Answers and Teachers' Notes

MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Contents

Introduction

The books in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand classrooms. In recent years, much of the Figure It Out student material has been aligned with Numeracy Development Project strategies, which are reflected in the *Answers and Teachers' Notes* where appropriate.

The level 2–3 Figure It Out *Statistics* book that these *Answers and Teachers' Notes* relate to is an extensive revision, in line with the achievement objectives of the mathematics and statistics learning area of *The New Zealand Curriculum*, of the level 2–3 *Statistics* book published in 1999.

Student books

The activities in the Figure It Out student books are written for New Zealand students and are set in meaningful contexts, including real-life and imaginary scenarios. The level 2–3 contexts reflect the ethnic and cultural diversity and life experiences that are meaningful to students in year 4. However, teachers should use their judgment as to whether to use the level 2–3 book with older students who are also working at this level.

The activities can be used as the focus for teacher-led lessons, for students working in groups, or for independent activities. You can also use the activities to fill knowledge gaps (hot spots), to reinforce knowledge that has just been taught, to help students develop mental strategies, or to provide further opportunities for students moving between strategy stages of the Number Framework.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* for the revised *Statistics* book includes full answers and explanatory notes. Students can use this section for self-marking, or you can use it for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, comments on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/r/maths/curriculum/figure/

Using Figure It Out in the classroom

Where applicable, each page of the students' book starts with a list of equipment that the students will need in order to do the activities. Encourage them to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to the problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Figure It Out

Statistics

Levels 2–3: Revised Edition

Answers

Pages 1–3: Choice Squares

Activity One

1. Those whose favourite fruit is not pictured can use the $\boxed{?}$. (The $\boxed{?}$ could also mean "don't know", but in this case, the graphs in the activity show that it means "other".)

2. Answers will vary. You may say the bar graph because it is easy to identify the tallest column or the pie chart because it is clear which is the biggest sector. If you ask yourself "Which graph most clearly shows *how many* voted for the most preferred fruit?", your answer should be the bar graph because the numbered vertical axis makes it possible to see in an instant how many people made each choice.

3. **a.–c.** Results and comments will vary. For **3 c ii**, a pie chart would probably show these differences more clearly.

4. **a.** Of those students who had fruit, some must have had more than one kind of fruit in their lunchbox.

   b. No. Everyone had a favourite fruit, yet 10 students (nearly half the class) had no fruit in their lunchbox. Also, it is not possible to tell if the person who, for example, had the orange in their lunchbox was one of those who said that oranges were their favourite fruit; neither graph identifies individuals.

   c. Possible answers may include:

   - While all students had a favourite fruit, almost half had no fruit in their lunchbox.
   - Students should be involved in making up their lunchboxes.
   - Parents should talk to their children about what goes into lunchboxes.

Activity Two

1. Possible answers:
   - Football is popular with both the girls and the boys (4 girls and 3 boys).
   - For the boys, rugby and football are both more popular than other sports (4 votes for rugby and 3 for football).
   - Netball is only popular with the girls (4 girls).
   - More girls than boys like hockey (3 girls but only 1 boy).
   - Only two people (1 girl and 1 boy) chose skiing.
2. Class activity. Graphs and discussion will vary.

Activity Three

Results will vary.

Pages 4–5: Greater Heights

Activity

1. Comments may vary. Possible comparisons that you might make are:
   - Most of the students in the class, both girls and boys, are between 127 cm and 132 cm tall.
   - The tallest girl and the two tallest boys in Room 7 are all 140 cm tall.
   - There are no boys shorter than 126 cm, but there are 4 girls shorter than this.
   - There are twice as many taller boys than girls, if you define "taller" as "more than 132 cm".
   - No matter where you make the cut-off point, there are more shorter girls than boys.

- There are more girls (15) than boys (13) in Room 7 (at least, there were on the day that the measuring took place).

2. The median (middle) girl is 129 cm tall and the median boy is 131 cm tall. Five out of the 15 girls and 7 out of the 13 boys are as tall or taller than 131 cm. Using this comparison, it could be said that the boys are taller than the girls.

3. Opinions and statements will vary. Some of the answers to question **1** could be used to argue against Ahere's view. Arguments that would support her view include:

- The tallest girl is the same height as the tallest boy.
- The girls' median height is only 2 cm less than the boys'.
- If you ignore the group of 4 girls who happen to be significantly shorter, the girls' data looks very similar to the boys' data.
- If there were equal numbers of boys and girls, the two groups might look even more similar [but then again, they might look even more different!].

Investigation

Investigations and dot plots will vary.

Pages 6–7: Too Much Telly?

Activity One

1. Possible comments include:

- All three students spent more time watching TV than doing homework.
- All watched more TV on Friday and Saturday than on other days.
- None did any homework on Friday or Saturday.
- All did half an hour of homework on Sunday.

2. Possible comments include:

- Jessie watched the most TV ($14\frac{1}{2}$ hrs) and did the least homework (2 hrs).
- Nina watched the least TV (10 hrs) and did the most homework (5 hrs).
- Jessie and Ruka watched some TV every night, whereas Nina had 2 nights where she didn't watch TV at all.
- Jessie did the least homework and only for 15–30 minutes at a time, whereas Nina and Ruka completed 2 hours' homework on at least 1 day.

3. On the basis of the data presented, "yes" (Jessie).

Activity Two

1. Comments might include:

- Most Room 6 students watched between 11 and 16 hours of TV, while most Room 5 students watched between 7 and 12 hours of TV.
- Most Room 6 students did between 1 and 3 hours of homework, while most Room 5 students did between 2 and 4 hours of homework.
- Overall, the students in Room 5 watched a lot less TV and did quite a bit more homework than those in Room 6.
- Both classes had students who watched considerably more or less TV than most of their classmates or did more or less homework than most of their classmates.
- Compared with those in Room 5, far more Room 6 students did only 1 hour or less of homework in the week.
- A lot more Room 5 students than Room 6 students did 4 or more hours of homework in the week.

Discussion points:

This data is for only 1 week. So:

- It may be that Room 5's teacher always sets more homework or simply that Room 5 was finishing a project that week.
- It may be that, if a different week were surveyed, Room 6 might have more homework than Room 5.
- Room 6 and Room 5 may be different year levels, which could make a big difference.

You might discuss why a few students watched no TV or did no homework, or why others watched far more TV (were they sick for a few days, during which time they watched lots of TV and did no homework?) or did more homework (were they slow or extra thorough, did they choose to do extra, or were they leaving an ongoing project until the last week?).

2. Opinions will vary, depending on your own experiences and values.

3. **a.–b.** Answers will vary.

Pages 8–9: Short and Sharp

Activity One

1.

Lengths of Pencils in Centimetres

Tens	Ones
0	3 4 5 6 7 8 8 8 8 9
1	0 1 2 3 4 5 5 6
2	6

2. **a.** Answers will vary. Try a pencil in your hand. How long at least does it need to be to rest comfortably between your first two fingers, supported by your thumb (or however you hold your pencil)? Probably somewhere between 8 and 10 cm.

   b. Answers will depend on your definition of too short. For example, if you decide that 6 cm is too short, there would be 4 pencils in Ms Smith's class that are too short.

3. **a.** Practical activity. Results will vary.

   b. Answers will vary. What is important is that you use the same definition of "too short" for the pencils in your room as you used for the pencils in Ms Smith's room.

Activity Two

1.–2. Graphs and answers will vary. The data your class collects will determine what you can say about it.

Activity Three

1. **a.–b.** Results and graphs will vary greatly. Here is a sample data set:

Products of Two Dice Numbers

0	3 4 4 5 6 8 8 8
1	2 2 2 2 2 5 5 5 8 8
2	
3	0 6

2. **a.** The data sets are likely to be very different because they are being decided by chance (probability). But the stem-and-leaf graphs will all have no more than four "branches", and the first two branches are likely to have a lot more leaves than the last two. (Can you explain why?)

   b. In **Activity Two**, everyone was working with the same data sets, so their graphs should have been the same. In this activity, even though everyone does an identical task, the data sets will differ greatly because they are determined by chance.

Pages 10–11: Open or Closed?

Activity One

1. The range of possible favourite colours is limited, so it would not be difficult to list them all.

2. **a.** The graph from the open question data gives a clear picture of which colours are most/least popular. The problem with the graph from the closed question data is the tall "other" bar. While the graph suggests that yellow is the most popular colour, we can't be sure. It may be, for example, that the six "other" votes were all from people who liked blue (not offered as a choice).

   b. The open question was better because everybody got to name their favourite colour and we are not left wondering what the "other" people would have chosen if they had had the chance.

Activity Two

1. **a.** Open. This question may be too open and get answers such as "often" or "hardly ever". You may get better information from a closed question with carefully chosen options such as "twice a day", "once every day", "twice a week", "once

a month", "only on Sunday", or "as a special treat".

b. Closed. The options could be improved. It seems odd to separate out auntie and uncle, brother and sister, but not mother and father. Also, the question could be confusing for those with family members who may buy fruit for their own eating rather than for the household as a whole. So a better question might be "Who buys the fruit for your household?"

c. Closed. It is unlikely that this question will get meaningful answers. Different types of fruit are plentiful and affordable at different times of the year and most people eat what is available at the time. Also, does it make any sense to include eating "most fruit" "all year round"?

2. The data might be useful in a class or school survey as the basis for discussion on healthy eating habits. However, it is unlikely that data gathered using these questions would be sufficiently detailed or accurate to be of much use to producers and suppliers. Surveys are often used to find out more about the people who buy certain products. Such surveying is called market research.

Investigation

Practical activity. Results and discussion will vary.

Pages 12-13: My Town

Activity

1.

House	People	TVs	Cars	Bedrooms
A	2	1	2	2
B	3	2	1	2
C	6	1	2	4
D	3	1	2	3
E	4	1	2	3
F	7	2	3	4
G	4	1	4	2
H	8	2	1	3

I	5	1	3	4
J	2	1	1	3
K	4	1	2	3
L	1	1	1	3
M	8	1	2	3
N	5	2	2	4
O	7	1	2	3
P	3	2	1	3
Q	2	1	2	2
R	1	1	1	1
S	7	1	1	3
T	2	1	2	1

In the spreadsheet below, the data has been sorted by number of people and bedrooms, using the Sort function. The bands of shading separate the data by number of people.

House	People	Bedrooms	TVs	Cars
R	1	1	1	1
L	1	3	1	1
T	2	1	1	2
A	2	2	1	2
Q	2	2	1	2
J	2	3	1	1
B	3	2	2	1
P	3	3	2	1
D	3	3	1	2
G	4	2	1	4
E	4	3	1	2
K	4	3	1	2
N	5	4	2	2
I	5	4	1	3
C	6	4	1	2
S	7	3	1	1
O	7	3	1	2
F	7	4	2	3
H	8	3	2	1
M	8	3	1	2

2. **a.–b.** The conclusion for Keni's first question should be "no". There are only 5 houses that have more than 1 TV (B, F, H, N, and P). Of these, only 2 (less than half) have more than 1 car (F and N).

Conclusions for Keni's second question will vary:

- If we assume that the houses with the most bedrooms are the "biggest" houses, then the biggest houses on Keni's map are the ones with 4 bedrooms. These houses are all occupied by between 5 and 7 people. Families of this size are definitely amongst the "biggest".
- The two biggest families actually have 8 people in them, and both these families are in 3-bedroom houses (I and N). By this measure, it is not true to say that "the biggest families live in the biggest houses".
- If we define "biggest families" to be those with 5 or more people in them, half of these families live in 3-bedroom houses and half live in 4-bedroom houses.

3. **a.** Dave: True. 14 out of the 20 houses have 3 or 4 bedrooms.

Silei: Not true. There are two 4-bedroom houses (I and N) that have only 5 people living in them.

Henare: There is no way to be sure, but this statement is very unlikely to be true. While there are only 2 houses that have more bedrooms than people (J and L), there are other houses where 2 adults or 2 children probably share a bedroom, leaving 1 bedroom spare.

Matt: True. There are only 2 houses with 1 bedroom (R and T); the first has 1 person in it, the second has 2.

Meng: Not true. Only 6 of the 20 houses (A, G, L, Q, R, and T) have 1 car per person. This is not "nearly half".

Fleur: True. 4 houses (H, M, O, and S) have 7 or 8 people living in them but only 3 bedrooms.

4. Keni should not try and make generalisations about the homes and community facilities (shops, library, churches, halls, and so on) in Rangipai as a whole, based on his data. The town has about 200 homes, and his data concerns only the 20 homes in his neighbourhood. We have no idea if, for example, there is a medical clinic in another part of town, or a library, hotel, or saleyard. We don't know what shopping is available locally, or whether the better-off homes are near Keni or in another part of the town. We don't know if the nearest big town is 10 or 100 km away.

Pages 14–15: Discussing Data

Activity

1. Answers will vary. Both graphs show clearly that bus and car are the most common ways of getting to school, so both are satisfactory for the purpose. But only the pictograph tells you how many students used each mode of transport. The pictograph also makes it very clear that bus was the most popular mode of all. For these reasons, the pictograph is probably the better of the two graphs.
2. The stem-and-leaf graph shows at a glance that most (in the sense of more than half) of the students usually begin dinner between 6 and 7. You can find this out from the other graph and the table, but it is much less obvious.
3. Answers will vary depending on how you interpret the question. If you take it to mean "how many more millimetres?", only the table will give you the precise information you need. If you take the question to be more about how rainfall varies over the year, the bar graph is best. The pie chart is not suitable: it is arranged by season, not month, and it has no millimetres information.

Investigation

1. Practical activity. Note that for many kinds of graphs, you should only select the number data (not the categories) and then choose a chart option. The category labels can be added as a second step. You will find that only some chart types will give you a graph that looks meaningful for a given set of data.
2. The various 3-dimensional bar graphs (including cylinders, cones, and pyramids) and pie charts (including donuts) look exciting, but it is very hard to read values off them; for this reason, they are not recommended. The best graphs are simple and clean.
3. Many graphing programs will not create pictographs, linear dot plots, or stem-and-leaf graphs.

Pages 16–17: Where on Earth?

Activity

1. All the graphs need titles so that it is clear what they show.

   You can't tell what's what in **i**. This graph needs labels on each axis and country names under the bars; bars need to be of equal width and equally spaced.

   Looking at the areas coloured red and green in **ii**, it appears that an equal number of those questioned were born in Sāmoa and New Zealand, but the data shows that this is not true. To get the areas of the different sectors correct, the divisions around the perimeter of the pie need to be equal.

   The icons in **iii** need to be the same size so that the reader can compare from the lengths of the lines how many people were born in each country.

   The line graph in **iv** is not a suitable choice for category data. A line graph shows increases, decreases, and trends in data *of the same kind*.

   In a strip graph, area matters: twice the area means twice the data. But there is too much going on in the strip graph in **v**, which gets thinner and curves, to be able to read or compare the areas of the different regions. 41 people were questioned about their country of birth, so a correct strip graph would be of constant width, 41 units long, and divided into lengths according to the number of people in each category.
2. Answers and graphs will vary.

Pages 18–19: Logo Appeal

Activity

1. Answers will vary. However, three out of the four groups surveyed ranked "learning" as their favourite or second favourite feature. Every group identified "happy students", "good teachers", and "tidy and clean" as favourite features (although with different rankings), so this could be a reason for including one or more of them in the new logo.
2. Possible answers are:
   - Pose a question (Problem):

     The students asked "What should be included in the new Te Paki School logo?"
   - Plan your investigation (Plan):

     The students considered "Who do we ask?" (students, parents, teachers, community)
   - Gather your data (Data):

     The students questioned samples of people from the groups identified in the planning stage.
   - Analyse your data (Analysis):

     The students collated and graphed the data and looked for patterns in the data (similarities and differences in the data from each sample).
   - Communicate your findings (Conclusion):

     Based on their analysis of the data, the students might decide on recommendations and report these to the principal and board of trustees. They might also have more questions as a result of their enquiries (which would start the cycle again!).
3. Practical activity

Page 20: Tally Ho!

Activity

1. Possible comments include:
   - Sandwiches are the most popular item.
   - Fruit and cake are equally popular.
   - Fruit and cake options are the second most popular purchases.
   - Few students buy nuts.
   - Fruit juice is the least popular item.
2. a. Fruit was the least popular purchase last year, but it is the second most popular purchase this year. Sandwiches were the fourth highest seller last year, but they are the highest selling item this year. The two most popular purchases from last year (pies and chips) do not appear in this year's data.

   b. Answers may vary. A likely explanation is that the school has been trying to promote healthier eating choices and is no longer selling pies, chips, and soft drink. Cake is not a healthy option but for some reason has been left on the menu and continues to sell well.
3. The data clearly indicates that the students would like the tuck shop to sell pies and soft drink. You could argue that pies would be a warm alternative in the winter months, or you may oppose pies and soft drink on the grounds that, even if the students want them, they are unhealthy and would tempt some people to make non-healthy purchases.
4. Answers will vary.

Page 21: Sneaky Snakes

Activity

1.–3. Results and discussion will vary.

Page 22: Way to Go

Activity One

The number of different ways for each student to walk from their home to school are: Tom (1), Rima (3), Suzie (4), Hong (6), and Margaret (20).

Activity Two

1.–2. Results and discussion will vary.

Page 23: Which When?

Investigation

1. Some will prefer one and some the other, but it actually makes no difference to the outcomes.
2. Answers will vary, but because the total angle (and area and circumference) allocated to each of the activities remains the same (one-third of the total, in each case), it makes no difference which spinner is used. (You may need to decide that if the spinner lands on a line, the "win" is the area that most of the spinner is in.)
3. a. 1 day in 3

   b. Answers will vary. While you might expect to get 1 or 2 days of each activity over a 5-day period, anything can happen. It may be that your 5 spins got you chores every time!

Page 24: In Between

Game

A game exploring probability

Activity

1. Marama
2. There are five cards in the pile that would give Marama a win. This is more than for any of the other players:

	Possible Winning Cards
Trevor	6, 7, 8
Katie	4, 5
Barbara	5, 6, 7
Marama	1, 2, 3, 4, 5

Figure It Out

Statistics

Revised Edition

Teachers' Notes

Overview of Levels 2–3

What Is Statistics About?

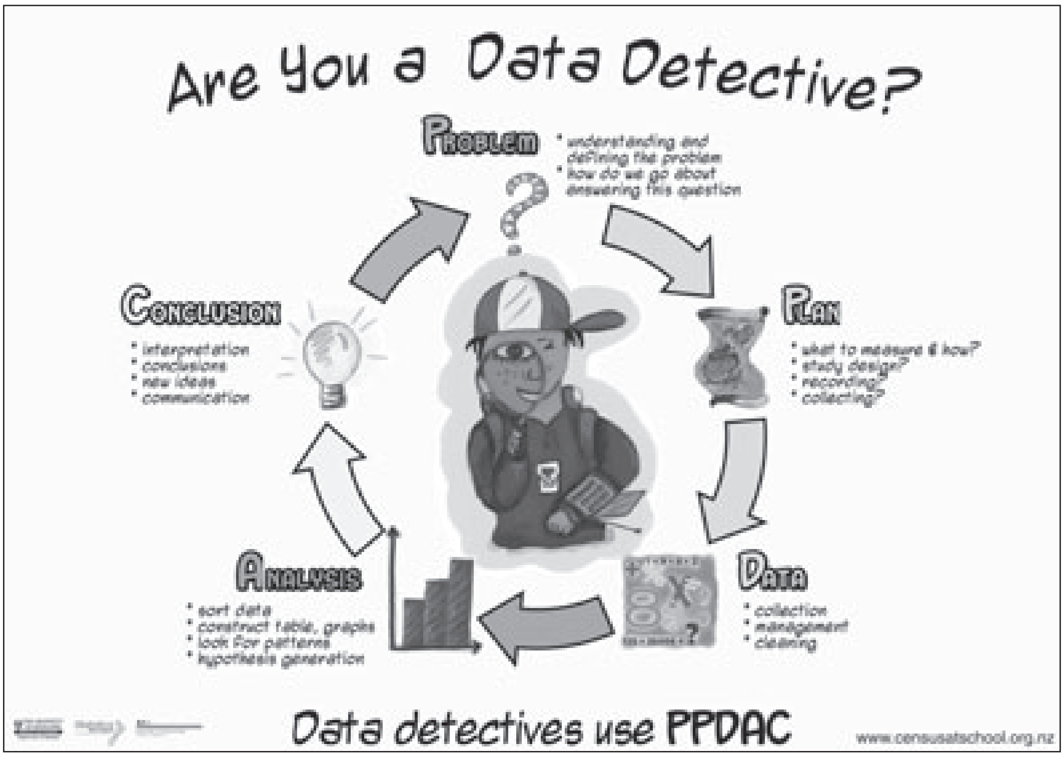
Statistics is defined in *The New Zealand Curriculum* as "the exploration and use of patterns and relationships in data". Like mathematics, it aims to equip students with "effective means for investigating, interpreting, explaining, and making sense of the world in which they live".

The New Zealand Curriculum goes on to say:

> Mathematicians and statisticians use symbols, graphs, and diagrams to help them find and communicate patterns and relationships, and they create models to represent both real-life and hypothetical situations. These situations are drawn from a wide range of social, cultural, scientific, technological, health, environmental, and economic contexts …
>
> Statistics involves identifying problems that can be explored by the use of appropriate data, designing investigations, collecting data, exploring and using patterns and relationships in data, solving problems, and communicating findings. Statistics also involves interpreting statistical information, evaluating data-based arguments, and dealing with uncertainty and variation.
>
> page 26

The **PPDAC** (Problem, Plan, Data, Analysis, Conclusion) statistical investigation cycle used for the CensusAtSchool New Zealand resources (see www.censusatschool.org.nz) provides an ideal model for statistical investigation. This approach is used in the revised level 2–3 Figure It Out *Statistics* book and in the *Answers and Teachers' Notes* that accompanies it.

CensusAtSchool New Zealand makes available two posters (aimed at different age levels) for the PPDAC cycle. One version is:



The five steps in this model are:

- Problem – deciding what to investigate, and why, and how to go about it;
- Planning – determining how to gather the necessary data;
- Data – collecting, managing, and preparing the data for analysis;
- Analysis – exploring the data with the help of graphs and statistical tools and asking what it says;
- Conclusion – determining how the data answers the original problem and deciding what to do next.

CensusAtSchool New Zealand provides this information in the form of a downloadable PDF.

Much of the information in the following sections is adapted (by permission) from information available on CensusAtSchool New Zealand and Statistics New Zealand (www.stats.govt.nz).

Types of Data

Category data classifies data according to a non-numeric attribute, such as gender, colour, style, model, opinion, type, feel, and so on. For example, foods could be sorted into categories such as meat, fish, vegetables, fruit, and cereal.

Numeric data classifies data according to an attribute that can be counted or measured. Numeric data may be either discrete or continuous. **Discrete data** is whole-number, countable data. **Continuous data** is data obtained using measurement, for example, time, height, area, mass, age. When continuous data is rounded to the nearest whole unit, it is effectively treated as discrete. **Time-series data** is data that is collected from a series of observations over time, with a view to discerning time-related trends.

A **variable** is an attribute or factor that can take on different values, for example, time, colour, length, favourite author, number of items, cost, age, temperature. When a number of pieces of data are collected for a single object or person, the result is a **bivariate** or **multivariate** data set. A list of movies by length is a **univariate** data set (that is, there is a single variable, time); a list of movies by length, genre, and country of origin is a multivariate data set. Multivariate data sets have much greater potential for exploration than univariate data sets.

Graphs

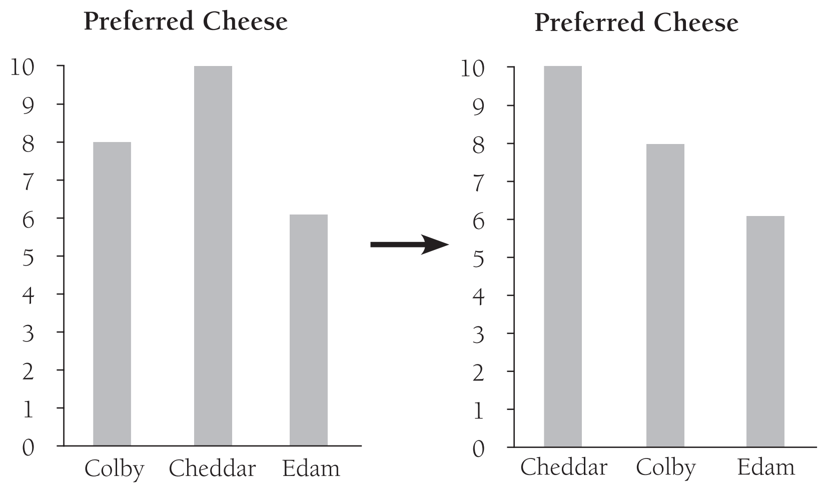
Graphs or **charts**? *Graph* is the more common usage in New Zealand (except in the case of pie chart) but *chart* is the term used by most graphing programs. In statistical contexts, these two terms are used virtually interchangeably.

The activities in the students' book promote graphs as a means of exploring data and communicating findings. It is important that students learn to "read" graphs, question what they read or see, find the stories in the data, and ask more questions. Especially in the early years, students can devise their own graphical representations. As they learn more, they need to become familiar with the standard types of graphs and associated conventions. All standard graphs should have a title that states the intent, axes (if used) should be labelled clearly, and the measures used should be consistent throughout. These basic conventions are designed to enhance the communicative power of graphs.

Bar graphs are used to show the frequency of category data or discrete numeric data. Unlike dot plots and strip graphs, they have two axes, one labelled with the category and the other with the frequency. There is always a gap between bars, showing that the categories are quite separate. The bars are normally vertical and, for category data, may be coloured or shaded differently.

On a well-constructed and labelled bar graph, it is easy to see which of the categories is most "popular" and to compare categories. Differences that appear insignificant in a pie chart or strip graph typically show up clearly in a bar graph.

Unless there is a good reason not to, the bars for **category** data are usually arranged in order of height, as in the example below.



In the students' book, bar graphs feature in activities on pages **1–3**, **10–11**, **14–15**, **16–17**, and **18–19**.

Pie charts and **strip graphs** show the relative size of the categories that make up a whole (whatever the whole may be). The categories are always labelled. The percentage value (and sometimes the actual data value) may also be shown on or alongside each region. Unlike bar graphs, pie charts and strip graphs do not show categories that contain zero data. Students find pie charts difficult to create by hand but easy to create in most graphing programs.

Pie charts or strip graphs feature in several of the activities in the students' book, including those on pages **1–3**, **14–15**, and **16–17**. While they have their place, these graphs can only be used for a single variable and can only tell the simplest of stories.

Pictographs are simple picture graphs. They can take the shape of a bar graph or a strip graph, or even a stem-and-leaf graph, but they use a number of little pictures of the item being counted to show how many items there are in a group (see pages **1** and **14** in the students' book).

Dot plots, found on pages **4–5** and **6–7** of the students' book, are very easy to construct and clearly show the spread of the data involved (that is, the way in which it is distributed and/or grouped). They suit discrete numeric data: each dot represents a single piece of data. The beginning and end of the scale are dictated by the least and greatest data value.

Stem-and-leaf graphs, explored on pages **8–9** and **14–15** of the students' book (see also the notes on page 23), are a convenient means of organising and displaying discrete numeric data. Each individual data value retains its identity at the same time as overall patterns emerge.

A stem-and-leaf graph is made by arranging numeric data in a display, using the first part of the number as the stem and the last digit as the leaf. For example, for 16 in the data set {16, 31, 25, 33, 27, 24, 14, 26, 31}:

- the tens digit (1) is the stem and appears to the left of the vertical line
- the ones digit (6) is the leaf and appears to the right of the vertical line.

Stem	Leaf
1	4 6
2	4 5 6 7
3	1 1 3

It is usual to sort the leaf data in number order, from least to greatest. This is often best done as a second step, particularly where there is quite a lot of data.

When constructing stem-and-leaf graphs, students should ensure that they space the digits equally. This is important because it makes it possible to observe features such as general shape, symmetry, gaps, and clusters. It also makes it easy to track down the median (and, in a larger data set, the quartiles).

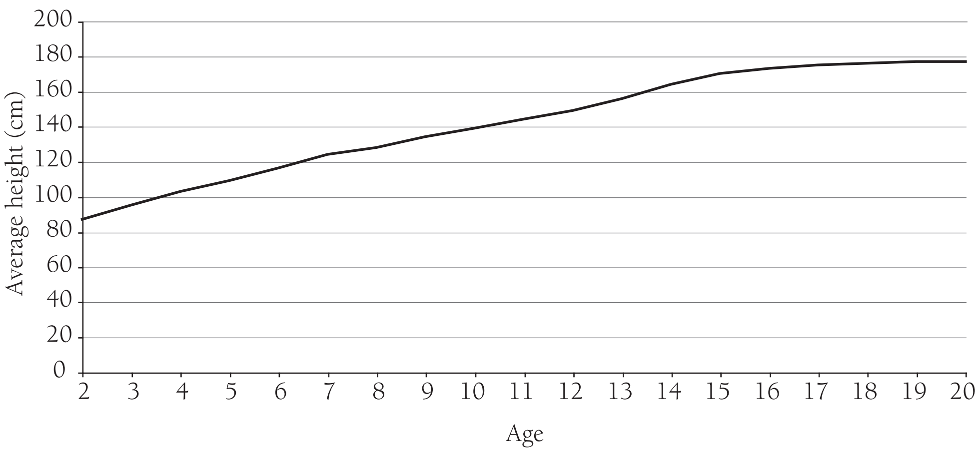
If the data collected is three-digit numbers, such as height in centimetres, the hundreds and tens digits make the stem of the graph, with the ones digits as the leaves. For example, if graphing the following heights recorded in centimetres, 114, 122, 142, 116, 125, 127, 142, 144, the stem represents the hundreds and tens digits:

Stem	Leaf
11	4 6
12	2 5 7
13	
14	2 2 4

Line graphs, an example of which can be found (used inappropriately) on page **17** of the students' book, compare two variables, one of which is plotted on the horizontal axis and the other on the vertical. Line graphs are useful for showing one variable in relation to another (for example, tree growth in relation to rainfall) and for showing how something changes over time. They should not be used for category data.

A **time-series graph** is a particular type of line graph in which time is measured on the horizontal axis and the variable being observed is measured on the vertical axis. This example shows how the average height for boys increases, beginning at 2 years and finishing at 20 years of age:

Height (Boys)



Other Statistical Terms

Axes (singular: **axis**) are the two lines, one horizontal and one vertical, that form the framework for most graphs. As a general principle, the vertical axis is used for frequency and the horizontal axis for categories, values, or time. This means that bars are equally spaced and vertical. (Pictographs and bar graphs do not always observe this rule.)

Choice squares are square cards that have a picture or symbol on them, representing a category. A student "votes" for a particular category by taking a card with the appropriate symbol on it (or by drawing or writing on a blank card). The cards are then easily collected, sorted, and manipulated to form strip and bar graphs and pie charts.

Collate means to collect and combine.

The number of data in a category or interval is known as its **frequency**. Frequency can be thought of as "number" or "total".

A **frequency table** is a table that organises data by category or interval and gives the frequency for each category or interval. For example:

Weeks between haircuts	2	4	6	8	10	12	14+
Number of classmates	4	6	9	5	2	3	1

"**I wonder**" questions are **investigative questions –** statistical questions or problems to be answered or solved. They consider the entire data set or population and do not involve locating an individual within a data set.

Two types of investigative question are of particular interest at this level:

- **Summary questions**, which usually involve a single variable and require the data to be described in some detail (for example, "I wonder how long it typically takes a year 6 student to run 100 metres?")
- **Comparison questions**, which involve comparing two or more subsets of data, for example, male and female, young and old, in relation to a common variable such as speed (for example, "I wonder whether year 6 girls are typically faster than year 6 boys?").

In the planning phase, we pose **survey questions**. These are questions asked to get the data (for example, "How many seconds did it take you to run 100 metres on athletics day?").

The **median** is the middle value in a data set when all the values are arranged in order from smallest to biggest or biggest to smallest.

Probability:

- **Probability** and **chance** are the same thing, although one or the other term may be more usual in a particular context.
- **Outcome**: the result of a trial (for example, a match or no match)
- **Trial**: performance of an action or actions where the outcome is uncertain (for example, the toss of a coin)
- **Experiment**: sometimes used interchangeably with trial; otherwise, a series of trials
- **Experimental probability**: the likelihood that something will happen, based on a number of trials
- **Theoretical probability (expectation)**: the likelihood that something will happen, based on reasoning or calculation.

Tally marks (|) are used when counting or categorising data by hand. Every fifth stroke is drawn across the previous four, facilitating skip-counting by 5s and 10s. For example, 𝍸 𝍸 || stands for 12.

In a **tally chart**, information is presented in three columns: category, tally, and frequency. For example:

Footware	**Tally**	**Frequency**
Shoes	𝍸	5
Sandals	𝍸 ||	7

Variation is the term used to refer to the differences between data, particularly differences from an expected pattern or trend. To illustrate: if a coin is tossed a very large number of times, we would expect that the numbers of heads and tails would be approximately equal (because the two outcomes are equally likely). But in practice, if we were to toss a coin 100 times and then repeat this experiment 10 times, we would almost certainly get widely differing results, for example:

Experiment	1	2	3	4	5	6	7	8	9	10
Heads	41	44	52	53	42	49	47	50	38	45
Tails	59	56	48	47	58	51	53	50	62	55

Variation can be described and, at later levels, measured, using a variety of measures of spread from the simple to the sophisticated.

Links to *The New Zealand Curriculum*

Achievement Objectives

Achievement objectives in the teachers' notes for existing, revised, or new material in the *Statistics* students' book are from the mathematics and statistics area of *The New Zealand Curriculum*.

Key competencies

The New Zealand Curriculum identifies key competencies that students will develop over time and in a range of settings. Schools can develop the key competencies within the mathematics and statistics learning area as well as encouraging and modelling values for students to explore.

The five key competencies identified in *The New Zealand Curriculum* are:

- thinking
- using language, symbols, and texts
- managing self
- relating to others
- participating and contributing.

The notes for the student activities in this revised *Statistics* book suggest one or more key competencies that the activities could help to develop. (You may, of course, decide to focus on key competencies other than those suggested.)

Activity Notes

Pages 1–3: Choice Squares

Achievement Objectives

Statistical investigation

- Conduct investigations using the statistical enquiry cycle:
  - posing and answering questions;
  - gathering, sorting, and displaying category and whole-number data;
  - communicating findings based on the data (Statistics, level 2).

Statistical literacy

- Compare statements with the features of simple data displays from statistical investigations or probability activities undertaken by others (Statistics, level 2).

Investigation	Literacy	Probability

P	P	D	A	C

Key Competencies

Choice Squares can be used to develop these key competencies:

- thinking
- using language, symbols, and texts
- participating and contributing.

Activity One

Before the students go on to answer the questions in this activity, discuss with them why Room 8 might want to find out which fruits were most preferred in their class. It is important that students see a reason for their own or others' investigations. A reason is suggested in question **4**, which implies that the students are dissatisfied with their lunchbox contents.

You could also discuss why the students in Room 8 decided to use **choice squares*** to gather their information. When your students do their own investigations, they can design suitable squares with the help of the blank copymaster (to keep the squares the same size) and then photocopy them for use.

The $\boxed{?}$ referred to in question **1** makes it possible to restrict the number of categories while ensuring that everyone can respond to the survey. As indicated in the Answers, the $\boxed{?}$ in some surveys can mean "don't know", but in this activity, it corresponds to the "other" category found in survey questions such as: "What is your favourite meat?"

☐ beef ☐ lamb ☐ pork ☐ chicken ☐ other

If a large number of people picked the "other" category, this would warrant further investigation. It may be, for example, that quite a few people would have selected venison if this option had been offered. [How might a vegetarian respond, given the above options?]

Question **2** involves comparing three different data displays: **strip graph**, **bar graph**, and **pie chart**. Discuss the characteristics of each type of graph (see the following page).

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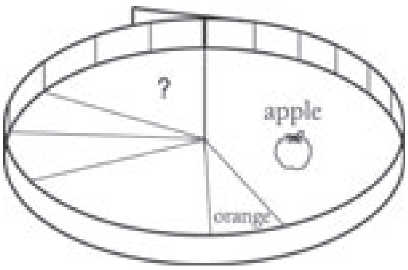
* For bolded terms, see introductory section.

Strip graph	All like objects are grouped together. There are no gaps between categories. The graph cannot show empty categories.
Bar graph	There are gaps between the columns. There are two **axes**: one showing categories, the other, the number in each category.
Pie chart	All like data is grouped together. There are no gaps between sectors. The chart cannot show empty categories. It is not usually clear how many items are included in each category.

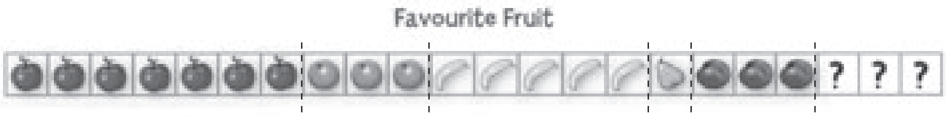
In question **2**, the strip graph is manipulated to form a pie chart and a bar graph. This makes it clear that the data displayed in the three examples is the same data displayed in three different ways, not three distinct lots of data.

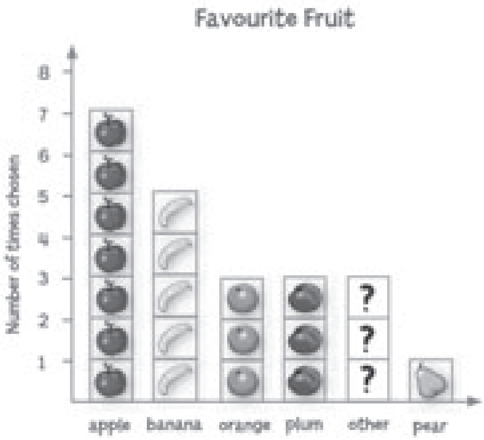
These diagrams may be helpful:



For the pie chart, bend the strip graph around to form a circle and draw line segments from the centre to the circumference to show each category.

For the bar graph, cut each section of fruit and use for the bars:





Ensure that the students understand the importance of labelling axes and giving each graph a succinct, descriptive title. These should not be viewed as technical requirements or "teacher says" rules but as keys to interpretation.

In each of the three graphs, the area allocated to each category is proportional to the number of items or values it represents: if one of the sectors in a pie chart (for example) is twice the area of another, it represents twice the number of items or values.

The students could collect the data for question **3a** as a class and then work through the questions either as individuals or as small groups working independently. If the students' own surveys bring a large number of $\boxed{?}$ responses, suggest they treat this as an opportunity for refining the options. They can then administer a revised survey.

After the data is collated and displayed in a graph, questions **3b** and **3c** will help the students interpret their results. Although the task is a simple one, encourage your students to state their findings in writing. By doing so, they learn how to make interpretative statements and then to put them up for scrutiny by classmates.

It is important that the students understand that question **4** introduces a new set of data, this time showing what is in Room 8's lunchboxes, not what the students would prefer to be there.

Question **4a** introduces the idea that survey questions can sometimes allow for multiple responses from the one person. A quick look at the students' own lunches would illustrate that some (perhaps many) contain more than one fruit.

Question **4b** asks students if they can link the "favourites" data with the "actuals" data. While some thoughtful comparisons can be made, the important idea here is that the data in the two graphs is not linked to particular individuals. It is impossible to tell whether a student who has put herself down as an apple lover in the first data set happens to have an apple in her lunchbox that day.

Activity Two

Note that although the vertical axes on the graphs in question **1** are not numbered, the graphs are easy to interpret.

Your students should be given the opportunity to answer question **2** with a minimum of assistance. Let them find a way of clearly representing the two sets of data in a single graph in such a way that none of the original data is lost. Avoid over-emphasising technical correctness, especially at this level. What matters is that the students are gaining confidence in their ability to create graphs that tell stories. They can test their graphs out on their peers.

Activity Three

This activity would make an excellent assessment task because it encompasses all aspects of the statistical enquiry cycle.

Pages 4–5: Greater Heights

Achievement Objectives

Statistical investigation

- Conduct investigations using the statistical enquiry cycle:
  - posing and answering questions;
  - gathering, sorting, and displaying category and whole-number data;
  - communicating findings based on the data (Statistics, level 2).

Statistical literacy

- Compare statements with the features of simple data displays from statistical investigations or probability activities undertaken by others (Statistics, level 2).

Investigation	Literacy	Probability

P	P	D	A	C

Key Competencies

Greater Heights can be used to develop these key competencies:

- using language, symbols, and texts
- relating to others.

Activity

This activity introduces the students to **dot plots*** and – most importantly – to the fact that data can be summarised to the extent that important details become invisible and stories lost.

Unlike the first dot plot, which graphs only one **variable** (height), the second and third dot plots graph two variables (gender and height). By comparing these latter two graphs, students are able to find points of similarity and difference and, potentially, answer a range of "**I wonder**" questions.

For question **1**, the Answers suggest some comparative statements. The students should have to justify their statements from the data. They could do this in small groups, followed by a report back. Particularly as they are learning how to conduct and resolve a mathematical or statistical argument, they need you to be involved.

Question **2** involves finding the "middle" or **median**. (The median is the number that comes in the middle of a set of numbers when they are arranged in order. Where two or more data sets are involved, it can be very useful to compare medians as part of the data exploration process.)

Question **3** challenges the students to think about whether the differences that they have found between the two data sets are actually significant and then to support their view.

Investigation

Use this investigation to reinforce the **PPDAC cycle**. It is of crucial importance that students are not just going through the motions of an investigation. The idea is that statistical investigation should acquire a specific shape in students' minds and that, over time, they come to rely less and less on teacher-provided scaffolding.

The PPDAC cycle makes it clear that graphing is not an end in itself but a means to an end. Graphs have an analytical purpose: they help the user find patterns and key features in a data set. They also have a communications purpose: they enable the user to explain, justify, and persuade. Students need to learn how to dig below the surface to uncover and tell the story or stories embedded within a graph. It is very important, therefore, that students don't get lost in technicalities and rules about what graphs must look like or have. The conventions are less important than the purposes.

Of all the graphs available, the linear dot plot is probably the easiest for a student to create (although not available in all graphing programs). One can be created as a communal exercise if each student is given a self-adhesive dot to represent themselves (in this case, their height), which they bring up to the front and stick on an appropriately labelled section of number line. By doing this, the link between themselves, the data, and the graph is emphatically made.

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* For bolded terms, see introductory section.

Pages 6–7: Too Much Telly?

Achievement Objectives

Statistical investigation

- Conduct investigations using the statistical enquiry cycle:
  - posing and answering questions;
  - gathering, sorting, and displaying category and whole-number data;
  - communicating findings based on the data (Statistics, level 2).

Statistical literacy

- Compare statements with the features of simple data displays from statistical investigations or probability activities undertaken by others (Statistics, level 2).

Investigation		Literacy		Probability
P	P	D	A	C

Key Competencies

Too Much Telly? can be used to develop these key competencies:

- thinking
- using language, symbols, and texts
- managing self.

Activity One

In this activity, the students are asked to interpret and compare three sets of data presented as double horizontal **dot plots***. They may not have encountered graphs exactly like these before. If this is the case, invite them to make sense of them without your assistance.

For questions **1** and **2**, it is important that the students write their statements down. This disciplines them to formulate ideas completely and commit to them. Once they have done this, their ideas can be challenged on the basis of the data and either justified, modified, or rejected.

The three data sets are **multivariate** in that, for each student, the **variables** of (i) day of week, (ii) hours watching TV, and (iii) hours doing homework all have to be considered.

For question **3**, the students could find the total amount of time each of the three Room 6 students spent watching TV and doing homework and compare the two amounts. Jessie watched $14\frac{1}{2}$ hours of TV and did 2 hours of homework; Nina watched 10 hours of TV and did 5 hours of homework; Ruka watched 11 hours of TV and did just over 4 hours of homework. So all three spent a total of 15–$16\frac{1}{2}$ hours on watching TV and doing homework. Jessie spent only about $\frac{1}{7}$ of this time on homework, Nina $\frac{1}{3}$, and Ruka about $\frac{1}{4}$. So yes, it appears that the more TV these students watched, the less homework they did.

It is important that students realise that they can't generalise from the very small data sample available about the relationship between TV watching and homework, or even about the habits of these three students.

* For bolded terms, see introductory section.

Activity Two

Again, challenge your students in pairs or small groups to interpret these two graphs without assistance from yourself. Dot plots are visually powerful, eminently accessible graphs. Once the students have had the opportunity to delve into the meaning of the graphs, encourage them to answer questions **1** and **2**. As before, they should express their points in the form of written statements, able to be contested.

Encourage the students to use phrases or statements such as "nearly all" or "hardly any". They need to be wary of statements that include "most". Students sometimes look for the most frequently occurring value (for example, 2 hours' homework in Room 6) and make incorrect statements, such as "Most people in Room 6 do 2 hours' homework each week." (In fact, although 11 people in that room do 2 hours' homework each week, the remaining 19 people do more or less than that). Another problem associated with the word "most" is that, to some people, it means "more than half" (a simple majority) while, to others, it means "substantially more than half".

Pages 8–9: Short and Sharp

Achievement Objectives

Statistical investigation

- Conduct investigations using the statistical enquiry cycle:
  - posing and answering questions;
  - gathering, sorting, and displaying category and whole-number data;
  - communicating findings based on the data (Statistics, level 2).

Statistical literacy

- Compare statements with the features of simple data displays from statistical investigations or probability activities undertaken by others (Statistics, level 2).

Investigation	Literacy	Probability

P	P	D	A	C

Key Competencies

Short and Sharp can be used to develop these key competencies:

- relating to others
- participating and contributing.

The focus of these pages is on **stem-and-leaf graphs***. As these graphs require and reinforce knowledge of place value, you may need to refresh this concept with your students before beginning the activities.

Activity One

In this scenario, Ms Smith thinks that the trouble she has reading her students' handwriting could be related to the size of the pencils they are using. In terms of the **PPDAC cycle**, a problem is recognised and an hypothesis formulated. Data is then collected and analysed and conclusions reached.

* For bolded terms, see introductory section.

The finished graph for question **1** is shown in the Answers.

Key features of stem-and-leaf graphs:

- The numbers on the stem are equally spaced and in numerical order.
- The digits down the stem represent tens (1 represents 10, 2 represents 20, and so on).
- When the data set contains numbers less than 10, these numbers appear as leaves alongside the 0 on the stem.
- Every number must be represented by its own leaf, even if there is a leaf of the same value already there.
- The stem only needs to be long enough to accommodate the least and greatest number in the data set. (If the least number is 27, the first number on the stem will be 2; if the greatest number is 81, the last number on the stem will be 8.)
- The numbers in the stem form an unbroken sequence, even if there are no leaves to attach to some of them.

Unless the data set is very small, it is usual to create a stem-and-leaf graph in two stages: first, enter the leaves as they appear in the data and second, reorganise them in numeric order.

When answering question **2**, remind the students of the original problem: Ms Smith thought that short pencils might be making her students' writing hard for her to read. How can they decide what "too short" means?

Question **3** involves students replicating the fictional scenario by gathering real data from their own classmates and then reaching a conclusion. Stickies are a convenient way to make the stem-and-leaf graph in this question. The students measure their pencils to the nearest centimetre and then write just the ones digit for the length on a sticky. The stem of the stem-and-leaf graph is drawn on the whiteboard, and the students come up with their stickies and locate them on the correct "branch". The stickies can easily be organised in numeric order as a second step in the process. Students should be able to identify exactly where their particular piece of data has got to in the finished graph.

Activity Two

These three investigations are designed to embed learning about stem-and-leaf graphs. As elsewhere, it is vital that the students do not stop with the creation of a graph. Even in presumably random data, such as house numbers and birth dates, there will be patterns to be discovered ("Look: eight of us have birthdays on the 12th of the month!"). As part of the Conclusion phase of the PPDAC cycle, the students need to think about what meaning, if any, lies behind these patterns.

Activity Three

Activity Three introduces the students to a simple **probability** task. Unlike the previous activity, where everyone was working with the same data sets, in this activity, everyone does the identical task but comes up with a different data set and a different graph. This does not mean, however, that there will be no similarities and patterns. For example:

- Everyone's data will certainly be between 1 and 36.
- The 20s and 30s branches are likely to be notably shorter than the units and 10s branches.
- The units branch is likely to be the longest of all.

If the students were to pool their data, it is likely that they would find that about half the leaves on their graph were units leaves.

Challenge your students to explain these patterns. To do so, they could create a product table like the one below and relate it to their data.

	1	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

Pages 10–11: Open or Closed?

Achievement Objectives

Statistical investigation

- Conduct investigations using the statistical enquiry cycle:
  - posing and answering questions;
  - gathering, sorting, and displaying category and whole-number data;
  - communicating findings based on the data (Statistics, level 2).

Statistical literacy

- Evaluate the effectiveness of different displays in representing the findings of a statistical investigation or probability activity undertaken by others (Statistics, level 3).

Investigation	Literacy	Probability

P	P	D	A	C

Key Competencies

Open or Closed? can be used to develop these key competencies:

- thinking
- relating to others.

The activities on these pages are designed to illustrate some simple principles relating to survey questions.

Activity One

This activity helps the students to differentiate between closed and open questions. The simplest closed questions are those that require a yes/no (or sometimes, a yes/no/don't know) response. A poll of class members, using a single closed question (for example, "Do you have a pet?") would immediately show the limited value of data gained in this way.

Question **1** shows that closed questions limit the kinds of responses people can make. This is often an advantage. It certainly makes the collation of **category data*** much easier. But care needs to be taken when deciding on the options that are to be offered. If the options offered do not cover the needs of most respondents, people will get frustrated and refuse to take the survey seriously, or they will choose the "other" category. If lots of people choose the "other" option, this undermines the value of the rest of the responses. The risk of this happening can be reduced by trialling survey questions before using them. This will usually reveal badly worded questions ("What is this question asking?") and shortcomings in the options being offered ("I can't tick any of these answers.").

Activity Two

Activity Two is designed to get students thinking about the wording of questions and the options made available. Questions **1** and **2** illustrate the importance of identifying the *purpose* for which information will be used rather than haphazardly dreaming up questions and options. The students get the opportunity to take this learning into account and to consolidate their understanding when they conduct their own small surveys in the **Investigation**.

Pages 12–13: My Town

Achievement Objective

Statistical literacy

- Evaluate the effectiveness of different displays in representing the findings of a statistical investigation or probability activity undertaken by others (Statistics, level 3).

Investigation		Literacy		Probability
P	P	D	A	C

Key Competencies

My Town can be used to develop these key competencies:

- thinking
- using language, symbols, and texts.

Activity

This self-contained activity revolves around the data depicted in the illustration on page **12** of the students' book. Students process and explore it in different ways as they determine the answers to questions and the validity of statements.

As a first task, the students enter the data into a spreadsheet. This is to get it into a form that can be easily sorted and re-sorted in whatever order is most helpful for the purpose. For example, to answer Keni's second "**I wonder**"* question, it is helpful to sort the data firstly by number of people and secondly by number of bedrooms, using the Sort function:

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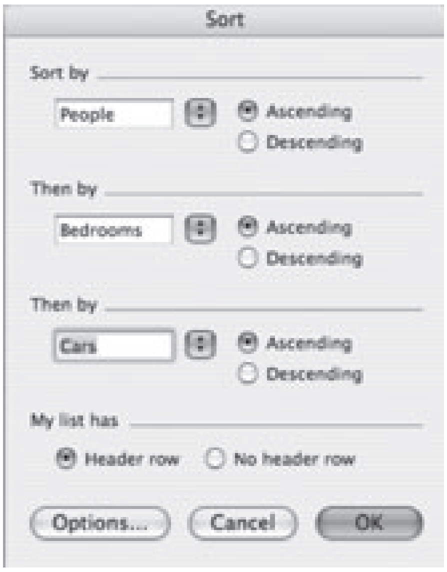
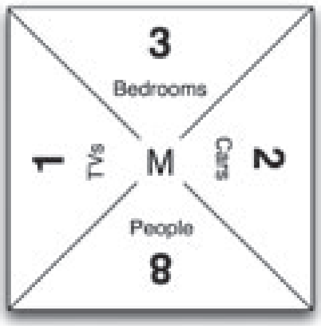
* For bolded terms, see introductory section.



The Answers show a table that has been sorted in this way. The grey bands group the lines according to household size.

An alternative method of sorting the data is to make a set of square data cards, one for each house, as in the example. Just as in the spreadsheet, the data cards can first be sorted by one **variable** (for example, number of people) and then by a second. They can easily be re-sorted to answer other queries.



Note that the answer to Keni's first "I wonder" question is very straightforward. This is not true of his second question, where the answer can be either yes or no, depending on how the question is interpreted. This invites mathematical disputation.

The statements in question **3** are designed to get the students to explore the database in some detail, to decide what data supports or refutes each statement, and to think about the personal circumstances that the statistics purport to represent.

Question **4** asks about generalising from the data. This challenges the students to think about whether Keni's data is likely to be representative of the whole town. If they think about their own community, town, or city, they are likely to realise that it is made up of different areas, each with its own distinct focus and character. They will realise also that amenities are spread around and that, just because there isn't (for example) a movie theatre in their immediate neighbourhood, this doesn't mean there isn't one on the other side of town.

This entire activity is best done in pairs or small groups. In this way, students are able to have unsubstantiated statements challenged and learn to use data as the basis of argument.

The students could be challenged to think of further "I wonder" questions or further statements, and to swap these with other students to answer or support/refute, using Keni's data.

Pages 14–15: Discussing Data

Achievement Objective

Statistical literacy

- Evaluate the effectiveness of different displays in representing the findings of a statistical investigation or probability activity undertaken by others (Statistics, level 3).

Investigation	Literacy	Probability

P	P	D	A	C

Key Competencies

Discussing Data can be used to develop these key competencies:

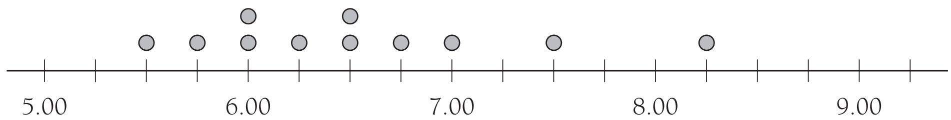
- thinking
- using language, symbols, and texts.

Activity

Note that the second part of this activity requires access to a computer spreadsheet/graphing program. See the introductory section for basic information on the various graphs encountered in this activity and elsewhere in this book.

The data gathered by Areta is **category data***, so she has used a **pictograph** and a **pie chart**. Both graphs are appropriate for Areta's data, and each has its strengths: the pictograph shows the *number* of people using each form of transport; the pie chart shows the *proportion* of people using each form of transport. Both clearly show that bus and car are the two main means of transport, but the pie chart does a poor job of distinguishing between the two (the two sectors look to be very similar in area). For this reason, the pictograph is probably the better graph.

Leilani has collected **numeric data**. Her displays are a **bar graph**, a table, and a **stem-and-leaf graph**. Neither the bar graph nor the table identifies patterns in the data, whereas the stem-and-leaf graph clearly shows that most (in the sense of more than half) of the students began dinner between 6.00 p.m. and 7.00 p.m. The bar graph could have been improved by arranging the bars in order of size, as is standard practice when the order of the categories is not relevant. A **dot plot** would have been another useful way of displaying this data:



Rory has collected category data, which he has displayed as a bar graph, a table, and a pie chart. The months are the categories. The bar graph shows a clear pattern in the rainfall, and it is easy to identify the wettest and driest months (August and January respectively). This information is all available in the table, but the patterns have to be searched for. The pie chart doesn't answer Rory's question at all because the data is collated by season instead of month.

Investigation

Computer graphing programs make it easy to experiment with different displays for any set of data without the time-consuming process of manually drawing them all. The students can enter the data as a table, select the number data, and choose a chart option from the menu. The same data can be dynamically linked to several different graphs at the same time. The students can

* For bolded terms, see introductory section.

experiment with changing the data in their table and observe how the various graphs change. Titles and **axis** labels can be added as a second step, when it has been decided what kind of graph best displays the data for the purpose in hand.

Encourage your students to explore the many different options available. It is important that they understand that different graphs will be appropriate for different situations and that sometimes more than one graph is needed to highlight different features of the data. This is particularly true of **multivariate data**.

Students also need to understand that many of the fancy chart options available in computer graphing programs are never going to be suitable for any of their purposes. This is true for most of the 3-dimensional charts. While they may look nice, their appearance comes at the cost of readability. The best graphs are simple and uncluttered.

Pages 16–17: Where on Earth?

Achievement Objective

Statistical literacy

- Evaluate the effectiveness of different displays in representing the findings of a statistical investigation or probability activity undertaken by others (Statistics, level 3).

Investigation	Literacy	Probability

P	P	D	A	C

Key Competencies

Where on Earth? can be used to develop these key competencies:

- using language, symbols, and texts
- relating to others.

Activity

Rather than start with a session on "the rules of graphs", challenge your students to look critically at the graphs in question **1** and:

- work out what information they do and do not convey;
- suggest changes that would improve their communicative power.

Once the students have read the context on page **16**, they can cover or fold this page back and not refer to it again until they have gone through the five graphs one by one and answered (as a series of statements) the question "What does this graph tell me?" Encourage them not to make assumptions (for example, not to assume that the numbers on a vertical scale are people, even though in this case they are).

They can then uncover page **16** and check each of the statements they have written against the actual data. What information got lost in each graph? What information got distorted?

Finally, they can make recommendations on steps that would improve each graph.

This is an appropriate pair or small-group activity. Each of the graphs presents its own issues. Some of these are discussed in the Answers.

Note that **iv** presents a particular issue that the students may not have enough experience to recognise; namely, the type of graph chosen is not suitable for the type of data.

Line graphs* are used to show increases, decreases, and trends in data *of the same kind*. In line graphs, the scale on the horizontal **axis** has a fixed order (for example, Jan, Feb, Mar, Apr, …) and the line segments that join the various data values (points) have meaning (for example, an upwards slope always means an increase of some kind).

In **iv**, the data is **category data**. The categories have no inbuilt order: they could be organised alphabetically, randomly, or according to **frequency**. (While the latter is usually preferred, this is for reasons of clarity, not mathematics.) In this graph, the lines joining the various data values have no meaning. (Why should Holland be linked to New Zealand and Cambodia, for example?) Whether the lines go up or down is entirely a result of the order in which the categories are placed.

In summary, line graphs should not be used for category data.

Pages 18–19: Logo Appeal

Achievement Objectives

Statistical investigation

- Conduct investigations using the statistical enquiry cycle:
  - gathering, sorting, and displaying multivariate category and whole-number data and simple time-series data to answer questions;
  - identifying patterns and trends in context, within and between data sets (Statistics, level 3)

Statistical literacy

- Compare statements with the features of simple data displays from statistical investigations or probability activities undertaken by others (Statistics, level 2).

Investigation	Literacy	Probability

P	P	D	A	C

Key Competencies

Logo Appeal can be used to develop these key competencies:

- managing self
- participating and contributing.

Activity

This activity illustrates some of the difficulties associated with drawing conclusions from different data sets in which the categories are not the same across all sets.

You could introduce the activity by asking your students what they can deduce about the students' approach. It seems likely that:

- their question was "What feature of our school is most important to you?" (or similar)
- they offered a range of options and asked each person to select just one (although each group votes for a slightly different range of features, they are all consistently named)
- they asked groups of about 30 students, parents, and other community members and half as many teachers.

For question **1**, the features that the students think should be included in a logo will vary, depending on how they analyse the information and, perhaps, their own values. What is important is that they can support their recommendations from the data.

* For bolded terms, see introductory section.

A possible aid to analysing the data is to enter the votes for each feature in a table:

	Community	**Teachers**	**Parents**	**Students**	**Total**
Aorangi River	9				**9**
Learning	9	5	9		**23**
Happy students	5	4	2	9	**20**
Good teachers	4	1	6	6	**17**
Tidy and clean	2	2	5	2	**11**
Safe		4	4	9	**17**
Lots of sport			3	3	**6**
Books and computers			1		**1**
Camps				2	**2**
Totals	29	16	30	31	106

The table reveals that:

- four features attracted the most votes: learning, happy students, good teachers, and safety. Of these, learning and happy students received the most votes of all.
- the highest-polling feature, learning, received no votes from the students surveyed.
- there are only half as many votes by teachers as the other groups. This means that the voting system gives more weight to the views of the other three stakeholder groups.
- even if the teachers' votes were doubled, this would not affect which features are the top four.

Two possible recommendations would be:

- learning, happy students, and good teachers (omitting safety on the grounds that students won't be happy unless they are safe);
- happy students, good teachers, and safety (omitting learning on the grounds that students didn't vote for this feature and/or if the school has good teachers, the students will be learning).

For question **2**, refer to the Data Detective poster (see the introductory section) and the Answers.

For question **3**, the students may need to check with you or a classmate to ensure that the question/issue they have selected is appropriate. The investigation needs to have a limited scope to ensure that the students can complete it.

Page 20: Tally Ho!

Achievement Objective

Statistical literacy

- Compare statements with the features of simple data displays from statistical investigations or probability activities undertaken by others (Statistics, level 2).

Investigation	Literacy	Probability

P	P	D	A	C

Key Competencies

Tally Ho! can be used to develop these key competencies:

- thinking
- managing self
- relating to others.

Activity

If students have not encountered **tally marks***, iceblock sticks provide an excellent visual model. Have the students lay down one stick for each item counted. When they get to the fifth item/count, they lay the stick across the others to make the group of 5. The convention of grouping tally marks in 5s facilitates skip-counting by 5s and 10s.

A tally chart normally sets data out in three columns: **category**, **tally**, and total (**frequency**).

For question **1**, encourage the students to get beyond simple statements ("Four people brought fruit juice") to statements that involve comparisons (see the Answers for suggestions).

Note also that, while students can say with validity that "sandwiches are the item most sold", they can't say "most of the students in our class buy sandwiches".

Questions **2** and **3** allow plenty of scope for class discussion. The issues raised go beyond mathematics and statistics and could usefully be linked to cross-curricular studies involving food and nutrition and even economics.

Question **4** provides another opportunity for students to practise the statistical enquiry cycle.

* For bolded terms, see introductory section.

Page 21: Sneaky Snakes

Achievement Objective

Probability

- Investigate simple situations that involve elements of chance by comparing experimental results with expectations from models of all the outcomes, acknowledging that samples vary (Statistics, level 3).

Investigation	Literacy	Probability

P	P	D	A	C

Key Competencies

Sneaky Snakes can be used to develop these key competencies:

- thinking
- using language, symbols, and texts.

Activity

This activity involves the students playing a simple game of **chance*** a minimum of 20 times. Using a **tally chart** (see copymaster), they record the number of throws of the dice it takes to complete each game. They then summarise their data in a second tally chart.

Before they start playing the game, challenge the students to think about these questions:

- *What is the least number of throws it could take to complete a game?* (2)
- *What is the maximum number of throws it could take to complete a game?* (12)

After they have played a few games, challenge the students to think about these questions:

- *Which numbers of throws (between 2 and 12) are likely to be very rare events?* (7 or more)
- *How many throws will it take to complete most games?* (3 to 5)

When the students have played the suggested 20 games, they summarise the **outcomes** in the second tally chart. This will almost certainly reveal that nearly all games are completed in 3 to 5 throws. If a number of groups are playing the game, they can pool their data and then see if the collated results confirm this pattern.

* For bolded terms, see introductory section.

The second tally chart might look like this when completed:

Number of throws	**Tally**	**Number of games (total = 20)**
1		
2	I	
3	卌 I	
4	卌 II	
5	IIII	
6	I	
7	I	
8		
9		
10		
11		
12		

It's not possible to complete the game with a single throw.

This means that in 6 of the 20 games, it took 3 throws of the dice to get to the finish square.

For your information, a computer simulation of 100 games produced this data:

Number of throws	**Number of games**
1	0
2	1
3	33
4	33
5	28
6	4
7	1
8	0
	100

The distribution of the data in this table is likely to be typical.

Encourage the students to try and explain the pattern they have found. They might say:

- "There are lots of ways to get 10 or over with three, four, or five throws."
- "If you get a low number first, you often get a high number next, and that gives you about 6 or 7 from two throws."
- "To take more than five throws, you have to get mostly ones or twos, and it's not often you get a run of ones or twos."

Students who are ready for a challenge could probe a bit deeper. They could start by recognising that, for a game to take 12 moves, they would need to get 11 ones (no matter what the 12th number is, the game finishes at that point). If just one of those 11 throws is a 2, not a 1, the game would finish in 11 moves, and if just one of those throws is a 3 (or two are 2s), the game would finish in 10 moves … Such sequences are likely to be very rare indeed.

Important ideas:

- For a given **probability** event, we can often predict the range of possible **outcomes**.
- If we repeat a probability **experiment** enough times, patterns will always emerge.
- From these patterns, we will be able to say that some outcomes are more likely than others.
- Using these patterns, we may be able to put numbers on the probabilities of the various outcomes.
- Even if we know the probabilities of the different outcomes, we can never be sure what will happen in any particular trial.

Page 22: Way to Go

Achievement Objective

Probability

- Investigate simple situations that involve elements of chance, recognising equal and different likelihoods and acknowledging uncertainty (Statistics, level 2).

Investigation	Literacy	Probability

P	P	D	A	C

Key Competencies

Way to Go can be used to develop these key competencies:

- thinking
- using language, symbols, and texts.

Activity One

Strictly speaking, this activity does not involve **probability*** in that there is no random event: the decision to take one route to school, rather than another, is made on the basis of choice, not chance. This choice will be based on a matrix of factors that may include, for example, weather, friends, lateness, mood, transport, safety.

But there are many chance situations that can only be explored by first setting out all the possible **outcomes**, and this activity gives students the opportunity to do this.

It is clear that Tom, who lives nearest the school, has only one south or east route he can take: east for one block. This route could be symbolised with a single arrow facing east (→).

Suzie's routes all involve one easterly block and three southerly blocks. The four possibilities can be symbolised in this way: →↓↓↓, ↓→↓↓, ↓↓→↓, and ↓↓↓→.

Similar reasoning can be used to find the number of routes for the other students:

Rima: →→↓, →↓→, and ↓→→ (three routes).

Hong: →→↓↓, ↓→↓→, ↓→→↓, →↓↓→, →↓→↓, ↓↓→→ (six routes).

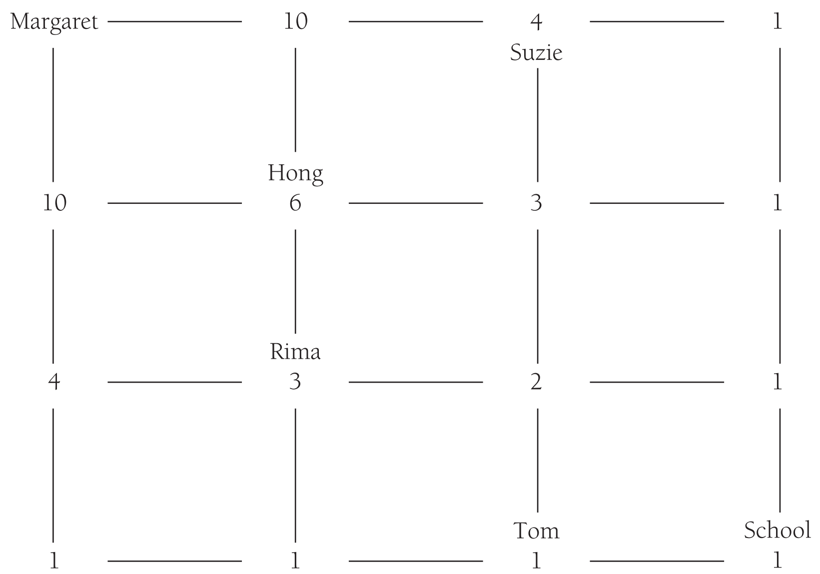
* For bolded terms, see introductory section.

Most students will find it a considerable challenge to write down all the possible routes for Margaret. They will need to be very systematic or they will have trouble working out which routes they have overlooked. There are 20 possibilities. If your students are up for the challenge, don't tell them how many routes there are; let them try and work it out for themselves:

1.	→	→	→	↓	↓	↓	11.	↓	↓	↓	→	→	→
2.	→	→	↓	→	↓	↓	12.	↓	↓	→	↓	→	→
3.	→	↓	↓	↓	→	→	13.	↓	→	→	→	↓	↓
4.	→	→	↓	↓	→	↓	14.	↓	↓	→	→	↓	→
5.	→	→	↓	↓	↓	→	15.	↓	↓	→	→	→	↓
6.	→	↓	→	→	↓	↓	16.	↓	→	↓	↓	→	→
7.	→	↓	→	↓	→	↓	17.	↓	→	↓	→	↓	→
8.	→	↓	→	↓	↓	→	18.	↓	→	↓	→	→	↓
9.	→	↓	↓	→	→	↓	19.	↓	→	→	↓	↓	→
10.	→	↓	↓	→	↓	→	20.	↓	→	→	↓	→	↓

Note that, in the table above, the second 10 routes can be obtained simply by swapping east and south. This kind of symmetry can often be observed when listing all possible outcomes for a probability **experiment**.

This diagram shows how many routes there are from each intersection:



The number at the top left-hand corner of a square is the number of possible routes to school. It is found by adding together the numbers at the top-right and bottom-left corners of the same square. So Margaret has 10 + 10 = 20 different routes (see two halves of the table above) to school. You may recognise that the grid is in fact a rotated Pascal's Triangle.

Activity Two

In the bottle-top throw, the only way to determine the likelihood of the outcomes is by **trialling**. This is because, unlike with a coin, there is no reason why a face-up and a face-down are equally likely and no way of **theoretically** determining the probability of an edge.

Encourage the students to record the results systematically in a **tally chart**. They are likely to find that the bottle top lands face up most of the time (due to the top's centre of gravity) and that an edge, while not impossible, is very rare. The reasons for this would make a good discussion point.

You could suggest that students pool their results to obtain a larger data set. If they do this, they will be able to reach more definite conclusions about the probabilities of the three outcomes. They should express these probabilities using numbers. For example, "You get a face-up about 12 times out of 20 trials."

Page 23: Which When?

Achievement Objective

Probability

- Investigate simple situations that involve elements of chance by comparing experimental results with expectations from models of all the outcomes, acknowledging that samples vary (Statistics, level 3).

Investigation	Literacy	Probability

P	P	D	A	C

Key Competencies

Which When? can be used to develop these key competencies:

- thinking
- relating to others
- managing self.

Investigation

This activity requires two spinners, for which patterns are provided in the copymaster. These are operated by flicking a paperclip around a pencil point. (An alternative is to make hexagonal spinners out of thin card and punch toothpicks through the centres. By using a hexagon instead of a circle, the paperclip will always stop in such a way that the **outcome*** is obvious.)

Questions **1** and **2** invite the students to think about and then **experiment** with the two spinners. Some may think that the spinner that is divided into 6 regions will give them more opportunities to get a preferred outcome. If this is the case, ask them to explain why and encourage students who hold the opposing view to engage with them in mathematical debate.

Unlike the previous activity involving the bottle top, the **probabilities** in this case can be determined mathematically: the sectors in the spinners are defined by identical angles. This means that the spinner has an identical chance of stopping at any of them. Each sector on the first disc is one-third of the total disc. The sectors on the second disc are half that size, but there are two for each option, so each option still "owns" one-third of the whole.

* For bolded terms, see introductory section.

Question **3** requests a 5-day simulation. This is too small an experiment to get meaningful data. But if students do a number of 5-day simulations and then pool their data with that of others, they will (a) see **variation** at work and (b) have enough data on which to make probability statements with some confidence.

When they have completed the set tasks, the students could be given the task of constructing spinners that favour particular outcomes, for example, a spinner that gives Sam a one to two (1:2) **chance** of getting to watch television (the spinner needs to have three equal sectors) or a one to three (1:3) chance of having to do homework (the spinner needs to have four equal sectors). (See the explanation of "ratio" in the activity below.)

Page 24: In Between

Achievement Objective

Probability

- Investigate simple situations that involve elements of chance, recognising equal and different likelihoods and acknowledging uncertainty (Statistics, level 2).

Investigation	Literacy	Probability

P	P	D	A	C

Key Competencies

In Between can be used to develop these key competencies:

- thinking
- participating and contributing.

Game

This simple game of **chance*** provides a scenario in which students can easily analyse the **probabilities** for any given hand. Only one set of digit cards is needed: each person completes their turn, returns their three cards to the pile, shuffles the pack, and hands the complete set to the next person. If preferred, however, each player could have their own set of 10 cards.

Activity

In the game shown, 4 and 9 are the two cards face up. There are 8 cards left in the pile. Out of the 8 cards, four {5, 6, 7, 8} will satisfy the "in between" requirement. This means that the probability that the player will pick a winning third card and gain a counter is 4 in 8 (or 4 out of 8).

"4 in 8" can be expressed as $\frac{4}{8}$ or $\frac{1}{2}$ or 0.5 or 50%.

"4 in 8" can also be expressed as a ratio, in which the number of ways of winning is compared with the number of ways of losing, in this case 4:4, which can be simplified to 1:1. Gamblers sometimes say such "odds" are "even" or "fifty-fifty".

* For bolded terms, see introductory section.

A number strip that is labelled for the remaining cards and has the winning **outcomes** shaded can be used to demonstrate these different ways of naming the probability in this situation.

0	1	2	3	5	6	7	8

Note that, if your students are using ratios, they must *not* make the mistake of thinking that $\frac{4}{8}$ = 4:8. 4:8 describes a situation in which the probability of losing is twice the probability of winning; in other words, a situation in which the probability of winning is *not* $\frac{4}{8}$ or $\frac{1}{2}$ but $\frac{4}{12}$ or $\frac{1}{3}$.

This table sets out the possible winning cards for Trevor, Katie, Barbara, and Marama, together with the probability of success in each case:

Player	**Possible winning cards**	**Probability of success**	**Probability as a fraction and ratio**
Trevor	6, 7, 8	3 out of 8	$\frac{3}{8}$ or 3:5
Katie	4, 5	2 out of 8	$\frac{2}{8}$ ($\frac{1}{4}$) or 2:6 (1:3)
Barbara	5, 6, 7	3 out of 8	$\frac{3}{8}$ or 3:5
Marama	1, 2, 3, 4, 5	5 out of 8	$\frac{5}{8}$ or 5:3

It can be seen that Marama has the greatest chance (more than even) of success.

Copymaster: Choice Squares

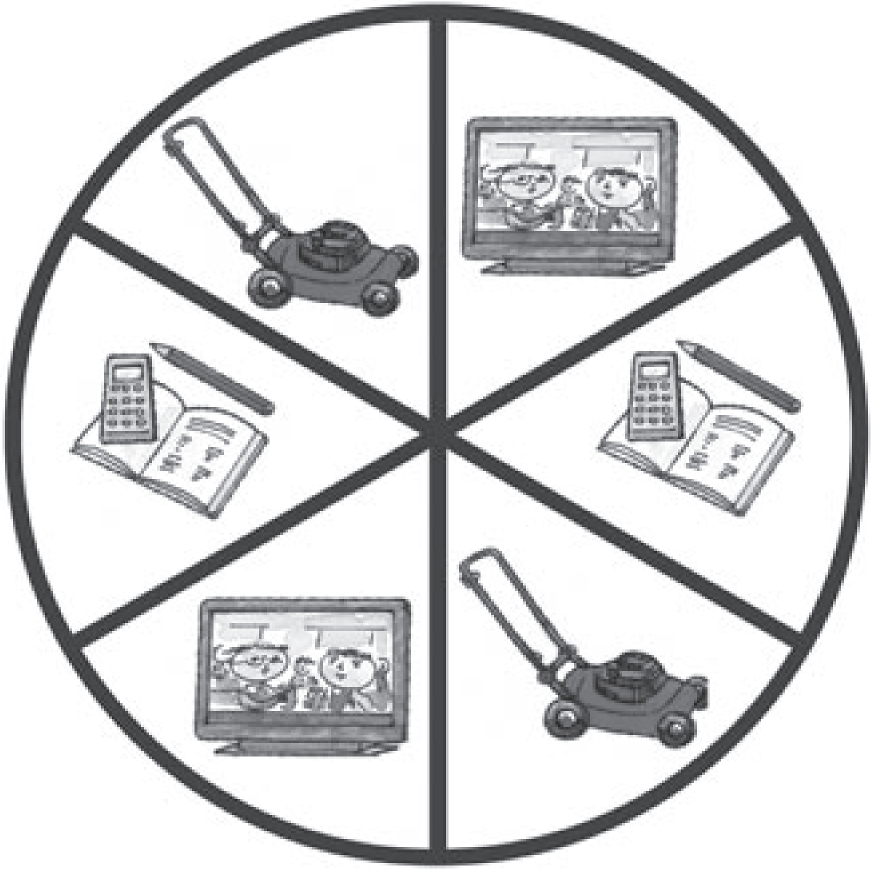
Copymaster: Sneaky Snakes

First tally chart

Game	Tally	Number of throws
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		

Second tally chart

Number of throws	Tally	Number of games (total = 20)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		



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