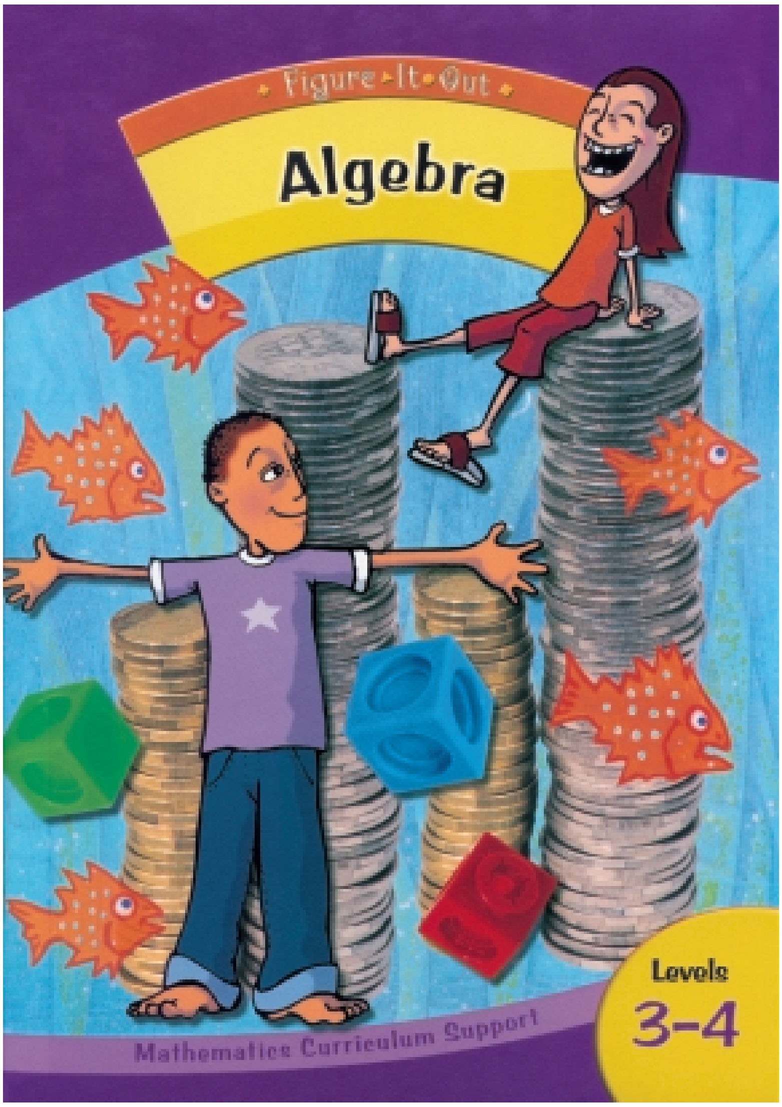
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

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Introduction

The Figure It Out series is designed to support *Mathematics in the New Zealand Curriculum.* The booklets have been developed and trialled by classroom teachers and mathematics educators. The series builds on the strengths of a previous series of mathematics booklets published by the Ministry of Education, the School Mathematics supplementary booklets.

Figure It Out is intended to supplement existing school mathematics programmes and can be used in various ways. It provides activities and investigations that students can work on independently or co-operatively in pairs or groups. Teachers can select particular activities that provide extension to work done in the regular classroom programme. Alternatively, teachers may wish to use all or most of the activities in combination with other activities to create a classroom programme. The booklets can be used for homework activities, and the relevant section in the teachers' notes could be copied for parents. These notes may also provide useful information that could be given as hints to students. The teachers' notes are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/community

There are eight booklets for levels 3–4: one booklet for each content strand, one on problem solving, one on basic facts, and a theme booklet. Each booklet has its own *Answers and Teachers' Notes.* The notes include relevant achievement objectives, suggested teaching approaches, and suggested ways to extend the activities. The booklets in this set (levels 3–4) are suitable for most students in year 6. However, teachers can decide whether to use the booklets with older or younger students who are also working at levels 3–4.

The booklets have been written in such a way that students should be able to work on the material independently, either alone or in groups. Where applicable, each page starts with a list of equipment that students will need to do the activities. Students should be encouraged to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Teachers could encourage students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Where possible, suggestions have been made to encourage discussion and oral presentation of answers, and teachers may wish to ask their students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. Successful ways of solving problems should be acknowledged, and where more effective or efficient processes can be used, students can be encouraged to consider other ways of solving the problem.

Figure It Out

Algebra

Answers

Page 1: Pattern Predictions

Activity

1. a. i. 7
   ii. 30
   b. 66
   c. 20

2. a. 7 orange and 44 purple
   b. 74
   c. A likely rule is: 6 x number of orange squares + 2.
   A "+ 6" rule (that is, 8 + 6 + 6 + 6 ...) is less powerful.

3. a. 57
   b. 93
   c. 13

Page 2: All Square

Activity One

1. a. 49
   b. 36
   c. 1
   d. 64
   e. 25
   f. 16
   g. 4
   h. 9

2. 100

3. a. 191
   b. Answers will vary. Many students will add up their **a–f** answers from question **1**.
   An easier way is to look at **a–f** as a rectangle (by adding four tiles).
   To find the answer, multiply length x width – 4.
   This will be 13 x 15 – 4 = 191.

Activity Two

1. a. 4
   b. 9
   c. 5
   d. 12

2. 17

3. Even-numbered width:
   number of tiles in width x 2
   Odd-numbered width:
   (number of tiles in width x 2) – 1

Page 3: Stacking Patterns

Activity

1. a. 320 mm
   b. 95 mm

2. a. 3.7 m
   b. 0.95 m

3. 5 m (assuming all legs are the same length)

Page 4: Where to Sit?

Activity

1. a. 60
   b. 30
   c. 42
   d. 24
   e. 42

2. Discussion will vary. In these suggested rules, c = the number of chairs and d = desks.
   a. A likely rule for 20 desks is: the number of seats = 3 x the number of desks (c = 3d).
   However, this only applies to an even number of desks.
   If there were 21 desks, for example, the rule would be c = 3d + 1.
   b. A likely rule is: six people for every four desks (6 ÷ 4 = $1^{1}/_{2}$ people per desk, so the rule can be expressed as c = $1^{1}/_{2}$d or c = 6d ÷ 4).

c. A likely rule is: the number of chairs = 2 x the number of desks + 1 for each end (c = 2d + 2).

d. A likely rule is: the number of chairs = the number of desks plus 4 (c = d + 4).

e. A likely rule is: the number of chairs = 2 x the number of desks + 1 for each end (c = 2d + 2).

3. Seating patterns and answers will vary.

4. The pattern of chair numbers increases by + 1, + 1, + 2, + 1, + 1, + 2, …

Page 5: Bits and Pieces

Activity One

1. Practical activity. It will have three creases and four sections.

2. a.

Folds	Creases	Sections
1	1	2
2	3	4
3	7	8
4	15	16

b. Pattern: add the creases and sections of each number of folds to get the next number of creases. For example, 1 + 2 = 3, 3 + 4 = 7.

Folds	Creases	Sections
5	31	32
6	63	64

Activity Two

1. a. 16 folds and 8 cuts

b. 24 folds and 12 cuts

2. Two rules are needed, one for an odd number of sheets and one for an even number of sheets. For an odd number of sheets, the number of sheets – 1 = folds; folds ÷ 2 = cuts. The rule could be expressed as: $(f = s - 1; c = f \div 2)$, where s is an odd number. For even numbers, the folds are found the same way, but the number of cuts is half the number of sheets minus one. The rule could be expressed as: $(f = s - 1; c = s \div 2 - 1)$, where s is an even number.

Pages 6–7: Waka Widths

Activity

1. Practical activity. Teacher to check

2. The approximate diameters are:

Circumference (arm spans)	Diameter (arm spans)
3	1
6	2
9	3
12	4

3. a. 5 arm spans

b. 6 arm spans

c. 10 arm spans

d. $3^1/_3$ arm spans

4. The circumference is approximately three times the diameter. This could be written as $c = 3 \times d$ or $c = 3d$.

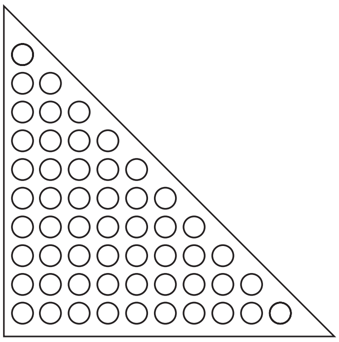
Investigation

Answers will vary, but a waka needs to be at least one arm span wide, so the totara must be at least three arm spans in circumference.

Page 8: Animal Antics

Activity One

1. The tenth triangular number is 55. One way of building the pattern is:



(Note: The number of counters in the bottom row is the same as the ordinal number of the triangle. For example, the tenth triangular number has 10 in the bottom row.)

2. The differences occur because each new line that is added to the triangle (to get the next triangular number) contains one number more than the previous line in the triangle does.

This means that the difference between successive triangular numbers increases by one each time.

Activity Two

Frog (9 jumps, including the jumps off the start bank and onto the bank on the other side. The rest can do it in 7 jumps.)

Page 9: Ten-storey Thomas

Activity One

1. 770
2. Two possible ways are:
   - Start by using multiplication to count the cubes in each section. The five sections shown in stage five of the building are 1 x 2, 2 x 4, 3 x 6, 4 x 8, and 5 x 10. From this, you can see a pattern, which will give you 6 x 12, 7 x 14, 8 x 16, 9 x 18, and 10 x 20. The answers to all these add up to 770.
   - Another way is to use square numbers. If you divide the fifth stage in half vertically, you can see that each section is a square number (1^2, 2^2, 3^2, 4^2, 5^2). This pattern continued to 10^2 gives a total of 385. Double this to get both sides, and you get 770.

Activity Two

a. 94

b. 132

Page 10: Double Rules

Activity

If this activity does not work on your calculator, try pressing the x operations key twice:

$\boxed{2}$ $\boxed{\text{x}}$ $\boxed{\text{x}}$ $\boxed{=}$

1. **a.** 7

   b. 23

   c. 9

   d. 11
2. Harmony realises that each number that comes out is twice the original number plus 3. So the first calculator has to have $\boxed{2}$ $\boxed{\text{x}}$ $\boxed{=}$ in it and the second has to have $\boxed{3}$ $\boxed{+}$ $\boxed{=}$.

Page 11: Seeing Dots

Activity One

1. **a.** Windmill 4: 45

   Windmill 5: 57

   b. Three possible rules are:

   i. Each windmill has 12 more pins on the outline than the previous windmill.

   ii. Number of pins for windmill 1 = (4 x 2) + 1

   Number of pins for windmill 2 = (4 x 5) + 1

   ↗

   This number increases by 3 for each successive windmill

   iii. Number of pins = (12 x windmill number) – 3

   c. 117
2. **a.** Windmill 4: 32 □s.

   Windmill 5: 50 □s.

   b. Area = (windmill number)^2 x 2. For example, for windmill 2, the area would be 2^2 x 2 = 8 □s.

   c. 200

Activity Two

1. Answers will vary. Patterns seen could include: The dots on the outline increase by 6 for each house. The dots inside increase by the next multiple of 6 (that is, + 6, + 12, …). The total number of dots also increases in multiples of 6 (+ 12, + 18, …).
2. Discussion will vary. (Some ideas are in the teachers' notes.)
3.

House number	1	2	3	4	5
Dots on outline	6	12	18	**24**	**30**
Dots inside	1	7	19	**37**	**61**
Total number of dots	7	19	37	**61**	**91**

Investigation

You may notice that each submarine increases by four dots (9, 13, 17). The base and the top deck increase by two dots each time, although the periscope stays the same. The area also increases by two □s each time.

Pages 12–13: Delicatessen Mathematics

Activity One

1. **a.** $^{1}/_{4}$
   b. **i.** 3
   ii. 9
   c. 30
   d. 1 200 g (1.2 kg)
2. **a.** Add $^{1}/_{2}$
   b. Add $^{1}/_{3}$
   c. Add $^{1}/_{4}$
   d. Subtract $^{1}/_{6}$
3. Answers will vary. One way is to work out the difference between the weight requested and the weight on the scales and write it as a fraction of the scale reading. For example, for question **2c**,

$$\frac{1\,000 - 800}{800} = \frac{200}{800}$$
$$= 200 \div 800$$
$$= 0.25 \text{ or } ^{1}/_{2}$$

So Hamish adds $^{1}/_{4}$ of the food (200 g).

Activity Two

a. $3.50
b. $2.90
c. Hamish's rule could be:
Amount on scales – 0.02 = amount of bean salad
(Amount of bean salad in kilograms x $4) + 50c = total cost.

Pages 14–15: Tongan Travel

Activity

1. Aunty Kuini is the second tallest.
2. Answers will vary. They could include: Uncle Pita is the shortest. Aunty Kuini has the least children. Uncle Loni is the youngest.
3. Answers will vary.

Pages 16–17: Patterns and Rules

Investigation

Conclusions will vary. (There are suggestions in the teachers' notes.)

Page 18: Dialling Dilemma

Activity

1. **a.** Kirsty: 7 a.m.
   Keyoko: 10 p.m.
   Kylie: 9 p.m.
   Brad: 1 p.m.
   Krisnan: 1.30 a.m.
   Andrew: 5 a.m.
   Anita: 10 a.m.
   b. Suggestions could include:

Penfriend	New Zealand time to call	Penfriend's time
Keyoko	10 a.m.–9.30 p.m. weekends or before 9.30 p.m. (allowing time for Keyoko to get home from school)	7 a.m.–6.30 p.m. weekends or from after school until 6.30 p.m.
Krisnan	1.30 p.m.–9.30 p.m. weekends	7 a.m.–3 p.m. weekends
Andrew	7–7.30 a.m. any day or 5–9.30 p.m. weekends	9–9.30 p.m. 7–11.30 a.m. weekends

Hirani can keep to her 7 p.m. plan by ringing Kirsty as soon as she gets up (7 p.m. Kirsty's time), Brad at 1 p.m. in the weekends (7 p.m. Brad's time), and Anita at 10 a.m. in the weekends (7 p.m. Anita's time).

2. Find the International Country and Area Codes section of the telephone book, find the country you want (they are listed alphabetically), and look under the column that says Hours behind NZ. Work backwards from the time showing on your watch.
3. The time is different in other countries so that the time of day matches when the sun rises in each country. (The planet Earth rotates, so different parts of the earth are facing the sun at different times.)

Page 19: Stacks of Money

Activity

1. **a.** $4
   b. $6
   c. $15
2. **a.** 39 mm
   b. 20

c. Two possible answers are:

- **i.** The line climbs evenly because the coins are shown on the horizontal axis in groups of 5. This means that the relationship between the number of coins and the height of the stack is constant or linear.
- **ii.** For every extra coin that is added, the height of the stack increases by the same amount (1.3 mm). This means that the relationship between the number of coins and the height of the stack is constant or linear and the graph of this relationship will be a straight line.

Investigation

The $1 and $2 coins are both 2.5 mm thick. So stacks of the same height will be worth twice as much when made with $2 coins as they are when made with $1 coins.

Page 20: Graphic Details

Activity

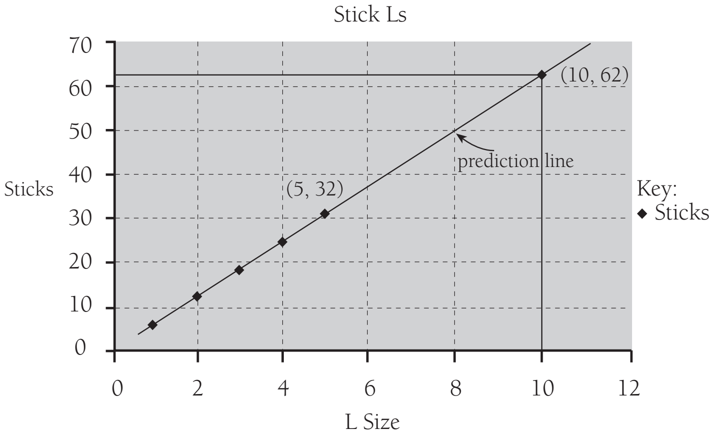
1. **a.** 51 sticks

b. 76 sticks

2. **a.** The computer spreadsheet should look like this:

	A	B
1	Ls	Sticks
2	1	8
3	2	14
4	3	20
5	4	26
6	5	32
7	6	38
8	7	44
9	8	50
10	9	56
11	10	62

The graph could look like this:

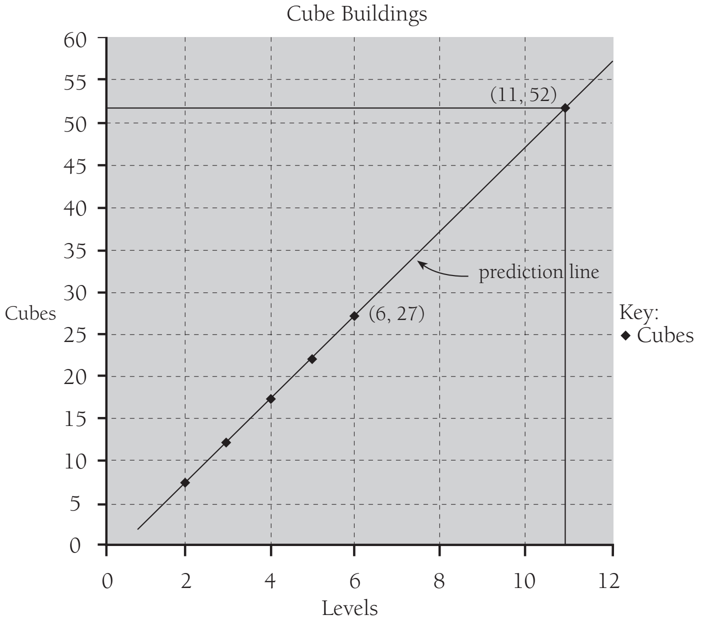


The next member of this pattern is 32 sticks and the tenth member is 62 sticks.

b. The computer spreadsheet should look like this:

	A	B
1	Levels	Cubes
2	2	7
3	3	12
4	4	17
5	5	22
6	6	27
7	7	32
8	8	37
9	9	42
10	10	47
11	11	52

The graph could look like this:

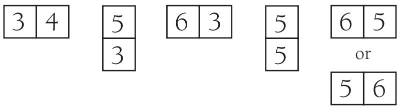


The next member of this pattern is 27 cubes, and the tenth member is 52 cubes.

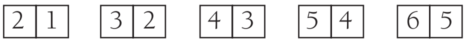
Page 21: Domino Delight

Activity

1. Starting from the domino at the bottom:

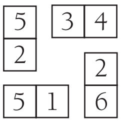


2. Starting from the domino on the bottom layer:

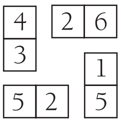


3. **a.** 9

b. The only other solution is:



This domino doughnut (and the one in the question) can be rotated, for example:



Page 22: Cup Capers

Activity

1. 3 in each yellow cup and 6 in the blue cup

2. 5

3. **i.** 2b + r

ii. 3g + 2y

iii. 4y + 2b + r

iv. 4r + 2y + b

v. 4y + 2b + r

4. **a.** **i.** 8

ii. 21

iii. 20

iv. 24

v. 20

b. Because they both have the same number of each coloured cup, even though they are shown in a different order

Page 23: Robot Rescue

Activity

1. **a.** 10 + 10 = 20

b. The first number plus itself equals double the first number: n + n = 2n.

2. **a.** **i.** next: 1 = 5 – 4
tenth: 1 = 10 – 9

ii. next: 4 + 5 = 5 + 4
tenth: 9 + 10 = 10 + 9

iii. next: $5 \times {}^1/_2 = 2{}^1/_2$
tenth: $10 \times {}^1/_2 = 5$

iv. next: 15 – 10 = 5
tenth: 30 – 20 = 10

v. next: 5 = 6 + 7 – 8
tenth: 10 = 11 + 12 – 13

vi. next: $1 - {}^1/_6 = {}^5/_6$
tenth: $1 - {}^1/_{11} = {}^{10}/_{11}$

b. Answers will vary. Possible answers are:

i. Take any number and then take away one less than it. The answer is always 1.
n – (n – 1) = 1.
For example, 10 – 9 = 1.

ii. Take any number. Add it to 1 more than itself. The answer is always the same as taking the next number and adding the number to it.
n + (n + 1) = (n + 1) + n.
For example, 10 + 11 = 11 + 10.

iii. A number multiplied by $^1/_2$ gives the same answer as the number divided by 2:

$n \div 2 = n \times {}^1/_2$.
For example, $10 \times {}^1/_2 = 10 \div 2$.

iv. Any number is equal to three times the number minus two times the number.
3n – 2n = n.
For example, 30 – 20 – 10.

v. Any number is equal to 1 more than itself added to 2 more than itself minus 3 more than itself:
n = (n + 1) + (n + 2) – (n + 3).
For example, 10 = 11 + 12 – 13.

vi. $1 - \frac{1}{n} = \frac{n-1}{n} \qquad 1 - \frac{1}{11} = \frac{10}{11}$

This is very cumbersome in words.
1 minus the fraction that has 1 as the numerator and the number itself as the denominator is equal to the fraction that has 1 less than the number as the numerator and the number itself as the denominator. (This shows how useful using letters can be!)

Page 24: Number Tricks

Activity One

Practical activity. Yes, you are always left with one cube.

Activity Two

1. Practical activity. The stop number is always 2. This happens because the middle section, number chosen x 2 – number chosen ÷ 2, done in this order, equals 2. The chosen number minus the chosen number equals 0. So you end up with 2.
2. Practical activity. This involves the 3 times table. After you have multiplied your number by 3, adding 3 to the answer will give you one more multiple of 3. So dividing this by 3 will always give you one more than the number you started with. For example, 5 x 3 + 3 = 18 is the same as 6 x 3 = 18. 18 ÷ 3 = 6, which is one more than 5. This can be written as a rule: ([number x 3] + 3) ÷ 3 = 1.

Figure It Out

Algebra

Teachers' Notes

Overview: Algebra

Title	Content	Page in students' book	Page in teachers' notes
Pattern Predictions	Finding and applying rules for sequential patterns	1	12
All Square	Describing relationships	2	13
Stacking Patterns	Finding relationships	3	15
Where to Sit?	Finding relationships	4	16
Bits and Pieces	Finding rules for patterns	5	18
Waka Widths	Recording and predicting relationships	6–7	20
Animal Antics	Using square and triangular numbers	8	21
Ten-storey Thomas	Continuing sequential patterns	9	22
Double Rules	Finding rules for number patterns	10	24
Seeing Dots	Finding rules to describe relationships	11	25
Delicatessen Mathematics	Describing relationships	12–13	27
Tongan Travel	Interpreting graphs of relationships	14–15	29
Patterns and Rules	Investigating relationships	16–17	30
Dialling Dilemma	Finding relationships	18	32
Stacks of Money	Investigating relationships	19	33
Graphic Details	Using tables and graphs	20	34
Domino Delight	Solving problems with possibilities and constraints	21	35
Cup Capers	Describing expressions using letters	22	37
Robot Rescue	Finding rules for patterns of equations	23	38
Number Tricks	Solving equation puzzles	24	39

Many of the activities in *Algebra,* Figure It Out, Levels 3–4 reflect the patterns approach suggested in *Mathematics in the New Zealand Curriculum*. For example, sequential geometric patterns are used to create number relationships. These relationships are used to describe and predict features within a pattern.

Teachers need to consider the process that students go through as they develop competence with these types of problems.

The following diagram shows what strategies students will use at the first four levels of the curriculum.

<table>
<tr><td>Strategy Type</td><td>Copies the pattern with materials and counts items</td><td>Continues the pattern with materials and uses strategic, systematic counting to find the number of items</td><td>Uses number rules to predict relationships in the pattern. The rules are additive or multiplicative for simple patterns.</td><td>Uses multiplication and division rules to predict relationships in the pattern</td></tr>
<tr><td>Level</td><td>Level One</td><td>Level Two</td><td>Level Three</td><td>Level Four</td></tr>
<tr><td>Examples</td><td>Builds blocks on top of pattern and counts, “one, two, three, ...”</td><td>Counts pattern items using skip counting and add ons for example, “two, four, six, eight, 10, and one more ... That’s 11!”</td><td>Uses a table<table><tr><td>black squares</td><td>1</td><td>2</td><td>3</td></tr><tr><td>white squares</td><td>8</td><td>14</td><td>20</td></tr></table>or an expression, for example, 8 + 6 + 6 … + 6, to add on to previous answers</td><td>Uses multiplication and addition rules to predict answers, for example, (9 x 6) + 8 or (10 x 8) – (9 x 2)</td></tr>
</table>

At levels 3–4, students should use tables to organise results and to look for patterns. Encourage them to use their knowledge of arithmetic to solve problems. Verbalising the process will help, for example, “What number times nine, plus three, equals 120?” (□ x 9 + 3 = 120).

Research shows that it is better to allow students to explore these problems without pre-teaching them algorithms such as flowcharting. In this way, they focus on the structure of the problems and thus develop their understanding of the processes involved.

Page 1: Pattern Predictions

Achievement Objectives

- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- make up and use a rule to create a sequential pattern (Algebra, level 3)
- state the general rule for a set of similar practical problems (Algebra, level 3)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)
- solve simple linear equations such as 2 x ☐ + 4 = 16 (Algebra, level 4)

Activity

In question **1**, the students must focus on the relationship between the number of hexagons and the number of triangles. To answer question **1b**, they will need to establish a rule for the number of green triangles used in relation to the number of hexagons. Then they can determine the triangles needed for different numbers of hexagons.

If the first hexagon stood alone, it would have six green triangles surrounding it. The next hexagon needs only four new green triangles because it shares two green triangles with the first hexagon.
So a general rule for this problem would be:
Green triangles needed = number of hexagons x 4 + 2 (for the first hexagon).

So for 16 hexagons, you need 16 x 4 + 2 = 66 triangles, and so on.

The students could approach question **2** in the same way as question **1**. However, question **2** does provide a good opportunity for levels 3–4 students to use tables to organise results and to look for patterns.

Some students may use spatial knowledge to solve the problem in the following way: Each 3 x 3 square within the pattern is made up of nine tiles. One square is orange, which means that eight squares are purple. Two purple squares are "lost" with each join, so 8 – 2 = 6 purple squares are added with each new orange square.

Although such reasoning is powerful and contains important skills involving spatial visualisation, it is also difficult. A numeric approach might be to organise the results in a table as the pattern is built up with tiles:

Orange squares	1	2	3	4
Purple squares	8	14	20	26

In doing this, the students may see the "growing by six" pattern of the numbers of orange squares.
A level 3 strategy would be either to continue the table by adding sixes along the bottom row of the table or to write an adding expression such as 8 + 6 + 6 + 6 + 6 + 6 …

Questions such as "Is there a quicker way to add up all those sixes?" will help the students to focus on the link between repeated addition and multiplication. This is a critical understanding about the processes of arithmetic that the students must acquire in order to use level 4 strategies, which in this case, for 12 orange squares, could be: "11 x 6 = 66, and I add on 8 for the first square. That means 12 orange squares will have 74 purple squares."

Whichever strategy they use, the students should be able to come up with a rule such as:
purple squares = 6 x the number of orange squares + 2.

An important idea in this activity is the reversal of rules, which may involve applying inverse operations. For example, in question **3**, students are asked how many crosses they could make with 120 sticks. The previous questions encouraged the students to develop some form of rule.

The students working at level 3 may use the following method:

Crosses	1	2	3	4	...
Sticks	12	21	30	39	

$$12 \underbrace{\quad}_{+9} 21 \underbrace{\quad}_{+9} 30 \underbrace{\quad}_{+9} 39$$

The students at this level may simply continue to add nines until 120 sticks are reached and the corresponding number of crosses is found. However, if you look at a rule with them, it could be written as: sticks needed = 9 x the number of crosses + 3.

The students working at level 4 may use multiplicative rules to find how many sticks were needed for 10 crosses. For example:

$$\underset{\substack{\uparrow \\ \text{9 for} \\ \text{every} \\ \text{other cross}}}{(9 \times 9)} + \underset{\substack{\uparrow \\ \text{first cross}}}{12} = 93 \qquad \text{or} \qquad (10 \times \underset{\substack{\uparrow \\ \text{9 sticks} \\ \text{added for} \\ \text{each cross}}}{9}) + \underset{\substack{\nwarrow \\ \text{extra sticks} \\ \text{from first} \\ \text{cross}}}{3} = 93.$$

Encourage the students to use their knowledge of arithmetic to solve the problems. They will probably come up with answers such as:
"I knew that 10 crosses needed 93 sticks, so I tried some numbers between 10 and 20."
"15 times 9 is 135 and 3 more is 138, so 15 crosses was too many."
"120 sticks was 27 more. 3 times 9 is 27, so there were 3 more crosses."

To encourage the students to reverse their rules, increase the number of sticks so that "trial and improvement" strategies become less profitable. For example, "246 sticks. How many crosses?" As an equation, this is ☐ x 9 + 3 = 246. The students can reverse this by subtracting 3 from each side to give ☐ x 9 = 243. They can then divide 243 by 9 to find the answer, 27.

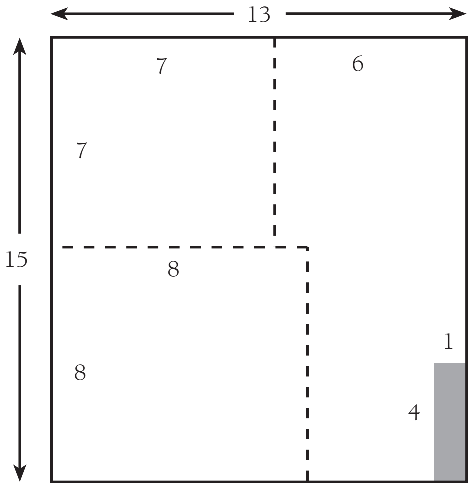
Page 2: All Square

Achievement Objectives

- state the general rule for a set of similar practical problems (Algebra, level 3)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)

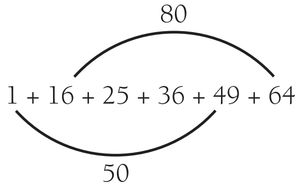
Activity One

For questions **1** and **2**, the students should use multiplication to find the number of tiles in each room rather than counting by ones or equal additions. In question **3**, looking at geometric patterns is more effective than just working with the numbers. As shown on the diagram on the following page, the students can find the sum of the tiles needed for rooms **a–f** by finding the area of the whole rectangle and subtracting the shaded piece (4). 13 x 15 – 4 = 191. So you would need 191 tiles for **a–f**. This is shown as a diagram on the next page.



The area of the whole rectangle
less the shaded piece, 4.
13 x 15 – 4 = 191

If the students do decide to add the room totals, they could simplify the task by looking for tidy pairs of numbers. Putting the numbers in order may help.

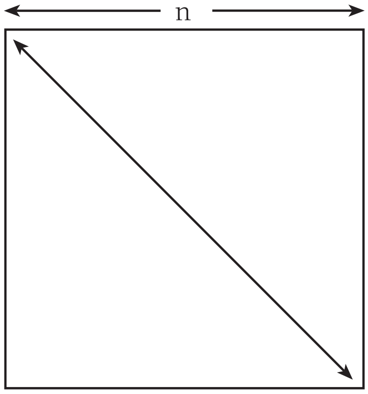


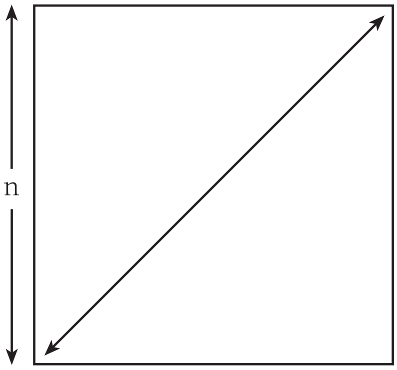
So the problem becomes: (50 + 80) + (25 + 36) = 130 + 61

= 191.

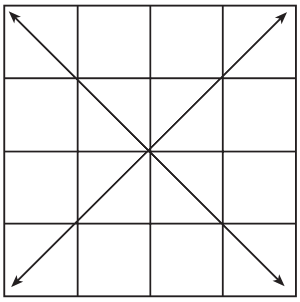
Activity Two

The students will need to see that the number of tiles a diagonal line passes through is the same as the number of tiles along any side.

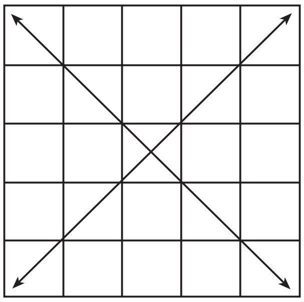




If n is even, the diagonals cross 2n squares:



But if n is odd, one square is crossed twice, so the rule is 2n – 1.



For example in room **b**, which is 6 x 6, the diagonals will cross 2 x 6 = 12 tiles because 6 is even. For a 9 x 9 room, the number of diagonals cut will be (2 x 9) – 1 = 17.

Page 3: Stacking Patterns

Achievement Objectives

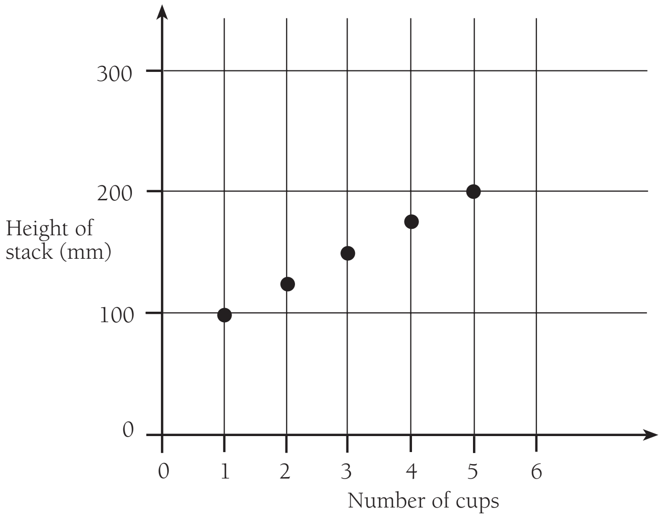
- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- make up and use a rule to create a sequential pattern (Algebra, level 3)
- state the general rule for a set of similar practical problems (Algebra, level 3)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)

Activity

The questions in this activity are based on patterns. Number relationships in which a variable grows by a constant amount are known as variable relationships.

Encourage the students to think about what strategies they could use to solve each question.

In question **1**, the addition of each extra cup increases the height of the stack by a constant amount (25 millimetres). If the relationship between the number of cups and the height of the stack is graphed on a number plane, the ordered pairs lie on a straight line.



The students could also work out the height of the cups using logic.

Five cups have a height of 195 millimetres, and three cups have a height of 45 millimetres. So the two extra cups in the five-cup stack add an extra 50 millimetres. This means that each cup, other than the first, must add 25 millimetres to the stack because 50 ÷ 2 = 25 millimetres.

This helps to find the height of the first cup as well: Three cups gives 145 millimetres, so 145 – 50 (the two extra cups after the first) = 95 millimetres.

The students can use this logic to find the height of any given stack of cups. In general words, it will be: 95 mm (for the first cup) + 25 mm x the rest of the cups. So for 21 cups, the height would be:

$$\begin{aligned} 95\text{ mm} + 25\text{ mm} \times (21 - 1) &= 95\text{ mm} + 25\text{ mm} \times 20 \\ &= 95\text{ mm} + 500\text{ mm} \\ &= 595\text{ mm}. \end{aligned}$$

Another useful strategy for students at levels 3–4 is to use a table to organise the data. For example, in question **2**:

Number of trolleys	1	2	3	4	5	6
Length of stack (m)		1.2		1.7		

When two trolleys are added to the first two, the length increases by 0.5 metres, so each trolley adds 0.25 metres to the length of the stack.

The students can use the constant (0.25 metres) to complete the table:

Number of trolleys	1	2	3	4	5	6
Length of stack (m)	0.95	1.2	1.45	1.7		

After the students have found the constant, they can use addition and (preferably) multiplication to find the stack length for any other number of trolleys. For example, 12 stacked trolleys will have a length of

$$\begin{aligned} \underset{\substack{\uparrow \\ \text{length of} \\ \text{a 4-trolley} \\ \text{stack}}}{1.7} + \underset{\substack{\nwarrow \\ \text{added length} \\ \text{from 8 trolleys}}}{(8 \times 0.25)} &= 1.7 + 2.0 \\ &= 3.7\text{ metres.} \end{aligned}$$

Page 4: Where to Sit?

Achievement Objectives

- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- make up and use a rule to create a sequential pattern (Algebra, level 3)
- state the general rule for a set of similar practical problems (Algebra, level 3)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)

Activity

This activity is similar to those on pages **1** and **3**. The students need to find and use rules to describe the relationships in geometric patterns. Using 20 desks is a deliberate strategy to make adding on (recursive) rules cumbersome. This should encourage the students to develop more efficient function rules.

Using tables will help the students to see connections between the number of desks and the number of chairs. A table for question **1c** could look like this:

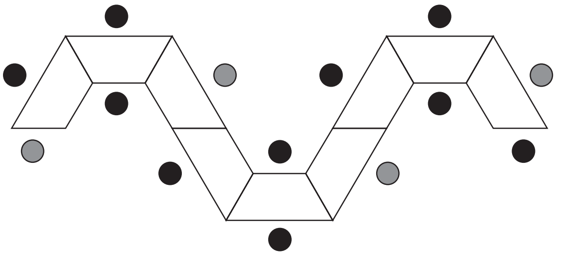
Desks	1	2	3	4
Chairs	4	6	8	10

+ 2 + 2 + 2

Two more chairs can be used for each new desk that is added to the line. Repeated addition of 2 is the same as multiplication by 2, so the students could try this as a possible function rule that links the number of desks with the number of chairs.

Desks	1	x 2	2	x 2	3	x 2	4	x 2
Chairs	4	+ 2	6	+ 2	8	+ 2	10	+ 2

The students will quickly see that they need to add 2 more to each "times 2" result to get the number of chairs. So a "times 2 plus 2" function rule will work. This could be written algebraically as $c = 2d + 2$, where c is the number of chairs and d is the number of desks.
As an extension, you could give the students this desk arrangement:



The relationship between desks and chairs in this arrangement is more complex.

Desks	1	2	3	4	5	6	7
Chairs	4	5	6	8	9	10	12

+ 1 + 1 + 2 + 1 + 1 + 2

This pattern is not linear because the increases in the number of chairs for each desk added are not constant. To describe the rules algebraically, the students would need to split the number of desks into three classes of numbers, {1, 4, 7, …}, {2, 5, 8, …}, {3, 6, 9, …}, and produce different rules for each class. This is beyond the ability of students at levels 3 and 4. For example, the rule for the numbers of desks that are multiples of three, {3, 6, 9, …}, is $c = {}^4/_3 d + 2$.

Students at this level need only look for patterns in the table numbers that will help them to predict the number of chairs. For example, they may see that increasing the number of desks by three increases the number of chairs by four. They can use this pattern to find out how many chairs would be around 18 desks.

Desks	3	6	9	12	15	18
Chairs	6	10	14	18	22	26

+ 4 + 4 + 4 + 4 + 4

From the + 1, + 1, + 2 of the previous table

Page 5: Bits and Pieces

Achievement Objectives

- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- state the general rule for a set of similar practical problems (Algebra, level 3)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)

Activity One

Most people find visualising the number of creases and sections very difficult. Give the students paper to fold to check their predictions and to complete the table. There are several interesting patterns in the table.

Folds	Creases	Sections
1	1	2
2	3	4
3	7	8
4	15	16
5	31	32
6	63	64

The creases and sections columns share the same pattern, based on subtraction. Here is the pattern for the creases column:

1
$3 - 1 = 2$
3
$7 - 3 = 4$
7
$15 - 7 = 8$
15
$31 - 15 = 16$
31
$63 - 31 = 32$
63

Note that the differences shown in the creases pattern are the figures found in the sections column.

The students are most likely to notice that each extra fold doubles the number of sections and that there is one less crease than there are sections:

Folds	Creases	Sections
2	3	4

These rules are recursive rules rather than functional ones. Note that with recursive rules, you cannot find the nth number in a pattern unless you know the previous number.

There are also functional rules that link the number of folds with the numbers of creases and sections. These rules use exponents. The number of sections doubles with each fold. So three folds will give $2 \times 2 \times 2$ (or 2^3) = 8 sections. If f is the number of folds, s is the number of sections, and c is the number of creases, then:

$s = 2^f$
$c = 2^f - 1$

With this functional rule, you can now find any number in the pattern. (You don't have to find the previous number.)

Students at levels 3 and 4 are not expected to be able to put together functional rules such as these.

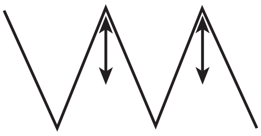
Activity Two

To clarify any confusion, you may need to point out to the students that one sheet of paper is two pages. In terms of the activity, they need to know that the number of folds includes the folds that are later cut. The comments and rules below are based on the first cut being the second fold.

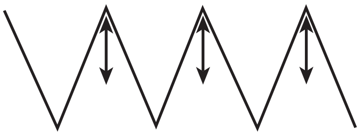
Question **1** concentrates on odd numbers of sheets. The students could experiment with small numbers of odd sheets and put their results in a table:

Sheets	1	3	5	7
Folds	0	2	4	6
Cuts	0	1	2	3

Diagrammatically, this can be shown as:



5 sheets
(4 folds and 2 cuts)



7 sheets
(6 folds and 3 cuts)

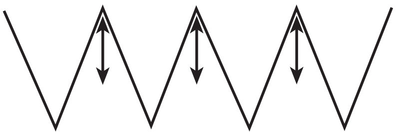
↕ shows a fold that is cut

The table shows that the number of folds for odd numbers of sheets is always one less than the number of sheets. A staple is put through half of the folds to make a spine, so the number of cuts is half the number of folds. So the rule for odd numbers of sheets is: folds = number of sheets – 1 and cuts = number of folds ÷ 2. This could be expressed, as in the Answers, as: $f = s - 1$; $c = f \div 2$, where s is an odd number.

To answer question **2**, the students will need to include even numbers of sheets. For even numbers of sheets, the table is:

Sheets	2	4	6	8
Folds	1	3	5	7
Cuts	0	1	2	3

Although the number of folds is still one less than the number of sheets, the rule for cuts is the number of sheets divided by two minus one. For example, for eight sheets, 8 (sheets) ÷ 2 = 4 – 1 = 3 (cuts). As a diagram, this could be represented as:



8 sheets
(7 folds and 3 cuts)

The rule for this can be expressed as (number of sheets ÷ 2) – 1.

Pages 6–7: Waka Widths

Achievement Objectives

- state the general rule for a set of similar practical problems (Algebra, level 3)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use graphs to represent number, or informal, relations (Algebra, level 3)

Activity

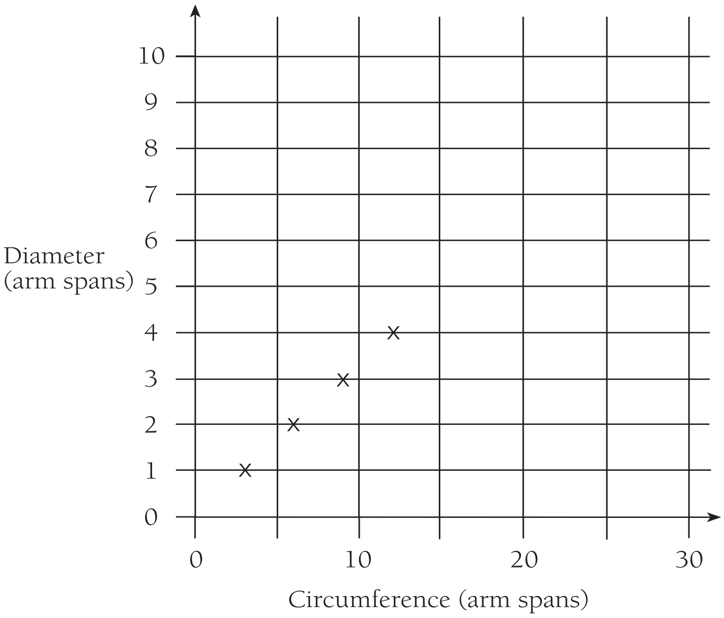
This activity explores the relationship between the circumference and diameter of a circle. Where the number of students forming a circle is a multiple of 3, (3, 6, 9, 12, …), they will find that the diameter is about one-third of the circumference. For example, a tree that has a circumference of 12 arm spans will have a diameter of 12 ÷ 3 = 4 arm spans. There is variation in the heights and arm spans of students, so the relationship will be approximate. To reduce this problem, try to put the students with peers who are about the same height.

The exact numbers of times that the diameter of a circle will go around the circumference is about 3.14 and is known as "pi". The symbol π is used to denote it.

The students can use tables and graphs to show the relationship that they find using their arm spans.

Circumference (arm spans)	3	6	9	12	…	30
Diameter (arm spans)	1	2	3	4	…	10

Relationship between Circumference and Diameter



Note that the relationship is linear because, when graphed, the points lie on a line.

Page 8: Animal Antics

Achievement Objectives

- state the general rule for a set of similar practical problems (Algebra, level 3)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)

Activity One

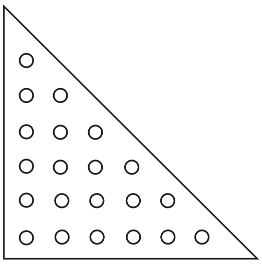
The triangular numbers are an interesting set of numbers. They are particularly significant for later work in probability.

Triangular numbers are not a linear pattern because the difference between successive terms is not constant. Therefore, level 3–4 students will need to use tables to find the tenth triangular number:

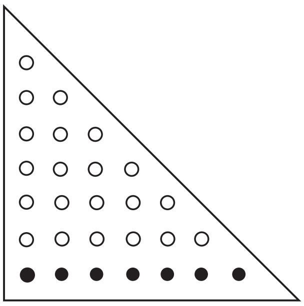
Bottom row of counters	1	2	3	4	5	6	7	8	9	10
Total number of counters	1	3	6	10	15	21	28	36	45	55

+ 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10

The differences grow by one each time because one more counter is needed to make the next row of counters. For example, here is the sixth triangular number, 21:



This must have a row of seven counters added to it to make the next triangle:



To make the triangles that follow, a row of eight will be added, followed by a row of nine, then 10, and so on.

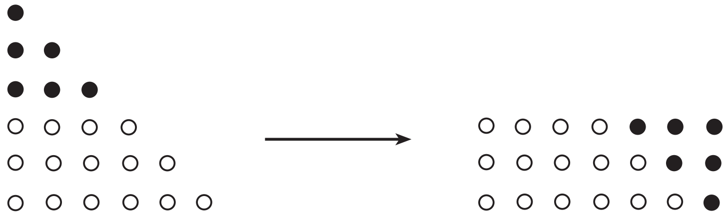
So the triangular numbers are the sums of sets of consecutive counting numbers.

1 + 2 = 3
1 + 2 + 3 = 6
1 + 2 + 3 + 4 = 10
1 + 2 + 3 + 4 + 5 = 15
1 + 2 + 3 + 4 + 5 + 6 = 21

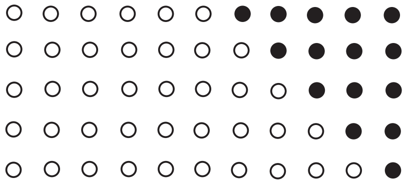
Consecutive counting numbers are always easy to add because they form pairs with the same total. For example, consider 1 + 2 + 3 + 4 + 5 + 6:

1 + 6 = 7, 2 + 5 = 7, 3 + 4 = 7. So the sum is 3 x 7 = 21.

The sum of consecutive counting numbers is always the sum of the first and last terms multiplied by half the number of terms (even if the number of terms is odd). Spatially, this is like rearranging a triangle to form a rectangle:



Following this pattern, the tenth triangular number can be calculated as (10 + 1) x 5 = 11 x 5
= 55.



Activity Two

Most of the animals could take several paths to cross the river, but the students need to find the path with the fewest jumps for each animal. Then they can find which animal has to take the most jumps.

For the triangular numbers, the students may need their table from **Activity One** to assist them because it is not easy to check these with mental calculation. All other sets of numbers can be confirmed using multiplication and division. For example, consider 36:

Is it odd?	No, because the ones digit is even, that is, an odd number is not divisible by 2.
Is it even?	Yes, because the ones digit is even, it is divisible by 2.
Is it square?	Yes, because a number times itself (6 x 6) gives 36.
Is it a multiple of 3?	Yes, because 36 ÷ 3 gives an exact answer (12).

Page 9: Ten-storey Thomas

Achievement Objectives

- state the general rule for a set of similar practical problems (Algebra, level 3)
- use a rule to make predictions (Algebra, level 4)

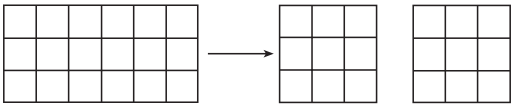
Activity One

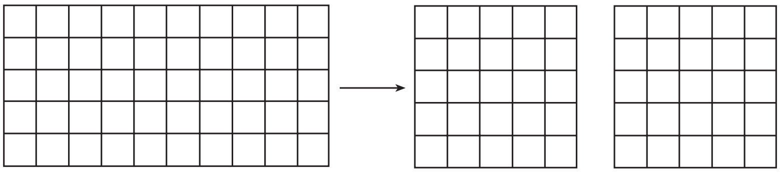
Thomas was a 9-year-old student from Silverdale Normal School in Hamilton when he created this delightful pattern using multilink cubes. His teacher challenged him to find the total number of cubes in stage 10 of his model.

He chose to focus on the rectangles that make up the sections of the model. This is a worthwhile strategy, but the students will need to organise their calculations carefully.

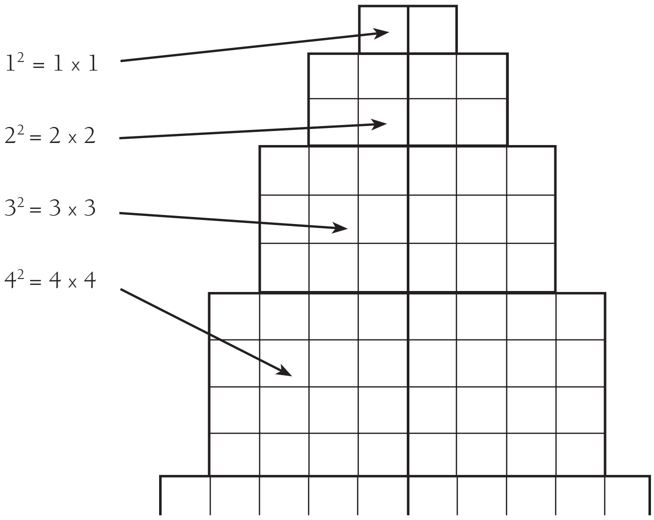
Section	Rectangle	Cubes
1 (top)	2 x 1	2
2	4 x 2	8
3	6 x 3	18
4	8 x 4	32
5	10 x 5	50
6	12 x 6	72
7	14 x 7	98
8	16 x 8	128
9 (bottom)	18 x 9	162
10	20 x 10	200
		770

The Answers also suggest using square numbers because each rectangular section is made up of a square number times two. For example, 6 x 3 is made of (3 x 3) + (3 x 3) and 10 x 5 is (5 x 5) + (5 x 5).





This approach uses the reflection symmetry of the figure.



As a table, the calculation for one half of the figure would be:

Section	Cubes
1 (top)	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10 (bottom)	100
	385

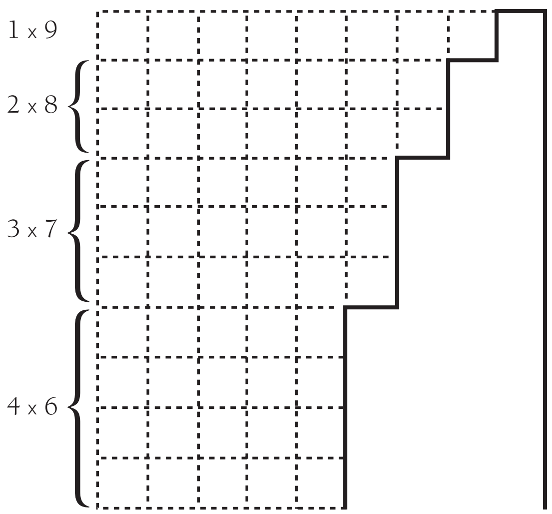
Another strategy is to focus on the rectangle that surrounds the whole figure and subtract the missing cubes. The students could do this by working with one side of the figure and doubling the result.

The rectangle that surrounds half of stage 10 of the model is 10 cubes across and 55 cubes high, which gives a total of 550 cubes. The missing cubes for one side are given by:

(1 x 9) + (2 x 8) + (3 x 7) + (4 x 6) + (5 x 5) + (6 x 4) + (7 x 3) + (8 x 2) + (9 x 1)
= 9 + 16 + 21 + 24 + 25 + 24 + 21 + 16 + 9
= 165.

550 – 165 = 385 gives the total number of cubes for one side.

Missing cubes for one side (top four levels):



As a table, the calculation for one half of the figure would be:

Section	Missing Cubes
1 (top)	1 x 9 = 9
2	2 x 8 = 16
3	3 x 7 = 21
4	4 x 6 = 24
5	5 x 5 = 25
6	6 x 4 = 24
7	7 x 3 = 21
8	8 x 2 = 16
9	9 x 1 = 9
10 (bottom)	10 x 0 = 0
Total	165

Activity Two

A possible strategy for solving these problems is to draw a bird's-eye view of each figure at its different stages. The numbers show how many cubes are used on each tower.

Model a

2	1	2
1	1	1
2	1	2

stage one

3	2	3
2	2	2
3	2	3

stage two

4	3	4
3	3	3
4	3	4

stage three

Model b

2	1
2	1

stage one

3	2	1
3	2	1

stage two

4	3	2	1
4	3	2	1

stage three

These diagrams help the students to see the number patterns in the models. Using the number patterns, the tenth stage of each model is:

Model a

11	10	11
10	10	10
11	10	11

Total: 94 cubes

Model b

11	10	9	8	7	6	5	4	3	2	1
11	10	9	8	7	6	5	4	3	2	1

Total: 132 cubes

Page 10: Double Rules

Achievement Objectives

- state the general rule for a set of similar practical problems (Algebra, level 3)
- find a rule to describe any number of a member sequence and express it in words (Algebra, level 4)
- solve simple linear equations such as 2 x □ + 4 = 16 (Algebra, level 4)

Activity

Most four-function calculators behave as described in the activity when a calculation is entered. However, Casio calculators need the operation button keyed in twice to activate the constant capability. For example, $\boxed{2}\,\boxed{x}\,\boxed{=}$ will set a doubling rule on most calculators, but on Casio calculators $\boxed{2}\,\boxed{x}\,\boxed{x}\,\boxed{=}$ must be entered.

The Find the Function game described in this activity is an excellent way for the students to find relationships between numbers without having to work with spatial patterns and practical tasks. Encourage the students to play the game regularly with a classmate.

Research indicates that it is unwise to train students to find the function by inputting 1 first, then 2, then 3, and so on. Students appear to learn more from having to apply the processes of arithmetic to discover the relationships. Consider the example given in the activity. Suppose that Harmony has asked Rāwiri to enter three numbers and has these in/out pairs:

In	6	2	9
Out	15	7	21

Harmony realises that the function makes the input number larger by more than double. She might try experimenting with x 2 and x 3 as rules and then working out how much she would have to add or subtract to get the out number:

Using x 2

In	6	2	9
x 2	12	4	18
Missing operation	+ 3	+ 3	+ 3
Out	15	7	21

Using x 3

In	6	2	9
x 3	18	6	27
Missing operation	– 3	+ 1	– 6
Out	15	7	21

The missing operation in the x 2 table is constant, so Harmony has discovered that Rāwiri is using the rule (□ x 2) + 3.

Page 11: Seeing Dots

Achievement Objectives

- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- make up and use a rule to create a sequential pattern (Algebra, level 3)
- state the general rule for a set of similar practical problems (Algebra, level 3)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)

The students will need photocopies of square dot paper (see the copymaster at the end of these notes) to draw the figures involved and to work out the number of pins.

Activity One

The students can use a numeric approach to find the dots in a windmill. The windmill has 12 sides, three for each of its four arms. For each successive windmill, a dot is added to each side, so the difference between the number of dots in successive windmills is a constant of 12.

The students can find the number of dots for any windmill by using either addition or multiplication. (See the notes for page **1** of the student booklet.)

Addition:

Windmill	1	2	3	4	5	...	10
Dots	9	21	33	45	57	...	117

+ 12 + 12 + 12 + 12

Multiplication:

The number of dots on the outside of windmill 10 is:

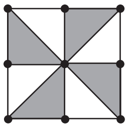
$9 + (9 \times 12) = 117$ or $(10 \times 12) - 3 = 117$

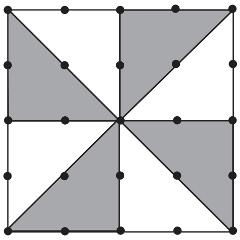
windmill 1

9 more extensions of 12

10 extensions of 12 less 3 because windmill 1 had 9 dots

The students can use spatial reasoning to find the area of the windmills. Each windmill can be enclosed within a square. The windmill takes up half the area of the square.





The square has an area of 2 x 2 = 4.
So the windmill has an area of 4 ÷ 2 = 2.

The square has an area of 4 x 4 = 16.
So the windmill has an area of 16 ÷ 2 = 8.

The areas for windmills of other sizes can be found in the same way.

Windmill	3	4	5	...	10
Enclosing square area	6 x 6 = 36	8 x 8 = 64	10 x 10 = 100	...	20 x 20 = 400
Area of windmill	18	32	50	...	200

Activity Two

The table of values for the house contains many interesting patterns and relationships:

House number	1	2	3	4	5
Dots on outline	6	12	18	24	30
Dots inside	1	7	19	37	61
Total number of dots	7	19	37	61	91

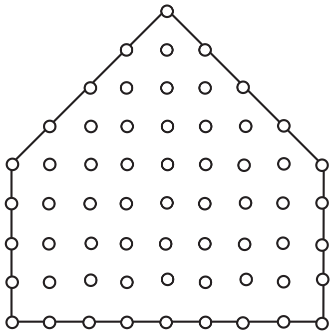
increases by 6

Previous total gives the number of dots inside the next house

1 7 9 37 61

+ 6 + 12 + 18 + 24 Differences increase by 6

Ask the students to explain why they think these patterns occur. Explanations might include: "The outline of the house has five sides, including the base. The base number of dots increases by two and the other sides gain another dot each time."



"Each larger houses surrounds the previous house, so the total number of dots on the previous house becomes the total for the dots inside the next house."

Investigation

The students may wish to investigate both the dots and the area of the submarine pattern. Here are some results:

Submarine number	1	2	3
Dots	9	13	17
Area in □s	$3\frac{1}{2}$	$5\frac{1}{2}$	$7\frac{1}{2}$

The students should be able to explain the increases with reference to the spatial pattern. For example, "The hull of the submarine is increased in length by two □s each time. This means the number of dots increases by four (two at the top of the hull and two at the bottom), and the area increases by two □s."

Pages 12–13: Delicatessen Mathematics

Achievement Objectives

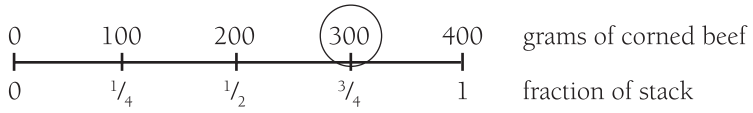
- state the general rule for a set of similar practical problems (Algebra, level 3)
- use a rule to make predictions (Algebra, level 4)
- solve simple linear equations such as $2 \times \square + 4 = 16$ (Algebra, level 4)

Activity One

This activity shows the students that many people use algebra in their daily lives. The rules in this example involve ideas about proportion. At levels 3–4, the students should have developed strong mental calculation strategies with multiplication and division. These strategies are an important foundation for this activity.

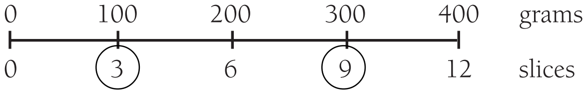
Diagrams may help some students to solve Hamish's problems. For example, consider question **1**. Here is a double number line showing the situation for question **1a**.

The students need to decide where 300 grams would be on the number line and therefore what fraction of the stack this corresponds to. They may use intermediate steps to do this.

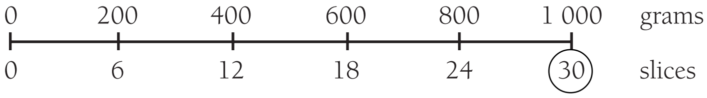


So Hamish will need to remove $^{1}/_{4}$ of the stack to leave 300 grams of meat. Similar strategies can be used with questions **1b**, **c**, and **d**.

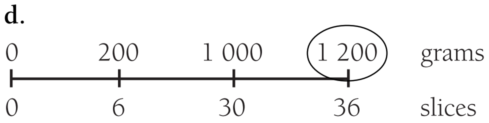
b.



c.



d.

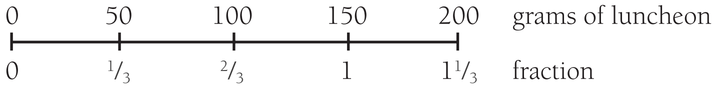


The students can use similar strategies to find the solutions for question **2**, but they will need to take their reasoning a bit further to find what fraction Hamish should add or subtract.

a.

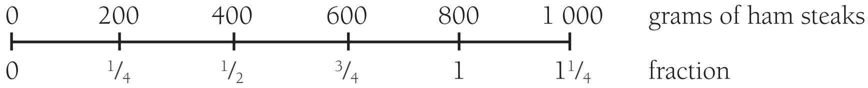
150 grams of olives is $1^{1}/_{2}$ times the current amount of olives, so Hamish will need to add $^{1}/_{2}$ of the current amount.

b.



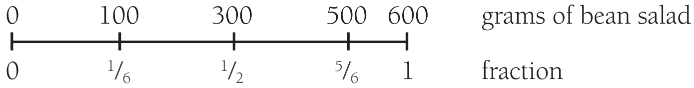
200 grams of luncheon is $1^{1}/_{3}$ times the current amount, so Hamish will need to add $^{1}/_{3}$ of the current amount.

c.



1 kilogram (or 1 000 grams) of ham steaks is $1^{1}/_{4}$ of the current amount, so Hamish will need to add $^{1}/_{4}$ of the current amount.

d.



500 grams of bean salad is $^{5}/_{6}$ of the current amount, so Hamish will need to take off $^{1}/_{6}$ of the current amount.

When the students are confident using the double number line approach, you could encourage them to calculate the answers using addition and multiplication. This is a quicker method than using a number line and will help the students to see the rules that Hamish is using (which they need to do for question **3**). For question **2a**, ask the students:
"How many more grams of olives does Hamish need?" (50 grams)
"What fraction of 100 grams is 50 grams?" (The students might write this as $^{50}/_{100}$
and then simplify it to $^{1}/_{2}$, so Hamish needs to add $^{1}/_{2}$).

When the students have done this several times, they should see the rule, which is to work out the difference between the weight requested and the weight on the scales and write it as a fraction of the weight on the scales, that is:

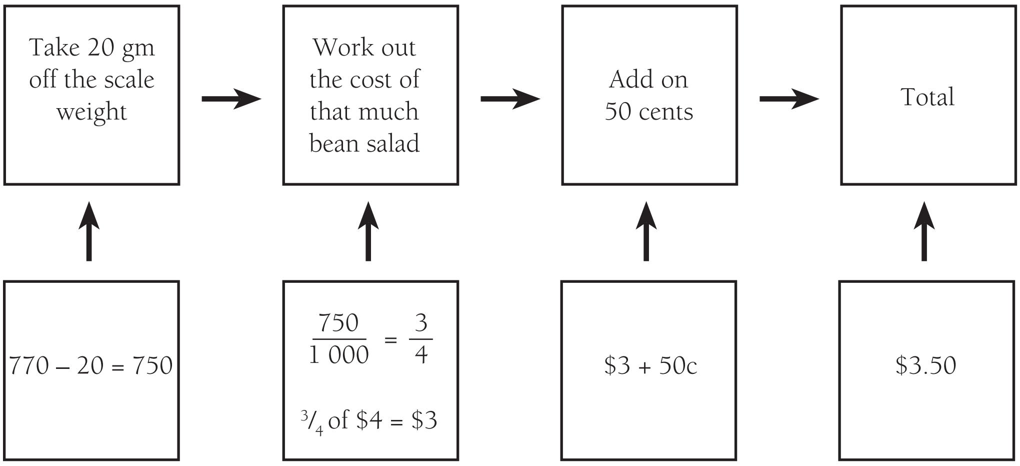
$$\frac{\text{weight requested } - \text{ weight on scales}}{\text{weight on scales}}$$

If the answer is positive, Hamish adds this fraction to the food on the scales. If the answer is negative, Hamish takes this fraction off the food on the scales. For example, for question **2d**,

$$\frac{500 - 600}{600} = \frac{-1}{6}$$ So Hamish takes off $^1/_6$ of the food.

Activity Two

This problem has some added information that requires the students to organise the steps needed to solve it. A flow chart might be useful here.



Pages 14–15: Tongan Travel

Achievement Objectives

- use graphs to represent number, or informal, relations (Algebra, level 3)
- sketch and interpret graphs on whole number grids which represent simple everyday situations (Algebra, level 4)

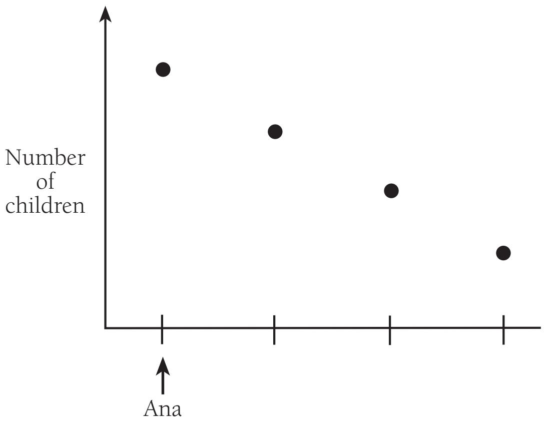
Activity

This activity draws on students' understanding of graphs. The students may take a while to get used to these graphs because they don't have units of measurement on the y axis. For example, the graph showing height doesn't give the actual height of the aunties and uncles, it just gives their relative heights. So the higher up the y axis the ordered pair is, the taller the person.

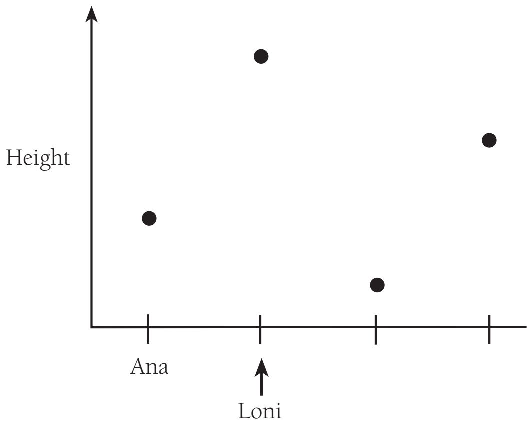
Acting out the problem may make it easier. The students can assume the characters of the aunties and uncles in the graphs (Ana, Loni, Pita, and Kuini).

The students can work systematically through the clues. A copymaster of the graph is at the end of these notes.

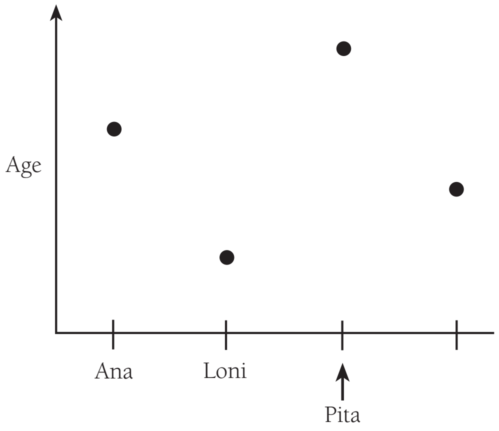
Clue: Who has the most children? (Ana)



Clue: Who is the tallest? (Loni)



Clue: Who is the oldest? (Pita)



Pages 16–17: Patterns and Rules

Achievement Objectives

- state the general rule for a set of similar practical problems (Algebra, level 3)
- use a rule to make predictions (Algebra, level 4)
- use graphs to represent number, or informal, relations (Algebra, level 3)
- sketch and interpret graphs on whole number grids which represent sample everyday situations (Algebra, level 4)

Investigation

These activities provide opportunities for students to apply their understanding of algebraic relationships to measurement. As measurement data always contains some error, the conclusions that the students come up with will be approximations and should express trends in the data rather than exact rules.

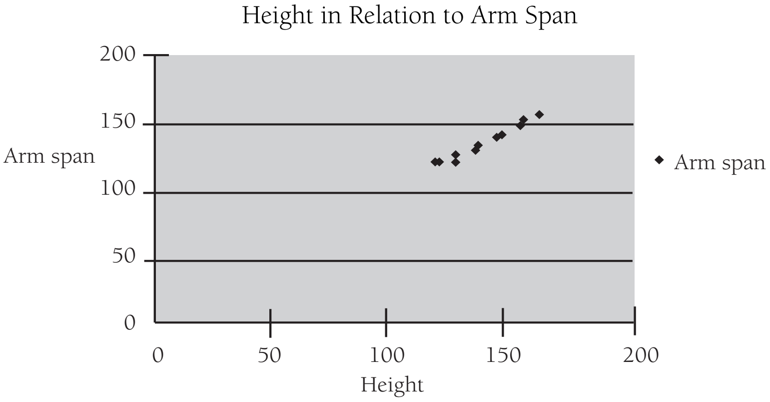
The students can express the data from each investigation in both table and graph forms, which will help them to see the patterns. The graphs will show that some of these relationships are linear and some are not.

Consider investigation **1**. Here are some hypothetical results:

As a table:

Height (cm)	Arm span (cm)
143	145
132	129
151	149
127	129
138	140
132	134
145	146
156	156
152	153
128	129
137	137

As a graph:

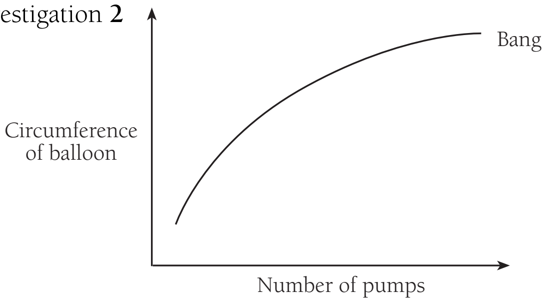


The data in the table shows that the heights of the people surveyed were close to their arm spans. This trend is confirmed in the graph because the points lie in a rough line.

In investigation **3**, a similar linear relationship exists between the volume of an amount of water and its mass. The mass of 1 millilitre of water is 1 gram. Although the inaccuracy of both the students' measurement techniques and the devices they use will create errors, there should be general trends in the data. (For this investigation, the scales will need to be accurate to 1 gram.)

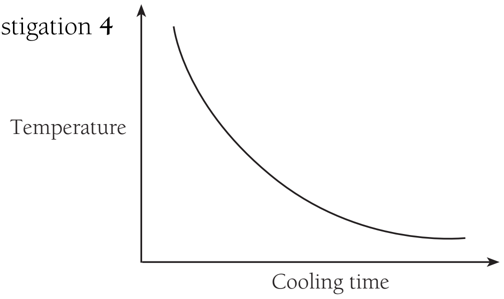
Some of the other investigations will produce non-linear relationships. Examples of these are shown below:

Investigation **2**



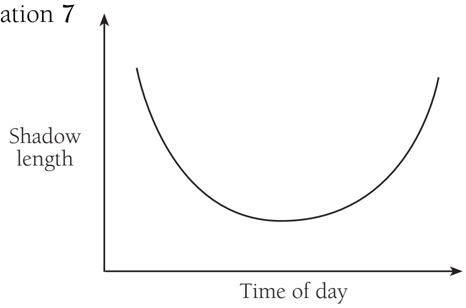
The circumference of the balloon should flatten out as the increased air pressure results in each pump having less effect. This is also due to the volume of the balloon being three-dimensional while the circumference measures two-dimensionally. The fact that air compresses is also significant. As the balloon gets tighter, the air within it compresses more until the skin of the balloon cannot stand the pressure and it bursts. (This compression accounts to a large degree for the drop in the rate of increase of the circumference. You would get a different result if you filled the balloon with water.)

Investigation **4**



Initially the temperature of hot water cools rapidly, but the rate of cooling flattens out as it nears room temperature. This is because the rate of cooling is related to the difference between the water temperature and the air temperature.

Investigation **7**



Shadows are longest at the start and end of the day when the sun is lowest in the sky. They get shortest about midday (or 1 p.m. during daylight saving) when the sun is highest.

Page 18: Dialling Dilemma

Achievement Objectives

- make up and use a rule to create a sequential pattern (Algebra, level 3)
- use a rule to make predictions (Algebra, level 4)

Activity

In this activity, the students apply rules in a practical situation. Have an analogue clock available for them to use to count the hours. It is easiest to do the problem without taking daylight saving into account. (As an extension, you could ask the students to do the problem again, this time with daylight saving. If New Zealand is in daylight saving time, our clocks are put forward by one hour. This means other countries are another hour behind us, assuming that they are not also on daylight saving.)

In non-daylight saving time, the differences are:

Person	Time behind in hours
Kirsty (Scotland)	12
Andrew (South Africa)	10
Krisnan (India)	$6^1/_2$
Keyoko (Japan)	3
Kylie (Sydney, Australia)	2
Brad (Texas, USA)	18
Anita (Argentina)	15

Hirani wants to phone Kirsty at 7 p.m. (or 1900 on a 24 hour clock) Scotland time. Scotland is 12 hours behind New Zealand, so Hirani will have to **add** 12 hours on to 7 p.m. to find the time in New Zealand. 12 hours on from 7 p.m. is 7 a.m., so Hirani will phone Kirsty at 7 a.m. New Zealand time. Using a 24 hour clock, you could calculate this as 1900 + 1200 = 3100, which is 700.

Brad in Texas is 18 hours behind New Zealand, so Hirani will have to add 18 hours on to 7 p.m.
18 hours on from 7 p.m. is 1 p.m. (1900 + 1800 = 3700, which is 1300) the next day.
That is, 7 p.m. for Brad on a Saturday will be 1 p.m. on Sunday for Hirani.

Hirani's bedtime means that she cannot call Keyoko, Krisnan, and Andrew at 7 p.m. their time. The students could work out what time in each country Hirani is awake. This time, the students will have to **subtract** the time difference because they are going from New Zealand time to another country's time.

For Krisnan, Hirani will be awake from:
7 a.m. New Zealand time: subtract $6^1/_2$ hours to get 12.30 a.m. Indian time
9.30 p.m. New Zealand time: subtract $6^1/_2$ hours to get 3 p.m. Indian time.

But 12.30 a.m. is the middle of the night, so Hirani can't phone Krisnan then. Supposing Krisnan gets up at 7 a.m., Hirani could phone him from 7 a.m. to 3 p.m. Indian time. This is from 1.30 p.m. to 9.30 p.m. New Zealand time.

The time differences between countries are due to the rotation of the Earth as it orbits the Sun. You can demonstrate this using an overhead projector and a globe of the Earth (or ball). Consider the position of each country and what time of day it will be as the globe rotates. For example, at 12 midday, the countries in longitude line with New Zealand are facing directly towards the sun. On the other side of the Earth (for example, the UK), it is midnight because they are facing directly away from the sun. The Earth rotates anticlockwise, so the dawning of a new day can be seen as the light enters countries, and the dusk in other countries can be seen as they lose the light.

Page 19: Stacks of Money

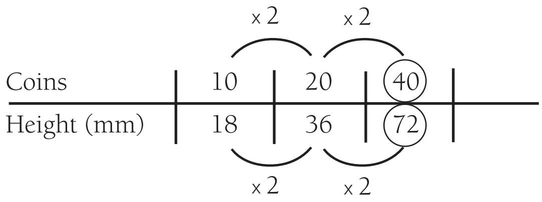
Achievement Objectives

- state the general rule for a set of similar practical problems (Algebra, level 3)
- use a rule to make predictions (Algebra, level 4)
- use graphs to represent number, or informal, relations (Algebra, level 3)
- sketch and interpret graphs on whole number grids which represent simple everyday situations (Algebra, level 4)

Activity

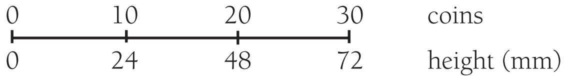
This activity gives students further experience with linear relationships and the way changing one variable (the number of coins) will effect the other variable (the height of the stack).

To answer question **1**, the students may find tables or double number lines useful.
For example, for **1a:**



There are 40 coins in the stack. 40 x $0.10 = $4.00

1b:



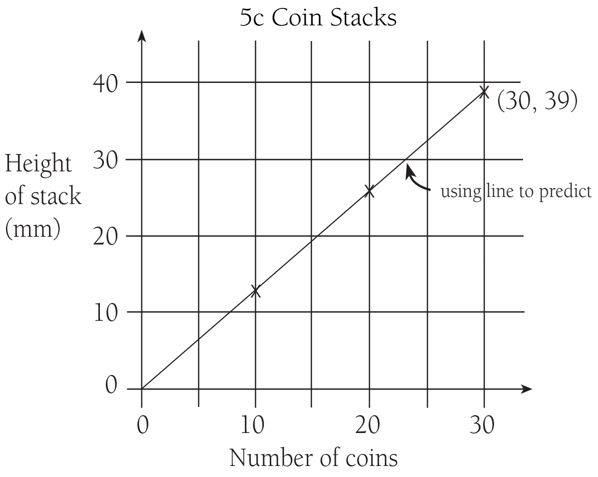
There are 30 coins in the stack. 30 x $0.20 = $6.00

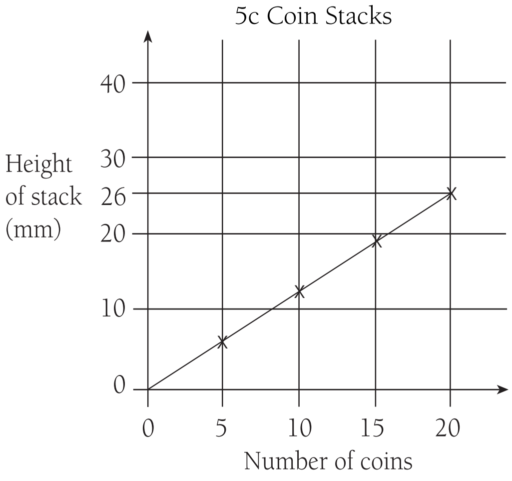
Alternatively, if the students use multiplication and division, they could answer **1c** in this way:
ten 50 cent coins is 24 millimetres high.
How many 24s in the 72 millimetre stack? 72 ÷ 24 = 3
So the stack has 3 x 10 = 30 coins, and 30 coins each worth 50 cents is $15.

In question **2**, the students extrapolate (work outside a data set) and interpolate (work within a data set) from a given graph.

For example, question **2a** can be answered by extrapolation. The students will need to extend the line of the graph until it is above 30 on the x axis. Then they will draw a vertical line directly up from 30 on the x axis until it meets the line of the graph. They will draw a horizontal line from the point above 30 on the line of the graph across to the y axis. The point at which the line hits the y axis is the height of the stack. The students will need to work carefully to ensure that their vertical line is at right angles to the x axis and their horizontal line is at right angles to the y axis.

The students will work in a similar way to answer question **2b** by interpolation. They will extend a horizontal line from the 26 millimetre point on the y axis until it meets the line of the graph. From this point on the line of the graph, they take a vertical line down to the x axis to find the number of coins.





The points on the graph lie on a straight line because the width of a coin is constant. As each extra coin is added to the stack, the height increases by a constant amount. The relationship between the number of coins and the height of the stack is linear.

Page 20: Graphic Details

Achievement Objectives

- use graphs to represent number, or informal, relations (Algebra, level 3)
- sketch and interpret graphs on whole number grids which represent simple everyday situations (Algebra, level 4)

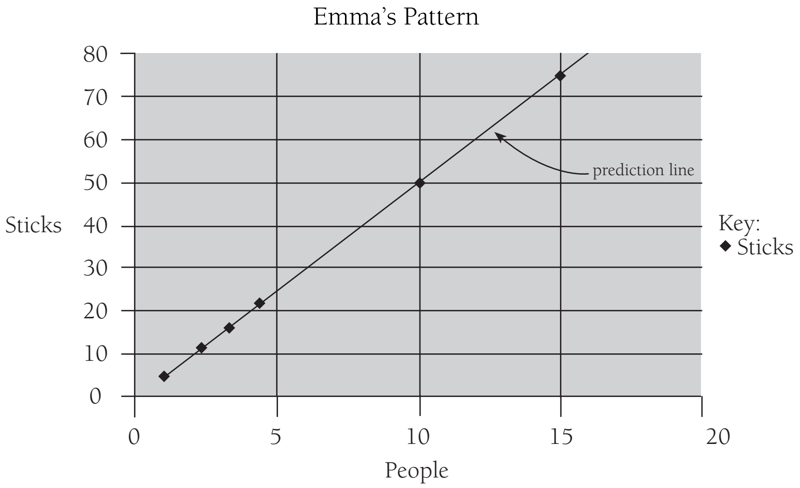
Activity

The students can use both the tables and the graphs to predict the values of further terms in each pattern.

The table below shows Emma's problem extended to the tenth and the fifteenth person.

People	Sticks	
1	6	+ 5
2	11	+ 5
3	16	+ 5
4	21	
5	26	
6	31	
7	36	
8	41	
9	46	
10	51	
⋮	⋮	
15	76	

The graph of the first four terms is a straight line (hence, the relationship is linear). This line can be extended to predict the values for the tenth and fifteenth terms.



The students can use formulas on a spreadsheet to find further values of a sequence. The spreadsheet table for Emma's sequence shows that when the number of people increases by one, the corresponding increase in the number of sticks is five. Instead of writing "2" in cell A3, the students could input the formula (rule) **=A2+1** and fill it down for as many cells as desired. Similarly, they could input the formula **=B2+5** in cell B3 and fill it down to give the matching number of sticks.

These rules can be adapted for the spreadsheets for question **2**. For example, for **2a**, the formula for the number of sticks in cell B3 is =B2+6 and for **2b**, the formula in cell B3 is =B2+5.

Page 21: Domino Delight

Achievement Objective

- find and justify a word formula which represents a given practical situation (Algebra, level 4)

Activity

These problems offer possibilities and constraints. A logical way of approaching question **1** is based on the information that the total of each of the five dominoes is one less than on the one behind it. So the numbers are 7, 8, 9, 10, and 11. The last domino can only be $\boxed{5 \mid 6}$ or $\boxed{6 \mid 5}$.

The possibilities and constraints both need to be considered when describing numeric problems algebraically. Although students at this level do not need to express the problems using algebraic expressions, the problems could be represented in that way.
For example for question **1**:

$$\underbrace{e + d - 4}_{\substack{\text{bottom}\\ \text{domino}}} = \underbrace{5 + c - 3}_{\substack{\text{fourth}\\ \text{domino}}} = \underbrace{6 + b - 2}_{\substack{\text{third}\\ \text{domino}}} = \underbrace{5 + a - 1}_{\substack{\text{second}\\ \text{domino}}} = \underset{\substack{\uparrow\\ \text{top}\\ \text{domino}}}{7}$$

where a, b, c, d, and e represent the unknown numbers on the domino halves.

To find the unknown number on the covered half of the second domino:

$$5 + a - 1 = 7$$
$$a + 4 = 7$$
$$a = 3$$

So the second domino is $\boxed{5}\boxed{3}$.

b = 3, so the third domino is $\boxed{6}\boxed{3}$.

c = 5, so the fourth domino is $\boxed{5}\boxed{5}$.

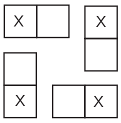
d = 11, so the total of dots on the bottom domino is 11, that is, $\boxed{5}\boxed{6}$ or $\boxed{6}\boxed{5}$.

For question **2**, let d be the total number of dots on the top domino.
We know two things:

- $? + 6 = d$
- $\underset{\substack{\uparrow \\ \text{top} \\ \text{domino}}}{d} + \underset{\substack{\uparrow \\ \text{second} \\ \text{domino}}}{(d - 2)} + (d - 4) + (d - 6) + \underset{\substack{\uparrow \\ \text{bottom} \\ \text{domino}}}{(d - 8)} = 35$ (because each domino has two fewer dots than the one above it)

Although this equation can be solved algebraically, students at levels 3–4 will probably look for an arithmetic method. This might be based on an idea of "average". There are five dominoes that total 35 dots, so the average number of dots is seven. This is the number of dots on the middle domino. So the five dominoes have 3, 5, 7, 9, and 11 dots respectively. Equation **i** becomes ? + 6 = 11, so ? is 5. The next domino will be 5 + ? = 9, and so on.

One way to solve domino doughnut puzzles such as the one in question **3** is to consider which halves of the dominoes could be in the corners of the doughnut.



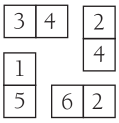
The totals of each side must be equal, so all four sides must add to a multiple of four. The halves marked x will be counted twice and the blank halves once.

The key to solving question **b** (which uses the same dominoes as the doughnut shown) is to remember that although the dominoes themselves do not have to add to a multiple of 4 (the ones in this puzzle add to 28), the four sides, with some dominoes halves counted twice, must add to a multiple of 4 because all four sides are equal. By looking at the possibilities, trying different halves at the corners, and checking to see if the total of the four sides is divisible by 4, solutions can be found.

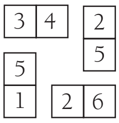
One way to work through the problem systematically is to use a table. Note that the multiples of 4 to be considered must be more than 28 because there are already 28 dots on the dominoes and the corner ones need to be counted twice.

Total of each side	Multiple of 4	Total of corner halves (multiple of 4 minus 28)	Possible corner combinations
8	32	4	none
9	36	8	(3–2–2–1)
10	40	12	(3–2–2–5), (3–2–6–1), (4–5–2–1)
11	44	16	(3–2–6–5), (4–5–2–5)
12	48	20	(4–5–6–5)

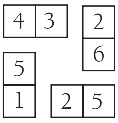
Only some of the possibilities in the table will work because some side totals do not work. For example, for the possibilities with a side total of 40, each side must add to 10. Experimenting with each possibility gives:



No arrangement works.

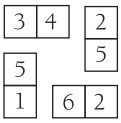


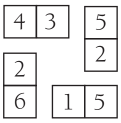
No arrangement works.



No arrangement works.

Of the other possibilities, only the side totals of 9 and 12 work for these dominoes:





The first solution has already been shown in the students' book.

Page 22: Cup Capers

Achievement Objectives

- solve problems of the type □ + 15 = 39 (Algebra, level 3)
- find and justify a word formula which represents a given practical situation (Algebra, level 4)
- solve simple linear equations such as 2 x □ + 4 = 16 (Algebra, level 4)

Activity

This activity introduces students to working with variables. Using cups that can be filled to represent unknowns helps the students to understand that letters in algebraic equations are variables rather than fixed numbers.

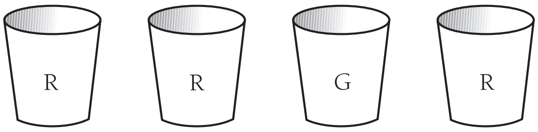
The students can answer question **1** using logical reasoning. Two yellow cups hold the same as one blue cup. So:

2 yellow cups + 1 blue cup = 1 blue cup + 1 blue cup
= 2 blue cups

This means that two blue cups would hold 12 cubes. In that case, one blue cup holds 12 ÷ 2 = 6 cubes. And that means that the two yellow cups hold six cubes, so they must hold three cubes each.

Question **2** is more complex.
Condition one: There are 25 cubes altogether.
Condition two: There are two cubes outside the cups.

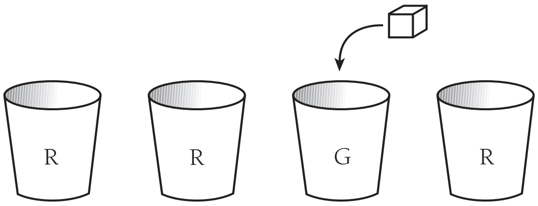
So



must have 23 cubes in total.

Condition three: Each red cup has one more cube in it than the green cup.

So if one extra cube were put in the green cup, there would be four equal cups, which would give a total of 24 cubes. Each red cup must hold 24 ÷ 4 = 6 cubes and the green cup holds 6 – 1 = 5.



Question **3** reinforces the students' understanding that letters are variables rather than fixed numbers.

In question **4**, confined values have been given for each letter (or cup). This gives the students practice in solving symbolic equations with certain values for the variables, and it also provides an opportunity to discuss equivalent symbolic expressions. For example, the students will find that both collection **iii** and collection **v** have 20 cubes. You could vary the number of cubes in each cup and calculate the number of cubes. Collections **iii** and **v** will still be equivalent. You could ask the students to explain why this is.

Their explanations for question **4b** should include the following points:

- Collection **iii** can be described as 2y + 2b + 2y + r.
- Altogether, there are four yellow cups, so **iii** can also be described as 4y + 2b + r.
- Collection **v** can be described as r + 4y + 2b.
- Addition is commutative, so the order of the addends does not matter. So collection **v** can also be described as 4y + 2b + r, which is the same as collection **iii**.

Page 23: Robot Rescue

Achievement Objectives

- state the general rule for a set of similar practical problems (Algebra, level 3)
- use a rule to make predictions (Algebra, level 4)

Activity

The strategies that the students use to continue equation patterns will give a strong indication of their ability to describe relationships. For example, those students who use lower level strategies will continue the patterns using recursive thinking until they find the required equation. In Maria's example, this would involve the following working:

1 + 1 = 2

2 + 2 = 4

3 + 3 = 6

4 + 4 = 8

5 + 5 = 10

⋮

9 + 9 = 18

10 + 10 = 20

A more powerful strategy is to look for relationships between the numbers in each equation in a pattern. In Maria's case, the relationship might be expressed in words, such as, "Take the first number, add it to itself, and the answer is double the first number". This could be expressed symbolically as n + n = 2n, where n is the first number. The first number in each successive equation increases by 1, so in the tenth equation, n = 10. This strategy is more efficient than writing the whole sequence.

When the students are expressing equation sets algebraically, they need to choose the number represented by n carefully. For example, in question **2a i**, choosing the number after the equals sign as n makes expressing the rule simple:

1 = 1 – 0
1 = 2 – 1
1 = 3 – 2
1 = 4 – 3
⋮
1 = n – (n – 1). Choose any number and take away one less than it.

The answer is always 1.

An important idea for the students to grasp is that the equals sign is a statement of symmetrical balance. Number balances such as those used in *Algebra*, Figure It Out, Level 3, page 19 can help reinforce this idea. Many students believe from their previous experiences that the equals sign means "works out to be" because the unknown was always shown at the right of the equals sign, for example, 4 + 3 = □, 9 x 7 = □. Activities such as these equation sets help to broaden the students' view of the equals sign.

Page 24: Number Tricks

Achievement Objectives

- state the general rule for a set of similar practical problems (Algebra, level 3)
- use a rule to make predictions (Algebra, level 4)

Activities One and Two

"Choose any number" problems like these help to develop the students' ideas about generalised properties of numbers and how these properties apply to variables rather than just to specific numbers.

Cups and cubes provide a useful model for these problems because they illustrate that the value of the starting number is insignificant. Consider **Activity Two**, question **1**. On the left are the instructions, and on the right is the matching model.

Choose any number:

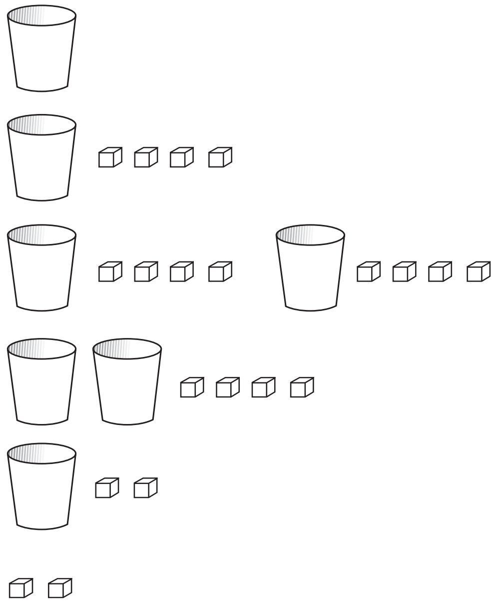
Add 4:

Multiply by 2:

Subtract 4:

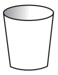
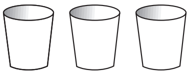
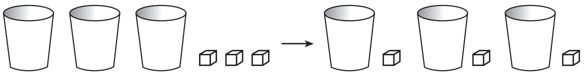
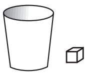
Divide by 2:

Subtract the starting number:



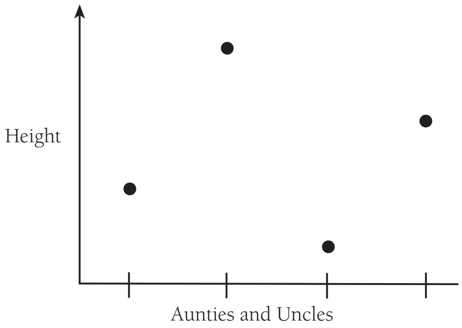
Irrespective of how many cubes are placed in the cup at first, that is, the starting number, the result is always 2.

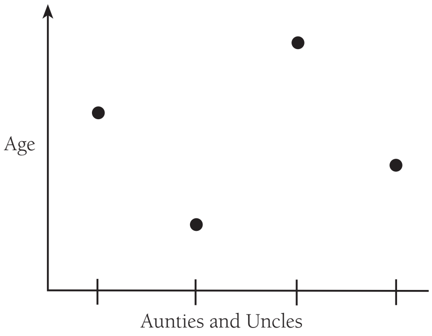
Some students may be able to grasp the use of algebraic symbols. Consider **Activity Two**, question **2**:

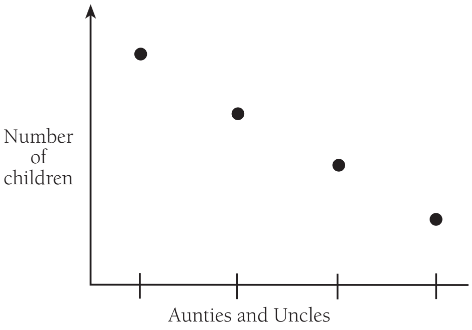
Instructions	Model	Symbols
Choose any number:		n
Multiply by 3:		$3n$
Add 3:		$3n + 3 = 3(n + 1)$
Divide by 3:		$n + 1$
Subtract the starting number:		1

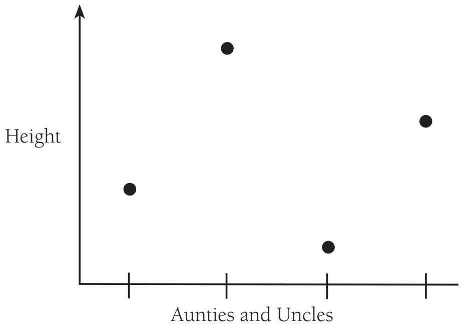
See also the notes for Activity Two in *Answers and Teachers' Notes: Algebra*, Figure It Out, Level 3, pages 33–34, which explain how puzzles like this work.

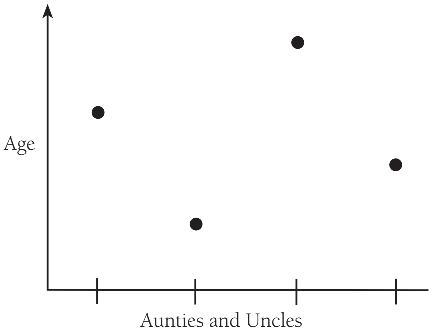
Copymaster: Tongan Travel

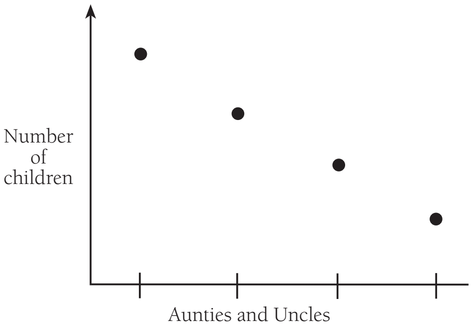


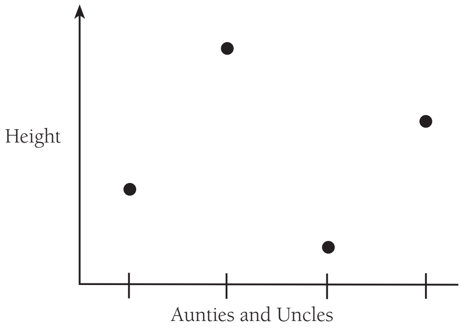


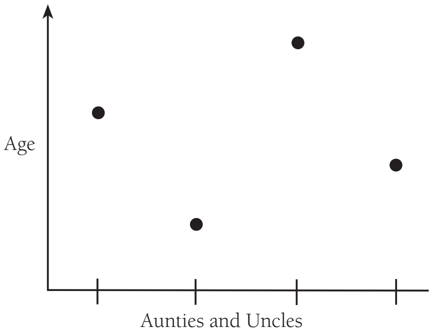


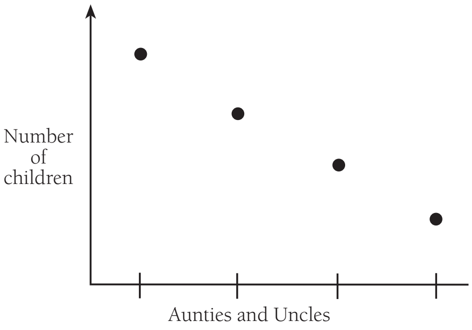












Copymaster: Seeing Dots

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In the main illustration on the cover and contents page, the people are by Ali Teo, the fish and the background are by Jeremy Clegg, and the photographs of the money and the multilink cubes are by Adrian Heke.

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