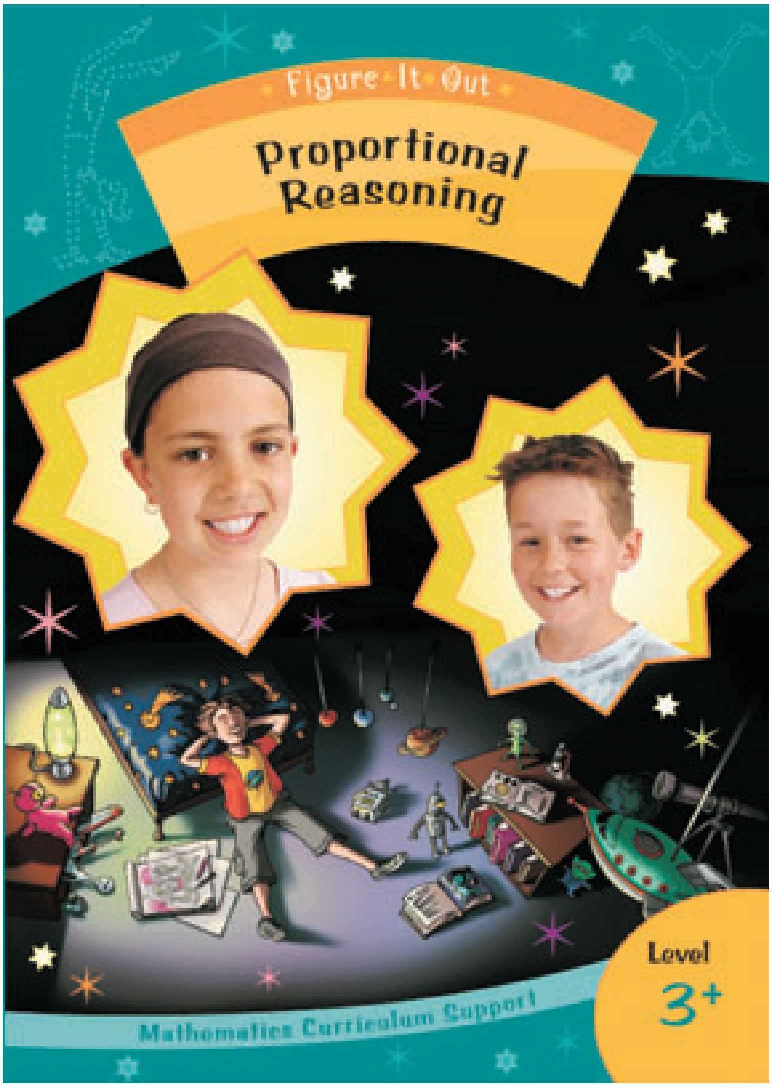
# Answers and Teachers' Notes




MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

## Contents

# Introduction

This book is one of three in the Figure It Out series that have proportional reasoning as their focus: *Proportional Reasoning*, Level 3+ and *Proportional Reasoning*, Level 3–4+ (Books One and Two). In these books, students explore the meaning of fractions and ratios and learn how to use them to make comparisons in a wide variety of contexts.

The books have been developed to support teachers whose students are moving on to the early proportional and advanced proportional stages of the Number Framework (stages 7 and 8). Like the other "plus" books in the Figure It Out series, these should be suitable for students needing extension. The level 3–4+ books are intended for use with students in year 6 but could be used at other levels at the discretion of the teacher.

The books aim to set activities in real-life and imaginary contexts that should appeal to students. The real-life contexts reflect many aspects of life in New Zealand, and the young people portrayed in illustrations and photos reflect our ethnic and cultural diversity.

The activities may be used as the focus for teacher-led lessons, for students working in groups, or for independent activities. But bear in mind that the Figure It Out series is a resource, not a set of textbooks. This means that if you are setting an activity to be done independently, you should check that you have done whatever prior teaching is needed.

Teachers sometimes say that their students have difficulty understanding the words on the page. We are very mindful of this and try to keep written instructions as brief and as clear as possible, but to create a context and pose questions, some words must be used. It is important that mathematical language and terminology be deliberately taught.

The Answers section of the *Answers and Teachers' Notes* that accompany each student book includes full answers and explanatory notes. Students can use them for self-marking, or you can use them for teacher-directed marking. The teachers' notes for each activity include achievement objectives, a commentary on the mathematics involved, and suggestions on teaching approaches. **Although the notes are directed at teachers, able students can use them as a self-help resource**. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/r/maths/curriculum/figure/

Where applicable, each page starts with a list of the equipment needed. Encourage the students to be responsible for collecting this equipment and returning it at the end of the session.

Ask your students to write down how they did their investigations or found solutions, drawing diagrams where appropriate. Discussion of strategies and answers is encouraged in many activities, and you may wish to ask your students to do this even where the instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage your students to consider their merits.

# Figure It Out

## Proportional Reasoning Level 3+

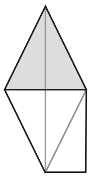
# Answers

## Page 1: Puzzling Patterns

**Activity**

1. i. $\frac{1}{3}$ ii. $\frac{3}{10}$ iii. $\frac{3}{5}$ iv. $\frac{3}{7}$
2. i. $\frac{1}{9}$ ii. $\frac{4}{10} = \frac{2}{5}$ iii. $\frac{2}{5}$ iv. $\frac{2}{7}$
3. a. $\frac{2}{5}$. (See the diagram below, which divides one part of the design into 5 equal small triangles. Looking at the whole design, there are 20 of these small triangles, of which 8 are orange. $\frac{8}{20} = \frac{2}{5}$)

   b. $\frac{3}{5}$. (See diagram at right.)



4. Problems will vary.

## Pages 2–3: Shaping Up

**Activity**

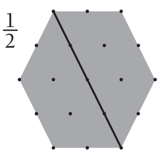
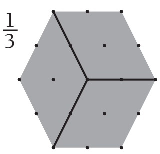
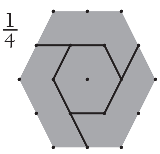
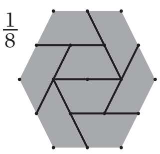
1. a. $\frac{1}{8}$. Explanations will vary. ($\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$)

   b. $\frac{1}{16}$. Explanations will vary. ($\frac{1}{4}$ of $\frac{1}{4}$)

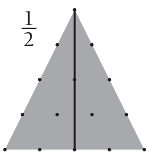
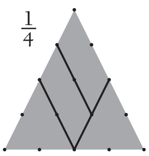
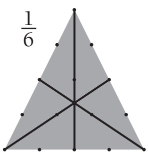
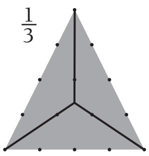
   c. $\frac{3}{4}$. Explanations will vary. ($\frac{18}{24}$, using triangle units)

   d. $\frac{1}{32}$. Explanations will vary. ($\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{1}{4}$)

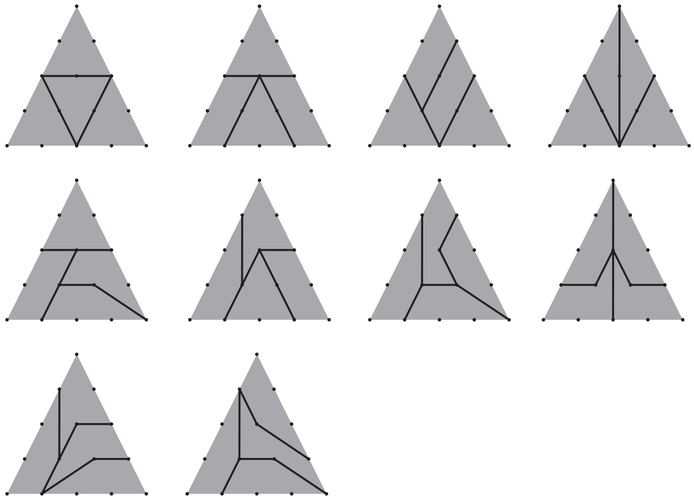
2. Answers will vary. Here are four:

$\frac{1}{2}$

$\frac{1}{3}$

$\frac{1}{4}$

$\frac{1}{8}$


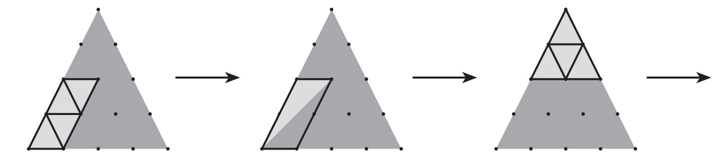
3. Answers will vary. Here are four:

$\frac{1}{2}$

$\frac{1}{4}$

$\frac{1}{6}$

$\frac{1}{3}$


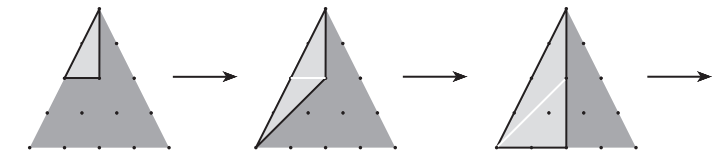
4. a. Here are 10 distinct ways:



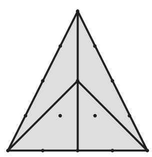
   b. Yes, the lines do divide the triangle into quarters. Here is one way of showing this:



Area = 4 small triangles
Area = 2 small triangles
Area = 4 small triangles



Area = 2 small triangles
Area = 2 + 2 small triangles
Area = 4 + 4 small triangles



Area = 4 + 4 + 4 + 4 small triangles

As the area of each of the triangles is the same, the lines must divide the large triangle into quarters.

## Page 4: Chocolate Choices

**Activity**

1. a. 500 g and 750 g

   b. Box A (250 g) should cost \$4.30.
   Box B (500 g) should cost \$8.60.

2. a. There are two possibilities: 2B or A + C. Both would give Tania 16 strawberry hearts.

   b. The ratio of favourites to non-favourites would be 16:32 = 1:2.

3. Atama would get most caramel circles (18) by choosing 2 x B. The ratio of favourites to non-favourites would be 18:30 = 3:5.

   Chloe would get most peppermint squares (20) by choosing 4 x A. The ratio of favourites to non-favourites would be 20:28 = 5:7.

4. If they buy boxes A and C, each person will get 16 of their favourite kind.

## Page 5: Star Clusters

### Activity

1. 3-star clusters:
   $\frac{1}{2}$ of a 6-pack; $\frac{3}{8}$ of an 8-pack; $\frac{1}{4}$ of a 12-pack; $\frac{3}{20}$ of a 20-pack.
   4-star clusters:
   $\frac{4}{6}$ or $\frac{2}{3}$ of a 6-pack; $\frac{1}{2}$ of an 8-pack; $\frac{1}{3}$ of a 12-pack; $\frac{1}{5}$ of a 20-pack.
   5-star clusters:
   $\frac{5}{6}$ of a 6-pack; $\frac{5}{8}$ of an 8-pack; $\frac{5}{12}$ of a 12-pack; $\frac{1}{4}$ of a 20-pack.

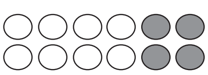
2. Many answers are possible. They include: twenty-four 6-packs; twelve 12-packs; six 6-packs + six 8-packs + three 20-packs.

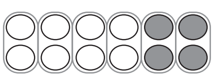
3. 16 x 3-star clusters. ($\frac{1}{3}$ of 144 = 48)
   9 x 4-star clusters. ($\frac{1}{4}$ of 144 = 36)
   12 x 5-star clusters. (144 – 48 – 36 = 60)

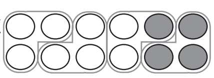
4. $\frac{3}{4}$. (12 x 9 = 108, and 108 is $\frac{3}{4}$ of 144.)

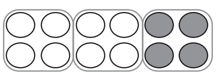
## Pages 6–7: What Do You See?

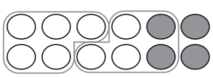
### Activity

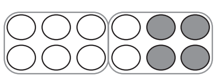
1. $\frac{8}{12}$ 

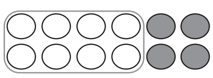
   $\frac{4}{6}$ 

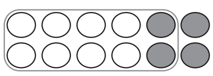
   $\frac{2\frac{2}{3}}{4}$ 

   $\frac{2}{3}$ 

   $\frac{1\frac{3}{5}}{2\frac{2}{5}}$ 

   $\frac{1\frac{1}{3}}{2}$ 

   $\frac{1}{1\frac{1}{2}}$ 

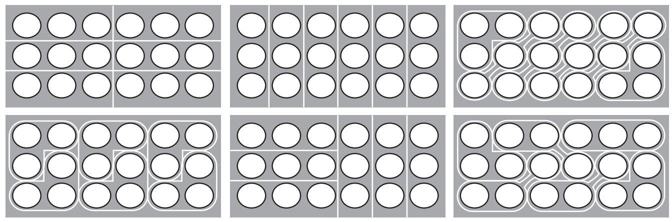
   $\frac{\frac{4}{5}}{1\frac{1}{5}}$ 

2. Answers will vary. There are many possibilities, including: $\frac{1}{4}$, $\frac{2}{8}$, $\frac{6}{24}$, $\frac{9}{36}$, $\frac{10}{40}$, $\frac{15}{60}$

3. a.

| | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{4}$ | $\frac{1}{6}$ | $\frac{1}{8}$ | $\frac{1}{9}$ | $\frac{1}{12}$ | $\frac{1}{16}$ |
|---|---|---|---|---|---|---|---|---|
| 18-tray | ✔ | ✔ | ✖ | ✔ | ✖ | ✔ | ✖ | ✖ |
| 24-tray | ✔ | ✔ | ✔ | ✔ | ✔ | ✖ | ✔ | ✖ |
| 32-tray | ✔ | ✖ | ✔ | ✖ | ✔ | ✖ | ✖ | ✔ |

   b. Answers will vary. For example, here are 6 ways of dividing the 18-egg tray into sixths:



## Page 8: Discount Deals

### Activity

1. Bookz 4 U is never the best because its discount of 20% is always beaten by the $\frac{1}{4}$ (25%) discount offered by Sam's Stationery.

2.

| Number | Cheapest at |
|---|---|
| 1 | Sam's Stationery |
| 2–6 | Mags & More |
| 7 | Mags & More or Bargain Books |
| 8–9 | Bargain Books |
| 10–11 | Books & Stuff |
| 12 | Bargain Books or Books & Stuff |
| 13–19 | Bargain Books |
| 20–21 | Books & Stuff |
| 22 | Bargain Books or Books & Stuff |
| 23–29 | Bargain Books |
| 30 | Books & Stuff |

3. Answers will vary. You could make points similar to these:
   - The way the discount at Mags & More works means that no further discount is available after 4 items have been purchased. But if these were expensive items, the discount would save you quite a bit.
   - Sam's Stationery offers an excellent discount on any number of items, but it can only beat the discount offered elsewhere for 1 item.
   - Bargain Books offers the best price most of the time – but only if you are buying quite a few of the same item.
   - Books & Stuff offers the best deal if the cost of your purchase is just a little over a multiple of $50.

## Page 9: Cereal Calculations

### Activity

1. Splitting the class of 30 in the 2:3:5 ratio gives 6, 9, and 15 students preferring each kind of cereal.

| | Needed | Buy |
|---|---|---|
| Wheat Bubbles | 6 x 30 x 3 = 540 | 1 x 750 g |
| Bran Flakes | 9 x 30 x 3 = 810 | 3 x 300 g |
| Banana Pops | 15 x 30 x 3 = 1 350 | 3 x 300 g + 1 x 500 g |
| Totals | 2 700 | 3 050 g |

2. Splitting the class of 20 in the 2:3:5 ratio gives 4, 6, and 10 students preferring each kind of cereal.

| | Needed | Buy |
|---|---|---|
| Wheat Bubbles | 4 x 30 x 2 = 240 | 1 x 500 g |
| Bran Flakes | 6 x 30 x 2 = 360 | 1 x 500 g |
| Banana Pops | 10 x 30 x 2 = 600 | 2 x 300 g |
| Totals | 1 200 | 1 600 g |

3. Splitting the class of 30 in the 2:3:5 ratio gives 6, 9, and 15 students preferring each kind of cereal.

| | Needed | Buy |
|---|---|---|
| Wheat Bubbles | 6 x 30 x 2 = 360 | 1 x 500 g |
| Bran Flakes | 9 x 30 x 2 = 540 | 2 x 300 g |
| Banana Pops | 15 x 30 x 2 = 900 | 3 x 300 g |
| Totals | 1 800 | 2 000 g |

4. Based on these estimates, Ms Leishman's class would waste least (200 g).

| | Needed | Buy | Excess |
|---|---|---|---|
| Mr Wehipeihana's class | 2 700 | 3 050 | 350 |
| Mrs Yeoman's class | 1 200 | 1 600 | 400 |
| Ms Leishman's class | 1 800 | 2 000 | 200 |

## Page 10: Speed Read

### Activity

1. Jack will take about 21 hours. (630 ÷ 30 = 21)
Atama will take about 31.5 hours.
(630 ÷ 20 = 31.5)
Mikey will take about 14 hours. (630 ÷ 45 = 14)
Charlotte will take about 35 hours.
(54 ÷ 3 = 18, 630 ÷ 18 = 35)
Miranda will take about 28 hours.
(630 ÷ 90 = 7, 7 x 4 = 28)

2. a. Atama will be on page 60, and Mikey will be on page 135.
(It would take 3 hours for Jack to reach page 90. In 3 hours, Atama would get to page 60 [3 x 20] and Mikey would get to page 135 [3 x 45].)

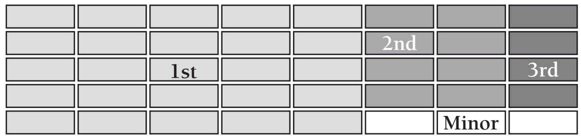
b. Mikey will take 4 hours to reach page 180.
After 4 hours:
Jack will be on page 120 (4 x 30)
Atama will be on page 80 (4 x 20)

## Page 11: Horse Sense

### Activity

1. a. \$125. (5 000 ÷ 40 = 125)

b. Here is one way the grid can be shaded:




c. First: $\frac{25}{40} = \frac{5}{8}$; second: $\frac{8}{40} = \frac{1}{5}$; third: $\frac{4}{40} = \frac{1}{10}$

d. $\frac{3}{40}$

2. a. $\frac{5}{8} \times \frac{1}{10} = \frac{5}{80} = \frac{1}{16}$ for the trainer and $\frac{1}{32}$ for the jockey.

b. $\frac{1}{5} \times \frac{1}{10} = \frac{1}{50}$ for the trainer and $\frac{1}{100}$ for the jockey.

c. $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$ for the trainer and $\frac{1}{200}$ for the jockey.

3. Both earn the same ($\frac{1}{100}$ of the total prize pool).

## Pages 12–13: Small Business

### Activity

1. As far as total profit goes, Five Alive is most successful. (\$30 is \$4 better than the next highest profit of \$26.)
On a profit-per-student basis, Carwash Ltd is the most successful. Each student earns \$7.
(21 ÷ 3 = \$7.00)

2. The order in which students do things makes no difference to the share that each gets. For example, for Necklace Co:
   - Share out first, then go to Mr Pinkerton:
   26 ÷ 4 = \$6.50 ⟶ \$6.50 x 3 = \$19.50.
   - Go to Mr Pinkerton first, then share out:
   26 x 3 = \$78 ⟶ \$78 ÷ 4 = \$19.50.

3.

| Team | Profit | Profit + IRD bonus | Individual earnings | Individual earnings as fraction of total profit |
|---|---|---|---|---|
| Carwash | \$21.00 | 21 + 42 = \$63 | 63 ÷ 3 = \$21.00 | $\frac{21}{21} = \frac{3}{3} = 1$ |
| Necklace | \$26.00 | 26 + 52 = \$78 | 78 ÷ 4 = \$19.50 | $\frac{19.5}{26} = \frac{3}{4}$ |
| Five Alive | \$30.00 | 30 + 60 = \$90 | 90 ÷ 5 = \$18.00 | $\frac{18}{30} = \frac{3}{5}$ |

4. a. \$48. Strategies will vary. Each person gets a $\frac{1}{8}$ share of the \$64 profit (\$8) plus $\frac{1}{8}$ of the \$320 (5 x 64) from the IRD, which is \$40.

   b. \$60. Strategies will vary. Each person gets a $\frac{1}{10}$ share of the \$75 profit (\$7.50) plus $\frac{1}{10}$ of the \$525 (7 x 75) from the IRD, which is \$52.50.

## Pages 14–15: Getting Tough

### Activity

1. a. Answers will vary. (Example: the maximum heart rate for a 10-yr-old would be 220 – 10 = 210 beats per min.)

   b. Answers will vary. For a 10-yr-old, the zones would be:
   Easy 126–147 beats per min
   Moderate 147–168 beats per min
   Race pace 168–179 beats per min

2. 28 hrs. (1 680 min ÷ 60 = 28 hrs)

3.

| Week | Training time per week (min) | Time at each zone: Easy | Moderate | Race |
|---|---|---|---|---|
| Phase 1: building fitness | | | | |
| 1 | 90 | 90 | | |
| 2 | 100 | 100 | | |
| 3 | 110 | 110 | | |
| 4 | 120 | 120 | | |
| 5 | 90 | 90 | | |
| Phase 2: building strength | | | | |
| 6 | 120 | 95 | 25 | |
| 7 | 130 | 95 | 35 | |
| 8 | 140 | 105 | 35 | |
| 9 | 150 | 110 | 40 | |
| 10 | 120 | 120 | | |
| Phase 3: building speed | | | | |
| 11 | 150 | 90 | 30 | 30 |
| 12 | 170 | 70 | 50 | 50 |
| 13 | 120 | 70 | | 50 |
| 14 | 70 | 40 | | 30 |
| Race | | | | |
| Totals | 1 680 | 1 305 | 215 | 160 |

4. 77.7% at easy, 12.8% at moderate, 9.5% at race.

## Page 16: Demolition Dollars

### Activity

1. \$5. (120 ÷ 24 = 5)

2. \$140. (7 x 4 x 5 = 140)

3. 6 hours. (150 ÷ 5 = 30; 30 ÷ 5 = 6)

4. 9 boys. (255 ÷ 5 = 45; 45 ÷ 5 = 9)

5. 8 boys. (Parents earn 3 x 10 x 4 = \$120; boys earn \$160. Each boy earns 4 x 5 = \$20. Number of boys is 160 ÷ 20 = 8)

6. \$280. (Parents earn 2 x 4 x 10 = \$80; boys earn 10 x 4 x 5 = \$200. 80 + 200 = \$280)

7. \$5 short

## Page 17: Painting by Numbers

### Activity

1. 3 hrs. 2 students take 12 hrs, so 4 students would take 6 hrs and 8 students would take 3 hrs. (Doubling and halving)

2. $\frac{1}{24}$ of the mural

3. 8 hrs. (24 ÷ 3 = 8)

4. 16 hrs. (There are many ways of working this out. Here is one:
   If 18 students are painting 12 murals, in groups of 3 students, each group has to paint 2 murals. It takes 24 x 2 = 48 student hrs to paint 2 murals. This means that it will take 48 ÷ 3 = 16 hrs for all the students to paint all the murals.)

## Pages 18–19: Make 1.5

### Game

A game involving decimal fractions

### Activity

a. Ben gets 2.5 points; Renee wins with 2.2 + 5 = 7.2 points.

b. Renee gets 1.9 points; Ben wins with 2.9 + 5 = 7.9 points.

c. No bonus. Ben wins with 3.6 points; Renee gets 1.9 points.

d. Ben gets 2.8 points; Renee wins with 1.8 + 5 = 6.8 points.

e. Ben gets 2.5 points; Renee wins with 1.7 + 5 = 6.7 points.

## Page 20: Pop Star Pics

**Activity**

1. a. 6 cm. (Compare bases: the base of the smallest picture is $\frac{4}{6} = \frac{2}{3}$ of the base of the medium picture. This means that height A will be $\frac{2}{3}$ of the height of the medium picture. $\frac{2}{3} \times 9 = 6$ cm.)

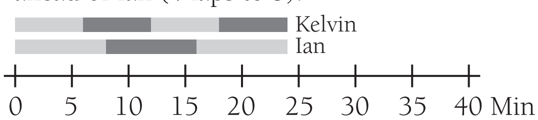
   b. 30 cm. (Compare bases: the base of the largest picture is $\frac{20}{6} = \frac{10}{3} = 3\frac{1}{3}$ times the base of the medium picture. This means that height B must be $3\frac{1}{3} \times 9 = 30$ cm.)

2. i. and vi. iii. and v.

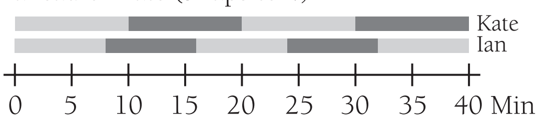
## Page 21: Tiring Teamwork

**Activity**

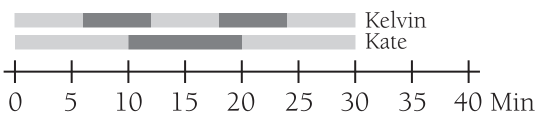
1. 24 min. A double strip diagram shows that it takes this long for Kelvin to get 1 complete lap ahead of Ian (4 laps to 3):




2. 40 min. After this time, Ian is 1 complete lap ahead of Kate (5 laps to 4):




3. 15 min. This is a little harder to see on a strip diagram, but after 30 min, Kelvin is exactly 2 laps ahead of Kate (5 laps to 3), so after 15 min, he must be exactly 1 lap ahead ($2\frac{1}{2}$ laps to $1\frac{1}{2}$).




## Pages 22–23: The Great Race

**Activity**

1. a. 25 km

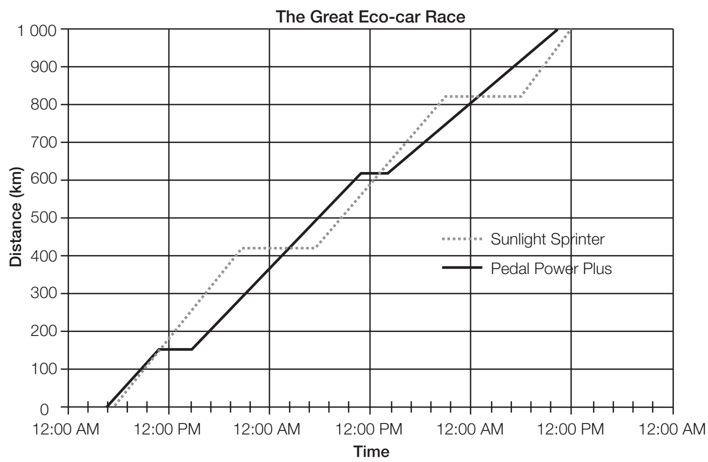
   b. Pedal Power Plus travels 150 km. Sunlight Sprinter travels 420 km. (PPP's first scheduled stop is at 11.00 a.m., which means that it has travelled for 6 hrs, covering 25 km each hr. SS starts at 6.00 a.m. and doesn't stop until 8.00 p.m., 14 hrs later. 14 x 30 = 420 km)

   c. PPP will travel 375 km (15 x 25 = 375). SS will travel 420 km. (See answer to question **1b** above.)

2. a. Practical activity

   b. PPP wins, by 1 hr and 20 min. (PPP arrives at 10.00 a.m. on Monday. SS arrives at 11.20 a.m.)

3. Graphs will vary in the way they are presented. Here is a suitable graph drawn using a computer spreadsheet program. You can tell which car wins by looking at where the two lines meet the 1 000 km line up in the top-right corner. The PPP line meets the 1 000 km line first, showing that it is the winner.




4. Answers will vary. Possible factors include mechanical problems, sickness or injury, accidents, weather conditions, road/track conditions.

## Page 24: A Dog's Breakfast

**Activity**

1. 26. (13 meals a day for 2 days)

2. Here is one way of setting out a chart:

| | Day 1 | | | Day 2 | | |
|---|---|---|---|---|---|---|
| | Morning | Midday | Evening | Morning | Midday | Evening |
| Pete | 110 g | 110 g | 110 g | 110 g | 110 g | 110 g |
| Benny | 100 g | | 100 g | 100 g | | 100 g |
| Spike | 210 g | | 210 g | 210 g | | 210 g |
| Molly | 720 g | | | 720 g | | |
| Danny | 590 g | | 590 g | 590 g | | 590 g |
| Sam | 520 g | 520 g | 520 g | 520 g | 520 g | 520 g |

Figure It Out

# Proportional Reasoning
# Teachers' Notes

## Overview of Level 3+

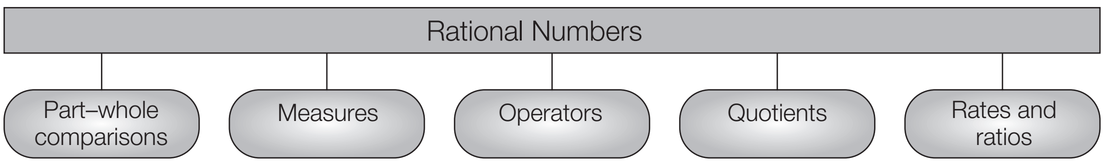
| Title | Content | Page in students' book | Page in teachers' book |
|---|---|---|---|
| Puzzling Patterns | Exploring fractions in relation to area | 1 | 10 |
| Shaping Up | Using spatial reasoning to find fractions of areas | 2–3 | 11 |
| Chocolate Choices | Using ratios to compare subsets | 4 | 12 |
| Star Clusters | Finding fractions of sets | 5 | 13 |
| What Do You See? | Exploring visuals models for fractions | 6–7 | 14 |
| Discount Deals | Comparing discounts written in different forms | 8 | 15 |
| Cereal Calculations | Solving problems that involve ratios and large numbers | 9 | 16 |
| Speed Read | Solving problems involving rates | 10 | 17 |
| Horse Sense | Renaming, adding, and multiplying fractions | 11 | 18 |
| Small Business | Finding and comparing proportions of whole number amounts | 12–13 | 19 |
| Getting Tough | Interpreting information expressed as percentages | 14–15 | 20 |
| Demolition Dollars | Solving problems involving rates | 16 | 21 |
| Painting By Numbers | Solving problems involving rates | 17 | 22 |
| Make 1.5 | Finding and comparing proportions | 18–19 | 24 |
| Pop Star Pics | Using ratio in an enlargement context | 20 | 25 |
| Tiring Teamwork | Comparing the relative positions of two moving objects | 21 | 26 |
| The Great Race | Working with the time/distance/speed relationship | 22–23 | 27 |
| A Dog's Breakfast | Using percentages in relation to body mass | 24 | 30 |

# Introduction

It is said that the rhinoceros beetle is the strongest animal on Earth; by some accounts, it can support up to 850 times its own weight on its back – the equivalent of a man supporting 75 cars. This does not mean that a rhinoceros beetle can lift heavier objects than any other animal; rather, that it is *proportionally* stronger than any other animal: the fairest measure of its strength is found by comparing what it can lift with its body weight.

Before they can make multiplicative comparisons of this kind, students need to extend their knowledge of numbers to include all rational numbers (those that can be written as fractions in the form $\frac{a}{b}$). With their two components (numerator and denominator), these numbers are able to express the relationship between two measures.

A difficulty for both teacher and student is that rational numbers can be used and interpreted in subtly different ways depending on the context. Kieran[1] suggested this helpful classification:




1. *Part–whole* comparisons involve finding the multiplicative relationship between part of a continuous space or of a set and the whole. For example, what fraction of a square has been shaded?

2. In a *measurement* context, a rational number is the answer to questions of the kind, "How many times does this fraction (or ratio) fit into that fraction (or ratio)?"

3. As *operators*, rational numbers perform operations on other numbers, for example, $\frac{1}{3} \times 12 = \square$.

4. As *quotients*, rational numbers provide the answers to sharing problems. It is important for students to recognise that $7 \div 4$ is an *operation* while $\frac{7}{4}$ (the quotient) is a *number* that is the result of that operation.

5. *Rates* involve a multiplicative relationship between two variables, each with a different unit of measurement (for example, kilometres and hours). Ratios are a special case of rates in that the units of measurement are the same for each variable (for example, 1 shovel of cement to every 5 shovels of builders' mix).

It is important that students are exposed to rational numbers in all their guises and that they learn to attribute different meanings to them, depending on the use and the context. It is also important that students learn a range of different ways of modelling situations that require proportional reasoning. This book will help in both areas. It should also help the teacher recognise that many everyday contexts can provide relevant and often intriguing rate and ratio challenges for their students.

[1]Kieran, T. (1976), "On the Mathematical, Cognitive, and Instructional Foundations of Rational Numbers", in Lesh, R. (ed.), *Number and Measurement: Papers from a Research Workshop*, Columbus, Ohio: ERIC/SMEAC.

# Page 1: Puzzling Patterns

**Achievement Objectives**

- find fractions equivalent to one given (Number, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)
- describe the features of 2-dimensional and 3-dimensional objects, using the language of geometry (Geometry, level 3)

| |
|---|
| AC |
| EA |
| AA |
| AM |
| AP |

**Number Framework Links**

Use this activity to help students consolidate and apply their knowledge of fractions (stages 6 and 7).

## Activity

In this whole-to-part activity, students are given a number of shapes and asked to work out what fraction of each has been shaded. This task is easier than part-to-whole problems of the kind "This triangle is one-sixth of my shape, so what might my shape look like?"

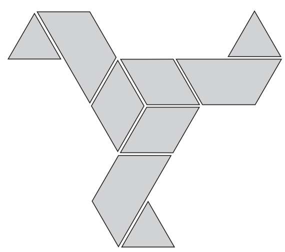
If your students need the support of materials, encourage them to use pattern blocks to make the designs in this activity. Using blocks, they can explore or confirm the simple equivalence relationships that exist between the areas of the different shapes. For example: 2 triangles = 1 rhombus, 1 rhombus + 1 triangle = 1 trapezium, 3 triangles = 1 trapezium, 2 rhombuses = 1 trapezium + 1 triangle. Once these relationships are understood, they can be reversed and expressed as fractions. For example: 1 triangle = $\frac{1}{3}$ of a trapezium; 1 rhombus = $\frac{2}{3}$ of a trapezium. Students need to understand also that the areas of the rhombus and the trapezium (and the hexagon) can be expressed in terms of multiples of the area of the equilateral triangle. This means that area problems involving these blocks can always be recreated or expressed using only the triangle. The parts of questions **1** and **2** can all be solved using this kind of approach, but some students will need the actual pattern blocks.

Question **3a** can be solved using two different approaches, and it would be useful to have the students share these. Some will image the whole design divided into equal triangles (each the size of half an equilateral triangle) and then count the number of orange triangles out of the total to give the fraction $\frac{8}{20}$. A variation on this approach would be to use the equilateral triangle as the smallest unit of area, in which case the total area of the design is 8 triangles + 4 half-triangles = 10. The orange area is therefore $\frac{4}{10}$.

Students using the other approach will recognise that the windmill design is made up of 4 identical blades, so whatever part of one is orange ($\frac{2}{5}$), that part of the whole will be orange.

Some students may view the question as a ratio problem. If they do this, the orange to non-orange ratio is $1:1\frac{1}{2}$ or 2:3, which is the same as 2 parts out of 5 or $\frac{2}{5}$.

Students can work out question **3b** using similar strategies to those for question **3a**, but encourage them to use their earlier answers and make the deduction that $1 - \frac{2}{5} = \frac{3}{5}$. (The whole minus the orange piece leaves the non-orange piece.)



Question **4** allows the students to take the ideas introduced in the first three questions and explore them further. Some will be happy to accept the open-ended challenge offered by the question; others may need you to pose a specific challenge. For example, have them make the design at right and then make up 10 fraction questions based on it.

*Book 9: Teaching Number through Measurement, Geometry, Algebra and Statistics,* Numeracy Project series, extends this concept to using number and geometry properties (page 21).

**Extension**

Question **4** can be extended further by considering part-to-whole shapes. Take a shape (for example, a hexagon) and ask *"If this shape is $\frac{1}{5}$ of another shape that I have hidden, what might my shape look like? What shape(s) could be made? How many different solutions are there? How do you know you have them all?"* The aim is to encourage the students to search for all solutions rather than stopping at one. Get them to make up a problem of their own, similar to the one above, and swap it with a classmate. But before doing this, they need to make sure that they have answers for their questions, particularly: "How many different solutions are there? How do I know?"

## Pages 2–3: Shaping Up

**Achievement Objectives**

- find fractions equivalent to one given (Number, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)

AC
EA
AA
AM
AP

**Number Framework Links**

Use this activity to help students consolidate and apply their knowledge of fractions (stages 7 and 8).

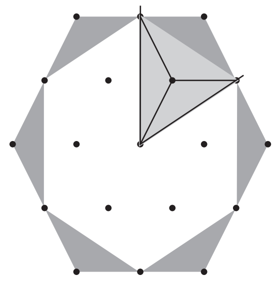
### Activity

In question **1** of this activity, students use the properties of geometrical shapes to work out what part of their areas are white. Dots and lines give additional structure to the shapes and allow for a variety of strategies. Students must explain how they arrived at their solutions. Strategies that involve multiplication of fractions belong at stage 8 on the Number Framework; strategies based on counting belong at a lower stage.

It is important that you allow for the sharing of strategies so that students who rely on counting are exposed to other, more efficient strategies. Give your students an opportunity to do this when everyone has had a chance to solve question **1**. Students need to know that any strategy that works is valid, while at the same time, they get a feel for what constitutes an efficient strategy. In this context, an efficient strategy is one that makes use of symmetry and multiplication to avoid unnecessarily counting. The extra lines on the question **1** diagrams are there to provide clues about what some of these more efficient strategies might be.

In question **1a**, students could simply count the number of smaller squares (16) that make up the large square. They could then look at the white part and see that it covers $\frac{1}{2}$ of 4 squares, which is the same as 2 smaller squares, so $\frac{2}{16}$ or $\frac{1}{8}$ of the large square is white. Alternatively, they could recognise that the square has been divided in half and then in half again and again (as shown by the lines). This means that the area of the white square is $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$ = $\frac{1}{8}$ of the area of the large square. Other multiplicative strategies are possible.

Question **1b** can be completed in the same way as question **1a**. If we count all the triangles, then the white triangle is $\frac{1}{16}$ of the large triangle. Alternatively, we can see that the large triangle has been divided into quarters and the middle quarter has been divided into quarters again, and one of those quarters is white: $\frac{1}{4}$ of $\frac{1}{4}$ = $\frac{1}{16}$.



Question **1c** can be broken into 12 rhombuses or 24 triangles and the white pieces added or the red pieces added and subtracted from 1. A more efficient strategy would be to divide the hexagon into quarters or, as in the diagram below, sixths. The diagram shows how the sixth can be divided into 4 equal triangles, of which 3 are white. In other words, $\frac{3}{4}$ of the sixth is white, but the same is also true of each of the other 5 sixths, making it true of the hexagon as a whole.

The rectangle in question **1d** has been divided into quarters, then each quarter has been divided in half and each half divided into quarters, one of which is white. This means that the area of the white triangle is $\frac{1}{4} \times \frac{1}{2} \times \frac{1}{4} = \frac{1}{32}$ of the whole rectangle.

In questions **2** and **3**, the students divide the shapes into equal-sized pieces and the name the parts as fractions.

Question **4a** poses a challenge that could occupy students for some time. They are likely to think that they have run out of possibilities once they have found four or five different ways of dividing the triangle into quarters, yet the question says that there are at least 10. The trick is to stop thinking of symmetrical divisions and to realise that a dot doesn't always have to be joined to its closest neighbour. The triangle in question **4b** is a further case of division into quarters. There are various ways of showing that the areas of the four parts are equal; one is demonstrated step by step in the Answers.

## Page 4: Chocolate Choices

**Achievement Objectives**

- find fractions equivalent to one given (Number, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)

| |
|---|
| AC |
| EA |
| AA |
| AM |
| AP |

**Number Framework Links**

Use this activity to help students learn to express fractions as ratios and to express ratios in their simplest form by dividing by common factors (stages 6 and 7).

### Activity

Question **1** involves straightforward proportional thinking and can be solved either by counting chocolates and comparing numbers or by making connections between the sizes of the boxes. In question **1a**, the students might notice that if they turned box A 90 degrees, it would fit twice into box B, so 2A = B. Box B would fit neatly over the top of box C, leaving 12 chocolates uncovered at the bottom, which is the number of chocolates in box A, so C = B + A or C = 3A. Once they know the relationship between the sizes of the boxes, it's easy for them to determine the weight and cost of each.

Question **2** requires the students to work out Tania's best option. This can be done by counting the strawberry hearts for each combination of boxes that will make up 1 kg. Box A gives Tania 3:9 (or $\frac{3}{12}$), box B gives her 8:16 (or $\frac{8}{24}$), and box C gives her 13:23 (or $\frac{13}{36}$).

If your students have had little experience working with ratios, you will need to show them carefully how the ratio 3:9 (favourites to non-favourites) can equally well be expressed as the fraction $\frac{3}{12}$ (favourites out of total number of chocolates in the box).

There are four combinations of boxes that will make up 1 kg: 4 of box A (4A), 2 of box A and 1 of box B (2A + B), 2 of box B (2B), or 1 of box A plus 1 of box C (A + C). The last two of these options will give Tania the same number of strawberry hearts: 16. Expressed as a ratio of favourites to non-favourites, this is 16:32 or 1:2.

Give your students time to explain their strategies. Make sure they understand that the ratios 16:32 and 1:2 are identical. They may think that the ratio 16:32 contains within itself the number of chocolates and 1:2 doesn't. This is a misconception. A ratio never means anything without its context. To make sense of the 16:32 (or 1:2), we need to know the "story" that goes with the numbers: in this case, the strawberry hearts and the number of chocolates in the boxes.

Question **3** doesn't introduce any new information or processes but gives students a chance to apply and consolidate the ideas they have met in question **2**. They may find it useful to collate the information and answers for both questions in a table.

In question **4**, the students need to work out which combination of boxes would give each of the three friends the same number of favourite chocolates. Encourage them to make use of the work they have already done. There are only four combinations to be considered, and they know from question **2** that both 2B and A + C give Tania 16 of her favourites. The only remaining task is to find which of these two combinations also gives Atama and Chloe 16 of their favourites.

## Page 5: Star Clusters

**Achievement Objectives**

- find fractions equivalent to one given (Number, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)
- find and justify a word formula which represents a given practical situation (Algebra, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

AC
EA
AA
AM
AP

**Number Framework Links**

Use this activity to help students consolidate and apply their knowledge of fractions (stages 6 and 7).

### Activity

Question **1** has 12 parts to it, so your students may find it helpful if they create a suitable table and then complete it as they work out each fraction:

| | Packets | | | |
|---|---|---|---|---|
| **Clusters** | **6 stars** | **8 stars** | **12 stars** | **20 stars** |
| 3 star | $\frac{3}{6} = \frac{1}{2}$ | $\frac{3}{8}$ | $\frac{3}{12} = \frac{1}{4}$ | $\frac{3}{20}$ |
| 4 star | $\frac{4}{6} = \frac{2}{3}$ | $\frac{4}{8} = \frac{1}{2}$ | $\frac{4}{12} = \frac{1}{3}$ | $\frac{4}{20} = \frac{1}{5}$ |
| 5 star | $\frac{5}{6}$ | $\frac{5}{8}$ | $\frac{5}{12}$ | $\frac{5}{20} = \frac{1}{4}$ |

Students should find it easy to come up with numerous answers for question **2** without needing any system. You may, however, like to suggest an approach that will help them find many solutions at the same time as they create meaning for algebraic representation and substitution.

Suggest using this shorthand: A (for one 6-star packet), B (8 stars), C (12 stars), and D (20 stars). We can see that 144 is divisible by 6 (24A packets make 144), and 8 (18B packets make 144), and 12 (12C packets make 144). Also, B + C = D and 2A = C, so by substitution, we can get many more solutions:
6D + 2C = 144
5D + 3C + B = 144 (replacing 1D with one B + C)
4D + 4C + 2B = 144 (replacing 1D with one B + C)
3D + 5C + 3B = 144 (replacing 1D with one B + C)
and so on until we end up with 8C + 6B = 144.

We know that 12C = 144 and 2A = C, so we can keep replacing 1C with 2A to get another group of solutions:
2A + 11C = 144 (replacing 1C with 2A)
4A + 10C = 144 (replacing 1C with 2A)
6A + 9C = 144 (replacing 1C with 2A)
and so on until we end up with 24A = 144.

Other combinations such as 6A + 6B + 3D can provide the starting point for further lists of possibilities.

Question **3** asks students to calculate $\frac{1}{3}$ of 144 and $\frac{1}{4}$ of 144. They should share their strategies for these calculations. Those who know their basic facts are at stage 7 and more likely to use the short form of division.

Question **4** requires students to express $\frac{108}{144}$ in its simplest form. Although these numbers lie outside the range of known facts for many students, they have been working with 144 in questions **2** and **3** and should have little trouble finding a strategy that they can use to simplify the fraction. Those who remember that 144 = 12 x 12 have access to the most efficient strategy of all, the one that avoids finding the product 12 x 9 and sees that Anita is in effect using just 9 stars out of every 12, so she uses $\frac{9}{12} = \frac{3}{4}$ of the total.

## Pages 6–7: What Do You See?

**Achievement Objectives**

- find fractions equivalent to one given (Number, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)

AC
EA
AA
AM
AP

**Number Framework Links**

Use this activity to help students consolidate and apply their knowledge of equivalent fractions (stage 7), using reunitising as a strategy (stage 8).

### Activity

This activity models a classroom situation in which a number of students share different ways of viewing the shaded part of a rectangle. It doesn't matter that three of the contributions seem unlikely; they make excellent subjects for discussion. Take particular care with the last two; your students may not have come across such fractions before, but if they study the illustrations, they will start to put meaning to the tricky notion of a fraction divided by a fraction. Study them carefully before introducing this activity to your students and think of ways of rewording or teasing out the speech bubbles. For example, you could expand the last speech bubble to read: *"I can see groups of 6 squares. There are $3\frac{1}{3}$ of these groups. 2 of them are shaded. That's 2 out of $3\frac{1}{3}$."*

When they come to do question **1**, the students should draw on the understanding that they have gained from the discussion of the ideas presented in the speech bubbles, particularly the last two.

In question **2**, the students write a number of equivalent fractions. Although they are only asked for "at least six", there are in fact an infinite number of possibilities. Encourage them to find at least one that is a fraction divided by a fraction (for example, $1\frac{1}{8} / 4\frac{1}{2}$). Students who have encountered algebraic notation may be able to say that all the equivalent fractions can be represented by the term $\frac{x}{4x}$.

The simplest way of finding out whether an egg tray can be divided into a particular kind of fraction is to check if the denominator is a factor of the number of eggs in the tray (stage 7). In question **3b**, the students need to draw egg trays and then divide them up accordingly. Some may like the challenge of finding just how many different ways the trays can be divided into sixths and eighths.

## Page 8: Discount Deals

**Achievement Objectives**

- increase and decrease quantities by given percentages, including mark up, discount, and GST (Number, level 5)
- express a fraction as a decimal, and vice versa (Number, level 4)
- express a decimal as a percentage, and vice versa (Number, level 4)

| |
|---|
| AC |
| EA |
| AA |
| AM |
| AP |

**Number Framework Links**

Use this activity to help students consolidate and apply their knowledge of fractions, decimals, and percentages (stages 7 and 8).

### Activity

Question **1** is designed to make students read all the information and think about it before getting into the detail. They should be able to look at $\frac{1}{4}$ and 20% and say that $\frac{1}{4}$ off is better than 20% off, so Bookz 4 U is never going to have the best deal.

The easiest way to answer question **2** is to create a spreadsheet or draw up a table, using a column for each number of notebooks and a column for each of the shops except for Bookz 4 U, which has been eliminated. If the students create a computer spreadsheet, they can use formulae to speed up the calculations, but they will need to be careful to apply them correctly: it is very easy to make mistakes. The students can shade or highlight the cells on the spreadsheet that have the best price. The finished and highlighted spreadsheet organises the information in a way that makes it easy to answer question **3**.

| Number | M & M | SS | BB | B & S |
|---|---|---|---|---|
| 1 | 5 | 3.75 | 5 | 5 |
| 2 | 5 | 7.5 | 10 | 10 |
| 3 | 10 | 11.25 | 13 | 15 |
| 4 | 10 | 15 | 16 | 20 |
| 5 | 15 | 18.75 | 19 | 25 |
| 6 | 20 | 22.5 | 22 | 30 |
| 7 | 25 | 26.25 | 25 | 35 |
| 8 | 30 | 30 | 28 | 40 |
| 9 | 35 | 33.75 | 31 | 45 |
| 10 | 40 | 37.5 | 34 | 30 |
| 11 | 45 | 41.25 | 37 | 35 |
| 12 | 50 | 45 | 40 | 40 |
| 13 | 55 | 48.75 | 43 | 45 |
| 14 | 60 | 52.5 | 46 | 50 |
| 15 | 65 | 56.25 | 49 | 55 |
| 16 | 70 | 60 | 52 | 60 |
| 17 | 75 | 63.75 | 55 | 65 |
| 18 | 80 | 67.5 | 58 | 70 |
| 19 | 85 | 71.25 | 61 | 75 |
| 20 | 90 | 75 | 64 | 60 |
| 21 | 95 | 78.75 | 67 | 65 |
| 22 | 100 | 82.5 | 70 | 70 |
| 23 | 105 | 86.25 | 73 | 75 |
| 24 | 110 | 90 | 76 | 80 |
| 25 | 115 | 93.75 | 79 | 85 |
| 26 | 120 | 97.5 | 82 | 90 |
| 27 | 125 | 101.25 | 85 | 95 |
| 28 | 130 | 105 | 88 | 100 |
| 29 | 135 | 108.75 | 91 | 105 |
| 30 | 140 | 112.5 | 94 | 90 |

Students may have difficulty interpreting the way the discounts are to be applied, and you may need to clarify these points by discussion:

- One notebook at Mags & More will cost \$5, but thanks to their discount, 2 books will also cost \$5. Three or 4 books will cost a total of \$10. All further books will cost a further \$5.
- The $\frac{1}{4}$ off discount at Sam's Stationery effectively means that the cost of each book is \$3.75.
- The 40% discount at Bargain Books means that for all books after the first 2, the cost is \$3. (40% of \$5 is \$2.)

The discount at Books & Stuff means a saving of \$20 on every 10 books bought. \$20 is the cost of 4 books at their regular price, so 10 end up costing the same as 6. Rather remarkably, it is cheaper to buy 10 books than it is to buy 7–9. When entering the information in a spreadsheet, the students should keep adding \$5 for each book until they get to 10. At that point, they deduct \$20 from the cost and then start adding \$5 to this new total for every additional book. They deduct a further \$20 when the number of books gets to 20 and 30.

As long as the students have constructed a spreadsheet or table, question **3** becomes a matter of looking at some well-organised information. After 10 books, a pattern emerges:

- For numbers that end in 0 or 1, Books & Stuff has the best price.
- For numbers that end in 2, Books & Stuff and Bargain Books are equally the best.
- For numbers that end in 3–9, Bargain Books has the best price.

**Extension**

Students could cut out and collect newspaper advertisements or flyers illustrating different types of discount or different discounts being offered on the same or a similar item. They could then compare them, making a display of their findings.

## Page 9: Cereal Calculations

**Achievement Objectives**

- explain satisfactory algorithms for addition, subtraction and multiplication (Number, level 4)
- share quantities in given ratios (Number, level 5)

AC
EA
AA
AM
AP

**Number Framework Links**

Use this activity to help students:

- consolidate their knowledge of basic facts (stage 6)
- gain confidence in using partitioning strategies for addition and multiplication (stages 6 and 7)
- apply their knowledge of ratio and proportion (stage 7).

### Activity

This activity has an unusual number of details and dimensions. These include:

- 3 classes of different size
- 3 camps of different length
- 3 cereals, each available in 2 sizes
- a standard serving
- a 3-part ratio to be applied to cereal consumption
- a requirement that wastage be minimised
- numerous calculations involving x and +
- the summarising and interpretation of results.

Given this complexity of information, students are likely to need help working out how the parts relate to each other and how to bring so many facts together to answer the questions.

Once they are started, however, they will find that the numbers are user-friendly and that the table on the page gives them an excellent pattern to follow. If your students need a lot of guidance, work through question **1** with them step by step and then leave them to do questions **2** and **3** by themselves or with a classmate. You may need to discuss what kind of table is needed for the final question.

Here is a suggested approach for question **1**:
1. Interpret the ratio.
2. Apply the ratio to the number of students in the class.
3. Work out quantities needed for each cereal and enter them in the table.
4. Decide what packet(s) to buy.
5. Add up the Needed and Buy columns. These two totals are required for question **4**.

As mentioned above, the numbers in this activity have been chosen so that students can multiply and add them using familiar strategies, such as place-value partitioning and rounding and compensation. Discourage them from using calculators.

**Extension**
Students could investigate the prices of different-sized packets of their favourite cereals in the supermarket and compare the price per unit. They should think about how to compare the prices and decide what the best deal is. They could also do a comparative pricing survey of several different cereals or brands.

## Page 10: Speed Read

**Achievement Objectives**
- share quantities in given ratios (Number, level 5)
- write and solve problems involving decimal multiplication and division (Number, level 4)

AC
EA
AA
AM
AP

**Number Framework Links**
Use this activity to help students consolidate and apply their knowledge of ratios and proportion (stages 6 and 7).

### Activity

In this activity, students need to calculate how long it will take Jack and each of his friends to read 630 pages, given information about their reading speeds. To help them, all speeds except Jack's are visualised as double number lines.

The book suggests that a calculator could be used for this activity, but there is nothing in it that students couldn't reasonably be expected to manage using familiar number strategies. If you decide that calculators are not to be used, you will need to make this clear from the beginning.

Each part of question **1** can be solved by working out the number of pages the person reads in 1 hour and dividing 630 by this number. But the parts can all be solved in a variety of ways, and your students are more likely to develop their proportional reasoning skills if you don't give them a formula to follow. Here are examples of possible strategies:
Jack: 30 x **20** = 600 and 30 x **1** =30. Time taken = 21 hours.
Atama: 20 x **30** = 600 and 20 x **1** = 20, leaving 10, which is half of 20. Time taken = 31.5 hours.
Mikey: 45 x **10** = 450 and 630 – 450 = 180, 45 x 2 = 90, so 45 x **4** =180. Time taken = 14 hours.
Charlotte: 630 ÷ 9 = 70, so the time taken would be half of this, which is 35 hours.
Miranda: 630 ÷ 90 = 7, so she must take 7 x **4** = 28 hours.

Students with more advanced proportional reasoning skills may realise that it is possible to use the reading time for one person to find the reading time for another. For example, Mikey reads 90 pages in 2 hours, which is twice Miranda's speed, so Miranda must take twice as long as Mikey: 2 x 14 = 28 hours. Jack reads $1\frac{1}{2}$ times as fast as Atama, so Atama must take $1\frac{1}{2}$ times as long as Jack: $1\frac{1}{2}$ x 21 = $31\frac{1}{2}$ hours.

The Answers demonstrate one way of solving question **2**. Another is to enter the known information in a table and fill in the empty cells. The result is a triple number line, slightly disguised. A strip diagram would be equally suitable.

| | 1 hour | 2 hours | 3 hours | 4 hours |
|---|---|---|---|---|
| **Jack** | 30 | 60 | 90 | 120 |
| **Atama** | 20 | 40 | 60 | 80 |
| **Mikey** | 45 | 90 | 135 | 180 |

From the table, we can see that after 3 hours, Jack has read 90 pages, Atama has read 60, and Mikey has read 135. When Mikey is on page 180, Jack is on page 120, and Atama is on page 80.

## Page 11: Horse Sense

**Achievement Objectives**

- find fractions equivalent to one given (Number, level 4)
- express quantities as fractions or percentages of a whole (Number, level 4)

AC
EA
AA
AM
AP

**Number Framework Links**

Use this activity to help students consolidate and apply their knowledge of fractions and proportions (stage 7).

### Activity

This activity is based on a real-life context: prize pools. Although the book suggests that a calculator can be used, students are likely to learn more about proportional reasoning if they manage without one. Answers expressed as fractions will picture the ratios better than decimals.

Question **1a** provides a visual model of the prize pool and asks students to work out the value of each cell ($125). They might do this using a partitioning strategy: 40 x **100** = 4 000, leaving 1 000; 40 x **20** = 800, leaving 200; 40 x **5** = 200. So one cell is worth 100 + 20 + 5 = $125. Alternatively, they might note that the grid has 5 rows worth $5,000, so 1 row of 8 cells must be worth $1,000. 8 x **100** + 8 x **20** + 8 x **5** = $1,000, so again 1 cell is worth 100 + 20 + 5 = $125.

In question **1b**, students shade blocks of cells to represent the value of the first three prizes. To do this, they first need to work out how many lots of $125 equate to the value of each prize. Perhaps the simplest strategy is to use repeated doubling to establish that 2 cells are worth $250, 4 are worth $500, and 8 are worth $1,000. Using this information, students should be able to see that the winner ($3,125) is represented by 3 x 8 + 1 = 25 cells, and the second and third place-getters ($1,000 and $500) by 8 and 4 cells respectively.

In question **1c**, students work out what fraction of the prize pool goes to each of the three place-getters. This is the same as working out what part of the grid has been shaded for each of the three placings. All three fractions can be expressed with 40 as the denominator, but you should be encouraging your students to always simplify fractions if possible.

The 3 unshaded cells should tell the students that the answer for question **1d** is $\frac{3}{40}$. At this point, you could ask them why the grid was divided into 40 rather than 20 or 50 or some other number. The reason is, of course, that 40 is the lowest common denominator of the four fractions, $\frac{5}{8}$, $\frac{1}{5}$, $\frac{1}{10}$, and $\frac{3}{40}$.

For question **2**, students need to calculate the trainer's fraction; they can then halve it for the jockey. They first must know that 10% is the same as $\frac{1}{10}$. The Answers show how the answers can be worked out by multiplying two fractions then simplifying.

Once students have solved question **2**, they can use their solutions to provide the answer to question **3**. No further calculation is needed.

## Pages 12–13: Small Business

**Achievement Objectives**

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- find a given fraction or percentage of a quantity (Number, level 4)

| |
|---|
| AC |
| EA |
| AA |
| AM |
| AP |

**Number Framework Links**

Use this activity to help students

- consolidate their knowledge of basic facts (stage 6)
- consolidate and apply their knowledge of fractions (stages 6 and 7)
- develop their ideas about proportion (stage 7).

### Activity

Using this imaginative (and unlikely!) business scenario, students explore the difference between group and individual profit and calculate and compare different profits.

Before they get far with question **1**, they need to address the question posed by the thought bubble: "How do you measure success?" It is clear that the Five Alive group had the largest group profit ($4 more than any of the other three teams), but when the $30 is shared between the 5 members of the team, this amounts to $6 each, which is less than the $7 per person earned by the members of the Carwash Ltd team. So if success is the profit earned by the individual members of the team, Carwash Ltd is most successful. Students are likely to see the second measure of success as the most valid one.

Question **2** is about order: does it make a difference whether we divide and then multiply or multiply and then divide? Students will find that no matter which of the four sets of results they use, they get the same answer regardless of the order of the operations. For example, Five Alive:

- Option 1: Each person gets 30 ÷ 5 = $6; when tripled by the 2-for-1 offer, this becomes 6 x 3 = $18.
- Option 2: The group profit, once tripled by the 2-for-1 offer, is 30 x 3 = 90; when shared between 5 people, this becomes 90 ÷ 5 = $18.

You could discuss with your students whether showing that this is true for one group is enough to prove that it is always true. (It's not. They could have a rogue result. They should check the figures for the other three groups for a greater degree of certainty.)

Question **3** asks students to look at the amount each person receives (including the bonus) and compare this with the original profit for the whole team. The result in each case is best expressed by a fraction. If the four fractions are left as improper fractions, an interesting pattern can be seen: $\frac{3}{2}, \frac{3}{3}, \frac{3}{4}, \frac{3}{5}$. You could challenge your students to think about why this pattern occurs. The reasons are these: the top number says that the profit has been tripled; the bottom number says that it is shared among that number of people. So the fraction is independent of the dollar value of the profit: if the profit is doubled and there are 7 people to share it, the fraction is $\frac{2}{7}$.

Students should share the strategies they use to answer question **4**. Here are three possibilities for question **4a**:

1. Each person earns 64 ÷ 8 = $8 profit. In addition to this, they get 5 x 8 = $40 from the IRD, making a total of $48.
2. Each person earns 64 ÷ 8 = $8 profit. In addition to this, the team gets 5 x 64 = $320 from the IRD. Each person gets $\frac{1}{8}$ of this bonus ($40), making a total of 8 + 40 = $48.
3. The team's total profit including the bonus from the IRD is 64 x 6 = $384. This needs to be split between the 8 team members, giving 384 ÷ 8 = 48. (8 x 50 = 400. 400 less (8 x 2) = $384. So 384 ÷ 8 = 50 – 2 = $48. [Rounding and compensating]).

Similar strategies can be used to solve question **4b**.

## Pages 14–15: Getting Tough

**Achievement Objectives**

- express a decimal as a percentage, and vice versa (Number, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)

AC
EA
AA
AM
AP

**Number Framework Links**

Use these activities to help students consolidate their knowledge of basic facts (stage 6) and use them in conjunction with percentages (stage 7).

### Activity

This activity illustrates the way that a training programme for an endurance event is often constructed. It gives students practice at interpreting information expressed as percentages, finding a percentage of a quantity (of time), and rounding. If your students are not very familiar with percentages, you should do some pre-teaching. You will also need to make a decision on whether you want them to use calculators for the first three questions. If not, discuss with them the kinds of strategies that they will need. If yes, make sure that they know how to enter percentages on a calculator. Even if theirs has a % key, expect them to enter percentages as decimals: 0.8 for 80%, 0.25 for 25%, 0.05 for 5%. By insisting on this, you will reinforce the understanding that the decimal and percentage forms are mathematically identical, you will be saving the students unnecessary keystrokes, and you will ensure that they are able to do percentage calculations on any calculator, not just one with a % key.

In question **1a**, students work out their own maximum heart rate. This is expressed as maximum heartbeats per minute. Using the given formula, a 9-year-old's maximum heart rate would be 220 – 9 = 211 beats/min.

Question **1b** is asking for the ranges of heartbeat that apply to the student answering the question. As most of those using the activity will be 9–11 years old, here are the ranges for your reference:

| Age | Easy (60–70%) | Moderate (70–80%) | Race Pace (80–85%) |
|---|---|---|---|
| 9 | 127–148 | 148–169 | 169–179 |
| 10 | 126–147 | 147–168 | 168–179 |
| 11 | 125–146 | 146–167 | 167–178 |

For question **2**, the students could either add all the minutes and then divide by 60 or convert time to hours and minutes and then add. Which method would be quicker and why? If students are not using calculators, ask them to share their strategies.

Question **3** asks students to convert the percentage amounts to their minute equivalents, rounded to the nearest 5 minutes. They should begin by drawing up a suitable table. This should be similar to the one in the student book but with the third column split into 3 so that there is a separate column for each training zone. If the information is organised in this way, it becomes very easy to answer the last question.

The calculations required for question **3** all involve user-friendly numbers, so there is no particular need for calculators. By using number strategies, students will gain experience and confidence at working with percentages and their fraction equivalents. For example: 25% of 140 is the same as $\frac{1}{4}$ of 140. 140 = 100 + 40. $\frac{1}{4}$ of 100 = 25 and $\frac{1}{4}$ of 40 = 10, so $\frac{1}{4}$ of 140 = 25 + 10 = 35.

Make sure that you remind students to round to the nearest 5 minutes. This is partly to keep the numbers simple, but it also reflects the fact that it is unlikely that a training programme of this kind would be organised in blocks of 19 minutes or 51 minutes (for example). When a number is exactly halfway between two 5s, it is standard practice to round up. In this situation, another number will have to be rounded down so that the total minutes remain the same. This means that the numbers in the Answers may be up to 5 minutes different from your students' answers, depending on the order in which they did the parts.

Question **4** asks for the proportion of time Maree will spend in each of the zones. As long as they have completed a table as suggested for question **3**, all that the students need to do is put the total for each of the three zones (in turn) over the total minutes for the entire training programme, calculate the results as decimals, and interpret these as percentages. A calculator is definitely needed for these calculations.

## Page 16: Demolition Dollars

**Achievement Objectives**

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- find and justify a word formula which represents a given practical situation (Algebra, level 4)

| |
|---|
| AC |
| EA |
| AA |
| AM |
| AP |

**Number Framework Links**

Use this activity to help students:

- consolidate their knowledge of basic facts (stage 6) and advanced multiplicative strategies (stage 7)
- develop strategies for solving problems involving rates and proportion (stage 7).

### Activity

This activity requires students to work through a series of problems, each of which has one missing detail that can be deduced using proportional reasoning. The speech bubble and the answer to question **1** contain information that is needed for answering questions **2–7**; questions **2–6** are not interdependent, but question **7** relies on correct answers for questions **2** and **6**.

The activity is rich with potential for proportional reasoning. Each part can be solved in a variety of different ways, and the numbers are such that there is no need for a calculator. Those who like the challenge of a puzzle may be able to work through each step independently, but you may prefer to discuss it first with your students before setting them to work in pairs or small groups. Make sure that you give them the opportunity to share their strategies.

The following paragraphs suggest one way of reasoning out the answer to each question:

For question **1**, we know that the boys worked 24 hours in total and earned \$120. Using a halving strategy, 120 ÷ 24 = 60 ÷ 12 = 5, so the pay rate is \$5 per hour.

The calculation for question **2** is 7 x 4 x 5 = \$☐. 28 x 5 = 14 x 10 = \$140 (doubling and halving). Alternatively, students may realise that 1 boy working for 4 hours earns \$20, so 7 boys working for 4 hours will earn 7 x 20 = \$140.

The weekly target mentioned in question **3** is \$150. This represents 30 hours' work. If 5 boys worked for a total of 30 hours, they each worked for 6 hours (6 x 5 = 30).

In week 4 (question **4**), the boys earned \$225 in 5 hours, so in 1 hour they must have earned 225 ÷ 5 = \$45. If the boys earned \$45 in 1 hour, there must have been 9 boys working (5 x 9 = 45).

In question **5**, we know how many parents worked, so one approach is to begin by finding out what the parent contribution was. The 3 parents worked for 4 hours at a rate of \$10 per hour, so they earned 3 x 4 x 10 = \$120 of the total of \$280. This means the boys earned 280 – 120 = \$160. The boys also worked for 4 hours, so each hour they earned 160 ÷ 4 = \$40. This means that 8 boys worked that day (5 x 8 = \$40).

In question **6**, we have all the details we need to go straight to the answer: the boys earn \$(10 x 4 x 5) and the parents earn \$(2 x 4 x 10). 10 x 20 + 8 x 10 = 200 + 80 = \$280.

Question **7** requires students to add together the totals for the 6 weekends:
120 + 140 + 150 + 225 + 280 + 280 = \$1,195. So Braydn's team is short of its target by \$5.

## Page 17: Painting by Numbers

**Achievement Objective**

- share quantities in given ratios (Number, level 5)

**Number Framework Links**

AC
EA
AA
AM
AP

Use this activity to introduce students to the concept of inverse proportion (stages 7 and 8).

### Activity

This activity introduces students to inverse proportion. This is the kind of relationship that exists between two quantities when an increase in one means a decrease in the other. Students know without being told that such relationships exist, and when you introduce this activity, you could begin by drawing out that knowledge.

Examples of inverse relationships include: the more fully the taps are turned on, the less time it takes to fill the bath; the faster you bike, the shorter the journey; the slower you type, the longer the document takes; the more expensive an item, the fewer you can buy; the bigger the files, the fewer that will fit on a disk; the thicker you apply the paint, the smaller the area you can cover.

An important idea about inverse relationships is that they can only exist when they relate to a third thing that is fixed in size or quantity: a bath to be filled, a journey to be completed, a document to be typed, an amount of money to be spent, a disk to be filled, a can of paint to be applied. If you had an endless supply of money, the relationship between cost and number would no longer be an inverse one: you could buy as much or as many of something as you wanted, no matter what the cost.

The dialogue in the student book provides an excellent starting point for class discussion: there is a job to be done (this is the thing that is "fixed in size or quantity") and the more people who help (within the limitations of common sense), the faster the task will be completed. The dialogue gives examples of how the ratio works but doesn't model how to to write it down. Students could start with a table like this:

| Students | Hours | Explanation |
|---|---|---|
| 2 | 12 | Given information |
| 4 | 6 | Double the number of students, halve the time |
| 8 | 3 | Double the number of students, halve the time |

The table answers question **1** and, if a line is added at the top, can provide the answer to question **2**.

If your students are familiar with ratio notation, you could get them to use it to describe the student:hours ratios. Emphasise that the ratios 2:12, 4:6, and 8:3, though connected, are *not* the same, so they must not have "=" signs placed between them. 2:12 → 4:6 → 8:3 is an acceptable way of linking the ratios and can be read "2:12 leads to the ratio 4:6, which leads to the ratio 8:3."

Students can solve question **3** by using the 1:24 ratio that they arrived at for question **2**. As they are interested in how long the job would take 3 student painters, they should multiply the 1 by 3 and divide the 24 by 3. Therefore, 3 painters would take 8 hours. Discuss whether it is a coincidence that 3 painters take 8 hours while 8 painters take 3 hours. It's not. It is important that students realise that inverse ratios always work in pairs like this: if a ratio is true for a given situation, when the two numbers are reversed that (different) ratio is also true. Students might notice that for all the inverse ratio pairs in this activity, the product of the two numbers is 24. This is not a coincidence either: 24 is the number of student hours 1 mural will take.

You should give your students every opportunity to solve question **4** by themselves or in small groups. As long as they have begun to think multiplicatively, they will have a number of different strategies they can use. They should share these with a classmate and with the group. Here are three possible strategies:

1. Divide the 18 students into 6 groups of 3 and set them all to work. This means that each group must paint 2 murals (6 x 2 = 12). We know from question **3** that it will take 3 students 8 hours to paint 1 mural, so it will take each group 16 hours (twice as long) to paint both their murals. So the 18 students will take 16 hours to paint 12 murals.
2. Using our answer from question **2**, we know that it would take 1 student 24 hours to paint a mural. So 18 students would paint a mural in $24 \div 18 = 1\frac{1}{3}$ hours. This means that it will take $1\frac{1}{3} \times 12 = 16$ hours for them to paint 12 murals.
3. If 18 students are painting 12 murals, that means that 3 students will be responsible for 2 murals. It takes 2 x 24 = 48 hours to paint 2 murals, so if 3 are sharing the work, it will take them 48 ÷ 3 = 16 hours.

# Pages 18–19: Make 1.5

**Achievement Objectives**

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- express quantities as fractions or percentages of a whole (Number, level 4)
- find fractions equivalent to one given (Number, level 4)

AC
EA
AA
AM
AP

**Number Framework Links**

Use this activity to:

- help students consolidate their basic addition facts (stages 5 and 6) and whole number strategies and apply these to decimal numbers (stage 7)
- introduce the idea that proportions can be compared (stage 8).

### Game

This game is designed to give students practice at adding decimals. The range of numbers used (all expressed to 1 decimal place) reinforces students' knowledge of basic number facts at the same time as it strengthens their understanding of place value.

If your students have had only a little experience of decimals, you may find it worthwhile to do some pre-teaching using The Value of Place Value (*Book 5: Teaching Addition, Subtraction, and Place Value*, Numeracy Project series, page 48).

The game is similar to games that students may already be familiar with and is straightforward to learn and play. But it is a good idea to introduce it to the whole group using another target number (for example, 1.7). If you do this, you can make sure that everyone understands the rules, knows the aim of the game, and is aware of the mathematical strategies needed. Some students will be able to use known facts to handle all the additions. Others may use tidy number and/or doubling strategies. For example: you have 0.9 and are trying to make 1.7. 0.9 + 0.1 = 1, and 0.1 + 0.7 = 0.8, so 0.8 is the card needed.

Make sure, too, that everyone knows how to score the game. Players can only earn points if they get rid of all their cards before the other player. The points they earn are the total of the numbers on the cards left in their opponent's hand plus the 5 bonus points. The 15 points total to win a game is only a suggestion. Working to that total, the bonus points alone will ensure that the first player to win 3 rounds wins the game. The winner may even reach the total of 15 in 2 rounds if their opponent is left with a number of high cards each time.

### Activity

The activity is an extension to the game (not a necessary part of it) that involves adding multiple decimal numbers and determining which of two proportions is greater. This latter skill will be new to a number of students, and you will need to think about how to introduce it. Mixing Numbers (*Book 7: Teaching Fractions, Decimals, and Percentages*, Numeracy Project series, pages 34–35) suggests one approach. Fraction Strips (Numeracy Project material master 7-7, available at www.nzmaths.co.nz) provides another approach. All the proportions encountered in questions **a–e** can be found on this material master and easily compared one with another. A third approach is to express each pair of proportions as fractions over a common denominator. It is then immediately obvious which is the greater.

# Page 20: Pop Star Pics

**Achievement Objective**

- There are no directly applicable achievement objectives, but enlargement appears under the suggested learning experiences for Geometry, levels 3 and 4.

AC
EA
AA
AM
AP

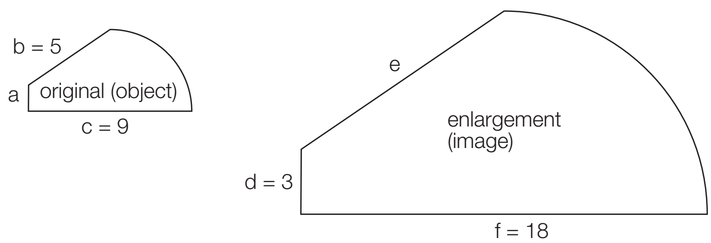
**Number Framework Links**

Use this activity to help students learn to solve problems involving fractions, proportions, and ratios (stage 8).

## Activity

This activity introduces ratio in an enlargement context. Students learn that by comparing known lengths, they can find missing lengths. They also learn how to test whether one figure is an enlargement of another.

The key idea behind enlargement is that all lengths grow or shrink according to the same ratio (or scale factor). If they didn't, the figure would lose its shape. If we put any length from an enlarged figure over its corresponding (matching) length in the original figure, we get the number (often a fraction) known as the scale factor. This principle can be described mathematically as $\frac{\text{image}}{\text{object}}$ = scale factor.




When two figures are related by enlargement, as in the example above, we can use ratio in two different ways:

1. To compare a length in the enlargement with its corresponding length in the original. These comparisons are made *between* figures and can be thought of as *external* comparisons. Any such comparison will give the scale factor for that enlargement.
   In the figure above, $\frac{f}{c} = \frac{d}{a} = \frac{e}{b}$ = scale factor. $\frac{f}{c} = \frac{18}{9} = 2$, so the scale factor is 2. This means that all lengths in the enlargement are twice what they are in the original. We can use this information to show that a = 1.5 and e = 10.

2. To compare the relationship between two lengths *within* the enlargement with the relationship between the two corresponding lengths within the original. These comparisons can be thought of as *internal* comparisons. In the figure above, $\frac{f}{d} = \frac{18}{3} = 6$. This means that the base of this figure is 6 times height d. The same must be true of c and a in the original. We can use this information to show that height a is 1.5 ($\frac{1}{6}$ x 9). In addition to $\frac{f}{d} = \frac{c}{a}$, other comparisons can be made, including $\frac{d}{e} = \frac{a}{b}$ and $\frac{e}{f} = \frac{b}{c}$. These ratios are all different, and none equals the scale factor. *You can't find the scale factor by comparing the lengths of sides* within the same figure – *only by making comparisons* between figures.

Some problems can be solved using either of these two kinds of comparison (for example, question **1**); most require one or the other (for example, question **2**).

The above ideas are included for teachers who are uncertain about the application of ratios to enlargement. There is no need to teach them formally at this level. Focus instead on building on intuitive understandings about enlargement and applying them to simple contexts with easy numbers, as in this activity.

In question **1**, students can use ratio in either of the two ways outlined above:

1. By comparing the base of the small rectangle with that of the medium rectangle, it can be seen that the scale factor must be $\frac{4}{6} = \frac{2}{3}$. In other words, all lengths in the small rectangle are $\frac{2}{3}$ of what they are in the medium rectangle. This means that height A must be $\frac{2}{3} \times 9 = 6$ cm. Similarly, by comparing the base of the large rectangle with the base of the medium rectangle, we can see that the scale factor for this enlargement is $\frac{20}{6} = \frac{10}{3} = 3\frac{1}{3}$. So height B must be $3\frac{1}{3} \times 9 = 27 + 3 = 30$ cm.
2. Comparing the height of the medium rectangle with its base, it can be seen that the height:base ratio is $\frac{9}{6} = \frac{3}{2} = 1\frac{1}{2}$. In other words, the height is $1\frac{1}{2}$ times the base. The same must be true in the small rectangle, so height A must be $1\frac{1}{2} \times 4 = 6$ cm. Similarly, height B must be $1\frac{1}{2} \times 20 = 30$ cm.

Students may need help to understand what question **2** means and to realise that they have to find which pairs of pictures have the same height:width or width:height ratio. Encourage your students to find their own strategies for solving this problem and to share them. The simplest strategy is likely to be to express each ratio in its simplest form (either using ratio notation, as below, or as a fraction):

| | | | | | |
|---|---|---|---|---|---|
| i. | 4:6 = 2:3 | ii. | 9:15 = 3:5 | iii. | 5:6 = 5:6 |
| iv | 7:8 = 7:8 | v. | 12.5:15 = 25:30 = 5:6 | vi. | 6:9 = 2:3. |

From the above, it is obvious that i and vi both have the same width:height ratio, as do iii and v, so these must be the pairs.

## Page 21: Tiring Teamwork

**Achievement Objective**

- use equipment appropriately when exploring mathematical ideas (Mathematical Processes, problem solving, level 4)

AC
EA
AA
AM
AP

**Number Framework Links**

Use this activity to help students learn to solve problems with the use of fractions, proportions, and ratios (stages 7 and 8).

### Activity

This problem-solving activity requires students to work out how long it would take walkers walking at different speeds to lap each other. Give your students every chance to solve the problems themselves, working in pairs or groups of no more than four. There are at least three strategies that they could use: a circular diagram, a double strip diagram, and a table.

The student book suggests using a circular diagram with counters to represent the walkers. Alternatively, students could mark a copy of the copymaster, using a pencil. A teaching clock face would serve the same purpose as long as the hands can be moved independently to represent the two walkers.

If they use a circular diagram, students may find it easier to focus on where the slower walker is when the fastest walker completes a lap rather than plot minute-by-minute progress as suggested in the student book. In question **1**, Kelvin takes 6 minutes and Ian 8 minutes, so where would Ian be when Kelvin has completed a circuit? Expressed differently, if Ian takes 8 minutes to do 1 lap, where is he after 6 minutes? After 6 minutes, he has completed $\frac{6}{8}$ or $\frac{3}{4}$ of a lap. After 12 minutes, Kelvin has completed 2 laps and Ian $2 \times \frac{3}{4} = 1\frac{1}{2}$ laps.

After 24 minutes, Kelvin will have completed 4 laps and Ian 4 x $\frac{3}{4}$ = 3 laps, so at that point, Kelvin is exactly 1 lap ahead of Ian.

The Answers show how the same information can be pictured using a double strip diagram.

Using the third suggested strategy, a table for question **1** could look like this:

| | Kelvin (min) | Ian (min) |
|---|---|---|
| Lap 1 | 6 | 8 |
| Lap 2 | 12 | 16 |
| Lap 3 | 18 | 24 |
| Lap 4 | 24 | 32 |

The table shows that, after 24 minutes, Kelvin has completed 4 laps and Ian 3.

A table for question **2** shows that, after 40 minutes, Ian has completed 5 laps and Kate 4.

| | Kate (min) | Ian (min) |
|---|---|---|
| Lap 1 | 10 | 8 |
| Lap 2 | 20 | 16 |
| Lap 3 | 30 | 24 |
| Lap 4 | 40 | 32 |
| Lap 5 | 50 | 40 |

A table for question **3** shows that, after 30 minutes, Kelvin has completed 5 laps and Kate 3. This means that Kelvin is now 2 laps ahead, so he must have lapped Kate at the 15 minute point:

| | Kelvin (min) | Kate (min) |
|---|---|---|
| Lap 1 | 6 | 10 |
| Lap 2 | 12 | 20 |
| Lap 3 | 18 | 30 |
| Lap 4 | 24 | 40 |
| Lap 5 | 30 | 50 |

## Pages 22–23: The Great Race

**Achievement Objectives**

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- read and interpret everyday statements involving time (Measurement, level 3)
- choose and construct quality data displays (frequency tables, bar charts, and histograms) to communicate significant features in measurement data (Statistics, level 4)

AC
EA
AA
AM
AP

**Number Framework Links**

Use this activity to help students:

- consolidate and apply a range of multiplicative strategies (stage 7)
- solve problems with the use of fractions, proportions, and ratios (stages 7 and 8).

### Activity

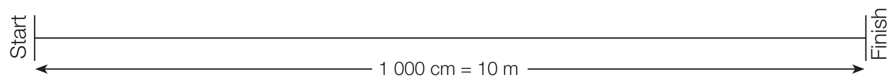
Like the previous activity, this one requires students to compare the progress of two objects (this time, cars) moving at different speeds. It is more straightforward in that the race takes place on a straight track but more complex in that each car stops and starts for various periods and one changes speed part-way through.

Because there are quite a few details to absorb, students need to make sure that they read the information carefully and interpret it correctly. One way of making sure that they do this is to put them in groups of three as for question **2** and have them read the activity through and then recount the details to each other without referring to the book. Other members of the group can then ask questions (especially about meaning), supply missing details, and correct errors. At the end, every detail is cross-checked against the book, and the group is ready to go. Alternatively, you may want to get everyone together for a final sharing and briefing.

The race starts at 5:00 a.m., but Sunlight Sprinter (SS) only operates from 6:00 a.m., so Pedal Power Plus (PPP) is able to get a 1 hour head start. As PPP travels 25 km in 1 hour, the answer to question **1a** is 25 km.

Questions **1b** and **1c** require students to work out how many hours each car spends travelling and to multiply these by the number of kilometres they cover each hour. Students should not use calculators for these calculations.

Question **2a** is a practical activity in which students model the race on a scaled-down track chalked on the school asphalt. The scale suggested is 1 cm to 1 km (1 : 100 000), which means that each group of students needs to measure and mark a track that is 1 000 cm (10 m) in length. A straight line will do for the track, with a start line at one end and a finish line at the other, like this:



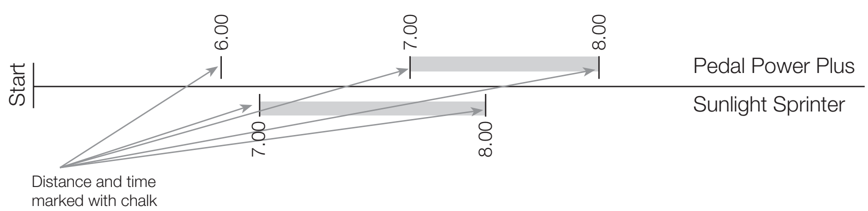

Given that the the two cars finish very close together, it is important that the 10 m is measured accurately.

Each group needs to cut out three strips of card as described in the student book. As for the track, their lengths need to be measured accurately. (The width doesn't matter.) The strips represent the scale distance that the cars will travel in 1 hour. Two are needed for PPP because it travels at one speed for the first part of the race and a slower speed for the second part.

Make sure that your students fully understand the relationship between the cars, the strips, and the length of the track.

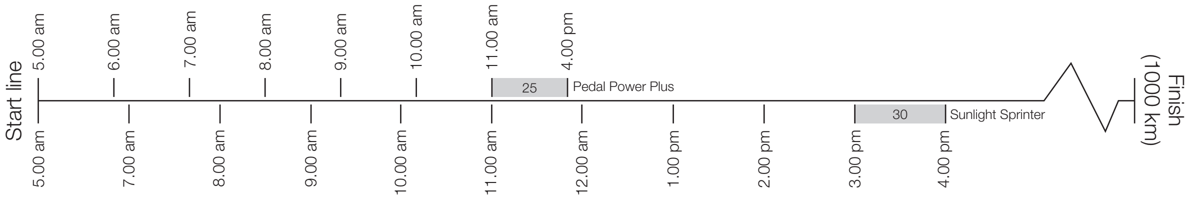
The race is controlled by the "time keeper" (one of the three people in each group), who gives the instructions and checks to see that they are followed. The other two members of the group are the "drivers" of the cars. The two cars follow the line that represents the track, one on either side of it.

When the time keeper says "6.00 a.m.", the driver of PPP lays their 25 km strip on their side of the track, marks off its length with chalk, and writes the time. The driver of SS does nothing, because, as at 6.00 a.m., their car has still not started. When the chalking is finished, the time keeper says "7.00 a.m.". At that point, the driver of PPP measures off a second 25 km length, marks the distance, and writes the time. At the same time, the driver of SS measures off a 30 km length and marks the distance and time. The chalk marks should now show clearly that, at 7.00 a.m., PPP has travelled 2 x 25 = 50 km and SS has travelled 1 x 30 = 30 km. Here is what the track should look like 1 hour later, at 8.00 a.m.:




The time keeper needs to keep a close eye on the script to make sure that the cars do not move when they are not meant to: PPP when resting and SS during the hours of darkness. The time keeper will also need to remind the driver of PPP when they need to swap over to the 20 km strip.

The diagram below shows the situation at 4.00 p.m. on the first day. SS is clearly well in the lead. Note how the time is marked for PPP. This car was stationary from 11.00 a.m. to 3.00 p.m., so has travelled only 25 km between 11.00 a.m. and 4.00 p.m.




The students should be able to see clearly from the practical exercise above that PPP wins the race but, due to measuring inaccuracies, it is unlikely that they will be able to say with confidence by how much. So to complete their answer for question **2b**, they will need to do some calculations. Each group of three should work on this together and then share their strategies with the wider group.

Many strategies are possible. Most involve breaking the race days into chunks of rest and and travel and keeping running records of both distance and time. For PPP, the reasoning might go: "PPP travels for 19 – 4 = 15 hours on day 1. At 25 km/h, this amounts to 375 km. On day 2, PPP travels 5 hours at 25 km/h, covering 125 km. It then travels 19 – 4 = 15 hours at 20 km/h, covering 300 km. The total distance covered by midnight on day 2 is 375 + 125 + 300 = 800 km. This leaves 200 km to be covered on day 3 at 20 km/h. 200 ÷ 20 = 10 hours. Travelling continuously from midnight, this means that PPP should cross the finish line at 10.00 a.m." A similar line of reasoning can be used to show that SS should arrive at 11.20 a.m.

An alternative strategy is to work with totals. "Beginning at 5.00 a.m., PPP travels 24 – 4 = 20 hours at 25 km/h and covers 20 x 25 = 500 km. It will take 500 ÷ 20 = 25 hours to travel the remaining 500 km. So the journey will take 20 + 25 = 45 hours plus 2 x 4 = 8 hours of breaks. That's 45 + 8 = 53 hours in all. 53 = 2 x 24 + 5, so PPP should arrive 2 days and 5 hours after it started, at 10.00 a.m." A similar line of reasoning will show that SS travels for $1000 \div 30 = 33\frac{1}{3}$ hours and spends 1 + (2 x 10) = 21 hours of darkness doing nothing, so it should finish $33\frac{1}{3} + 21 = 54\frac{1}{3}$ hours after starting. $54\frac{1}{3} = (2 \times 24) + 6\frac{1}{3}$, so SS should finish at $5 + 6\frac{1}{3} = 11.20$ a.m.

Question **3** provides an excellent opportunity for teaching your students how to draw graphs of continuous data. The speech bubble suggests suitable dimensions for a hand-drawn graph. Alternatively, students can create the graph using a computer spreadsheet and charting program. The final part of the question involves interpreting the picture. Ask your students to "read the picture and tell the story".

Question **4** asks students to consider the realities of racing and to think about some of the many factors that could influence the outcome.

# Page 24: A Dog's Breakfast

**Achievement Objectives**

- express a decimal as a percentage, and vice versa (Number, level 4)
- solve practical problems involving decimals and percentages (Number, level 5)

| |
|---|
| AC |
| EA |
| AA |
| AM |
| AP |

**Number Framework Links**

Use this activity to help students consolidate and apply their knowledge of fractions, proportions, and ratios (stage 7).

## Activity

This activity gives students practice at finding percentages of amounts using a calculator and rounding.

In question **1**, they need to add up the number of meals the dogs have in 1 day and then double this to give the number needed for 2 days.

In question **2**, they need to calculate the daily food allowance for each dog and divide this by the number of times the dog is to be fed. For example, Danny the Dane's daily allowance is 2% of 59 kg. 0.02 x 59 = 1 180 g. 1 180 ÷ 2 = 590 g per meal. A more efficient strategy would be to use the fact that Danny's daily allowance is 2% of his body weight and he gets this in two instalments. This means that he must get 1% (half of 2%) of his body weight each morning and evening. 1% of 59 kg is 0.59 kg or 590 g.

Make sure that your students know how you expect them to solve percentage problems on a calculator. Insist that they enter percentages directly as their decimal equivalents. This reinforces the fact that, for example, 4% = 0.04 and means that the students will not be at a loss when they have to use a calculator without a % key. Teach them how to read a decimal as a percentage (for example, 1.2 as 120%). Discourage them from multiplying by $\frac{100}{1}$. By learning good practices at this stage, they will come to understand the meaning of percentages and how to work with them as numbers.

Note that the activity uses kilograms as the unit for mass while the answers use grams. Ask your students for their views on the reason for this apparent inconsistency. The reason is that the gram unit is most suited to small weights, particularly those less than a couple of kg, while the kg unit is most suited to bigger weights, particularly those that are over a couple of thousands of grams. This is partly because all those zeros on bigger weights become a nuisance (Sam the St Bernard with a weight of 78 000 g!) and partly because we are not usually interested in finding or writing the weights of heavier things accurate to the last gram.

The question calls for answers to be rounded to the nearest 10 g. Check that everyone understands how to do this, making sure that they understand the convention of rounding up when the number is halfway between two 10s.

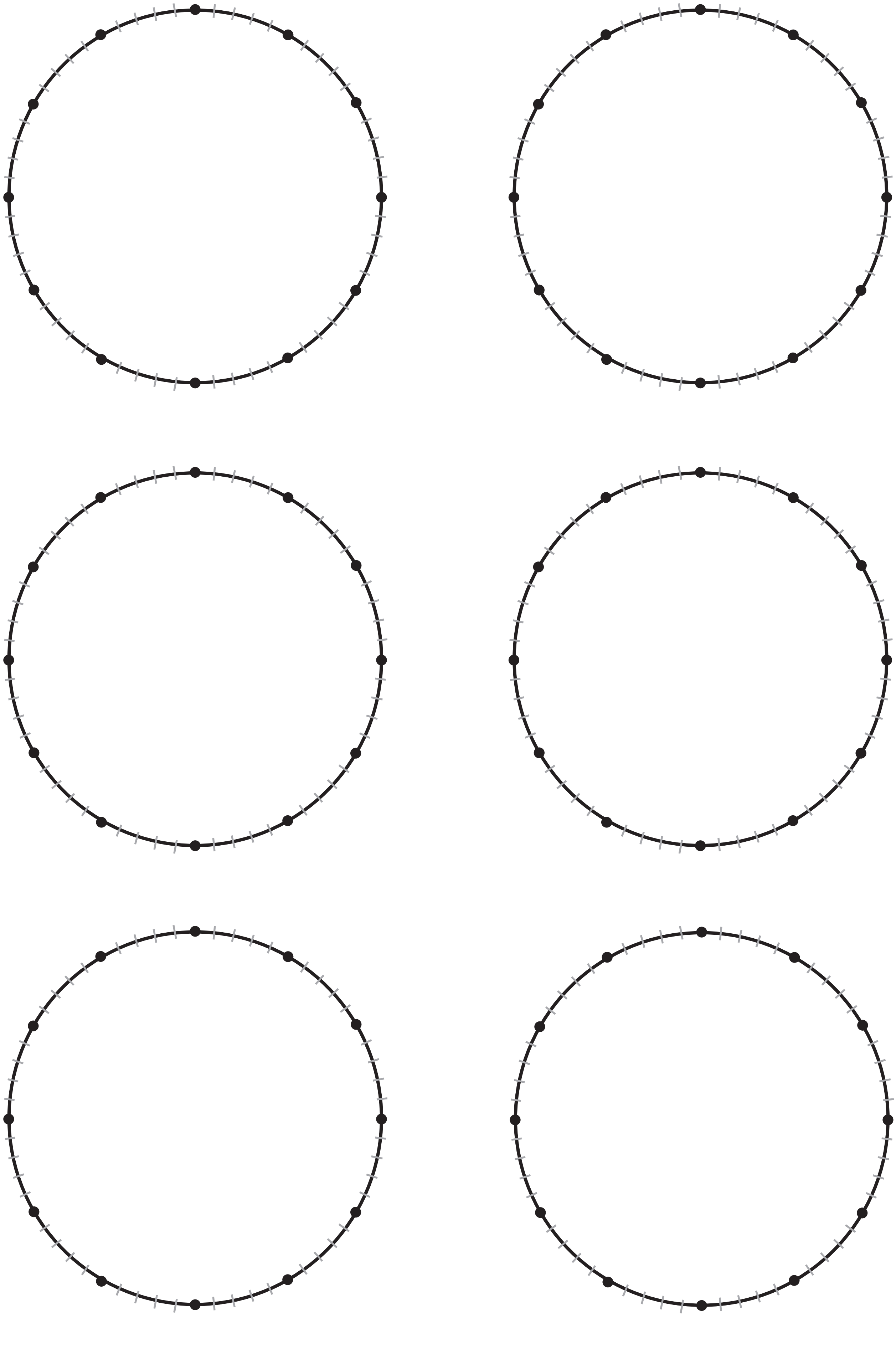
Many students are very interested in pets and may like to use this activity as a springboard for extension work. They could investigate local kennels, including how they manage the feeding process, ensure hygiene, allow for exercise, and what they charge. Or they could investigate what daily food allowance is recommended for cats and survey friends and classmates to find out how much their cats weigh and how much food they are fed.

## Copymaster: Make 1.5

To play the game, four sets of these cards are needed.

| | |
|---|---|
| | 0.1 |
| 0.2 | 0.3 |
| 0.4 | 0.5 |
| 0.6 | 0.7 |
| 0.8 | 0.9 |

## Copymaster: Tiring Teamwork



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