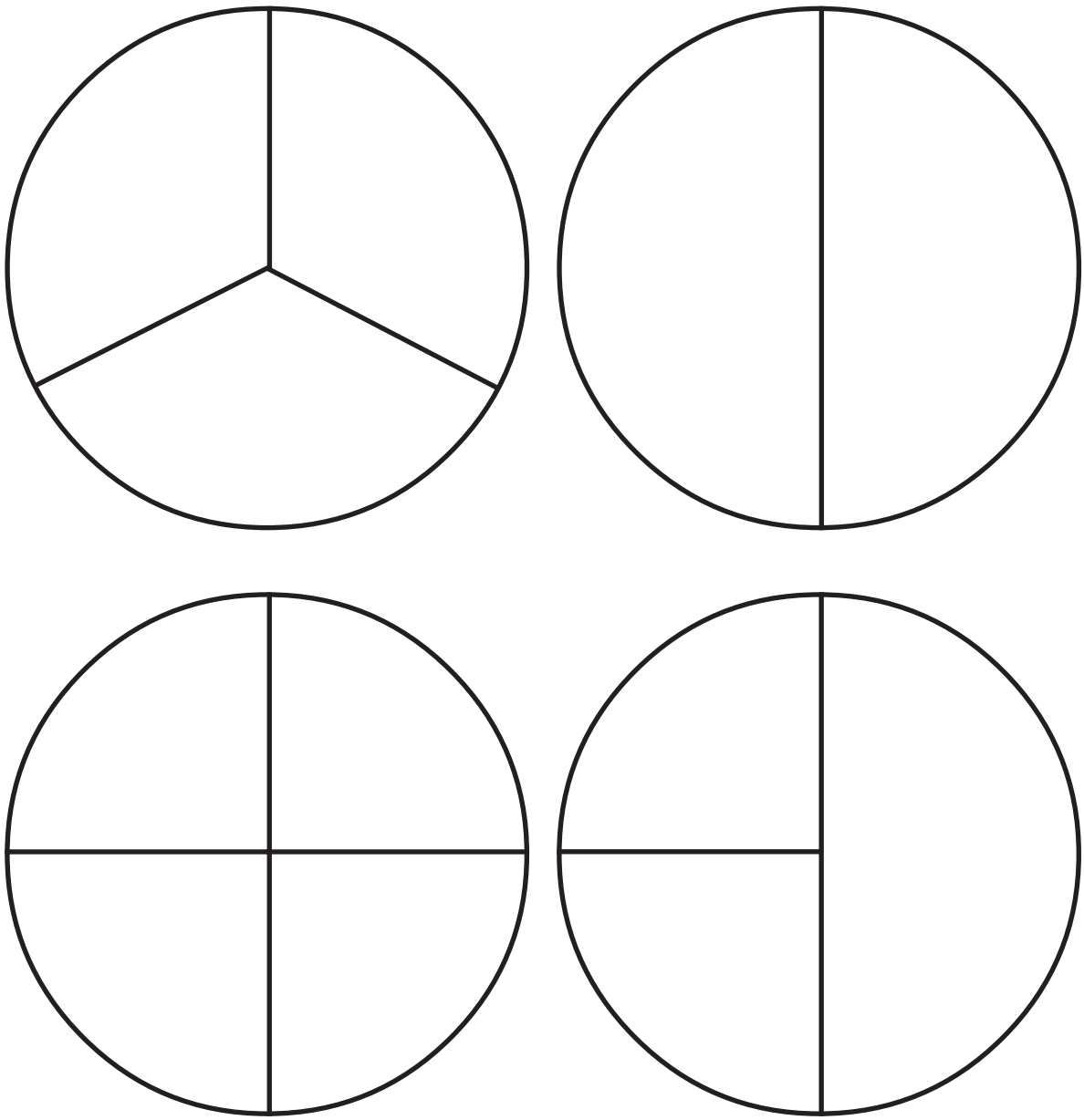
Question 7



Each day on the life raft, 22 litres of water is shared equally between the 8 survivors.
How much water, in litres, does each person get each day?

Question 8

Which of these cakes has been cut into thirds?



Question 9

What is $\frac{3}{4}$ of 28?

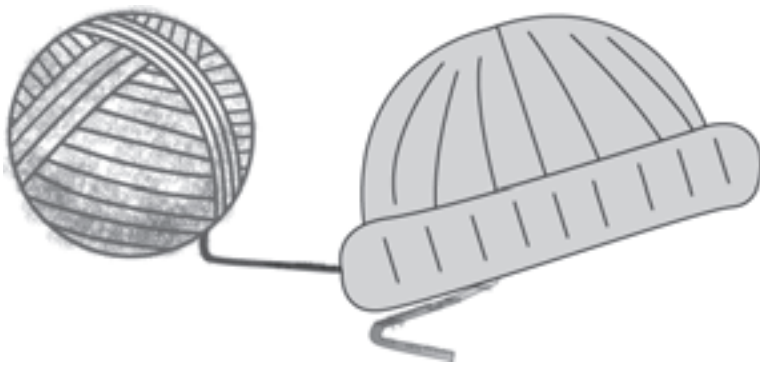
Green

Question 10

12 is $\frac{2}{3}$ of a number. What is the number?

Question 11

It takes 10 balls of wool to make 15 beanies. How many balls of wool does it take to make 6 beanies?



Question 12

There are 21 boys and 14 girls in Ana's class.

What percentage of Ana's class are boys?

Question 19

Which of these numbers are the same as $\frac{8}{6}$?

$\frac{6}{8}$ $1\frac{2}{6}$ $1\frac{1}{3}$ 1 $\frac{2}{14}$

Question 21

A radio costs $230.

How many $10 notes do you need to pay for it?

Question 22

What number is the arrow pointing to? How do you know?

Question 23

You have $26,700 in $100 notes.

How many notes do you have?

Question 24

What number is three tenths less than 2? How do you know?

Question 25

How many tenths are in all of this number? 4.67

Question 27

How many hundredths are in all of this number? 2.097

Question 28

Round 7.649 to the nearest tenth.

Question 29

Give three numbers that are between 7.59 and 7.6. If you had time, how many numbers could you find?

Question 30

Name 137.5% as a decimal.

Question 31 **8 + 6**	Question 32 **6 + 9**
Question 33 **8 × 5**	Question 34 **5 × 7**
Question 35 **17 – 9**	Question 36 **15 – 6**
Question 37 **6 × 7**	Question 38 **8 × 4**
Question 39 **56 ÷ 7**	Question 40 **63 ÷ 9**

Question 41

Name all the factors of 81.

Question 42

What is the highest common factor of 72 and 81?

Question 43

What is the least common multiple of 8 and 12?

NumPA Form A Individual Assessment Sheet

* denotes cards needed # test booklet needed

Child's Name: _____ DoB: _____ Year: _____ Ethnicity: E M P A O Gender: M F Date: _____

Operational Strategy Questions

Addition and Subtraction (Strategy Windows) #

(1) Please get 8 counters for me.

(2) Please hold out your hands for me. Here are 4 counters. Here are another 3 counters. How many counters have you got altogether?

(3) There are 8 counters under this card and 5 counters under this card. How many counters are there altogether?

Stage 0	Stage 1	Stage 2	Stage 3	Stage 4
Emergent Does not count up to 10 objects	Counts one to one	Counts from One on Materials	Counts from One by Imaging	Advanced Counting Counts on

Comments

Knowledge Questions

Forwards Number Word Sequence (FNWS)

(1) Start counting from 1. I will tell you when to stop. *(Stop at 32.)*
What's the next number after ...?
The next number after 2 is 3.
So if I say 2, you say 3. What is the next number after ...?

(2) **5** (3) **9**
(4) **13** (5) **19** (6) **12** (7) **15**
(8) **29** (9) **46** (10) **69** (11) **80** (12) **139** (13) **899**

Stage 0	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5
Emergent FNWS Does not say FNWS up to 10	Initial FNWS up to 10 Says FNWS up to 10	FNWS up to 10 Says FNWS up to 10 and number after	FNWS up to 20 Says FNWS up to 20 and number after	FNWS up to 100 Says FNWS up to 100 and number after	FNWS up to 1 000 Says FNWS up to 1 000 and number after

Comments

Backwards Number Word Sequence (BNWS)

(14) Count backwards from 10. I will tell you when to stop. *(Stop at 0 or 1.)*

(15) Count backwards from 24. I will tell you when to stop. *(Stop at 11.)*

What number comes before ...? The number that comes before 2 is 1. So if I say 2, you say 1. What number comes before ...?

(16) **3** (17) **9** (18) **5** (19) **8**

(20) **16** (21) **20** (22) **17** (23) **11** (24) **13**

(25) **31** (26) **47** (27) **70** (28) **236** (29) **600**

Stage 0	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5
Emergent BNWS Does not say BNWS from 10	Initial BNWS back from 10 Says BNWS back from 10	BNWS back from 10 Says BNWS back from 10 and number before	BNWS from 20 Says BNWS back from 20 and number before	BNWS back from 100 Says BNWS back from 100 and number before	BNWS back from 1 000 Says BNWS back from 1 000 and number before

Comments

Numeral Identification *

What is this number?

(30) **3** (31) **9** (32) **5** (33) **1** (34) **8**

(35) **6** (36) **0** (37) **4** (38) **2** (39) **7** (40) **10**

(41) **13** (42) **19** (43) **11** (44) **16** (45) **12**

(46) **66** (47) **43** (48) **80** (49) **38** (50) **137** (51) **702**

Stage 0	Stage 1	Stage 2	Stage 3	Stage 4
Emergent Numeral Identification	Identifies numerals to 10	Identifies numerals to 20	Identifies numerals to 100	Identifies numerals to 1 000

Comments

Place Value *

(52) Here are four dots. Here are ten more dots. How many dots are there now?

14, 24, 34, 44, 54, 64, 74.

Stage 0–1	Stage 2	Stage 3	Stage 4
Cannot count the number of objects	Counts in ones	Counts in fives and ones	Counts in tens, knows number of tens, knows place value of tens digit

Comments

Basic Facts #

Tell me the answer to ...

(53) **2 + 3** (54) **5 + 4** (55) **6 and what makes 10?**

(56) **6 + 6** (57) **9 + 9** (58) **10 + 4** (59) **7 + 10**

Stage 0–1	Stage 2	Stage 3	Stage 4
Unable to recall addition facts to five	Instantly recalls facts to five	Instantly recalls facts to ten	Recalls doubles and teen facts

Comments

NumPA Form B Individual Assessment Sheet

* denotes cards needed # test booklet needed

Child's Name: _____ DoB: _____ Year: _____ Ethnicity: E M P A O Gender: M F Date: _____

Operational Strategy Questions

Addition and Subtraction (Strategy Windows)

(3) There are 8 counters under this card and 5 counters under this card. How many counters are there altogether?

(4) There are 9 counters under this card and 8 counters under this card. How many counters are there altogether?

(5) You have 37 lollies, and you eat 9 of them. How many lollies have you got left?

(6) There are 53 people on the bus. 26 people get off. How many people are left on the bus?

Stage 4 Advanced Counting		**Stage 5** Early Additive Part-Whole	
Counts on		Derives addition and subtraction facts	

Comments

Multiplication and Division

(1) Here is a forest of trees. There are 5 trees in each row, and there are 8 rows. How many trees are there in the forest altogether?

If I planted 15 more trees, how many rows of 5 would I have then altogether?

(2) What is 3×20? If $3 \times 20 = 60$, what does 3×18 equal?

(3) What is 5×8? If $5 \times 8 = 40$, what does 5×16 equal?

Stage 2–3		**Stage 4**		**Stage 5**		**Stage 6**	
Count from One		Advanced Counting		Early Additive Part–Whole		Advanced Additive–Early Multiplicative Part–Whole	
Counts all the objects		Uses skip-counting		Uses repeated addition and/or uses known multiplication facts		Derives multiplication facts	

Comments

Proportions and Ratios

(4) Which of these cakes has been cut into thirds? Here are 12 jelly beans to spread out evenly on top of the cake. You eat one-third of the cake. How many jelly beans do you get?

(5) What is $\frac{3}{4}$ of 28?

Stage 1		**Stage 2–4**		**Stage 5**		**Stage 6**	
Unequal Sharing		Equal Sharing		Early Additive Part–Whole		Advanced Additive–Early Multiplicative Part–Whole	
Unequally shares objects		Shares objects physically or by imaging		Uses addition facts		Uses addition and multiplication facts	

Comments

Knowledge Questions

Forwards Number Word Sequence (FNWS) *

(6) Start counting from 10. I will tell you when to stop. (*Stop at 32.*)

For each number I show you, read the number then tell me the number that comes just after it, the number that is one more. For example, if I show you 4, you say 5.

(7) **12** (8) **17** (9) **29** (10) **99** (11) **209**

(12) **999** (13) **3 049** (14) **989 999**

Stage 2		**Stage 3**		**Stage 4**		**Stage 5**		**Stage 6**	
Says FNWS up to 10		Says FNWS up to 20		Says number after up to 100		Says number after up to 1 000		Says number after up to 1 000 000	

Comments

Yellow

Backwards Number Word Sequence (BNWS) *

(15) Start counting backwards from 23. I will tell you when to stop. (*Stop at 10.*)

For each number I show you, read the number then tell me the number that comes just before it, that is, the number that is one less. For example, if I show you 4, you say 3.

(16) **13** (17) **19** (18) **30** (19) **100** (20) **680**

(21) **900** (22) **2 400** (23) **603 000**

Stage 2	Stage 3	Stage 4	Stage 5	Stage 6
Says BNWS back from 10	Says BNWS back from 20 and number before	Says number before up to 100	Says number before up to 1□00	Says number before up to 1□00□00

Comments

Fractional Numbers # *

(24) Here are some fractions. Say each fraction as I show it. ($\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{2}$, $\frac{1}{6}$)

(25) Put these fractions (*from question 24*) in order from smallest over here to largest over here. (*If correct ask ...*) Why do you think one-quarter is less than one-third?

(26) Which of these numbers is the same as $\frac{8}{6}$ (*pointing to* $\frac{8}{6}$)? (*Show the numbers,* $\frac{6}{8}$, $1\frac{2}{6}$, $1\frac{1}{3}$, 1, $\frac{2}{14}$, *in the test booklet*) Explain how you know this.

Stage 2–3	Stage 4	Stage 5	Stage 6
Does not recognise unit fractions	Recognises unit fractions	Orders unit fractions	Co-ordinates numerators and denominators

Comments

Place Value * #

Tell me the answer to ...

(27) A toy costs $80. How many $10 notes do you need to pay for it?

(28) A radio costs $230. How many $10 notes do you need to pay for it?

(29) What number is the arrow pointing to? How do you know?

(30) You have $26,700 in $100 notes. How many notes do you have?

(31) What number is three tenths more than 4.8? How do you know?

(32) How many tenths are in all of this number? 4.67

(33) Put these decimals (0.39, 0.478, 0.8) in order from smallest over here to largest over here.

Stage 4	Stage 5	Stage 6	Stage 7
Counts in tens	Knows tens in numbers to 1000, tenths among whole numbers	Knows hundreds in whole numbers, connects tenths and ones	Knows number of tenths in decimals, orders decimals

Comments

Basic Facts #

What is the answer to...?

(34) **2 + 3** (35) **5 + 4** (36) **6 and what makes 10?**

(37) **6 + 6** (38) **9 + 9** (39) **10 + 4** (40) **7 + 10**

(41) **8 + 6** (42) **6 + 9** (43) **8 × 5** (44) **5 × 7**

(45) **17 – 9** (46) **15 – 6** (47) **6 × 7** (48) **8 × 4**

Stage 2	Stage 3	Stage 4	Stage 5	Stage 6
Instantly recalls facts to five	Instantly recalls facts for ten	Recalls doubles and teen facts	Addition facts and multiplication facts for 2, 5, 10	Subtraction and multiplication facts

Comments

NumPA Form C Individual Assessment Sheet

* denotes cards needed # test booklet needed

Child's Name: _____ DoB: _____ Year: _____ Ethnicity: E M P A O Gender: M F Date: _____

Operational Strategy Questions

Addition and Subtraction (Strategy Windows)

(6) There are 53 people on the bus. 26 people get off. How many people are left on the bus?

(7) Sandra has 394 stamps. She gets another 79 stamps from her brother. How many stamps does she have then?

(8) Marija has a 5.3 metre length of fabric. She uses 2.89 metres of it to make a tracksuit. How much fabric has she got left?

(9) Harry and Sally buy two pizzas. Harry eats $\frac{3}{4}$ of a pizza while his friend Sally eats $\frac{7}{8}$ of a pizza. How much pizza is left over?

Stage 6	Stage 7
Advanced Additive–Early Multiplicative Part–Whole Uses at least two different advanced mental part-whole strategies	Advanced Multiplicative–Early Proportional Part–Whole Uses at least two different strategies with decimals and fractions

Comments

Multiplication and Division

(1) Here is a forest of trees. There are 5 trees in each row, and there are 8 rows. How many trees are there in the forest altogether?

If I planted 15 more trees, how many rows of 5 would I have then altogether?

(2) What is 3×20? If $3 \times 20 = 60$, what does 3×18 equal?

(3) What is 5×8? If $5 \times 8 = 40$, what does 5×16 equal?

(4) There are 24 muffins in each basket. How many muffins are there altogether?

(5) At the car factory, they need 4 wheels to make each car. How many cars could they make with 72 wheels?

(6) Ivan has 2.4 kilograms of mince. Each pattie takes 0.15 kilograms of mince. How many patties can Ivan make?

(7) Each day on the life raft, 22 litres of water is shared equally among the 8 survivors. How much water, in litres, does each person get each day?

Stage 4	Stage 5	Stage 6	Stage 7	Stage 8
Advanced Counting Uses skip-counting	Early Additive Part–Whole Uses repeated addition and/or uses known multiplication facts	Advanced Additive–Early Multiplicative Part–Whole Derives multiplication facts	Advanced Multiplicative –Early Proportional Part–Whole Uses at least two different advanced mental strategies	Advanced Proportional Part–Whole Uses two different advanced strategies for decimals and fracitons

Comments

Proportions and Ratios

(8) Which of these cakes has been cut into thirds? Here are 12 jelly beans to spread out evenly on top of the cake. You eat one-third of the cake. How many jelly beans do you get?

(9) What is $\frac{3}{4}$ of 28?

(10) 12 is $\frac{2}{3}$ of a number. What is the number?

(11) It takes 10 balls of wool to make 15 beanies. How many balls of wool does it take to make 6 beanies?

(12) There are 21 boys and 14 girls in Ana's class. What percentage of Ana's class are boys?

Stage 2–4	Stage 5	Stage 6	Stage 7	Stage 8
Equal sharing of objects physically or by imaging	Early Additive Part–Whole Uses addition facts	Advanced Additive–Early Multiplicative Part–Whole Uses addition with multiplication and division facts	Advanced Multiplicative–Early Proportional Part–Whole Finds fractions of numbers using multiplication and division	Advanced Proportional Part–Whole Uses at least two different advanced mental strategies

Comments

Green

Knowledge Questions

Forwards and Backwards Number Word Sequence *

For each number I show you, tell me the number that comes just after it, the number that is one more. Also tell me the number that comes just before it, the number that is one less.

(13) **2 400** (14) **3 049** (15) **603 000** (16) **989 999**

Stage 5	Stage 6
Says number before or after up to 1 000	Says number before or after up to 1 000 000

Comments

Fractional Numbers # *

(17) Here are some fractions. Say each fraction as I show it. ($\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{2}$, $\frac{1}{6}$)

(18) Put these fractions (*from question 17*) in order from smallest over here to largest over here. (*If correct ask ...*) Why do you think one-quarter is less than one-third?

(19) Which of these numbers is the same as $\frac{8}{6}$ (*pointing to* $\frac{8}{6}$)? (*Show the numbers,* $\frac{6}{8}$, $1\frac{2}{6}$, $1\frac{1}{3}$, 1, $\frac{2}{14}$, *in the test booklet*) Explain how you know this.

(20) Here are some fractions ($\frac{2}{5}$, $\frac{7}{16}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{6}{9}$, $\frac{3}{4}$). Put them in order from smallest over here to largest over here.

Stage 2–3	Stage 4	Stage 5	Stage 6	Stage 7	Stage 8
Does not recognise unit fractions	Recognises unit fractions	Orders unit fractions	Co-ordinates numerators and denominators	Recognises equivalent fractions	Orders fractions with unlike denominators and numerators

Comments

Place Value # *

Tell me the answer to?

(21) A radio costs $230. How many $10 notes do you need to pay for it?

(22) What number is the arrow pointing to? How do you know? (6.8 or 6 and 8 tenths)

(23) You have $26,700 in $100 notes. How many notes do you have?

(24) What number is three tenths less than 2? How do you know?

(25) How many tenths are in all of this number? 4.67

(26) Put these decimals (0.39, 0.478, 0.8) in order from smallest over here to largest over here.

(27) How many hundredths are in all of this number? 2.097

(28) Round 7.649 to the nearest tenth.

(29) Give three numbers that are between 7.59 and 7.6? If you have time, how many numbers could you find?

(30) Name 137.5% as a decimal.

Stage 4	Stage 5	Stage 6	Stage 7	Stage 8
Counts in tens	Knows tens in numbers to 1000, tenths among whole numbers	Knows hundreds in whole numbers, connects tenths and ones	Knows number of tenths in decimals, orders decimals	Knows hundredths in decimals, decimals between others, names percentages as decimals and vice versa.

Comments

Basic Facts

What is the answer to...?

(31) **8 + 6** (32) **6 + 9** (33) **8 × 5** (34) **5 × 7** (35) **17 – 9** (36) **15 – 6**
(37) **6 × 7** (38) **8 × 4** (39) **56 ÷ 7** (40) **63 ÷ 9**

(41) Name all the factors of 81.

(42) What is the highest common factor of 72 and 81?

(43) What is the least common multiple of 8 and 12?

Stage 4	Stage 5	Stage 6	Stage 7	Stage 8
Recalls doubles and teen facts	Addition facts and multiplication facts for 2, 5, 10	Subtraction and multiplication facts	Division facts	Common factors and multiples

Comments

Answers to Form C

Operational Strategy Questions

Addition and Subtraction

(6) **27** people (7) **473** stamps (8) **2.41 metres** (8) $\frac{3}{8}$ **pizza**

Multiplication and Division

(1) **40** trees
3 more rows making **11** rows in total

(2) $3 \times 18 = \mathbf{54}$ (60 – 6)

(3) $5 \times 16 = \mathbf{80}$ (double 40)

(4) **144** muffins

(5) **18** cars

(6) **16** patties

(7) **2.75 litres** or **2** $\frac{3}{4}$ **litres**

Proportions and Ratios

(8) **4** jellybeans

(9) $\frac{3}{4}$ of 28 is **21**

(10) $\frac{2}{3}$ of **18** is 12

(11) **4** balls of wool makes 6 beanies

(12) **60**% of Ana's class are boys

Knowledge Questions

Forwards and Backwards Number Word Sequence

(13) **2 401, 2 399** (14) **3 050, 3 048** (15) **603 001, 602 999** (16) **990 000, 989 998**

Fractional Numbers

(17) **one-third, one-quarter, one-fifth, one-half, one-sixth**

(18) $\frac{1}{6}$ $\frac{1}{5}$ $\frac{1}{4}$ $\frac{1}{3}$ $\frac{1}{2}$

(19) **1** $\frac{2}{6}$ **and 1** $\frac{1}{3}$ **are the same as** $\frac{8}{6}$

(20) $\frac{2}{5}$ $\frac{7}{16}$ $\frac{1}{2}$ $\frac{6}{9}$ $\frac{3}{4}$

($\frac{2}{3}$) ($\frac{6}{9} = \frac{2}{3}$)

Place Value

(21) **23**

(22) **6.8 or 6 and 8 tenths**

(23) **267 $100 notes**

(24) **1.7 or 1 and 7 tenths**

(25) **46 or 46.7**

(26) **0.39 0.478 0.8**

(27) **209 or 209.7**

(28) **7.6**

(29) **7.591 – 7.599, etc. Infinite number of decimals are possible**

(30) **1.375**

Basic Facts

(31) **14** (32) **15** (33) **40** (34) **35**

(35) **8** (36) **9** (37) **42** (38) **32**

(39) **8** (40) **7** (41) **1, 3, 9, 27, 81** (42) **9**

(43) **24**

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