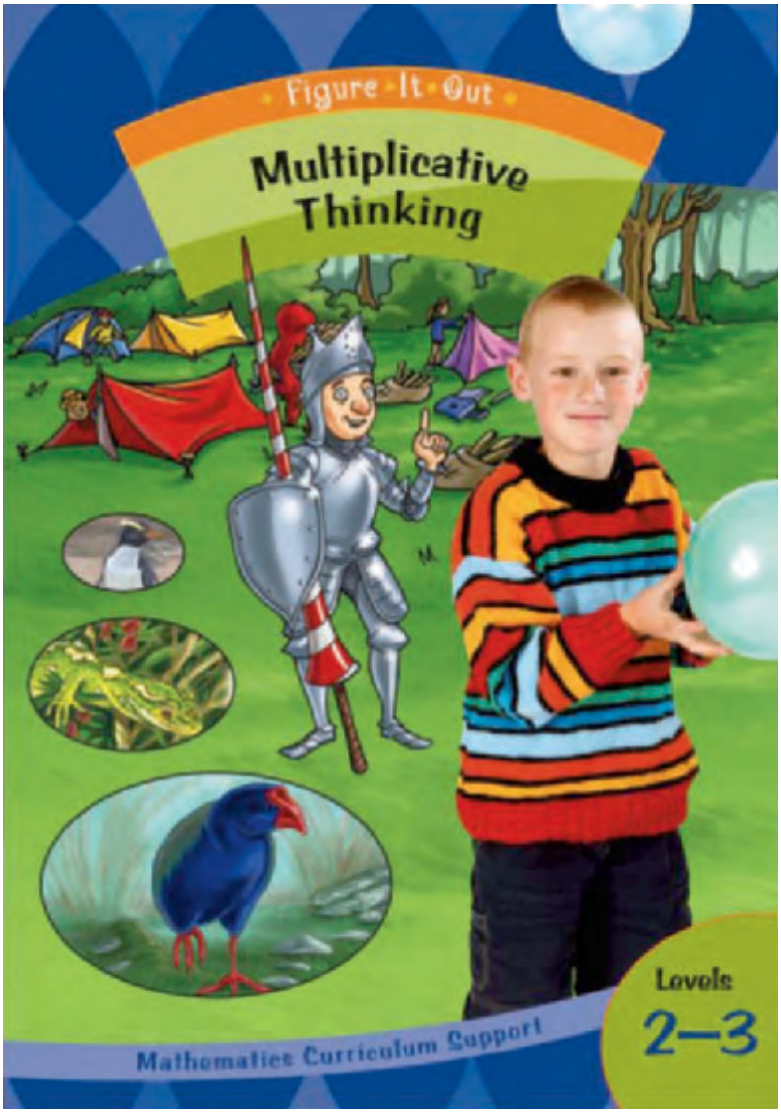
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Contents

Introduction

The books in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand classrooms. In recent years, much of the Figure It Out student material has been aligned with Numeracy Development Project strategies, which are reflected in the answers and in the teachers' notes.

Student books

The activities in the student books are written for New Zealand students and are set in meaningful contexts, including real-life and imaginary scenarios. The level 2–3 contexts reflect the ethnic and cultural diversity and the life experiences that are meaningful to students in year 4. However, teachers should use their judgment as to whether to use the level 2–3 books with older or younger students who are also working at these levels.

The activities can be used as the focus for teacher-led lessons, for students working in groups, or for independent activities. Also, the activities can be used to fill knowledge gaps (hot spots), to reinforce knowledge that has just been taught, to help students develop mental strategies, or to provide further opportunities for students moving between strategy stages of the Number Framework.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the *Multiplicative Thinking* student books includes full answers and explanatory notes. Students can use this section for self-marking, or you can use it for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, Number Framework and other links, comments on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/r/maths/curriculum/figure/

Using Figure It Out in the classroom

Where applicable, each page starts with a list of equipment that the students will need in order to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to the problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Figure It Out

Multiplicative Thinking Levels 2–3 Answers

Pages 1–3: Party to the Max

Activity One

1. Fruit Nectar. (4 packs of Fruity Fun is 4 x 6 = 24 cartons, compared with 25 in the Fruit Nectar special.)
2. About 20 apples: 4 kg can be bought for $8 ($8 ÷ $2 = 4 or 2 x ☐ = 8), and 4 kg with approximately 5 apples in each kg is about 20 apples (4 x 5 = 20).
3. 4 tubs. (12 ÷ 3 = 4 or 3 x ☐ = 12; or $3 + $3 + $3 + $3 = 4 x $3 = $12.)
4. 2 packs. (Even though 2 x 10 = 20 is too many blowers, they need 2 packs because 1 pack is 10 blowers and that wouldn't be enough.)
5. **a.** 3 packs. (There are 5 in each pack: 3 x 5 = 15 or 15 ÷ 5 = 3.)

   b. i. 6 packs. (Either double the last answer or use 2 x 15 = 30. 30 ÷ 5 = 6 or ☐ x 5 = 30.)

   ii. $18. (6 x $3 = $18)

Activity Two

1. **a.** 20. (10 x 2 = 20)

   b. 70. (10 x 7 = 70)

   c. No, he needs 2 more grapes because he needs 10 lots of 3 grapes, 10 x 3 = 30. 30 – 28 = 2 or ☐ + 28 = 30.

2. **a.** No. There are 15 people at the party, and there are only 12 different vege and dip combinations possible: 4 vege bites x 3 dips = 12 different combinations, 4 x 3 = 12. A possible diagram is:

Vege bites

Dips	Carrot	Celery	Capsicum	Cauliflower
Onion	✓	✓	✓	✓
Tomato salsa	✓	✓	✓	✓
Guacamole	✓	✓	✓	✓

   b. i. 16. 4 vege bites x 4 dips = 16 different combinations (4 x 4 = 16).

   i. 15. 5 vege bites x 3 dips = 15 different combinations (5 x 3 = 15).

3. Answers will vary.

Pages 4–5: Marble Mania

Activity

1. **a.** Each child gets 6 marbles. Possible methods include using counters or drawing the marbles and separating them into groups; counting on and multiplication (4 + 4 + 4 + 4 + 4 + 4 = 24, 6 x 4 = 24); subtraction and multiplication (24 – 4 – 4 – 4 – 4 – 4 – 4 = 0, 6 x 4 = 24); multiplication (4 x ☐ = 24); or division (24 ÷ 4 = 6).

   b. Possible groups are: 2, 3, 4, 6, 8, or 12 children. A group of 24 children is also possible, although they would get only 1 marble each.

2. Methods will vary. Possible answers include:

   a. 7 marbles each. Sharing out the marbles in groups of 3 gives: 3 + 3 + 3 + 3 + 3 + 3 + 3 = 21, that is, 7 x 3 = 21; or 21 – 3 – 3 – 3 – 3 – 3 – 3 = 0, 7 x 3 = 21; or 3 x ☐ = 21; 21 ÷ 3 = 7

   b. 5 marbles each: 5 + 5 + 5 + 5 + 5 = 25, 5 x 5 = 25; 25 – 5 – 5 – 5 – 5 – 5 = 0, 5 x 5 = 25; 5 x ☐ = 25; 25 ÷ 5 = 5

c. 3 marbles each: 3 + 3 + 3 + 3 + 3 + 3 = 18, 6 x 3 = 18; 18 – 3 – 3 – 3 – 3 – 3 – 3 = 0, 6 x 3 = 18; 3 x ☐ = 18; 18 ÷ 6 = 3

3. **a.** 3 m. (4 ÷ 2 = 2, 2 + 1 = 3)

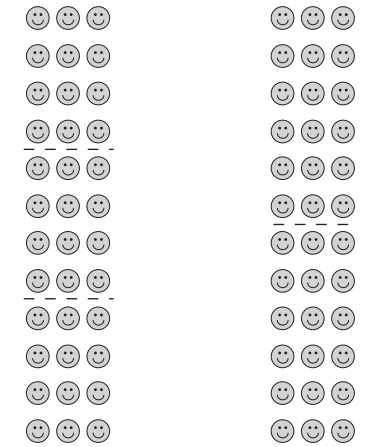
b. 2 m. (6 ÷ 3 = 2; 3 x ☐ = 6)

4. Problems will vary.

Pages 6–7: Face the Facts

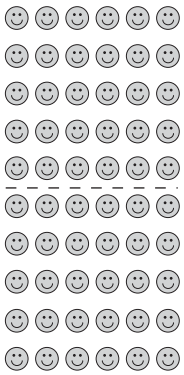
Activity

1. Yes, Elton's thinking is right. 6 x 3 = 18, 12 x 3 = 36, and 2 x 18 = 36.



12 x 3 = 36 2 x 18 = 36

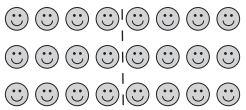
2. He could make a cut after 5 rows because there are 6 faces in each row.



10 x 6 = 60

5 x 6 = 30 or half of 60 is 30.

3. **a.** You could double each row of 4 to make 8. 3 x 8 = 24.



3 x 8 = 24

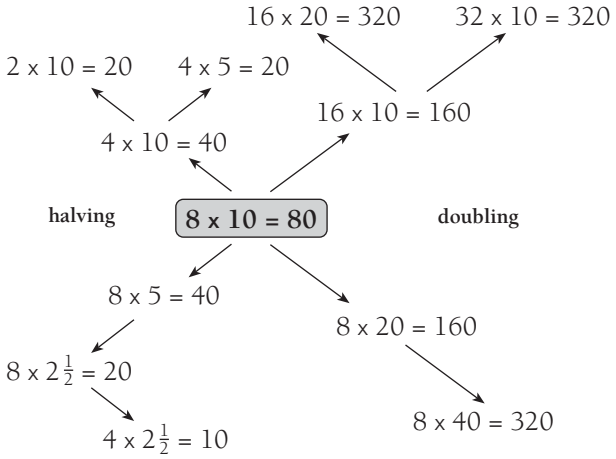
b. 3 x 8 = 24, so 3 x 16 = 48 because 16 is double 8 and so the answer is double 24. 2 x 24 = 48.

3 x 16 = 48

4. **a.** **i.** Doubling: 2 x 8 = 16, so 4 x 8 = 32, 8 x 8 = 64, 16 x 8 = 128. (Another doubling sequence, using only the first part of the train, would be: 2 x 8 = 16, 2 x 16 = 32, 2 x 32 = 64, 2 x 64 = 128.)

ii. Halving: 4 x 20 = 80, so 4 x 10 = 40 (10 is half of 20, and 40 is half of 80), 4 x 5 = 20 (5 is half of 10, and half of 40 is 20), 4 x $2\frac{1}{2}$ = 10 ($2\frac{1}{2}$ is half of 5, and half of 20 is 10).

b. Some problems that could be solved are:



Pages 8–9: Sushi Surgery

Activity

1. **a.** 16. (2 x 8)

b. 32. (4 x 8)

2. **a.** 8 boxes. (1 roll fills 2 boxes. 2 x 4 = 8. Or: 4 rolls is 4 x 8 = 32 pieces. 32 ÷ 4 pieces per box = 8 boxes)

b. 16 boxes. (4 rolls in **a** makes 8 boxes. 8 rolls is double 4 rolls, so 2 x 8 boxes = 16 boxes.)

c. 12 boxes. 1 roll fills 2 boxes. 2 x 6 = 12. Or: 6 rolls is 6 x 8 = 48 pieces; 48 ÷ 4 = 12 boxes.

3. Answers will vary. For example, to work out the pieces: 8 + 8 + 8 + 8 = 32 pieces or 4 x 8 = 32 pieces. To work out the boxes: 32 ÷ 4 = 8 boxes, or 32 – 4 – 4 – 4 – 4 – 4 – 4 – 4 – 4 = 0, or 4 x ☐ = 32.

4. **a.** **i.** 4 x 5 = 20 or 5 x 4 = 20

ii. 3 x 5 = 15 or 5 x 3 = 15

iii. 7 x 6 = 42 or 6 x 7 = 42

b. **i.** 12

ii. 8

iii. 30

Strategies may vary, for example:

i. 3 x 4 or 4 x 3; or 20 – 8

ii. 2 x 4 or 4 x 2; or 15 – 7

iii. 6 x 5 or 5 x 6; or 42 – 12

5. **a.** **i.** 32 pieces.
4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 = 32;
8 x 4 = 32

ii. 36 pieces. 6 + 6 + 6 + 6 + 6 + 6 = 36;
6 x 6 = 36

iii. 30 pieces. 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 30; 10 x 3 = 30

iv. 32 pieces. 8 + 8 + 8 + 8 = 32;
4 x 8 = 32

b. Salmon sushi (based on the number of pieces sold) or vegetarian sushi (based on the number of boxes sold)

6. Problems will vary.

Pages 10–11: Class Farewell

Activity

1. 12 packets. (1 packet is enough for 2 students. 24 ÷ 2 = 12, so 12 packets is enough for 24 students.)

2. **a.** 3 packets. (8 + 8 + 8 = 24 or 24 – 8 – 8 – 8 = 0; 3 x ☐ = 24)

b. 4 packets. (48 ÷ 12 = 4. Or, using the hint given, 48 ÷ 12 is the same as ☐ x 12 = 48. If you don't know that fact, you could go: ☐ x 6 = 24; 4 x 6 = 24, so 4 x 12 = 48.)

3. **a.** 10 packets of pencils and 8 packets of pencil cases. (They need 48 pencils and 24 pencil cases. 5 x 10 = 50 pencils, and 3 x 8 = 24 pencil cases.)

b. 2 pencils. (50 – 48 = 2)

4. Yes. The total cost is $74, which is $1 less than $75. One possible way to work out your answer is: (12 x $2) + (3 x $2) + (4 x $2) + (10 x $2) + (8 x $2) = 24 + 6 + 8 + 20 + 16 = 74.

5. **a.–b.** Answers will vary. Lists and total costs will vary depending on what items you choose and how many of each.

Pages 12–13: Fishy Business

Activity

1. **a.** 4 x 10 = 40 orange fish

b. 4 x 3 = 12 starfish

c. 4 x 2 = 8 crabs

2. **a.** 30 fish. (10 x 3 = 30)

b. 40 snails. (10 x 4 = 40)

3. **a.** 30 fish. (5 x 6 = 30)

b. 12 fish. (2 x 3 = 6 plus 3 x 2 = 6. 6 + 6 = 12 or 2 x 6 = 12.)

4. **a.** 5 fish in each tank. (The tanks hold 6 fish, but 6 x 5 = 30, and Arnold only has 27 fish. 5 x 5 = 25)

b. 2 fish left over: 5 x 5 = 25, 27 – 25 = 2.

5. **a.–c.** Answers will vary. You could use your 5 times table to help you. For example, if you had 5 tanks with 2 orange fish in each one, you could say you had 5 x 2 = 10 orange fish altogether.

6. Problems will vary.

Pages 14–15: Survivor Challenge

Activity

1. 10 groups. (3 x ☐ = 30; 30 ÷ 3 = 10)

2. **a.** **i.** 6 m. (18 ÷ 3 = 6; 3 x ☐ = 18)

ii. 2. (5 – 3 = 2)

b. 1.3 m or $1\frac{1}{3}$ m. (4 m is 1 m each and 1 m left over to share: $1 \div 3 = \frac{1}{3}$ each)

c. **i.** 2. (There will be 2 + 2 + 1 = 5 fires altogether. 10 ÷ 5 = 2)

ii. 4. (20 ÷ 5 = 4)

iii. 25. (5 x 5 = 25)

3. **a.** 5. (15 ÷ 3 = 5)

b. **i.** 60. (3 x 2 = 6 slices per group. 6 x 10 groups = 60 slices)

ii. 4 loaves. 18 x 2 = 36, 36 x 2 = 72 slices. 72 – 60 = 12. So 4 loaves are needed, with 12 slices left over. Or: 18 is close to 20. 20 x 3 = 60, which is enough for the 10 groups, but that's counting 2 slices per loaf that aren't there, so an extra loaf is needed. 20 x 4 = 80. 80 – 6 = 72 (that is, minus the 6 "tidy number" slices), 72 – 60 = 12 spare slices.

Pages 16–17: Fiordland Holiday

Game

A game using multiplicative strategies

"Adventures and mishaps" game card answers and some possible methods:
3 x 27 = 9 x 9 = 81
12 x 5 = 60. 12 x 5 is 2 x 5 = 10 more than 10 x 5. 10 x 5 = 50 and 50 + 10 = 60, so 12 x 5 = 60.
20 x 7 = 140. 2 x 7 = 14. 14 x 10 = 140, so 20 x 7 = 140.
5 x 36 = 180. 5 x 30 = 150 and 5 x 6 = 30. 150 + 30 = 180, so 5 x 36 = 180.
565 x 0 = 0
6 x 11 = 66. 6 x 11 is 6 more than 6 x 10. 6 x 10 = 60 and 60 + 6 = 66, so 6 x 11 = 66.
53 x 3 = 159. 50 x 3 = 150; 3 x 3 = 9. 150 + 9 = 159, so 53 x 3 = 159.
3 x 99 = 297. 3 x 100 = 300. 300 – 3 = 297. (Or: 3 x 99 is 3 less than 3 x 100. 300 – 3 = 297)
5 x 18 = 90. 5 x 18 is 5 x 2 = 10 less than 5 x 20. 5 x 20 = 100 and 100 – 10 = 90, so 5 x 18 = 90.
5 x 44 = 220. 5 x 44 = 10 x 22 = 220
3 x 300 = 900. 3 x 3 = 9; 9 x 100 = 900, so 3 x 300 = 900.
39 x 0 = 0
5 x 19 = 95. 5 x 19 is 5 less than 5 x 20. 5 x 20 = 100. 100 – 5 = 95
6 x 9 = 54. 6 x 10 = 60. 60 – 6 = 54
45 ÷ 5 = 9. 5 x ☐ = 45; 5 x 9 = 45
102 x 4 = 408. 102 x 4 = 4 x 102. 4 x 100 = 400 and 4 x 2 = 8; 400 + 8 = 408
36 ÷ 4 = 9. 4 x ☐ = 36; 4 x 9 = 36
7 x 198 = 1 386. 7 x 198 is 7 x 2 = 14 less than 7 x 200. 7 x 200 = 1 400. 1 400 – 10 = 1 390; 1 390 – 4 = 1 386
56 ÷ 7 = 8. 7 x ☐ = 56; 7 x 8 = 56, so 56 ÷ 7 = 8.
4 x 18 = 72. 4 x 18 is 4 x 2 = 8 less than 4 x 20. 4 x 20 = 80. 80 – 8 = 72. (Or use doubling and halving: 8 x 9 = 72.)
5 x 80 = 400. 5 x 8 = 40 and 40 x 10 = 400
120 ÷ 5 = 24. 120 ÷ 5 = 240 ÷ 10 = 24
9 x 6 = 54. 10 x 6 = 60; 60 – 6 = 54

"Rare species" game card answers and some possible methods:
12 x 8 = 96. 10 x 8 = 80 and 2 x 8 = 16. 80 + 16 = 96
21 x 3 = 63. 21 x 3 = 3 x 21; 3 x 20 = 60 and 3 x 1 = 3; 60 + 3 = 63
7 x 8 = 56. 5 x 8 = 40 and 2 x 8 = 16. 40 + 16 = 56
15 x 10 = 150. 15 x 10 = 10 x 15; 10 x 10 = 100 and 10 x 5 = 50; 100 + 50 = 150
2 x 231 = 462.
(2 x 200) + (2 x 30) + (2 x 1) = 400 + 60 + 2 = 462
4 x 20 = 80. 4 x 20 = 8 x 10 = 80
7 x 4 = 28. 7 x 4 = 4 x 7 = 28
Or: 4 x 4 = 16. 3 x 4 = 12. 16 + 12 = 28
17 ÷ 1 = 17. 1 x ☐ = 17; 1 x 17 = 17
42 ÷ 6 = 7. 6 x ☐ = 42; 6 x 7 = 42
26 x 5 = 130. 26 x 5 = 13 x 10 = 130
24 x 2 = 48. 24 x 2 = 12 x 4 = 6 x 8 = 48

Pages 18–19: Clever Clues

Activity

1. Hannah is 16 yrs old (8 x 2 = 16), and Joe is 8 (16 ÷ 2 = 8).

2. Joe weighs 45 kg (3 x 15 = 45 or [3 x 10] + [3 x 5] = 30 + 15 = 45), and Hannah weighs 60 kg (4 x 15 = 60 or [4 x 10] + [4 x 5] = 40 + 20 = 60).

3. **a.** They are both right. (Joe's answer uses additive thinking [50c + $1.50 = $2], and Hannah has compared the amounts multiplicatively [4 x 50c = $2].)

b. 5 times. (Hannah gets $10 a week [2 + 8 = 10], which is 5 times as much as Joe: 5 x $2 = $10.)

c. **i.** $9.50.

ii. Hannah gets $9.50 more than Rion: 0.50 + $9.50 = $10; *and* Hannah gets 20 times as much as Rion: 50c x 20 = $10.

4. Joe has 12 people in his family, Hannah has 6, and Rion has 3. (Joe must have between 9 and 14 people in his family because they can't fit in one 8-seater van, but if they go in two 8-seater vans, there are at least 2 seats to spare [Joe says there are seats to spare, so that means at least 2]. Joe has 4 times as many people in his family as Rion does, so it must be a multiple of 4. 12 is the only multiple of 4 between 9 and 14, so Rion must have 3 people in his family and Joe must have 12. Hannah has twice as many as Rion: 2 x 3 = 6, which is also half of Joe's family: $\frac{1}{2}$ of 12 = 6.)

5. Problems will vary.

Pages 20–21: Friends Forever

Activity

1. **a.** Liata's pattern goes in groups of 5: each group has 2 blue beads then 3 orange beads.

b. Orange. (There would be 7 groups of 5, and the 35th bead would be the fifth bead in the seventh group of 5. The fifth bead in each group of 5 is always orange.)

c. Blue. (There would be 4 groups of 5 beads making 20, and the first bead in the next group of 5 would be the 21st one. The first bead in each group of 5 is always blue.)

2. **a.** 8 blue beads and 12 orange beads. To make a 12 cm string of beads, Liata will need to repeat her pattern 4 times. 4 x 2 = 8 blue beads, and 4 x 3 = 12 orange beads.

b. $1.00. 8 + 12 = 20 beads. 20 beads at 2 for 10c is 10 x 10c = 100c or $1.00.

3. **a.** Miria's plan has shapes in groups of 3 (cube, cylinder, sphere [round bead]) and colours in groups of 4 (2 red beads and 2 green beads).

b. A red cube

c. The 20th bead would be green. The colour pattern repeats in groups of 4. 5 x 4 = 20. The 20th bead is the last bead in the 5th group of 4. The last bead in each group of 4 is always green.

d. The 3 times table, because the shapes repeat in groups of 3.

e. The 30th bead would be a sphere. 10 x 3 = 30. The 30th bead is the last bead in the 10th group of 3. The last bead in each group of 3 is always a sphere.

4. **a.** 16 red beads and 14 green beads. Each group of 3 shapes is 2 cm long. There would be 30 beads altogether in a 20 cm string because 3 shapes x 10 repeats of the shapes pattern makes 20 cm = 30 beads. There are 2 red and 2 green beads in every 4 beads. 4 x 7 = 28, which is 14 red and 14 green beads. The last 2 beads, to make 30, are red. This makes a total of 16 red beads and 14 green beads.

b. 10. There are 10 lots of 2 cm in 20 cm, so there would be 10 spheres, one in each group of the shapes.

5. **a.** Practical activity

b. Answers will vary. You could use your 5 times table to help you. For example, if you have 2 triangle beads in your pattern and you repeat it 5 times, you could work out how many triangular beads you need by using 5 x 2 = □.

Pages 22–23: Joust Away!

Activity

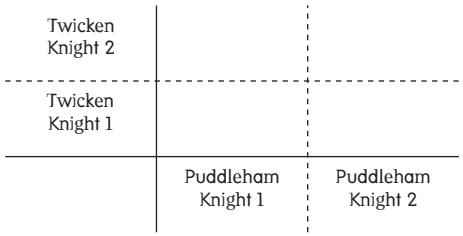
1. **a.** 6. (Each Dipshire knight has 2 jousts. 3 x 2 = 6)

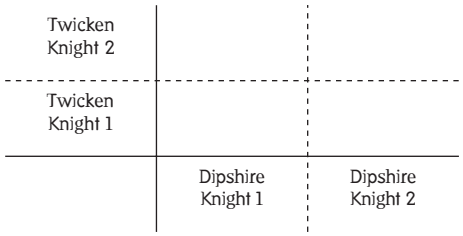
b. **i.** 8. (2 Puddleham knights x 4 knights in Dipshire team is 2 x 4 = 8 jousts.)

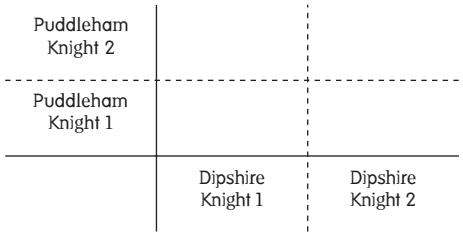
ii. 9. (3 knights in Puddleham team x 3 Dipshire knights is 3 x 3 = 9 jousts.)

2. The greatest number of jousts possible is by splitting into a team of 3 and a team of 4: 3 knights x 4 knights is 12 jousts. Other options are 1 knight x 6 knights (6 jousts) or 2 knights x 5 knights (10 jousts).

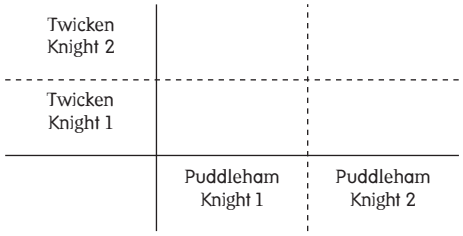
3. **a.** Based on Sir Lancelot's diagram, you could have 3 or 2 diagrams.
As 3 diagrams:

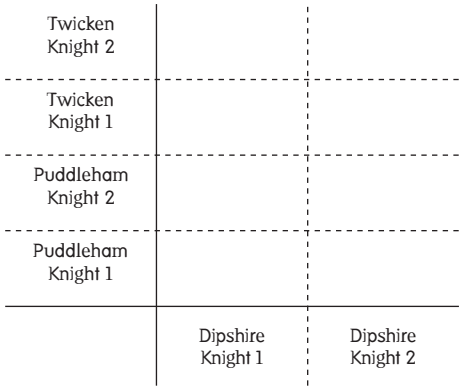






As 2 diagrams:





b. There will be 12 jousts altogether. 2 teams of 2 is 2 x 2 = 4 jousts; each of those 4 knights has to joust with 2 more knights as well:
4 x 2 = 8; 4 + 8 = 12 jousts.

Page 24: Hydroslides

Activity

1. 2 times as long. ($\square$ x 12 = 24 or 24 ÷ 12 = 2)

2. **a.** 36 m. (3 x 12 = 36 or [3 x 10] + [3 x 2] = 30 + 6 = 36)

b. 18 m. (36 ÷ 2 = 18 or $\frac{1}{2}$ of 30 is 15 and $\frac{1}{2}$ of 6 is 3: 15 + 3 = 18)

3. Speeder: 1 min; Ultimate: 3 min

4. Answers may vary. If you had 4 Ultimate slides, you would finish 2 min early (starting from the top: 3 + 2 + 3 + 2 + 3 + 2 + 3 = 18 min, with 3 x 4 = 12 min actually sliding). You would also get 12 min on the slides if you had 3 Ultimate Slides, 1 Twister slide, and 1 Speeder slide: 3 + 2 + 3 + 2 + 3 + 2 + 2 + 2 + 1 = 20 min. (Note that, in real life, you would often do more waiting than sliding!)

Figure It Out

Multiplicative Thinking Teachers' Notes

Overview: Levels 2–3

Title	Content	Page in students' book	Page in teachers' book
Party to the Max	Using multiplication to solve problems	1–3	11
Marble Mania	Solving division problems	4–5	12
Face the Facts	Using halving and/or doubling to solve multiplication problems	6–7	14
Sushi Surgery	Using multiplication strategies	8–9	15
Class Farewell	Solving multiplication problems using groups	10–11	16
Fishy Business	Using multiplication to solve equal-groups problems	12–13	17
Survivor Challenge	Solving division problems	14–15	19
Fiordland Holiday	Using multiplicative strategies to solve problems	16–17	20
Clever Clues	Using multiplication to solve repeated addition problems	18–19	22
Friends Forever	Using multiplication to solve equal-groups problems	20–21	23
Joust Away!	Using multiplication to solve combination problems	22–23	24
Hydroslides	Solving comparison problems	24	26

Introduction to Multiplicative Thinking

Multiplicative thinking is the term used to describe thinking that employs the mathematical power of multiplication. Students need to use the properties of multiplication in order to understand many areas of mathematics, such as area and volume, the metric measurement system, fractions, and algebra.

The Number Framework outlines the way that children seem to build their multiplicative understandings on their additive ones, just as they previously built additive understandings out of their knowledge of counting. Understanding multiplication as repeated addition seems to be an important step for many students and is often a starting point for instruction.

Many problems are multiplicative, and while the most commonly used strategy is repeated addition or equal groups, we also need to include other models of multiplication to extend and deepen students' understanding. The Figure It Out *Multiplicative Thinking* books include activities that explore multiplication as a comparison, a rate, an array, and a Cartesian product. From each of these problem types, students will begin to extract the key ideas of multiplication as commutative, distributive, and associative. Understanding how multiplication works helps students to use the strategies outlined in the Numeracy Development Project booklets and in these Figure It Out books.

Division is the inverse of multiplication in the same way as subtraction is the inverse of addition. Traditionally regarded as the most difficult of the operations to understand and perform, division, research suggests, is best learned alongside multiplication. Situations involving division can sometimes be solved using multiplication, particularly when the numbers are in the basic facts range. Recognising the relationship between multiplication and division is a very important task for students.

To work effectively with multiplicative strategies, students need to know their basic facts. In the initial stages, they will use skip-counting and materials to develop a conceptual understanding of what multiplication statements mean. This emerging concept feeds into, and is fed from, knowledge of basic facts. Recall of basic multiplication facts does not equate with multiplicative thinking, but multiplicative thinking will be facilitated by a sound knowledge of these facts.

A change in students' thinking is needed if they are to fully understand multiplication. In multiplication, they encounter for the first time numbers that are telling them to perform an operation rather than numbers that represent an amount. For example, in 4 x 3 (which can be read as four sets of 3 or as three sets of 4), one number tells how many times the other should be repeated rather than representing an amount (a set of 4 plus a set of 3). However, solving multiplication problems additively like this can be unwieldy; students who continue to use this method will have problems with more sophisticated concepts.

The activities in the Figure It Out *Multiplicative Thinking* books use a range of problem types, consider strategies based on the properties of multiplication, ask students to explore these properties, and include division as the inverse of multiplication. In doing so, the books aim to support teachers in developing a robust and deep understanding of multiplication in their learners.

Pages 1–3: Party to the Max

Achievement Objectives

- demonstrate the ability to use the multiplication facts (Number, level 2)
- recall the basic multiplication facts (Number, level 3)
- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)

Number Framework Links

AC
EA
AA
AM
AP

Use this activity as a guided learning experience with groups at stage 4 or stage 5. It could be used as an independent activity by groups at stages 5–6.

Activity One

This activity presents simple multiplication and division problems in a range of ways. It provides a context for students to use and share their strategies and will also provide teachers with assessment information through observation.

The party context provides opportunities for exploring different multiplication problems and strategies. The problems emphasise different aspects of multiplication and are designed to encourage students to develop multiplicative strategies for solving them rather than relying on their additive strategies. While the numbers involved in all the problems are of a similar magnitude, students may find some more difficult than others because of the way in which a particular problem is posed.

For question **1**, ask the students to identify what they need to know to answer Max's question (How many cartons of each brand would he get for $20?), then how they will work it out. If the students want to make an array with materials, use cardboard to shield their array (or the ones on the student page) as you discuss how they could work out the answer quickly, without skip-counting. Collect students' strategies on a shared chart and discuss them.

There are two parts to question **2**: "How many lots of $2 are in $8?" and "How many lots of 5 apples will we therefore get?" The question looks very simple in its presentation but contains a lot of thinking. Ask the students to share their ideas and look for students who explain the multiplicative relationship between $2 and $8 and 5 and 20. For those who find this difficult, pose some further problems based on these numbers. For example: *How many apples would we get for $10? For $4?,* and so on. *If we got 4 apples per kilo, how many would we get for $8?*

The problem in question **3** could be solved using any of the four operations. Have the students share their strategies for this problem and record them in equations. Challenge the students who are using addition or subtraction to explain the multiplication/division methods that others may have used.

Question **4** adds the complication of "some left over". You can extend it by asking how much Max and his mother will pay for the party blowers they need or how many blowers there would be if they bought 3 packs.

Question **5** is a two-step problem, like question **2**. Watch to see if students can extend their solution methods to a new context. Part **b** adds further steps. For students who are struggling to use number properties or imaging, use containers or pieces of card to model the packs and counters for the balloons. Let the students use skip-counting or repeated addition to find a solution and then challenge them to find a pattern that could help them.

Activity Two

Question **1** is multiplication posed as a rate problem: "how many per?" Using the pattern for 1 fruit kebab, the students need to extrapolate to 10 kebabs. If they are having trouble solving the problem without materials, they could explore this using beads to represent the items on the kebab sticks. If they are using materials, ask them to make 5 kebabs and then see if they can find the solution without having to make all 10. Students who need further work could make their own kebab pattern, using beads or shape blocks, and then pose similar questions to others within the group.

Question **2** uses a Cartesian product presentation of multiplication, which many students find difficult to understand. In this problem type, the question is essentially "How many different pairs will these objects make?" Each of one type of object (in this case, vege bites) is paired with each of the other type (dips) to form a set of pairs. The students can complete the table on the student page to make a matrix showing the pairs. If students want to list the pairs, encourage them to be systematic as they form them: *How will you know if you have got everything?*

For question **3**, students may struggle to structure a problem that is multiplicative. To get them started, you could use the problem types on these pages to help them frame a new problem, keeping the mathematical structure and changing the numbers and/or the context. Students will usually need a lot of support to write mathematics questions that "work". Have a group discussion and sharing session before asking the students to complete this task. Perhaps write a few questions yourself and ask the students to comment on them. A common mistake that students at this level make is to write addition situations thinking they are multiplications, for example: "I had 3 bananas, and I bought 4 more. How many do I have?" Help students by giving them suggestions of objects that have groupings embedded in them (for example, 12 biscuits per packet, 6 drinks per bottle).

Links

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 6: Teaching Multiplication and Division*
  Party to the Max would be a useful follow-up and practice activity for any of the lessons suggested for advanced counting and early additive learners in Book 6.

Figure It Out

- *Basic Facts*, levels 2–3
  Students will find it easier to solve the different problem types in this activity if they are secure in their basic facts. Games such as Dicing Times (page 2), Magnificent Multiples (page 14), Loopy (page 23), and Six Shooters (page 24) would be useful independent activities to encourage basic facts recall.

Pages 4–5: Marble Mania

Achievement Objectives

- write and solve story problems which involve halves, quarters, thirds, and fifths (Number, level 2)
- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- demonstrate the ability to use the multiplication facts (Number, level 2)
- recall the basic multiplication facts (Number, level 3)
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

The use of numbers within the basic facts range makes this activity appropriate for students at stage 4 to solve in a guided group and for students at stage 5 to attempt with some guidance. Students at stage 6 should be able to complete this independently.

Activity

This activity uses marbles, a game with which most students are familiar, as a context for division problems. It presents the two types of division problem: sharing division and measurement division (see page 5 of Numeracy Development Project *Book 6: Teaching Multiplication and Division*). In sharing, discrete objects are shared out into a number of groups (for example, biscuits into packets). In measurement division, continuous amounts are divided into equal parts (for example, fabric into lengths). In this activity, the students share the marbles and then look at dividing metre lengths. The students in your class may find the discrete sharing problems easier to conceptualise and solve than the measurement problems. Both problem types can be solved using concrete models to aid understanding. Remember to encourage students to apply the number facts they know to these problems to develop the use of number properties.

For question **1a**, share the methods that the students devise. Make a shared record and discuss the different strategies. *Which do you think is the "smartest" way to solve this problem?* Encourage the students who are using addition or subtraction to explain the multiplication/division methods that others may have used.

For part **b**, challenge the students to look for a pattern and to organise their answers in a way that shows that they know they have found all the possibilities. This leads to a worthwhile discussion of how we know we have found all the factors of a number. Although students at this stage will not be able to form an abstract generalisation for this, they will be able to explore aspects of it. For example, using a Happy Hundreds sheet (material master 6-5) or counters, the students can make rectangles that have 24 objects in them; the sides will then be factors. Encourage them to be systematic: *Can you make a rectangle that uses all 24 counters in rows? Which rectangle has a side of 2 counters? 3 counters?* When you get to 6, prompt the students to see how many counters are on the other side. If they record the equations, they will see a pattern and can predict what other pairs may be left.

Question **2** is a sharing problem that can be modelled if necessary. Encourage the students to explore how they can use what they know to help them solve the problem, for example, "I know that if it was 3 marbles for 3 people, we would need 9 marbles, so it's more than 3 …"

Question **3** involves measurement division. For **3a**, the students need to partition a continuous 4 metre length. They may need to make a 4 metre length (or a scale model of that length), which they could then fold and label, using sticky-backed notes to give them reference points. Again, draw links between the mathematics of this question and facts that the students know. For **3b**, emphasise the connection between 3 twos in 6 and one-third of 6 being 2. These two expressions mean the same thing in this context but often become disconnected in students' thinking.

For question **4**, see the suggestions in Party to the Max about helping students to write problems. They may tend to write addition problems or subtractions such as "I lost some marbles." Use containers or pieces of cardboard to represent bags of marbles and get them to write a problem that matches the bag of marbles you have modelled.

Links

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 6: Teaching Multiplication and Division*
  For students at stages 4–5:
  Biscuit Boxes (using repeated addition to solve division problems), page 8
  Pirate Crews (sharing into equal sets), page 7

For students at stages 5–6:
Long Jumps (solving division problems, using repeated addition and known facts), page 19
Goesintas (solving division problems, using multiplication), page 20

Pages 6–7: Face the Facts

Achievement Objectives

- demonstrate the ability to use the multiplication facts (Number, level 2)
- recall the basic multiplication facts (Number, level 3)
- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)

AC
EA
AA
AM
AP

Number Framework Links

This activity would be suitable for guided instruction with students at stages 5 or 6. Students at stage 7 may be able to do this independently.

Activity

These pages use a powerful model of multiplication, the array, to help students develop multiplicative rather than additive strategies. The array embodies the structure of multiplication, with one factor represented by the length and the other by the width. The structure may seem obvious to us, but research suggests that students do not automatically recognise the pattern of rows and columns. It is important to bear this in mind as you watch your students work with the happy faces (see also material master 6-5). Get them to talk about what they can see in the pattern to determine if the mathematics of these activities makes sense to them. There is a lot of mathematics embedded in these activities; this page could form the basis of rich work for a week with a group at stages 5 or 6.

You could introduce this activity with some everyday objects that have an array structure, such as egg trays/cartons, muffin pans, or inside trays from boxes of chocolates. Ice cube trays and boxes of apples are used on page 9 of *Basic Facts*, Figure It Out, levels 2–3. This could be used to support students who are having difficulty with the larger collections on these pages.

For question **1**, it's important that students are able to manipulate the faces to see the new arrangement of rows and columns. This will encourage multiplicative thinking. The reasoning that Elton is describing requires thinking about the factors and the product in relation to each other and predicting what will happen if the amounts change. To extend this activity, ask the students to make predictions like Elton's, based on what they have observed: "I know … so would …?" Use the faces to explore these predictions.

Question **2** again confronts students with multiplicative reasoning. Encourage lots of discussion to see if the students can follow what Elton is doing and provide additional examples such as 10 rows of 4 or 10 rows of 8 if further reinforcement is needed.

To solve question **3**, students are asked to employ the reasoning from the earlier questions. Listening to your students' answers and observing who can solve this without the faces or who is using additive methods will give you useful assessment information.

Question **4** further extends the use of halving and doubling because it moves away from the faces model to working with just the numbers. This reflects the Numeracy Development Project teaching model and seeks to get students working with number properties. As above, it will provide a useful checkpoint for your students' thinking. Extend this by setting up some trains of your own in which halving and doubling would be a useful strategy and then challenge the students to think of some other examples. Part **b** is an open-ended opportunity for students to apply this strategy. Students who use fractions, as in part **a ii**, are showing effective multiplicative thinking.

Consider finishing this work with a discussion of when halving and doubling might be "the best" strategy to use. Ask:

- *Does halving and doubling work with any multiplication?*
- *Why is halving and doubling more appropriate in some situations than in others?*
- *What sorts of numbers does halving and doubling work well with?*

Students need to learn when to apply particular strategies as well as knowing how they work.

Links

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 6: Teaching Multiplication and Division*
  Twos, Fives, and Tens, page 9
  Cut and Paste (halving and doubling, thirding and trebling), page 25
  Students working on the Twos, Fives, and Tens lesson in Book 6 will be exploring ideas that will help them with Face the Facts, although they would need to be using number properties to understand the more difficult questions. Face the Facts would be a good independent follow-up to the Cut and Paste lesson.

Figure It Out

- *Basic Facts*, levels 2–3
  An Apple a Day (introduces arrays), page 9

Pages 8–9: Sushi Surgery

Achievement Objectives

- demonstrate the ability to use the multiplication facts (Number, level 2)
- recall the basic multiplication facts (Number, level 3)
- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)

AC
EA
AA
AM
AP

Number Framework Links

This is a useful activity for guided instruction with groups at stage 5. It would also be useful for challenging additive thinking for students using stage 4 strategies, but the students would need the support of materials to visualise the groupings. Students at stage 6 should be able to attempt this independently.

Activity

This activity builds multiplicative thinking by presenting students with a context in which "wholes" (rolls of sushi) are split and regrouped (boxes). It also uses the array model discussed in Face the Facts. The students are asked to record their ideas as equations to help them see the pattern (that 4 rolls of 8 pieces will become 8 boxes of 4 pieces). The grouping changes, but the total number of pieces of sushi remains the same.

For question **1**, you could help students who are struggling by "making" sushi rolls out of multilink blocks, forming sticks that could be broken up and regrouped as needed. Alternatively, sushi rolls could be made of playdough and cut into rounds. While it may be useful to start off with physical models, remember to challenge the students to move towards using number patterns to solve subsequent problems. Continue the pattern by asking them how many pieces of sushi there are in 6, 8, 10 rolls and so on.

Questions **2** and **3** are designed to get the students to visualise the groupings, changing from rolls to pieces to boxes. In multiplicative thinking, students need to see numbers as comprising different groupings and to develop flexibility in changing them around. Use concrete materials, as suggested above, to aid students who become confused by this.

Question **4** uses the array structure but deliberately focuses students on the length and width of the rectangles. It prompts the students to use basic facts to solve the problems rather than counting the squares or skip-counting. Encourage this and look for students who can apply the strategy to solving part **b** of the question.

Question **5** also asks the students to formalise their thinking in equations. Look for students who write addition equations and those who write multiplications. Use a discussion of strategies, during which students explain their thinking, to help the students writing additions.

Encourage the writing of multiplication problems for question **6** by providing materials to model problems, such as boxes to put pieces in and blocks to use as sushi pieces. Ask the students to make up some "sushi boxes" and then write a problem that matches what they have made. This is quite challenging. Pairing students will allow them to discuss their ideas. (See the notes for question **3** of Party to the Max for ways to help students when they are making up their own problems.)

Links

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 6: Teaching Multiplication and Division*
  For students at stages 4–5:
  Biscuit Boxes (using repeated addition to solve division problems), page 8
  Pirate Crews (sharing into equal sets), page 7
  For students at stages 5–6:
  Long Jumps (solving division problems, using repeated addition and known facts), page 19
  Goesintas (solving division problems, using multiplication), page 20
  Turn Abouts (exploring the commutative property), page 17

Figure It Out

- *Basic Facts*, levels 2–3
  An Apple a Day (array structure), page 9
  A Sticky Problem (sharing out into packets), page 20

Pages 10–11: Class Farewell

Achievement Objectives

- demonstrate the ability to use the multiplication facts (Number, level 2)
- recall the basic multiplication facts (Number, level 3)
- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- make sensible estimates and check the reasonableness of answers (Number, levels 2–3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity as a guided learning experience with groups at stage 4 or stage 5. It could be used as an independent activity by groups at stages 5–6.

Activity

This activity provides a context for applying skills of deriving facts from known facts. The context will be familiar to many students (the $2 bargain shops) and is used to pose questions with a range of groupings. The use of 24 students in the class allows for groupings of 2, 3, 4, 6, 8, and 12. The questions build to finding the total cost for all the items. For this, the students will list the items chosen and multiply by $2.

There are two steps to question **1**. Students may work out how many packets are needed for 1 popper each and then multiply that number by 3, or they may work out how many poppers 3 per person would be and then find the number of packets. The prompt on the page suggests finding out how many students 1 packet will provide for, which is a third method. Encourage your students to begin by describing how they are going to tackle the problem; get them to rephrase the question in their own words to show that they understand the multiplicative structures that are implicit within it. Then ask them to record their solution strategies and to share them in discussion.

Questions **2** and **3** extend the ideas described in question **1**, using new amounts. To extend your students further, ask additional questions: *What if there were 30 people in the class? What if we wanted twice as many balloons?*

For students who are struggling, model the groupings using beans or counters and snap-top bags for the products. To encourage imaging, make 1 bag of each "product" and then let the students use the single bags to work out the problems rather than using multiple bags of each grouping. Discuss with the students how they could use the 1 bag to help them solve the problems.

Question **4** focuses on money rather than on item amounts, although the students will need to use all their earlier answers. Ask them to estimate how much they think the total might be so that they have an idea of a reasonable answer before they do any actual working out.

You could use the activity in guided instruction up to this point and then ask the students to use what they have found to complete question **4** independently or in pairs. This helps to develop number sense in calculation. The resulting work, and their own lists from question **5**, could be shared at your next group session.

Links

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 6: Teaching Multiplication and Division*
  Fun with Fives (deriving unknown facts from known 5 times facts), page 12

Figure It Out

- *Basic Facts*, levels 2–3
  Students will find it easier to solve the different problem types in this activity if they are secure in their basic facts. Games such as Dicing Times (page 2), Magnificent Multiples (page 14), Loopy (page 23), and Six Shooters (page 24) would be useful independent activities to encourage basic facts recall.

Pages 12–13: Fishy Business

Achievement Objectives

- demonstrate the ability to use the multiplication facts (Number, level 2)
- recall the basic multiplication facts (Number, level 3)
- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)

AC
EA
AA
AM
AP

Number Framework Links

This activity would be ideal for students at stage 4 who are being helped to move on from using additive strategies. Students at stage 5 may still need some guidance, while students at stage 6 should be able to use this as an independent activity.

Activity

This activity provides a rich context for seeing division and multiplication as inverse operations and encourages students to use multiplication to express an "equal-groups" model of multiplication. When equal-groups problems are posed to students, they often respond with a repeated addition or skip-counting strategy rather than seeing it as a multiplication. The structure of an equal-groups situation prompts this response because it is a repetition of one group: an obvious response is to start with the first group and add on the next and so on. Multiplication, however, provides a more powerful and easier way to solve these problems, one that will work when the numbers are too big for efficient addition.

Arnold's log book in question **1** explicitly shows the repeated addition strategy. However, Hint Crab shows a shorter way, using multiplication instead. Discuss with your students what Hint Crab is saying. *Is this true? Can you explain why? Use materials to help you explain …* The students are then given the opportunity to apply this to the other critters to see if they can generalise the idea. They may need to write the additions first and then say the groupings aloud, as in the hint, in order to remember that what they are looking for is "How many groups?" You may need to remind the students that each tank contains 1 group of a particular critter.

Question **2** works the other way, with critters going into the tanks. The multiplication is times 10 (because there are 10 tanks). This should be straightforward, but because there are two groupings of critters in each tank, discussion may be necessary to establish this. Arnold's speech bubble provides a clue. Going back to using materials might help students to model the situation, but remember that the aim of this is to use multiplication rather than to skip-count or add, so once students have sorted out their thinking by modelling the tanks, you could shield the model to encourage using multiplicative strategies.

For question **3**, discuss how the students might solve the problem and then use modelling if necessary. Encourage the students to include a multiplication equation as a part of their answers.

Students at stages 4 and 5 may need to use counters for question **4**. However, before they do use materials, prompt them to think about the numbers to see if they can solve each part of the question without modelling. Point out that they know the total number of fish and the number of groups (tanks) to share them into; their task is to find the number of fish in each group.

Questions **5** and **6** are useful summary activities that a group of students could go and do independently or in pairs following your discussion. Their answers will vary, so you could begin your next session with them by having them share their ideas. You might want to use a small magnetic whiteboard or piece of card to be a "fish tank". Ask the students to draw some critters and put magnets or something sticky on the back of them. They can then put some critters "in" the tank. If you then specify how many tanks there are, they can work out how many critters altogether. In this way, you can generate lots of multiplications to work out, perhaps as part of a whole-class warm-up.

Links

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 6: Teaching Multiplication and Division*
  Long Jumps (solving division problems using repeated addition and known facts), page 19
  Goesintas (solving division problems using multiplication), page 20

Figure It Out

- *Basic Facts*, levels 2–3
  A Sticky Problem, page 20 (similar "sharing out" in a different context)

Pages 14–15: Survivor Challenge

Achievement Objectives

- demonstrate the ability to use the multiplication facts (Number, level 2)
- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)

Number Framework Links

AC
EA
AA
AM
AP

This activity is suitable for group instruction at stage 5. Some of the questions will challenge students at stage 4 in terms of the strategies they might think of using, but that may make it a worthwhile activity for seeing how these students cope with problems involving fractions and continuous models for division. Students at stage 6 could attempt this independently and bring it to a group session for discussion.

Activity

This activity presents simple multiplication and division problems in a variety of ways. It provides a context for students to use and share their strategies. You could also use it for getting assessment information through observation.

Some of the objects to be shared are discrete items, such as sticks and fish hooks. Others are continuous, such as fishing line and groundsheets. This provides students with the opportunity to solve different types of divisions. Dividing something like a long string into pieces is different from sharing out items like sticks. With the items, you can see the things to be shared and can keep giving them out until they are gone. With the string, you can't see the "bits"; they have to be formed by splitting the length into equal pieces.

Encourage the students to use known facts to solve question **1**. They could then move from $3 \times \square = 30$ to exploring $30 \div 3 = 10$. If necessary, you could link this to $10 + 10 + 10 = 30$ to make it clearer.

Parts **a** and **b** of question **2** involve the continuous model. In **2a i**, the fishing line is continuous. If the students need help with solving this problem, you could use a number line or a measuring tape to represent the fishing line so that the students can visualise dividing it into segments. Similarly, in **2 a ii**, the groundsheet problem involves dividing a continuous length. The numbers here are also tricky because the answer involves a fraction. This further emphasises the continuous model. When sharing objects, we end up with some "left over" (such as in the fish-hook question) rather than dividing into fractional parts. In the case of the groundsheet, the whole groundsheet is used rather than bits being chopped off, so the students have to work out that each person will get 1 metre and a share of the leftover metre. The other difficulty with this problem is that the groundsheet is an oblong, with two dimensions. Discuss this with the students and agree on dividing the 4 metre length to find out how much space each child would get. You could extend this problem by measuring some of your students and seeing how much space they take up in two dimensions – how big an oblong they would occupy. The students could then explore other ways to fit 3 people on the groundsheet. A powerful illustration of this would be to use centimetres for measuring the students and then convert the groundsheet dimensions into centimetres as well.

Question **2c** reverts to discrete items. The students should realise, from $2 + 2 + 1$, that they need to use their 5 times table.

The meat patties in question **3** are discrete objects to be shared among the group. Encourage the students to try to use what they know to solve the problem in **a** rather than modelling the sharing. The next two problems (**b i** and **ii**) use the same context to pose multiplication problems. Although addition and subtraction can be used, encourage the students to think multiplicatively. The numbers are useful for encouraging students to use doubling strategies or tidy numbers. You could use this question to discuss remainders with the students: *What might Ms Brook do with the leftover slices of bread?* Students who are beginning to think proportionally could consider what fraction of a loaf is left.

Links

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 6: Teaching Multiplication and Division*
  Survivor Challenge is a useful follow-up and practice activity for any of the lessons suggested for advanced counting and early additive learners in Book 6.

Figure It Out

- *Basic Facts*, levels 2–3
  Secret Codes, page 17

Pages 16–17: Fiordland Holiday

Achievement Objectives

- demonstrate the ability to use the multiplication facts (Number, level 2)
- recall the basic multiplication facts (Number, level 3)
- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)

AC
EA
AA
AM
AP

Number Framework Links

The strategy prompts included with this game suggest that it will be most appropriate for students at stage 6. Students at stage 5 may be inspired to try new strategies through playing the game, although they may need help to make these connections.

Game

This board game uses multiplication strategies to calculate a score. The context is a Fiordland holiday, and the game features native birds, plants, and animals. (The information on the game cards may stimulate students to use resources such as the Internet to do some research, for example, on the endangered or rare species or places of interest in Fiordland.)

Note that a colour version of the game board is provided in the centre of these notes. You could use this as it is or photocopy it on a colour photocopier and laminate it. A black and white version for photocopying is also provided (see copymaster), along with the game cards, which could be photocopied onto coloured paper for added appeal.

The game instructions explain the basics of the game, and the facing page provides students with strategy suggestions. The multiplications in the game have been chosen to fall within the range of what students at stage 6 should be able to do mentally. The score is important, so the responsibility for checking whether or not the multiplications are correct will fall to other players. (The added incentive for players to check each other's scores is that the player whose turn it is forfeits points for an incorrect answer!) This gives all the players plenty of practice in using multiplicative strategies. To ensure that the extra practice is in fact taking place, you might want to emphasise that the checking has to be done without the aid of a calculator.

You could change the checking and forfeiting instruction to suit the groups using the game. For example, in a mixed-ability group, those better at using multiplicative strategies could help the weaker players to work out their score, without any penalty. A group of lower stage students could all be helping each other, again without penalty. However, for a higher stage group, catching out another player and seeing them lose points may well be a good incentive to do the extra working out!

The game could be introduced to the whole class initially. It could be played by the class divided into two teams, using an enlarged version of the game board as a shared resource. This would allow you to demonstrate the rules and to show how to keep a running total.

To help students keep a running total while they are playing, you could provide them with a thousands book or a hundreds square to help them calculate their total. Alternatively, you could allow the students to keep the running total on a calculator, emphasising mental calculation of the multiplications on the game squares and cards. (They may need to jot down their scores on paper as well after their turn, in case they lose track or press a wrong button on the calculator.)

If you are playing with an instructional group, you could ask the students to refer back to the list of strategies on page **17** when they have a problem to solve and to say which they think would be best for the problem they have before them. This encourages students to make meta-level choices about appropriate strategies and to think about the numbers they have been given and how those numbers dictate what will be a useful method. Sometimes a strategy is suggested; discuss why the numbers make this strategy particularly useful. We don't want students to memorise particular strategies because this can become like learning an algorithm. The aim is for them to develop a range of strategies so that they can select the best one for a particular situation. Some strategies (such as doubling and halving) are only effective with some combinations of numbers. This game prompts the use of some strategies that students are less likely to use spontaneously, although they suit the numbers given. This is an attempt to broaden the students' strategies rather than to tell them to master a particular method. As always, discussion about strategy selection is vital.

You could simplify the game by making a set of cards with easier multiplications on them. Groups of students could add to the game cards with facts and ideas of their own. Adaptations of the game could be made to match topics of study, such as the Rocky Shore or Space.

Links

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 6: Teaching Multiplication and Division*
  The Fiordland Holiday game provides an opportunity to use, discuss, and apply various multiplication strategies. As such, it could be used as an adjunct to any of the lessons in Book 6.

Figure It Out

- *Basic Facts*, levels 2–3
  The activities in this book are designed to reinforce basic facts recall and will help children play this game and be able to strategise. Games such as Dicing Times (page 2), Magnificent Multiples (page 14), Loopy (page 23), and Six Shooters (page 24) would be useful independent activities to encourage basic facts recall.

Pages 18–19: Clever Clues

Achievement Objectives

- demonstrate the ability to use the multiplication facts (Number, level 2)
- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- write and solve comparison problems (Number, level 2)

AC
EA
AA
AM
AP

Number Framework Links

Students at stage 6 will be most able to see the relationships and use the strategies needed in this activity.

Activity

This activity presents multiplication as multiplicative comparison problems. Students at all levels may need discussion to get them "into" the problems, which encourage doubling and halving and seeing a multiplicative relationship between amounts.

While the basic arithmetic of all multiplication problems is the same, the different situations that call for multiplication to be used have quite different structures. In this level 2–3 book, the Fishy Business problems are equal-groups problems, the most common way that we present multiplication to students. The Joust Away! pages are Cartesian product problems, in which two factors are combined to make pairs. Face the Facts uses an array model.

Multiplicative comparison problems express the relationship between things as "times as many", comparing the two amounts with a multiplicative relationship. Although the arithmetic in these problems is simple, this presentation of multiplication is harder for students to decode. The multiplication is describing relationships between numbers rather than an operation to be performed, making it more difficult to extract what should be done. The students may read the clues and think that they don't have enough information to solve the problem.

For question **1**, discuss with the students where to start: *What do we know? What do we need to work out next?* They need to use Rion's age to work out Hannah's age, and then they can work out Joe's age. The only clue for Joe's age is what it is in relation to Hannah's age.

Question **2** is slightly simpler because both unknown quantities are described in comparison to a known amount. Encourage your students to try this and see if they can apply what they learned while solving the first question.

The students need to be able to generate their own statements in question **3**. The "more than" statement is an additive comparison (sometimes solved as subtraction, but it is an additive process). The "times as much as" is a multiplicative comparison. This may require lots of discussion and support to generate. Modelling the amounts using 50 cent pieces might be helpful.

Encourage the students to use what they have already learned when they tackle the problem in question **4**. (There are several pairs of numbers that could fit Rion's and Hannah's clues, but Joe's clue tells you which to select.)

Question **5** involves comparison problems. The students will probably find writing these difficult. First, they need to identify a multiplicative relationship, and then they need to encode it in a problem format. They will probably think of additive problems instinctively: "I have 4 brothers and 2 sisters: how many siblings do I have?" Help them to alter this to a multiplicative statement: "I have twice as many brothers as sisters, and I have 6 siblings altogether … How many brothers and how many sisters do I have?"

Preparing some pictures of different families for them to write about may help. Height comparison is a useful context ("I am half as tall as … and twice as tall as …") or perhaps the number of pets in a family ("We have 4 times as many fish as guinea pigs …"). Drawing the relationship will give them something to refer to when they are trying to make it into a word problem.

Links

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 6: Teaching Multiplication and Division*
  Twos, Fives, and Tens, page 9
  Cut and Paste (halving and doubling, thirding and trebling), page 25

Figure It Out

- *Basic Facts*, levels 2–3
  Flying Fish Soup (uses doubling and halving as a multiplicative comparison in the context of a recipe), page 14

Pages 20–21: Friends Forever

Achievement Objectives

- demonstrate the ability to use the multiplication facts (Number, level 2)
- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- write and solve comparison problems (Number, level 2)

AC
EA
AA
AM
AP

Number Framework Links

This would be a useful guided instruction lesson with students at stage 5. Their preference for using addition to solve equal-groups problems can be challenged through discussion and exploration.

Activity

The calculations in these problems are straightforward, but the context makes them more difficult. Each pattern of beads in this activity forms an equal-groups structure, but the repetition of the pattern in a continuous form makes it harder for students to pick this out. The students will need to use their knowledge of multiples to help them work out the problems. They may begin by modelling or drawing the pattern. The clues with the questions prompt them to use their number knowledge.

You can generate additional examples by asking the students to form a repeating pattern by threading beads or using counters or attribute blocks. Then you can ask other questions based on the students' patterns. With attribute blocks, you can make the questions more complex by "overlaying" patterns. This would be a good activity to do before the students make their own designs in question **5**. For example, the attribute-block pattern might go thick-thin-thick-thin, square-circle-hexagon-triangle-square-circle-hexagon-triangle, and red-blue-yellow-red-blue-yellow at the same time. Students could predict whether a thin square would ever appear and so on.

For question **1**, explain the context, allowing students to thread some repeating patterns if necessary. Make sure they can see the pattern in the beads. Use the prompts to guide discussion about ways to find out the colour of the later beads. Encourage answers that use number properties, especially tables and multiples.

Question **2** asks the students to see the pattern "chunk" as a unit to be repeated to a certain length. At the same time, they have to apply this "chunking" to the two parts of the pattern, which is a useful encouragement of part–whole thinking.

The pattern in question **3** is more complex because it varies in both shape and colour. The question asks the students to explain how their times tables might help them, thus making explicit the number properties solution strategy we want students to move towards. These questions, especially the "How do you know?" part, will stimulate explanations of strategies. Recording these as a group and developing a shared understanding would be a powerful way to help those students who are solving the problem by drawing the pattern.

In question **4**, students need to use the information in the diagram (3 shapes = 2 centimetres) and their 10 times tables. This question extends question **2** by including shape and colour elements.

For question **5**, look carefully at the patterns the students design to see who can manage to make a pattern with shape and colour elements. Ensure that they can see the chunks of pattern that will be repeated. If they can't discern the equal-groups structure, they won't be able to use multiplicative thinking to solve the problems. Making the patterns with beads and putting some questions on cards would make a good display that other students could use as an independent activity. Bead patterns could hang from the ceiling with questions attached (for example, "What colour will the *x*th bead be?"), and other groups could discuss and record their ideas.

Links

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 6: Teaching Multiplication and Division*
  Fun with Fives (deriving unknown facts from known 5 times facts), page 12
  Multiplication Smorgasbord, page 27

Figure It Out

- *Basic Facts*, levels 2–3
  Use games such as Dicing Times (page 2), Magnificent Multiples (page 14), Loopy (page 23), and Six Shooters (page 24) to improve students' basic facts recall. This will increase their ability to use number properties to solve the types of problems in Friends Forever.

Pages 22–23: Joust Away!

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- record, in an organised way, and talk about the results of mathematical exploration (Mathematical Processes, communicating mathematical ideas, levels 2–3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)

AC
EA
AA
AM
AP

Number Framework Links

This activity is suitable for encouraging students at stages 5 and 6 to use multiplicative rather than additive thinking. Students at stage 4 could use the page in a guided instruction group, but they may not recognise the problem structure as multiplication.

Activity

This activity represents another situation in which multiplication is the means of working out the problem, but the structure of the problem differs from the common equal-groups structure. Combination problems can be a difficult presentation of multiplication, so while the calculations in this activity are within students' basic facts range, the structure they are embedded in might make it hard for them to recognise this.

It is very important that students are exposed to a variety of problems that embody different structures so that they develop a fully robust understanding of the power of multiplication. Equal-groups problems can readily be solved by repeated addition, but problems such as the ones in this activity require a different sort of thinking. There are not obvious groups to be repeated. In this structure, two factors are combined to form a set of pairs. The number of pairs is the solution to the problem. They are also known as Cartesian product problems and are thought to be the most difficult for students to understand. (Students need to know the concept to do this activity, but they do not need to worry about the terminology.)

A motivating way to introduce these pages would be with model knights and horses (or you could use chess pieces: the black and white knights could represent two different teams, and the black and white horses could represent two more teams). These could be used to "act out" the jousts, emphasising to the students the pairings of knights required in the tournament.

One way to start the activity is to pose the question in **1a** without the students looking at the actual page that it is on and, in particular, without looking at the diagram that relates to this question. This will enable you to see how the students might approach the problem. Ask the students to keep a systematic record of their answers. Then you can look at the diagram on the page and discuss how it works: *How does it help us to find the answer?*

As the students move to question **1b**, ask them to predict what they think the effect of the knight from Bilbury will be. They may think it will be the same whichever team he joins. Use the diagram or other recording format to explore the answer.

Question **2** is a difficult problem. Students could work in pairs to provide an opportunity for support and discussion. Ask the students to present their answer to the group or class using diagrams or other means that show why their solution is correct. Trial and error may be the most common strategy for solving this. Look for students who recognise the multiplicative structure and employ known facts.

Question **3** adds another level to the problem: there are jousts between knights, but there are also groups of jousts between different pairings of teams. The prompt for students suggests using diagrams to help solve the problem. There is a lot of problem solving and thinking to be done here. Encourage the students to record and share their thinking.

These problems could be extended by making up different-sized jousting teams or different numbers of counties competing in a big tournament. Modern contexts such as tennis tournaments or inter-school sports could also be used to provide further practice with this concept.

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 6: Teaching Multiplication and Division*
  Multiplication Smorgasbord, page 27

Page 24: Hydroslides

Achievement Objectives

- demonstrate the ability to use the multiplication facts (Number, level 2)
- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- write and solve comparison problems (Number, level 2)

AC
EA
AA
AM
AP

Number Framework Links

The problem structure of this activity will encourage students at stages 4 and 5 to use multiplication rather than addition. It would be a useful independent activity for students at stage 6.

Activity

This activity asks students to apply their multiplication knowledge to comparison problems. It could be linked with Numeracy Project lessons that seek to get students to apply their number fact knowledge in multiplicative situations.

This activity is another example of multiplicative comparison, like Clever Clues. The first hydroslide is 12 metres long, the second is twice as long, and the third 3 times as long. This is used as a basis to compare other related measurements, such as how long it will take to get down each slide. The activity could be used as an independent activity for students who have already worked on Clever Clues with you as a group to see how they have picked up the concept of "times as many" (multiplicative comparison).

Question **1** asks students to identify the multiplicative relationship between the two slides. Watch for students who say 12 metres. They are seeing the additive relationship, not the multiplicative one.

Questions **2** and **3** further explore the multiplicative relationships and extend them to how long it takes to go down the slide. These numbers are needed to work on question **4**.

In question **4**, students can explore different ways of using their 20 minutes, using the times found in question **3**. The context of the problem means that there are lots of possible solutions and explanations here. Students can present their answers and justifications to the group. They will probably repeat certain slides, so encourage them to use multiplication to express this. For example, 3 turns on the Speeder is 3 minutes (3 x 1), with 3 lots of 2 minutes to get to the top (3 x 2). So 3 turns on the Speeder is 3 + 6 = 9 minutes.

Extend this activity by asking the students to think of other contexts and problems that involve multiplicative comparison. A useful example is Goldilocks and the Three Bears. Mama Bear's belongings can be 3 times as big as Baby Bear's, and Papa Bear's can be 4 times as big (or any other factor). The students can make up measurements and work out the sizes for the other bears' belongings. For example, Baby Bear's bowl may hold $\frac{1}{2}$ a cup of porridge, or Mama Bear's bed may be 3 metres long.

Links

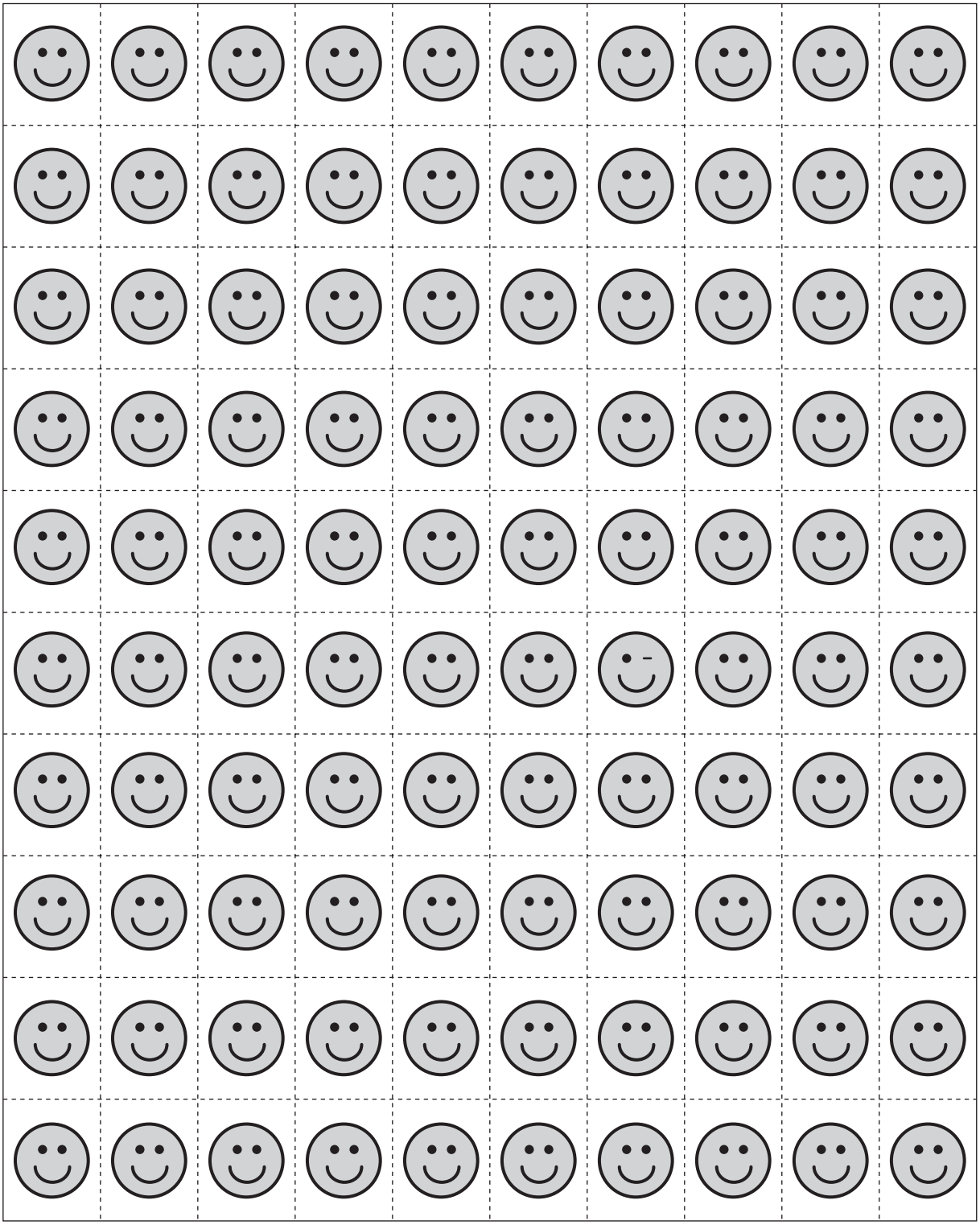
Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.aspx)

- *Book 6: Teaching Multiplication and Division*
  Fun with Fives (deriving unknown facts from known 5 times facts), page 12
  Multiplication Smorgasbord, page 27

Figure It Out

- *Basic Facts*, levels 2–3
  Flying Fish Soup (doubling and halving a recipe), page 14

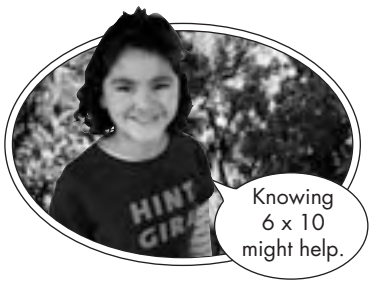
Copymaster: Face the Facts



Copymaster: Fiordland Holiday

Adventures & mishaps	Adventures & mishaps	Adventures & mishaps
Go on a helicopter ride in Te Anau. Holiday highlight! **Use triples and thirds to solve 3 x 27.**	Visit the Milford Deep Underwater Observatory. **Solve 12 x 5.**  	On a scenic coach tour, see dense rainforest and glacial lakes. **Score 20 x 7 points.**
Adventures & mishaps	**Adventures & mishaps**	**Adventures & mishaps**
Find a small piece of pounamu (greenstone) in a Fiordland river. **Get 5 x 36 points for putting it back.**	Get water in your gumboots while fishing in the Monowai River. **Score 565 x 0 points.**	Spend $2\frac{1}{2}$ hours in the glow-worm caves at Te Anau. **Use 6 x 10 to solve 6 x 11.**
Adventures & mishaps	**Adventures & mishaps**	**Adventures & mishaps**
Go on a Te Anau nature cruise. **Score 53 x 3 points.**	Go on a Milford Sound cruise.  **Score 3 x 99 points.**	You forgot to put sunburn cream on. **Score 5 x 18 points to make it less painful but miss your next turn.**

Copymaster: Fiordland Holiday

Adventures & mishaps	Adventures & mishaps	Adventures & mishaps
Feed pet lambs and try to milk a cow on a farm visit.	On a Milford Sound day tour, see the famous Mirror Lakes.	On a bush walk, you trip over a rotten log.
Double and halve to solve 5 x 44.	**Score 3 x 300 points.**	**Score 39 x 0 points.**
Adventures & mishaps	**Adventures & mishaps**	**Adventures & mishaps**
A cheeky kea nibbles at the windscreen wipers on your car.	Go on a horse trek in Te Anau. **Solve 6 x 9.** 	Go on a guided 1 day walk on the Milford Track.
Score 5 x 19 points.		**Solve 45 ÷ 5.**
Adventures & mishaps	**Adventures & mishaps**	**Adventures & mishaps**
Go for a drive over alpine roads in a glass-roofed coach.	Go kayaking in Milford Sound.	Go on a scenic bush walk in Fiordland National Park. 
Score 102 x 4 points.	**Work out 36 ÷ 4.**	**7 x 200 = 1 400, so what is 7 x 198?**

Copymaster: Fiordland Holiday

Adventures & mishaps	Adventures & mishaps	Adventures & mishaps
Get badly bitten by namu (sandflies). **Solve 56 ÷ 7 to stop the itching. Now have an extra turn to take your mind off sandflies!**	Hire a tandem bike. **Double all the points you have already earned!**	Play minigolf in Te Anau. **Score 4 x 18.**
Adventures & mishaps	**Adventures & mishaps**	**Adventures & mishaps**
Go for a jet boat ride on Lake Hauroko. **Score 5 x 80.**	See a pod of bottlenose dolphins in Doubtful Sound. **Use multiplication to solve 120 ÷ 5.**	See an albatross off the Fiordland coast. **Use 10 x 6 to solve 9 x 6.**
Fiordland is the only place in the South Island where brown teal ducks live. **Solve 12 x 8.**	People thought the takahē was extinct until it was rediscovered in Fiordland in 1948.  **Solve 21 x 3.**	You can see rare species of black and red coral at the Underwater Observatory in Milford Sound. **Solve 7 x 8.**
Rare species	**Rare species**	**Rare species**

Copymaster: Fiordland Holiday

The rare mohua (yellowhead) lives in some Fiordland valleys. You see one!

Double all the points you have already earned!

Rare species

The Fiordland crested penguin is one of the world's rarest species.

Solve 15 x 10.

Rare species

The kākāpō, which is nocturnal, is the world's only flightless parrot.

Solve 2 x 231.

Rare species

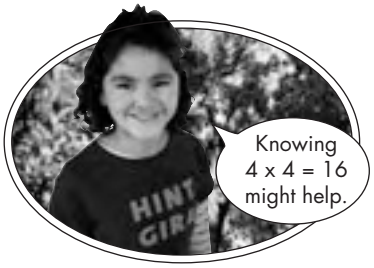
Fiordland is home for the threatened blue duck.

Work out 4 x 20.

Rare species

You find a rare native iris plant on one of your tramps.

Solve 7 x 4.



Rare species

The New Zealand falcon is an endangered species found in Fiordland.

Solve 17 ÷ 1.

Rare species

You need to camp overnight to see brown kiwi.

Use multiplication to solve 42 ÷ 6.

Rare species

Weka and the yellow-crowned parakeet are endangered species found in Fiordland.

Solve 26 x 5.

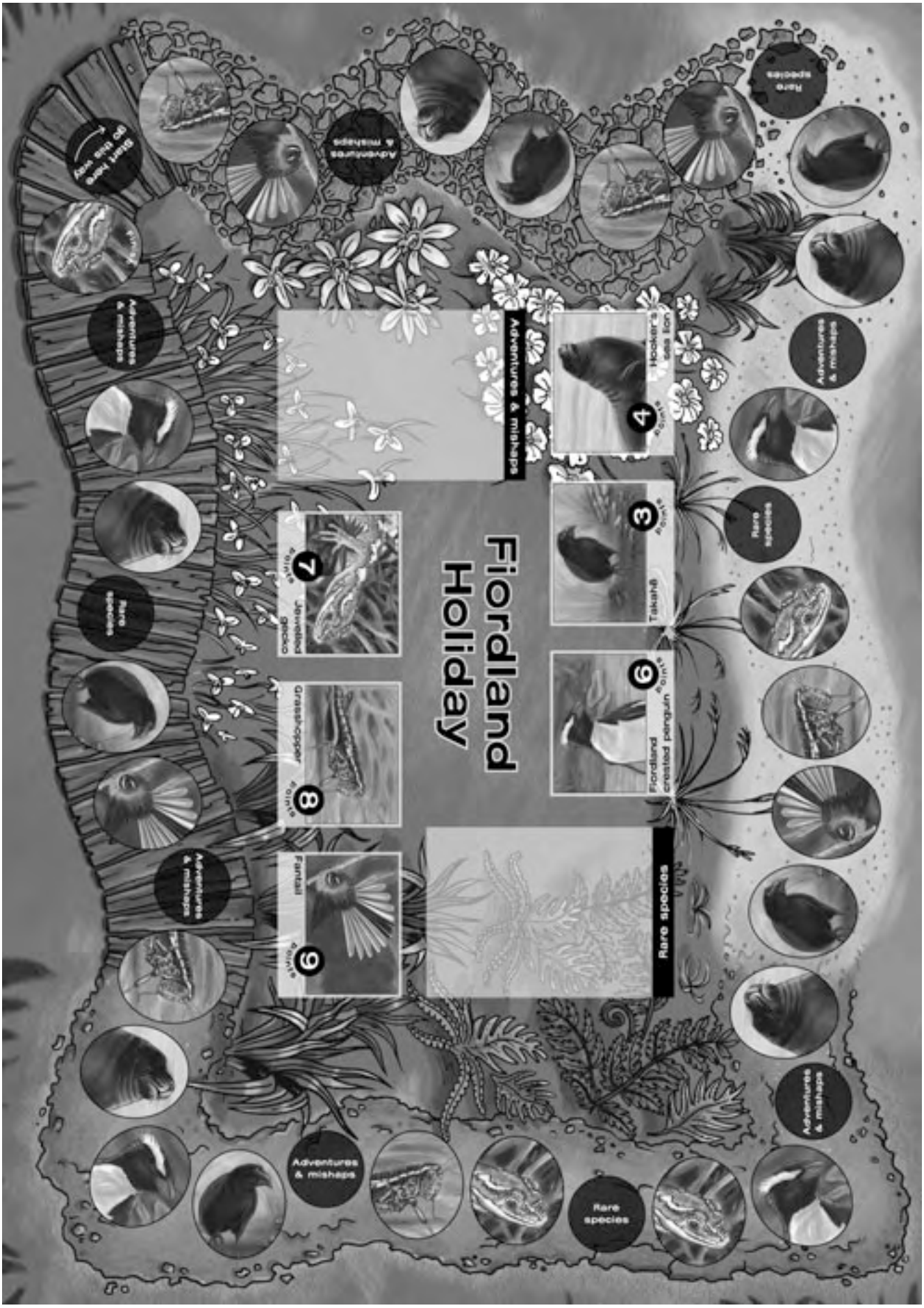
Rare species

You are lucky enough to see some rare Hector's dolphins from the Tuatapere Hump Ridge Track.

Solve 24 x 2.

Rare species

Copymaster: Fiordland Holiday



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