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Number in Probability

We are solving problems in probability contexts
We are exploring probability concepts and language
We are exploring the properties of numbers in probability contexts

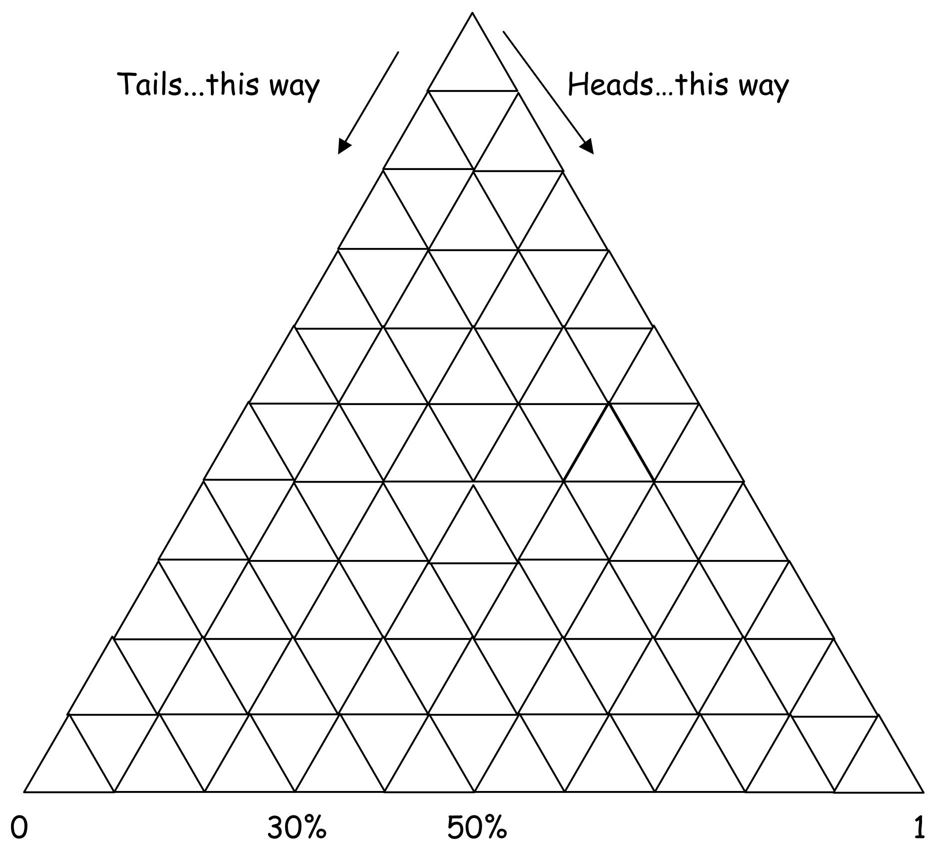
Exercise 1 – Trials

You will need a coin.

1) Before starting this exercise, predict what you think the chance is of getting a head when you toss a coin.

This exercise is about experimenting with what happens when a coin is tossed fairly. To be a fair toss, you must throw or flick the coin high so it has lots of flips then let it land without interfering one way or the other. Do this 10 times and see where your trials end up on the grid below.

Using the grid to record the results of your coin tosses.
With a highlighter or a felt tip pen, start at the tip and if you get a tail move left down the side of the first small triangle. (If you get a head, move down the right side of the triangle.) For the next toss, record your results on the next triangle down. By the time you finish you should have a continuous path from the top of the grid to the bottom. Where you end up is the likelihood of throwing a head.



Do the experiments several times. Each time record your results on a different grid (your teacher should have handed out several to each of you).

2) Is the path you take always the same?

3) Do you always end up in the same place?

4) Based on what you have done in these experiments, make a guess about how likely it is to toss a head with your coin. Explain your answer.

My guess is _______________

Exercise 2 – Tribulations

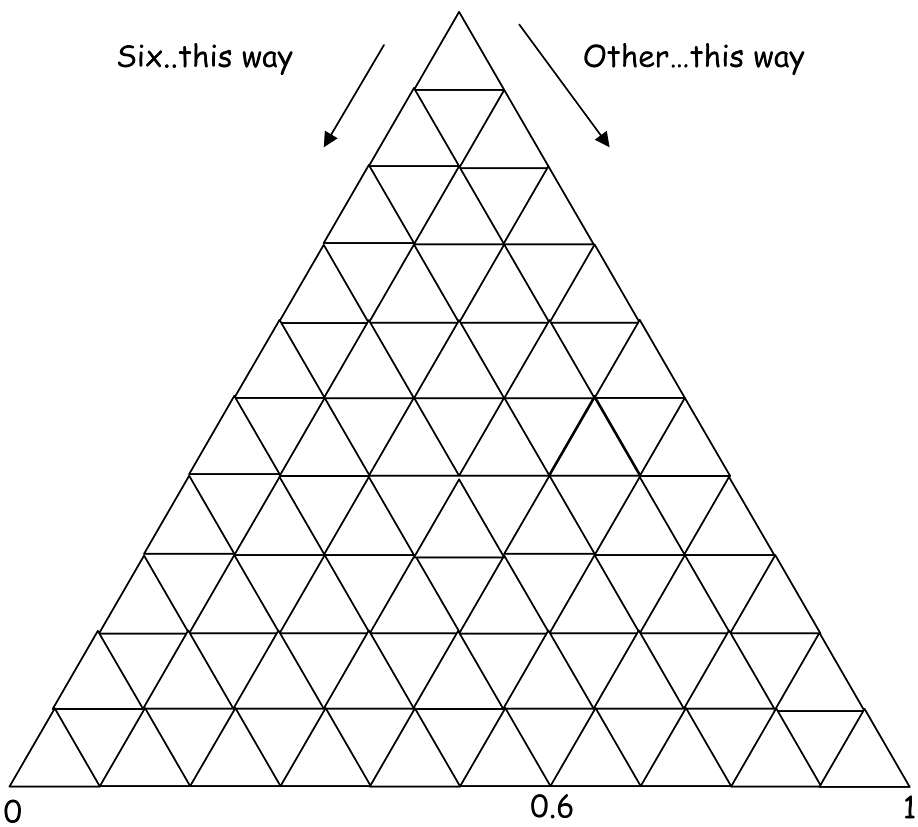
You will need a die.

This exercise is about discovering the likelihood of getting a six when a die is tossed fairly.

Use the grid below to record the results of your tosses. To be fair you must shake the die well and let it settle without interfering with it. Do this 10 times and see where your trials end up on the grid. Start at the top and if you get a six move left and down, a 1,2,3,4 or 5 move right and down. Where you end up is the likelihood of throwing a six.

1) Before you start, what do you think the probability of getting a six should be?

My answer is ____________



2) Also before you start, predict what you think will happen if you do the experiment a number of times.

3) Do the experiment several times to see if you get to the same place each time. Make an estimate about how likely it is to toss a six with your die. Explain your answer

My estimate is ______________

Exercise 3 – Drawing Pins

You will need a thumb tack (drawing pin).

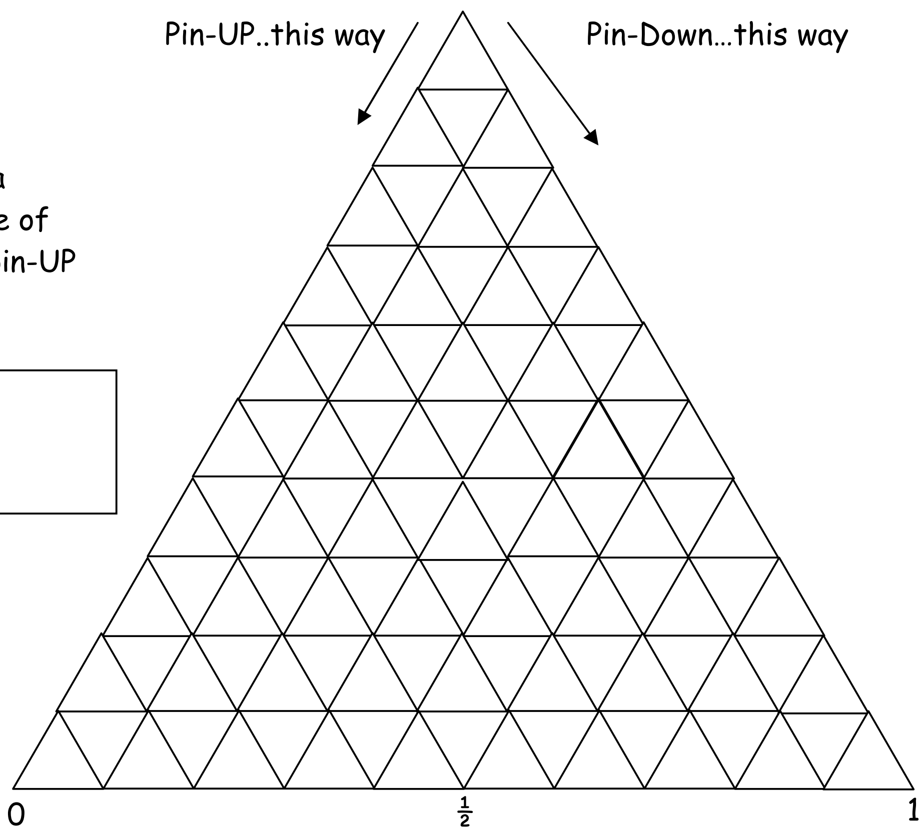
This exercise is about discovering the likelihood of getting the pin to land pointing upwards when tossing a thumb tack tossed fairly.

1) Make a prediction about how likely it get the drawing pin to land with the pin facing up. Your answer will be a number between 0 and 1. Justify your prediction (say why you think you are right).

My guess is _______________

Use the grid below to record the results of your drawing pin tosses. To be fair you must toss the pin high with lots of turns and let it settle without interfering with it. Do this 10 times and see where your experiment ends up on the grid. Start at the top and if you get a pin-up (a pin pointing up) move left and down. For a pin-down move right and down. Where you end up is the likelihood of throwing a pin-up.

Draw a picture of what pin-UP means!



2) Do the experiment several times to see if you get to the same place each time.
3) By combining the results from all of the trails, calculate the average number of times the drawing pin lands pin-up.
4) Is this result what you expected? If not, try to explain why the drawing pin gets different results to the coin or the die.

Exercise 4 – The Language of Probability

You will need a buddy, your maths book or an A3 sheet of paper and pens.
This exercise is about developing your ideas of what all the words mean.

1) Words describing the probability of an event

These words or phrases are all estimates of chance. We should have similar meanings of them if we are to communicate clearly. Cut them out and talk about which means more likely to happen and then order them from least likely on the left to most likely on the right. Two words can mean the same thing.

never	sometimes	certain	almost certain
very likely	impossible	100% Chance of happening	zero chance
possible	might happen	even chance	average chance
very probably	unlikely	very unlikely	nearly always
slim chance	50/50	highly likely	a good bet
equally likely	rare occasion	probable	better than average

2) Words describing the world of probability.

These words just need to be given definitions. Write the words and what they mean as clearly and briefly as you can.

chance	probability	success
failure	event	experiment
bias	fair	random
likelihood	likely	outcome
die, dice	trial	mathematical probability
heads	prediction	sample space

Trick your friends…

If you want to make sure you win when you toss a coin say "Heads I win, tails you lose!"

Exercise 5 – Mathematical Probability 1

You will need a fair coin.

Mathematical probability of an event happening is based on knowing the likelihood of all the outcomes in an experiment. For a coin the outcomes are very simple and both of them are equally likely. A coin is symmetrical with two distinct sides.

Philippa says the only possible outcomes when a coin is tossed are a head (H) or a tail (T). We can write this

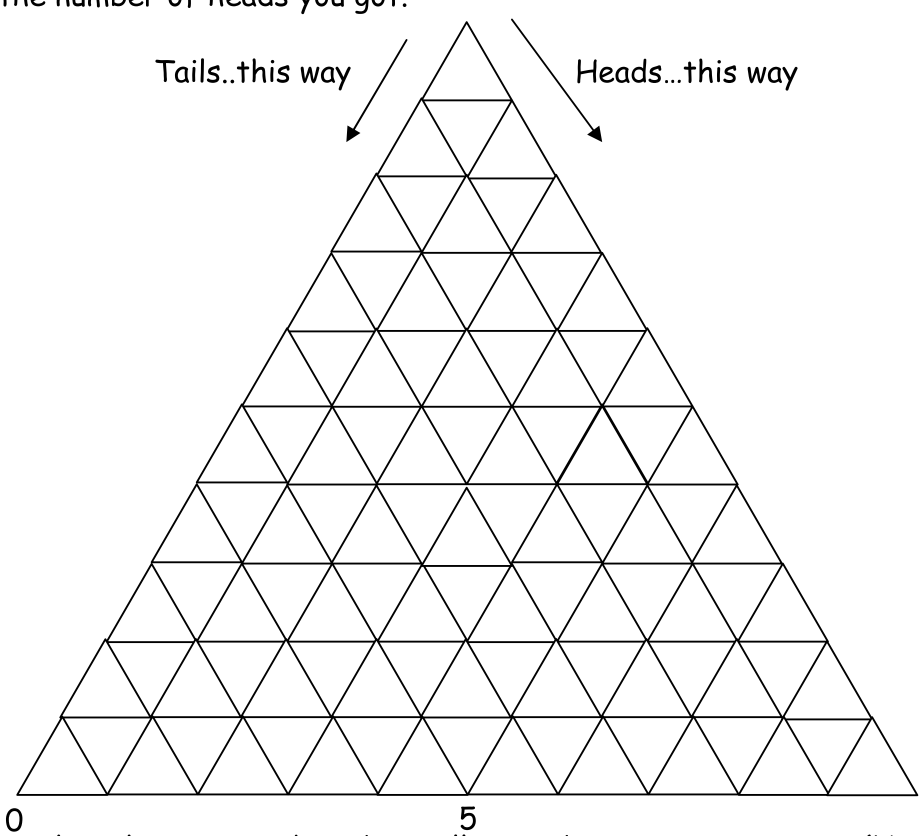
Sample Space for a Coin Toss = {H, T}

She says the coin is fair, so each of these two events are equally likely. This means she can work out the chance of one of them happening when a coin is tossed as one out of two. We can write this as:

Probability (Head) = $\frac{1}{2}$ and probability (Tail) = $\frac{1}{2}$

Philippa then predicts if she tosses a coin ten times, half of them will be heads, so she expects to get 5 heads.

1) Is she correct?
2) Use the pyramid to help explain what can happen if she tosses the coin ten times and records the results. Start at the tip or apex and move left and down when you toss a tail and right and down when you toss a head. Where you end up is the number of heads you got.



3) Explain what the expected number tells you about an experiment. (Hint: you may want to collect the results from around the class to help you with this).

Exercise 6 – Mathematical Probability 2

You will need a fair six sided die or a fair ten sided die.

Mathematical probability of an event happening is based on knowing the likelihood of all the outcomes in an experiment. For a die (one of many dice) the outcomes are simple and all sides are equally likely to come up. Dice are made symmetrically so that equal likelihood is a property they possess.

1) List all the possible outcomes when a die when is tossed.

Sample Space for the toss of a die = { __, __, __, __, __ }

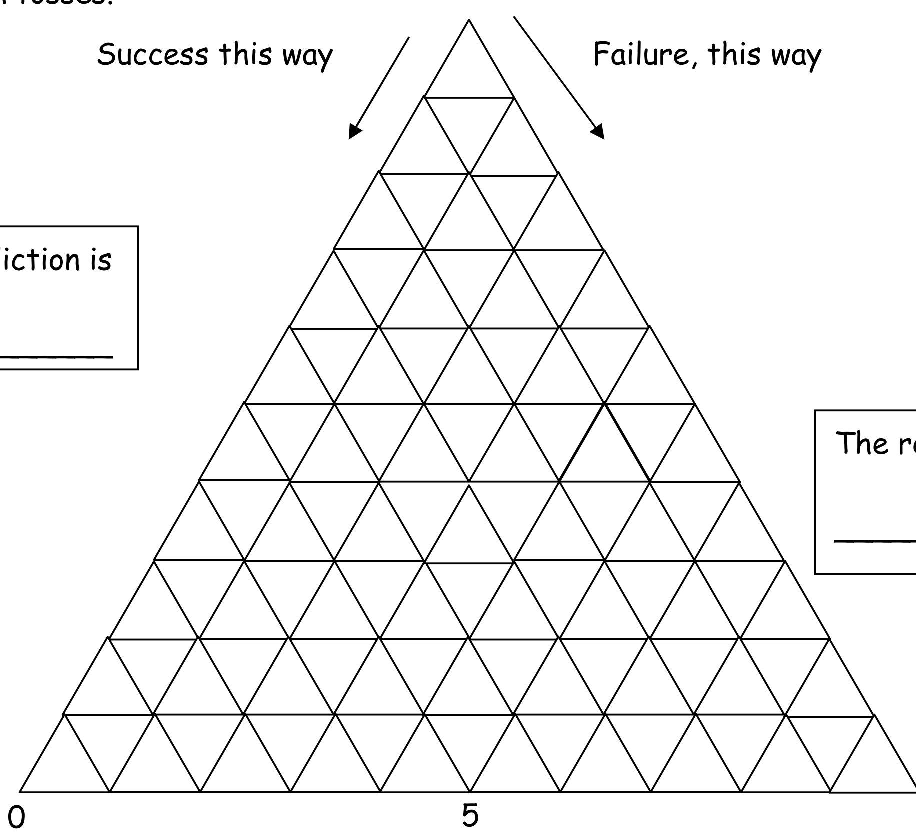
2) Write the probability for each of the outcomes you named above.

Probability of _____ =	Probability of _____ =	Probability of _____ =
Probability of _____ =	Probability of _____ =	Probability of _____ =

3) Choose an outcome (eg getting a 3) and predict how many you expect to throw in ten tosses.

My prediction is

____



The result was

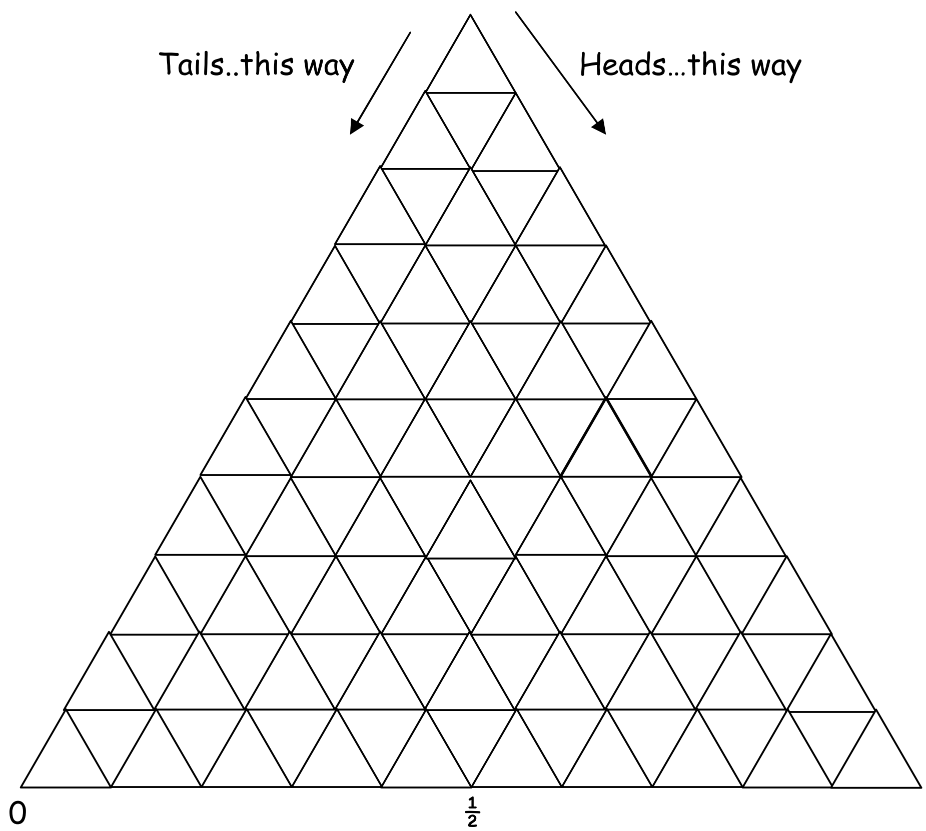
____

Exercise 7 – Experimental Probability 1

You will need a fair coin.

Experimental probability of an event happening is doing an experiment many times to establish the probabilities of the outcomes. This is an intuitive notion but is formally recorded as The Law of Large Numbers which is a major part of probability theory.

Your task is to toss a coin 100 times and record the result of each throw. You may use a (very big) calculating pyramid like this one or use a table to record H and T.



An interesting way to summarise the results is to compare the number of heads thrown to the number of tosses, writing the answer as a fraction. (So if I have thrown the coin 4 times and have got one head, I record $\frac{1}{4}$. If the next throw is a head, I now have 2 heads in 5 tosses so I have 2/5). The graph waves wildly at first but after a while it starts to become more stable and trends towards a certain number. Put your fractions into an excel spreadsheet and graph it to see this. Also compare your answer with the mathematical probability of this event.

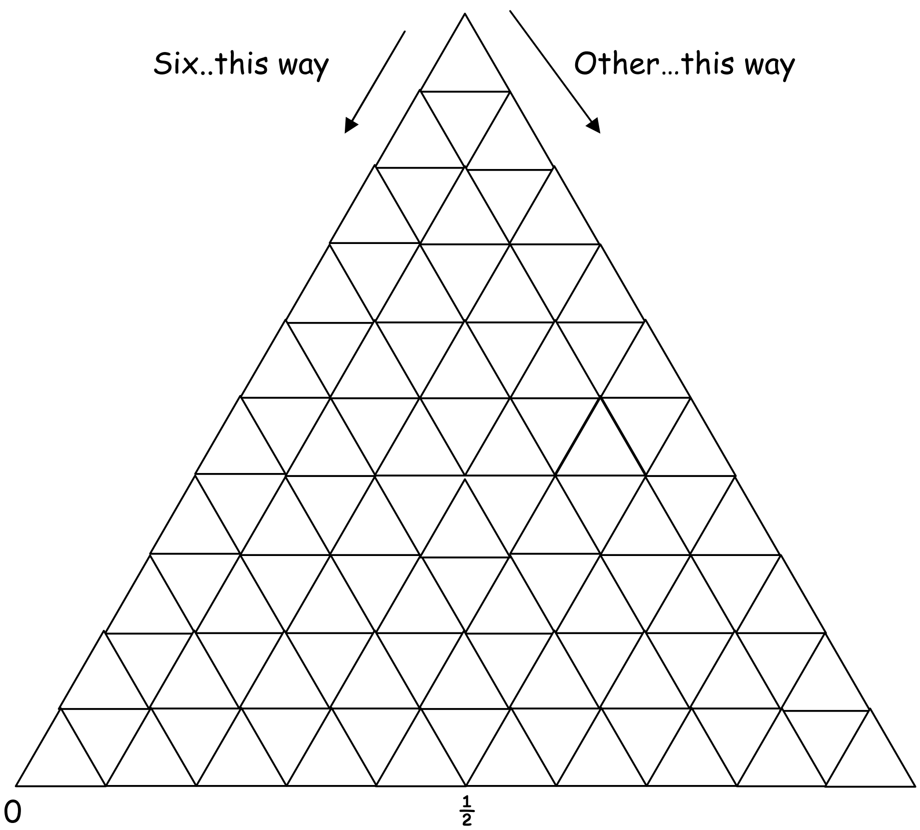
There are good website links simulating tossing a coin and a variety of other things many thousands of times. Explore a few sites and write up what you have discovered as an investigation. Arrange a time with your teacher to present your findings to the class. This is a very good mathematics investigation for science fairs.

Exercise 8 – Experimental Probability 2

You will need a fair die.

Experimental probability of an event happening is doing an experiment many times to establish the probabilities of the outcomes. This is an intuitive notion but is formally recorded as The Law of Large Numbers which is a major part of probability theory.

Your task is to toss a die many times, recording as you do so the number of sixes and the others. You may use a calculating pyramid like this one or record the results in a table.



An interesting way is to graph the proportion of heads to total after every toss of the die. The graph waves wildly at first but after about 20 tosses the graph becomes more stable and trends towards a certain number. Compare your answer with the mathematical probability of this event.

How many trials do you need in an experiment to get a reliable result? Use a graph to help you decide.

Exercise 9 – Simulation Design

You will need a tool for generating random outcomes such as a ten sided die.

Your Task
You are to design an experiment that will simulate a situation that happened in real life. Follow the example below to see the steps you can do to make the experiment work and to get some valid and reliable outcomes.

Example Simulation of Twin Lambs being born

Barbara notices that there are quite a few twin lambs in the paddock after lambing is over. She counts them all up and gets 24 sets of twins out of a flock of 112 ewes. She says this shows that 24 out of 112 is the probability of getting twin lambs. Dad tells her that twins should happen about once in every 8 births. She decides to simulate the lambing to compare what Dad says with what she sees in the paddock.

Step 1: Choose a tool to simulate the situation.

Barbara chooses a white ten sided die to simulate a ewe.

Step 2: Decide what it is that simulates the event you are investigating.

Barbara says 1 out of every 8 ewes will have twins if Dad is correct. There are ten sides on this die so I am going to use 8 of them. I will use the 1,2,3,4,5,6,7 and 8 and ignore the 0 and 9. The 2 can be "twins" because it is the number 2.

Step 3: Do the simulation many times.

Barbara decides to toss the die 30 times. She records the tosses in a table and ignores any throw with 0 or 9. Her results are:-

7	1	6	4	4	2	5	7	2	4
3	3	2	2	6	2	3	4	1	1
1	1	5	7	1	3	8	2	6	8

Step 4: Calculate the proportion from the results.

Barbara counts up the number of "twins" or 2s and calculates the proportion by comparing this with 30. She says this is 6 out of 30 or 20% are twins.

Step 5: Compare the simulated results with what is in the problem.

Barbara says "24 out of 112 is 21.4% and the results of the simulation are very close to this proportion, but I think I will do this again just to be sure because we have had good weather this year so we should have got more twins."

Your task is to repeat her simulation twice to see if you get a similar result and then think up a different task to simulate.

Exercise 10 – Random Numbers

You will need a table of random numbers.

Task 1: How random are you?

Choose numbers at random from 1, 2, 3, 4, 5 or 6 and write them down in the table.

Count up the numbers of each type and see if they are about the same.

Make a comment about how random you are.

Task 2: Is a die random?

Toss a six sided die 30 times and record the results in this table.

Count up the numbers of each type and see if they are about the same.

Make a comment about how randomness of a die.

Task 3: Are the tables random?

Select 30 numbers from the random number tables.

Count up the numbers of each type and see if they are about the same.

Make a comment about how randomness of a the tables.

Which is the most random?

Number in Probability
Answers

Exercise 1: Trials

1) 1/2
2) It would be rare to find two identical paths with only a few experiments
3) The end result is not always the same
4) Guess on the chance of getting a head – the result should still be 1/2

Students should do the simulation several times and be able to explain that this improves reliability of estimating the likelihood.

Exercise 2: Tribulations

1) 1/6, because there are six sides on a die, and each side has the same chance of coming up if it is thrown fairly
2) With ten throws each experiment I would expect different pathways to be taken, and that I would not always end up in the same place. What happens is random – it cannot be predicted in advance.
3) My estimate is still 1/6, because in a fair die each side has the same chance of coming up

Exercise 3: Drawing Pins

1) From experience, different types of drawing pin have different probabilities – depending on the size and the weight of the back of the pin. None of them have a chance of one half
2) As expected, you do not always end up in the same place.
3) The result will depend on the pin and the number of trials combined. In general, if the results from the whole class are combined (and all the pins are the same) the probability will be somewhere around 0.7
4) The result is not about 0.5 as not all of the outcomes have the same chance of occurring. Here most of the weight is in the back of the pin, so it tends to land downwards (with the pin-up) more often than happens with the coin. The drawing pin is an example of what happens when the results are not equally likely. (Similar things can happen when a die is weighted, or when one side is made heavier or lighter).

Exercise 4: The Language of Probability

1) Different people interpret the words in different ways, and some change meanings in different contexts. Having said that, a possible order is Never = impossible -= zero chance, rare occasion, slim chance, very unlikely, sometimes, unlikely, might happen, possible, average chance, 50/50 = even chance = equally likely, probable, better than average, very likely, highly likely, a good bet, nearly always, almost certain, certain = 100% chance of happening.

2) Word meanings

Chance – the likelihood or probability of something happening. Usually described in words or as a number between 0 and 1
Probability – the branch of mathematics that looks at the chance of things (events) happening or when referring to an experiment, the chance of an event happening
Experiment – a probability test, like the toss of a coin ten times
Trial – one repetition in an experiment – like a single toss of a coin
Outcome – one of the results of a trial, like getting a 6 when tossing a die
Event – something happening in a trial. It can be one outcome or a number of outcomes combined, like getting a prime number when tossing a die
Success – an outcome or event that you want to happen

Failure – an outcome or event that you don't want to happen (the opposite event to a success)
Bias – the favouring of an outcome over another
Fair – a tool that delivers expected outcomes. In general, a fair coin is one that produces equal numbers of heads and tails, or the outcomes are equally likely
Random – something that is unpredictable or a series with no pattern
Likelihood – another word for or chance or probability.
Likely – the event happens more than half of the time
Die, Dice – one die is a die and several die are dice. (Die is the singular of dice).
Sample space – the set of all possible outcomes
Mathematical probability – is the expected probability confirmed by experiment
Heads – one outcome from a coin
Prediction – a calculation or estimation of the outcome of some future event based on what is known or expected

Exercise 5: Mathematical Probability 1

1) Philippa is correct. The expected number is how many I will get if everything is fair (that each outcome is equally likely, and that the same number of each outcome happens - even though we know that things do not always happen like this in real life).
2) In real life, if we toss a coin twice we do not always get one head and one tail. Sometimes we can two heads or two tails. The results are unpredictable or random in nature, so can take any path from the top of the grid to the bottom (though the end point is more likely to be near the middle than the edge as there are lots of different paths that end up at the middle and only a few that end up at the edge.)
3) If we take everyone's results, and average out the number of heads everyone got, the answer is likely to be very close to five. This is because in a lot of trials, if one person got a lot of heads, another may have got a lot of tails, so in the long run these things balance out.

Exercise 6: Mathematical Probability 2

For a six sided die

1) {1,2,3,4,5,6}
2) all are 1/6
3) Prediction and result can vary. In the toss of a six sided die, you can expect 1/6 of the throws to come up with a 3 (or 4 or 5…). This means in ten throws you can expect 1/6 of ten threes – which means one or two threes (two being the closest whole number)

Exercise 7: Experimental Probability 1

Various answers all written up as an investigation.

Exercise 8: Experimental Probability 2

Various answers all written up as an investigation. The graphs tend to stabilise around 30 trials

Exercise 9: Simulation Design

Various answers all written up as an investigation.

Exercise 10: Random Numbers

Various answers.
Task 1. Students will discover a propensity to select a favourite number.
Task 2. A well tossed dice is random.
Task 3. Tables a random.
Making a simple bar graph of these results should show an even height of all bars if the tool is random.