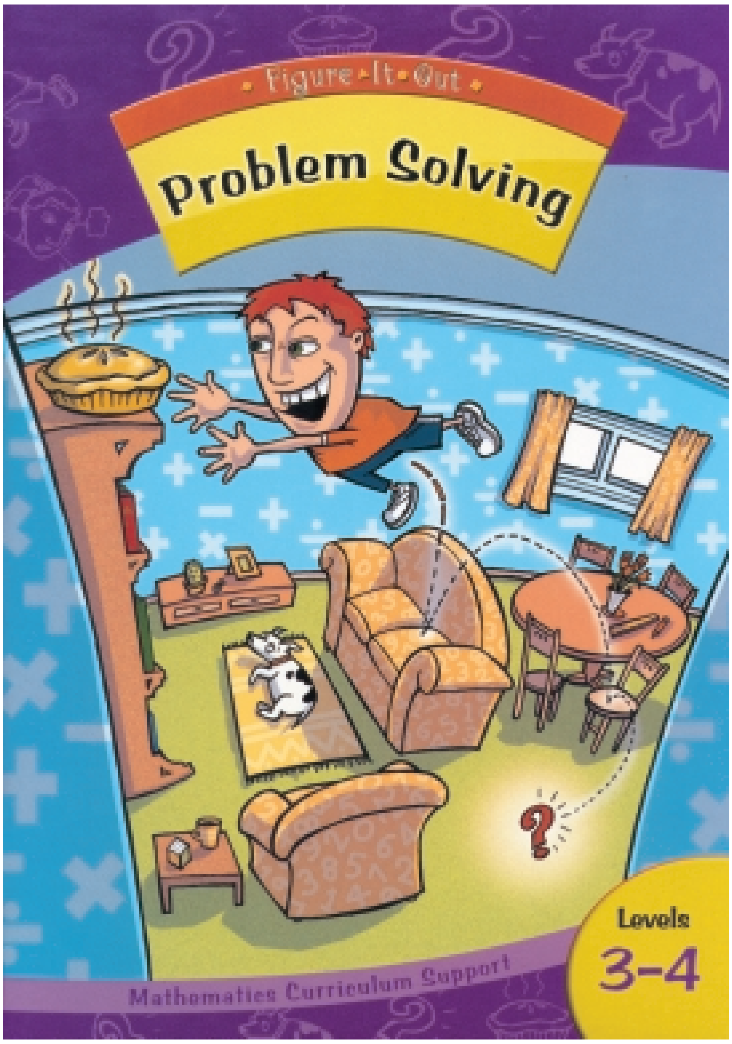
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Contents

Introduction

The Figure It Out series is designed to support *Mathematics in the New Zealand Curriculum.* The booklets have been developed and trialled by classroom teachers and mathematics educators. The series builds on the strengths of a previous series of mathematics booklets published by the Ministry of Education, the School Mathematics supplementary booklets.

Figure It Out is intended to supplement existing school mathematics programmes and can be used in various ways. It provides activities and investigations that students can work on independently or co-operatively in pairs or groups. Teachers can select particular activities that provide extension to work done in the regular classroom programme. Alternatively, teachers may wish to use all or most of the activities in combination with other activities to create a classroom programme. The booklets can be used for homework activities, and the relevant section in the teachers' notes could be copied for parents. These notes may also provide useful information that could be given as hints to students. The teachers' notes are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/community

There are eight booklets for levels 3–4: one booklet for each content strand, one on problem solving, one on basic facts, and a theme booklet. Each booklet has its own *Answers and Teachers' Notes*. The notes include relevant achievement objectives, suggested teaching approaches, and suggested ways to extend the activities. The booklets in this set (levels 3–4) are suitable for most students in year 6. However, teachers can decide whether to use the booklets with older or younger students who are also working at levels 3–4.

The booklets have been written in such a way that students should be able to work on the material independently, either alone or in groups. Where applicable, each page starts with a list of equipment that the students will need to do the activities. Students should be encouraged to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Teachers could encourage students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Where possible, suggestions have been made to encourage discussion and oral presentation of answers, and teachers may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. Successful ways of solving problems should be acknowledged, and where more effective or efficient processes can be used, students can be encouraged to consider other ways of solving the problem.

Figure It Out

Problem Solving

Hints for Students

Page 1: To the Wire

1. Start by finding the sum of the numbers from 1 to 12. Then work out the total that the numbers in each region should add up to. What goes with 12?
2. How much profit did Melody make on *each* of her deals?
3. You need to work systematically here. First count the number of triangles made up of one triangle, then two, then three, and so on.
4. What might you hope to do every time you cut this cheese? (Think of fractions.) Can you always do it?

Page 2: Money and More

1. You might use trial and improvement here. You can be more systematic if you use a table.
2. Try drawing a net diagram, which you can use for constructing a cube. Now fill in the icons on each face of the net.
3. Sometimes it pays to work backwards through the list of clues. Try this.
4. Try to solve this problem in your head by doing it backwards. If this is difficult, do the problem with a square of paper and scissors.

Page 3: Using Your Head

1. a. This is really about numbers. How can you get 6 using 2, 5, and 9? Or is there some other piece of information that you know about Cuisenaire rods that might be useful?

   b. What is the longest possible length? Can you make all the lengths 1, 2, 3, and so on up to that length? Why? Why not? You might not need all three rods.
2. Look for the pattern in the left-side factors (3, 6, 9, 12, …) and the pattern in the products (111 111, 222 222, …). Can you see a connection between the left-side factors and the products?
3. Work systematically. A tree diagram could help.
4. Look for a systematic way to calculate the total.

Page 4: Snap!

1. Try working backwards. If that doesn't help, try using a table and being systematic through trial and improvement.
2. Try drawing a diagram.
3. Try using a table, being systematic, trial and improvement, or working backwards.
4. What different-shaped rectangular chocolate blocks of eight pieces could you make? Try drawing a diagram.

Page 5: Finding a Way

1. How many days of bed making does it take for Aroha to make 30 cents? How many days of making and not making her bed does it take for her to break even?
2. You could guess and check, but there are a lot of places on the map to check. How could you cut the guessing down?
3. How many of his crew have a peg leg? How many boots will they need? Now work out the boots for the rest of the crew.
4. How long is one bead of each shape?

Page 6: Lining Up

1. If you made 6 by adding some of these top numbers, which cards would the 6 be on?
   How many ways can 15 be made by adding these top numbers? What cards is 15 on?
   Can you see a pattern in the top numbers of the cards?
2. How many different letters are there?
   How many different digits are there?
   How can you find out how many combinations there are of all these letters and digits?
   If you are still stuck, look at **Problem Three** on page **3**.
3. Spend some time at first making sure you understand the question.

   For example, what exactly does "the girls have twice as many sisters as brothers" mean?
   You might find it easier to say "the boys have three times as many sisters as brothers" rather than saying "one-third".

   You could guess and check this one or use a table.
4. Try working upwards from four to five and then to six streets.

Page 7: Following On

1. Experiment with different groups of three consecutive numbers. What happens every time?
2. You could draw the next figure or you could look for patterns in the number of squares.
3. You could work backwards, draw a diagram, or practise with a photocopy of the shape.
4. Think of this in terms of rounds of calls.
   How many people will be contacted after the first round? After the second round? How many rounds will be needed? (This is a good problem to try a smaller case first.)

Page 8: Eye Antics

1. Experiment with some sticks.
   How can you be sure in **b** that you have moved the smallest number of sticks?
2. How big are the pieces? What shape can they be?
3. Experiment and work systematically.
4. Ways that are the same as others when rotated or reflected are not counted as different ways.

Page 9: Filling the Gaps

1. Experiment and work systematically.

   Try placing two more trees to start with.
   Draw as many lines as you can. Can you see a place to put the last two trees that means there will be 10 lines?

   The arrangement will have two lines of reflectional symmetry (mirror lines) that are at right angles to each other.
2. How much would 100 grams from each packet cost?
3. How can you get the next answer from the last answer?
4. There are many answers to this problem, but in all the answers, the digits in the sum add up to 18.
   Remember that you will probably have to carry over digits from one column to the next.
   Try guess and check.

Page 10: Working Together

1. Try working backwards.
2. How many cubes were used to make each side?
   Are any of the cubes used for two sides?
   You could try a smaller case first.
3. How many stamps have they got altogether?
   How many stamps have they doubled up on?
4. Can you work out the total number of cats and parrots, the total of dogs and parrots, and the total of cats and dogs? This will help you as you use trial and improvement.

Page 11: Looking Ahead

1. What does $^{1}/_{7}$ look like as a decimal?
   Try a smaller case than the 50th place.
   Can you see a pattern?
2. You could try working backwards, use equipment, or perhaps use trial and improvement.

3. Use equipment such as play money.
Then use trial and improvement and record your results on a table.
Remember that you are looking for the minimum number of coins.

4. Experiment with small cases first, for example, a square. Then look for a pattern.

Page 12: In and Up

1. Use trial and improvement and record your results on a table.

2. How much will it cost to park for 15 minutes?

3. Experiment with taking the bags and barrels up in different combinations.
Modelling the problem or using a table could help.

4. How can you use multiplication and division basic facts to help you?

Page 13: Back to Back

1. Try a smaller case first and try using equipment.

2. Answers will vary depending how the paper is folded and cut.
You could check your work by putting page numbers on a piece of paper, folding it into a book, stapling and cutting the pages, and checking that the page numbers are in the right order.

3. Find all possible round trips – there aren't that many.

4. This problem is really about numbers.
See **Problem Three**, page **3**.

Page 14: Creases and Cuts

1. You could use trial and improvement and record your results on a table.

2. Try using equipment or drawing a diagram.
Work systematically and experiment with making cuts from different parts of the square.
Use a similar method for the hexagon.

3. Think. Try to visualise the problem.
Fold a strip of paper or try a smaller case first if you are having difficulty.

4. Try a smaller case first (for example, if just rice and soup were served). Can you find a pattern that will help you work out the larger problem?

Page 15: In Line

1. Take each piece of information step by step and build up the queue.
It might be useful to use equipment or draw a diagram to do this.

2. Use equipment if you can't imagine what is going on by looking at the model, or you could use a diagram.

3. You could find the sums of the two lots of consecutive numbers given in the question (3, 4, 5 and 29, 30, 31).
Can you see a pattern?
Why do you think this works?

4. Use trial and improvement and record your results on a table.

Page 16: Dates and Shapes

1. Think carefully about all the numbers you can use and then work systematically.

2. What do you need to know? How many days are there in a year? What happens in leap years?

3. Use a diagram, or use equipment to find different-shaped tetrominoes.
Then experiment (but work systematically) to make the rectangle.

4. Try a smaller case first. For example, how can you measure 2 litres with these buckets?

Page 17: Out for the Count

1. Use each clue step by step. Be systematic.

2. Identify the rhombuses and trapezia in the hexagon.
Now find a systematic way to count them.

3. How many of each letter are there?
Can you see a pattern?

4. Can you see a right angle? For part **b**, make sure you have a perfect right angle each time.
You could experiment with equipment or a diagram.

Page 18: You're All Heart!

1. How can you find an amount that can be divided evenly by 2, 3, or 4?
Remember that you have to be able to make the amount in coins.
2. Try using equipment or drawing a diagram.
Can you see a pattern in the number of cubes in each layer?
3. Try an easier problem first. For example, how much blood does your heart pump in a minute?
Build up.
4. What distance does Sam's bike wheel travel when it goes around once?

Page 19: What Goes Around ...

1. Where is Billy after Millie's first lap, second lap, and so on?
You could act this problem out, draw a diagram, or use equipment.
2. Can you find a more efficient strategy than trial and improvement?
Try adding all the numbers together.
What will the sum of each set of four numbers be?
3. Can you find a pattern in the number of counters in each side and the total number of counters in each pentagon?
4. Work systematically to find all the possible numbers.

Page 20: Going Around

1. Try placing the 1 first. Work systematically to fill in the other squares.
Experiment with different arrangements of numbers.
2. Can you find a quick way to count the tracks?
Try a smaller case first.
3. Start by counting all the marbles. What will you need to do to find out how many marbles need to go in each jar?
You could use trial and improvement to find the answer for part **b**, but can you find a quicker way to find the answer?
4. Experiment. Possibly try a smaller number of patties first.
For part **a**, cooking four patties on both sides and then the remaining two patties on both sides is *not* the quickest way to cook the patties.

Page 21: Round Robins

1. How can you make up 10 with even numbers?
How many cartons will go in each column?
How many will go in each row?
2. Use equipment or a drawing.
Cut up the shapes. Start with the fractions that you do know.
3. Notice that the page numbers are consecutive numbers.
4. You could draw a diagram.
Can you see a quick way to count the games?
Remember the Circuit Island tracks in **Problem Two**, page **20**?

Page 22: Dining on Digits

1. How wide is a 5 cent coin?
If you put the two 5 cent coins side by side, how wide is that? What about ten 5 cent coins?
How many 5 cent coins in $25?
2. Look carefully at the 9 in the picture.
What would happen if you turned it upside down?
Work systematically to put the digits on the cubes.
You could try a smaller case first.
3. To get the largest number of pieces, what do you want to do each time you make a cut?
4. How many tables will each group need?
Work systematically.

Page 23: Waste Not ...

1. Make a table.
2. How many times does 12 go into 30?
3. Work through this problem systematically and carefully.
How many bins can six boys fill in 12 hours?
4. You could use equipment or a drawing.
Check that you have counted all the bottles.

Page 24: Mystery Amounts

1. You could use trial and improvement.
Think about what happens when you multiply two two-digit numbers. Does this help you find the answer?

2. Use equipment or draw diagrams to experiment with different arrangements.
Work systematically. Two arrangements are considered the same if they are rotated or can be reflected onto each other.

3. Use a number line to model what is happening in this problem.
Think about how many CDs Tyron can buy when he has the smallest amount of money and how many he can buy when he has the largest amount of money.

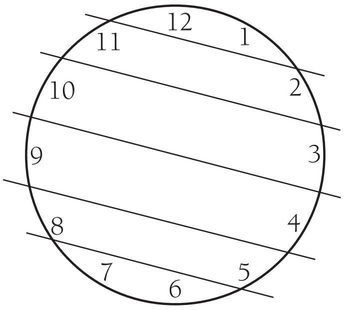
4. You can use trial and improvement.
Another strategy is to add all the weights together. This will give the total of each person weighed twice. Can you use this to find each person's weight?

Figure It Out

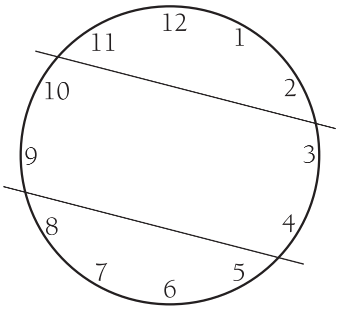
Problem Solving Answers

Page 1: To the Wire

1. a. The clock can be divided into six regions with numbers that add up to 13:



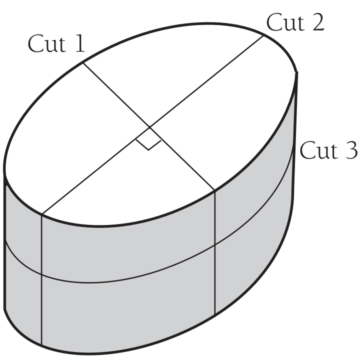
b. Yes. Each of the three regions adds up to 26.



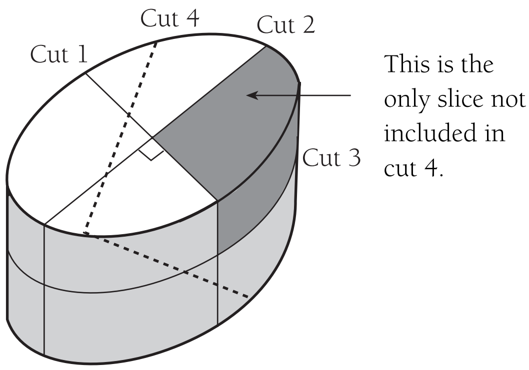
2. $35. (The original $75 was not part of the money that she "made".)

3. 16

4. a. 8 (two vertical cuts and one horizontal cut)



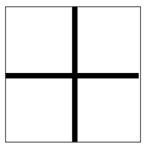
b. 15. The fourth cut is an angled (slanted) cut that goes through seven of the eight pieces in **a.**



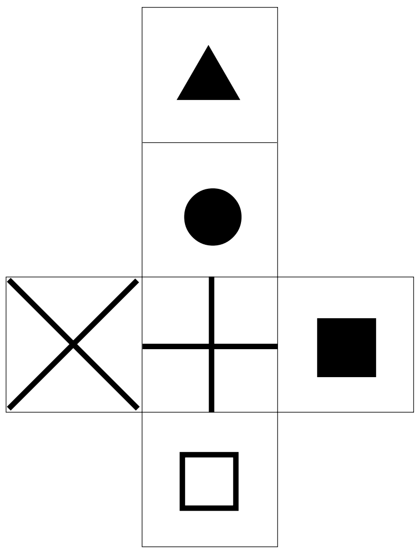
Page 2: Money and More

1. Six 20c coins and seven 5c coins

2.



A net of the cube could look like this:

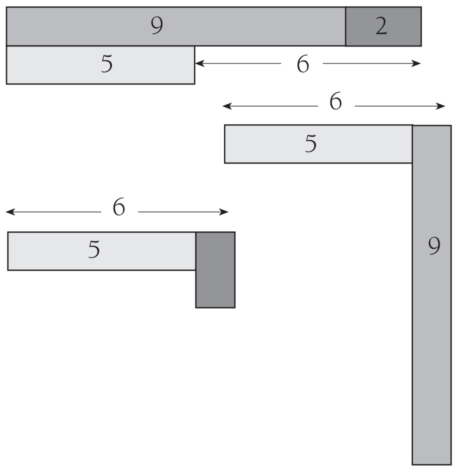


3. 55c

4. A square

Page 3: Using Your Head

1. a. Some possible ways are:



b. If you include the 1 cm width of the rods, you can get all the other lengths between 1 and 16:
1 cm (using the width of a rod), 2 cm,
3 cm (5 – 2 or 2 + the width of a rod),
4 cm (9 – 5 or 5 – the width of a rod),
5 cm, 7 cm (5 + 2), 8 cm (5 + 2 + the width of the 9 rod), 9 cm, 10 cm (9 + the width of a rod), 11 cm (9 + 2), 12 cm (9 + 5 – 2),
13 cm (9 + 5 – the width of the 2 rod),
14 cm (9 + 5), 15 cm (9 + 5 + the width of the 2 rod), and 16 cm (9 + 5 + 2).

2. a. **iii.** 333 333

iv. 444 444

b. 21

c. You may notice that the left side factors are multiples of 3. The digit that is repeated six times in the product is the left-side factor divided by 3.

3. 27

4. $27.20

Page 4: Snap!

1. 32

2. $5^{1}/_{3}$ m

3. Three ways, as shown in the following table:

5 cent coins	10 cent coins	20 cent coins	50 cent coins	Total
5			1	25 + 50 = 75
3		3		15 + 60 = 75
1	3	2		5 + 30 + 40 = 75

4. The only possible rectangular design is an 8 x 1 block.

Page 5: Finding a Way

1. 31 days

2. The first sign is in Rotorua and the second is in Dunedin.

3. 40 single boots (16 single boots for those with wooden legs and 12 pairs of boots for $^{3}/_{4}$ of the rest)

4. 18.5 cm

Page 6: Lining Up

1. a. 11

b. 10

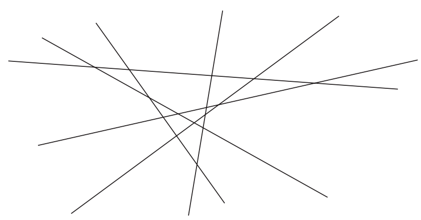
c. The first number on each card is added to the first number on one or more of the other cards to give the rest of the numbers on each card. For example, green 4 + red 1 = green 5. Green 4 + blue 2 = green 6. Green 4 + blue 2 + red 1 = green 7. You can't have green 8, because no other first numbers add up to 4.

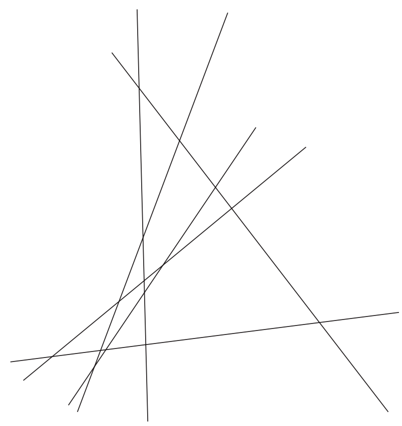
d. Answers will vary, but they will be based on adding the top numbers on the cards that contain the mystery number.

2. 67 600 (26 x 26 x 10 x 10)

3. 13 children (nine girls and four boys)

4. 15. Two of the ways to show this are:





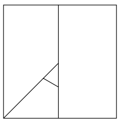
Page 7: Following On

1. **a.** 1

   b. The sum is always three times the middle number. (Also, the first and last numbers add up to twice the middle number.)

2. 11 white, 4 blue

3. **a.** the picture labelled **ii.**



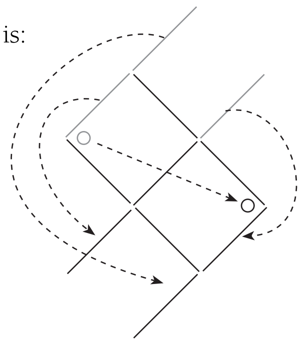
   b. Including the one shown in the problem, there are 24 ways in which the paper shape can be folded. (The four letters refer to the fold sequence: for example, ABCD means first fold A, second fold B, then C, then D.)

ABCD	ABDC	ACBD	ACDB	ADBC	ADCB
BACD	BADC	BCAD	BCDA	BDAC	BDCA
CABD	CADB	CBAD	CBDA	CDAB	CDBA
DABC	DACB	DBAC	DBCA	DCAB	DCBA

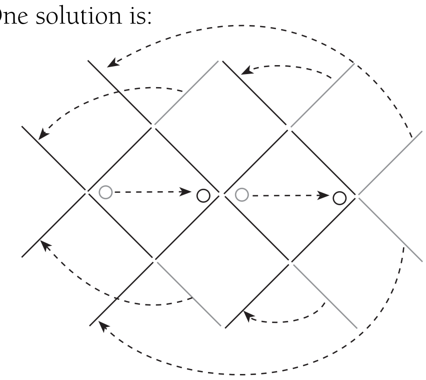
4. 364 families (1 → 3 → 9 → 27 → 81 → 243 → 729. The last 729 families did not have to pass the message on. 1 + 3 + 9 + 27 + 81 + 243 = 364. 364 + 729 = 1 093.)

Page 8: Eye Antics

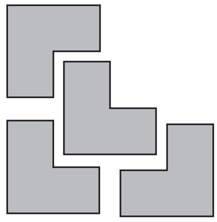
1. **a.** One solution is:



   b. 6. One solution is:



2. There is only one solution:



3. There are many ways of doing this.
   Some possible expressions are:

   $0 = 3 - 2 - 1$

   $1 = 1 \times (3 - 2)$

   $2 = 3 - 2 + 1$

   $3 = 3 \times (2 - 1)$

   $4 = 3 + 2 - 1$ or $2 \times (3 - 1)$

   $5 = 3 \times 2 - 1$ or $2 \div 1 + 3$

   $6 = 1 + 2 + 3$ or $3 \times 2 \times 1$ or $3 \div 1 \times 2$

   $7 = 3 \times 2 + 1$

   $8 = 2 \times (3 + 1)$

   $9 = 3 \times (2 + 1)$

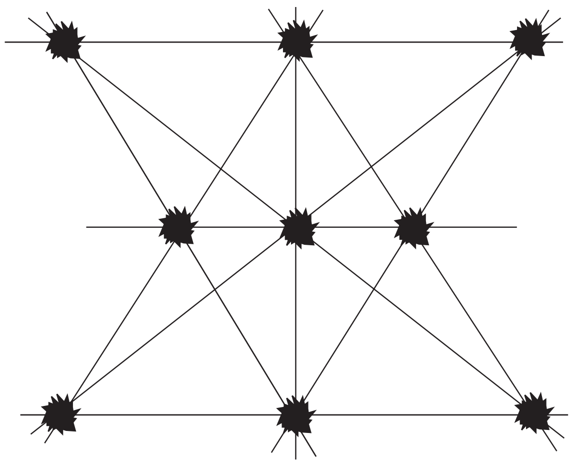
   Note that some expressions can be rewritten in a different way. For example, $1 = 1 \times (3 - 2)$ is the same as $1 = (3 - 2) \times 1$.

4. **a.** Three ways. (Ways that are the same through rotation or reflection are not included.)

   b. Eight ways. (Ways that are the same through rotation or reflection are not included.)

Page 9: Filling the Gaps

1.



2. The 300 g packet

3. **a.** 2, 6, 12, 20

   b. 5 x 6 = 30

   6 x 7 = 42

   7 x 8 = 56

   8 x 9 = 72

   9 x 10 = 90

   c. The pattern of the differences between consecutive terms is 4, 6, 8, 10, 12, and so on. In other words, the difference between consecutive terms increases by 2 each time.

4. There are many answers, but eight possible solutions are:

   314 + 658 = 972

   341 + 586 = 927

   134 + 658 = 792

   243 + 576 = 819

   735 + 246 = 981

   215 + 748 = 963

   271 + 593 = 864

   142 + 695 = 837

   (Look at the digits in the sum part of the first three answers. That should open up some other possibilities with the rest of the answers given.)

Page 10: Working Together

1. The three possible solutions are:
   G, F, B, A, H, D, E, C
   G, F, B, A, D, H, E, C
   G, F, B, D, A, H, E, C

2. 204

3. 55

4. Six pets (two dogs, three cats, one parrot)

Page 11: Looking Ahead

1. 4

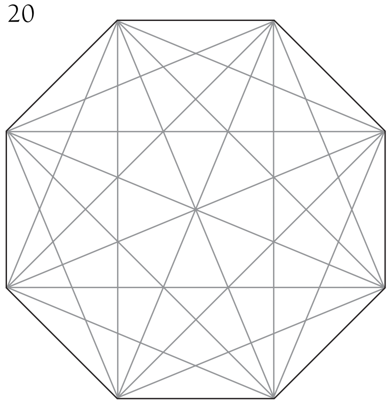
2. 7 days. You could show this in a diagram:

   Day 1: • $\xrightarrow{\text{20 km}}$ hides 3 at start, carries 4, uses 1
   Day 2: ← • hides 2 at 20 km site, uses 1
   Day 3: → carries 3, uses 1
   Day 4: → carries 4, uses 1
   Day 5: → carries 3, uses 1
   Day 6: → carries 2, uses 1
   Day 7: → uses 1

3. Five coins: 5c, 10c, 20c, 20c, and 50c

4. **a.** 9

   b. 20

   c. A possible rule is: Half the number of sides multiplied by three less than the number of sides. This can also be written as: $^1/_2$ x (number of sides) x (number of sides – 3).

Page 12: In and Up

1. 10

2. $1^3/_4$ hours

3. 25 times

4. **a.** The smallest number of shots needed to score 30 points is six. There are four ways of doing this:

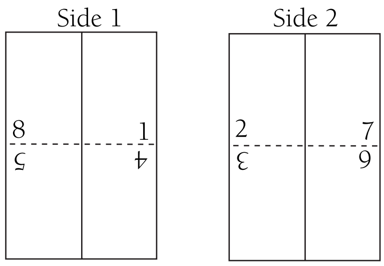
	Points per hole			
	2	**5**	**8**	Total
Number	1	4	1	30
of shots	2	2	2	30
per	3		3	30
hole		6		30

   b. 45

Page 13: Back to Back

1. 1, 6, 2, 10, 3, 7, 4, 9, 5, 8

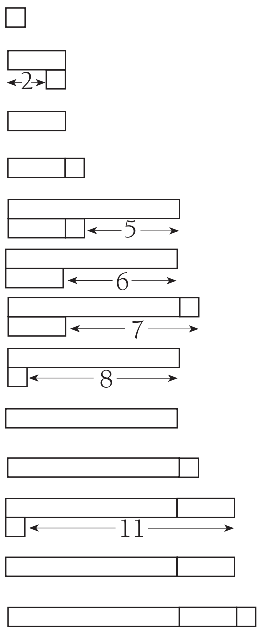
2. With the cut made at the bottom:



3. **a.** There are essentially only two routes, but you can go either forward or backward in the cycle. If you started at Paki, the two routes would be: Paki, Wainui, Te Poi, Nīkau, Kōwhai, Paki or Paki, Wainui, Te Poi, Kōwhai, Nīkau, Paki.

b. The first route given in **a** is the shortest, at 29 km. (The second route is 31 km.)

4. Answers will vary.
Some ways are:



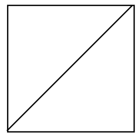
1:
2: (3 – 1)
3:
4: (3 + 1)
5: (9 – 3 – 1)
6: (9 – 3)
7: (9 – 3 + 1)
8: (9 – 1)
9:
10: (9 + 1)
11: (9 + 3 – 1)
12: (9 + 3)
13: (9 + 3 + 1)

Page 14: Creases and Cuts

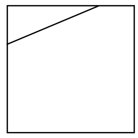
1. Four letters and 12 cards

2. **a.** You can also get:

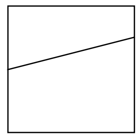
- two congruent right-angled triangles, for example:



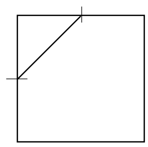
- a right-angled triangle and a pentagon, for example:



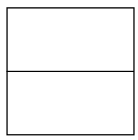
- two trapezia, for example:



- an isosceles triangle and a pentagon, for example:

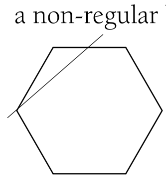


- two rectangles, for example:

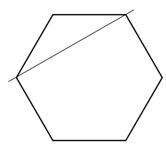


b. By cutting through a corner, you can get:

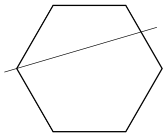
- a triangle with one angle of 120° and a non-regular hexagon, for example:



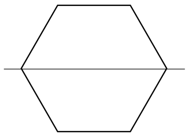
- an isosceles triangle with one angle of 120° and a non-regular pentagon, for example:



- a quadrilateral and a pentagon (neither of these will be regular), for example:

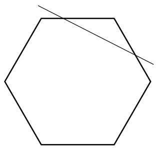


- two trapezia, for example:

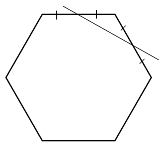


By making a cut through a side, you can get:

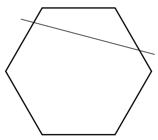
- a triangle with one angle of 120° (which could be an isosceles triangle) and a non-regular heptagon, for example:



or



- a quadrilateral (which could be a trapezium) and a hexagon (neither of these will be regular), for example:



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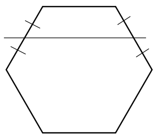
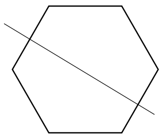


- two pentagons (which may be congruent), for example:



3. 7 creases
4. 36 guests

Page 15: In Line

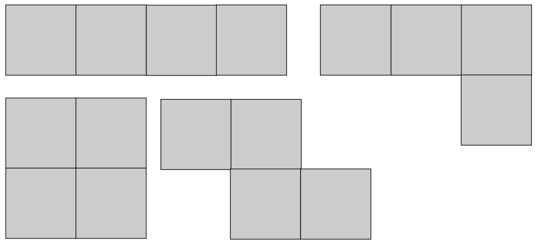
1. Ripeka
2. **a.** 2

   b. 12

   c. 10
3. 26, 27, 28
4. 16

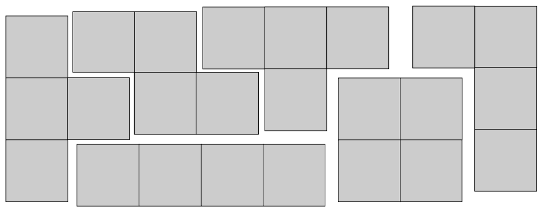
Page 16: Dates and Shapes

1. Possible answers are:
   1 + 1 + 1 + 9
   1 + 1 + 3 + 7
   1 + 1 + 5 + 5
   1 + 3 + 3 + 5
   3 + 3 + 3 + 3
2. One way of doing this is to track back. You need to know how many days are in each year (including leap years) and how the days change from year to year.
3. **a.**

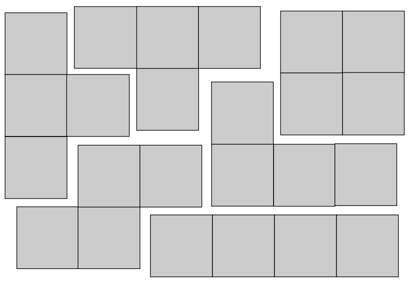


   b. You can have a 3 x 8 rectangle or a 4 x 6 rectangle. One way of constructing each of these is shown below.

3 x 8:



4 x 6:



4. Step 1: Fill the 7 L bucket.

   Step 2: Fill the 5 L bucket from the 7 L bucket. (There is now 2 L remaining in the 7 L bucket.)

   Step 3: Empty out the 5 L bucket.

   Step 4: Empty the 2 L in the 7 L bucket into the 5 L bucket.

   Step 5: Refill the 7 L bucket.

   Step 6: Top up the 5 L bucket from the 7 L bucket. (This will take 3 L of water, which means 4 L remain in the 7 L bucket.)

Page 17: Out for the Count

1. a. Hearts
   Eagles
   Eagles

   b. Rovers
   Hearts
   Hearts

   c. Eagles
   Rovers
   Rovers

2. a. 6

   b. 6

3. 25

4. a. The angle between the hour and minute hands is 90° or a right angle.

   b. The only other right-angled o'clock time is 3 o'clock.

Page 18: You're All Heart!

1. There are three possible answers: 65c, $1.25, or $1.85.

2. a. 20

   b. Explanations may vary. The hidden part of each layer is the same as the row above. So the number is:
   1 + (1 + 2) + (1 + 2 + 3) + (1 + 2 + 3 + 4)
   = 1 + 3 + 6 + 10
   = 20.

3. Answers will vary, depending on your pulse rate. For example, if your pulse rate is 100 per minute, your heart would pump
   80 x 100 x 60 x 24 = 11 520 000 mL of blood in one day. (This is 11 520 L.)

4. a. 531 m

   b. Approximately 444 times (the exact answer is 444.4)

Page 19: What Goes Around ...

1. After 12 minutes (at the end of Billy's second lap and Millie's third lap)

2. There are four possibilities:
   2 + 3 + 8 + 9 and 4 + 5 + 6 + 7
   2 + 4 + 7 + 9 and 3 + 5 + 6 + 8
   2 + 5 + 6 + 9 and 3 + 4 + 7 + 8
   2 + 5 + 7 + 8 and 3 + 4 + 6 + 9

3. 16

4. a. 100 200 300 400 500 600 700 800 900
   111 221 331 441 551 661 771 881 991
   122 242 362
   133 263 393
   144 284
   155
   166
   177
   188
   199

   b. 101 202 303 404 505 606 707 808 909
   110 211 312 413 514 615 716 817 918
   220 321 422 523 624 725 826 927
   330 431 532 633 734 835 936
   440 541 642 743 844 945
   550 651 752 853 954
   660 761 862 963
   770 871 972
   880 981
   990

Page 20: Going Around

1. One way of setting out the solution is:

4	1	4
3	3	3
2	5	2

Note that the order of the rows and columns could change with rotation or reflection.

2. 15, including existing tracks
3. **a.** Two from the yellow jar and one from the green jar go in the blue jar, making eight marbles in each.

   b. The red jar has 5, the blue jar has 7, the yellow jar has 9, and the green jar has 11.
4. **a.** 8 minutes

   b. 13 minutes

Page 21: Round Robins

1. There are 60 possible solutions. All will have one row that has only two cartons in it. Three of the solutions are:

X		X	X	X
X	X	X	X	
	X			X

X				X
X	X	X	X	
	X	X	X	X

X	X		X	X
		X	X	
X	X	X		X

2. **a.** **i** is $\frac{1}{2}$, **ii** is $\frac{1}{8}$, and **iii** is $\frac{3}{8}$.

   b. **i** is $\frac{1}{2}$, **ii** is $\frac{1}{8}$, and **iii** is $\frac{3}{8}$.

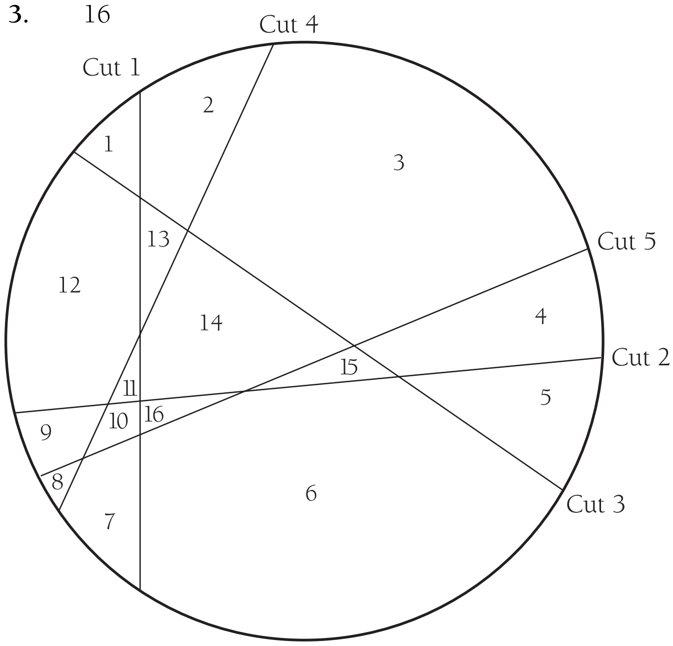
   c. **i** is $\frac{1}{2}$, **ii** is $\frac{1}{12}$, and **iii** is $\frac{5}{12}$.
3. 32 and 33
4. 15 games

Page 22: Dining on Digits

1. **a.** 9.5 m (9 500 mm)

   b. 5.75 m (5 750 mm)
2. A possible answer (based on the cubes in the first illustration in the students' book) is:
   One cube: 0, 1, 2, 3, 4, and 6. (The 6 is used upside down as a 9.)
   Other cube: 0, 1, 2, 5, 7, and 8.

   On two blank cubes, each cube would have 0, 1, and 2. The 3, 4, 5, 6, 7, and 8 could go on either cube.
3. 16



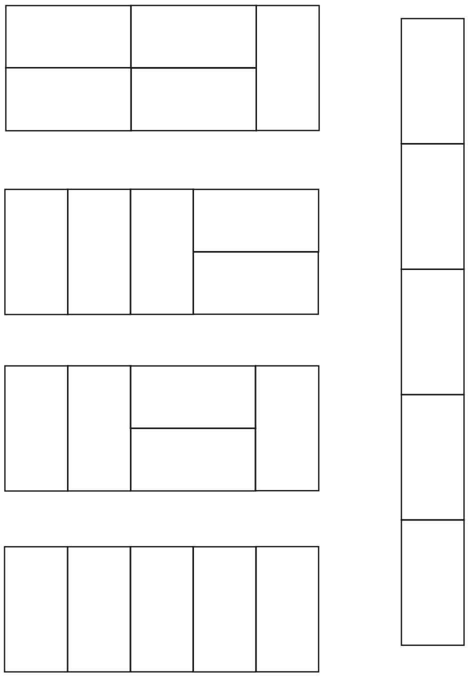
4. 20

Page 23: Waste Not ...

1. Tania: hockey
   Jerome: archery
   Sonny: golf
   Marie: netball
2. $3\frac{3}{4}$ cups of flour, $1\frac{7}{8}$ cups of milk, 5 eggs, $2\frac{1}{2}$ teaspoons of baking powder, $1\frac{1}{4}$ cups of oatmeal
3. 60 bins
4. 6

Page 24: Mystery Amounts

1. ○ = 1, △ = 2, □ = 5. The equation is 12 x 21 = 252.
2. There are five other different arrangements apart from the one on the page:



3. **a.** \$15

   b. \$80
4. Trish: 48 kg
   John: 42 kg
   Zoe: 34 kg

Figure It Out

Problem Solving

Teachers' Notes

Overview: Problem Solving

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Snap!	Applying problem-solving strategies	4	25
Finding a Way	Applying problem-solving strategies	5	27
Lining Up	Applying problem-solving strategies	6	28
Following On	Applying problem-solving strategies	7	30
Eye Antics	Applying problem-solving strategies	8	32
Filling the Gaps	Applying problem-solving strategies	9	35
Working Together	Applying problem-solving strategies	10	37
Looking Ahead	Applying problem-solving strategies	11	38
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Going Around	Applying problem-solving strategies	20	54
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About Problem Solving

The nature of this booklet is different from the other level 3–4 Figure It Out booklets in that it focuses on students' ability to solve non-routine problems rather than focusing specifically on mathematical content. Some ways in which the problems could be used are:

- "Problem of the day": examples for the introductory part of a lesson
- Homework examples for students and parents to work on together
- Examples that students can use to write their own problems.

The hints and answers included at the front of these notes are for students to use.

These teachers' notes contain suggestions about how each problem can develop effective problem-solving strategies, powerful reasoning, and communication. The notes do not list the achievement objectives for each problem because effective problem solving involves the combined application of all of the mathematical processes.

In the notes, important strategies are shown as a way of describing how a particular problem might be solved. These are not necessarily the only productive methods, and you will need to be receptive to the ideas of your students.

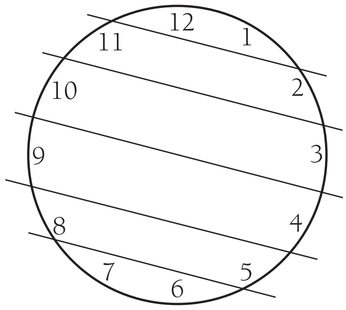
Page 1: To the Wire

Problem One

The solutions for **a** and **b** below show that there is only one way to break the clock into either six or three regions so that the sums of the numbers in each region are the same.

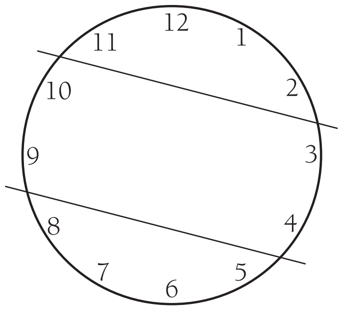
a. The sum of all the numbers from 1 to 12 is 78. So the sum in each of the six regions has to be 78 ÷ 6 = 13.

What goes with 12 to get 13? Clearly, it has to go with the 1. In the same way, 11 goes with 2, 10 with 3, 9 with 4, 8 with 5, and 7 with 6. These are the six regions.



b. With three regions, the sum of each region has to be 78 ÷ 3 = 26. What does 12 go with? It can't go with 10 and 11 because 10 + 11 + 12 = 33, which is too big. It can't go without the 11 because 12 + 1 + 2 + 3 + 4 = 22 (not enough) and 12 + 1 + 2 + 3 + 4 + 5 = 27 (too big). But 11 + 12 = 23, and you can get the extra 3 you need by adding in 1 + 2. So the region containing 12 also contains 11, 1, and 2.

What numbers are in the region with 10? This region can't contain 8 because 10 + 9 + 8 = 27, and that's too big. On the other hand, 9 must be in with 10 because 10 + 3 + 4 + 5 = 22 (not enough) and 10 + 3 + 4 + 5 + 6 = 28 (too big). So you have to have 10 + 9 + 3 + 4. That leaves 5 + 6 + 7 + 8 = 26.



There is a method for adding consecutive numbers together. For example, to add the numbers 1 to 12 together:

sum = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12
= (1 + 12) + (2 + 11) + (3 + 10) + (4 + 9) + (5 + 8) + (6 + 7)
= 6 x 13
= 78.

The sum of the first and last numbers is the same as the sum of the second and second to last numbers, which is the same as the third and third to last numbers, and so on. There are six sums that add up to 13, so the sum is 78.

The sum of consecutive numbers is always the sum of the first and the last terms multiplied by half the number of terms (even if the number of terms is odd). For example, 12 + 1 = 13, 13 x 6 = 78.

You could show the students this method as a way of making them more aware of the power of maths.

Extension

a. "Are there other regions (for example, two or four) that you can break the clock up into, with the numbers in each region adding to the same total? If you can, can this be done in only one way?"

b. You might like to ask the students to explore other possible "clocks". "What happens if you have a 10 hour clock or a 24 hour clock? In how many ways can the numbers be divided into regions then?"

Problem Two

Melody did two deals and made $15 on the first one and $20 on the second. Altogether she made a profit of $35. (The fact that she paid $75 to start with is only relevant when calculating her profit on the first guitar.)

Students can easily make up their own problems like this. You can also rearrange the problem to make it more difficult. For example, suppose Melody bought her first guitar for $55. She bought her second guitar for $70 and sold it for $90. If she made a total profit of $60, how much did she sell her first guitar for?

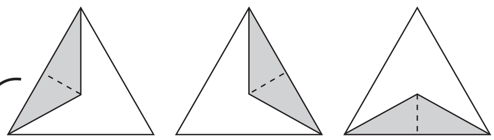
There are essentially five numbers involved in this problem: the cost of each of the two guitars, the sale price of each of the two guitars, and Melody's profit. The problem can be arranged so that any of these is the unknown.

Extension

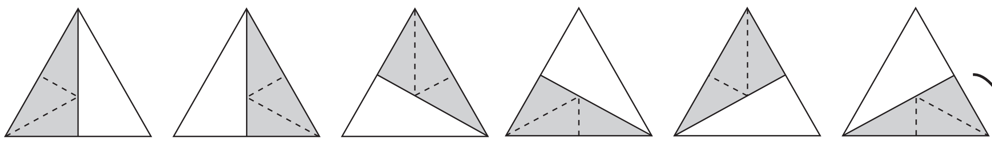
"In the initial problem, how do you know that Melody must have had at least $85 before she started?"

Problem Three

Students need to work systematically to count the number of triangles. First, they could count the number of triangles made up of one small triangle. Then they could count the number of triangles made up of two small triangles, and so on up to the number of triangles made up of six small triangles. They could record their results on a table.



(two small triangles)



(three small triangles)

Number of small triangles in triangle	Number of triangles
1	6
2	3
3	6
4	0
5	0
6	1
Total	16

Problem Four

Provide the students with Plasticine or play dough and a plastic knife so that they can experiment with different cuts. Some students will find this easier than drawing the cuts on paper.

a. It is possible to cut the cheese so that each cut slices every existing piece in half. You then get 2 x 2 x 2 = 8 pieces.

To get the eight pieces, first cut vertically through the centre of the cheese (thus cutting through the diameter). Then cut vertically again, but this time make the cut at 90° to the first cut. Then make a horizontal cut. (See the diagram in the Answers.)

b. You can get 15 pieces by making a diagonal cut through seven of the eight pieces from **a**. There seems to be no way that a single slice can go through all of the eight pieces from **a**. You may need to model this with your class because it is not easy to draw in a diagram. The diagram in the Answers may help.

Extension

"How many pieces can you make with the next (fifth) cut? And the next?"

This problem is similar to the pizza problem in **Problem Three**, page **22**.

Page 2: Money and More

Problem One

The students can solve this problem by using trial and improvement and recording the results on a table that shows different combinations of 5 cent and 20 cent coins, always with a total of 13 coins.

Number of 5c coins	Number of 20c coins	Total value	
5	8	$1.85	too high
6	7	$1.70	still too high
9	4	$1.25	too low
7	6	$1.55	✔

Encourage the students to look for patterns that will improve the accuracy of their trials. They may notice that increasing the number of 5 cent coins by one and decreasing the number of 20 cent coins by one reduces that total by 15 cents. The original guess in the table above is too high. This means that you need to increase the number of 5 cent coins by two and decrease the number of 20 cent coins by two to reduce the total by 2 x 15c = 30c.

Here is another way to look at the problem:
The nearest multiple of 20 to 155 is 7 x 20 = 140. This means seven 20 cent coins, which is no good because you would then need six 5 cent coins to make the 13 coins. 6 x 5 = 30 and 140 + 30 = 170, which is too much. The next lowest multiple of 20 is 120, which is six 20 cent coins. You would then need seven 5 cent coins to make the 13 coins. 6 x 20 + 7 x 5 = 155, so six 20 cent coins and seven 5 cent coins = $1.55, the amount that Titus had.

Problem Two

One way to think about this is to look at the first and the third drawings of the cube. Notice that these show all four of the sides that are next to the ✚, that is, the ✖, ☐, ●, and ■. The only shape that hasn't been accounted for is the ▲, so the ✚ must be opposite the ▲.

Some students will find it very hard to visualise this. Give them blank cubes and let them draw the shapes on the faces. This does not mean that these students are necessarily poor at maths. There are a lot of bright people who have trouble visualising things like this.

Making a model of the cube using the net shown in the Answers is a simple way of demonstrating this problem.

Problem Three

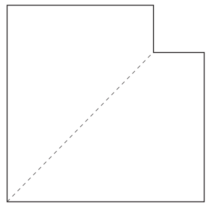
This problem can be solved very simply, as long as the students think carefully about the information they have been given.

Heinz needs 25 cents more to buy the ruler. If Phoebe gives Heinz the 25 cents he needs, she will have 30 cents left. (We are told that together they have enough money to buy a ruler and have 30 cents change.) So Phoebe initially had 30 + 25 = 55 cents. Heinz must have had 40 cents at the start because Phoebe started with 15 cents more than him (55 – 15 = 40 cents).

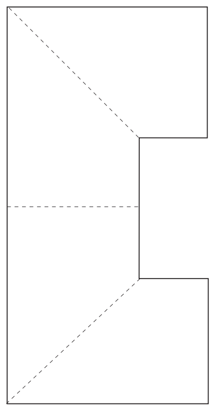
Problem Four

This problem can be solved by working backwards.

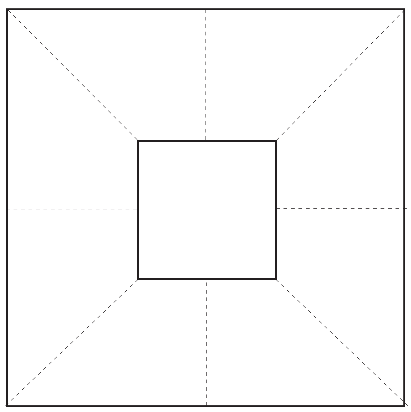
Start at the last picture. Going back one step will open up the cut triangle. This means that the cut piece will still be partly horizontal, but it will also be partly vertical. So at this stage, there is a small square cut out of the folded square shape.



If you unfold to go back another step, the cut-out square opens up to become a cut-out rectangle. Its horizontal sides are the same length as the original cut. Its vertical sides are twice this length.



As you unfold to get back to the original square, the cut-out rectangle folds out to make a square whose sides are the same length as the rectangle. So the hole is a square whose sides are twice as long as the original cut.



Extension

As in **Problem Two,** many students will have difficulty visualising what is going on. Give them paper and scissors and let them experiment with their cuts. "What happens if you cut off another corner?" Get them to guess and then do it. "What if you try a zigzag cut? What if you fold the paper some other way?" This will give the non-visualisers practice in visualising.

Page 3: Using Your Head

Problem One

a. 6 = 9 + 2 – 5. With the plus meaning "putting together" and the minus meaning "taking away", you can get variations of the first diagram shown in the Answers.

Encourage the students to think about the lengths of the other dimensions of Cuisenaire rods. They'll probably know that they are 1 centimetre wide. So they can also make 6 by putting either the 2 or the 9 sideways against the 5.

b. The answers allow for using the width (1 centimetre) of any of the rods.

Challenge the students to try to make all the lengths from 1 to 16 centimetres without using the width of the Cuisenaire rods. You may need to remind them of the rules below to make their working more efficient:

odd + odd = even	odd – odd = even
even + even = even	even – even = even
odd + even = odd	odd – even = odd or even – odd = odd

For example, if they are trying to make 8:
Combining even and odd lengths (2 + 5 or 2 + 9) will give an odd length. Combining 5 and 9 gives only 4 or 14. So 8 cannot be made by combining two of the rods. The possible combinations of three rods are (9 + 5) – 2, 9 + (5 – 2), (9 – 5) + 2, 9 – (5 + 2), 9 + 5 + 2, or (9 – 5) – 2, none of which give 8, so 8 cannot be made from 2, 5, and 9.

Working systematically in this way, the students will find that they can't make 10, 13, or 15 either.

Extension

Give the students a different combination of three Cuisenaire rods to make into various lengths.

Problem Two

The students will probably see the patterns in this set of equations fairly quickly. The left-side factors are multiples of 3, and the digit that is repeated six times in the product increases by one for each successive equation.

From these observations, they will be able to answer question **a**: 333 333 and 444 444.

To answer question **b**, they might just continue the pattern and write out the equations until they get to the product 777 777. But they can solve the problem far more efficiently if they look for a relationship between the left-side factors and the products.

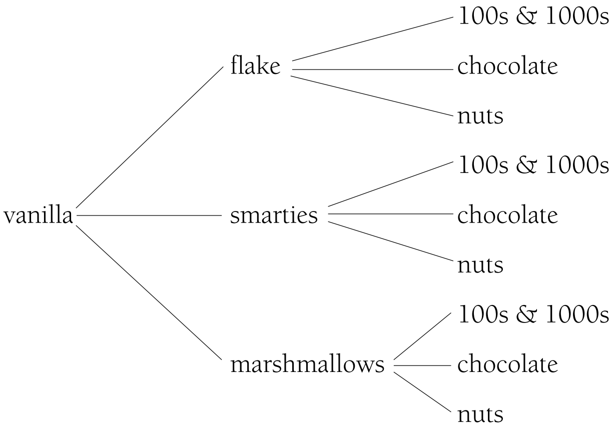
The digit that is repeated six times in the product is the left-side factor divided by 3, or conversely, multiply the digit repeated six times in the product by 3 to get the factor. For example, 7 x 3 = 21, so 21 x 37 037 = 777 777.

Another way of approaching this problem is to look at the relationship between the first equation and successive equations:
3 x 37 037 = 111 111
9 x 37 037 is 3 x (3 x 37 037), so 9 x 37 037 is 3 x 111 111 = 333 333.

$\boxed{}$ x 37 037 = 777 777, so $\boxed{}$ must be 7 x 3, or 21.

Problem Three

The students should be able to confidently use a tree diagram to solve this problem. The vanilla ice cream section should look like this:



After the students have drawn a tree diagram, discuss with them more efficient ways of solving the problem. Is there any way they could calculate the answer without having to draw a diagram?

Using the tree diagram to illustrate your points, explain that there are three flavours of ice cream. Next, they have three choices of confectionery. Altogether, this is three lots of three choices, or 3 x 3 = 9 different types of ice cream and confectionery. Finally, they have another three choices of topping, which is 9 x 3 = 27 different types of ice cream with confectionery and topping.

Students working at level 4 will be starting to use exponents and should see that 3 x 3 x 3 is the same as 3^3.

number of choices $\rightarrow 3^3 \leftarrow$ number of items put on each ice cream cone

Give them several different examples so that they understand which number is the base number and which is the exponent. For example, there are four flavours of ice cream and four toppings. How is this written using an exponent? (4^2) There are two flavours of ice cream, two types of confectionery, and two toppings. How is this written using exponents? (2^3) There are only five flavours of ice cream and no confectionery or topping. How is this written? (5^1)

Extension

Ask the students to make up a problem of this type with 64 as the answer (64 is 4^3, 2^6, or 8^2).

Problem Four

This can be calculated by noting how many Saturday, Sunday, and weekday papers there are and how much each of these three types of paper costs. The students need to know that September has 30 days. If they don't know this, they could look at a calendar or recall the rhyme that tells the lengths of all the months.

If 3 September is a Saturday, then so is September 10, 17, and 24. This means four Saturday papers, and they cost 4 x 90 = 360 cents or $3.60.

Similarly, there are four Sunday papers, and they cost 4 x $1.50 = $6.00.

So you have 30 – 4 – 4 = 22 weekdays. Papers on these days will cost
22 x 80 = 1 760
= $17.60.

The total cost of papers is therefore $3.60 + $6.00 + $17.60 = $27.20.

Page 4: Snap!

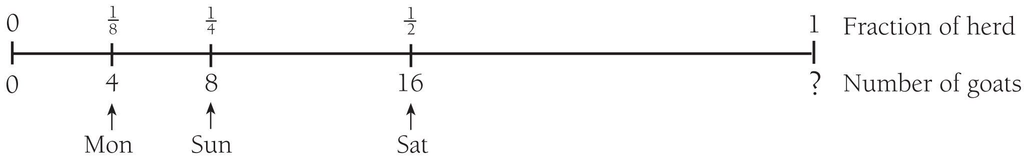
Problem One

Trial and improvement is one strategy that the students might use, but it is inefficient. The students will probably find the answer more quickly by working backwards:
Felicity ended up with four goats. She lost half in the third escape. So she must have had 2 x 4 = 8 before that.
Half escaped the second time, so she must have had 2 x 8 = 16 before that.
And half escaped the first time. So she must have had 2 x 16 = 32 at the start.

These results could be recorded in a table:

Escape	Amount escaped	Number left
Third escape	$^1/_2$	4
Second escape	$^1/_2$	8
First escape	$^1/_2$	16
Original number		32

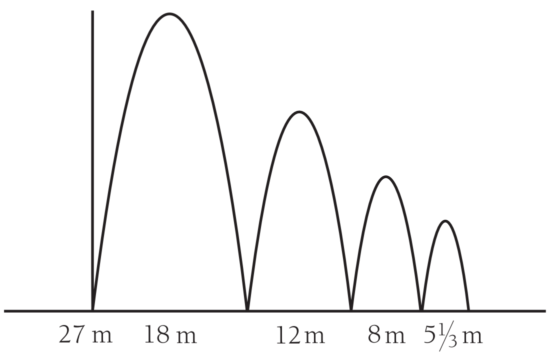
A double number line could also be used to solve the problem.



Four goats were left on Monday, which was one-eighth of the herd, so the starting number of goats must have been 4 x 8 = 32.

Problem Two

This problem gives the students practice in finding fractions of whole numbers (which is the same as multiplying fractions and whole numbers, for example $^1/_2$ x 8). The problem is straightforward as long as they accurately calculate the fractions. Drawing a diagram will help them visualise the problem.



One simple way of working out the problem is:

$^1/_3$ of 27 = 9, so $^2/_3$ = 18.

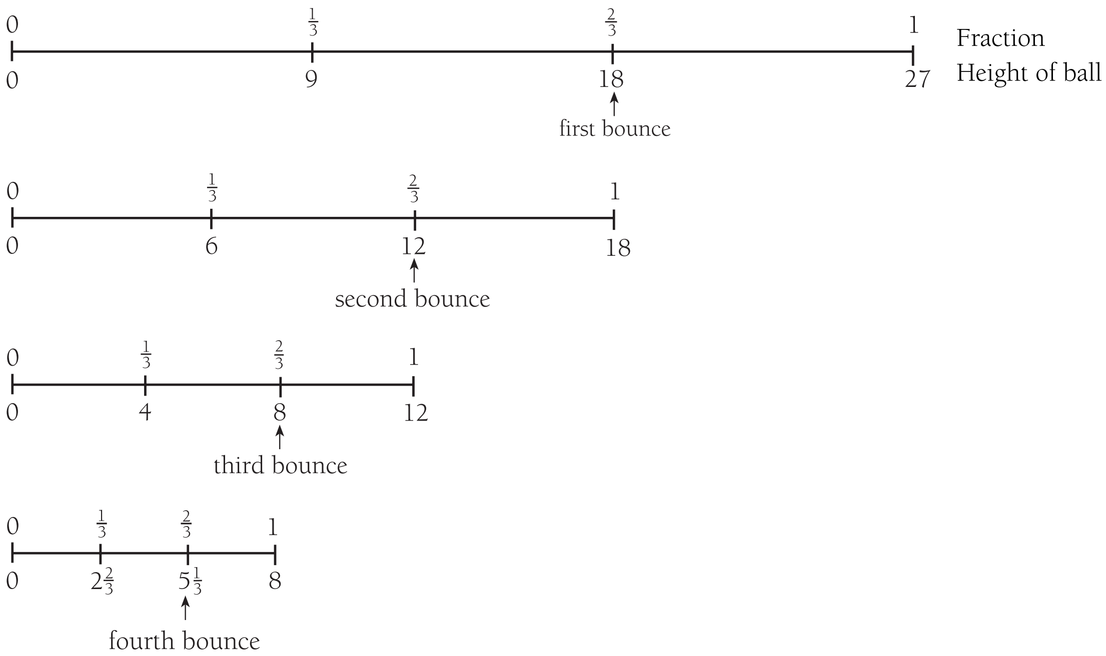
$^1/_3$ of 18 = 6, so $^2/_3$ = 12.

$^1/_3$ of 12 = 4, so $^2/_3$ = 8.

$^1/_3$ of 8 = $2^2/_3$, so $^2/_3$ = $5^1/_3$.

So the ball bounces to a height of $5^1/_3$ metres (5.33 metres) on the fourth bounce.

Progressive double number lines could also be used to solve this problem:



Problem Three

The best strategy is to build up a table, systematically starting with the largest coins and working downwards.

Using one 50 cent coin, you need five more coins to make a total of six. The only way to make the additional 25 cents with five coins is to use 5 cent coins. Working systematically in this way, there are three possible answers:

5 cents	10 cents	20 cents	50 cents
5			1
3		3	
1	3	2	
		1	not possible
	6		not possible

Problem Four

The students need to think about the different ways of making a rectangular, eight-square chocolate block. They will find that there are only two ways: 2 x 4 or 1 x 8 rectangles. It is not possible to get a single square by snapping a 2 x 4 block along one crease, but it is by snapping a 1 x 8 block. You can also get every other number of squares from 1 to 7 inclusive by snapping along one crease.

Page 5: Finding a Way

Problem One

The students need to look carefully at the information provided and see what they can deduce from that information.

Overall, Aroha made 30 cents, so she must have made her bed for at least 6 days (6 x 5c = 30c). For the rest of the month, the amount she earned making her bed must have been cancelled out by the non-making days. Four days of making are cancelled out by 1 day of non-making, which is a set of 5 days. So the number of days in the month must be 6 plus a multiple of 5, that is, 6, 11, 16, 21, 26, 31, 36, and so on. Months can be 28, 29, 30, or 31 days. The only number of days that fits is 31 days, so there must have been 31 days in the month.

Extension

You can extend this problem by changing the amounts of pocket money in the question. The students could suggest some amounts.

You can also turn the question around. "If there are 30 days in the month, what amounts of money can Aroha earn?" (You can introduce negative numbers this way.) "Is there a pattern in these amounts?" "If Aroha earned 80 cents in a month, what could you say about the month?"

"Suppose it was a 31 day month and Aroha got 5 cents for making her bed. If she earned 65 cents that month, how much did she lose for a non-bed-making day?"

Another scenario is to have neutral days. "On the days when the family went over to see Granny, they had to leave early in the morning, and so they didn't have time to make the beds. In a 30 day month when Aroha earned 70 cents, how many Granny days were there?"

Problem Two

To solve the problem, the students could use the map's scale to find what distance is equivalent to 107 kilometres. They then set a compass to this distance and draw a circle on the map with Hamilton at the centre. (They will need a photocopy of the relevant pages of an atlas to draw on.) They repeat this with an 86 kilometre radius circle from Tauranga and an 82 kilometre radius circle from Taupo. The three circles will have an area in common. The biggest town in the common area will be where the road sign is.

They can use the same approach for the second road sign.

Problem Three

Half the crew (16) had only one leg and wore a boot on that leg. So the captain needed to buy 16 boots for these crew members.

Three-quarters of the rest of the crew wore two boots: $^3/_4$ x 16 = 12. These 12 crew members needed two boots, a total of 24 boots.

Altogether therefore, the pirate captain needed to buy 16 + 24 = 40 boots.

Extension

Boots are usually sold in pairs. Students could consider what difference it would make if the captain could only buy pairs of boots. They could consider a scenario where the pirates had an equal number of left and right feet or one where there were more right feet needing boots than left feet. Would shoe size be an issue?

You could reverse the problem and tell the students that the captain bought 35 boots. "What fraction of his crew had only one leg?" (All the other data could stay the same.)

This problem can also be used to give the students practice with fractions other than $^3/_4$.

Problem Four

A long bead is 14 ÷ 4 centimetres. A round bead is 10 ÷ 5 centimetres. So the new string of beads is (3 x 14 ÷ 4) + (4 x 10 ÷ 5) = 10.5 + 8 = 18.5 centimetres.

Extension

The students could make up their own questions for other students to solve.

Page 6: Lining Up

Problem One

The students could find out the mystery number by a process of elimination, but they need to understand how the cards work for question **c**. The explanation in the Answers should clear up any uncertainty and enable the students to choose their own mystery numbers for a classmate to guess. You could use the examples in the Answers as a starting point to check whether all the other numbers follow the same pattern. The mystery number in question **b**, for example, is made up of blue 2 + yellow 8 = 10.

To answer question **d**, the students only need to be told which cards the mystery number is on. All they need to do is add the top numbers on those cards to find the mystery number.

You may want to take the students beyond looking at the top numbers as random numbers to be added together. The cards are actually based on 2 being multiplied by itself (powers of 2). Look at the numbers in the top row of the cards: Red has 1; blue has 2, that is, 1 x 2; green has 4, that is, 1 x 2 x 2; yellow has 8, that is, 1 x 2 x 2 x 2.

Problem Two

This problem is similar to **Problem Three**, page **3**, in that it asks how many different combinations there are of a certain number of items. (On page **3**, the items were ice cream flavours and toppings. On this page, the items are letters and digits.)

Ensure that the students know how many letters there are in the alphabet and how many digits there are so that they know how many items they are combining. If the students try to solve this problem using a tree diagram, they will quickly see that the diagram is far too large to be practical. They will have to look for a more efficient method, which in this case is multiplication.

There are 26 choices for the first letter and 26 for the second letter. So altogether there are 26 x 26 = 676 possible letter combinations. There are 10 possible digits for the first number and 10 for the second. This gives 10 x 10 = 100 possibilities. (Alternatively, the numbers run from 00 to 99, which is a total of 100 different numbers.)

This means that for every possible letter combination, you have 100 number combinations. So there must be 676 x 100 = 67 600 possible number plates.

Extension

Ask the students to work out how many different number plates there are using two letters followed by four digits. How many different number plates would there be if there was just one more letter (three letters and four digits)?

Problem Three

Some students may find this problem difficult because the number of boys and girls in a family is expressed differently to the number of brothers and sisters. This is because brothers and sisters are always expressed in relation to another sibling, whereas expressing the number of boys and girls in a family is not complicated by a relationship to someone else.

For example: In a family of five girls and two boys, each girl will have four sisters and two brothers and each boy will have five sisters and one brother. (The number of boys and girls remains constant.) So the girls will have twice as many sisters as brothers, and the boys will have five times as many sisters as brothers.

To work out the number of girls and boys in the family in the problem, the students could draw up a table (see below).

The girls in this family have twice as many sisters as brothers. The minimum number of children in the family for this to be true for each girl is one boy and three girls. (The three girls are the one girl and her two sisters.) The next smallest number of children for this to be true for each girl is two boys and five girls.

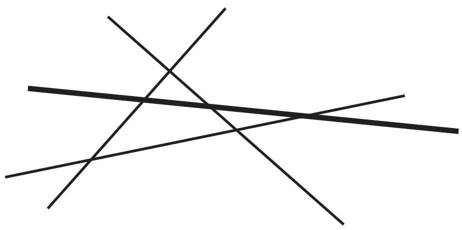
The students could continue to calculate the number of children in the family until they find the total that is true for both boys and girls. They could record their results in a table:

Children	Family total	Number of sisters	Number of brothers	Correct in terms of the problem?
1 boy	4	3	0	no
3 girls		2	1	yes
2 boys	7	5	1	no
5 girls		4	2	yes
3 boys	10	7	2	no
7 girls		6	3	yes
4 boys	13	9	3	yes
9 girls		8	4	yes

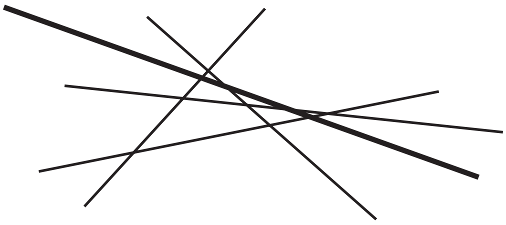
Problem Four

The students are looking for the greatest number of lamp posts for six streets, which means they need to arrange the six streets so there are the greatest number of intersections. You may need to point out to the students that arranging the streets in a conventional right-angled grid will not give the greatest number of intersections.

How many lamp posts can you have if there are four streets? Build this up from the three-street example. If you add one more street, that street can cross all the original three streets. So you get three extra lamp posts or six altogether.



Now try five streets. Starting from the four-street case, you can add another street that will cut all the previous four. This has to add another four lamp posts to give a total of 10. (Of course, from a town planning point of view, this layout may not be very practical. You could use this for a discussion about road planning.)



Now the sixth street can be made to cut each of the other five to give five more lamp posts. At this point, there are 15 lamp posts altogether. There are lots of ways to do this. Two ways are shown in the answers.

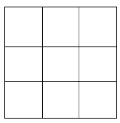
This question is similar to **Problem Four,** page **1**. See also **Problem Three,** page **22.**

Page 7: Following On

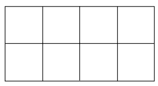
Problem One

a. One way of setting this out is:
(3 x 3) – (2 x 4) = 9 – 8
= 1.
Using cubes to model it is another useful way for students to get a sense of what's happening:

"You've got



and



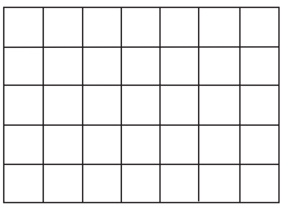
.

Rearrange one set of blocks to get the other set of blocks. How many more or fewer blocks do you need?"

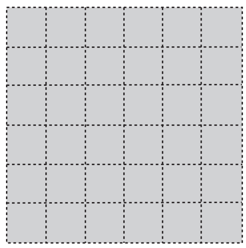
Extension

You could investigate with the students whether this always works with three consecutive numbers.

A useful way to explain this to the students is to draw a diagram. For example, for the numbers
5, 6, and 7, investigate the difference between a 5 x 7 rectangle and a 6 x 6 square:

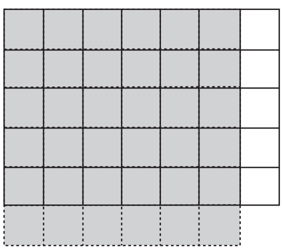


5 x 7 rectangle



6 x 6 square

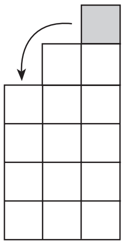
If you lay the shapes over the top of each other, you will see the difference:



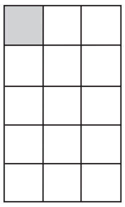
Both diagrams have a 6 x 5 rectangle in common. What is the difference between the leftover areas, that is, the 6 x 1 horizontal strip and the 1 x 5 vertical strip? The difference is 1. The students should find that the same difference will apply using any three consecutive numbers.

b. This pattern works because the first number is one less than the second number and the third number is one more than the middle number. The "one more than" and "one less than" cancel each other out to make three numbers that are the same as the middle number.

Modelling this with cubes is a useful way to explain it to the students. For 4, 5, and 6, put rows of four cubes, five cubes, and six cubes next to each other.



Take a cube from the six-cube pile and add it to the four-cube pile. This makes three equal rows, each with five cubes in them.



So the sum is 3 x 5 = 15. The sum will always be three times the middle number.

The "one less than" and "one more than" cancel each other out because the operation used is addition.

Problem Two

There are several patterns in these figures that students can use to find the number of squares in the next figure. They may notice that the total number of squares in the figures are triangular numbers. They should also see that the number of blue squares increases by one for each consecutive figure and that the number of blue squares is one less than the position of the figure in the pattern (for example, for the fourth figure, there are 4 – 1 = 3 blue squares), or one less than the number of squares along the bottom line, or one less than the longest diagonal line of white squares.

One way to find the number of blue and white squares in the next (fifth) figure is as follows:
The number of blue squares is either one more than in the previous figure (3 + 1) or one less than the number of the figure (5 – 1), both of which are 4.
The total number of squares is the next triangular number (15).
Therefore the number of white squares = total squares – blue squares
= 15 – 4
= 11

Some students may prefer to draw a table to record the number of black, white, and total squares in each figure and then find and continue the pattern for each number of squares.

Extension

Challenge the students to find the number of black and white squares in the hundredth figure. They'll need to find a quick way to add the numbers from 1 to 100. A method for doing this is explained in the notes for **Problem One**, page **1**.

Problem Three

You will find a copymaster of this shape at the back of these notes.

a. This problem can be solved by working backwards. The last piece that was used was piece **B**, so nothing can be over the top of that piece. And when **B** comes down, it will form a rectangular shape on the right of the square. The only one with a rectangle down the right side is **ii**.

b. Rather than the students experimenting by folding the shape parts, encourage them to look for strategies. The basic question is how many different combinations and orders are there of the four letters. The students could solve this in the same way as **Problem Three**, page **3** (see the notes for this problem), by using a tree diagram and calculating 4 x 3 x 2 x 1.
A systematic list, such as that shown in the Answers, is one way of showing the folds.

Problem Four

At each round, the number of families being contacted increases by a factor of 3. The brackets show the total number of families contacted after each round, including the kaumātua's family.

1	→	3	→	9	→	27	→	81	→	243	→	729
(1)		(4)		(13)		(40)		(121)		(364)		(1 093)

Therefore you need six rounds to get in touch with any number of families from 365 to 1 093.

The students will probably recognise that this is another problem that uses exponents. The number of families called in each round are the powers of 3 (that is, $3^0 = 1$, $3^1 = 3$, $3^2 = 9$, $3^3 = 27$, and so on). So the total number of families called after each round is the sum of the powers of 3. Students may be able to use this as a more efficient way to solve the problem.

Page 8: Eye Antics

Problem One

a. The students can model this problem with sticks and experiment to find the answer. If the students are having difficulties, they could try Problem One, page 3, in *Problem Solving*, Figure It Out, Level 3, as a warm-up.

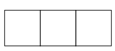
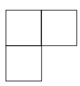
b. To find the smallest number of sticks, you need to turn the fish around, make two drawings, one with the fish facing left and the other with them facing right. Put one drawing on top of the other. Move this drawing around until as many sticks as possible match up. Only eight sticks can be the same. There are 14 sticks altogether, so six sticks have to be moved. This shows that you can't re-orientate the fish by moving less than six sticks.

Extension

Ask the students to find a pattern in the number of fish and the minimum number of sticks moved. What is the smallest number of sticks that they need to move to turn around three fish swimming together? How about 90 fish swimming together?

Problem Two

There are 12 squares that need to be divided into four pieces, so each piece has to have three squares. There are only two possible smaller shapes:

i.  **ii.** 

Four copies of shape **i** will not fit into the original shape, but four copies of shape **ii** will.

Problem Three

This problem uses the students' knowledge of basic facts and encourages them to think about the effects of the four operations. The students will probably start with a wildcat approach to this. They'll see what they can make without too much work and then concentrate on the more difficult ones that don't come out quickly.

One way to be slightly more systematic is to take the numbers 1, 2, and 3 in order and then put plus and/or minus signs between them. For example:
1 + 2 + 3 = 6 1 + 2 – 3 = 0 (1 – 2) + 3 = 2

Then the same sort of thing can be done with multiplication and addition or subtraction:
(1 x 2) + 3 = 5 1 + (2 x 3) = 7
(1 + 2) x 3 = 9 (–1 + 2) x 3 = 3

After exhausting all possibilities with 1, 2, and 3 in order, students can experiment with the numbers in different orders. For example:
(1 x 3) – 2 = 1 (1 + 3) x 2 = 8

The solutions in the Answers show other possibilities with the numbers in different orders.

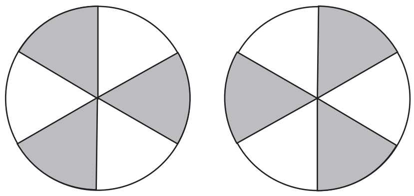
Extension

Can the students find ways to get numbers bigger than 9?
What numbers can the students get using 2, 3, 4? (You could send this problem home for the students to try with their families.)

Problem Four

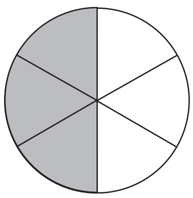
Many of the students will probably do this using trial and improvement. This is fine to get an idea of what the problem is about. But they need to work systematically to be sure they have found all the different ways. If they use trial and error in **a**, encourage them to be systematic in **b**.

The students need to remember that some circle drawings are the same. For example, in the two shown below, you can get the second one from the first one by rotating it through 60 degrees.

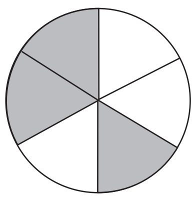


a. To get one half, three sectors need to be shaded. How many ways can this be done?

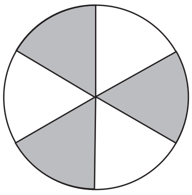
- Three sectors next to each other. This can be done only one way:



- Two sectors next to each other, with the other one separated by one or two white sectors. Again there is only one way:



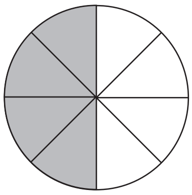
- No two sectors next to each other. This can also be done only in one way:



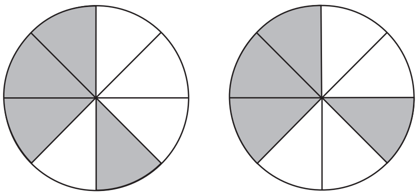
Hence there are only three ways to shade the sectors so that they show one half.

b. For this question, four sectors make up a half.   The different ways of arranging the sectors are:

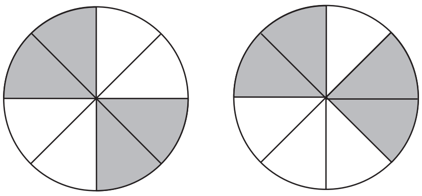
- All four sectors together. This can be done in only one way.



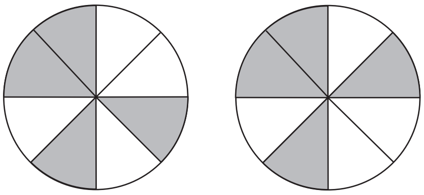
- Three sectors together. There are two ways here:



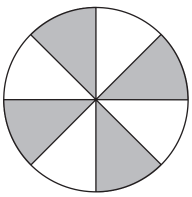
- Two lots of two sectors together. There are two ways here:



- Two sectors together and two not together. There are two ways here:



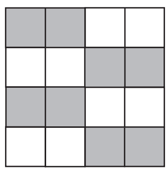
- No two sectors next to each other. There is only one way here:



Hence there are eight ways to shade the sectors so that they show one half.

Extension

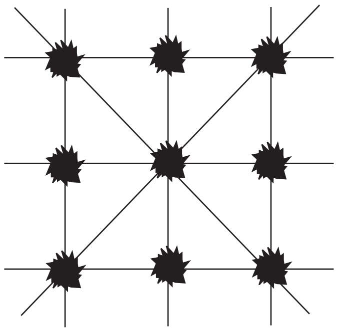
How does this problem work if you divide a square into an even number of smaller squares and shade half the squares?



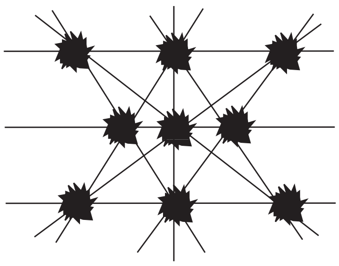
Page 9: Filling the Gaps

Problem One

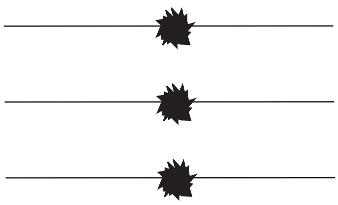
This is a reasonably difficult problem. If the students start with the arrangement shown, they are likely to extend it to:



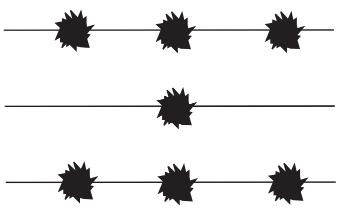
But this gives only eight lines of three. It's not possible to add any other straight lines that go through three trees, as the diagram below shows:



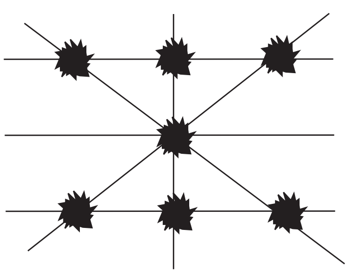
So this is obviously not the correct arrangement. However, this arrangement can be varied slightly to give 10 lines of three trees. Start with three parallel lines. Put one tree on each line so that the trees are one on top of each other:



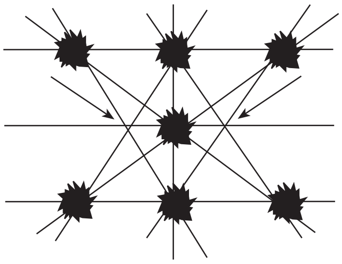
Then put two more trees on the top line, one on either side of the existing tree and the same distance from the existing tree. Do the same on the bottom line. Make sure the trees on the bottom line are the same distance from the middle tree as the trees on the top line.



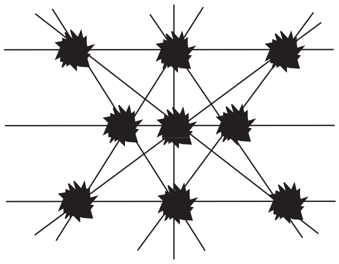
Draw as many lines as you can through three trees.



Now you have six lines, and you still have two trees left to place. Drawing lines from the middle top tree through each of the outside bottom trees and two more lines from the middle bottom tree to each of the outside top trees will give 10 lines altogether and also show where on the middle horizontal line to put the last two trees.



Place the last two trees:



Problem Two

This is a good problem for the students to discuss among themselves and compare their methods. One method is to find what a smaller common value, for example, 100 grams, is worth. Divide the price of packet A by 3 (because the volume is also divided by 3 to get 100) to get 93.33 cents per 100 grams, divide packet B by 5 to get 94 cents per 100 grams, and divide packet C by 8 to get \$1.19 per 100 grams. Clearly, packet A is the cheapest per 100 grams and so is the best value.

Another method is to find a higher common value, that is, a multiple of all three volumes. The lowest common multiple is 12 kilograms or 12 000 grams. Multiply packet A by 40, packet B by 24, and packet C by 15 to find the cost of 12 kilograms of each packet and then compare the values.

Extension

Students could investigate the prices of different volumes of items at a supermarket or use an online supermarket on the Internet.

"Take three different volumes of the same item. Without calculating, which one seems the best value? Does it work out to be the best value after calculating?" The students may notice that generally, with items of the same brand, the larger the volume, the better the value. They could discuss why this is. They could also look at how some items are priced. For example, the price of fish is usually given per kilogram, but sometimes it is given per 100 grams. "Which seems cheaper: salmon at \$21.40 per kilogram or salmon at \$2.14 per 100 grams? Why might supermarkets decide to price by the kilogram or per 100 grams?"

Problem Three

The students should be able to see that the difference between consecutive products increases by 2 each time. Something else that you might like to note is that the first product is 2 x 1, the second is 2 x (1 + 2), the third is 2 x (1 + 2 + 3), the fourth is 2 x (1 + 2 + 3 + 4), and so on. The difference between the second and third product will be the difference between 2 x (1 + 2) and 2 x (1 + 2 + 3), in other words, 2 x 3 = 6. Similarly, the difference between the third and fourth products is 2 x 4 = 8. The differences between the subsequent consecutive terms are 10, 12, 14, and so on. This gives what is known as a recurrence relation between consecutive products.

Another pattern is that the final digit in the answer cycles round. It goes 2, 6, 2, 0, 0 and then round again, 2, 6, 2, 0, 0. Do the other digits follow any pattern?

Problem Four

One way to go about this is to use the hints and pick a set of numbers that adds up to 18, for example, 1 + 8 + 9. But 189 can't be the sum because two other three-digit numbers would have to add up to 189. What about 819?

How can you get 9? Try starting with 3 + 6. Then you have to get 1 in the second place. Try 4 + 7. Actually, there's no other way to get 11 since you've used up 6 (that could have gone with 5), 8 (that could have gone with 3), and 9 (that could have gone with 2). At this point, the equation is □43 + □76 = 819. The only two numbers that you haven't used yet are 2 and 5:
243 + 576 = 819 and 543 + 276 = 819.

Once you have one answer, you can rearrange the digits to get many other answers. For example, 243 + 576 = 819 and so does 246 + 573, 276 + 543, and 273 + 546.

Why the sum of the digits in the answer is always 18 and why there is a carry-over are analysed in full on the nzmaths website: www.nzmaths.co.nz

Page 10: Working Together

Problem One

As the students work backwards from the top notice (**c**), they need to think "What notice can I take off next? What notice has no other notice on top of it?" Answering these questions will give them the order in which they could take the notices off. The notices would have been put up in the reverse order.

Problem Two

There are several ways of solving this problem. The students could find the number of cubes in each side of the model and add them together and then subtract the cubes on the corners that are common to two sides (which can't be counted twice). Two of the sides are 11 x 6, and the other two are 8 x 6. There are four lots of six cubes on the corners that are common to two sides:

(2 x 11 x 6) + (2 x 8 x 6) – 24 = 132 + 96 – 24
= 204

Another way is to count all the cubes on both sides (2 x 11 x 6) and leave off a row at each end of the top and the bottom (2 x 6 x 6):

(2 x 11 x 6) + (2 x 6 x 6) = 132 + 72
= 204

Yet another way is to calculate how many cubes there would be if the model was solid and then subtract the number of cubes that have been taken out to make the model hollow. The solid model would have 11 x 8 x 6 = 528 cubes. Then calculate the number of cubes to be taken out to leave a hollow shell, one cube thick. The side that is 11 cubes long would need nine cubes taken out to leave one cube at each end. The side that is eight cubes long would need six cubes taken out to leave one at each end. All the cubes in the six-cube width need to be taken out. So the number of cubes to be taken out is 9 x 6 x 6 = 324. The number of cubes in the hollow model is 528 – 324 = 204.

Extension

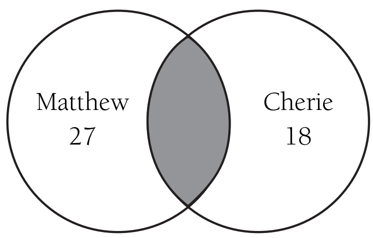
Vary the thickness of the shell of the hollow model. The shell is one cube thick in the problem given. Ask the students to find the number of cubes in a hollow model of dimensions 5 x 8 x 9 but with sides that are two cubes thick.

Problem Three

Here are two ways to find the number of different stamps:

- Add Matthew's 37 stamps and Cherie's 28 stamps and then subtract the 10 they have in common: 37 + 28 – 10 = 55.
- Matthew has 37 – 10 = 27 stamps that Cherie doesn't have. Cherie has 28 – 10 = 18 stamps that Matthew doesn't have. There are the 10 stamps they have in common: 27 + 18 + 10 = 55.

A diagram could look like this:



Problem Four

This problem can be solved using reasoning.

From the first condition, four of the pets are cats and parrots. This means there can be:

- one parrot and three cats or
- two parrots and two cats or
- three parrots and one cat.

Condition two says that three of the pets are parrots and dogs, so this means that you can have:

- one parrot and two dogs or
- two parrots and one dog.

This cancels one of the alternatives in the first condition because you cannot have three parrots.

Condition three shows that five of the pets are cats and dogs. From condition one, you know that you cannot have more than three cats and from condition two, you know that you cannot have more than two dogs. So, to fulfil condition three, there must be three cats and two dogs. Inserting these figures into conditions one and two shows that there must be two dogs, three cats, and one parrot, a total of six pets.

Page 11: Looking Ahead

Problem One

The students should be used to using a calculator to convert fractions to decimals. The illustration in the students' book shows what a calculator display would show when 1 is divided by 3. It clearly shows that this is a recurring decimal.

When students divide 1 by 7, the standard calculator display will show 0.1428571. This could be a recurring decimal with the string 142857 recurring infinitely, but it's impossible to know for sure without finding a calculator that displays more than eight digits or by doing long division. If students use long division to check, they should quickly see that ${}^{1}/_{7}$ does convert to a recurring decimal.

To show that 142857 recurs, use the notation $0.\dot{1}4285\dot{7}$, where the dots over the 1 and the 7 show that the section between and including these numbers repeats over and over again.

That string will in fact repeat eight times in the first 50 decimal places because 6 x 8 = 48. There will be two places still left after the eight lots of 142857. So the next two numbers will be 1 and 4. This means that the 50th decimal place is occupied by 4.

Problem Two

Most students will find the solution to the problem outlined below easier to understand if they have worked with cubes first.

Wiha can't carry enough food and water to walk straight across the desert, so she will have to leave a stash somewhere on the way. Use one cube to represent 1 day's food and water.

To survive walking the last 80 kilometres, Wiha must have four bags at the 20 kilometre mark (or the mark that was 80 kilometres from the end). How does she get the four bags there?

She can get three bags there by starting off with four bags, but that isn't enough to get her all the way across the desert, so she has to go back to get some more. She needs another bag to get back to her starting point, so she can leave only two bags hidden at the 20 kilometre mark on that first foray. It will take her 2 days to return to her hidden bags: 1 day to get there and 1 day to get back.

If she starts at the beginning of the desert again with four bags, she will have three bags when she gets to the 20 kilometre mark, where she has stashed two bags. That means she has five bags altogether. But she only needs four bags to get across the rest of the remaining 80 kilometres of desert and she can carry only four bags at a time, so she has one bag more than she needs. Therefore it would be sensible for her to take only three bags on her second "first 20 kilometre" trip: one to use on the way and two to add to the store at the 20 kilometre point.

She now has four bags at the 20 kilometre mark and can use one of these during each of the next four 20 kilometre walks. So Wiha can get across in 7 days: 3 days to get four bags to the 20 kilometre mark and 4 days more to go the final 80 kilometres.

This process is shown as a diagram in the Answers.

Extension

You could vary the conditions. Perhaps Wiha has a companion who can only carry three bags. How long will it take the two of them to cross the desert?

Problem Three

The students can use a table to find ways to make all the amounts from 5 cents to $1.05. They should always look for the minimum number of coins needed to make each amount. Consider the amounts from 5 cents to 95 cents:

Amount	5 cents	10 cents	20 cents	50 cents
5c	✔			
10c		✔		
15c	✔	✔		
20c			✔	
25c	✔		✔	
30c		✔	✔	
35c	✔	✔	✔	
40c			✔✔	
45c	✔		✔✔	
50c		✔	✔✔	
55c	✔	✔	✔✔	
60c		✔		✔
65c	✔	✔		✔
70c			✔	✔
75c	✔		✔	✔
80c		✔	✔	✔
85c	✔	✔	✔	✔
90c			✔✔	✔
95c	✔		✔✔	✔

They can make all these amounts with a minimum of five coins: one 5 cent coin, one 10 cent coin, two 20 cent coins, and one 50 cent coins. The students might now see that they can make the final two amounts ($1 and $1.05) with the five coins that they have already used. And these five coins add up to $1.05, so they will use all the coins when they buy something for $1.05. So this is the correct answer.

If the students continued the table to find the minimum number of coins needed for $1 and $1.05, they would use two 50 cent coins. Added to the coins used previously, this would bring the minimum number of coins up to six, and it would also mean that they won't use all the coins when they buy something for $1.05. Obviously, this can't be the correct answer.

Amount	5 cents	10 cents	20 cents	50 cents
$1				✔✔
$1.05	✔			✔✔

Problem Four

The students will have already encountered the relationship between the number of sides of polygons and the number of diagonals in Problem Three, page 7, in *Problem Solving*, Figure It Out, Level 3. This problem, which uses more sophisticated reasoning, should extend their understanding.

The students could count the number of diagonals in a hexagon. But it is more efficient to notice that there are three diagonals from every corner of the hexagon. So 3 x 6 = 18. But this counts each diagonal twice. So the answer is 9. Similarly, the students could draw and count for question **b**. They will see that there are five diagonals from every corner of the octagon, and 5 x 8 = 40. But once again, this counts each diagonal twice. So the number of diagonals is 40 ÷ 2 = 20.

This can be used to find the number of diagonals for any polygon. The rule is ($^1/_2$ x the number of sides) x (the number of sides – 3). The number of sides in a polygon is the same as the number of vertices. The number of sides minus 3 equals the number of diagonals from each vertex. Why subtract 3? If you join a vertex of a polygon to every other vertex, you use the number of sides minus 1. In this question, you have to count only the diagonals. There is no diagonal from a vertex to the two adjacent vertices, so another 2 is subtracted. Altogether, 3 is subtracted.

To write this algebraically, with n being the number of sides or vertices, the formula is:
diagonals = $^1/_2$ n x (n – 3).

Most students at this level won't be ready for this algebraic notation, but more able students may be comfortable using it.

Page 12: In and Up

Problem One

Students can use trial and improvement to solve this problem. Recording their trials on a table will help.

Hine's age now	Paora's age now	Hine's age in 4 years	Paora's age in 4 years	Is Hine twice Paora's age?
8	1	12	5	no
9	2	13	6	no
10	3	14	7	yes

There is always a 7 year difference in the ages, so if Hine is 10 years old now, in 4 years' time she will be 14 and her brother will be 7.

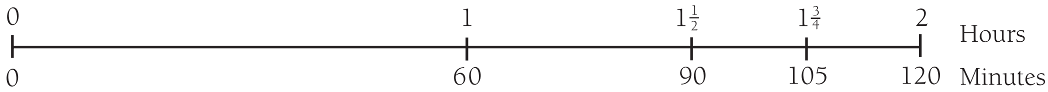
Problem Four, page 3, in *Problem Solving*, Figure It Out, Level 3, is a similar problem that could be used as a warm-up.

Problem Two

This problem requires students to convert dollars, which divide into 100 cents, to hours, which divide into 60 minutes. They need to recognise that the 15 minutes shown on the parking meter as 00:15 is not the same as 0.15 hours.

Parking costs $1 per hour, and Cindy put in $2, which will pay for 2 hours. What proportion of 1 hour is 15 minutes? ${}^{15}/_{60}$ is ${}^{1}/_{4}$ of an hour, so Cindy has shopped for 2 hours minus the 15 minutes she has left, which is $1{}^{3}/_{4}$ hours.

Alternatively, the students might work out that if Cindy had let her money run out, she would have been away for 2 hours. But she has 15 minutes left, so she must have been away for 2 hours minus 15 minutes which is $1{}^{3}/_{4}$ hours.
A double number line could also be used to solve this problem:



Problem Three

The students could model this problem with equipment.

The first load up in the pulley bucket can either be one barrel or two bags. Some students may try to take one of the barrels up first, but then they won't have anything to put in the bucket to go down (unless they just take the barrel back down, which achieves nothing). So the first load up must be two bags. One of the bags will go back down on the return journey. That will leave one bag at the top and six barrels and one bag at the bottom.

What should they send up in the next load? It could be the one bag, but that wouldn't achieve anything because the bag would have to come straight back down again. But if they send one barrel up, they can leave it there and send the one bag at the top down on the return journey. That will leave one barrel at the top and five barrels and two bags at the bottom. The next move will have to be the two bags again, because if they send another barrel, they will have to send down the barrel at the top down in the return journey, which won't achieve anything. So the two bags will go up again, and one will come down. Then another barrel can go up, and the bag at the top will come down.

Students will probably notice that this is a recurring pattern. To get one barrel up takes four trips up and down:
Trip 1: up with two bags
Trip 2: down with one bag
Trip 3: up with one barrel
Trip 4: down with one bag.

There are six barrels to go up, so after 6 x 4 = 24 trips, all six barrels will be at the top and the two bags will be at the bottom. One more trip is needed to get the two bags to the top, giving a total of 25 trips.

This problem is a slightly more complex version of the farmer and corn problem in Problem Three, page 10, in *Problem Solving*, Figure It Out, Level 3.

Problem Four

The students may try to solve this using trial and improvement, but using multiplication and division basic facts and their knowledge of multiplying and adding odd and even numbers is more efficient.

a. The students are looking for the smallest number of shots, so they could start by dividing 30 by the highest scoring hole (8). But 30 doesn't divide evenly into 8s. They could try three shots of 8 to get 24 points and then another three shots of 2 to make 30. That's six shots altogether. If they divide by the next highest scoring hole (5), they get exactly six shots

(30 ÷ 5 = 6). The students can continue to experiment with different combinations of shots to check that six is the smallest number of shots. Obviously, most of the score of 30 will need to be made up of shots of 5 or 8; if most of the score is made up of shots of 2, they will need a lot more than six shots. The answers give the other two ways of scoring 30 from six shots.

b. The highest number of points gained by scoring 8 each time gives a total of 48. But, of course, 48 is even. To get an odd number, you have to score an odd number on one shot. So the highest odd number is scored by shooting five 8s and a 5, for a total of 45.

Page 13: Back to Back

Problem One

Encourage the students to experiment with cards and to try to solve a smaller case before you go over this method with them.

To get the cards 1, 2, 3, 4, and 5 in the right order, you have to start off with the order 1, ■, 2, ●, 3, ♣, 4, ♠, 5, ♦, where the symbols stand for an unknown card.

The ■ has to be 6 because Derek will have put the ♦ on the bottom of the pile and the ■ will be dealt next. This tells you that the original order is 1, 6, 2, ●, 3, ♣, 4, ♠, 5, ♦.

It's tempting to think that the ● in this revised line up is 7. But that ● goes on the bottom of the pack. So it's the ♣ that is 7. The original order is now 1, 6, 2, ●, 3, 7, 4, ♠, 5, ♦.

The ♠ goes on the bottom of the pack, and so the ♦ must be 8. This gives 1, 6, 2, ●, 3, 7, 4, ♠, 5, 8.

The ● has to be missed again because it is put on the bottom of the pack. So the ♠ is 9. The ● has to be 10. The original order is therefore 1, 6, 2, 10, 3, 7, 4, 9, 5, 8.

The students can check this with a pack of 10 cards.

Discuss other methods that the students come up with for solving the problem. For example, they may put 10 cards in order from 1 to 10, deal them out in the order Derek did, and then work backwards from the way the cards were dealt to get the original order.

Extension

"What if Derek had had 20 cards to start with? What order would they have been in? Can you do 50?"

Problem Two

This could be a good problem to include in a unit of work about books, including how books are written and edited, the history of printing, current printing technology, and book design.

The students will need some understanding of how sheets of paper are folded and cut to make up books. They will probably have made their own books in which they've put several pages on top of each other, stapled them down the middle, and folded the pages over the staples. They could experiment by making books like this and seeing where the page numbers are positioned when the pages are unstapled.

This is a difficult problem to visualise. Model it by folding an A4 piece of paper in half twice, cutting most of the way across the fold (which is at the top or the bottom, depending on how you folded it), and numbering the pages in order. (Cutting most of the way and leaving a bit of the pages still attached means you can open the paper up again and see where the page numbers are positioned.) The position of the page numbers will vary depending on whether your fold was at the top of the page or the bottom of the page.

Problem Three

There aren't many possible routes in this problem, so trying all possible routes is still an efficient strategy. You could begin by asking the students what they think will be the shortest route.

The 9 kilometre road from Nīkau to Kōwhai is the longest leg in the journey, and students may try to find a way to avoid travelling this road. They will find, though, that if they avoid it, they will have to travel another road twice, which will make the overall journey longer.

Whichever route the students choose, there is only one road going through Wainui. So they could start with Paki, 7 kilometres, Wainui, 5 kilometres, Te Poi. At this point, they either go to Nīkau or Kōwhai. The first way gives Paki, 7 kilometres, Wainui, 5 kilometres, Te Poi, 3 kilometres, Nīkau, 9 kilometres, Kōwhai, 5 kilometres, Paki, a round-trip distance of 29 kilometres. Or the second way is Paki, 7 kilometres, Wainui, 5 kilometres, Te Poi, 5 kilometres, Kōwhai, 9 kilometres, Nīkau, 5 kilometres, Paki, a round-trip distance of 31 kilometres. The students should realise that it would be shorter overall to use the 3 kilometre trip between Te Poi and Nīkau.

Problem Four

This looks a lot like **Problem One**, page **3**. How can you get all the numbers from 1 to 13 using 1, 3, and 9 and addition and subtraction?

The answers show how you can find all the numbers. In the equations, + essentially means put the sticks end to end and – means put as a second row and use the difference between the lengths of the two rows.

Page 14: Creases and Cuts

Problem One

This question is very similar to **Problem One**, page **2**.

A good strategy is to use a table with trial and improvement. In the table, the number of letters and cards is always 16.

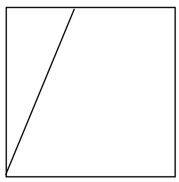
Letters	Cards	Cost
5	11	$5.55
6	10	$5.70
4	12	$5.40

The first row is a guess, and the second row shows the change in cost if you add on one letter. An extra letter costs an extra 15 cents. The first guess totalled $5.55, and you know that Irene spent $5.40. You can reduce the cost by 15 cents if Irene sent one fewer letter and one more card than in the first guess. So Irene sent four letters and 12 cards.

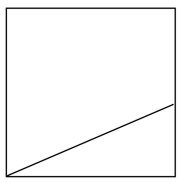
Problem Two

a. Students could experiment with different cuts and then come together as a class or group to compare their different cuts. Students who are working systematically will probably have tried all the cuts from one corner, then all the cuts from another corner, and so on, and then all the cuts from one side, all the cuts from another side, and so on. You may need to point out to them that all the cuts from the other corners will just give reflections or rotations of the cuts from the first corner. The same applies for the sides: all the cuts from the last three sides are reflections or rotations of cuts from the first side. So there are only two places the cuts can start from: a corner (it doesn't matter which) or a side (it doesn't matter which).

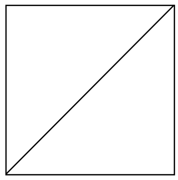
From a corner, you can either go to an opposite side (it doesn't matter which – a cut to one side will be a reflection of a cut to another side) to get a right-angled triangle and a trapezium:



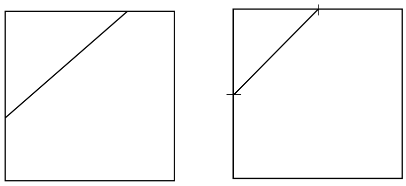
or



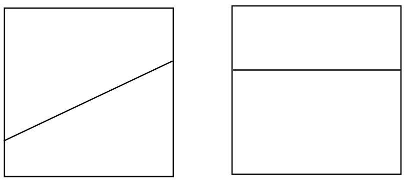
or to the opposite corner to get two right-angled, isosceles triangles:



From a side, you can either go to an adjacent side to get a right-angled triangle (that may be isosceles) and a pentagon:



or to the opposite side to get two trapezia or two rectangles:



b. With a hexagon there are more possibilities, so you have to be a bit more careful. However, you can still only start a cut from a corner or a side. As a result, you get:

- a cut through a corner
  - **i.** a triangle with one angle of 120° and a non-regular hexagon;
  - **ii.** an isosceles triangle with one angle of 120° and a non-regular pentagon;
  - **iii.** a quadrilateral and a pentagon (neither of which can be regular);
  - **iv.** two trapezia.
- a cut through a side
  - **i.** a triangle with one angle of 120° (which may or may not be isosceles) and a non-regular heptagon;
  - **ii.** a quadrilateral (which may be a trapezium) and a hexagon (neither of which can be regular);
  - **iii.** two pentagons (which may be congruent).

These diagrams are provided in the Answers.

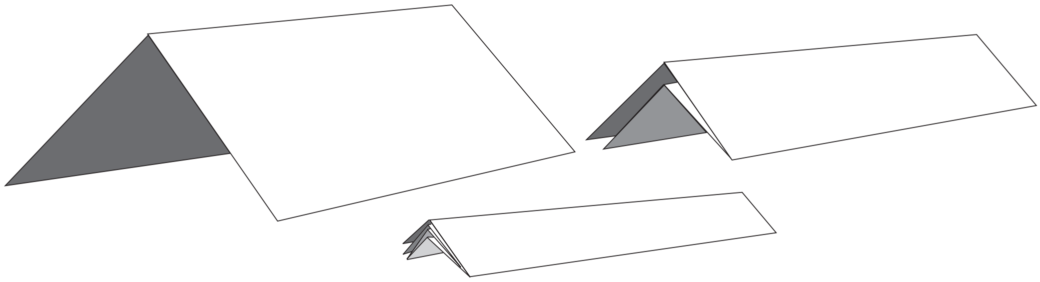
Problem Three

Most students will find this hard to visualise. Start by asking them to visualise the paper with one fold. Ask them how many creases there are.

If they visualise folding it again, they would be putting a crease into each of the two previously unfolded sections. So there are now three creases (the original crease plus the two new ones). The strip is now in four sections.

The third fold puts a crease in each of the four previously unfolded pieces. So now there are seven creases.

The students can then check by folding a strip of paper.



Problem Four

One bowl of rice is served for two guests, one bowl of soup for three guests, and one bowl of meat for four guests, so the number of guests must be divisible by 2, 3, and 4. 2 goes into 4, so you only have to find numbers that are divisible by 3 and 4. These could be 12, 24, 36, 48, … or in fact any multiple of 12.

Now check how many bowls of rice, soup, and meat these numbers would give and stop when you find that you are using 39 bowls.

For 12 guests, you would need six bowls of rice, four bowls of soup, and three bowls of meat. This gives 13 bowls, which is not enough. But 13 x 3 = 39, the number of bowls used at the banquet, so you need three times as many bowls and also three times as many guests.
3 x 12 = 36, so there are 36 people.

Page 15: In Line

Problem One

The best strategy is to build up the queue as you feed in the information. It is done here in words, although the students might find it easier to use marked bottle-tops or pieces of paper with names on.

i. Only Zoe is between Albert and Ripeka, so you have:
either Albert, Zoe, Ripeka or Ripeka, Zoe, Albert.
There is no one between these three shoppers.

ii. Katrina is next to Emil, so you have:
either Katrina, Emil or Emil, Katrina.
There is no one between these two shoppers.

iii. There are two shoppers between Ripeka and Emil. There are only five shoppers in the queue, so Emil comes either before or after Albert in the queues you started in **i**.
Hence you have:
either Emil, Albert, Zoe, Ripeka or Ripeka, Zoe, Albert, Emil.

From **ii**, Katrina is next to Emil, so you have:
either Katrina, Emil, Albert, Zoe, Ripeka or Ripeka, Zoe, Albert, Emil, Katrina.

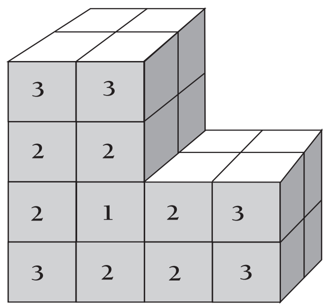
The last fact given is that Emil is near the end of the queue, so the order has to be Ripeka, Zoe, Albert, Emil, Katrina. Ripeka is first in line.

The students should check their final answer by making sure that all of the information in the problem has been used and used correctly.

This is a good problem to test the students' logic. They could also write their own problems about queues. They will see the need to include sufficient information to produce an answer but only just enough to make it unique. (In the problem given, there would have to be two answers if the last fact was not given.)

Problem Two

The students will need to work systematically to ensure that they count all the cubes but also that they count each cube only once. It's probably easier to draw a diagram and label each cube with the number of painted faces. Then they can count all the cubes with one painted face, two painted faces, and so on.



Counting will be quicker if they realise that the model is the same front and back, so they only need to count the painted faces of the cubes on the front of the model and then double these numbers to get the totals.

The students can check their answers by counting the number of cubes in the diagram (24) and then adding all the numbers of cubes that have got one, two, and three painted faces (2 + 12 + 10 = 24). This way, they can be sure that they've counted all the cubes.

Problem Three

Students can approach this in various ways and with varying degrees of sophistication. They could use trial and improvement, or they could make their trial and improvement more efficient if they notice that starting with the consecutive number that is one higher than in their previous trial increases the sum by 3. For example, 18 + 19 + 20 = 57 and 19 + 20 + 21 = 60. The sum increases by 3. How far are they away from 81? 81 – 60 = 21, so they need another 21 in the sum. 21 ÷ 3 = 7, so they need to add 7 to the first consecutive number. This gives 26 + 27 + 28 = 81.

The most sophisticated approach requires students to understand that multiplication is repeated addition. Adding three consecutive numbers is like multiplying the middle number by 3 (because multiplication is repeated addition). It is the middle number that is multiplied by 3 because the first number is one less than the middle number and the last number is one more and they cancel each other out. To find the middle number, divide 81 by 3. 81 ÷ 3 = 27, so the three consecutive numbers are 26, 27, and 28.

You might like to begin by referring to **Problem One b** on page **7** and then asking the students an easier question. For example, what three consecutive numbers add to 21?

Problem Four

This problem involves finding unknown variables. For students at this level, using a table and trial and improvement are good strategies. They need to make sure that, in each row, the numbers of one- and two-scoop ice creams add up to 20.

When they get to this point in the table, the students should be able to work out the correct answer:

Number of one-scoop	Number of two-scoop	Cost of one-scoop	Cost of two-scoop	Total cost
5	15	$5	$22.50	$27.50

At this point, they are only 50 cents short in the total cost, so they need to add one two-scoop and subtract one one-scoop.

The correct number of ice creams is 4 one-scoops and 16 two-scoops.
Check: (4 x $1) + (16 x $1.50) = $4 + $24
= $28

The students could also work through the problem like this:
If all 20 children had one-scoop ice creams, the total cost would be $20. But if one child had a two-scoop ice cream, the cost would have been $19 + $1.50 = $20.50. In fact, each child who has a two-scoop ice cream pushes the $20 one-scoop bill up by 50 cents. The final bill was $28, which is $8 above $20. $8 is 16 x 50 cents, so 16 children had two-scoop ice creams and four children had one-scoop ice creams.
Check: (4 x $1) + (16 x $1.50) = $4 + $24
= $28

Page 16: Dates and Shapes

Problem One

The students need to think first about what odd numbers they can use. They can use 1, 3, 5, 7, and 9, but they can't use 11 or above because adding three more odd numbers will give a total higher than 12.

The best strategy is to start with three ones and work systematically. Some students may need to write down all the options and cross out those that do not work, for example:

1 1 1 1 ✘
1 1 1 3 ✘
1 1 1 5 ✘
1 1 1 7 ✘
1 1 1 9 ✔
1 1 3 3 ✘
1 1 3 5 ✘
1 1 3 7 ✔

Encourage the students to move as quickly as possible to a list of only those that do add up to 12, by adding in their heads as they go and leaving out those that do not fit the criteria.

They will then end up with a list of five:
1 + 1 + 1 + 9
1 + 1 + 3 + 7
1 + 1 + 5 + 5
1 + 3 + 3 + 5
3 + 3 + 3 + 3

Problem Two

To solve this problem mathematically, students need to work out how the days of the week change for a certain date in preceding years. (The other way to solve the problem is to look up a calendar for the year they were born at a library or on the Internet. Once the students have solved the problem mathematically, they could use this alternative method to check their answer.)

Working out how days change from year to year would be simple if it weren't for leap years. In an ordinary year, there are 365 days, which is 52 weeks and 1 day. So from one non-leap year to the next year that is also a non-leap year, any date will be one day of the week later. For example, 2 March 2001 is a Friday; 2 March 2002 is a Saturday. Conversely, going from one non-leap year back to the previous year that is also a non-leap year, any date will be one day of the week earlier. For example, 24 January 2002 is a Thursday; 24 January 2001 is a Wednesday.

Leap years occur every year that is a multiple of 4. Leap years are 366 days long or 52 weeks and 2 days. The extra day is 29 February. So for birthdays on or before 28 February, moving forwards from a non-leap year to a leap year, the day of the week will move forward by 1 day. This is because they have not yet got to the extra day in the year (29 February). Moving from a leap year to a non-leap year, the day of the week will move forward by 2 days. This is because the leap-year birthday is before the extra day (29 February).

For birthdays on 1 March and after, moving from a non-leap to a leap year, the day of the week will move forward by 2 days because the leap-year birthday is after the extra day (29 February). Moving from a leap to a non-leap year, the day of the week will move forward by 1 day because the extra day (29 February) occurred before the leap-year birthday. As above, these rules are reversed when working backwards.

Another way of looking at this is asking "Is there a leap day (29 February) in the 1 year period from one birthday to the next?" If yes, the day of the week will move forward by 2 days. If no, the day of the week will move forward by 1 day. Working backwards, if there is a leap day in the 1 year period from one birthday to the previous birthday, the day of the week will move back 2 days. If there isn't a leap day in the 1 year period from one birthday to the previous birthday, the day of the week will move back 1 day.

Below are examples of the changing days for a birthday before and a birthday after 28 February for people born in 1992.

Birthday on 24 January			
Year	Number of days between previous birthday and this birthday	Go back 1 or 2 days	Day
2002			Thurs
2001	365	1	Wed
2000	366	2	Mon
1999	365	1	Sun
1998	365	1	Sat
1997	365	1	Fri
1996	366	2	Wed
1995	365	1	Tues
1994	365	1	Mon
1993	365	1	Sun
1992	366	2	Fri

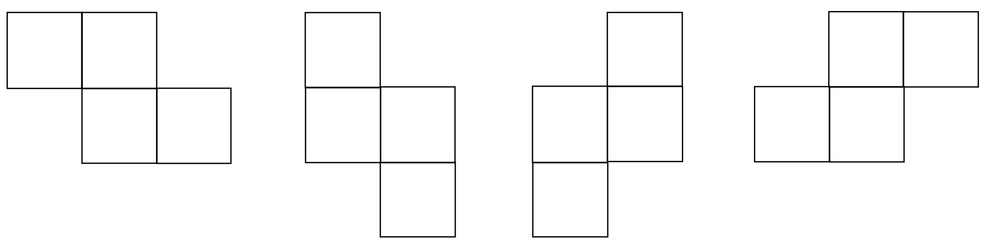
Birthday on 2 March			
Year	Number of days between previous birthday and this birthday	Go back 1 or 2 days	Day
2002			Sat
2001	365	1	Fri
2000	365	1	Thurs
1999	366	2	Tues
1998	365	1	Mon
1997	365	1	Sun
1996	365	1	Sat
1995	366	2	Thurs
1994	365	1	Wed
1993	365	1	Tues
1992	365	1	Mon

Note that for birthdays on or before 28 February, the day of the week moves back 2 days on the leap year, but for birthdays on or after 1 March, the day of the week moves back 2 days the year before the leap year.

The students will need to work very carefully through this problem because of the effect of a leap year. Those who were born on 29 February will have to decide whether they celebrate their birthday in non-leap years on 28 February or 1 March.

Problem Three

a. There are only five tetrominoes that can be made without involving reflection or rotation. Examples of these are shown in the Answers. The students may have reflections or rotations of these shapes and think that they are different tetrominoes. You may need to show them that they are actually the same as one of the tetrominoes given in the Answers. For example, the following shapes are all the same tetromino under rotation or reflection:



b. The six tetrominoes combined will give 24 squares (4 x 6 = 24). So the rectangle could have dimensions of 1 x 24, 2 x 12, 3 x 8, or 4 x 6. Some of the tetrominoes take up two rows, so you can't make a 1 x 24 rectangle.

A 2 x 12 rectangle is also not possible. Experiment, and you will see why. The only two rectangles left with 24 squares are 3 x 8 and 4 x 6. These are shown in the Answers.

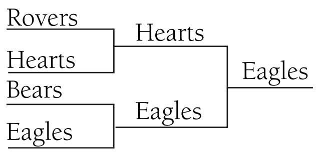
Problem Four

This problem can be solved very simply. The solution is outlined in the Answers.

Page 17: Out for the Count

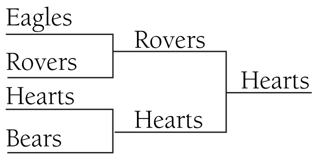
Problem One

a. Based on the information given, the Hearts must have beaten the Rovers in this tournament and the Eagles won the other preliminary game (because the Bears lost all their games). The Eagles did not lose to the Hearts, so they had to win the final.

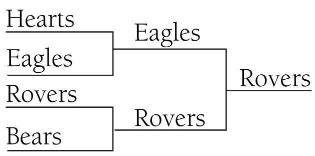


b. The first game in the top half of the draw could go either way. There is no information about how the Eagles and the Rovers fare when they play each other. So either outcome is possible.

The second game is a definite win for the Hearts against the Bears, who lost all of their games. If you assume that the Eagles beat the Rovers, they would have played the Hearts in the final. This would make the Eagles the winners because they always beat the Hearts. However, the Eagles won the first tournament, so they cannot have won the second one (a different team won each tournament). So the only possibility, based on the information given, is that the Rovers won their first game and had to play the Hearts (for the reason above). In the final, the Hearts won, based on the information given (the Rovers lost to the Hearts).



c. In the last tournament, the Rovers beat the Bears and the Eagles beat the Hearts (based on the information given). The final is the Eagles versus the Rovers. The Eagles have already won a tournament (see **a**), so the Rovers must have won this final.



Problem Two

The students need to first identify the rhombuses and trapezia in the shape. There is only one type of each: the rhombuses have side lengths of one stick and the trapezia are half the hexagon and comprise three triangles. The students can work systematically to count the rhombuses and trapezia. Drawing a diagram and marking the shapes may help to avoid counting any shape twice.

Problem Three

This question can be done (the long way) by writing out the alphabet backwards up to N and then assigning odd numbers from 1 up, starting at Z:

Z	Y	X	W	V	U	T	S	R	Q	P	O	N
1	3	5	7	9	11	13	15	17	19	21	23	25

You'll probably find that it's slightly quicker to write the alphabet out starting at N and ending at Z. Then write down the odd numbers in order going backwards from 1 at Z.

Another way is to calculate how many letters there are from N to Z, inclusive. There are 13. The 13th odd number is 25.

Problem Four

Students who have trouble with this problem could first try Problem Three, page 9, in *Problem Solving,* Figure It Out, Level 3.

At any "o'clock", the hour hand is always on the 12. The only way that the minute hand can make an angle of 90° with the hour hand is if it is on the 3 or the 9. So the only other right-angled o'clock is 3 o'clock. Some students may have suggested other times where the hour and the minute hands are 15 minutes away from each other, for example, 5 past 4 or quarter to 6. You may need to remind them that they have to find "o'clock" times, that is, where the hour hand is on 12. Also, at times such as 5 past 4 or quarter to 6, the hands don't make a true right angle because the hour hand is not exactly on the hour number; it will be before or after that number.

Extension

"You could explore right-angled time. Can you have the two hands at right angles at some time other than 3 o'clock and 9 o'clock? Can you have a right-angled half hour?"

Page 18: You're All Heart!

Problem One

Like **Problem Four**, page **14**, this problem involves common multiples. Begin by considering the problem without the extra 5 cents. This can be added on later. The amount must be made up in coins, so it must be able to be divided by 5 (the smallest coin). Find the lowest common multiple of 2, 3, 4, and 5. 2 x 3 x 4 x 5 = 120. Check to see if this is the lowest common multiple by halving it and seeing whether it can still be divided evenly by all the numbers. 60 ÷ 2 = 30, 60 ÷ 3 = 20, 60 ÷ 4 = 15, 60 ÷ 5 = 12. Can it be halved again? No, 30 ÷ 4 = 7.5, so 30 is not a common multiple. Can 20 be the lowest common multiple? No, 20 ÷ 3 = $6.\dot{6}$. So 60 is the lowest common multiple. So the amount of money is 60 cents + 5 cents or any multiple of 60 cents + 5 cents that is less than $2, that is 65 cents, 125 cents, or 185 cents.

Problem Two

The students should be able to see the pattern in this building. The number of cubes added to each new bottom layer increases by one each time. So the number of cubes in each layer is:

first layer	1
second layer	1 + 2 = 3
third layer	1 + 2 + 3 = 6
fourth layer	1 + 2 + 3 + 4 = 10

Altogether, there are 1 + 3 + 6 + 10 = 20 cubes.

Students may recognise these numbers as triangular numbers or they may see that from a bird's-eye view, the diagram is a triangle and hence the numbers will be triangular.

Students can check their calculations by making the building with cubes and counting how many they needed.

Problem Three

This problem links to the Measurement strand of the curriculum because students will need to learn how to accurately measure their pulse, and it could also form part of a health unit.

Suppose that a student's heart beats 100 times a minute. (The normal range for a 10- to 11-year-old is 90–110 beats per minute.) Then their heart will pump 80 x 100 millilitres of blood per minute.

The student's heart pumps 80 millilitres of blood every beat and beats 100 times per minute. There are 60 minutes in an hour and 24 hours in a day. The amount of blood pumped is 80 x 100 x 60 x 24 = 11 520 000 millilitres per day. There are 1 000 millilitres in a litre, so this is 11 520 litres per day. To help the students to visualise how much 11 520 litres is, they could calculate the amount of water in a full bath or a swimming pool, calculate how many litres would fit in their classroom (remembering that 1 litre is 1 cubic metre), or calculate how far 11 520 1-litre milk containers would stretch if they were laid end to end.

Problem Four

The wheel has gone around 236 times on Sam's way to school, and each time it goes around, it covers 225 centimetres. Sam lives 236 x 225 centimetres from school. This is 53 100 centimetres or 53 100 ÷ 100 = 531 metres (just over half a kilometre). If the students have difficulty with this concept, you could model it with a trundle wheel or have the students measure the circumference of a round object, roll it along a surface for one revolution, and measure the distance it has covered (which will be the same as the circumference).

To answer part **b**, the students need to find out how many 225 centimetres there are in 1 kilometre. 1 kilometre = 1 000 metres = 100 000 centimetres. So the wheels turn 100 000 ÷ 225 = $444.\dot{4}$ times.

Page 19: What Goes Around …

Problem One

The students can approach this problem in various ways. More able students will recognise this as another lowest common multiple problem. What is the lowest common multiple of 6 and 4? It's 12. At 12 minutes, Millie will have completed exactly three laps at 4 minutes per lap and Billy will have completed exactly two laps at 6 minutes per lap. (Millie will lap Billy at the point where they started racing.)

A slightly less efficient approach is to record Millie's and Billy's progress on a table. Students could use several different tables, depending on what is clearest for them. This table shows where Billy is each time Millie completes a lap.

Time	Millie's laps	Billy's laps
4	1	$^2/_3$
8	2	$1^1/_3$
12	3	2

If the students are having trouble grasping these concepts, this is a good problem to act out, model with equipment, or draw as a diagram.

Millie obviously takes the lead. When Billy completes his first lap, Millie has done a lap and a half. So when Billy completes two laps, Millie completes three and catches him up. This takes 2 x 6 or 3 x 4 minutes. In other words, Millie catches up in 12 minutes.

Another way to look at this is that they meet again when a multiple of 6 equals a multiple of 4.

Problem Two

This problem is similar to **Problem One**, page **1**. The students can use the same method to find the total of the string of eight numbers, that is: sum $= (2 + 9) \times (8 \div 2)$

$$= 11 \times 4$$
$$= 44$$

The students have to find sets of four numbers that add up to 22. Once they have found one, the remaining four numbers add up to 22 as well, giving two sets of four numbers. It is best to do this systematically by considering 2 with every other possible number.

2 and 3: 2 + 3 = 5, so you need to make up 17 with the other two numbers in that set. You can only make up 17 with 8 and 9, so the first two sets are {2, 3, 8, 9} and the numbers that are left, {4, 5, 6, 7}.

2 + 4 = 6, so you need to make 16 with the other two numbers. You can do this with 8 + 8, 7 + 9, 6 + 10, 5 + 11, and so on. Clearly you can't have 8 + 8 because you only have one 8 to use. You can't have 6 + 10 because there is no 10. So the only possibility here is 7 + 9. The two sets here are {2, 4, 7, 9} and {3, 5, 6, 8}.

2 + 5 = 7, so you need 15. 15 can only be made up here from 6 + 9 and 7 + 8. So this gives two possibilities: {2, 5, 6, 9} and {3, 4, 7, 8}; or {2, 5, 7, 8} and {3, 4, 6, 9}.

2 + 6 = 8, so you need 14. 14 can be made up from 5 + 9, 6 + 8, and 7 + 7. 5 + 9 is no good because you already had that combination {2, 5, 6, 9} in the last case. And 6 + 8 or 7 + 7 would both mean using a number twice. There are no answers here.

2 + 7 = 9, so you need 13. 13 = 4 + 9, 5 + 8, 6 + 7. You have already considered all these possible sets.

2 and 8: You have considered all these possibilities.

2 and 9: You have considered all these possibilities.

The four pairs of sets that satisfy the conditions of the question are given in the Answers.

Problem Three

The students could use trial and improvement to solve this problem, but encourage them to look for a pattern in the length of the sides and the total number of counters. A table will help them see the pattern:

	Side length	Total
First pentagon	2	5
Second pentagon	3	10
Third pentagon	4	15

There are five sides in a pentagon, so the students might assume that to get the total number of counters they just need to multiply the side length by 5. But this doesn't work because each corner counter is part of two sides, and multiplying the side length by 5 will mean the corner counters are counted twice. (This is a similar situation to **Problem Two**, page **10**.) To avoid counting counters twice, the students need to subtract 1 from the side length and then multiply by 5. This will give the general rule for any side length:

	Side length	Total
First pentagon	2	5
Second pentagon	3	10
Third pentagon	4	15
	n	(n – 1) x 5

Joseph used 75 counters to make a pentagon. Applying the rule gives an answer of 16:

$$(n - 1) \times 5 = 75$$
$$n - 1 = 75 \div 5$$
$$n - 1 = 15$$
$$n = 16$$

Problem Four

The students can find all the possible numbers by working systematically. There can only be one digit in the tens column, so the only ones and hundreds digits will be digits that multiply to give a one-digit product. For example, there can't be a number with 3 in the hundreds column and 4 in the ones column because the product of 3 and 4 is 12, and this won't fit in the tens column.

Take each hundreds digit in turn, starting with 1, and work out what the ones digit would be to give a tens digit that is the product of the hundreds and the ones digits. For example, a hundreds digit of 1 and a ones digit of 0 would give 0 as the tens digit, so that house number is 100. A hundreds digit of 1 with a ones digit of 1 would give 111, and so on. Work systematically through the remaining hundreds digits.

A full list is given in the Answers.

Work in a similar way for part **b**, although this time, the only possible ones and tens digits are those that *add* to a one-digit sum.

Page 20: Going Around

Problem One

The information that the students have to work with is that a 3 goes in the centre square and that there is only one 1 used somewhere else. The 1 can go either in a corner or in the middle of one of the four sides of the square. Putting it in a corner will mean that a 5 will go in the opposite corner.

1		
	3	
		5

The two missing numbers in the top row must add up to 8, the two missing numbers in the middle row must add up to 6, and in the bottom row to 4. The bottom row is the best one to fill in next because there is only one possible option for this row:

1		
	3	
2	2	5

The numbers must be 2 and 2. The other two pairs of numbers that add up to 4 are 1 + 3, which is not possible because you can use only one 1, or 4 + 0, which is not possible because you can't use 0 (you have to use positive numbers).

But from here, you come to a dead end for several reasons. The only option for the right column is another two 2s, which means the middle square in the top row must be 6, which makes the middle column add up to 11, which is too high. Also, the only option for the middle square in the left column is 6, which would mean the middle square in the right column would have to be 0, which is not possible.

There is no point trying the 1 in another column because, due to the symmetry of the square, you will end up at the same dead end, with the numbers being a rotation or reflection of the attempt above.

With the 1 in a middle square you can get this (and rotations and reflections of this):

	1	
	3	
	5	

Following the reasoning given above, the other two numbers in the bottom row must be 2 and 2, and this allows you to fill in all the other squares:

4	1	4
3	3	3
2	5	2

Explained like this, the problem seems fairly straightforward, but it will probably take the students quite a lot of trial and improvement to work it out. Carefully thinking through all the options and the effects of all the options will make their working more efficient. For example:

- When there are several squares to fill in, first fill in the squares where there is only one possible option (such as the two 2s either side of the 5 in the examples above).
- Think about the numbers that you can use. Using another 5 will mean using another 1, and this is not possible, so there must only be one 5. As explained above, 6 and above are not possible because it would mean using a 0 as well and you can use only positive numbers.
- Consider the effect of rotation and reflection (such as trying a 1 in a different corner). As explained above, this will also lead to a dead end.

Problem Two

This question is similar to **Problem Four**, page **11**. There must be five tracks from any village to any other village. There are six villages, so it looks as if there are going to be 5 x 6 = 30 tracks. However, this method has counted every track twice. (The one from Zeebe to Uba was counted when you counted the tracks from Zeebe, and it was also counted when you counted the tracks from Uba.) So there are actually 15 tracks on Circuit Island.

Extension

"What if there were 100 villages on the island?"

Problem Three

a. There are 32 marbles altogether. If there is to be an equal number in each jar, there has to be eight marbles in each. The simplest way to do this is to take two marbles from the yellow jar and one from the green jar and put all three marbles into the blue jar.

b. The students can build on their working for part **a** to find the answer for part **b**. There are 32 marbles and 4 jars, so there is an average of 8 marbles in each jar. The two middle jars (blue and yellow), which have a difference of two marbles, will have the numbers either side of 8, that is, 7 in the blue jar and 9 in the yellow jar. The red jar will have 5 marbles, and the green jar will have 11 marbles. Check: 5 + 7 + 9 + 11 = 32.

Note that because there are an even number of jars, none of the jars in this scenario will contain the average number of marbles. This might be slightly difficult for the students to grasp. They would probably find the problem easier if there were an odd number of jars, in which case the middle jar would contain the average number of marbles. For example, if there were three jars: the first has two fewer marbles than the second and the second has two fewer marbles than the third. Altogether, there are 30 marbles in the jars. How many in each jar? There'll be an average of 10 marbles in each jar, so there'll be 10 in the second jar, 8 in the first jar, and 12 in the third jar.

Some students may approach the problem algebraically:
r = number of marbles in red jar
$b = r + 2$
$y = r + 4$
$g = r + 6$

$r + b + y + g = 32$
$r + (r + 2) + (r + 4) + (r + 6) = 32$
$4r + 12 = 32$
$4r = 20$
$r = 5$
so $b = 7$, $y = 9$, and $g = 11$

If the students have trouble with these strategies, they could use trial and improvement to find four consecutive odd numbers that add up to 32.

Problem Four

This problem is very similar to Activity One, page 23, in *Measurement*, Figure It Out, Level 3. You could give the students this activity first as a warm-up. Refer also to the teachers' notes for this activity.

a. If you cooked four patties on both sides and then the remaining two, the time taken would be 5 minutes plus another 5 minutes = 10 minutes. However, the following steps show a different story:

- Place four patties on the griller.
- After 3 minutes, flip two of the patties and remove two (place in a warmer). Add the two remaining patties.

- After 2 minutes, remove the two patties that are now cooked on both sides.
- After another minute, flip the two remaining patties and add the two "half-cooked" patties that were placed in the warmer.
- Remove the four patties after a further 2 minutes.

Total time elapsed is 3 + 2 + 1 + 2 = 8 minutes.

Notice that if there were five patties, the same method would work if you split the patties into one group of three and another of two. After 3 minutes, two of the cooking patties would be flipped and the other placed in the warmer while the two remaining raw patties would be started up. If, on the other hand, there were seven patties, nothing would be gained. It would take 10 minutes to cook seven patties, just as it would for eight.

b. It would take 13 minutes to cook 10 patties. This is because 10 = 6 + 4.
It takes 8 minutes to cook six patties (shown above) and another 5 minutes to cook the remaining four. So the time needed is 8 + 5 = 13.

Extension

An extension question could be: "How long do you need to cook 25 patties?"

The most efficient patty cook-up is when you have multiples of 4. 25 is not a multiple of 4. The situation that wastes time is when one patty has to be cooked alone, so breaking 25 up into six lots of four with one remaining is not efficient. But by cooking five lots of four and then the remaining 5, you can do it in 5 x 5 minutes + 1 x 8 minutes = 33 minutes.

The same could be achieved by grouping as follows:
Four lots of four, one lot of six, and then the remaining three patties gives
4 x 5 + 1 x 8 + 1 x 5 = 20 + 8 + 5
= 33.

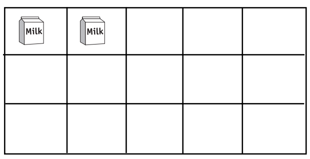
Page 21: Round Robins

Problem One

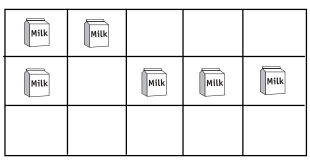
There are many possible answers for this. The students can experiment with equipment or diagrams and discuss their approaches with one another. For example, they could discuss how many cartons there are in each row and column and whether they find a rule for arranging the cartons.

There isn't room for more than five cartons in each row, so the possible even numbers in each row are two and four. There are 10 cartons altogether, so there must be two rows of four and one row of two. Similarly, the only possible even number of cartons in a column is two, so there must be two cartons in each column.

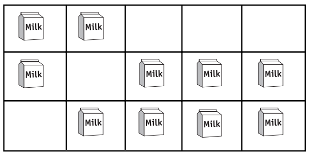
Here's one answer, starting with two cartons in the top row:



When you put four cartons in the second row, you can't have two of them in the same columns as the first two cartons. If you do, you'll force three in a column when you put the last four cartons in. Therefore, you must have something like the situation below:



Then the final four cartons go in like this:

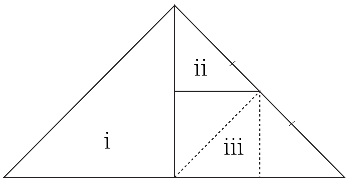


Every other possible answer is obtained in the same way. Put two cartons in any row. Then add the remaining cartons. In the last two rows, you'll have three cartons in the columns not occupied by the first two cartons. You'll also have one carton in each of these last rows aligned in a column with one of the original two cartons.

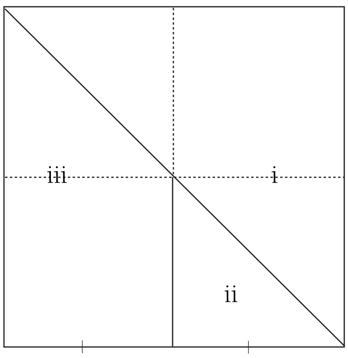
Problem Two

This problem draws on the students' knowledge of geometry and symmetry and requires them to visualise fractions of shapes. Drawing diagrams or cutting up the shapes will help this visualisation. The students will visualise the fractions in different ways depending on what makes most sense to them. The way they visualise does not matter as long as they check that their visualisation is correct. In the methods discussed below, each is visualised in a slightly different way.

The half in the triangle should be easy for students to see. The vertical line runs down the centre of the triangle and cuts it in half. What is the next part that is easy to recognise? The students may notice that part **ii** will fit into half the triangle four times, so it must be $^1/_4$ of $^1/_2$, which is $^1/_8$. Parts **ii** and **iii** make up $^1/_2$, so **iii** must be $^3/_8$ ($^1/_8 + {}^3/_8 = {}^4/_8$, and $^4/_8 = {}^1/_2$).

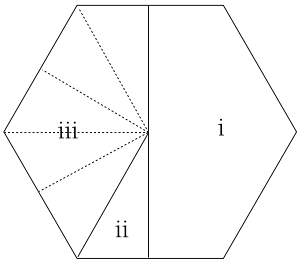


Once again, the half of the square should be easy to see. The diagonal line is a mirror line that cuts the square in half. Part **ii** must be $^1/_8$ because it is half of $^1/_4$ of the square. Part **iii** must be $^3/_8$ because it is a whole quarter plus another $^1/_8$.



The diagram shows the square divided into quarters.

Part **i** of the hexagon is also a half. (This may be slightly harder to see than a hexagon divided in half with a line from vertex to vertex and resulting in two trapezia.) A student cutting up the shape will find that **ii** fits into the hexagon 12 times, so it must be $^1/_{12}$. Part **iii** must be $^5/_{12}$ because $^1/_2 - {}^1/_{12} = {}^5/_{12}$.



Problem Three

The students need to look for two consecutive factors of 1 056. They could use trial and improvement by starting from a known multiplication fact that gives a product that is close to 1 056. For example, if they look at 30 x 30 = 900, this will tell them that the two numbers will be a bit higher than 30. However, as factors form an important part of mathematical thinking, it is better for the students to use them to solve this problem.

To find the prime factors of any large number, divide by the smallest possible factor (2 if it is an even number). Divide the result again by the smallest possible factor. Keep doing this until you arrive at **1.** A table layout is a very useful way to do this:

The number	Divide by
1 056	2
528	2
264	2
132	2
66	2
33	3
11	11
1	

The right column shows that 1 056 = 2 x 2 x 2 x 2 x 2 x 3 x 11. The diagram below shows these factors as two groups:

$$1\,056 = \underbrace{2 \times 2 \times 2 \times 2 \times 2}_{(= 32)} \times \overbrace{3 \times 11}^{(= 33)}$$

The pages are 32 and 33.

Another check is to note that the left-hand page of a book always has an even page number and the right hand always has an odd page number, so the even number in the answer should be smaller than the odd.

Extension

Challenge the students to find the consecutive pages that have a product of 2 352 (48 and 49). The prime factors of 2 352 are 2 x 2 x 2 x 2 x 3 x 7 x 7.

Problem Four

This is the same problem as **Problem Two**, page **20**, but the tracks have been changed to games. There are six teams, so there will be $\frac{1}{2}$ x 6 x 5 = 15 games.

Page 22: Dining on Digits

Problem One

Note that there will be small errors depending on the way that the coins are measured. The measurements given here are as accurate as possible.

a. There are 20 5-cent coins in $1, so there are 20 x 25 = 500 5-cent coins in $25.
A 5 cent coin has a diameter of about 1.9 centimetres.
So the trail was 500 x 1.9 = 950 centimetres or 9.5 metres long.

b. There are 10 10-cent coins in $1, so there are 10 x 25 = 250 10-cent coins in $25.
A 10 cent coin has a diameter of about 2.3 centimetres.
So the trail was 250 x 2.3 = 575 centimetres or 5.75 metres long.

Problem Two

The students need to work systematically to arrange the digits on the cubes. For example:

- There are six faces on each cube.
- Which digits do they need two of? They need two of 1 and two of 2 to get 11 and 22. They also need a 0 to go in front of all the other nine digits (1–9), and because the nine digits won't fit on one cube, they'll need a 0 for both cubes. So far, this makes:

0	1	2			

0	1	2			

- 6 can go upside down as a 9.
- There are six digits left to be placed: 3, 4, 5, 6, 7, and 8. There are six places left on the cubes. It doesn't matter which cube each digit goes on although, as noted in the Answers, there is only one solution if the cubes are based on the ones in the first illustration in the students' book.

0	1	2	3	5	7

0	1	2	4	6	8

Problem Three

This problem is similar to **Problem Four**, page **1**, but this time it is dealing with cuts in two dimensions rather than the three dimensions of the cheese problem. The same principle applies, though, that with every cut that the students make, they should try to divide as many existing pieces as possible. They should keep in mind that the pieces can be any shape or size.

After a bit of experimenting, the students should be able to get 16 pieces. But is this the largest number of pieces with five straight cuts? The pattern below suggests that it is:

Number of cuts	Number of pieces	Difference
1	2	
2	4	2
3	7	3
4	11	4
5	16	5

The pattern above also indicates that 22 cuts are possible with six straight cuts. In reality, some pieces may be so small as to be insignificant!

Problem Four, page 14, in *Problem Solving*, Figure It Out, Levels 2–3 and the corresponding teachers' notes could be a useful warm-up for students who are having difficulty with this problem.

Problem Four

This is not a difficult problem, but the students will have to work systematically through the information to find the answer. Firstly, they need to work out how many tables are needed for each size group. Groups of one, two, three, or four will need one table each. Groups of five or six will need two tables each. (The illustration in the students' book shows how six people sit around two tables.) Groups of seven or eight will need three tables.

Group size	Tables needed	Number of groups	Total tables
8	3	2	6
6	2	3	6
5	2	1	2
4	1	3	3
2	1	2	2
1	1	1	1
		Total	20

Page 23: Waste Not ...

Problem One

Drawing a table and working through the clues systematically to eliminate possibilities is a good way to solve this problem. Put an X in a square where the person doesn't play the game.

Tania and Marie play team sports, so put an X against them in the golf and archery columns.

	Netball	Hockey	Golf	Archery
Tania			✘	✘
Jerome				
Sonny				
Marie			✘	✘

Sonny and Tania hit the ball, which eliminates netball and archery for both of them. There is now only one space left in Tania's row, so she must play hockey. As a result, no one else can play hockey, and Sonny must play golf.

	Netball	Hockey	Golf	Archery
Tania	✘	✔	✘	✘
Jerome		✘	✘	
Sonny	✘	✘	✔	✘
Marie		✘	✘	✘

This shows that Marie plays netball and Jerome must be the archer.

	Netball	Hockey	Golf	Archery
Tania	✘	✔	✘	✘
Jerome	✘	✘	✘	✔
Sonny	✘	✘	✔	✘
Marie	✔	✘	✘	✘

See also Problem Four, page 16, in *Problem Solving*, Figure It Out, Level 3.

Problem Two

In this problem, the students need to work out how many times 12 goes into 30. $30 \div 12 = \frac{5}{2}$. Multiply the amount of each ingredient by $\frac{5}{2}$, as shown below.

Flour: $1\frac{1}{2} \times \frac{5}{2} = \frac{3}{2} \times \frac{5}{2} = \frac{15}{4} = 3\frac{3}{4}$ cups
Milk: $\frac{3}{4} \times \frac{5}{2} = \frac{15}{8} = 1\frac{7}{8}$ cups
Eggs: $2 \times \frac{5}{2} = 5$
Baking powder: $1 \times \frac{5}{2} = \frac{5}{2} = 2\frac{1}{2}$ teaspoons
Oatmeal: $\frac{1}{2} \times \frac{5}{2} = \frac{5}{4} = 1\frac{1}{4}$ cups.

Problem Three

Ensure that the students read this problem carefully. It's easy to make the mistake of assuming that one boy can fill one bin in 1 hour, two boys can fill two bins in 2 hours, three boys can fill three bins in 3 hours, and so on. This is not correct.

If six boys can fill six bins in 6 hours, they can fill 12 bins in 12 hours and 12 boys can fill twice as much in the same time. So 12 boys can fill 24 bins in 12 hours.

If four girls can fill four bins in 4 hours, they can fill eight bins in 8 hours and 12 bins in 12 hours. So eight girls can fill 24 bins in that time and 12 girls can fill 36 bins in that time.

Altogether, 24 + 36 = 60, so 60 bins of apples were filled.

Problem Four

This is another problem that the students have to approach carefully. It's easy to miss out the last bottle.

If the leftover liquid from five bottles can fill one bottle, then the leftover liquid from 25 bottles can fill five bottles. But the leftover liquid in the five refilled bottles can be used to fill another bottle, so altogether, six bottles were refilled (with a little bit left over).

Page 24: Mystery Amounts

Problem One

The students will probably find the answer to this problem by experimenting, but they can use logic and reasoning to narrow down the scope of their experiments. The method outlined below shows what the answer is and also shows that there is only one possible answer. The students might want to follow it through to the end, but it is probably quickest to use the first step to eliminate some options and then experiment to find the correct answer.

Problem Two, page 16, in *Problem Solving*, Figure It Out, Level 3, could be a useful warm-up for this problem.

○△ and △○ are "mirror numbers" because the digits are reversed. Mirror numbers are number pairs such as 23 and 32, 45 and 54, 19 and 91, and so on.

In this problem, you are looking for a mirror number where the product of the two reflections is less than 999. Writing out the calculation as long multiplication helps to visualise the problem:

```
   ○△
 x △○
 ----

 ----
 △□△
```

There is no point in looking at numbers less than 10. Using numbers less than 10 (for example, 09 x 90 = 810) will always result in a 0 as the last digit of the answer and a non-zero number as the first digit. This does not fit in the pattern you are looking for, where the product is △□△.

Other trials:

- 10 x 01 = 10: no good.
- 11 can't be used because the digits in ○△ x △○ must be different.
- 12 x 21 = 252: this is it. It fits the pattern.

But is this the only solution? Look at how the numbers work in 12 x 21 = 252.

The units digit of 252 is made up from the multiplication of the units digits of each number in the mirrored pair (2 x 1). The hundreds digit is made up by the tens digits of the reflections (1 x 2). This only works for a mirror number where one of the digits is a 1. So this limits you to the range 13 to 19 and then 31, 41, and so on.

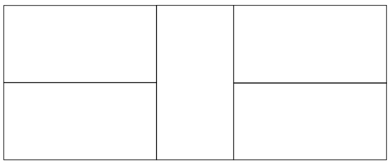
Now you need to look at the middle digit of the product. It is formed using the digits of just one of the reflections. For example, for 12, square the units digit and add it to the square of the tens digit: 1 x 1 + 2 x 2 = 1 + 4 = 5. (Look back at your long multiplication model if you are unsure how this works.) This means that you cannot have a number where the sum of these two squares is greater than 9, otherwise there will be a carry over (for example, for 13, 1 x 1 + 3 x 3 = 1 + 9 = 10), and the pattern is spoiled. This eliminates all the possibilities except 12 and 21, so the solution 12 x 21 = 252 is unique.

Problem Two

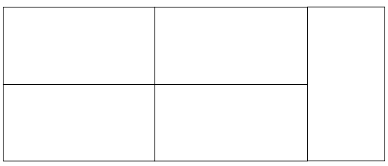
The mats are twice as long as they are wide. Each mat can be arranged horizontally or vertically. The students should work systematically to find the different possible arrangements of mats.

If all of the mats are horizontal, you can only make one rectangular arrangement out of them:

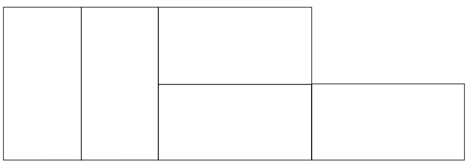
If one of the mats is vertical and the rest are horizontal, there are three possible rectangles:

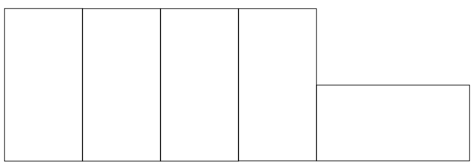


But note that the first rectangle is the same as the one shown in the problem, and the third one is a rotation of the second one, so there is only one new arrangement of the mats:

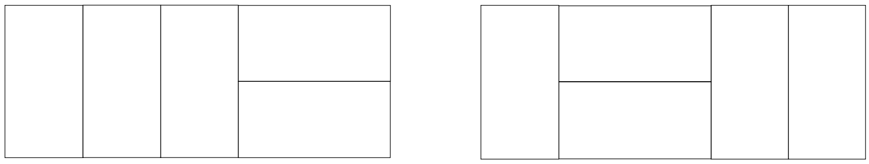


If two of the mats are vertical and three are horizontal, there will be one horizontal mat sticking out, so it's impossible to make a rectangle. The same is the case with four vertical mats and one horizontal mat:

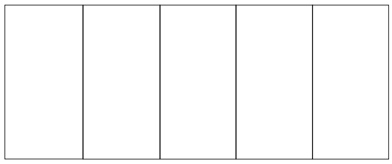




If three mats are vertical and two horizontal, there are two new mat arrangements:



If all five mats are vertical, this gives one more new arrangement:



So there are five ways of arranging the mats in addition to the arrangement in the students' book.

Problem Three

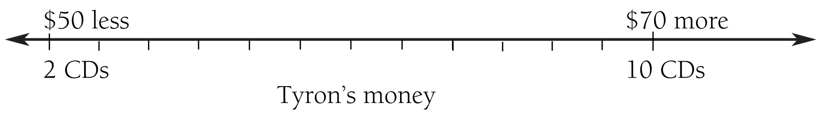
Tyron must have had more than \$50, because the second clue says that if he had \$50 less, he could still buy two CDs. Recording their trials on a table will help the students to find the correct answer quickly.

In the first column, put the trial amount for Tyron's cash. In the next column, add \$70 to the trial cash amount and divide the total by 10. That will give the cost of one CD. In the third column, subtract \$50 from the trial cash amount and divide the total by 2. This will also give the amount of one CD. When the cost of one CD is the same in the last two columns, the students have found out how much cash Tyron had and the cost of one CD.

Tyron's cash	Plus \$70 ÷ 10	Minus \$50 ÷ 2
\$60	\$13	\$5
\$70	\$14	\$10
\$90	\$16	\$20
\$80	\$15	\$15

A CD costs \$15, and Tyron had \$80.

Another way of solving this is as follows:
Regardless of how much cash Tyron had to begin with, the difference between having \$70 more and having \$50 less is \$120. The students could check this by plotting it on a number line.



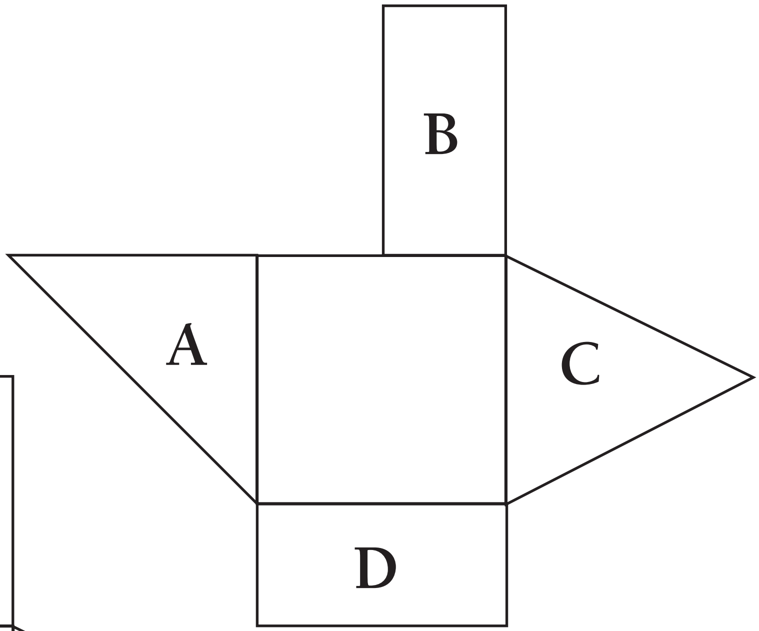
The number line shows there is a difference of eight CDs being bought, so eight CDs cost \$120.
One CD costs \$120 ÷ 8 = \$15.
If Tyron had \$50 less, he could buy 2 CDs, which cost 2 x \$15 = \$30.
So he had \$30 + \$50 = \$80 to begin with.

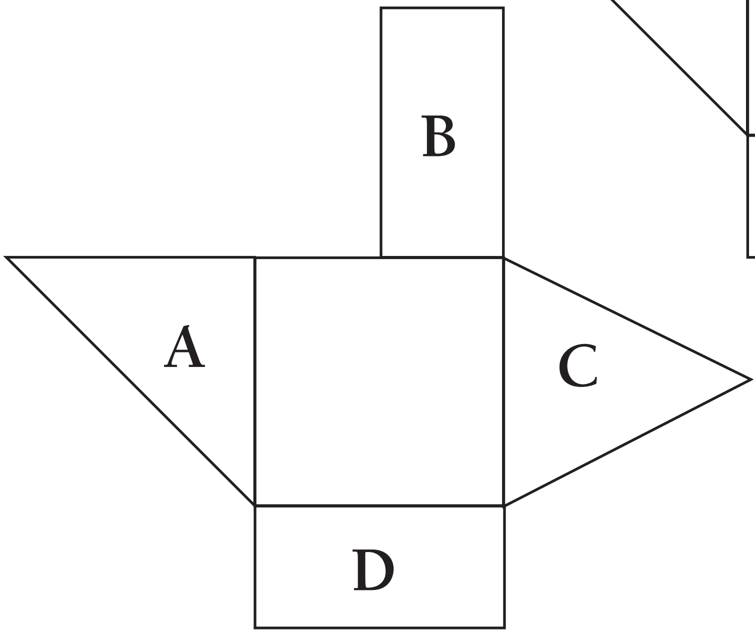
Problem Four

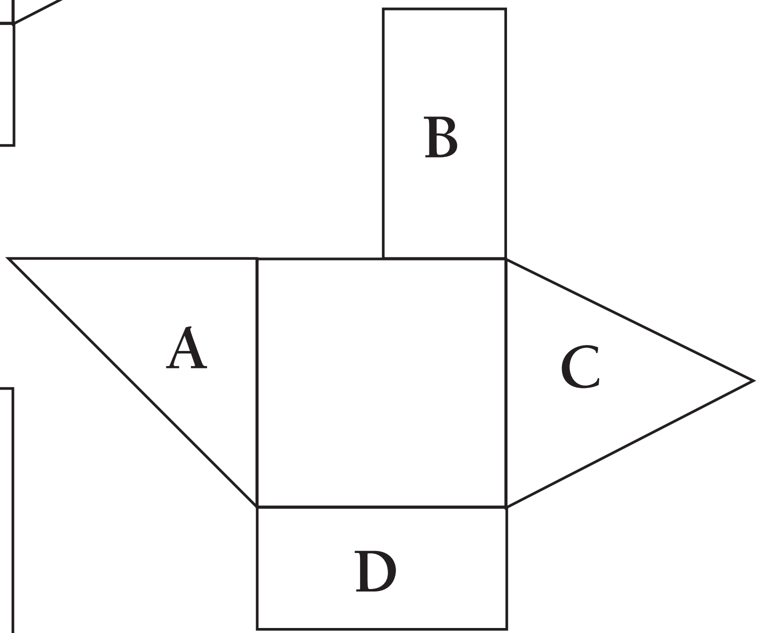
Most students will need to use trial and improvement. However, if you add 90 and 76 and 82, you'll get 248, which is the combined total when each person is weighed twice, as in the illustration. Half of this is 124 kilograms, the combined mass of the three friends.

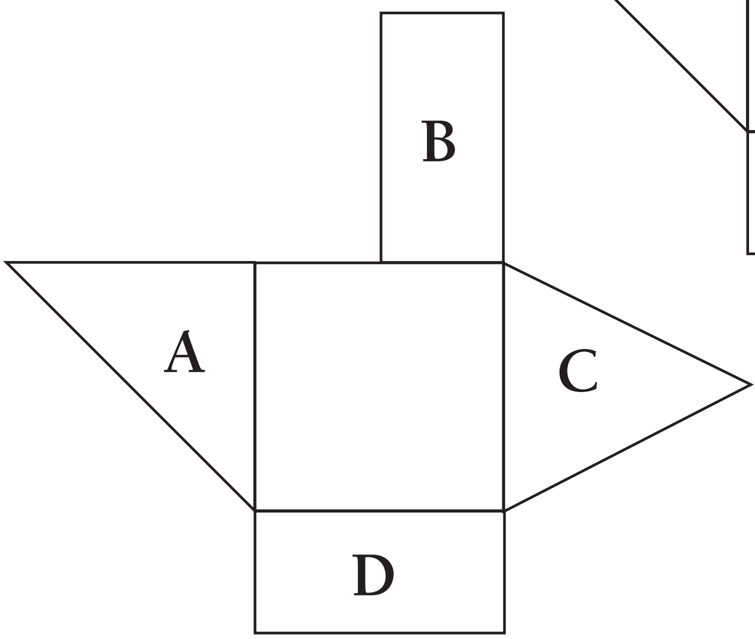
John and Trish weigh 90 kilograms. 124 – 90 = 34. So Zoe weighs 34 kilograms.
John and Zoe weigh 76. 124 – 76 = 48. So Trish weighs 48 kilograms.
Zoe and Trish weigh 82. 124 – 82 = 42. So John weighs 42 kilograms.

Copymaster: Page 7, Problem Three









Acknowledgments

Learning Media would like to thank Derek Holton, Department of Mathematics and Statistics, University of Otago, for developing the teachers' notes. Thanks also to Ken Benn and Kathy Campbell for reviewing the answers and notes.

The main illustration on the cover and contents page and the line art on the cover, contents page, and pages 2, 3, 8, and 17 are by Ali Teo.

Series Editor: Susan Roche
Designer: Dawn Barry

Published 2001 for the Ministry of Education by
Learning Media Limited, Box 3293, Wellington, New Zealand.
www.learningmedia.co.nz

Dewey number 510.76
ISBN 0 478 12982 3
Item number 12982
Students' book: item number 12981