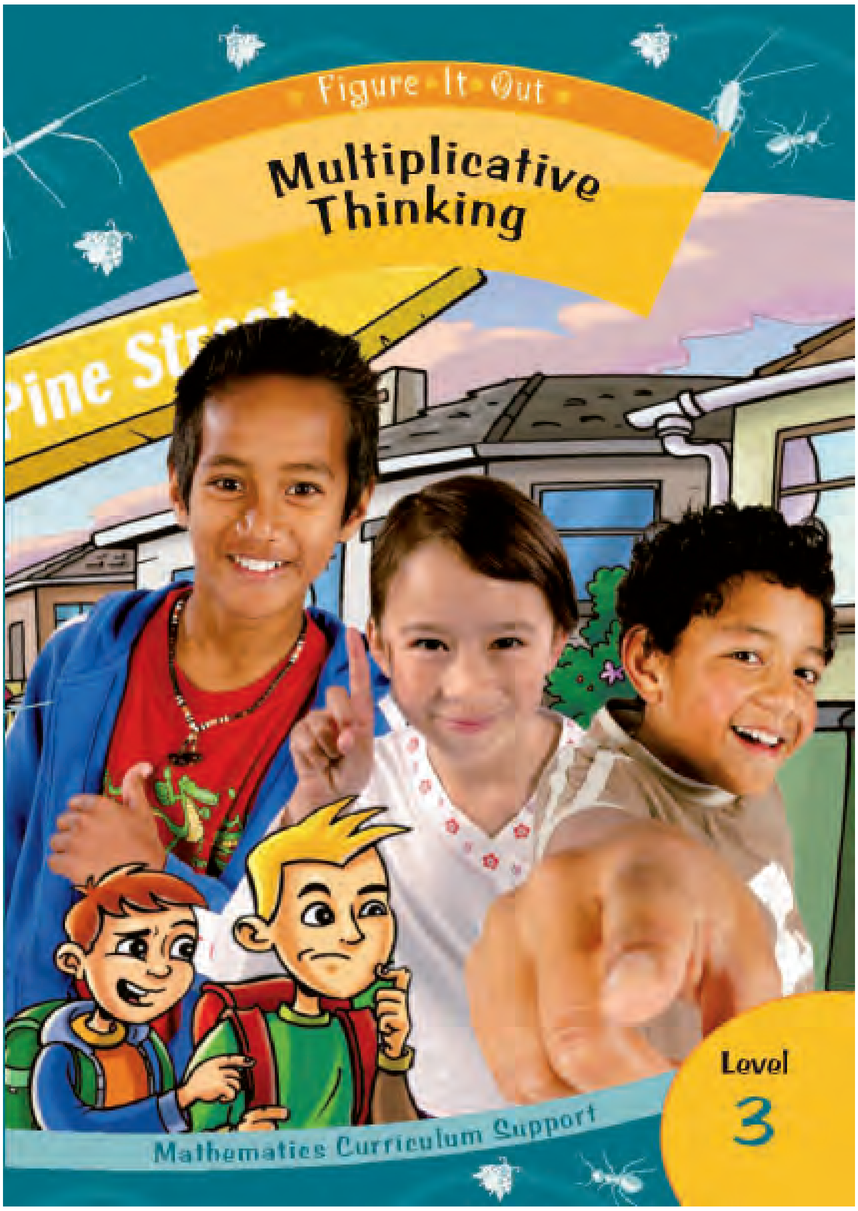
Answers and Teachers' Notes



Contents

Introduction

The books in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand classrooms. In recent years, much of the Figure It Out student material has been aligned with Numeracy Development Project strategies, which are reflected in the answers and in the teachers' notes.

Student books

The activities in the student books are written for New Zealand students and are set in meaningful contexts, including real-life and imaginary scenarios. The level 3 contexts reflect the ethnic and cultural diversity and the life experiences that are meaningful to students in year 5. However, teachers should use their judgment as to whether to use the level 3 books with older or younger students who are also working at this level.

The activities can be used as the focus for teacher-led lessons, for students working in groups, or for independent activities. Also, the activities can be used to fill knowledge gaps (hot spots), to reinforce knowledge that has just been taught, to help students develop mental strategies, or to provide further opportunities for students moving between strategy stages of the Number Framework.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the *Multiplicative Thinking* student books includes full answers and explanatory notes. Students can use this section for self-marking, or you can use it for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, Number Framework and other links, comments on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/r/maths/curriculum/figure

Using Figure It Out in the classroom

Where applicable, each page starts with a list of equipment that the students will need in order to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to the problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Figure It Out

Multiplicative Thinking

Level 3

Answers

Page 1: Train Teasers

Activity

1. 45. (A multiplicative strategy is to break the 15 into smaller groups and then use known facts. For example: [3 x 10] + [3 x 5] = 30 + 15 = 45.)
2. 90. (Using place-value partitioning, 45 + 45 = [40 + 40] + [5 + 5] = 80 + 10 = 90.)
3. 225. (A multiplicative strategy is to break the 45 into smaller groups and multiply each group by 5. For example, 5 x [40 + 5] is [5 x 40] + [5 x 5] = 200 + 25 = 225. Another way to break up 45 is 10 + 10 + 10 + 10 + 5. So 5 x 45 = [5 x 10] + [5 x 10] + [5 x 10] + [5 x 10] + [5 x 5] = 50 + 50 + 50 + 50 + 25 = 225.)
4. a. 5

   b. Methods will vary. If you know that 5 x 8 = 40, you could use this to solve 40 ÷ 8 = □.
5. Problems will vary.

Pages 2–3: The Hundreds Board Hunt

Activity

1. a. Descriptions may vary. The pattern is a diagonal one, with two uncoloured diagonals after each coloured diagonal.

   b. Practical activity

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

   c. 3 + = = = = … (or 3 + + = = = = …) will give you all the multiples of 3. Alternatively, if you divide each of the coloured-in numbers by 3 on your calculator, the answer will be a whole number.
2. a. 102. (102 ÷ 3 = 34. There is no remainder.)

   b. 198. The last multiple must be one of the last 3 numbers on the square. 198 is divisible by 3 with no remainder, but 199 and 200 aren't.

   c. Practical activity

101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

3. Answers will vary. One simple rule (generalisation) is: a number is a multiple of 3 if all its digits add up to a number that is a multiple of 3. For example, for 147,
1 + 4 + 7 = 12. 12 is divisible by 3
(12 ÷ 3 = 4), so 147 is a multiple of 3
(147 ÷ 3 = 49).

Note: If your addition gives you a number with two or more digits and you are not sure if it is divisible by 3, add up the new digits: if the number is divisible by 3, your final answer must be 3, 6, or 9. For example, for 198:
1 + 9 + 8 = 18, 1 + 8 = 9.

4. Numbers tried will vary. The rule and the note in question **3** still apply. For example, for 2 835:
2 + 8 + 3 + 5 = 18, 1 + 8 = 9; and for 9 864:
9 + 8 + 6 + 4 = 27, 2 + 7 = 9. On the calculator, all multiples of 3 will have whole-number answers when divided by 3.

Pages 4–6: Problems on the Way to School

Activity

1. 9 tails and 36 paws. (Lachlan must have seen 9 animals if he saw 18 eyes: 18 eyes ÷ 2 eyes on each animal = 9 animals. Tails: 9 x 1 = 9.
Paws: 9 x 4 = 36)

2. 42. (Using place value partitioning:
14 x 3 = [10 x 3] + [4 x 3] = 30 + 12 = 42
Using halving and doubling: 14 x 3 = 7 x 6 = 42
Using tidy numbers and compensating:
[15 x 3] – [1 x 3] = 45 – 3 = 42.)

3. Methods may vary. (There are 32 palings.) Each post is followed by 4 palings, with an extra post at the end. Lachlan could count the posts, take 1 off the total, and multiply that number by 4. So, for this fence, 9 posts – 1 = 8 groups of 4 palings. 8 x 4 palings = 32 palings. Or Lachlan may have seen the posts and palings as groups of 5: (8 x 5) – (8 x 1) = 40 – 8 = 32

4. **a.** 2 hrs. (There are 60 min in 1 hr.
2 x 60 = 120 min or 120 ÷ 60 = 2)

   b. 28. (7 x 4 = 28)

5. 2 groups of 12, 3 groups of 8, 4 groups of 6, 6 groups of 4, 8 groups of 3, 12 groups of 2.

6. **a.** 21 bins. (7 x 3 = 21)

   b. 11 houses. (33 ÷ 3 = 11)

7. **a.** 1 533 steps. (Half of 3 000 is 1 500, and half of 66 is 33. 1 500 + 33 = 1 533)

   b. **i.** 1 022. ($\frac{1}{3}$ of 3 000 is 1 000, and $\frac{1}{3}$ of 66 is 22. So $\frac{1}{3}$ of 3 066 is 1 022.)

   ii. 2 044. (1 022 x 2)

8. Maps and problems will vary.

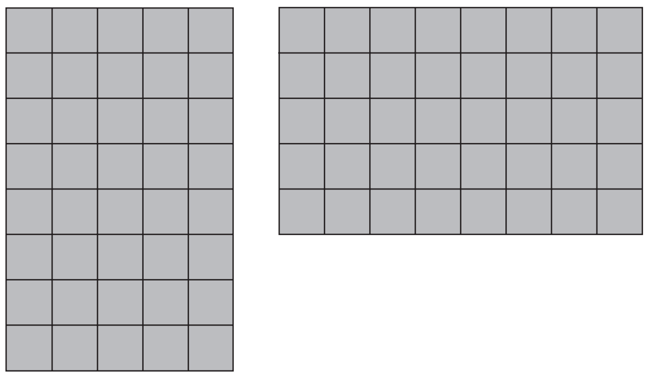
Page 7: Card Arrays

Activity

1. **a.** 40 cards. (Using the ace as 1, 1 to 10 is 10 cards, 4 suits x 10 cards = 40 cards. Or: 4 suits x 3 face cards = 12 face cards taken out: 52 – 12 = 40.)

   b. There are 8 ways: 1 by 40, 40 by 1, 2 by 20, 20 by 2, 4 by 10, 10 by 4, 5 by 8, and 8 by 5. (Note that each way has a matching rectangle, for example 1 by 40 and 40 by 1. The rectangles are rotated.)

   Drawings: Note that no matter which way the pairs of rectangles are facing, each rectangle contains 40 units. For example, for 5 by 8 and 8 by 5:



2. **a.** Predictions will vary. The correct answer is fewer.

   b. Practical activity. (There are 6 possible rectangles with 52 cards [1 by 52, 52 by 1, 2 by 26, 26 by 2, 4 by 13, 13 by 4] and 8 possible rectangles with 40 cards [see question **1b**].)

   c. **i.** Yes, if you said “fewer”. You may have said “more” because there are more cards, but 40 has more factors than 52, and therefore, more rectangular arrays can be made with 40 cards than with 52 cards.

ii. You know you have found all the rectangles when there are no more pairs of possible factors to use. (For example, with 40, after you have found *1* x 40, *2* x 20, *4* x 10, and *5* x 8, the next factor is 8, which you have already paired with 5. [40 ÷ 3, 6, 7, 9, 11, or 12 has a remainder.] With 52, only 1, 2, 4, 13, 26, and 52 are factors.)

3. 8. (The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24, so the possible rectangles are 1 by 24, 24 by 1, 2 by 12, 12 by 2, 3 by 8, 8 by 3, 4 by 6, and 6 by 4.)

4. 4 cards long. (11 x 4 = 44)

5. There are 7 possible square arrays that you can make using up to 52 square cards: 1 by 1, 2 by 2, 3 by 3, 4 by 4, 5 by 5, 6 by 6, and 7 by 7.

Pages 8–9: Bean Counters

Activity

1. Aronui knows that 20 is 2 tens (2 x 10 = 20). She uses 5 x 2 = 10 to work out 5 x 2 tens = 10 tens. 10 x 10 = 100, so 5 x 20 = 100.

2. **a.** 17. (20 – 3 = 17)

b. 15. (3 x 5 = 15)

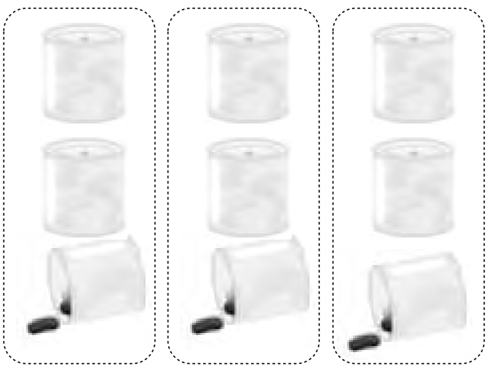
c. Aronui knows that 5 x 20 = 100 beans.
She took out 5 x 3 = 15 beans.
(5 x 20) – (5 x 3) = 100 – 15
= 85
So 5 x 17 = 85.

3. **a.** 60. (3 x 2 tens = 6 tens or 60 beans)

b. Aronui had 3 x 20 = 60 beans, and she took out 3 x 2 = 6 beans.
(3 x 20) – (3 x 2) = 60 – 6
= 54
So 3 x 18 = 54.

4. **a.** 90. (3 x 3 tens = 9 tens or 90 beans)

b. A possible picture is:



c. Aronui had 3 x 30 = 90 beans, and she took out 3 x 1 = 3 beans.
(3 x 30) – (3 x 1) = 90 – 3 = 87
So 3 x 29 = 87.

5. 5 x 98 = (5 x 100) – (5 x 2)
= 500 – 10
= 490

6. Problems will vary. One of the numbers in the problem should be a single digit and the other close to (but just a little bit less than) a tidy number. Some examples are: 3 x 68 = ☐, 5 x 77 = ☐, 6 x 999 = ☐. Check your answer with a calculator before you give it to your classmate.

Pages 10–11: Sticking Together

Activity

1. **a.** 4 x 2 = 8, so 4 x 2 tens = 8 tens, or 80.

b. 4 x 3 = 12 because there are 4 groups of 3 single sticks.

c. 92. ([4 x 20] + [4 x 3] = 80 + 12 = 92.
So 4 x 23 = 92.)

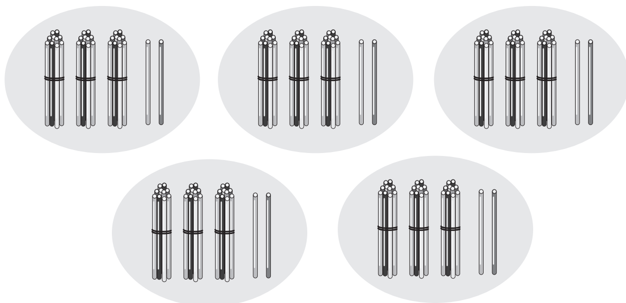
2. **a.** 3 groups of 42 or 3 x 42 = ☐

b. 120. (3 x 4 tens = 12 tens, which is 120. Or, using Paul's hint, 10 x 10 = 100 and 2 x 10 = 20. 100 + 20 = 120)

c. 6. (3 x 2 = 6)

d. 126 sticks. 120 + 6 = 126, so 3 x 42 = 126.

3. **a.** A possible picture is:

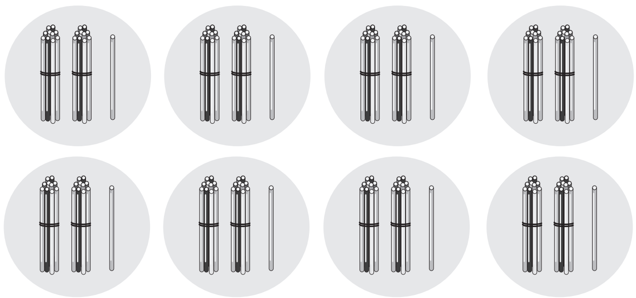


b. 150. (5 x 3 = 15, so 5 x 3 tens = 15 tens, which is 150.)

c. 10. (5 groups of 2 = 10)

d. 160. (150 sticks in the bundles + 10 single sticks = 160, so 5 x 32 = 160.)

4. Paul could use his sticks like this:



Then he could go: 8 x 2 = 16, so 8 x 2 tens = 16 tens or 160. (Or he could go: 8 x 2 tens = 8 x 20 = 160.) 8 x 1 = 8. 160 + 8 = 168. So 8 x 21 = 168.

5. The 1 in 152 represents 1 hundred, the 5 represents 5 tens, and the 2 represents 2 ones. Paul could think of the digits as being "hundreds", "tens", and "ones".
3 x 1 hundred = 300
3 x 5 tens = 15 tens, which is 150
3 x 2 ones = 6 ones
300 + 150 + 6 = 456. So 3 x 152 = 456.

6. a. i. (4 x 2 000) + (4 x 100) + (4 x 50) + (4 x 3) = 8 000 + 400 + 200 + 12 = 8 612

ii. (4 x 800) + (4 x 90) + (4 x 8) = 3 200 + 360 + 32 = 3 592

b. Paul's strategy was easiest to use with the first problem. The second problem is harder to keep track of in your head because there are lots of numbers that need renaming.

c. 898 is close to a tidy number, so a useful strategy would be to use the tidy number and then compensate: (4 x 900) – (4 x 2) = 3 600 – 8 = 3 592

7. Problems will vary. Your problem might have one number that has 1 digit, and the other number might have 2, 3, or more digits. If you use small digits, the strategy will be easier to use. Examples could be:
3 x 34 = (3 x 30) + (3 x 4) = 90 + 12 = 102, or
5 x 162 = (5 x 100) + (5 x 60) + (5 x 2) = 500 + 300 + 10 = 810. Use a calculator to check that your answer is correct before you give your problem to your classmate.

Pages 12–13: Multiple Multiplication Methods

Activity

1. a. i. Manua

ii. Sophie

iii. Larry

b. Answers may vary. Manua's method involved the simplest calculations and was very efficient in this case.

2. a. i. (5 x 100) – (5 x 2) = 500 – 10
= 490

ii. (5 x 90) + (5 x 8) = 450 + 40
= 490

iii. 5 x 98 = 10 x 49
= 490

b. Answers may vary. Sophie's method involved the simplest calculations and was very efficient.

3. a. Discussion and choice of methods will vary.

i. This would be a good problem for using doubling and halving, because doubling 5 times makes it into 10 times, which is easy to calculate, and the other factor is an even number, which is easy to divide. The other two methods would also work well but involve slightly more calculations (using tidy numbers and place value).

ii. This would be a good problem for using place value to break up the 43. Using tidy numbers is also possible, but 43 is not very close to a tidy number so the calculation is a little harder. Doubling and halving is not useful in this case: both numbers are odd, so it doesn't help to halve one of them.

iii. This would be a good problem for using tidy numbers because the 79 is very close to a tidy number. Doubling and halving won't help. 79 could be broken up using place value, but the calculations will be more difficult than calculations using a tidy number.

b. Discussion about the most efficient method will vary. Possible working for each problem is:

i. Tidy numbers
5 x 26 = □
5 x 26 = (5 x 30) – (5 x 4)
= 150 – 20
= 130

Place value
5 x 26 = (5 x 20) + (5 x 6)
= 100 + 30
= 130

Doubling and halving
5 x 26 = 10 x 13
= 130

ii. Tidy numbers
43 x 7 = □
43 x 7 = (50 x 7) – (7 x 7)
= 350 – 49
= 301

Place value
43 x 7 = (40 x 7) + (3 x 7)
= 280 + 21
= 301

Doubling and halving
43 x 7 = 86 x 3.5 = 301
or 21.5 x 14 = 301
so 43 x 7 = 301. (In both cases, this strategy makes the calculation harder, not easier.)

iii. Tidy numbers
6 x 79 = □
6 x 79 = (6 x 80) – (6 x 1)
= 480 – 6
= 474

Place value
6 x 79 = (6 x 70) + (6 x 9)
= 420 + 54
= 474

Doubling and halving
6 x 79 = 12 x 39.5
= 474
(This strategy makes the calculation harder, not easier.)

4. a. Problems will vary.

i. The problem should have one number that is close to a tidy number to make it easy to solve.

ii. This strategy suits many problems, but it's the easiest to use if one number in the problem is a single-digit number and there isn't much renaming involved.

iii. This strategy suits 5 times or 50 times problems because these numbers double to make 10 times or 100 times, which are easy to calculate with. Other problems suit this strategy if they can be changed into known facts, for example:
4 x 16 = 8 x 8 = 64.

b. Practical activity. See points in **4a**.

Pages 14–15: Rubber Band Rectangles

Activity

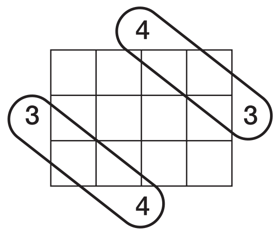
1. a. 24 squares. (4 rows of 6 squares)

b. 15 squares. (5 rows of 3 squares)

c. 18 squares. (3 rows of 6 squares)

2. a. Discussion will vary. Elsie's likely method could be: Work out how many geoboard units "long" the two different sides are. Add those two numbers together and multiply the answer by 2.

b. Diagrams will vary. The diagram below shows how Elsie would add 3 + 4 and then double that because there are 2 sides of 3 and 2 sides of 4. (3 + 4) x 2 = 7 x 2
= 14 geoboard units long



c. (4 + 6) x 2 = 10 x 2
= 20 geoboard units long
(3 + 5) x 2 = 8 x 2
= 16 geoboard units long
(3 + 6) x 2 = 9 x 2
= 18 geoboard units long

3. **a.** Practical activity. There are 5 different rectangles with a perimeter of 24 units possible on a 10 by 10 geoboard:

Width	2	3	4	5	6
Length	10	9	8	7	6

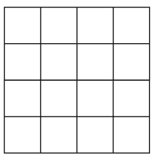
Note that these rectangles each have a matching rectangle. For example, a rectangle that is 3 wide and 9 long is the same as one that is 9 wide and 3 long; it's just rotated. A square (for example, 6 by 6) is a special sort of rectangle: one with all its sides the same length.

b. Yes. A 1 by 11 rectangle would be too big for a 10 by 10 geoboard.

4.

Width	1	2	3	4	5	**6**	**7**	**8**	**9**
Length	17	16	15	14	13	**12**	**11**	**10**	**9**

5. **a.** Practical activity. A 2 by 8 and a 4 by 4 both have an area of 16 squares and can be made on a 10 by 10 geoboard.



b. 1 by 16 won't fit on a 10 x 10 geoboard.

c. The 1 by 16 rectangle: its perimeter is 34 geoboard units long.

d. The 4 by 4 rectangle: its perimeter is 16 geoboard units long.

6. The 3 by 4 rectangle has an area of 12 square units and a perimeter of 14 geoboard units. There are 5 possible answers with a longer perimeter and a smaller area:

width	length	perimeter	area
1	7	16	7
1	8	18	8
1	9	20	9
1	10	22	10
1	11	24	11

Note that even though a 5 by 2 rectangle has a smaller area, it has the same perimeter as the 3 by 4. And although a 6 by 2 rectangle has a longer perimeter than a 3 by 4, it has the same area.

Pages 16–17: Multiples and Factors

Activity One

1. 7, 14, 21, 28, 35, 42, 49, and so on
2. 1, 2, 3, 4, 6, 8, 12, 24

Game

A game involving factors and multiples

Activity Two

1. **a.** 8, because 8 is the highest factor that 16 and 24 have in common (highest common factor). The dice throws would be 1 and 8 or 2 and 4.

   b. 4 is the highest common factor of 12, 16, and 60. (The other common factors are 1 and 2.)

2. **a.** 21. (3 x 7 = 21)

   b. 30. (2 x 3 x 5 = 30. 15 is a multiple of 3 and 5 but not of 2, and the only even numbers less than 30 that are multiples of 5 are 10 and 20, neither of which is a multiple of 3.)

Page 18: Buttons Galore

Activity

1. Answers will vary (you may count different numbers of buttons in each layer than other classmates). Possible answers include:

   a. About 100 buttons. (5 x 2 x 10)

   b. About 300 buttons. (The bottle shape is triangular, with about 5 or 6 buttons on each of the front two sides. If this was a rectangle, this would be about 5 x 6 = 30 buttons. A triangle is half a rectangle, so 30 ÷ 2 = 15 for the top layer. There are about 20 layers, so 20 x 15 = 10 x 30 = 300.)

   c. About 120 buttons. (There are about 6 x 4 x 5 buttons. 6 x 4 = 24. 24 x 5 = 12 x 10 = 120)

   d. About 252 buttons. (There are about 8 buttons showing on the top layer, with probably 4 hidden at the back of the circle, and 21 layers. 12 x 21 = 10 x 21 + 2 x 21 = 210 + 42 = 252)

2. Yes, as long as the shells of different shapes and sizes are evenly mixed throughout the jar.

Page 19: Adventure Tokens

Activity

1. a. 96 tokens left. (6 x 8 tokens = 48 tokens, which is one-third of her tokens. So she has two-thirds left:
2 x 48 = 2 x 50 – 4 = 96 tokens)

   b. Answers will vary. One way is for Naraini to go on the dodgems 4 times (4 x 7 = 28), the river ride 6 times (6 x 6 = 36), and the merry-go-round 8 times (8 x 4 = 32).
28 + 36 + 32 = 30 + 36 + 30 = 3 x 30 + 6 = 96 tokens

2. 111 tokens. (Possible methods:
multiply each ride by 3: [3 x 4] + [3 x 5] + [3 x 6] + [3 x 7] + [3 x 7] + [3 x 8] = 111;
add up adjacent pairs of rides and multiply by 3: [3 x 9] + [3 x 13] + [3 x 15] = 111;
find tidy pairs and multiply by 3: [3 x 10] + [3 x 12] + [3 x 15] = 111;
add up all the token values and multiply by 3:
4 + 5 + 6 + 7 + 7 + 8 = 37; 37 x 3 = 111;
or add up in groups of 2 and multiply by 3:
9 + 13 + 15 = 37; 37 x 3 = 111)

3. 6 rides each. (Using multiplication: If the 3 friends all have 1 ride on the dodgems, this will cost them 3 x 7 = 21 tokens altogether. 21 x $\square$ = 126; 21 x 6 = 126. Using division: 126 ÷ 3 = 42; 42 ÷ 7 = 6.)

4. 6 times on the river ride and 2 times on the bumper boats. (6 x 6) + (2 x 7) = 36 + 14 = 50

5. Problems will vary.

Pages 20–21: Team Schemes

Activity

1. All of the schools have about 30 students, so Ms Kelly could work out how much more or less than 30 each school's total is, add up all these positive and negative numbers, and then add or subtract the result from 210 (7 x 30).
(– 4 + 1 + 0 – 2 – 2 + 5 + 2 = 0, so there are exactly 210 students.)

2. 105 teams for bowls: 210 ÷ 2 = 105
35 teams for mini-soccer: 210 ÷ 6 = 35
30 teams for touch rugby: 210 ÷ 7 = 30
42 teams for basketball: 210 ÷ 5 = 42
70 teams for triathlon: 210 ÷ 3 = 70
53 teams for beach volleyball: 210 ÷ 4 = $52\frac{1}{2}$.
(2 students would probably have to play beach volleyball twice so that the 2 in the half team get a game.)

3.

Bowls	2 players	(2 schools have an odd number of students.)
Mini-soccer	18 players	(5 + 2 + 4 + 0 + 2 + 1 + 4)
Touch rugby	14 players	(0 + 5 + 0 + 2 + 4 + 3 + 0)
Basketball	10 players	(0 + 1 + 3 + 0 + 2 + 1 + 3)
Triathlon	9 players	(2 + 2 + 1 + 0 + 2 + 1 + 1)
Beach volleyball	10 players	(3 + 2 + 0 + 2 + 0 + 3 + 0)

This information is shown as a table on the following page.

	Number of students in a team	Number of students left out of a team							Number of students in mixed-school teams
		Waikino School 35 students	Hampton School 26 students	Kahunui School 28 students	St Joseph's School 30 students	Te Horo School 32 students	Sherwood School 31 students	Mapui School 28 students	
Bowls	2	1	0	0	0	0	1	0	**2**
Mini–soccer	6	5	2	4	0	2	1	4	**18**
Touch rugby	7	0	5	0	2	4	3	0	**14**
Basketball	5	0	1	3	0	2	1	3	**10**
Triathlon	3	2	2	1	0	2	1	1	**9**
Beach volleyball	4	3	2	0	2	0	3	0	**10**

4. Answers will vary. For example, if each student on a winning team earns 2 points for their school, then the available points for each sport would be:
   - 4 points for a win in bowls (because there are 2 players in a team)
   - 12 points for a win in mini-soccer
   - 14 points for a win in touch rugby
   - 10 points for a win in basketball
   - 6 points for a win in the triathlon
   - 8 points for a win in beach volleyball.

This system is fair because each winning player earns points for their school.

This would also work very well for mixed teams, and the points could be halved for a draw.

Page 22: Square Skills

Activity

1. Grandma could make a 1 by 24, a 2 by 12, a 3 by 8, or a 4 by 6 rectangular shape. (Note that a rectangle that is 4 wide and 6 long is the same as one that is 6 wide and 4 long.)

2. a. Note that all the squares have 10 cm sides. So: 500 cm for 1 by 24 (because the perimeter is 10 + 240 + 10 + 240 = 500 cm); 280 cm for 2 by 12 (the perimeter is 20 + 120 + 20 + 120 = 280 cm); 220 cm for 3 by 8 (the perimeter is 30 + 80 + 30 + 80 = 220 cm); 200 cm for 4 by 6 (the perimeter is 40 + 60 + 40 + 60 = 200 cm)

   b. The areas are all 2 400 cm^2, but the perimeters are all different.

3. a. 1 by 36; 2 by 18; 3 by 12; 4 by 9; 6 by 6

   b. i. The 6 by 6 rectangle uses the least binding. (The perimeter of a 6 by 6 rectangle is 6 + 6 + 6 + 6 = 24 x 10 cm = 240 cm. The perimeters of the other rectangles are all more than 240 cm.)

   ii. The 1 by 36 rectangle. (1 + 36 + 1 + 36 = 74 x 10 cm = 740 cm)

Page 23: Munching Monarchs

Activity

1. a. 12 caterpillars. (3:4 = 9:**12**.
9 small plants ÷ groups of 3 = 3 groups,
3 groups x 4 caterpillars = 12 caterpillars)

   b. 15 caterpillars. (2:5 = 6:**15**.
6 large plants ÷ groups of 2 = 3 groups,
3 groups x 5 caterpillars = 15 caterpillars)

2. Buying all large plants would be the cheaper option ($44 compared with $45 for small plants). 20 caterpillars would need 15 small plants (3:4 = 15:20), 15 small plants at $3 cost $45. 20 caterpillars would need 8 large plants (2:5 = 8:20), 8 large plants at $5.50 cost $44.

3. a. i. 22 small plants. ($66 ÷ $3 = 22)

   ii. About 29 caterpillars. (21 small plants would feed 28 caterpillars because 3:4 = 21:28. The 22nd plant would feed $1\frac{1}{3}$ caterpillars [$\frac{1}{3}$ of 4 = $1\frac{1}{3}$], so the extra plant would be enough for 1 extra caterpillar, but not for 2.)

   b. i. 12 large plants. ($66 ÷ $5.50 = 12. A possible strategy for this is to solve [66 ÷ 11] x 2 = 6 x 2 = 12. $5.50 x 2 = $11, so this is easier to calculate. Alternatively, you could reverse the problem and use multiplication: 10 x $5.50 = $55, $66 – $55 = $11, 2 x $5.50 = $11. 10 + 2 = 12 plants)

   ii. 30 caterpillars. (2:5 = 12:30)

4. Answers will vary. To spend all her $66, Mrs McPherson would have to buy 6 large plants and 11 small ones (6 x $5.50) + (11 x $3) = $33 + $33 = $66. There are 11 possible options:

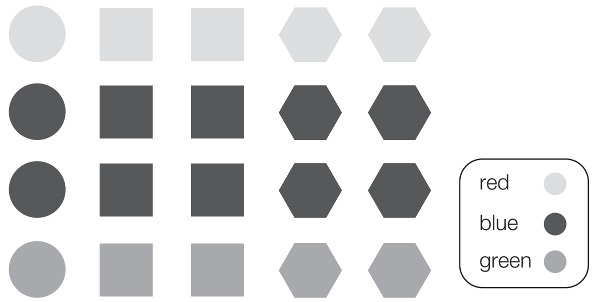
Number of large plants	Cost of large plants ($5.50 each)	Number of small plants	Cost of small plants ($3 each)	Total cost
1	$5.50	20	$60	$65.50
2	$11	18	$54	$65
3	$16.50	16	$48	$64.50
4	$22	14	$42	$64
5	$27.50	12	$36	$63.50
6	$33	11	$33	$66
7	$38.50	9	$27	$65.50
8	$44	7	$21	$65
9	$49.50	5	$15	$64.50
10	$55	3	$9	$64
11	$60.50	1	$3	$63.50

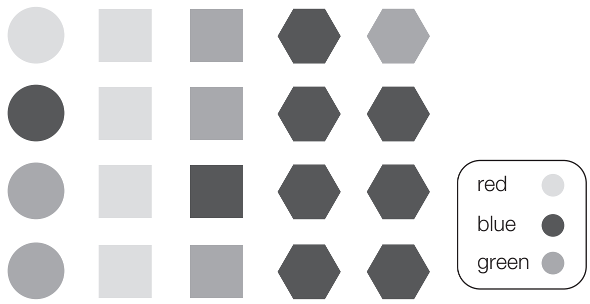
5. Practical activity

Page 24: Fractions in Room 7

Activity

1. a. Practical activity. The smallest set will be 20 blocks. The colours need to be 5 red, 10 blue, and 5 green, and with the shapes 4 circles, 8 squares, and 8 hexagons. There are many ways of distributing the colours across the shapes. Here are two ways:





   b.–c. Practical activity. Sets and fractions will vary. For example, $\frac{1}{6}$ red, $\frac{2}{3}$ blue, $\frac{1}{6}$ green; $\frac{1}{4}$ circles, $\frac{1}{2}$ squares, $\frac{1}{4}$ hexagons. The smallest set that matches these fractions is 12 blocks: 2 red, 8 blue, 2 green; 3 circles, 6 squares, and 3 hexagons.

2. a. Marama is 10 yrs old. Explanations may vary. One possible way to work this out is: $\frac{1}{4}$ plus $\frac{1}{4}$ is $\frac{2}{4}$ or $\frac{1}{2}$, so Marama's 5 yrs in Auckland is also $\frac{1}{2}$. If $\frac{1}{2}$ is 5 yrs, then $\frac{1}{4}$ is $2\frac{1}{2}$ yrs. $2\frac{1}{2} + 2\frac{1}{2} = 5$, so Marama is 2 x 5 = 10 yrs old.

   b. 8 yrs old. ($\frac{1}{4} = \frac{2}{8}$. $\frac{1}{8} + \frac{2}{8} = \frac{3}{8}$. $\frac{3}{8} + \frac{5}{8} = 1$ whole, so the 5 years in Napier is $\frac{5}{8}$ of Sio's age and 1 yr = $\frac{1}{8}$. 5 + 1 + 2 = 8)

3. Fraction challenges will vary.

Figure It Out

Multiplicative Thinking
Teachers' Notes

Overview: Level 3

Introduction to Multiplicative Thinking

Multiplicative thinking is the term used to describe thinking that employs the mathematical power of multiplication. Students need to use the properties of multiplication in order to understand many areas of mathematics, such as area and volume, the metric measurement system, fractions, and algebra.

The Number Framework outlines the way that children seem to build their multiplicative understandings on their additive ones, just as they previously built additive understandings out of their knowledge of counting. Understanding multiplication as repeated addition seems to be an important step for many students and is often a starting point for instruction.

Many problems are multiplicative, with the most commonly used strategy being repeated addition or equal groups. In these problems, an amount can be added repeatedly to solve the problem.

However, we also need to include other models of multiplication to extend and deepen students' understanding. The Figure It Out *Multiplicative Thinking* books include activities that explore multiplication as a comparison, a rate, an array, and a Cartesian product. From each of these problem types, students will begin to extract the key ideas of multiplication as commutative, distributive, and associative. Understanding how multiplication works helps students to use the strategies outlined in the Numeracy Development Project booklets and in these Figure It Out books.

Division is the inverse of multiplication in the same way that subtraction is the inverse of addition. Traditionally regarded as the most difficult of the operations to understand and perform, division, research suggests, is best learned alongside multiplication. Situations involving division can sometimes be solved using multiplication, particularly when the numbers are in the basic facts range. Recognising the relationship between multiplication and division is a very important task for students.

To work effectively with multiplicative strategies, students need to know their basic facts. In the initial stages, they will use skip-counting and materials to develop a conceptual understanding of what multiplication statements mean. This emerging concept feeds into, and is fed from, knowledge of basic facts. Recall of basic multiplication facts does not equate with multiplicative thinking, but multiplicative thinking will be facilitated by a sound knowledge of these facts.

A change in students' thinking is needed if they are to fully understand multiplication. In multiplication, they encounter for the first time numbers that are telling them to perform an operation rather than numbers that represent an amount. For example, in 4×3 (which can be read as four sets of 3 or as three sets of 4), one number tells how many times the other should be repeated rather than representing an amount (a set of 4 plus a set of 3). However, solving multiplication problems additively like this can be unwieldy; students who continue to use this method will have problems with more sophisticated concepts.

The activities in the Figure It Out *Multiplicative Thinking* books use a range of problem types, consider strategies based on the properties of multiplication, ask students to explore the properties of multiplication, and include division as the inverse of multiplication. In doing so, the books aim to support teachers in developing a robust and deep understanding of multiplication in their learners.

Page 1: Train Teasers

Achievement Objectives

- recall the basic multiplication facts (Number, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity:

- in a guided instruction situation, to practise multiplication by repeated addition (stage 5)
- to develop your students' ability to derive unknown multiplication facts by applying their addition and subtraction strategies to multiplication facts they already know (stage 6)
- as an independent activity for students already using advanced additive strategies (stage 6).

Activity

You can use this activity to encourage your students to use a combination of known facts and mental strategies to derive answers to multiplicative problems. Key mathematical concepts to focus on are:

- multiplication and division involve making equal groups and then working with those groups to speed up counting
- unknown multiplication facts can be derived from facts that students already know, by applying addition and subtraction strategies. Students need to know their 5 times and 10 times basic facts in order to use stage 6 thinking and derive answers from those facts in this activity.

Vocabulary that you may need to discuss when introducing the activity:

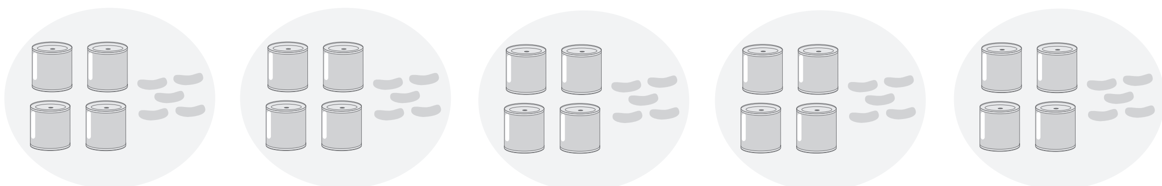
- commuter: a person who travels some distance to and from work
- peak times: busiest times
- identical: exactly the same.

For question **1**, encourage students who are transitioning to stage 6 to look for alternative ways of grouping and counting 3 x 15 rather than just skip-counting in threes or adding 15 + 15 + 15. This could be done by using counters and blocks to represent the people on each seat and then arranging and grouping them in different ways to make them quicker to count or by using place value materials such as canisters and beans.

Possible multiplicative strategies include:

- working out 3 lots of 15 rather than 15 lots of 3. This requires students to understand that changing the order of the factors gives the same result (the commutative property of multiplication).
- breaking up the 15 seats into smaller groups and then using known facts to work out those groups: (3 x 10) + (3 x 5) = 30 + 15 = 45. This strategy requires students to use the distributive property of multiplication over addition: the sets of 15 are distributed into sets of 10 and 5.

Question **3** provides another opportunity for your students to work multiplicatively rather than additively; in this case, viewing the problem as 5 x 45 rather than 45 + 45 + 45 + 45 + 45. You can encourage this by asking them to think of alternative ways they could break up the groupings of 45 to make them quicker to count. Place value materials could be used to represent the 5 lots of 45:



Possible multiplicative strategies include:

- Using place value partitioning to split the 45 into 40 + 5 and then working out (5 x 40) + (5 x 5). The students could use known facts to solve 5 x 40. You could prompt them by asking: *There are 5 groups of 4 canisters. How many canisters are there altogether? How many beans would there be in 20 canisters (20 tens)?*
- Breaking 45 into 10 + 10 + 10 + 10 + 5 and then using easier multiplication facts to solve the problem: (5 x 10) + (5 x 10) + (5 x 10) + (5 x 10) + (5 x 5).

How your students solve question **4** will depend on what basic facts that they already know. Prompt them to use those facts by asking: *Do you know any multiplication facts that have 40 and 8 in them? How could you use that fact to solve this problem?* Students who do not know that 40 ÷ 8 = 5 or 5 x 8 = 40 may need to use repeated subtraction and/or materials. Encourage these students to make the link between repeated subtraction and division by discussing the meaning of the ÷ symbol ("share into sets/groups of") and asking them how they could record what they did by putting numbers into the boxes: □ people shared into □ groups makes □ in each group, □ ÷ □ = □ (for example, 40 ÷ 8 = 5).

For question **5**, encourage the students to make up both "groups of" (multiplication) problems and "sharing" (division) problems. Remind them that they have to be able to solve and explain the solutions to their problem themselves. This makes it unlikely that they will use numbers that are unfriendly for mental calculations and numbers that cannot be shared without remainders.

Links

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 4: Teaching Number Knowledge*
  Dividing? Think about Multiplying First (using multiplication facts to solve division problems), page 37
- *Book 6: Teaching Multiplication and Division*
  Biscuit Boxes (using repeated addition to solve division problems), page 8
  Twos, Fives, and Tens (deriving unknown facts from known 2 times, 5 times, and 10 times facts), page 9
  Fun with Fives (deriving unknown facts from known 5 times facts), page 12
  Turn Abouts (recognising that 3 x 5 gives the same result as 5 x 3; the commutative property), page 17
  Long Jumps (solving division problems using repeated addition and known facts), page 19
  Goesintas (solving division problems using multiplication), page 20

Pages 2–3: The Hundreds Board Hunt

Achievement Objective

- recall the basic multiplication facts (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to develop your students' knowledge of multiples of 3. They can use this knowledge to help them devise multiplicative part–whole strategies (stages 6–7).

Activity

In this activity, students colour a hundreds square to explore patterns in the 3 times table. They then discover that if they add the digits of any multiple of 3, the result is a number that is also divisible by 3. (For example: 54 is a multiple of 3, and 5 + 4 = 9, which is divisible by 3.)

For large numbers, they may need to repeatedly add the digits until they get a single digit, known as the *digital root* of that number. (For example, the digital root of 756 is 9 because 7 + 5 + 6 = 18 and 1 + 8 = 9, which is divisible by 3.)

Before the activity, discuss what multiples are. (You can use the definitions and examples on the student page.) A good introduction is to play a game of Yes or No, where the students decide whether given numbers are multiples of 5 or not. Talk about what you would expect to show up on the calculator if you entered ☐ ÷ 5 = and if ☐ was a multiple of 5. (The calculator would show a whole number.) Compare this to the result if ☐ wasn't a multiple of 5. (The calculator would display a decimal fraction.) (See the Numeracy Development Project [NDP] activity, In and Out, page 36, *Book 4: Teaching Number Knowledge*.)

Nines and Threes (*Book 6: Teaching Multiplication and Division,* page 38) uses place value materials to help students understand why it is that the digit total of a multiple of 3 is also a multiple of 3.

Extend the activity by investigating other divisibility rules. For stage 7:

- multiples of 2 have 0 or an even number in the ones place
- multiples of 5 have 0 or 5 in the ones place
- for multiples of 9, if you keep adding the digits in the multiple until you get a single digit, it will be 9 (this is its digital root), for example, 693: 6 + 9 + 3 = 18, 1 + 8 = 9
- multiples of 10 have 0 in the ones place.

For stage 8:

- multiples of 4 are always even, and if you look at only the tens and the ones places, that number will be a multiple of 4; for example, 1 328 is a multiple of 4 because 28 is divisible by 4. This works because all digits in the hundreds place and above represent multiples of 100, which is itself divisible by 4.
- multiples of 6 are always even and have a digital root that is divisible by 3 (that is, they are divisible by both 2 and 3)
- multiples of 8 are always even, and if you look at only the hundreds, tens, and ones places, that number will be a multiple of 8. For example, 7 624 is a multiple of 8 because 624 is divisible by 8. This works because all digits in the thousands place and above are multiples of 1 000, which is itself divisible by 8.

Links

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 4: Teaching Number Knowledge*
  In and Out, page 36
- *Book 6: Teaching Multiplication and Division*
  Nines and Threes (tests for multiples of 3 and 9), page 38
- *Book 8: Teaching Number Sense and Algebraic Thinking*
  Divisibility Tests (tests for multiples of 2, 3, 5, and 9), page 33
  Factor Trees (using factor trees to find all the prime factors of a number), page 33

Figure It Out

- *Basic Facts,* Level 3
  Digital Delights (digital sums and multiples of 5, 6, 7, and 8), page 13

www.nzmaths.co.nz

- Guzzinta (divisibility rules unit), www.nzmaths.co.nz/number/Operating%20Units/guzzinta.aspx

Pages 4–6: Problems on the Way to School

Achievement Objectives

- recall the basic multiplication facts (Number, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity:

- in a guided instruction situation, to practise multiplication by repeated addition (stage 5)
- to develop your students' ability to derive unknown multiplication facts by applying their addition and subtraction strategies to multiplication facts they already know (stage 6)
- as an independent activity for students already using advanced additive strategies (stage 6).

Activity

In this activity, students solve multiplicative problems drawn from everyday contexts.

Key mathematical concepts to focus on are:

- multiplication and division involve making equal groups and then working with those groups to count more quickly
- by applying addition and subtraction strategies, unknown multiplication facts can be derived from facts that students already know.

Working out what mathematics is needed to solve a word problem can be challenging for students. As a first step, encourage the students to discuss what maths they will need, and as a second step, ask them to write what the problem is asking as a number sentence, for example, $14 \times 3 = \square$ or $120 \div 60 = \square$.

The students will need to work out how many animals there are in question **1** from the fact that they have 18 eyes. An extension question could be to find out how many claws there could be. (Dogs have 5 claws on each paw [although often the small dew claw on each back foot is removed for safety reasons] and cats have 5 claws on each front paw and 4 claws on each back paw.) Ask: *You don't know exactly how many cats and dogs there are, only that there are 9 in total: what's the least and most number of claws there could be?*

For question **2**, encourage students who are transitioning to stage 6 to look for multiplicative ways of grouping and counting 14 x 3 rather than just skip-counting in threes or adding 14 + 14 + 14. Using place value materials such as canisters and beans to represent 3 x 14, the students may be able to find alternative ways of grouping by moving the materials into different arrangements.

Possible multiplicative strategies include:

- working out 3 lots of 14 rather than 14 lots of 3. This involves understanding that changing the order of the factors gives the same result (the commutative property of multiplication).
- breaking the 14 into smaller groups and using known facts to work with those smaller groups: $(3 \times 10) + (3 \times 4) = 30 + 12 = 42$. This strategy involves using the distributive property of multiplication over addition. The sets of 14 are distributed into sets of 10 and 4.

- halving one factor and doubling the other to create a known fact: $14 \times 3 = 7 \times 6 = 42$
- multiplying by a tidy number and then compensating: working out 15 lots of 3 and then subtracting 1 lot of 3: $(15 \times 3) - (1 \times 3) = 45 - 3 = 42$.

Talk about which parts of the fence in question **3** are posts and which are palings. Encourage students who are transitioning to stage 6 to look for multiplicative ways of solving the problem rather than just skip-counting in 4s. One way to promote this is to record the numbers of posts and palings as $1 + 4 + 1 + 4 + 1 + 4 + 1 + 4 + 1 + 4 + 1 + 4 + 1 + 4 + 1 + 4 + 1$ = total number of posts and palings. Ask the students:

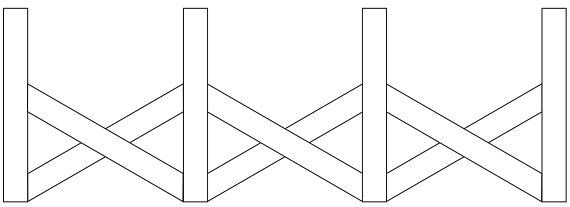
- *If you know that there are 9 posts (the 1s), how many groups of 4 palings would there be?* (8)
- *If the fence was shorter and there were only 5 posts, how many groups of 4 palings would there be?* (4)
- *If the fence was longer and there were 16 posts, how many groups of 4 palings would there be then?* (15)
- *What pattern can you see that helps you work out how many groups of 4 palings there are if you know how many posts there are?* (There is always one fewer group of 4 palings than there are posts.)

Possible multiplicative strategies for working out 8×4 could be:

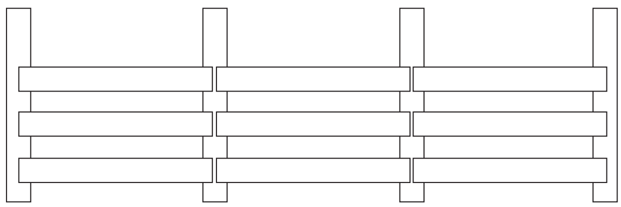
- breaking the 8 into smaller groups and using known facts to work with those smaller groups, for example: $(4 \times 4) + (4 \times 4) = 16 + 16 = 32$
- doubling repeatedly: $2 \times 4 = 8$, $4 \times 4 = 16$, $8 \times 4 = 32$
- using a known fact and compensating: $(8 \times 5) - (8 \times 1) = 40 - 8 = 32$.

Promote algebraic thinking by asking how many palings there would be if the fence had 6 posts ($5 \times 4 = 20$ palings) or 21 posts ($20 \times 4 = 80$ palings).

Extend the question by using iceblock sticks to create different fence structures. Ask the students: *If you had □ (a given number of) posts, how many palings would you need? How could you work this out without having to build the whole fence?*



$(4 - 1) \times 2 = 3 \times 2 = 6$ palings



$(4 - 1) \times 3 = 3 \times 3 = 9$ palings

In question **4**, some students may notice a link between $120 \div 60 = 2$ and the basic fact $12 \div 6 = 2$. Ask them to investigate whether this is true for similar examples, such as $150 \div 30 =$ □ and $200 \div 50 =$ □.

Extend part **b** of the problem by asking *What about if you counted a spare tyre for each car as well?* ($7 \times 5 = 35$)

The students could use counters to help solve question **5**. You could introduce the term "factor": a number that is multiplied by another number to create a product. (See also the activity on pages **16–17**.) If you divide a number by one of its factors, there will be no remainder. All the equal-sized groups must be factors of 24, except that there are no children walking by themselves.

Encourage your students to use the multiplication facts they know to help them solve the problems in question **6**. Challenge them to find alternatives to skip-counting by asking them to find another way to solve each problem and then to share and compare solution paths with the group.

Students who don't know their 7 times table could solve 7×3 by reversing it to become a known fact, 3×7. However, you could also use 7×3 to demonstrate a multiplicative strategy that is useful for more difficult multiplications: that is, solving 7×3 by breaking the 7 down into smaller groups

to create known facts and then adding the products together, for example:
$7 \times 3 = (3 \times 3) + (3 \times 3) + (1 \times 3) = 9 + 9 + 3 = 21$.

To solve 33 ÷ 3, prompt the students to use relevant facts they may know by asking: *Do you know any multiplication facts that have 33 and 3 in them? How could you use that fact to solve this problem?* Students who do not know that 33 ÷ 3 = 11 or 3 x 11 = 33 might use repeated subtraction to solve the problem, keeping track of how many groups of 3 they subtract, or they may need to use materials. Encourage these students to make the link between repeated subtraction and division by discussing the meaning of the ÷ symbol ("put into sets/groups of") and asking them how they could record what they did by putting numbers into the boxes in the following: ☐ bins in groups of ☐ makes ☐ groups, ☐ ÷ ☐ = ☐ (33 ÷ 3 = 11).

To solve the problems in question **7**, students need to recognise the links between finding half of something and dividing by 2 and finding a third of something and dividing by 3.

If your students have difficulty halving or thirding 3 066, ask them how they could break it up to make the calculations easier, perhaps by using place value partitioning to find half or a third of 3 000 and then of 66 or by breaking it up further to work with 3 000 + 60 + 6.

Question **8** asks students to make up their own problems. To avoid impossibly difficult problems being created, remind the students that they need to be able to solve (and explain the solutions to) all the problems on their map.

Links
Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 4: Teaching Number Knowledge*
  Dividing? Think about Multiplying First (using multiplication facts to solve division problems), page 37
- *Book 6: Teaching Multiplication and Division*
  Biscuit Boxes (using repeated addition to solve division problems), page 8
  Fun with Fives (deriving unknown facts from known 5 times facts), page 12
  Turn Abouts (recognising that 3 x 5 gives the same result as 5 x 3; the commutative property), page 17
  Long Jumps (solving division problems using repeated addition and known facts), page 19
  Goesintas (solving division problems using multiplication), page 20

Page 7: Card Arrays

Achievement Objectives

- recall the basic multiplication facts (Number, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

These problems can be solved using stage 4 through to stage 6 strategies. They are particularly suitable for encouraging transition from stage 5 to stage 6 thinking in the domain of multiplication and division. Students will need a reasonable knowledge of multiplication basic facts in order to use number properties and factors to solve the problems rather than always relying on manipulating the cards or skip-counting (stage 4).

Activity

This activity is intended to develop the concept of multiplication as an array. A key idea underpinning the questions is that of factors. The activity is suited to a co-operative problem-solving approach, with students working in small groups of 3–4 and then reporting back and comparing ideas and solutions with other groups.

Encourage the students to use vocabulary such as "arrays" and "factors" by defining them together, asking for examples, and including the words in your questions and discussion.

Definitions:

- Factor: a number that is multiplied by another number to create a product. If you divide a number by one of its factors, there will be no remainder. (See the activity on pages **16–17**.)
- Array: an ordered arrangement of objects by rows and columns.

Some students may need to learn how a deck of cards is organised before beginning the activity. A completed game of Patience both sorts the cards and emphasises the organisation of the pack.

For question **1b**, ask the students: *How do the arrays you have drawn relate to the factors of 40? Can you use your drawings to make a list of all the factors of 40?* (1, 2, 4, 5, 8, 10, 20, and 40)

The answers for question **2c ii** describe a process for finding all the factors of 40. The same process can be used to find all the factors of 52 (or any other number). Have the students compare the factors of 40 with those of 52. Extend the question by asking: *Can you use what you know about factors to predict which of these numbers of cards would give you the most rectangles?*
12 or 17 cards? (12) *15 or 11*? (15) *12 or 18?* (same)

For question **3**, encourage students who have a good knowledge of multiplication basic facts to use number properties before checking with their cards or drawings. Ask: *How could listing all the factors of 24 help you to describe all the possible rectangles that could be drawn?*

Encourage the students working on question **4** to use imaging or number properties to connect the division operation to arrays. Ask: *If you know there are 11 cards across one row, how many rows do you think there will be? Are there any multiplication facts that could help you with this problem?*
(4 x 11 = 44)

Extend the question by getting each student in the group to make up their own "How long is the other side of the array?" problem to share with the group. Remind them that they must be able to solve their own problem and explain the solution. This would show you which students are able to apply their knowledge of basic facts to arrays and division problems.

After they have solved the problem in question **5**, extend it by telling the students: *The numbers of cards you used in your square arrays can be called "square numbers". The highest square number you got was 49 (an array with sides 7 by 7). What would the next three square numbers be if you had more cards?* (8 x 8 = 64, 9 x 9 = 81, 10 x 10 = 100)

After the activity

Encourage reflective thinking and the discussion of generalisations by asking:

- *How did knowing about arrays help with the problems in this activity?*
- *How did knowing about factors help with this activity?*
- *What advice and helpful tips could you give someone about to start this activity?*

Links

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 6: Teaching Multiplication and Division*
  Animal Arrays (solving multiplication problems using arrays), page 6
- *Book 8: Teaching Number Sense and Algebraic Thinking*
  Squaring, page 28, and Square Roots, page 29 (relating the area of squares to square numbers and square roots)
  Prime Numbers (finding primes by representing numbers as rectangular arrays), page 32

Figure It Out

- *Basic Facts,* Levels 2–3
  An Apple a Day (relating arrays to multiplication), page 9
- *Basic Facts,* Level 3
  Factor Puzzles (using basic facts to identify factors), page 11
  Stars and Students (game using multiplication facts to identify factors), page 12

- *Basic Facts*, Levels 3–4
  Matrix (finding factors), page 10
  A Matter of Factor (factor game), page 12
  How Many Factors? (factor investigation), page 15
  Primates (finding prime factors), page 22

www.nzmaths.co.nz
Arrays Hooray (exploring multiplication concepts through arrays),
www.nzmaths.co.nz/number/Operating%20Units/Arrays.aspx

Pages 8–9: Bean Counters

Achievement Objectives

- recall the basic multiplication facts (Number, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to promote the development of multiplicative part–whole strategies (stages 6–7) in the domain of multiplication and division. It is suitable for students transitioning to stages 6 or 7.

Activity

In this activity, students solve multiplication problems by multiplying by a tidy number and then compensating. This strategy works well for problems where one of the two numbers involved is close to a tidy number, for example, 4 x 38 or 197 x 3. It's easier to work out 4 x 40 than 4 x 38, but we must then compensate because we've got 4 groups of 2 more than we need. We compensate by subtracting the extras:

$$\begin{aligned} 4 \times 38 &= (4 \times 40) - (4 \times 2) \\ &= 160 - 8 \\ &= 152 \end{aligned}$$

It's important that students can use a broad range of multiplicative strategies. They can then select the most efficient strategy for a particular problem rather than always relying on one familiar method. Encourage your students to use Aronui's method as they work through the activity, even if they want to solve the questions in other (more familiar) ways. If they wish, they can use their existing strategies to check their answers. Once the students have acquired a number of strategies that they can use flexibly, they need to learn how to pick the one that will be most useful and efficient for a particular problem. You can help them develop this skill by getting them to discuss and compare different strategies and activities such as Multiple Multiplication Methods on pages **12–13**.

This activity could be used as a guided teaching session with a group. Align it with the strategy teaching model from the NDP, encouraging your students to use materials, then imaging, and finally number properties. Alternatively, it could be used as an independent activity for complementing and following up the learning experiences covered in the NDP activities, A Little Bit More/A Little Bit Less, page 15, and Multiplication Smorgasbord, page 27, in *Book 6: Teaching Multiplication and Division.*

When they encounter question **1**, some students may not yet have made the connection between their basic facts and how they can be applied to multiples of 10. Emphasise this link by saying "2 tens" rather than "twenty" when discussing how the knowledge that 5 x 2 = 10 could be useful when working out 5 x 2 tens. The word "twenty" does of course mean "2 tens", but this is not necessarily obvious. See the NDP links at the end of the notes for this activity for learning experiences that help students decode English number words such as "-teen" and "-ty" (which both mean "ten").

To promote generalisations, ask questions such as *How does knowing 5 x 7 = 35 help you to work out 5 x 70 or 50 x 7 or 50 x 70?* Have the students check their predictions on a calculator and talk about patterns they can see. Ask them to try to generalise a rule.

Encourage the students to manipulate place value materials to solve the problems in questions **2** and **3**. Bundles of 10 iceblock sticks could be used in place of beans and canisters.

Promote imaging for question **4** by asking students to describe what they would do with the canisters to solve 3 x 29 using Aronui's method. Expect responses such as: "I would have 3 groups of 3 canisters, so there would be 30 beans in each group. I'd take 1 bean out of each group and lay it on the table, leaving 29 beans in the canisters. To know how many beans I had to start with, I'd work out 3 x 3 tens is 9 tens, so I had 90 beans altogether. I took out 3 lots of 1 bean, or 3 beans. 90 beans take away 3 beans is 87 beans." Go back to using materials if needed.

Use question **5** as a formative assessment opportunity to see if the students are able to use number properties to solve the problem using the tidy numbers and compensating strategy. Go back to imaging if needed by asking the students to describe what the materials would look like and what they would do with them to solve the problem.

Before the students write their own problems, talk about the sorts of numbers that this strategy is useful for in order to encourage them to see this strategy as one tool in their mental toolbox of strategies.

Ask them to compare the use of this strategy to solve the two problems 7 x 34 and 7 x 199.
Ask *Why is this strategy easier to use with the second problem?*

Look back at the problems in the activity and record them in a list: 5 x 17, 3 x 18, 3 x 29, 5 x 98.
Ask *What is similar about all of these problems?* (One of the numbers is a single digit, and the other one is close to a tidy number.)

Extension
Encourage the students to name the strategy with a descriptive name (such as using tidy numbers and compensating). Create a display that they can use as a reference by getting them to write in their own words the sorts of problems this strategy is most efficient for. Each time the group learns a new strategy, add it to the display, which can then be used to remind students of alternative strategies that they could try. In discussions, expect the students to justify the selection of a strategy to suit a particular problem.

The metaphor of a toolbox can be very useful. Write the name of each strategy on a picture of a tool, such as a hammer or a saw, and discuss how different tools (strategies) are suited to different situations and are able to be used in combination with each other to do a task. This metaphor can be used to justify the flexible use of a broad range of strategies: although you could probably do many jobs with a screwdriver if you really persisted (perhaps even banging in a nail), it's much more efficient to have a hammer when you need one.

Links
Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)
- *Book 4: Teaching Number Knowledge*
  "Teen" and "Ty" Numbers (identifying them), page 3
  Tens and Ones (using place value material to show 2-digit numbers), page 23
- *Book 6: Teaching Multiplication and Division*
  Multiplying Tens (multiplying tens, hundreds, thousands, and other tens numbers), page 14
  A Little Bit More/A Little Bit Less (using known facts and compensating), page 15
  Multiplication Smorgasbord (using a variety of strategies), page 27

Figure It Out
- *Number Sense and Algebraic Thinking: Book 1,* Level 3
  What's Best? (assessing and using different strategies to solve multiplication problems), page 6

Pages 10–11: Sticking Together

Achievement Objectives

- recall the basic multiplication facts (Number, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to promote the development of advanced multiplicative part–whole strategies (stage 7) in the domain of multiplication and division. It is suitable for students transitioning from stage 6 to stage 7.

Activity

The aim of this activity is to explore the strategy of partitioning a factor in a multiplication problem to make it easier to solve. This is most often done using place value to break the number into smaller chunks.

To solve 4 x 132 using this strategy, 132 is partitioned into 100 + 30 + 2, each of these parts is multiplied by 4, and the products added together:

$$\begin{aligned} 4 \times 132 &= (4 \times 100) + (4 \times 30) + (4 \times 2) \\ &= 400 + 120 + 8 \\ &= 528 \end{aligned}$$

See the notes for Bean Counters (page 21) for a rationale for students learning and using a broad range of multiplicative strategies and for links to the NDP strategy teaching model.

Encourage the students to manipulate place value materials to solve the problems in questions **1** and **2**. Beans and canisters or tens money ($1 coins and $10 notes) could be used in place of bundles of sticks. Watch that the students are seeing the bundle of 10 as a unit rather than treating them as 10 single sticks.

Challenge students who simply skip-count the bundles in the picture (10, 20, 30, …) to come up with an alternative way of finding out how many sticks there are in the bundles, using their multiplication facts as Paul does.

Some students may not have made the connection between their basic facts and how they can be applied to multiples of 10 or 100. Emphasise this link by saying "2 tens" rather than "twenty" when talking about how knowing 4 x 2 = 8 might help Paul with working out 4 x 2 tens. "Twenty" does mean "2 tens", but this is not necessarily obvious. See the NDP links at the end of the notes for this activity for learning experiences that help students to decode English number words such as "-teen" and "-ty" (which both mean "ten").

To promote generalisations, ask questions such as *How does knowing 5 x 7 = 35 help you to work out 5 x 70 or 50 x 7 or 50 x 70?* Have the students check their predictions on a calculator and talk about patterns they can see. Ask them to try to generalise a rule.

Promote imaging for questions **3–4** by asking the students to describe what they would see and do with the sticks to solve 5 x 32 using Paul's method. Expect responses such as: "I would have 5 groups, and each group would have 3 bundles and 2 single sticks. I'd find out how many sticks there are in the bundles by working out 5 groups of 3 tens, which is 15 tens or 150. Then I'd find out how many single sticks there are by working out 5 groups of 2, which is 10. So the total number of sticks in the bundles and singles is 150 + 10, which is 160." Go back to using materials if needed.

Use question **5** as a formative assessment opportunity to see if your students are able to use number properties to solve the problem using place value partitioning. Go back to imaging if needed by asking the students to describe what the materials would look like and what they would do with them to solve the problem. 100 sticks can be shown by putting 10 bundles of 10 in a ziplock bag or an ice cream container.

Use question **6** as a lead-in to the students writing their own problems in question **7**. Talk about the sorts of numbers that Paul's strategy is useful for, encouraging the students to see this strategy as one tool in their toolbox of strategies to be used as needed.

After the students have worked on question **6**, ask: *Even though 898 is a smaller number than 2 153, it was more complicated to solve 4 x 898 using this strategy than it was to solve 4 x 2 153. Why was this?* (For 4 x 898, there were several numbers that needed renaming when adding, so it was harder to keep track of the mental calculations.)

Look back at the previous problems in the activity and record them in a list: 4 x 23, 3 x 42, 5 x 32, 8 x 21, 3 x 152. Ask *What is similar about all of these problems?* (One of the numbers is a single digit, and the other uses only small digits.)

Extension
See the strategy toolbox reference display idea outlined in the notes for "Bean Counters" (page 22).

Links
Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 4: Teaching Number Knowledge*
  "Teen" and "Ty" Numbers (identifying them), page 3
- *Book 5: Teaching Addition, Subtraction, and Place Value*
  Ones and Tens ("-ty" numbers with place value materials), page 11
- *Book 6: Teaching Multiplication and Division*
  Multiplying Tens (multiplying tens, hundreds, thousands, and other tens numbers), page 14
  Multiplication Smorgasbord (using a variety of strategies), page 27

Figure It Out

- *Number Sense and Algebraic Thinking: Book 1,* Level 3
  What's Best? (assessing and using different strategies to solve multiplication problems), page 6

Pages 12–13: Multiple Multiplication Methods

Achievement Objective

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links
Use this activity to help students consolidate and compare advanced multiplicative part–whole strategies (stage 7) in the domain of multiplication and division.

Activity

See the notes for Bean Counters (page 21) on the reasons for students having and using a broad range of multiplicative strategies.

For this activity, students need to be already familiar with the following multiplicative strategies:

- Place value partitioning: partitioning a number (using place value), multiplying the parts separately, and then adding the products.
  5 x 48 = (5 x 40) + (5 x 8)
  = 200 + 40
  = 240
  (see Sticking Together, pages **10–11**)
- Using tidy numbers and compensating: working out 5 x 50 instead of 5 x 48 because 50 is a tidy number and easy to multiply. But then we have to compensate because we have got 5 groups of 2 more than we need, so we subtract them again.
  5 x 48 = (5 x 50) – (5 x 2)
  = 250 – 10
  = 240
  (see Bean Counters, pages **8–9**)
- Doubling and halving (making proportional adjustments): doubling one factor and halving the other gives you the same answer and may make the calculation easier.
  5 x 48 = 10 x 24
  = 240
  (see Face the Facts in *Multiplicative Thinking*, Figure It Out, Levels 2–3, page 6).

This activity is suited to a guided teaching session with a group because there are lots of opportunities for discussion. Introduce the activity by getting your students to solve Mr Mannering's problem before they look at how the students in the book solve it. Discuss the different strategies your group used and use this discussion as an opportunity to remind the students of the meaning of the terms "place value", "tidy numbers", and "doubling and halving".

The aim of this activity is to help the students learn to make wise decisions about which strategy to use in a particular situation. In order to do this, they need to be able to look at a problem before they solve it and identify features that would prompt them to use one strategy rather than another. Promote generalisations by asking questions such as:

- *Look at the problem before you start working it out; which strategy do you think would be most efficient here?*
- *What is it about this problem that makes you want to use that strategy?*
- *List all the problems in the activity and group them according to the strategy you thought was most efficient for each problem.* (It may be valid to have some problems in two groups if both strategies are equally efficient for those problems.)
- *What do all the problems in each group have in common?*

Useful responses will include these ideas:

Doubling and halving: This strategy is useful if it can change the calculation into a known fact or one that is easy to work out, such as 10 times. At least one of the factors should be even so that halving it gives us a whole number.

Tidy numbers and compensating: One of the factors has to be close to a tidy number so there isn't too much compensating to do.

Place value partitioning: This strategy suits a wide range of problems, but it can be difficult to keep track of if there's a lot of renaming to do when you're adding up the parts. It's easier to use if the digits are small.

Extension

Students could create a flow diagram that shows what they would look for when confronted with a problem and how their strategy decision-making process might progress. They could start by asking "Is the answer to this problem a known fact?" and then ask questions such as "Is the problem

5 times or 50 times something?" If the answer is yes, "Use doubling and halving to create a 10 times or 100 times problem and solve it"; if the answer is no, "Would doubling and halving the problem turn it into a known fact?" and so on.

Links

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 6: Teaching Multiplication and Division*
  A Little Bit More/A Little Bit Less (using known facts and compensating), page 15
  Cut and Paste (halving and doubling, thirding and trebling), page 25
  Multiplication Smorgasbord (using a variety of strategies), page 27

Figure It Out

- *Number Sense and Algebraic Thinking: Book 1,* Level 3
  What's Best? (assessing and using different strategies to solve multiplication problems), page 6

Pages 14–15: Rubber Band Rectangles

Achievement Objectives

- perform measuring tasks, using a range of units and scales (Measurement, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to give students opportunities to apply multiplicative part–whole strategies (stage 6).

Activity

In this activity, students use multiplication to solve area and perimeter problems. A co-operative problem-solving approach could be used, with the students working in small groups of 3–4 and then reporting back and comparing ideas and solutions with other groups. Square Skills (page **22**) complements this activity.

Discuss these key words with your students:

- Perimeter: the distance around the outside of a shape (imagine an ant walking around it).
- Area: the surface that a shape covers, measured in squares (imagine a squared blanket covering the floor).

Introduce question **1** by giving each student a geoboard and a rubber band and asking them to make a rectangle with their rubber band. Then ask:

- *How many geoboard squares are there inside your rectangle?*
- *What's a quick way of counting the squares that the area covers, instead of counting one by one? How could you use multiplication to make it quicker?*

Expect responses such as: "I can count how many squares are along the bottom row and multiply that by the number of rows. If there are 3 rows of 4 squares, that's 12 squares."

For question **2**, encourage the students to find a way of working out the perimeter that uses multiplication rather than simply adding the 4 sides. Expect responses such as: "I look at how many squares there are in 1 row and how many rows there are. I add those two numbers together, and then I double the total."

When they are doing questions **3–4**, the students may not realise that a square is also a rectangle and so may not include a 6 by 6 square in their list. If they haven't encountered this idea before, have them try to define the qualities of a rectangle and then discuss with them whether a square also has these qualities. It does, so a square is a special sort of rectangle: one with all its sides the same length, just as an equilateral triangle is a special sort of triangle with all its sides the same length.

Once the students have completed their tables, ask:

- *What patterns can you see in your tables? If you add each width and length, what do you notice?* (They always add up to 12 with the 24-unit string and 18 for the 36-unit string. This is half of the perimeter in each case.)
- *Could you use this pattern to predict all the possible combinations of length and width if you made rectangles with a 14-unit piece of string?* (Each pair would add up to half of 14, which is 7. The possibilities are 1 and 6, 2 and 5, and 3 and 4.)

Extend the problem by asking *Could you make a geoboard rectangle with a perimeter length that is an odd number of units?* (No, because you can't make a corner halfway between 2 nails. It would, however, be possible to draw such a rectangle.)

A key idea underpinning question **5** is that of factors. A factor is a number that is multiplied by another number to create a product. If you divide a number by one of its factors, there will be no remainder. (See also Multiples and Factors, pages **16–17**.) Finding all the different rectangles that have an area of 16 squares involves using all the pairs of factors of 16: namely, 1 and 16, 2 and 8, and 4 and 4.

After the students have completed **5a** and **5b**, talk about factors and define them together and then ask:

- *What are the factors of 16?*
- *How do they relate to the rectangles that you drew with an area of 16 squares?*
- *How could factors help us to find all possible rectangles with an area of 24 squares?* (You could list all the factors and then pair them up with another factor so that each pair multiplies to make 24: namely, 1 by 24, 2 by 12, 3 by 8, and 4 by 6.)
- *How could factors help us to find all possible rectangles with an area of 36 squares?* (1 by 36, 2 by 18, 3 by 12, 4 by 9, and 6 by 6)

For **5c** and **5d**, ask: *Given that these rectangles all have the same area, what do you notice about the rectangles with shorter perimeters compared with those that have longer perimeters?* (They are closer to being squares. A square rectangle has the smallest perimeter for any rectangle with the same area.) This is an important and interesting mathematical idea.

Extend the problem in question **6** by getting the students to draw two more rectangles in which the one with the smallest area has the longest perimeter. (Rectangles with a shorter perimeter compared to their area will look squarer, while rectangles that are long and thin have longer perimeters compared to their area.) Interested students may like to see how far they can push this idea: "What is the greatest length the long side of the rectangle could have?" The answer introduces the important idea of a limit.

Links

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 8: Teaching Number Sense and Algebraic Thinking*
  Squaring (relating squaring numbers to square shapes), page 28
  Square Roots (using square roots to find the length of the side given the area of a square), page 29
  Prime Numbers (representing prime and other numbers by arranging tiles into rectangles), page 32
- *Book 9: Teaching Number through Measurement, Geometry, Algebra and Statistics*
  Investigating Area (comparing and measuring areas), page 11

Pages 16–17: Multiples and Factors

Achievement Objectives

- recall the basic multiplication facts (Number, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to develop knowledge about factors and multiples to complement advanced multiplicative part–whole strategies (stage 7).

Activity

In this activity, students identify multiples and factors and solve problems that involve finding highest common factors and lowest common multiples. Students will need a good recall of multiplication basic facts in order to be able to do these activities.

Activity One and Game

Discuss the definitions of multiples and factors (on the student book page) before your students begin this activity. Make sure that they understand that every number is a factor of itself, because if they divide a number by itself, there is no remainder. For example, 12 ÷ 12 = 1 without a remainder, so 12 is a factor of 12.

A prime number is a number that has only two factors, itself and 1, for example: 5, 7, 13, and 29. (Note that 1 itself is not considered to be a prime number.)

Before the students play the game, ask the following questions:

- *Imagine you threw a 4 and a 6. Which squares could you choose to cover with your counter?* (a number with more than two factors, a factor of 24, a multiple of 2, a multiple of 3, a multiple of 4, a multiple of 8, an even number, or a multiple of 6)
- *Imagine you need a multiple of 5 to get four counters in a row. Which throws of the dice would give you a multiple of 5?* (1 and 5, 2 and 5, 3 and 5, 4 and 5, 5 and 5, 6 and 5, 7 and 5, 8 and 5, or 9 and 5)

This game could be extended by asking:

- *What are all the different products you could throw with the two game dice, one labelled 1–6 and the other 4–9?* (4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 24, 25, 27, 28, 30, 32, 35, 36, 40, 42, 45, 48, 54)
- *There are two different ways of getting a product of 12: throwing a 3 and a 4 or a 2 and a 6. Which other products can you throw more than one way using the game dice?* (8: 1 x 8 or 2 x 4; 16: 2 x 8 or 4 x 4; 18: 2 x 9 or 3 x 6; 20: 4 x 5 or 5 x 4; 24: 3 x 8 or 4 x 6 or 6 x 4; 30: 5 x 6 or 6 x 5; 36: 4 x 9 or 6 x 6)
- *What's the probability of throwing a double?* (There are 36 possible combinations that can be thrown with these dice, and only 3 of these are doubles: double 4, 5, or 6. So the probability of throwing a double is $\frac{3}{36}$ or $\frac{1}{12}$.)
- *Which squares in the game are easier/harder to cover? Can you use the information you have about the possible products that can be thrown to explain why?* (Easier to cover: a number with more than two factors [34 out of 36 possible combinations have more than 2 factors; only 5 and 7 don't], an even number, a multiple of 2 [27 out of 36 possible combinations are even and are therefore also multiples of 2], and a multiple of 3 [20 out of 36 possible combinations]. Harder to cover: a prime number [only 2 out of 36 combinations] and a multiple of 7 [only 6 chances out of 36].)

Activity Two

These problems ask students to find highest common factors and lowest common multiples. An understanding of these ideas is important for working with problems involving fractions and in algebra.

It may help the students if they make a list of all the possible products that can be thrown with the two game dice so that they can then compare this list with the factors and multiples needed in the questions.

Links

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 8: Teaching Number Sense and Algebraic Thinking*
  Prime Numbers (finding primes by representing numbers as rectangular arrays), page 32
  Factor Trees (using factor trees to produce prime factors of a number), page 33
  The Sieve of Eratosthenes (finding prime numbers), page 34
  Highest Common Factors, page 39
  Lowest Common Multiples, page 40

Figure It Out

- *Basic Facts,* Level 3
  Factor Puzzles (using basic facts to identify factors), page 11
  Stars and Students (game using multiplication facts to identify factors), page 12
- *Basic Facts,* Levels 3–4
  Matrix (finding factors), page 10
  A Matter of Factor (factor game), page 12
  How Many Factors? (factor investigation), page 15
  Primates (finding prime factors), page 22

www.nzmaths.co.nz

A Prime Search (making arrays to explore prime numbers),
www.nzmaths.co.nz/number/Operating%20Units/Aprimesearch.aspx

Page 18: Buttons Galore

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to help students consolidate and apply advanced multiplicative part–whole strategies (stage 7) in the domain of multiplication and division.

Activity

This activity gives students opportunities to use a range of multiplicative strategies when finding the capacity (volume) of prisms, in this case using non-standard units.

An important concept introduced in this activity is that to find the capacity of a prism, you can measure what will fit in one layer and then multiply this by the number of layers.

Although the word "prism" is not used in the activity, this is a good opportunity to introduce the term. This may be the first time that many of your students have met the word prism, at least in a mathematical context. If this is the case, discuss its meaning with them. Ask *Apart from being full of buttons or shells, what do all the (5) containers pictured in questions* ***1*** *and* ***2*** *have in common?* (They all have completely regular cross sections. For example, if you "sliced" container **1c** anywhere parallel to the base, you would reveal a triangle identical to the ones that form its top and bottom. It is this quality that makes the containers prisms.) If the students have trouble seeing where the question is leading, introduce some containers that are not prisms (for example, bottles that are spherical or tapering at the top) and ask how these containers are different from the ones pictured.

There are lots of opportunities in this activity to discuss the merits of knowing a range of different multiplicative strategies. See the notes for Bean Counters (page 21) for reasons.

Before the students start the activity, you could remind them of the multiplicative strategies that they already know. Ask *How many different ways can you solve 5 x 28 =* ☐? Expect responses such as:

- doubling and halving: 5 x 28 = 10 x 14 = 140
- using a tidy number and compensating: 5 x 28 = (5 x 30) – (5 x 2) = 150 – 10 = 140
- using place value partitioning: 5 x 28 = (5 x 20) + (5 x 8) = 100 + 40 = 140.

Throughout the activity, ask questions such as:

- *Before you start working out the problem, look at the numbers and decide which strategy is likely to be most efficient for this particular problem. Could you explain why to a classmate?*
- *Share your solution path with the group. Did someone else use a more efficient method? What is there about the problem that suggested the use of this strategy?*

Extension investigation

How could you use Matilda's method to measure the volume of a square or rectangular prism in square centimetres? Measure a box and explain what you did.

Links

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 9: Teaching Number through Measurement, Geometry, Algebra and Statistics*
  Fitting In (comparing volumes of boxes), page 13

Page 19: Adventure Tokens

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to help your students consolidate and apply multiplicative part–whole strategies (stages 6–7) in the domain of multiplication and division.

Activity

This activity gives students opportunities to use a range of multiplicative strategies to solve rate problems involving multiplication and division. Students will need a reasonable knowledge of multiplication basic facts if they are to use multiplicative strategies rather than repeated addition.

If your students use repeated addition to solve the problems, encourage them to use multiplicative part–whole strategies by asking them to find alternative solution paths. Share strategies for each problem with the group and talk about which was most efficient.

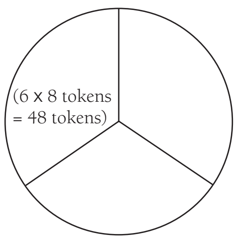
Once students have acquired a range of part–whole strategies that they can use flexibly, it is important that they recognise which one would be most useful and efficient when faced with a particular problem situation. This activity provides opportunities to encourage this skill if used as a guided teaching session. Throughout the activity, ask questions such as:

- *Before you start working out the problem, look at the numbers and decide which strategy is likely to be most efficient for this particular problem. Could you explain why to a classmate?*
- *Share your solution path with the group. Did someone else use a more efficient method? What is there about the problem that suggested the use of this strategy?*

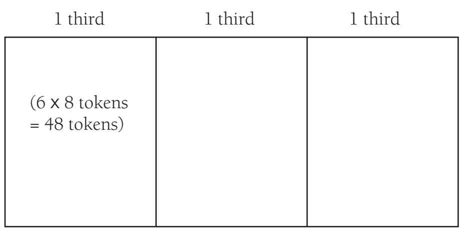
If students have difficulties using the one-third information to solve Naraini's problem in question **1**, ask:

- *How many tokens has Naraini used having 6 rides on the roller coaster?* (6 x 8 = 48 tokens)
- *Draw a diagram that shows that Naraini has used one-third of her tokens and that this one-third is 48.*

Possible diagrams could be:



or



- *If there are 48 tokens in one-third of the diagram, how many tokens are in the other two-thirds?* (2 x 48 = 96)

Possible part–whole multiplicative strategies to explore and compare for 2 x 48 = ☐ include:

- using tidy numbers and compensating: 2 x 48 = (2 x 50) – (2 x 2) = 100 – 4 = 96
- using place value partitioning: 2 x 48 = (2 x 40) + (2 x 8) = 80 + 16 = 96
- doubling and halving: 2 x 48 = 4 x 24 = 8 x 12 = 96.

Possible solution paths to explore for question **2** are noted in the Answers.

Compare different solution paths for question **3**, such as:

- 126 tokens ÷ 3 people = 42 tokens each; 42 tokens each ÷ the 7 tokens it costs to go on the dodgems = 6 rides each: 126 ÷ 3 = 42, 42 ÷ 7 = 6
- 126 tokens ÷ 7 token dodgem cost = 18 turns for 3 people; 18 turns ÷ 3 = 6 turns each: 126 ÷ 7 = 18, 18 ÷ 3 = 6
- 1 turn on the dodgems for 3 people costs 7 x 3 = 21 tokens. (☐ x 20) + (☐ x 1) = 120 + 6 = 126; ☐ = 6.

Possible part–whole multiplicative strategies to explore and compare include:

- 126 ÷ 3 = ☐
  - partitioning: 126 ÷ 3 = (120 ÷ 3) + (6 ÷ 3) = 40 + 2 = 42
  - making proportional adjustments: 126 ÷ 3 = 2 x (63 ÷ 3) = 2 x 21 = 42
- 126 ÷ 7 = ☐
  - making proportional adjustments: (126 ÷ 7) = 2 x (63 ÷ 7) = 2 x 9 = 18
  - using known facts and the distributive property: 70 ÷ 7 = 10, 126 – 70 = 56, 56 ÷ 7 = 8, 10 + 8 = 18.

There is only one solution for question **4** if Chloe is to use all her tokens. A possible solution path is to start with a guess and then work systematically to adjust the levels of tickets until exactly 50 tokens are spent. One way of doing this is to create two lists: one for the cost of trips on the river ride (6, 12, 18, …) and another for the cost of rides in the bumper boats (7, 14, 21, …). The students then see if they can pair 6 (12, 18, …) in the first list with a number in the second list that will make 50. Ask the students how they can be sure that their solution is the only one.

To ensure that the students don't create problems without solutions in question **5,** remind them that they must be able to solve their own problems and explain the solutions to others.

After the activity

Encourage reflective thinking by asking questions such as:

- *Was there a strategy that you used today that you thought was particularly efficient for one of the problems?*
- *Did someone else share a solution path or strategy today that you might use if you were doing the same problem again? What was it about the problem that might tell you that this would be an efficient strategy?*

Links

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 6: Teaching Multiplication and Division*
  A Little Bit More/A Little Bit Less (using known facts and compensating), page 15
  Cut and Paste (halving and doubling, thirding and trebling), page 25
  Multiplication Smorgasbord (using a variety of strategies), page 27
  Proportional Packets (solving division problems by making proportional adjustments), page 28
  The Royal Cooking Lessons (solving division problems by making proportional adjustments), page 30
  Remainders (solving division problems by using known facts and the distributive property), page 32

Pages 20–21: Team Schemes

Achievement Objective

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to help your students consolidate and apply multiplicative part–whole strategies (stages 6–7) in the domain of multiplication and division.

Activity

This activity gives students opportunities to use a range of multiplicative strategies to solve practical problems involving multiplication and division.

For question **1**, challenge students who simply add the numbers of students at each school to use a multiplicative method by asking *How could you use Ms Kelly's 7 x 30 statement to solve the problem in a different way?*

Encourage the students to share their strategies for each problem in question **2** with the group and to talk about which was most efficient.

Possible part–whole multiplicative strategies to explore and compare include:

- using known facts and place value knowledge. For example, when solving 210 ÷ 3 = ☐: if you know that 21 ÷ 3 = 7, you can use knowledge of place value to work out that 210 ÷ 3 = 70.
- using known facts and the distributive property. For 210 ÷ 6 = ☐ : if you know that 18 ÷ 6 = 3, you can use this to work out that 180 ÷ 6 = 30. There are still another 30 students to divide into 6-player teams: 30 ÷ 6 = 5. 30 + 5 = 35 teams.
- using partitioning. For 210 ÷ 4 = ☐ : you can partition 210 into 200 + 10 and then divide each part separately by 4. 200 ÷ 4 = 50, 10 ÷ 4 = 2 remainder 2, so 210 ÷ 4 = 52 remainder 2.
- making proportional adjustments. For 210 ÷ 5 = ☐: 210 ÷ 5 = 420 ÷ 10 = 42.
- reversing and using multiplication. 210 ÷ 2 = ☐ can be changed to 2 x ☐ = 210.

For question **3**, encourage the students to design a table to help record all the information they need in a clear way. One possibility might be:

	Number of students in a team	Number of students left out of a team							Number of students in mixed-school teams
		Waikino School 35 students	Hampton School 26 students	Kahunui School 28 students	St Joseph's School 30 students	Te Horo School 32 students	Sherwood School 31 students	Mapui School 28 students	
Bowls	2	1					1		2
Mini–soccer	6	5							
Touch rugby	7								
Basketball	5								
Triathlon	3								
Beach volleyball	4								

A key concept underpinning this problem is that of factors (a number that is multiplied by another number to create a product; if you divide a number by one of its factors, there will be no remainder. See Multiples and Factors, pages **16–17**). Define the term "factor" together, then ask:

- *There are 35 students at Waikino School. What are the factors of 35?* (1, 5, 7, and 35)
- *How does knowing about the factors of a number help you to know whether some students will need to play in mixed-school teams?* (If the number of students in a team is a factor of the number of students at the school, there won't be any remainders, and so no students from that school will need to be in mixed teams.)
- *Which schools have fewer students who will need to go in to mixed teams? Why is this?* (St Joseph's School only has 4 students who need to go into mixed teams because they have 30 students and 30 has more factors than numbers such as 26 or 31.)
- *A prime number only has two factors, itself and 1. Do any schools have a prime number roll?* (Sherwood, with 31 students) *How does this affect the numbers of students who will need to play in mixed teams?* (A school with a roll that is a prime number will have a lot more leftover students than a school such as St Joseph's with its 30 students. For example, unless a team has 31 students in it, a school with 31 students will always have leftover students.)

If students have difficulty coming up with a fair way of scoring for question **4**, direct their attention to the speech bubble hint and ask questions such as:

- *If you are in a winning team, how many points do you think each player in the team should get?*
- *What about if you draw?* (You could get half the winning amount of points.)
- *Would this system be equally fair to students who are in mixed teams?*
- *Would this system be fair to schools who have fewer students at the sports day?*
- *If each player in the mini-soccer team got, say, 5 points, how many points would the whole team have?* ($6 \times 5 = 30$ points) *Would every sport be the same?* (No, because it would depend on how many are in the team.)

Extend the problem by getting the students to design a syndicate sports extravaganza, using the numbers in your syndicate's classes and coming up with their own team sports and team numbers (note that small teams are easier to work with for this problem):

- *How many students are there altogether in our syndicate? Which multiplication fact would give you an estimate?* (For example, $5 \times 30 = 150$)
- *How many teams will be needed for each sport in your sports extravaganza?*
- *How many students will need to play twice so that everyone gets to be in a team?*
- *For each sport, how many players will need to be in mixed-class teams to make up numbers?*

Links

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 6: Teaching Multiplication and Division*
  Long Jumps (solving division problems using repeated addition and known facts), page 19
  Goesintas (solving division problems using multiplication), page 20
  Remainders (solving division problems that have remainders, using known facts and the distributive property), page 32

Page 22: Square Skills

Achievement Objectives

- perform measuring tasks, using a range of units and scales (Measurement, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

This activity gives students opportunities to apply multiplicative part–whole strategies (stage 6).

Activity

In this activity, students use multiplication to solve area and perimeter problems. The activity usefully complements Rubber Band Rectangles (pages **14–15**) by revising the main ideas. It could also be used as an assessment task.

A co-operative problem-solving approach could be used to work on this activity, with students working in small groups of 3–4 and then reporting back and comparing ideas and solutions with other groups.

Key vocabulary to discuss before the activity:

- Perimeter: the distance around the outside of a shape (imagine an ant walking around it).
- Area: the surface that a shape covers, measured in squares (imagine a square blanket covering the floor).

Some students may need to solve question **1** by manipulating materials such as multilink cubes or squares of card to represent Grandma's quilting squares.

A key idea underpinning this problem is that of factors. (See Multiples and Factors, pages **16–17**.) The students need to use factors if they are to solve these problems using number properties rather than by manipulating materials. Finding all the different rectangles that have an area of 24 squares involves finding all the pairs of factors of 24: namely, 1 and 24, 2 and 12, 3 and 8, and 4 and 6.

After the students have completed the problem, discuss factors and define them together and then ask:

- *What are the factors of 24? How do they relate to the rectangular quilts with an area of 24 squares that you drew?*
- *How could using factors help us to find all the possible different rectangles that have an area of 30 squares?* (We could list all the factors and then pair them up with another factor that will multiply together to make 30: namely, 1 and 30, 2 and 15, 3 and 10, and 5 and 6.)

For question **2**, encourage the students to find a way of working out the perimeter that uses multiplication rather than simply adding the four sides. Expect responses such as: "I look at how many squares there are in one row and how many rows there are. I add those two numbers together, and then I double the total."

Return to the idea of using factors to solve question **3a**: *What rectangular shapes could Grandma make using 36 squares?*

When they are doing question **3**, your students may not realise that a square is also a rectangle, and for this reason they won't include a 6 by 6 square in their list. Have them try and define the qualities of a rectangle, and then discuss with them whether a square also has these qualities. It does, so a square is a special sort of rectangle: one with all its sides the same length, just as an equilateral triangle is a special sort of triangle.

To promote generalisations, ask:

- *What do you notice about rectangular shapes that need more binding (have a longer perimeter) compared to rectangular shapes that use less binding?* (Rectangles that are long and thin have longer perimeters, while rectangles that have the same area but look more square have a shorter perimeter.)
- *If Grandma was running short of binding, what would you advise her to do when arranging her patchwork squares?* (Try to make a square rectangle, or as close to one as possible, because a square has the smallest perimeter of any rectangle.)

Links

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 8: Teaching Number Sense and Algebraic Thinking*
  Squaring (relating the squaring of numbers to square shapes), page 28
  Square Roots (using square roots to find the length of the side given the area of a square), page 29
  Prime Numbers (representing prime and other numbers by arranging tiles into rectangles), page 32
- *Book 9: Teaching Number through Measurement, Geometry, Algebra and Statistics*
  Investigating Area (comparing and measuring areas), page 11

Figure It Out

- *Basic Facts,* Level 3
  Factor Puzzles (using basic facts to identify factors), page 11
  Stars and Students (game using multiplication facts to identify factors), page 12
- *Basic Facts,* Levels 3–4
  Matrix (finding factors), page 10
  A Matter of Factor (factor game), page 12
  How Many Factors? (factor investigation), page 15
  Primates (finding prime factors), page 22

www.nzmaths.co.nz

A Prime Search (making arrays to explore prime numbers),
www.nzmaths.co.nz/number/Operating%20Units/Aprimesearch.aspx

Page 23: Munching Monarchs

Achievement Objective

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to help your students consolidate and apply advanced multiplicative part–whole strategies (stage 7) in the domain of proportions and ratios.

Activity

In this activity, students use multiplication to solve ratio problems. It would suit a guided teaching session or could be used as an independent activity for a group. To work independently, students will need a reasonable range of multiplicative strategies, a good recall of multiplication basic facts, and the ability to look carefully at a question to work out what they need to do.

For question **1**, some students may have difficulty with the idea of ratios and how they can use the information on the signs to solve the problem. If so, they may find it a help to use a double number line or a table to show how the number of caterpillars that can be fed increases with each group of swan plants bought.

Number of small swan plants	0	1	2	3	4	5	6	7	8	9
Number of caterpillars that can be fed	0			4			8			?

This table shows similar information, but for more plants and caterpillars and for both grades of plant:

Small swan plants		**Large swan plants**	
Number of swan plants	**Number of caterpillars you could feed**	**Number of swan plants**	**Number of caterpillars you could feed**
0	0	0	0
3	4	2	5
6	8	4	10
9	12	6	15
12	16	8	20
15	20	10	25
…	…	12	30
		…	…

Encourage the students to use proportional strategies by asking *If you know that it takes 3 small swan plants to feed 4 caterpillars, can you use that information to work out how many caterpillars could feed on 9 small plants?* Record this as 3:4 = 9:$\square$. Expect the students to use multiplication or equivalent fractions to work out that a ratio of 9:$\boxed{12}$ is the same as 3:4. Similarly, for the large swan plants, record: 2:5 = 6:$\square$.

The double number lines and tables can be extended for question **2** until each shows how many swan plants are needed for 20 caterpillars.

To encourage the students to use proportional strategies, ask *If you know that it takes 3 small swan plants to feed 4 caterpillars, can you use that information to work out how many small swan plants are needed for 20 caterpillars?* Record this as 3:4 = $\square$:20. Expect the students to use multiplication or equivalent fractions to work out that a ratio of $\boxed{15}$:20 is the same as 3:4. Similarly, for the large swan plants, record: 2:5 = $\square$:20.

Alternatively, the students could first work out how many plants of each size would feed 20 caterpillars by dividing the 20 caterpillars into smaller groups that suit each type of swan plant (small plants: 20 caterpillars ÷ groups of 4 = 5 groups, 5 groups of 3 plants = 15 plants; large plants: 20 caterpillars ÷ groups of 5 = 4 groups, 4 groups of 2 plants = 8 plants) and then working out how much those plants would cost (small plants: 15 plants at $3 each = $45, large plants: 8 plants at $5.50 each = $44).

In question **3**, some students may have difficulty working out how many caterpillars Mrs McPherson could feed with 22 small plants because 22 does not divide evenly into groups of 3 swan plants. Encourage them to estimate the number of caterpillars by asking questions such as:

- *What's the nearest multiple of 3 to 22 that you could use?* (21)
- *How many caterpillars would 21 small plants feed?* Record: 3:4 = 21:$\square$ (28)
- *How many extra plants does Mrs McPherson have?* (1)
- *How many caterpillars would you estimate 1 small plant will feed if you know that 3 plants will feed 4 caterpillars?* ($1\frac{1}{3}$ of a caterpillar, but since you can't have a third of a caterpillar, Mrs McPherson will only be able to have 1 caterpillar for that plant, meaning that she can raise 29 caterpillars on 22 plants.)

Some students may have difficulty multiplying and dividing by $5.50 or 5.5 in question **4**. If so, encourage them to make the link between 5.5 and 11, which is easier to use in calculations. Alternatively, they could reverse the problem and use multiplication to build up to $66 using facts they know, such as $10 \times 5.5 = 55$.

For question **5**, remind the students that they need to be able to solve and explain their own problems. This will help ensure that the numbers they choose work well with mental strategies.

After the activity

Encourage reflective thinking and the discussion of generalisations by asking questions such as:

- *Which problems did you find hardest? Share how you solved them.*
- *Was there a strategy that someone else used that you could use another time? What is it about the problem that would remind you that this would be a good strategy to try?*
- *What can you tell me about ratios? How does knowing about ratios help with this activity?*

The photograph on the student page indicates that the mathematics on this page does not accurately reflect the reality of the rate at which monarch caterpillars eat (or destroy!) swan plants. The students could do some Internet research on this topic.

Links

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 7: Teaching Fractions, Decimals, and Percentages*
  Seed Packets (solving simple ratio problems by repeated copying), page 17
  Mixing Colours (comparing ratios and proportions), page 34
- *Book 8: Teaching Number Sense and Algebraic Thinking*
  Ratios with Whole Numbers (using equivalent ratios), page 42

nzmaths.co.nz

Mixing Paint and Other Problems (using ratios to solve problems),
www.nzmaths.co.nz/number/Operating%20Units/mixingpaint.aspx

Page 24: Fractions in Room 7

Achievement Objective

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to help your students to consolidate and apply advanced multiplicative part–whole strategies (stage 7) in the domain of proportions and ratios.

Activity

In this activity, students need to use multiplicative part–whole strategies to solve problems involving finding the lowest common multiple and simple equivalent fractions.

Students doing this activity will need to:
- have a reasonable recall of multiplication basic facts
- know how to find fractions of groups of objects, for example, $\frac{2}{5}$ of 20
- be able to find simple equivalent fractions, for example, $\frac{2}{4} = \frac{1}{2}$, $\frac{1}{4} = \frac{2}{8}$.

This activity lends itself to co-operative problem solving. The students, in groups of 3–4, can discuss and solve each problem before they come back together with the whole group to share and compare solution paths.

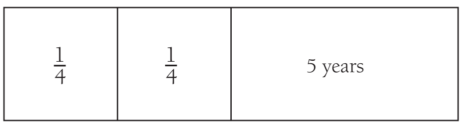
Introduce question **1** by making attribute blocks available for the students to demonstrate their solutions. Look for students who use number properties rather than materials and, in the group-sharing time, ask them to explain what they were doing.

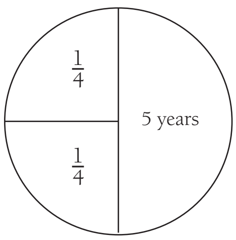
If the students have already done Multiples and Factors, pages **16–17**, this question provides an opportunity for reinforcing the idea of lowest common multiples: if you know that 20 is the lowest common multiple of 2, 4, and 5, you can solve the problem easily.

The students should be familiar with the term "multiple" from earlier activities. The lowest common multiple (LCM) is the smallest number that is a multiple of two (or more) numbers, for example, 10 is the smallest number that is a multiple of both 2 and 5. Ask:
- *How does knowing about lowest common multiples help with solving this problem?* (You have to find the smallest number that can be divided evenly into halves, quarters, and fifths; this is also the LCM of 2, 4, and 5.)
- *What is the LCM of 2, 5, and 6?* (30)
- *What strategies do you use to find the LCM? Is it more efficient to count up in multiples of 2 and check whether they are also multiples of 5 and 6 or to count up in multiples of 6 and check whether they are also multiples of 2 and 5?* (Multiples of 6, because only every third multiple of 2 will be a multiple of 6, whereas every multiple of 6 is also a multiple of 2)

For question **2**, clarify with the students that $\frac{1}{4} + \frac{1}{4}$ + 5 years = the whole of Marama's life.

If students are having difficulty, ask scaffolding questions such as:
- *Could you draw a diagram to help you solve the problem?*





- *If you know that* $\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$ *and that the other two quarters together equal 5 years, how many years is one quarter worth?*
- *Sio spent one-quarter of his life in Tokoroa and one-eighth of it in Hamilton. How many eighths is this altogether?* (three eighths)
- *What do you have to do to the quarters before you can add them to the eighths?* (Use equivalent fractions to change the one quarter into two eighths.)
- *How many eighths are left to cover Sio's time in Napier?* (five eighths)
- *If you know that five eighths = 5 years, how many years are equal to 1 whole?* (Each eighth = 1 year, so 8 eighths [1 whole] = 8 years)

Extend the problems by challenging the students to describe their own lives in fractional terms. If they haven't moved towns, they could describe their family or something they own, for example:
- "One-third of my family is male, four-sixths of my family is under the age of 15, and I have 1 brother."
- "I collect Super 14 cards. Three-eighths of my collection is Highlanders players, one-quarter is the Blues, one-eighth is the Chiefs, and I have 12 Hurricanes cards. How many cards do I have altogether?"

Links

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)
- *Book 4: Teaching Number Knowledge*
  Packets of Lollies (creating a whole fraction from a given part), page 8
  The Same but Different (equivalent fractions), page 30
- *Book 7: Teaching Fractions, Decimals, and Percentages*
  Birthday Cakes (using multiplication to find a fraction of a set), page 14
  Deci-mats (identifying equivalent fractions), page 25
  Comparing Apples with Apples (adding and subtracting fractions), page 38
- *Book 8: Teaching Number Sense and Algebraic Thinking*
  Lowest Common Multiples (using stage 8 strategies to find LCMs), page 40

Copymaster: 1–100 and 101–200

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