Acknowledgments

This resource has been developed to support
The New Zealand Curriculum

Teacher Support Material
(including Answers)

MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Contents

Introduction

The books in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand classrooms. The achievement objectives for mathematics and statistics and for science and the key competencies referred to in this *Teacher Support Material (including Answers)* are from *The New Zealand Curriculum.*

Student books

The activities in the Figure It Out student books are written for New Zealand students and are set in meaningful contexts, including real-life and imaginary scenarios. The contexts in the level 3+–4+ *Using Resources* book reflect the ethnic and cultural diversity and the life experiences that are meaningful to students in years 5–8. However, you should use your judgment as to whether to use the students' book with older or younger students who are also working at these levels.

Figure It Out activities can be used as the focus for teacher-led lessons, for students working in groups, or for independent activities. You can also use the activities to fill knowledge gaps (hot spots), to reinforce knowledge that has just been taught, to help students develop mental strategies, or to provide further opportunities for students moving between strategy stages of the Number Framework.

Teacher Support Material (including Answers)

In this new format, the answers are placed with the support material that they relate to. The answers are directed to the students and include full solutions and explanatory notes. Students can use these for self-marking, or you can use them for teacher-directed marking. The teacher support material for each activity, game, or investigation includes comments on mathematics and science ideas, processes, and principles, as well as suggestions on teaching approaches. The *Teacher Support Material (including Answers)* for *Using Resources* can also be downloaded from the nzmaths website at www.nzmaths.co.nz/node/1995

Using Figure It Out in the classroom

Where applicable, each page of the students' book starts with a list of equipment that the students will need in order to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to the problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Figure It Out

Using Resources

Mathematics in science contexts

Teacher Support Material
(including Answers)

Introduction to Science

> Science is a way of investigating, understanding, and explaining our natural, physical world and the wider universe.
>
> *The New Zealand Curriculum,* page 28

Inquiry in science is called investigating. Science investigations can take many forms, including classifying and identifying, pattern seeking, exploring, investigating models, fair testing, making things, and developing systems. Investigating in science may involve more than one type of investigation; each investigation can share elements with other investigations. Scientists choose the appropriate type of investigation to answer their question(s). Science investigations also provide students with rich contexts for mathematical opportunities as they decide what and how to measure, what units to use, and how to record findings as they identify trends and patterns and describe relationships. See www.tki.org.nz/r/science/science_is/dssa/focus_07_approach_e.php for examples of different types of science investigations and activities that illustrate them.

The *Using Resources* students' book emphasises making and testing statements about data. The progression of activities requires students to interpret and manipulate increasingly sophisticated representations of data, from simple categorisation to bar graphs, pie charts, ratios, line graphs, composition of percentages, and relative rates, ending with a graphical representation of flood data (year, water height, water flow, and water travel time). Explanations are only scientific if they are testable and evidence based; gathering and interpreting data – a major emphasis in the mathematics and statistics learning area of *The New Zealand Curriculum* – is therefore directly relevant to science investigations.

This book provides rich opportunities for students to practise working with percentages, ratios, and large numbers. In so doing, it supports the requirement for students to demonstrate increasingly sophisticated understanding of place value, fractions, percentages, and proportions as they advance from level 3 to level 4 and higher in the mathematics and statistics learning area. The activities also target number sense. Many of the quantities in the book intentionally require approximation – several calculations will be too large for most calculators, and percentages and corresponding values will need to be carefully rounded to suit the precision of the given data or measuring tool. Over the course of the book, students should become increasingly comfortable with ambiguity, whether in extrapolating trends, interpolating in tables, or finding a range of solutions to the game on pages **16–17**.

Several activities in the students' book require use of the Internet. The teacher support material for the students' book includes conceptual background material and suggestions for wider and deeper exploration, especially in using web resources. For example, Pareto's rule can be tested against other contexts or the activity broadened to include a general evaluation of the usefulness of many different rules-of-thumb. The game on pages **16–17** introduces students to the core economics concepts of scarcity, supply, demand, and competition.

Internet links: Note that on the downloadable version of this support material (www.nzmaths.co.nz/node/1995), all the Internet links can be activated by clicking on a hyperlink.

Investigating in Science

EXPLORING A SITUATION

A problem, question, situation, phenomenon, artefact, model, event, story

I wonder why ...?

I wonder how ...?

OBSERVE

What is happening?

What changes happen?

What materials are involved?

What are the main parts?

What are the key aspects?

What do these parts do?

ASKING QUESTIONS

Why? How? What? When? Where?

PLANNING AN INVESTIGATION

What will we observe/measure? What do we want to find out? How can we find out? What can we do to find out? Are there things we need to change or keep the same?

MAKING PREDICTIONS

What do we think will happen and why?

GATHERING DATA

Find out, measure, compare, verify, test, clarify, identify

PROCESSING AND INTERPRETING

Creating personal explanations supported by evidence

What is the cause and effect of change?

What patterns keep occurring?

What parts are interacting with other parts?

What are the outcomes of these interactions?

EVALUATION

What do the results mean?

Does our investigation answer our question?

Do our results support or challenge our predictions?

Is there another explanation for what we found out?

What do other people think about our investigation?

What have other people found?

APPLICATION AND SHARING OF NEW THINKING

How could we use our idea or explanation?

Does it explain other things?

What next? Where to now?

FURTHER INVESTIGATION

Do we have another question to investigate?

Is there another idea or explanation that needs testing?

Pages 1-3: Using Resources and Classifying Resources

Mathematics and Statistics Achievement Objectives

- Number strategies and knowledge: Understand addition and subtraction of fractions, decimals, and integers (Number and Algebra, level 4)
- Statistics:
  - Investigate simple situations that involve elements of chance by comparing experimental results with expectations from models of all the outcomes, acknowledging that samples vary (Probability, level 3)
  - Use simple fractions and percentages to describe probabilities (Probability, level 4)

Mathematics standards. The approaches and thinking that students demonstrate as they engage with these tasks and problems can provide evidence in relation to the mathematics standards.

Science Achievement Objectives

- Investigating in science:
  - Build on prior experiences, working together to share and examine their own and others' knowledge
  - Ask questions, find evidence, explore simple models, and carry out appropriate investigations to develop simple explanations (Nature of Science, levels 3–4)
- Communicating in science: Begin to use a range of scientific symbols, conventions, and vocabulary (Nature of Science, levels 3–4)
- Participating and contributing:
  - Use their growing science knowledge when considering issues of concern to them
  - Explore various aspects of an issue and make decisions about possible actions (Nature of Science, levels 3–4)
- Earth systems: Appreciate that water, air, rocks and soil, and life forms make up our planet and recognise that these are also Earth's resources (Planet Earth and Beyond, levels 3–4)
- Properties and changes of matter: Group materials in different ways, based on the observations and measurements of the characteristic chemical and physical properties of a range of different materials (Material World, levels 3–4)

Mathematics and statistics context

Students will:

- explore how probability influences the outcomes in a game of chance.

Students should discover that:

- probabilities can sometimes be determined theoretically by considering all possible outcomes
- probabilities can be expressed as simple fractions.

Science context

Students will:

- explore the meanings of the terms renewable and non-renewable in relation to resources
- find examples of renewable and non-renewable resources and be able to state why particular resources belong in each category
- explore how the use of resources impacts on the environment.

Students should discover that:

- we can better manage the resources we have if we participate in reducing, recycling, or reusing the resources we use in our everyday life.

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Answers

Activity (Using Resources)

1. **a.** Definitions of "resource" will vary, but may include:
   - something that people use, especially in the manufacture of goods
   - a raw material
   - a person, thing, or action needed for living or to improve the quality of life
   - anything that is both naturally occurring and of use to humans.

   b. Discussion will vary but should arise from the definitions in **a**. For example, many companies employ a human resources manager, who looks after the needs of the people in the company (the resources), schools have art resources in their classrooms or in a separate art room, and oil is a resource.

2. Possible points include:
   - Renewable resources are resources that can be replaced (replenished), such as trees and plants.
   - A renewable resource is a natural resource that can be replaced in a relatively short time, for example, wood, water, wind, and solar energy.
   - A non-renewable resource is a natural resource that is not replaceable after it has been removed from its source, for example, coal or mineral ores.

     (A natural resource is a material that is found naturally in the environment, for example, coal or timber, and is used for food or energy or to produce other materials.)
   - Soil is often classified as a renewable resource (www.ehow.com/about_5200783_soil-renewable-resource_.html), but it can also be classified as non-renewable (http://urbanext.illinois.edu/world/nres.html).

Renewable	Non-renewable
Living things that are not endangered	Fossil fuels: coal, oil, natural gas
Sustainable forests	Metals
Geothermal energy	Minerals
Landfill gas	Nuclear fuel
Hydroelectricity	
Solar energy	
Tidal energy	
Wind energy	
Biofuels	
Soil?	Soil?

Game (Classifying Resources)

A game that involves identifying renewable and non-renewable resources

Activity (Classifying Resources)

1. **a.** Ethanol should be classified as a renewable resource because it can be obtained from biological sources, such as dairy products, sugar cane, potatoes, and corn.

    b. Answers will vary. Electricity can be either renewable or non-renewable depending on its source. Hydroelectric, solar, and wind energy are regarded as renewable energy resources. Geothermal energy is usually regarded as a renewable resource, although overuse will result in its depletion. Electricity in New Zealand is also produced from coal and natural gas, both of which are non-renewable.

2. **a. i.–ii.** There are 16 resources on the spinner. 6 of them (coal, natural gas, aluminium, petroleum, plastic, and gold) are non-renewable, so the probability of moving forwards (3 spaces) by landing on one of the 10 renewable resources is 10:16 and the probability of moving backwards (1 space) is 6:16.

    b. There are 3 bonus spaces and 3 penalty spaces, but the reward (forward 2) for landing on a bonus space is greater than the penalty (back 1) for landing on a penalty space, so the overall impact of these spaces is to move the game forward.

    Note that two of the bonus spaces are recyclable rather than renewable, but the principle of helping our environment still applies.

Notes

Points of entry: Mathematics

The activities and game in Using Resources and Classifying Resources challenge students to think about what part of the resources they use is renewable and what the implications are if current patterns of use continue (or accelerate). Students may not realise that every discussion of resources – especially resources that are non-renewable – inevitably comes back to mathematical understandings, particularly inverses, rates, and ratios: if a non-renewable resource is consumed at a faster rate, it will run out sooner; if some consume more than their share, others have to make do with less.

Question **2** in Classifying Resources shifts the focus to the mathematics behind the game: integers and probability. Integers are used to describe the direction of each move (whether forwards or backwards); probability concerns the likelihood of particular outcomes.

The speed at which the students complete the game is largely determined by the probability of landing on a renewable or non-renewable sector of the spinner. The students explore this in question **2a** (see the answers). In terms of probability, the "expected value" for a move is forward 1.5 squares ($\frac{10}{16} \times 3 + \frac{6}{16} \times {}^-1 = \frac{15}{8} - \frac{3}{8} = \frac{12}{8} = 1.5$). Question **2b** looks at the impact of forwards and backwards movement that results from landing on bonus or penalty squares. The expected value is $\frac{3}{6} \times 2 + \frac{3}{6} \times {}^-1 = \frac{1}{2} = 0.5$.

Not all students will be ready for calculations such as those shown for expected value, but all should understand that, on any given spin of the spinner, the probability of moving forwards is greater than the probability of moving backwards. They should also be able to see that, on balance, the bonus or penalty spaces are helpful because, although the number of bonus spaces is the same as the number of penalty spaces, the bonus spaces provide for double the movement of the penalty spaces.

Challenge the students to suggest alterations to the spinner and then describe how those alterations would affect the probabilities. For example: *If the wind segment was replaced with uranium, would that make the game longer or shorter? What would make the game harder or easier?* (Including uranium instead of wind would make the game harder and longer because uranium would add another negative move.)

Points of entry: Science

Determine students' prior understandings of the terms and support the class to explore a variety of sources to add to their understanding. The point of the game is not winning as such but providing the students with opportunities to realise that classifications of renewable and non-renewable can vary.

To get the most out of the activity on page **1**, students will need to work co-operatively with a classmate. This gives them an opportunity to develop the key competency *participating and contributing*. The Classifying Resources game and activity require students to interact, share ideas, and work effectively with others, so the key competency *relating to others* is a suitable focus for development.

As the students play the game, they could record their decisions about resources on a renewable/non-renewable table.

After the game, discuss the concept that there is not always a clear right answer. Some resources that seem renewable can get used up if we use them faster than they can replace themselves, as was the case with the moa.

Pages 4–5: Ecological Footprints

Mathematics and Statistics Achievement Objectives
- Number strategies: Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages (Number and Algebra, level 3)
- Statistical investigation: Plan and conduct investigations using the statistical enquiry cycle:
  - determining appropriate variables and data collection methods
  - gathering, sorting, and displaying multivariate category, measurement, and time-series data to detect patterns, variations, relationships, and trends
  - comparing distributions visually
  - communicating findings, using appropriate displays (Statistics, level 4)

Mathematics standards. The approaches and thinking that students demonstrate as they engage with these tasks and problems can provide evidence in relation to the mathematics standards.

Science Achievement Objectives
- Earth systems: Appreciate that water, air, rocks and soil, and life forms make up our planet and recognise that these are also Earth's resources (Planet Earth and Beyond, level 3)
- Investigating in science:
  - Build on prior experiences, working together to share and examine their own and others' knowledge
  - Ask questions, find evidence, explore simple models, and carry out appropriate investigations to develop simple explanations (Nature of Science, levels 3–4)
- Participating and contributing:
  - Use their growing science knowledge when considering issues of concern to them
  - Explore various aspects of an issue and make decisions about possible actions (Nature of Science, levels 3–4)

Mathematics and statistics context

Students will:

- gather and enter personal lifestyle data
- interpret data displays in order to identify patterns and trends
- change input variables and observe the effect on outputs
- estimate sources of variation in eco-footprints.

Students should discover that:

- outputs are a function of inputs.

Science context

Students will:

- calculate their own ecological footprint
- consider and investigate issues of sustainability
- explore what actions they could take to make their lifestyle more sustainable
- test out their ideas about actions, using the ecological footprint calculator, and consider the consequences.

Related information

Building Science Concepts: Book 60, *Rubbish*; Book 61, *Recycling*; Book 47, *Insulation*

Bio-capacity: www.wwf.org.au/footprint/

Programs to estimate eco-footprints:
www.earthday.net/footprint2/flash.html
www.myfootprint.org/

Answers

Activity

1. Quiz results will vary.

2. **a.** Answers will vary.

    b. Reasons may include: transport used (public transport, car, bike, walking), size of house, whether the family recycles and reuses, the amount of food grown and prepared at home. Resource-consuming activities, such as eating food that has travelled a long way, living in a bigger house than needed to house the occupants, using a fuel-inefficient vehicle, and flying, will increase your eco-footprint. Resource-conserving activities, such as walking instead of driving and recycling waste, will lower your eco-footprint.

3. **a.** Compared with Matthew, Adena uses a lot more transport and goods. Because Adena is part of a family, the reasons for this will mostly relate to decisions that have been made by the adults in her family. This means that to reduce her eco-footprint by much, her family may need to do some things differently. Possibilities might include putting the dishwasher on only when full, not upgrading appliances that still work well, getting rid of a second car or replacing a fuel-hungry car with an economical one, consistently recycling and reusing, growing

some vegetables, preparing more food at home, making do with 1 computer only, making clothes last longer, fixing items instead of replacing them.

Adena definitely could do some things herself to reduce her eco-footprint, for example, drinking tap water instead of bottled water, taking shorter showers, not leaving the heater running in her bedroom, not asking for the latest gadgets, wearing clothes longer before replacing them, walking instead of asking for a ride, sharing more things with siblings. She could also take initiatives to reduce the family's eco-footprint, for example, organising the recycling bin and taking responsibility for composting organic waste.

b. Results will vary, but changing one variable at one time will help to show the effect of that change on an eco-footprint and help you make decisions about lifestyle choices.

c. Targets and ideas will vary. See answers **2b** and **3a** for ideas for possible changes.

d. Results will vary. For other ideas, look at less obvious factors that affect the family's footprint, such as detergent use (some are designed to need only cold water), laundry (do less often and only when there is a full load), or home heating and insulation.

Preparation and points to note

Choose a suitable eco-footprint calculator and, if possible, take the quiz yourself to get a feel for the data required. Some of the calculators are more interesting than others, and some ask for quite detailed information. Identify what information the students will need before they can calculate their own footprint. Allow the students time to gather this information; they will need to discuss the questions and may need to collect some information from home, for example, the fuel consumption of the family car(s).

In this activity, students need to make sense of information and their own experiences and come up with ideas, so the key competency *thinking* is applicable.

If your students need practice in interpreting visual data displays, the Figure It Out level 3–4 books on statistics are a good place to find suitable resources.

Support the students to make links between the information required for the ecological calculator and their own impact on the environment. Explore the meanings of carbon footprint, food footprint, and goods and services footprint.

Note that eco-footprint calculators are unlikely to be specific to New Zealand and may not even have New Zealand on the list of selectable countries. If necessary, the students can pick the country or place that is likely to provide the closest match, for example, Australia.

Points of entry: Mathematics

This activity explores 1 idea using 4 different representations: a bar graph, a pie chart, an index (49.41 gha), and a ratio (3.6 Earths if everyone on Earth lived like Adena). The representations on page **5** of the students' book are an excellent starting point for investigating how personal lifestyle contributes to humankind's overall eco-footprint.

The students need to understand that pie charts don't show the size of the "cake" (Adena's 3.6 Earths pie is the same size as Matthew's 2 Earths pie), only how it is "carved up" or divided by category. This should lead to discussion as to whether, for example, Matthew is actually using more food than Adena, as appears at first glance (he is using less). A useful activity is to convert the information in the 3 pie charts into a grouped bar chart like Oromia's on page **4** of the students' book. To do this, the students will need to use a protractor to measure the angle of each sector (for example, 35°), then multiply this angle over 360° by the number of global hectares represented by the pie (for example, $\frac{35}{360}$ x 58.6 = 5.70 gha). By doing this, the students will gain a powerful insight into the meaning of the information in the two kinds of graph.

Carefully unpack the meaning of the unit Earth. The statement "Adena consumes 3.6 Earths" doesn't mean that she personally consumes 3.6 Earths but that if everyone were to consume resources like her, they would need 3.6 Earths to supply those needs/wants (1 Earth wouldn't be anywhere near enough). This understanding is important for the next activity, Pareto's Rule.

What matters more than actual magnitude is that the students have a qualitative understanding of the direction of change so that they can link cause and effect. When they are discussing the various factors that can increase or decrease an eco-footprint, encourage them to think not only of things that lie outside their present sphere of influence, responsibility, or possibility (for example, making more use of wind generation) but of things that they can do themselves (for example, walk more, not ask for or buy the latest gadgets, make clothes last longer, take shorter showers).

Note that if, in question **3**, the students change more than 1 variable at a time, they will be unable to separate the effect of each. This is why question **3b** specifically says to change only 1 variable. For question **3c**, too, encourage the students to think about 1 variable at a time and to explore the consequences. Get them to record the changes they make and their impact. One way of doing this could be to print the screen that shows the results at the end of the quiz and annotate it with the changes made; alternatively, the students could capture their eco-footprint "journey" in a table or other data display.

Global hectares also use really big numbers (1 Earth = 13 billion global hectares). Find ways of giving meaning to these numbers. Set the students to find out the area of New Zealand in hectares. Ask: *How large is 1 billion hectares compared with all of New Zealand?* (New Zealand has about 27 million hectares; 1 billion hectares is about 37 New Zealands or 1.3 Australias.) *What other quantities are expressed in terms of billions?* (Human population, national budgets, distances in space)

As an extension, discuss how the website calculators illustrate the use of variables, both independent and dependent. Help them to realise that Earth's future is the dependent variable: it's dependent on the independent choices we all make about how we live our lives.

Points of entry: Science

The mathematics of eco-footprints and "Earths" is based on the global hectare (gha) concept. This needs careful discussion and explanation.

A global hectare is not the same as 1 hectare of farmland; rather, the unit is a measure, based on the productive capacity of a standard land hectare, of the productive capacity of the entire Earth, including the resources to be found in the sea, in the sky, and under the ground. Different eco-footprint calculators use different methodologies to convert from gha to Earths. For example, the Earth Day Network website assumes that 1 Earth = 13 billion gha of renewable resources (about 2 gha per person), while the My Footprint website factors in multiple uses of the same area of Earth to get a ratio of about 15 gha per person or about 102 billion gha of renewable resources for 1 Earth.

Some students may be concerned at the lack of exactitude surrounding the gha concept. This variability simply reflects the uncertainty and complexity of any calculations about Earth's productive capacity. Every system or formula used involves lots of assumptions.

Have the students compare what we need to sustain life and what we actually use in maintaining our lifestyles. Ask *How can we manage resources better to ensure the ongoing survival of the planet?* This discussion will inevitably come back to values and, often, to competing values. Link this discussion to the values in *The New Zealand Curriculum*.

Science is a social activity, done by people. Link your students' choices to real outputs; encourage them to think of their inputs into the calculator as decisions they make about how to live their lives and to see that these decisions, if carried out, have a real effect on our planet.

Mathematics and Statistics Achievement Objectives

- Number strategies and knowledge:
  - Use a range of multiplicative strategies when operating on whole numbers
  - Find fractions, decimals, and percentages of amounts expressed as whole numbers, simple fractions, and decimals
  - Apply simple linear proportions, including ordering fractions
  - Know the relative size and place value structure of positive and negative integers and decimals to three places (Number and Algebra, level 4)

Mathematics standards. The approaches and thinking that students demonstrate as they engage with these tasks and problems can provide evidence in relation to the mathematics standards.

Science Achievement Objectives

- Investigating in science: Build on prior experiences, working together to share and examine their own and others' knowledge (Nature of Science, levels 3–4)
- Participating and contributing:
  - Use their growing science knowledge when considering issues of concern to them
  - Explore various aspects of an issue and make decisions about possible actions (Nature of Science, levels 3–4)
- Earth systems: Appreciate that water, air, rocks and soil, and life forms make up our planet and recognise that these are also Earth's resources (Planet Earth and Beyond, level 3)

Mathematics and statistics context	*Science context*
Students will:	Students will:
- work with very large numbers - express large number fractions as percentages - use ratios in different contexts - identify the appropriate numerical information and operations needed to solve a problem or answer a question. Students should discover that: - it is possible to have a percentage of a percentage - it is possible to convert from ratio to percentage, and vice versa.	- apply what they have found out about ecological footprints as they explore the implications on a larger scale - use mathematics to test a rule of thumb against actual data. Students should discover that: - a rule of thumb is a useful tool for estimation, but they should undertake a scientific investigation to collect actual data to back up or refute the rule.

Related information

Pictures of relative consumption in different countries:
http://efs.tki.org.nz/Curriculum-Resources-and-Tools/ (search under Food and Families)
www.worldmapper.org/display.php?selected=322 (world map)

Answers

Activity One

1. 17%. (1 168 530 000 ÷ 6 778 070 000 x 100)

   b. 0.61 Earths. (If 100% of the world lived like Adena, in a developed country, 3.6 Earths are needed. But only 17% of people live in developed countries, so they are using "just" 17% of 3.6 Earths. $17\% = \frac{17}{100} = 17 \div 100$. So 17% x 3.6 = 17 ÷ 100 x 3.6 = 0.61 Earths.)

   c. 0.39 Earths. (1 Earth – 0.61 = 0.39 Earths)

   d. 83%. (100% – 17% = 83%)

e. At a rate of 0.47 Earths. (83% have the use of 0.39 Earths; 0.39 x [100 ÷ 83] = 0.47 Earths)

2. a. 80% or 0.8 Earths

   b. According to the information in question **1**, 17% of people live in developed countries and consume 0.61 Earths, leaving 0.39 Earths for the 83% who live in developing countries. This is a 60:40 split rather than Pareto's 80:20. So Pareto's rule is not exact, but it does give some indication of the imbalance.

3. Pareto's rule provides a quick, reasonable approximation in many circumstances. Nobody pretends that it is a substitute for careful data gathering and calculation.

Activity Two

1. a. 10 444 million hectares. (80% x 13 056 million hectares)

   b. 2 612 million hectares. (13 056 – 10 444)

2. We are already consuming more resources than Earth can renew, and the supply of land is fixed (we can't make more). If people in developing countries were to consume more, those in developed countries would have to consume less and/or new resources would have to be found. We may be able to find new resources by discovering new methods of generating power, by farming the sea, or by inventing new foods, fuels, and fabrics.

Notes

Preparation and points to note

These activities work best if students have completed Ecological Footprints on pages **4–5** and understand the concept of resource use and Adena's eco-footprint.

As well as fostering *thinking*, the challenges in Pareto's Rule will build new knowledge and require the use of precise language and appropriate tools – all part of the key competency *using language*, *symbols*, *and texts*.

Pareto's rule is an example of a rule of thumb. The questions in the activities compare his rule with actual data and discuss the usefulness of such rules. These activities can also help students deal with the numbers and operations they need to understand and find meaning in actual situations.

Introduce and discuss rules of thumb that the students may be familiar with, for example, the New Zealand Police slogan "The faster you go, the bigger the mess". A rule of thumb is simply a shorthand way of predicting an outcome.

In these activities, students need to use percentages and ratios in various combinations (percentages are actually ratios too: so many out of 100) to make meaningful comparisons possible. Percentages and ratios require students to think proportionally.

Points of entry: Mathematics

Given that proportional measures depend on the context for their meaning, you need to emphasise reading and interpretation to your students. Often the most important question to be asked is "What is the whole?"

Review your students' understanding of percentages and give them practice at converting percentages to fractions (and vice versa) and applying percentages in different situations.

The quantities in these activities are large: in the millions and up to hundreds and thousands of millions. Have the students practise reading them. Discuss the strategies that people use to break up and manage big numbers; for example, thinking of 6 778 070 000 as 6 778 millions.

The answer to **Activity One**, question **1a**, is found by dividing the number of people living in the developed world by the number of people in the whole world and expressing this as a percentage. Basic calculators may not accept this number of digits. If this is a problem, the students can divide both numbers by 1 000 or 10 000 before they enter them. Encourage them to think about why this works. Link back to previous discussions about really big numbers, for example, global hectares in Ecological Footprints.

Question **1b** requires the students to use their answer for the previous question. If the entire human population were to live like Adena, 3.6 Earths would be needed to support them. But "just" 17% live, like Adena, in the developed countries. 17% x 3.6 = 0.61. So this minority segment of Earth's population consumes the bulk of its resources.

Questions **1c–d** involve simple subtractions, but question **1e** involves a level of abstraction that will challenge most students. They may need help to interpret and answer it. One way of working this out is shown in the answers. Another way of describing the mathematics is: 83% of Earth's population live in developing countries; these people consume resources at the rate of 0.39 Earths; 1% of the population would consume at just $\frac{1}{83}$ this rate (0.39 x $\frac{1}{83}$); 100% of the population ("everyone") would consume at 100 times this rate (0.39 x $\frac{1}{83}$ x 100 = 0.47 Earths).

Questions **2–3** compare the rule of thumb (Pareto's rule) with the data from question **1**.

The diagram at the top of page **7** of the students' book illustrates the ratios used in this activity. People in developed countries have a large eco-footprint; those in developing countries have a much smaller footprint. Compared with developing countries, there are relatively few people in the countries that are developed. The diagram invites questions such as: *What happens when a small number of people have a large footprint? What happens when a large number have a small footprint? Which is large: a lot of small amounts or a few big amounts?*

The answer to the last question is "It depends on the actual figures." This can be easily demonstrated with pairs of examples such as $\frac{1}{5}$ of 40 and $\frac{3}{4}$ of 8. In this case, the small fraction of the larger number is greater (8 versus 6), but with $\frac{1}{5}$ of 40 and $\frac{3}{4}$ of 12, the large fraction of the smaller number is greater (8 versus 9).

Points of entry: Science

Many of the resources that sustain our lifestyle are finite, and even renewable resources (for example, fresh water, food, and wood) are often only available in strictly limited supply. These resources are typically unevenly distributed and unequally shared. A graphic illustration of the inequities in resource use is "the world seen in terms of use of resources". (See the map URL in Related information above.)

Compare this map with a globe and focus on countries that the students are familiar with, for example, New Zealand, Australia, the Pacific, and other countries of students' births. Discuss the shape and size of these countries as they appear in this map. Ask *Why does New Zealand appear to be its correct size and shape but Australia looks smaller, even though both are developed countries that use a large per-person share of Earth's resources?* (Encourage them to think about population density.)

The students could research the area of land used by people in different countries, developed and developing. The pictures at the bottom right of page **7** of the students' book depict graphically the difference between the amount of land required to support 1 New Zealander and that for 1 person from a developing country. You could use them as a starting point for discussion on diet and lifestyle. (For example, in New Zealand, we have become used to lots of space. We like plenty of personal space at home [including our own bedroom, if possible] and expect to be able to "get away from it all" on holiday. We have so much space outside cities and towns that we can allow animals such as sheep and cows to occupy huge parts of it!) The students could select a densely populated developing country, do a point-to-point comparison, and present their findings to the class. Have them discuss whether their investigation suggests actions that they could take to make their lifestyle more sustainable.

Mathematics and Statistics Achievement Objectives

- Measurement: Use appropriate scales, devices, and metric units for ... temperature ... and time (Geometry and Measurement, levels 3–4)
- Statistical investigation: Plan and conduct investigations using the statistical enquiry cycle:
  - determining appropriate variables and data collection methods
  - gathering, sorting, and displaying multivariate category, measurement, and time-series data to detect patterns, variations, relationships, and trends
  - comparing distributions visually
  - communicating findings, using appropriate displays (Statistics, level 4)

Mathematics standards. The approaches and thinking that students demonstrate as they engage with these tasks and problems can provide evidence in relation to the mathematics standards.

Science Achievement Objectives

- Investigating in science:
  - Build on prior experiences, working together to share and examine their own and others' knowledge
  - Ask questions, find evidence, explore simple models, and carry out appropriate investigations to develop simple explanations (Nature of Science, levels 3–4)
- Communicating in science: Begin to use a range of scientific symbols, conventions, and vocabulary (Nature of Science, levels 3–4)
- Participating and contributing:
  - Use their growing science knowledge when considering issues of concern to them
  - Explore various aspects of an issue and make decisions about possible actions (Nature of Science, levels 3–4)
- Physical inquiry and physics concepts: Explore, describe, and represent patterns and trends for everyday examples of physical phenomena, such as ... light ... and heat (Physical World, levels 3–4)

Mathematics and statistics context

Students will:

- collect and display times-series data
- use a scale to read temperature in degrees Celsius
- interpret time-series graphs and use them for reasoned predictions
- draw a trendline (line of best fit).

Students should discover that:

- graphs are an important means of showing relationships and trends
- the shape of a graph is based on the real-world data it depicts.

Science context

Students will:

- carry out an investigation to find out the relationship between time in sunlight and the temperature rise of water
- design their own investigation to test their ideas about increasing the rate of temperature rise
- use their findings to design a solar shower.

Students should discover that:

- sunlight heats water, but the rise in temperature slows and reaches a maximum (how long this takes is a function of the conditions, for example, wind direction, shade, and ambient temperature)
- the rate at which the water heats up can be increased by enlarging the area exposed to the sun and using different colours and materials.

Related information

Building Science Concepts: Book 36, *Heat on the Move*; Book 46, *Keeping Warm*; Book 47, *Insulation*.
Making Better Sense of the Physical World: Heat, pp. 55–64

Answers

Activity

1. i.–iv. Practical activity

2. a. Results will vary, but your graph should show the temperature of the water increasing over time.

   b. Find 2 points on the graph that are 5° apart and then find the time difference between them. For example, if the temperature was 17° at 10 minutes and 22° halfway between 20 and 30 minutes, it probably takes about 15 minutes to increase by 5°. If possible, use several sets of points on the graph to find a consistent or average time interval because the rate of increase is likely to change as the water gets warmer.

   c. Extend the trend to 60 minutes to estimate the temperature after an hour in sunlight. For example, draw a trendline through the middle of your points in the direction that the graph is heading and see what temperature it crosses at 60 minutes on the time axis:

Water Temperature

23°C

Degrees Celsius

20

15

0   10   20   30   40   50   60

Minutes
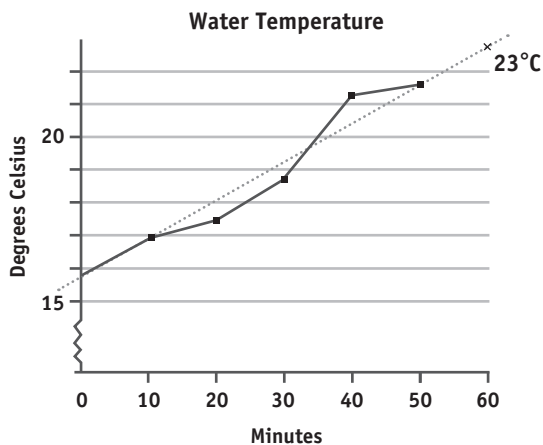
   d. The line on your temperature graph should flatten out to look something like this over 3 hours:

No matter how long the hose is left in sunlight, the water is not going to get anywhere near boiling point (100°). Eventually, the increase will stop and the water will start radiating heat.

3. Ideas for heating the water faster might include: using smaller or larger hoses, putting the hoses on concrete, putting the hoses on a mirror or reflective foil, putting the hoses in an open box (or one covered with glass) with reflective or insulated sides, putting the hoses together or spreading them out, coiling the hoses, using hoses of a different colour.

   a.–b. Practical activity. Results will vary.

   c. Use your evidence to determine which ideas worked better than others and how these ideas might be incorporated into your shower design.

4. a. Sketches will vary; make sure you label the ideas you include.

   b. Estimates will vary, but they should be based on your earlier experiments.

   c. Your maximum temperature should probably be hotter than that obtained in your experiment. Solar showers sold in stores can heat water to about 70° on a 25° day. In fact, the water can get so hot that it scalds, but it won't get to boiling point (100°).

   d. Your graph should be similar to the graph of your experimental data, but as you are combining your best ideas, hopefully you should expect an even better outcome. This will be reflected in a graph that ends at a higher point.

5. Practical activity

Preparation and points to note

This activity requires a sunny day. If more than one group is involved, you will need multiple sets of materials.

Tables and graphs are ways of representing information and ideas and, as such, are part of the key competency *using language*, *symbols*, *and texts* that can be applied to this activity.

The results of the investigation will be influenced by the conditions at the time, for example, cloud obscuring the Sun for a time or an occasional cool breeze. For this reason, the students should keep a record of any changes in conditions and take account of this information when evaluating their results and comparing their findings with those of other groups. Before they begin, they should plan how they will record and organise their data. If possible, the data should be transferred to a computer spreadsheet program.

Additional resources are likely to be needed for the second part of the experiment; these should be the students' responsibility, so they will need to plan ahead.

Be aware that although the water won't get to boiling point, it may get very hot. Solar shower manufacturers warn that water can reach 70º in a couple of hours.

Points of entry: Mathematics

The initial phase of this activity involves students in gathering measurement data. Make sure that your students are able to read the scale, including any gradations between the numbered units. They will need to be as quick and as accurate as possible when taking measurements.

Question **2** asks them to create a graph of their data. This should be a time-series graph, something like the one on page **8** of the students' book. If the students have entered their data into a spreadsheet, they can create the graph using the computer. As long as the measurements have been done well and there have been no changes in the conditions, the graph should be a smooth curve, not a straight line. It is important that the students "read" the story their graph is telling them and, if it looks erratic, that they explain the likely causes.

Question **2d** asks the students to predict what would happen if they continued to take measurements for a full 3 hours. Check their graphs to make sure that they realise the water won't get hotter and hotter indefinitely. Ask them to try and explain why it won't.

Question **3** involves making a change and seeing what the impact of this change is on the rate of heating and the temperature of the water in the hoses. If the students enter their data for this second experiment into the spreadsheet used for question **2**, they will be able to graph both sets of data (series) on the same axes and easily compare them. The important thing is to relate the effect of the experimental change to the shape of the graph. For example, the use of reflective material should increase the slope of the graph and may give it a higher maximum.

Points of entry: Science

Discuss how the properties of different materials conduct, radiate, and reflect heat or insulate against temperature change. Ask *How do the colour and transparency (of the hose material) influence heat absorption?* Have them investigate how radiant heat travels through the vacuum of space to Earth and the different ways in which heat energy is transferred (conduction, radiation, convection).

Reinforce the importance of method and the risk of experimental error. There are numerous potential sources of error that may invalidate conclusions or make them harder to reach. For example, unequal quantities of water in the hoses, thermometers left in sunlight, hoses entering shade as the Sun moves across the sky, clouds obscuring the Sun, some hoses getting more or less than their 10 minutes due to slowness or carelessness. The second experiment will necessarily take place at a different time of day and/or under slightly different conditions. The students should try to control the experiment and account for changing conditions as best they can.

Encourage the students to think about why the investigation has been set up with 5 equal lengths of hose. Ask *What would be different if you were to keep taking temperature readings from a single length?*

When the students compare notes, they should find that some practices are much more effective than others, for example, using narrow diameter black hose in a box lined with reflective material and covered with a well-sealed glass lid. Although each student group tested only 1 idea, for question **4** they can combine changes to get the most effective solar shower. The students' designs for a solar shower should incorporate ideas that have been shown to improve heat gain. The estimates in question **4** will necessarily be speculative, but they should be grounded in the experimental data that the students have available. They may be interested in making a prototype of their solar showers. This could become a joint mathematics, science, and technology project.

Pages 10–11: Invisible Resources

Mathematics and Statistics Achievement Objectives

- Number strategies and knowledge: Find ... percentages of amounts expressed as whole numbers ... (Number and Algebra, level 4)
- Measurement:
  – Use appropriate scales, devices, and metric units for ... volume and capacity ... and time (Geometry and Measurement, level 4)

Mathematics standards. The approaches and thinking that students demonstrate as they engage with these tasks and problems can provide evidence in relation to the mathematics standards.

Science Achievement Objectives

- Investigating in science:
  – Build on prior experiences, working together to share and examine their own and others' knowledge
  – Ask questions, find evidence, explore simple models, and carry out appropriate investigations to develop simple explanations (Nature of Science, levels 3–4)
- Life processes: Recognise that there are life processes common to all living things and that these occur in different ways (Living World, levels 3–4)

Mathematics and statistics context	*Science context*
Students will:	Students will:
• perform calculations with percentages involving time and volume.	• investigate the gas composition of their breath
	• use evidence from their investigations to draw conclusions about the process of breathing.
Students should discover that:	Students should discover that:
• one ratio (for example, burn times) may be used to discover another (for example, relative amounts of gases in air).	• a candle will burn for a longer time in a high-oxygen environment and a shorter time in a low-oxygen environment
	• there is less oxygen in the air we breathe out than in the air we breathe in.

Related information

Building Science Concepts: Book 64, *Candles*; Book 39, *Is This an Animal?*; Book 35, *Is This a Plant?*

Making Better Sense of the Living World: Structure and Function, pp. 43–76

Composition of gases in air:
www.engineeringtoolbox.com/air-composition-d_212.html

From http://science.howstuffworks.com/question83.htm: "Exhaled air contains about 4.5 percent carbon dioxide."
In other words, we convert about 4% of air (or 20% of the oxygen in air) into CO_2 by breathing.

Interesting background reading on breathing and CO_2:
www.abc.net.au/science/articles/2005/06/15/1392360.htm

Activity

1. **a.** Practical activity. Answers will vary according to the size of the jars and amounts of reactants used.

    b. The candle needs oxygen to burn, so it should burn longest in the high-oxygen atmosphere. In the low-oxygen, high-carbon-dioxide (baking soda and vinegar) atmosphere, the candle should go out much more quickly.

    c. Estimates will vary depending on how long the candles burn. (It's hard to get accurate data.)

    If, for example, the candle burned for 40 seconds (s) in the normal atmosphere (21% oxygen), 44 s in the high-oxygen atmosphere, and 32 s in the low-oxygen, high-carbon-dioxide atmosphere, you could estimate the different percentages in this way:

    - In the high-oxygen atmosphere, the candle burns 10% longer (44 s instead of 40), which suggests that this atmosphere contains 10% more oxygen. 110% of 21% (the oxygen in normal air) is 23%.

    - In the low-oxygen, high-carbon-dioxide atmosphere, the candle burns for 80% of the time that it burns in the normal atmosphere (32 s instead of 40 or $\frac{32}{40} = \frac{4}{5} = 80\%$), which suggests that this atmosphere contains only 80% of the oxygen in normal air. 80% of 21% = 17%.

2. **a.** Practical activity. The candle should burn out in about the same time as in the low-oxygen atmosphere, but this will depend on how successful you have been at replacing the normal air with your breath. Note that the water has no impact (it doesn't add any significant CO_2 or O_2 to the atmosphere by itself). It and the container are there as a control, to occupy the same air space as the container and contents in the other parts of the experiment.

    b. Breath is low in oxygen and high in carbon dioxide, so the burn time should be most similar to that in the low-oxygen experiment. If it is not, your attempt to replace the normal air with your breath was not very successful!

Extension

Practical activity. You could estimate the amount of oxygen converted into carbon dioxide by comparing the time the candle burned in the normal atmosphere with the time it burned in the atmosphere provided by your breath.

For example, if the candle burned for 40 s in the normal atmosphere (21% oxygen), and only 28 s using your breath, you could estimate like this:

"Using my breath, the candle burned for $\frac{28}{40} = 70\%$ of the time that it did in the normal atmosphere, which suggests my breath contained just 70% of the 21% oxygen present in normal air. 70% x 21 is approximately 15%.

If normal air contains 21% oxygen but my breath contains only 15%, then 21 – 15 = 6% of the air has been converted to other gases. If that 6% is completely converted to carbon dioxide, my breath must contain about 78% nitrogen, 15% oxygen, 1% argon, and 6% carbon dioxide."

Research has found that our breath typically contains 4–5% carbon dioxide, so if your estimate is around 5%, you've done well!

Preparation and points to note

It's a good idea to do the activity yourself first so that you will know what is likely to happen when the students do it. It's also a good idea to work through the calculations in question **1c** and the **Extension** beforehand to familiarise yourself with the thinking involved and work out how you can scaffold the mathematics for your students.

The science language and concepts in this activity will probably be new to students: exposure to different experiences is one of the aims of the key competency of *using languages*, *symbols*, *and texts*.

To compare the time the candles take to go out, the jars and the candles need to be the same size.

Review safety procedures with your students before doing any experiment involving fire or hydrogen peroxide. Carefully demonstrate safe handling and candle-lighting procedures. Emphasise care with the peroxide: it can affect the skin and eyes.

If possible, use high-concentration hydrogen peroxide from a pharmacy. Lower strength versions used as bleaches will inevitably produce less oxygen.

Points of entry: Mathematics

The table on page **10** of the students' book provides an excellent context for furthering students' understanding of percentage. They know that 100% is "the whole"; ask *What is the whole in this case?* This is always a crucial question when working with percentages; as the students work through this activity, "the whole" changes. In the table, the whole is "(normal) air"; in question **1c**, the whole becomes "oxygen content of normal air (21%)", then "burn time in normal air (in the example, 40 s)", then back to "oxygen content of normal air (21%)"; in the **Extension**, the whole moves from "burn time in normal air" to "oxygen content of normal air" to "normal air".

As the calculations in question **1c** and the **Extension** involve proportional reasoning, students who are not at stages 6 or 7 of the Number Framework will need careful scaffolding. See the answers for suitable approaches.

To successfully carry out this experiment, the students will need to measure time accurately and record their data. Accurate timing is likely to require good teamwork.

Points of entry: Science

It is not possible to get precise results from a simple experiment such as this, which is why the students are asked to estimate, not calculate, the answers.

The experiment involves relative (not absolute) quantities, so set expectations appropriately. Results are likely to vary widely. What matters is that the students discover that their breath has more CO_2 in it than the normal atmosphere rather than that they determine the actual percentage.

There are numerous potential sources of error that will affect the results. For example, because CO_2 is heavier than oxygen, some of the oxygen may not be consumed: CO_2 may accumulate in the bottom while unburned oxygen accumulates at top. Discuss experimental error and the principle that results should be repeatable. Carrying out multiple trials will increase the reliability of data and the accuracy of estimates. (The answer for question **2a** explains why a container with water is used in the "using air you have breathed" experiment.)

Use this as an opportunity to discuss the fact that all living things need oxygen to survive. It's a common misconception that animals produce carbon dioxide and plants produce oxygen. In fact, all living things respire and convert oxygen to carbon dioxide. But additionally, plants photosynthesise, a process that uses carbon dioxide to produce sugars and other chemicals for growth. Oxygen is a by-product of photosynthesis.

When a candle burns, oxidation takes place: oxygen combines with the hydrocarbon fuel that makes up the candle to produce carbon dioxide and water. This may cause moisture in the jar.

Discuss the reactions that take place and where the gases come from. You could make important links to the idea of conservation of matter.

Mathematics and Statistics Achievement Objectives

- Number strategies: Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages (Number and Algebra, level 3)
- Number strategies and knowledge:
  - Use a range of multiplicative strategies when operating on whole numbers
  - Find fractions, decimals, and percentages of amounts expressed as whole numbers, simple fractions, and decimals (Number and Algebra, level 4)
- Statistical investigation: Plan and conduct investigations using the statistical enquiry cycle:
  - determining appropriate variables and data collection methods
  - gathering, sorting, and displaying multivariate category, measurement, and time-series data to detect patterns, variations, relationships, and trends
  - comparing distributions visually
  - communicating findings, using appropriate displays (Statistics, level 4)

Mathematics standards. The approaches and thinking that students demonstrate as they engage with these tasks and problems can provide evidence in relation to the mathematics standards.

Science Achievement Objectives

- Investigating in science:
  - Build on prior experiences, working together to share and examine their own and others' knowledge
  - Ask questions, find evidence, explore simple models, and carry out appropriate investigations to develop simple explanations (Nature of Science, levels 3–4)
- Participating and contributing:
  - Use their growing science knowledge when considering issues of concern to them
  - Explore various aspects of an issue and make decisions about possible actions (Nature of Science, levels 3–4)

Mathematics and statistics context

Students will:

- perform calculations with percentages
- gather and interpret statistical data.

Students should discover that:

- they can create a statistical map that shows the air pollution of different parts of the school environment.

Science context

Students will:

- gather data about particulates in air
- predict and verify where air pollution occurs most.

Students should discover that:

- most air pollution is caused by motor vehicles.

Related information

A good discussion of how to detect air-borne particles using white petroleum jelly:
www.education.com/reference/article/Ref_Counting_Air/?page=2

Background information on air pollution for students:
www.windows.ucar.edu/tour/link=/earth/Atmosphere/airpollution_intro.html&edu=elem

Statistics on asthma in New Zealand:
www.asthmanz.co.nz/in_new_zealand.php

Activity One

1. Practical activity. You should notice some soot on the mirror or mug.

2. **a.** About 460 litres (L). (11 000 L ÷ 24 hrs = 458.3 L)

   b. About 640. (58% of 1 100 = 638)

   c. About 60. (13% of 450 = 58.5)

   (Answers have been rounded to reflect the approximate nature of the given statistics.)

Activity Two

1. **a.–b.** Answers will vary, but pollution is likely to be greatest in locations close to busy roads, sources of dust (such as an unsealed road), or plants or trees (such as pines) that are releasing lots of pollen.

2. Practical activity. Predictions will vary, but you should have good reasons for them. For example, it's likely that a collector next to a heating vent will have more particles on it than a collector hung in a clean, unused cupboard.

3. **a.–c.** Results and comments will vary.

Notes

Preparation and points to note

Ensure that protocols are clearly established and proper safety procedures in place before doing any experiment that involves lighting candles. In this investigation of air pollution, the students need to make sense of information, draw on their own experiences, and explain their ideas, all of which use and further develop the key competency *thinking*.

For the demonstration in **Activity One**, a ceramic tile or a plain white coffee mug is a good substitute for a mirror.

For **Activity Two**, you will need a reasonably accurate map of the school. If one is not already available, use a satellite view from, for example, Google Earth, Google Maps, Terralink, or your local body website.

The collectors should hang undisturbed for about a week. Decide on a good time of year, and alert the rest of the school to the investigation and the importance of leaving the collectors undisturbed. Work with the students to find ways to protect the collectors from rain, accidental removal, or interference.

Note that pollution is not always visible to the naked eye. Make sure the students understand that even if they don't see pollution, it may be there (the candle demonstration will hopefully make this clear). A common misconception is that only those cars that are emitting black smoke are polluters. Cars produce at least some carbon monoxide and nitrous oxide even if recently tuned, and landfills emit methane (another invisible greenhouse gas).

Particulate pollution (and experimental results) will vary depending on the location in the school, weather conditions, and the time of year. For example, the presence of pollens and other naturally occurring particles is very seasonal, as is the operation of the school boiler. Minute particles can travel a long way from their source, depending on wind direction and air currents.

Points of entry: Mathematics

Relate the statistics on page **12** of the students' book to the students themselves. For example, ask: *How many students in the class suffer from asthma or hay fever? How does this compare with the Auckland data? Would the area in which our school is situated be likely to have more or less vehicle-generated pollution than Auckland?*

Discuss the application of rates (for example, "13 out of every 100") and rules of thumb (for example, Pareto's rule). Ask: *Can we be sure that 58.5 students in an Auckland school of 450 have asthma?* (No.) *So what is the value of a statistic like this?* Statistics such as this enable us to make reasonable estimates of incidence, but they give no certainty – for example, a small Auckland school might be entirely asthma-free (while very unlikely, this is not impossible).

Reliable statistical rates are based on large-scale surveying/sampling. Relate this to the present activity. Sampling is necessary because it is impractical to collect pollution data for every square centimetre of the school. But the sampling is small-scale and the methodology fairly crude, so the results will be tentative and limited in their application.

Ideally, this activity would be done as a whole-class activity. If so, challenge the students to come up with a way of systematically parcelling out the territory to be covered. They should also be challenged to predict the areas of the school that will have the least and greatest particulate counts. They should record these predictions.

Students will need to use their estimation skills when comparing and matching samples. Different students may judge samples differently, so you may wish to do a calibration exercise or have a group of students agree on each reading.

When the results from all groups have been collected and collated, the students could make a "heat" map in which high-pollution areas are coloured red and low-pollution areas blue. Ask: *What are the implications, if any, of your findings? Who could be interested in a presentation of these findings?*

Points of entry: Science

Ask the students where and when they have been able to observe air pollution. Have them brainstorm probable local sources of pollution. (Particulate air pollution will be heaviest near heavy traffic, machinery exhaust outlets, or close to naturally occurring sources, such as wheat fields, orchards, and flowering plants.)

Highlight what particles are and their sources. In **Activity One**, question **1**, the candle should appear to be non-polluting when burning in open air, but it should blacken the mirror, mug, or tile when held over the flame. The students should observe soot; if the tile or mirror is cold, they should be able to observe condensation as well – incomplete combustion creates the soot and carbon monoxide. (Complete combustion, yielding only carbon dioxide and water, is almost impossible to achieve.)

This activity reinforces the use of the scientific method for testing predictions. Encourage your students to think logically about where and why they will place the collectors. They should be placed in a variety of locations: inside, outside, near pollution sources (pollen-shedding trees, heater vents, car exhausts), and in clean areas. Confirm that they will be suitably sheltered (weather conditions may be a factor!) and secure for a week. Also, make sure that there is a control collector in a place that is isolated and still (perhaps a classroom cupboard). If the students are very motivated, suggest they take some collectors home and use them to gather additional data.

Have the students test the predictions against the data and link the findings back to the original ideas: *Why is site X the most polluted?* Discuss the results as a class. Ask: *Are there actions that you can or should take? Do you need to do further research first?*

Mathematics and Statistics Achievement Objectives

- Number strategies: Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages (Number and Algebra, level 3)
- Number strategies and knowledge:
    - Use a range of multiplicative strategies when operating on whole numbers
    - Find fractions, decimals, and percentages of amounts expressed as whole numbers, simple fractions, and decimals (Number and Algebra, level 4)

Mathematics standards. The approaches and thinking that students demonstrate as they engage with these tasks and problems can provide evidence in relation to the mathematics standards.

Science Achievement Objective

- Participating and contributing: Use their growing science knowledge when considering issues of concern to them (Nature of Science, levels 3–4)

Mathematics and statistics context	*Science context*
Students will:	Students will:
• extract appropriate data from tables	• analyse traffic data
• perform calculations with percentage and rate data	• use national statistics to make reasonable estimates in a local context.
• convert between percentages and absolute quantities.	
Students should discover that:	Students should discover that:
• significant amounts of time and money are wasted in traffic jams.	• simple changes, like improved public transport and carpooling, can have a big impact on traffic.

Related information

Detailed information for how people in New Zealand get to work is available from Statistics New Zealand:
www.stats.govt.nz/methods_and_services/schools_corner/activities/primary/teacher-how-do-people-get-to-work.aspx

Data on United States delays sourced from:
http://mobility.tamu.edu/ums/congestion_data/

Information on the cost and pollution effects of traffic jams:
www.edf.org/documents/9236_Idling_Nowhere_2009.pdf

Sustainable traffic management:
www.newurbanism.org/Frame-496683-sustainabilitypage496683.html?refresh=1250098301875

Answers

Activity One

1. **a.** 16 710 people. ($\frac{6}{100}$ x 278 500)

    b. 172 670 people. ($\frac{62}{100}$ x 278 500)

    c. Up to 86 335 cars. (172 670 people are driving. If each driver drove 1 other driver, there would be half as many cars.)

2. **a.** 12.6%. (35 000 ÷ 278 500 x 100)

    b. The national percentage for workers who used public transport was 6%. In the Wellington region, the percentage was 12.6%, which is more than double. This suggests that Wellington may have more to offer in terms of public transport options than other major cities (for example, Wellington's train system is bigger than Auckland's, and Christchurch does not have trains). Those who live outside the major cities, particularly those who live in rural areas, often have no realistic alternative to driving when it comes to getting to work.

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Activity Two

1. **a.** Los Angeles

    b. 84 hours. (1.2 x 70. In other words, 70 hrs plus 20% more, which is 120% of 70)

    c. If each year the time increases by 20%, that would be 84 hrs in 2008, 101 hrs in 2009 (1.2 x 84), 121 hrs in 2010 (1.2 x 101), and 145 hrs in 2011 (1.2 x 121). To find the amount of time for any year, just multiply the previous year's figure by 120% or 1.2. (Note: the hours for each year have been rounded to the nearest whole number.)

2. **a.** Answers will vary depending on the city but will be the number of commuting hours ÷ 8. For example, for New York, 44 ÷ 8 = 5.5 workdays.

    b. Answers will vary depending on the city but should be that city's hours from the table doubled (for the $2) and multiplied by 3.5 (for the litres). For example, for New York, 44 x 2 x 3.5 = $308.

Notes

Preparation and points to note

A useful introductory exercise is to discuss as a class the different ways that people travel to work and the pros and cons of these alternatives.

Decide in advance whether your students should work individually or in groups or pairs. Ask them to identify in the two tables the information they need to answer the different questions. No matter how they work, encourage them to discuss their results with other students. This will reinforce the key competency *thinking* by providing opportunities for students to learn from their mistakes, improve their processes, and refine their conclusions.

Points of entry: Mathematics

This activity approaches percentages and rates from three slightly different angles: (i) given a percentage, calculate the number/quantity; (ii) express one number as a percentage of another; and (iii) calculate a compounding increase (using repeated multiplication). Note that in **Activity Two**, question **1b-c**, the students need to multiply by 120% per year. This is very different from **Activity One**, where they are calculating a single percentage or quantity.

Reinforce the key understanding that percentage is "parts out of 100" by, for example, asking the students to add up the figures in the table on page **14** of the students' book. Also reinforce that percentages are fractions: $75\% = \frac{75}{100} = \frac{3}{4} = 0.75$, $120\% = \frac{120}{100} = \frac{6}{5} = 1.2$, and so on. The usefulness of the percentage form lies in its standardisation: because the denominator is always 100, it is very easy to compare the relative size of different percentages. Once students have this equality clear, they can be weaned off the practice of writing a percentage as a number over 100 and encouraged to use the decimal form that is so convenient in most percentage calculations. (Double strip diagrams are a useful tool for teaching the meaning of percentage [see NDP *Book 7: Teaching Fractions, Decimals, and Percentages*]. Another access point is using key percentages such as 10%, 1%, 50%.)

Note that **Activity Two** requires students to equate different types of unit (hours, workdays, litres, and dollars). Ask: *Which of these units gives you the best idea of the cost of traffic jams? Why?*

Points of entry: Science

This activity highlights what happens when people overuse a cheap resource (in this case, roads): overuse always generates costs, although these costs are not always obvious or taken into account. Encourage the students to reflect on the time, money, and fuel that is used to no purpose. Relate their comments from **Activity One** to the answers in **Activity Two**.

This activity makes a very suitable transition to the next activity, Common Pasture. Drivers share the available road just as calves share the available pasture.

Ask your students to think about why people often choose to drive their car instead of using the public transport options that may be available and to think creatively about solutions to traffic congestion. The focus for this could be a map of the area in which the students live. They may feel that there is no alternative to a car. If so, ask them to project themselves into a future where petroleum resources have been exhausted and no suitable substitute has been found. Ask *How would we have to change the way we live?*

Another approach would be to compare the different scenarios of a city that grows up around a well-designed public transport system and the city that tries to retrofit a good public transport system after having developed without one.

The questions in **Activity One** illustrate that change is possible; they ask students to compare Wellington's public transport usage with the national average and to think about the impact of carpooling on the number of vehicles. You could extend this to a consideration of sustainability and how we could reduce our use of resources overall.

Pages 16–17: Common Pasture

Mathematics and Statistics Achievement Objectives

- Number strategies: Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages (Number and Algebra, level 3)
- Number strategies and knowledge: Use a range of multiplicative strategies when operating on whole numbers (Number and Algebra, level 4)

Mathematics standards. The approaches and thinking that students demonstrate as they engage with these tasks and problems can provide evidence in relation to the mathematics standards.

Science Achievement Objectives

- Participating and contributing:
    - Use their growing science knowledge when considering issues of concern to them
    - Explore various aspects of an issue and make decisions about possible actions (Nature of Science, levels 3–4)
- Life processes: Recognise that there are life processes common to all living things and that these occur in different ways (Living World, levels 3–4)

Mathematics and statistics context	*Science context*
Students will:	Students will:
• use a variety of multiplicative and additive strategies when playing the game	• relate a game about pasture to the sustainability of resource use on Earth
• use mathematical knowledge to strategise and predict how to win the game.	• discuss how resources should be managed to ensure continuation of the species.
Students should discover that:	Students should discover that:
• social games in which one player's actions affect everyone require multiple-move strategies and predictions such as tit-for-tat or co-operation.	• competition for resources impacts on sustainability
	• there is an optimum population that a certain environment can sustain and a threshold of resource use above which a species will not survive
	• life cycles and the length of time to reach maturity will also affect survival as a species if a population is under threat, for example, by over-fishing.

Related information

A description of the tragedy of the commons applied to fish stocks:
www.textbookleague.org/34cmmns.htm

A variation on the commons game, using M&Ms:
www.heart.siu.edu/Modules/Module1_Commons

Answers

Game

A game of strategy

Activity One

1. **a.–b.** Answers will vary. In general, the way to win is to raise more calves when other players are raising fewer and to choose only a few more calves than the others so that the return per calf doesn't fall below the profit line.

   c. If most of the players chose to have lots of calves, there was not enough food to go around and all the animals lost value.

2. **a.** Comments will vary. Winning strategies will involve thinking about what is working and why and planning ahead. If you choose too few calves, other people can have more and you are effectively subsidising them. If everybody chooses too many calves, no one will make a profit and everybody may make a loss.

   b. Based on the market table, a total of either 6 or 8 calves gives the largest payout for the group as a whole.

6 x $300 = $1,800. $1,800 – $600 = $1,200;
8 x $250 = $2,000. $2,000 – $800 = $1,200.
(For 7 calves, it's 7 x $250 = $1,750.
$1,750 – $700 = $1,050)

Activity Two

1. **a.** Fish stocks will be depleted if the fish are caught in large numbers. For some species, fish stocks can decline to the point where they can't replenish themselves. Eventually, there would be no fish of that species to catch.

   b. Fishermen could make sure that they don't take more fish than can quickly be replaced. In New Zealand, fishermen are bound by a government quota that limits the amount of fish they can take. They can trade portions of their quota, but they can't exceed it.

2. Discussion will vary. Points may include people only taking enough resources to cover their needs (for example, seafood such as paua), customary fishing rights, and the role, rights, and obligations of tangata whenua.

Notes

Preparation and points to note

Play the game before you introduce it to your students. There are subtleties of strategy that can make it quite competitive. For example, if one player consistently chooses to farm lots of cows, they can ruin the outcome for everyone. Establish any ground rules and expectations relating, for example, to volume and interactions (*It's just a game!*). Consider asking the students to try playing the game in silence so that they can't co-operate or collude.

For **Activity One**, group and regroup the students strategically so that they are exposed to a variety of examples and are asked to justify their thinking to a variety of classmates. In this way, you are deliberately making the students *relate to others*, the key competency in which students recognise different points of view and share ideas.

Note that 3 "farmers" are less likely to overgraze the pasture than larger groups; the game works best if they have 4, 5, or even 6 players – the students are more competitive, and the results are more varied and provide scope for more discussion.

If, in their first game, the students have trouble grasping the steps (for example, forgetting to subtract the cost of the calves), you might consider working through a few rounds of the game as a class, scoring hypothetical or random numbers of calves. In this way, the rules will become clearer to players, but it still leaves them scope to find out for themselves what happens in various scenarios. If you don't model the game, you should go through the rules carefully and answer questions about the rules or procedures. Don't answer questions about what will happen.

In **Activity Two**, the learning from the game is applied to fish stocks. Students may not understand that, like all animals, fish take time (sometimes a long time) to mature and reach breeding age. For this reason, the consequences of current fishing activity may become apparent only with the passage of time.

Points of entry: Mathematics

This game requires lots of multiplication and addition. If you decide to model the game to the whole class, you can discuss number strategies that are likely to be useful. Later, as well as asking the students what conclusions they have reached, ask them to explain how they did the mathematics. *Did you, for example, subtract the cost of each calf from its selling price before multiplying, or did you calculate the profit and cost separately?*

Once the students have played the game long enough for patterns and strategies to emerge, ask groups to stop and discuss what they have found. They can then share their insights with the whole class. Press for understanding: *Did you discover a strategy that always worked to your advantage? Did you discover a strategy that maximised the return for the group as a whole? Under what circumstances does choosing a large number of calves turn out to be a good move?* (Only if the others choose small numbers)

Different-sized groups function differently and require different strategies. For example, in a 5-player game, one player will always either make less money or cause overgrazing. Challenge the students to discover and describe these patterns and strategies.

Once the game is thoroughly understood, the students can experiment with variations designed to regulate the market and discourage unfair or excessive behaviours. For example, to investigate the effect of transfer payments, have the students play the game again, but this time if a student decides to raise no calves, the person who raises the most has to pay them $100. Ask *What impact is there if sanctions are imposed on the player who overgrazes the most?*

The students can also investigate the effects of both collusion and co-operation. Ask *Can subgroups secretly pursue a strategy designed to shut others out of the market?* You could challenge same-sized class groups to compete to create the greatest collective profit.

Those who become fascinated with this game may like to look at other classics of game theory, such as the Prisoner's Dilemma (see Related information).

Points of entry: Science

The Tragedy of the Commons (as this activity is best known) is a much-discussed scenario that has wide application to the use of resources. It is applicable in such diverse contexts as fish stocks (as in **Activity Two**), traffic delays (if every vehicle slows just slightly to look at an accident, a jam soon ensues), or the Aral Sea (a textbook tragedy, see Related information).

This activity provides scope for a lot of student-generated thinking and investigation. Your role is to ensure that the connections are made between the game, limited resources, and human activity. The game models a situation in which, if people take more than their share, they gain a short-term personal advantage at the cost of others; ultimately, their unsustainable behaviour impacts on everyone. Ask your students to consider whether the game also models situations in which people "put in" instead of take out. For example, ask *What if everyone were to put as much carbon dioxide as they wanted into the atmosphere?* (We would reach a tipping point for climate change, just as, in the game, there is a tipping point between calf weight gain and starvation.)

This activity should lead to discussion on the general issue: *How, as a nation, can we use resources responsibly while allowing people to earn a living?* Discussion should also come back to the personal: *So what should I do?*

Pages 18-19: Clean Enough to Drink?

Mathematics and Statistics Achievement Objectives

- Number strategies: Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages (Number and Algebra, level 3)
- Number strategies and knowledge:
  - Use a range of multiplicative strategies when operating on whole numbers
  - Find fractions, decimals, and percentages of amounts expressed as whole numbers, simple fractions, and decimals (Number and Algebra, level 4)
- Measurement:
  - Use appropriate scales, devices, and metric units for ... volume and capacity ... (Geometry and Measurement, level 4)

Mathematics standards. The approaches and thinking that students demonstrate as they engage with these tasks and problems can provide evidence in relation to the mathematics standards.

Science Achievement Objectives

- Participating and contributing:
  - Use their growing science knowledge when considering issues of concern to them
  - Explore various aspects of an issue and make decisions about possible actions (Nature of Science, levels 3–4)
- Properties and changes of matter: Compare chemical and physical changes (Material World, levels 3–4)
- Earth systems: Appreciate that water, air, rocks and soil, and life forms make up our planet and recognise that these are also Earth's resources (Planet Earth and Beyond, level 3)
- Interacting systems: Investigate the water cycle and its effect on climate, landforms, and life (Planet Earth and Beyond, levels 3–4)

Mathematics and statistics context	*Science context*
Students will:	Students will:
• decide on appropriate ways to measure the amount of water lost in **Activity Two**	• use a simple simulation to explore an issue
• calculate percentages of large numbers and percentages of percentages.	• use real data to give meaning to an issue.
Students should discover that:	Students should discover that:
• a small percentage of a large number can itself be a large number	• there is a finite amount of water in the world
• a familiar context can help give meaning to a large number.	• human actions and natural processes can reduce the amount of water that is fit to drink
	• only a small proportion of Earth's water is accessible and suitable for drinking.

Related information:

Building Science Concepts: Book 1, *Waterways*; Book 15, *Where's The Water?*; Book 31, *Water and Weather*; Book 58, *Ice Making Better Sense of the Material World*: pp. 33 and 39–40

Connected 2 1998: "Testing the North River"

Connected 2 2002: "An Interview with a Glass of Water", "The Water Wardens", "The Water Cycle"

Information on distribution of water resources:
http://ga.water.usgs.gov/edu/earthwherewater.html

Water-rich and water-poor areas:
http://whyfiles.org/131fresh_water/2.html

Answers

Activity One

1. Lists will vary. Possible sources include: tap water, stream, river, pond, sea, wetland, rainwater tank, and well. Which of these are safe to drink will also vary (for example, only some streams and rivers are safe to drink from).

2. **a.** Discussion will vary. In general, the water in Antarctica is locked up in ice, snow, and glaciers, and it would be very difficult getting it to where it is needed.

   b. 0.25%. (97.5% of water is salty, leaving 2.5% fresh; 90% of this fresh water is locked up in Antarctica, leaving 10% available; 10% of 2.5% is 0.25% [one quarter of a percent].)

   c. Approximately 529 cubic kilometres (km^3) per person. (For ease of calculation, work in millions: 0.25 % x 1 433 854 549 millions = 3 584 636 million km^3 of potential drinking water; divide this by the world's 2009 population in millions to get 3 584 636 ÷ 6 778 = 529 million km^3 per person.)

Activity Two

1. **a.** Practical activity

   b. Answers will vary. One way to work out (in litres [L]) how much water is lost is: subtract the amount that reached the end from the amount you started with; divide the total amount by the total starting amount and then multiply the result by 100 (to give a percentage). For example, if the class started with 7.5 L and transported just 4 L to the other end, that means 3.5 L was lost. 3.5 ÷ 7.5 x 100 = 46.7% of the water was lost.

2. Discussion should cover reasons relating to distribution, pollution, and wastage. Water may not be available where people live (a distribution issue), filtering and pumping stations may have limited capacity (a wastage issue), holding tanks or dams may not be efficient for conserving water (a wastage issue), water sources may be contaminated (a pollution issue), or water might be used to grow thirsty crops that are not suited to dry areas (a wastage issue). The issue of deforestation might also come up: deforestation can change rainfall distribution patterns.

3. About 976 042 080 people. (18% of 80% [0.18 x 0.8 = 14.4%] of the world's population doesn't have access to clean drinking water. Using the information provided on page **18**, 14.4% of the world's population is 0.144 x 6 778 070 000 = 976 042 080.)

Preparation and points to note

Before starting this activity, consider collecting samples of water from different sources and using these as a basis for initial group and/or class discussion: *What makes water drinkable? Why can't we drink sea water?* Students will have relevant experiences to share and build on. Some may come from countries or areas where drinking water is scarce.

To get the most out of this activity, students will need to work co-operatively. This gives them an opportunity to develop the key competency *participating and contributing*.

The activity involves percentages and large numbers. Students need to bring to it some experience and confidence with percentages.   They need to know, for example, that if 90% of fresh water is in Antarctica, only 10% is to be found elsewhere; that 0.25% is one-quarter of 1%, not 25%; that 15% is entered into a calculator as 0.15; that "80% of …" involves multiplication; and that it is possible to calculate a percentage of a percentage. Calculators are appropriate and necessary.

Students could work collaboratively to identify and extract the information that is relevant for each question.

Points of entry: Mathematics

There are several different usages of percentage in this activity; for example, percentage of a whole, percentage lost, and percentage of a percentage. This highlights the importance of students learning that, when working with percentages, they must pay careful attention to the context. Avoid teaching "rules" about percentage because they encourage students to ignore the context and go straight to the numbers. Discourage use of the percentage key found on some calculators – it is unnecessary and obscures the process you want the students to understand.

Because of the very large numbers involved, many students will find question **2c** in **Activity One** difficult, even if they understand what needs to be done. Most calculators express large numbers using scientific notation (the estimated volume of water on Earth will appear as 1.433854549^{15} km^3), which will create an unnecessary distraction. Using "millions" as the unit avoids this. See the answers for a straightforward calculation and explanation.

Note, however, that saying Earth's population in 2009 was 6 778 millions is not quite the same as saying that it was 6 778 070 000. It would be worth discussing this with your students. Ask *Is such rounding justified?* (Yes. These figures are all very approximate and, anyhow, world population changes by the minute.) Students usually find it difficult to understand that, in some circumstances, a rounded figure (for example, 458 000) might more accurately represent a situation than one with more significant digits (such as 458 437).

A single cubic kilometre of water is a huge amount, so the students will probably conclude that there is plenty of fresh water for everyone. This is mathematically true, but it does not take account of distribution: water is often not where it is needed. Even within New Zealand, the West Coast may be experiencing constant rain while the Canterbury Plains, the Wairarapa, or Gisborne is suffering severe water shortages.

Points of entry: Science

Review the water cycle. Your students may not realise that water can exist in different forms or be purified by relatively simple means.

While fresh water is a renewable resource, access to clean fresh water is steadily decreasing. Have your students brainstorm the reasons an area might run out of drinkable water. These should include changes in weather patterns, over-irrigation, lack of suitable storage, diversion by a neighbouring country, alternating wet and dry seasons (monsoon), deforestation, pollution, urban development that increases run-off, lack of effective distribution network, poverty, corruption, or political turmoil. They should note that most of these reasons stem one way or another from human decision making and activity.

In **Activity One**, the students will work out that, in theory, there is plenty of fresh water for everyone. There are enormous amounts of fresh water in the North American Great Lakes, in the world's great rivers, in huge groundwater aquifers, and in countless smaller rivers and lakes, including those in New Zealand. The problem is that many people live in places where water is not plentiful and/or where there is no effective distribution network to get it to the people who need it.

In **Activity Two**, the students carry out a simple simulation that is designed to demonstrate how easily water is lost in distribution. Relate the findings of this simulation to the earlier discussion on why so many people don't have the water they need.

Pages 20–21: Too Much Water

Mathematics and Statistics Achievement Objectives

- Number strategies: Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages (Number and Algebra, level 3)
- Number strategies and knowledge:
  – Use a range of multiplicative strategies when operating on whole numbers
  – Find fractions, decimals, and percentages of amounts expressed as whole numbers, simple fractions, and decimals (Number and Algebra, level 4)
- Measurement: Use appropriate scales, devices, and metric units for ... volume and capacity ... and time (Geometry and Measurement, level 4)

Mathematics standards. The approaches and thinking that students demonstrate as they engage with these tasks and problems can provide evidence in relation to the mathematics standards.

Science Achievement Objectives

- Investigating in science:
  – Build on prior experiences, working together to share and examine their own and others' knowledge
  – Ask questions, find evidence, explore simple models, and carry out appropriate investigations to develop simple explanations (Nature of Science, levels 3–4)
- Participating and contributing:
  – Use their growing science knowledge when considering issues of concern to them
  – Explore various aspects of an issue and make decisions about possible actions (Nature of Science, levels 3–4)
- Properties and changes of matter:
  – Group materials in different ways, based on the observations and measurements of the characteristic chemical and physical properties of a range of different materials (Material World, levels 3–4)

Mathematics and statistics context	*Science context*
Students will:	Students will:
• collect and accurately measure amounts of water	• investigate which soil type holds most water
• calculate percentages for retention of water.	• draw conclusions about the likelihood of flooding, based on soil type.
Students should discover that:	Students should discover that:
• rates of dry and saturated (wet) water retention are independent of each other.	• different soils have different capacities to retain water
	• saturated (wet) soil holds less water than dry soil
	• some soils require extensive drainage to reduce the risk of flooding.

Answers

Activity

1.–2. Practical activity

3. Answers will vary.

    a. The soil with the *least* amount of water dripped through when dry (probably a clay soil)

    b. The soil with the *greatest* amount of water dripped through when wet (probably a sandy soil)

    c. The soil with the *least* amount of water dripped through when wet is most likely to flood. For example, a clay soil doesn't let much water through (or even none), so water will tend to collect on the surface and flood.

4. If houses are built on a type of soil that drains poorly, extra drains need to be installed. If a house is to be built on very badly draining soil, large-capacity stormwater drainage should be installed and/or the house should be built on raised foundations.

5. Your experiment will have shown you that sand retains water initially (because of the large spaces between the sand particles) but that once saturated, most of the water passes through it. Sandy regions tend to have little natural groundwater or year-round sources of fresh water. In other regions with different soils, water that passes through topsoil collects in the subsoil or lower layers or flows into streams and rivers.

Notes

Preparation and points to note

As this is an outside activity involving dirt and water, a level of mess is inevitable. Establish protocols designed to minimise mess and prevent "accidents" and horseplay.

Experiments should be structured so that students can draw meaningful results. Prompt the students to apply the results of their experiment. (This can be linked to reasoning in flexible ways, an aspect of the key competency *thinking*.)

Finding genuinely dry soil with high clay content is almost impossible because of its ability to retain water. This doesn't matter, and nor does it matter if the other soils are somewhat damp to start with because it is only when they are saturated that the difference in carrying capacity becomes really marked. Note that, when damp, pure clay is virtually waterproof (so much so that it is sometimes used to line reservoirs).

Any same-sized plant pots (plastic or fired or unfired clay pots) can be used for this investigation as long as they have holes in the bottom. The students will need to stop the sand and fine soil disappearing through these without interfering with the drainage. A little scrunched-up newspaper works well. 2 litre ice cream containers with holes poked in the bottom could be used instead of pots. If the pots aren't filled right to rim level, this allows water to pond on the surface of poor-draining soils.

The second part of the investigation uses the same pots and soil as the first. There is a time lag (overnight) before the pots are ready for this second part.

Points of entry: Mathematics

Twice in this activity, the students pour a measured amount ($\frac{1}{2}$ litre, which is 500 mL) of water into each pot, wait until what can't be absorbed has stopped dripping through the holes, measure the amount that has dripped through, and do some calculations using percentages. It is important that the students understand what the process is designed to find out rather than blindly follow a set of steps because they have been told to. Fundamentally, they are trying to compare what happens when rain falls on different types of dry ground with what happens when it falls on the same ground when saturated (following earlier rain). They do this by measuring the water-holding capacity of 4 soils in the 2 different states. By translating the amount dripped through into percentages, they can easily compare results for the different soils.

Ensure that the students understand that, in the end, they are interested in how much water has been retained by the soil, not how much has dripped through. This means that, for each pot, they need to measure the amount that has dripped through and subtract this from 500 mL (the amount poured in). They then convert this difference (the amount of water held) to a percentage. By comparing the 4 percentages for dry soil and the 4 percentages for the wet soil, they should be able to reach some conclusions about the water-retention properties of the different soils. You could ask them to find a way of representing their findings graphically.

Points of entry: Science

Like any scientific experiment, standardised procedures need to be established so that variables can be controlled, results compared, and conclusions reached. If doing this experiment as a whole-class activity, have the students work in small groups. Challenge them to identify the various variables (independent and dependent) and any problems they can see in controlling the independent variables. Can they suggest ways of managing these problems?

One variable that is difficult to control is volume of soil: how can the students ensure that each pot contains the same amount when soils compress differently? (Soil that is mostly gravel will compress very little; loam will compress quite a bit. A small piece of concrete or something similar could be used to apply standardised pressure to the different soils in the pots.) Ask the different groups to reach agreement on a common methodology so that their results can be compared.

Before they pour any water, ask each group to make and state a prediction (for example, "Gravel will drain best whether dry or wet."). They can then test their hypothesis against the data.

The following is a set of hypothetical data:

Retention of Water in Soil						
Soil	Dry soil		Wet soil		Percentage	
	Amount lost (mL)	Amount retained (mL)	Amount lost (mL)	Amount retained (mL)	Water held by dry soil	Water held by wet soil
Sandy soil	140	360	450	50	72	10
Clay soil	180	320	25	475	64	95
Potting mix	260	240	405	95	48	19
Stony soil	400	100	480	20	20	4

Conclusions that might be drawn from the above "data":

- Stony soil drains best overall (allows water to simply pass through it, no matter how wet it gets). When saturated, the sample held only 4% of the new water.

- Sandy soil holds the most water initially. (Water collects in the large spaces between the sand particles; sandy soil dries out quickly and never becomes waterlogged.) When saturated, 90% of the new water just passed through it.

- Potting mix struggles to absorb water when dry (it retained about 50%). It also drains fairly freely when saturated (it retained just 19% of new water). (This is a problem for plants in pots, as the minerals they need drain out with the water.)

- Clay soil absorbs water well initially. However, when saturated, the clay let very little of the new water through (just 5%). Soil like this would be very prone to flooding or to retaining surface water.

Pages 22–24: How High's the Water Now, Mum?

Mathematics and Statistics Achievement Objectives
- Number strategies: Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages (Number and Algebra, level 3)
- Statistical investigation: Plan and conduct investigations using the statistical enquiry cycle:
  - comparing distributions visually
  - communicating findings, using appropriate displays (Statistics, level 4)

Mathematics standards. The approaches and thinking that students demonstrate as they engage with these tasks and problems can provide evidence in relation to the mathematics standards.

Science Achievement Objectives
- Investigating in science:
  - Build on prior experiences, working together to share and examine their own and others' knowledge
  - Ask questions, find evidence, explore simple models, and carry out appropriate investigations to develop simple explanations (Nature of Science, levels 3–4)
- Interacting systems: Investigate the water cycle and its effect on climate, landforms, and life (Planet Earth and Beyond, levels 3–4)

Mathematics and statistics context	*Science context*
Students will:	Students will:
• use data and tables to make predictions in an authentic context.	• relate data to a real-life scenario
	• use data to justify, question, or negate conclusions.
Students should discover that:	Students should discover that:
• multiple factors need to be taken into account when estimating or predicting outcomes in the natural world	• although water is a valuable resource, too much can cause problems and damage
• an everyday benchmark can help make sense of large quantities (for example, 1 cumec = 5 full bathtubs of water).	• damage can be reduced by flood-protection mechanisms, early-warning systems, and control of land use.

Related information

Building Science Concepts: Book 1, *Waterways*; Book 2, *Weathering and Erosion*; Book 15, *Where's the Water?*; Book 31, *Water and Weather*; Book 50, *Storms*; Book 52, *The Land Changes*

Making Better Sense of Planet Earth and Beyond: Landforms, pp. 23–62

Connected 2 1998: "Testing the North River"

Connected 2 2002: "An Interview with a Glass of Water"; "The Water Cycle"

Connected 3 2004: "Wonderful Water"

River-flow activities:

www.lpb.org/education/classroom/itv/envirotacklebox/teacherguide/module4/4fldlsn1.htm

Regional websites discussing why and how they monitor river flow and levels:

www.ew.govt.nz/environmental-information/River-levels-and-rainfall/#Heading2

Answers

Activity

1. **a.–b.** Decisions may vary. Information that can be read from the data in relation to each statement is:

    i. True. The river was running at a height of about 19 m instead of a typical 9 m (see white line on green bar).

    ii. False. Flow (cumecs) and height (metres) are two completely different variables (that's why the bars have different scales) so they cannot be compared in this way. (Likewise, it makes no sense to say that a person's weight is more than their height.)

    iii. True. The individual flows of the rivers (both about 700 cumecs) combine at Ngāruawāhia to give a flow of about 1 400 cumecs.

    iv. Not enough evidence. Although the increase in height was greatest here, the water may have been effectively contained by a deep valley.

    v. Not enough evidence (though probably true). Mercer is the last monitoring station before the river reaches the sea, so the height must be lowest here. Rivers usually become shallower as they get near the sea, but the height of a river tells us nothing about its depth.

    vi. True. Once the Waipā and the Waikato merge, the flow does not increase by much (from about 1 500 to 1 600 cumecs) as the river completes its journey to the sea.

    vii. True. According to the data in the diagram, the water takes approximately 70 hrs to cover this distance. This is just marginally less than the 3 x 24 = 72 hrs in 3 days.

    viii. Not enough evidence. The diagram gives no information on damage.

    ix. True. The flow at these last two monitoring stations (and at Ngāruawāhia, for that matter) is much greater than at the first two, yet the increase in height (above the "typical" or normal level) is no greater. This can only be because the river is wider (and its capacity is therefore greater).

    c. Statements will vary. For example: "The flow at Ngāruawāhia looks like the sum of the flows at Whatawhata and Hamilton"; "It takes almost 2 days for water from Lake Taupō to reach Huntly"; "The river at Whatawhata must be in a narrow channel because a relatively small flow caused an almost 10 metre increase in the height of the river"; "Normal river levels are deepest at Hamilton".

2. **a.** Two: 1907 and 2007

    b. Yes, the house will definitely be in danger of flooding. But it is impossible to say for certain if the water will reach the house. What we can say is that a fairly small increase in flow can cause a big increase in height (for example, by comparing the data for 1907 and 2007, we can see that an increase of just 50 cumecs meant an increase in height of 0.7 m).

3. 30 hrs. (5 hrs from Ngāruawāhia to Huntly, 8 hrs to Rangiriri, and 17 hrs to Mercer. 5 + 8 + 17 = 30)

Preparation and points to note

This activity requires students to gather together pieces of information from a single, complex diagram. Closely examine the diagram before introducing it to your students; some students may need careful scaffolding.

As with any activity involving groups, select members of groups so that students of different ability and temperament can take advantage of each other's strengths.

The language and concepts in this activity will be new to most students: exposure to a rich range of experiences is one of the aims of the key competency of *using languages*, *symbols*, *and texts*.

Points of entry: Mathematics

This activity is primarily about interpreting statistical data.

The diagram contains three different sets of data: height (of river above sea level), flow (in cumecs), and time (for water to get from one monitoring station to the next). The diagram also shows the spatial relationship of the 5 stations. The height and flow data relates specifically to the flood of 2007.

Students who are used to dealing with a single data set and 1 graph at a time may be challenged by the apparent complexity of a diagram that brings together several sets of data and multiple graphs. In a situation like this, they should start by focusing on 1 data set and try to fully understand what it can tell them before moving on to another data set and doing the same. In this case, they could begin with the time (hours) data. After focusing on the data sets one at a time, they can begin thinking about the interplay between them.

Students may also have difficulty with the height and flow data because the bars for each are included on the same small graphs but represent totally different measures (this practice is often found in infographic-type diagrams in newspapers and magazines). Because the measures are different (cumecs versus metres), the two bars can't be directly compared: it makes no sense, for example, to say that flow at Rangiriri is greater than height. You may need to explain this carefully, pointing out that the two bars have their own, entirely different scales. Although both happen to have 10 seemingly identical divisions, this is the result of a design decision, not a mathematical relationship.

It is, however, entirely appropriate to compare either the height or the flow of the river at different monitoring stations.

The horizontal white line on the river height bars (the green bars) is an important piece of information that you may need to draw your students' attention to (see boxed information on page **23** of the students' book). It represents a "typical" or "normal" height. Without this information, it is impossible to say how much the river has risen in flood because levels at monitoring stations further from the sea are always higher than those closer to the sea (naturally – the water flows down to the sea). In practice, the level of a river typically fluctuates within a range of approximately 1 or 2 m. The white line roughly represents the median.

By adding together the average water travel times between the different monitoring stations, the students can estimate the time the flood peak will take to reach a certain station. Give these times meaning by relating them to the students' day.

Students may have noticed river height scales by major rivers. These scales suggest a question: Height above what? The simple answer is "sea level". But this suggests another question: What is sea level given that the level of the sea fluctuates by time of day, phase of moon, atmospheric conditions, and every wave? So that genuine comparisons can be made, all measurements made in the Waikato river system use a common reference point known as the Moturiki Datum.

The following table contains height and flow data for the 2007 flood (on which the diagram was based) as well as height and flow data for other major floods. As an extension activity, the students could create graphs for another flood or bar charts for height and flow across the 5 floods.

| Flood Peaks										
Year	Hamilton		Whatawhata		Ngāruawāhia		Rangiriri		Mercer	
	Level	Flow	Level	Flow	Level	Flow	Level	Flow	Level	Flow
1907	20.21	1 350	22.19	1 203	14.71	1 870	9.23	1 700	6.76	1 700
1958	18.28	905	21.96	1 130	14.31	1 482	8.44	1 128	5.74	1 260
1995	15.15	527	19.94	539	12.59	1 020	8.24	1 115	5.23	1 213
1998	15.68	586	17.16	517	12.68	1 050	8.23	1 111	5.03	1 134
2007	16.71	706	19.56	776	13.79	1 475	9.04	1 490	6.06	1 650
Level = metres above sea level			Flow = cubic metres per second (cumecs)							

Points of entry: Science

This activity focuses on aspects of the water cycle and on human attempts to explain and control it.

Some students may have prior knowledge of floods; discuss events that they may have witnessed or read about and collect media coverage if possible.

Encourage the students to think about the impact of fast-flowing water on the landscape and riverbanks. Discuss the different ways in which humans try to reduce the destructive impact of major floods, particularly through the use of storage lakes, dams, stopbanks, and early-warning systems. If your school is in the vicinity of a major hydroelectric scheme, ask the students to investigate how the dams that supply electricity are managed to also minimise flooding. They could also investigate the negative impacts of very large-scale river control schemes. Some of the most dramatic examples are to be found overseas.

It is important that students understand that while previous data can be used to predict what is likely to happen in the future and as a basis for schemes to reduce harmful impact, such predictions are only best guesses. As in any naturally occurring event, there are always unknowns. For example, severe rainfall events are often surprisingly localised, so even when level and flow at one point suggest what will happen downstream, other water may flow into the river from streams between the two points and complicate the predictions. Also, heavy rain falling on ground that is already saturated will have a very different impact from the same rain falling on dry ground (see the previous activity, Too Much Water).

As an extension, the students could do their own research into the causes of floods and how scientists predict them.
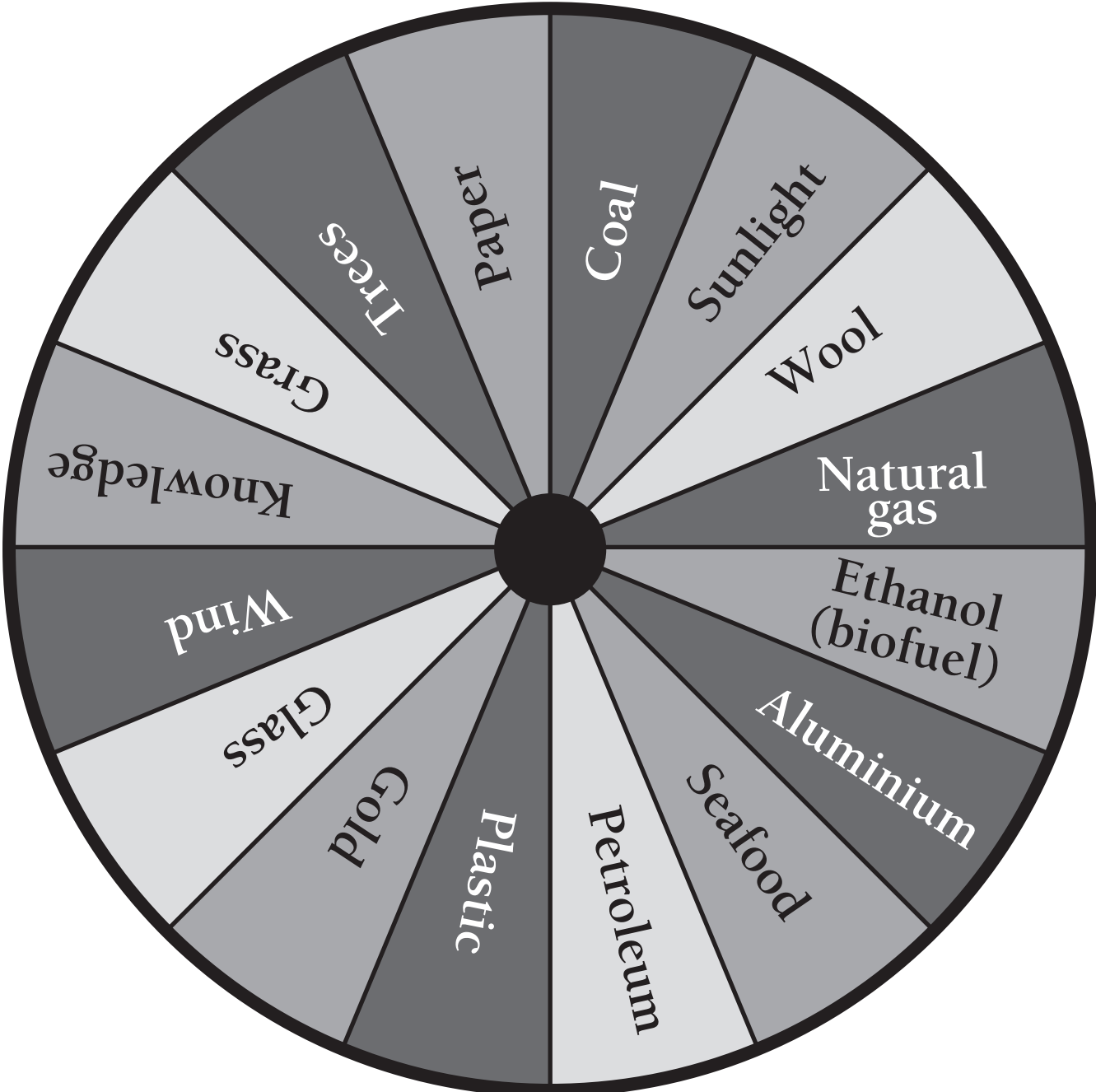
Calf market table and recording sheet

Total number of calves	1 or 2	3 or 4	5 or 6	7 or 8	9 or 10	11 or 12	13 or 14	15 or 16	17, 18, or 19	20 or more
Selling price	$400	$350	$300	$250	$200	$150	$125	$100	$50	$0

Year 1								
Player	1	2	3	4	5	6	Total calves	Selling price
Number of calves								
Player's profit								

Year 2								
Player	1	2	3	4	5	6	Total calves	Selling price
Number of calves								
Player's profit								

Year 3								
Player	1	2	3	4	5	6	Total calves	Selling price
Number of calves								
Player's profit								

Year 4								
Player	1	2	3	4	5	6	Total calves	Selling price
Number of calves								
Player's profit								

Profit after 5 years						
Player	1	2	3	4	5	6
Player's profit after 5 years						

Retention of Water in Soil					
Soil	Dry soil		Wet soil		Percentage	
	Amount lost (mL)	Amount retained (mL)	Amount lost (mL)	Amount retained (mL)	Water held by dry soil	Water held by wet soil
Sandy soil						
Clay soil						
Potting mix						
Stony soil						

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Soil	Dry soil		Wet soil		Percentage	
	Amount lost (mL)	Amount retained (mL)	Amount lost (mL)	Amount retained (mL)	Water held by dry soil	Water held by wet soil
Sandy soil						
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Notes

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Notes

Notes

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