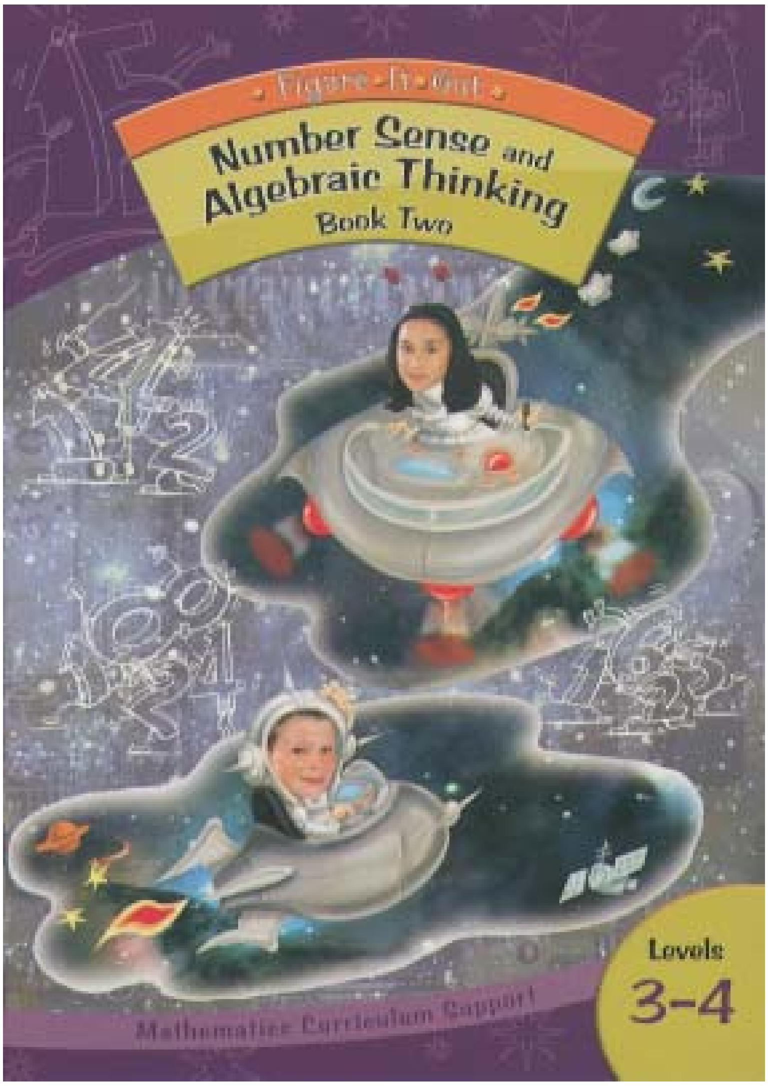
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Contents

Introduction

The books for levels 3–4 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand classrooms. These books are most suitable for students in year 6, but you should use your judgment as to whether to use the books with older or younger students who are also working at levels 3–4.

Student books

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and the life experiences that are meaningful to students in year 6.

The activities can be used as the focus for teacher-led lessons, for students working in groups, or for independent activities. Also, the activities can be used to fill knowledge gaps (hot spots), to reinforce knowledge that has just been taught, to help students develop mental strategies, or to provide further opportunities for students moving between strategy stages of the Number Framework.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the *Number Sense and Algebraic Thinking* student books includes full answers and explanatory notes. Students can use this section for self-marking, or you can use it for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, Number Framework links, comments on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/r/maths/curriculum/figure

Using Figure It Out in the classroom

Where applicable, each page starts with a list of equipment that the students will need in order to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to the problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Figure It Out

Number Sense and Algebraic Thinking Levels 3–4, Book Two Answers

Page 1: Factor Patterns

Activity

1.

1	1
2	1, 2
3	1, 3
4	1, 2, 4
5	1, 5
6	1, 2, 3, 6
7	1, 7
8	1, 2, 4, 8
9	1, 3, 9
10	1, 2, 5, 10
11	1, 11
12	1, 2, 3, 4, 6, 12
13	1, 13
14	1, 2, 7, 14
15	1, 3, 5, 15
16	1, 2, 4, 8, 16
17	1, 17
18	1, 2, 3, 6, 9, 18
19	1, 19
20	1, 2, 4, 5, 10, 20

2. a. 2, 6, 10, 14, 18

   b. The numbers are going up in fours.

   c. 22 and 26

   d. Yes. The pattern starts at 2 and goes up in fours. 102 – 2 = 100, which is divisible by 4, so 102 is part of the pattern.

3. a. 2, 4, 8, 16

   b. They are doubling (being multiplied by 2) each time. (Each number in the pattern is $2^{\text{place in pattern}}$. For example, 8 is the third number in the pattern. $2^3 = 2 \times 2 \times 2$)

   c. 32. (16×2 or 2^5)

   d. 1 024. ($2^{10} = 1\ 024$)

4. a. 1, 4, 9, 16

   b. They are square numbers. (The next one is 5 x 5 = 25.) The differences between the numbers form an odd number sequence:

   1 4 9 16 25 36

   +3 +5 +7 +9 +11

Pages 2–4: Mobilised

Activity

1. a. \$19.80. (10 + [20 x 0.49])

   b. i. 99c. (19.80 ÷ 20)

   ii. 74c. (10 + [40 x 0.49] = 29.60. 29.60 ÷ 40 = 74c.)

2. Discussion will vary. Based on Koleti's current usage, the monthly cost of each plan for her would be: Prepaid: \$28.50; Earbasher: \$34.50; Long Talk: \$30. Prepaid is the cheapest option for Koleti. However, you might recommend Long Talk because it is almost the same cost but Koleti can double her calls and triple her texts with the Long Talk plan without any extra cost.

3. a. \$43.90. (30 + [10 x 1.39])

   b. No. The number of minutes doubles, but the \$30 monthly fee stays the same. (30 + [20 x 1.39] is \$57.80, not \$87.80 or \$43.90 x 2.)

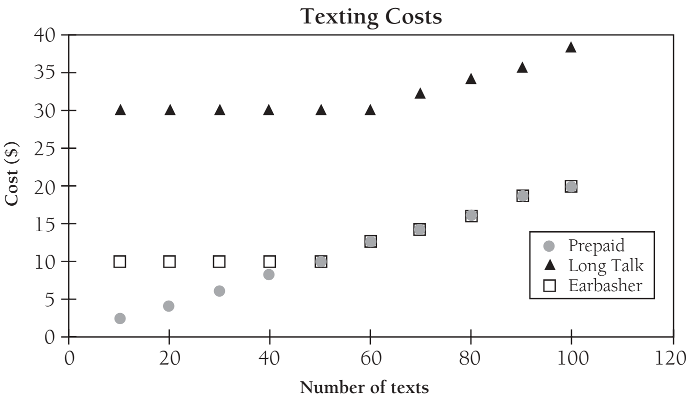
   c. Monthly cost = 30 + (number of min x 1.39)

   d. Based on his current usage, the monthly cost of each plan for Koleti's father would be: Prepaid: \$95.40; Earbasher: \$95.40; Long Talk: \$113.40. The Prepaid and Earbasher plans are the same cost, but you might recommend that Koleti's father use the Prepaid plan because it would be the cheaper plan for any month in which he made fewer calls. (The \$10 Earbasher monthly fee is the equivalent of just over 7 minutes of peak-time calls.)

4. a.

	A Number of texts	B Prepaid ($)	C Earbasher ($)	D Long Talk ($)
1	10	2	10	30
2	20	4	10	30
3	30	6	10	30
4	40	8	10	30
5	50	10	10	30
6	60	12	12	30
7	70	14	14	32
8	80	16	16	34
9	90	18	18	36
10	100	20	20	38

b.



c. i. Prepaid

ii. Prepaid or Earbasher

d. Prepaid if she sends fewer than 50 texts in any month. For 50 texts or more in any month, Prepaid and Earbasher would cost her the same.

5. Cost in dollars = number of texts x 0.2
Cost in dollars = number of texts ÷ 5

Page 5: Missing Digits

Activity

1. □ = 5

2. Strategies will vary, but the answers are:

a. □ = 7

b. ○ = 5

c. ⬠ = 8

d. △ = 6

e. ✧ = 4

f. ⬡ = 3

g. ▽ = 2

3. Answers will vary. Possibilities include:

In addition problems, if the □ is in the ones place, the ones digit in the answer and in the other number will tell you what the □ is.

The size of the answer in a multiplication problem can tell you how big the digit in the □ must be (when each □ is the same digit). For example, in □6 x □ = 18□, the □ could not be more than 4 because 50 x 5 = 250, which is more than 180.

4. Problems will vary.

Pages 6–9: Picking Patterns

Activity One

1. a. The numbers go up in 3s.

b. i. 3 times the row number gives the number in the pattern.

ii. 18

iii. 13. (39 ÷ 3)

2. a. Practical activity

b. You need to change the 3 to a 4. (The formula will now be =4*A1)

3. a. i. The 8 times table

ii. Your first 10 rows should look like this:

	A	B
1	1	8
2	2	16
3	3	24
4	4	32
5	5	40
6	6	48
7	7	56
8	8	64
9	9	72
10	10	80

24 x 8 = 192. (192 ÷ 8 = 24)

b. **i.** Practical activity

ii. 61. (3 x 20 + 1 = 61)

iii. 67. (202 – 1 = 201. 201 ÷ 3 = 67)

Activity Two

1. The formula for cell B1 is: =2*A1+3

2. **a.** 4 x number + 2 = number in B column. (So the formula is 4*A1+2)

b. A game using generalisations and formulae to make predictions

3. The formula for cell B1 is: =A1/2–2 or =(A1–4)/2
Practical activity

Pages 10–11: Growing Avocados

Activity One

1. **a.** 1 800. Methods will vary. One method is:
(150 x 10) + (150 x 2) = 1 500 + 300
= 1 800

b. **i.** 3 600 from 24 trees

ii. 7 200 from 48 trees

c. 40. (6 000 ÷ 150. You might see this as 12 000 ÷ 300.)

2. 120 trees. (They need 3 times as many trees. 3 x 40 = 120)

3. Number of avocados
= (number of trees x 200) ÷ 2
or number of trees x 100

Activity Two

1. **a.** Small: 20c; medium: 45c (rounded); large: 65c (rounded)

b. 16 bags of small avocados; 9 bags of medium avocados; 8 bags of large avocados

c. Answers will vary. One possible answer is: 5 bags of large avocados ($12.50) + 5 bags of medium avocados ($11.00) + 5 bags of small avocados ($6.00) = $29.50

2. $1,342.50. That is, 600 small avocados in bags of 6 at $1.20 per bag = 1.20 x 100
= $120

1 500 medium avocados in bags of 5 at $2.20 per bag = 2.20 x 300
= $660

900 large avocados in bags of 4 at $2.50 per bag = 2.50 x 225
= $562.50

120 + 660 + 562.50 = $1,342.50

Activity Three

1. **a.** 5 avocados and 1 lemon

b. 20 avocados and 4 lemons

2. **a.** 3 lemons. ($2\frac{1}{2}$ x 6 = 15. 6 x $\frac{1}{2}$ = 3)

b. 24 people. (6 x 4)

Pages 12–13: Non-stop Ninths

Activity

1. Practical activity

2. Discussion will vary. For example: "We keep getting 1 piece each and 1 piece left over that has to be divided. The decimal for 1 share goes 0.111111 … with the 1 going on forever."

3. **a.** 0.11

b. 0.11

4. **a.** **i.** $1.78. (2 – 0.22)

ii. $2.67. (3 – 0.33)

iii. $3.56. (4 – 0.44)

iv. $4.44. (5 – 0.56)

v. $5.33. (6 – 0.67)

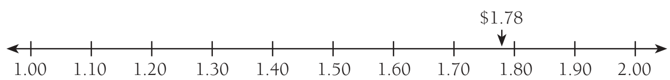
vi. $6.22. (7 – 0.78)

vii. $7.11. (8 – 0.89)

viii. $8.00. (9 – 1)

b.

i.



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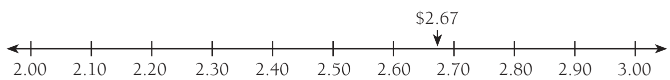

iii.

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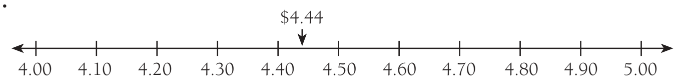

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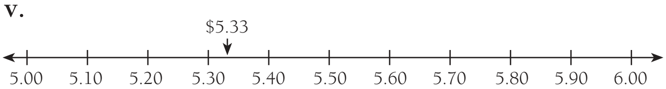

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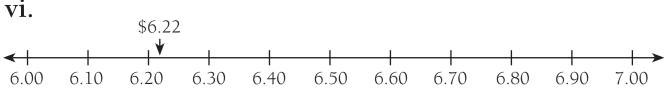

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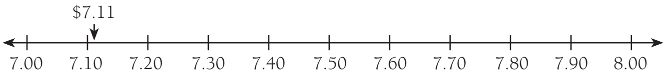

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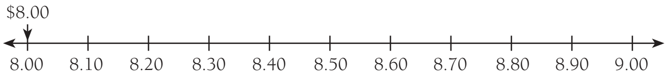


Pages 14–15: Animal Antics

Activity

1. **a.** The elephant should win. 2 rhinoceros = 3 yaks; 1 elephant = 3 yaks + 1 rhinoceros; therefore the elephant is a whole rhinoceros stronger.

b. The 5 yaks should win. 3 rhinoceros would equal $4\frac{1}{2}$ yaks (and half a yak isn't possible).

c. 1 elephant + 3 yaks should win.
4 rhinoceros = 6 yaks, and
1 elephant + 3 yaks = 6 yaks + 1 rhinoceros.
So the left side is stronger by 1 rhinoceros.

2. **a.** **i.** (30 + 5) bananas and (20 + 15) apples
= 35 bananas and 35 apples

ii. (45 + 20) bananas and (30 + 60) apples
= 65 bananas and 90 apples

b. **i.** 2 chimpanzees and 2 monkeys.
([6 + 2] bananas and [4 + 6] apples)

ii. 8 chimpanzees and 5 monkeys.
([24 + 5] bananas and [16 + 15] apples)

c. A possible short cut for bananas is:
3 x number of chimpanzees + 1 x number of monkeys.
A possible short cut for apples is:
2 x number of chimpanzees + 3 x number of monkeys.

3. **a.** $2

b. $4

c. A possible short cut is: Cost = the number of salad rolls x $4 + the number of drinks x $2.

d. $3.50. ($28.50 – $12 – $6 = $10.50.
$10.50 ÷ 3 = $3.50)

Pages 16–17: Using Mates

Activity

1. Discussion will vary.

a. 8

b. 400

c. 28

d. 20 000

2. Answers will vary. For example:

a. 8 (8 x 0.25 = 2) or 40 (40 x 0.25 = 10)

b. 2 (2 x 2.5 = 5) or 4 (4 x 2.5 = 10)

c. 5 (5 x 0.6 = 3) or 20 (20 x 0.6 = 12)

d. 4 (4 x 1.25 = 5) or 8 (8 x 1.25 = 10)

3. Discussion will vary. 20 x 0.5 = 10 and 10 x 97 = 970 is much easier to do than 20 x 97 or 97 x 0.5.
Possible methods include:

a. 20 x 0.5 = 10. 806 x 10 = 8 060

b. 0.5 x 6 = 3. 3 x 89 = 3 x 90 – 3
= 270 – 3
= 267

c. 0.25 x 8 = 2. 2 x 193 = 2 x 100 + 2 x 90 + 2 x 3
= 200 + 180 + 6
= 386

d. 4 x 5 = 20. 20 x 326 = 10 x 326 x 2
= 3 260 x 2
= 6 520

4. Methods may vary. Possible steps (with the expressions reorganised) are:

a. 22 x (8 x 0.5) = 22 x 4
= 88

b. (5 x 0.6) x 312 = 3 x 312
= 936

c. 165 x (0.2 x 5) = 165 x 1
= 165

d. (4 x 2.5) x (7 x 3) = 10 x 21
= 210

e. 13 x (1.25 x 4) = 13 x 5
= 65

f. (6 x 5) x 71 = 30 x 71
= 2 130

g. (25 x 2) x 36 = 50 x 36
= 100 x 18
= 1 800

h. (4 x 15) x 81 = 60 x 81
= (60 x 80) + (60 x 1)
= 4 800 + 60
= 4 860

5. Discussion will vary. Wiha's strategy extends the strategy she used in question **3**. Possible steps are:

a. 23 x (8 x 25) = 23 x 200
= 4 600
Or: 23 x 100 x 2 = 4 600

b. (150 x 2) x 43 = 300 x 43
= 12 900

c. (40 x 5) x (5 x 70) = 200 x 350
= 100 x 350 + 100 x 350
= 35 000 + 35 000
= 70 000

d. (125 x 4) x (60 x 7) = 500 x 420
= 1 000 x 420 ÷ 2
= 1 000 x 210
= 210 000

6. Answers will vary.

Pages 18–20: Lunchtime Mardi Gras

Activity One

1. $6

2. a.

Group	Amount made ($)	Hoops thrown
Eru	60	150
Simon	45	135
Mere	93	186

b. Mere's group

c. Although Mere's group charged more per hoop, their basic charge of $1 was less than the other groups charged. This may have attracted more people to their group.

d. $198. (60 + 45 + 93)

Activity Two

1. a. 90 (3 bullseyes), 84 (2 bullseyes and a triple 8), 81 (2 bullseyes and a triple 7)

b. Answers will vary. For example,
21 (triple 7) + 12 (double 6) + 5,
24 (triple 8) + 9 (triple 3) + 5, or
6 + 18 (triple 6) + 14 (double 7)

2. a. 18. (6 x 3)

b. A possible rule is: number of darts = 6 x number of dollars paid.

c. 120 darts. (20 x 6)

d. $51. (306 ÷ 6)

3. **a.–b.** Answers and some possible strategies are:

i. 44. Possible strategy: compatible numbers (5 + 15)

ii. 17. Possible strategies: compatible numbers (7 + 3) or double 7

iii. 46. Possible strategies: compatible or tidy numbers (using 18 + 12 for both)

iv. 49. Possible strategies: double 21 or place value (20 x 2 + 1 x 2 + 7)

v. 45. Possible strategies: compatible or tidy numbers (14 + 16), compensation (3 x 15)

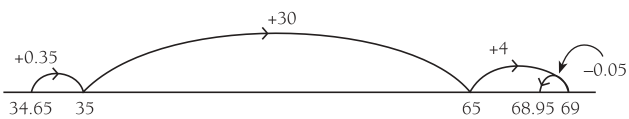
vi. 30. Possible strategy: known facts (9 + 6 = 15) and double 15

Activity Three

1. Answers will vary. For example: $12.50 doubled is $25. 25 + 10 = $35, which is 35 cents more than $34.65. So $12.50 + $10 – 35c = $22.15. (12.50 + 12.50 + 10 – 0.35 = $34.65)

2. Strategies will vary. Answers and a possible strategy for each are:

a. $34.30. 68.95 – 34.65 = □
or 34.65 + □ = 68.95.



0.35 + 30 + 4 – 0.05 = $34.30

b. $13.15. 82.10 – 68.95 = 82 – 70 + 0.1 + 1.05
= $13.15
(Using tidy numbers and compensation)

c. $20.70. 102.80 – 82.10: 102 – 82 = 20 and 0.80 – 0.10 = 0.70. 20 + 0.70 = $20.70
(Using place value)

d. \$18.55. 121.35 – 102.80
= 122 – 102 – 0.65 – 0.80
= 20 – 1.45
= 18.55
(Using tidy numbers and compensation)

Activity Four

1. Answers may vary. Possible combinations include:
48.65 + 51.35 = \$100
121.35 + 121.15 = \$242.50
162.50 + 242.50 = \$405.00. (100 + 200 = 300 and 62 + 42 = 104; 300 + 104 + 1 = \$405.00)
Then 405 + 100 = \$505; 198 = 200 – 2, so
505 + 198 = 505 + 200 – 2
= 705 – 2
= \$703

2. \$703

Page 21: Compatible Multiples

Activity

1. a. 7. 5 x 0.2 = 1, so 5 x 7 x 0.2 = 1 x 7
= 7

b. 62. 0.5 x 4 = 2, so 0.5 x 31 x 4 = 2 x 31
= 62

c. 54. 1.5 x 2 = 3,
3 x 18 = 6 x 9 (double and halve)
= 54

d. 7. 0.25 x 4 = 1, so 7 x 0.25 x 4 = 1 x 7
= 7

e. 7. $\frac{1}{3}$ x 3 = 1, so $\frac{1}{3}$ x 7 x 3 = 1 x 7
= 7

f. 370. $\frac{1}{2}$ x 20 = 10, so $\frac{1}{2}$ x 37 x 20 = 10 x 37
= 370

g. 68. 18 x $\frac{1}{9}$ = 2, so 18 x 34 x $\frac{1}{9}$ = 2 x 34
= 68

h. 250. 8 x $\frac{1}{4}$ = 2, so 8 x 125 x $\frac{1}{4}$ = 2 x 125
= 250

i. 26. $\frac{1}{7}$ x 14 = 2, so 13 x $\frac{1}{7}$ x 14 = 2 x 13
= 26

j. 69. $\frac{1}{6}$ x 18 = 3, so 23 x $\frac{1}{6}$ x 18 = 3 x 23
= 69

k. 6. 15 x 0.2 = 3 and 0.25 x 8 = 2, so
15 x 0.25 x 0.2 x 8 = 3 x 2
= 6

l. 10. $\frac{1}{8}$ x 16 = 2 and 25 x $\frac{1}{5}$ = 5, so
$\frac{1}{8}$ x 25 x 16 x $\frac{1}{5}$ = 2 x 5
= 10

2. There are many possibilities. Here are some examples:
8 x 0.25
12 x $\frac{1}{3}$ x $\frac{1}{2}$
9 x 2 x $\frac{1}{9}$
20 x 0.1
4 x 25 x 0.2 x 0.5 x 0.2
$\frac{1}{12}$ x 6 x 4

3. There are many possibilities. Here are some examples:
28 x 0.25 x 27 x $\frac{1}{9}$
120 x 0.1 x 0.5 x 8
0.3 x 16 x 40 x $\frac{1}{8}$

4. Answers will vary. For example: 8 x 5 x 0.5 or 30 x 4 x $\frac{1}{6}$

Pages 22–23: Even Pictures Talk

Activity

1. Raj noticed that 8 students preferred caramel, while 16 (that is, 1 + 2 + 3 + 4 + 6) chose other options. So there were 24 students in Room 1, and $\frac{8}{24}$ or $\frac{1}{3}$ of these preferred caramel.
Keiran noticed that 4 preferred peanut and 2 preferred dark chocolate, so twice as many preferred peanut. (Or Keiran may have noticed that the peanut bar on the graph is twice the height of the dark bar.)
Bridie noticed that 1 person didn't like chocolate at all. That was 1 out of 24. 1 out of 20 ($\frac{1}{20}$) is 5%, so 1 out of 24 ($\frac{1}{24}$) is less than 5%. So Bridie is correct in saying that more than 95% like chocolate.
Reece noticed that 3 out of 24 preferred white chocolate. $\frac{3}{24} = \frac{1}{8}$

2. Answers will vary for all graphs, but each answer should discuss/cover the fact that:

a. Half of Room 7 students were born in the Pacific and $\frac{1}{4}$ of them in Asia. About $\frac{1}{6}$ were born in Europe and the rest in Africa. (In a class of 36 students, 18 would have been born in the Pacific, 9 in Asia, 6 in Europe, and 3 in Africa.)

b. Taller people tend to have larger hand spans, although this is not true for every individual. The students in Room 6 are mostly between 140 and 160 cm in height, and their hand spans are mostly in the 14–18 cm range. One student in the class is much taller than everyone else (170 cm) and has a much larger hand span (21 cm). The average height would be about 150 cm, and the average hand span would be about 16 cm.

c. Most students in Room 2 watched between 10 and 20 hours of TV or video in the week. 1 student watched 41 hours, and 1 student watched none.

d. Explanations may vary as to why the levels altered, but the times should follow those in the following scenario. From 3.00 until 3.15, the water in the paddling pool rose from nothing to 30 cm, so for each 5 min, it rose 10 cm. The water level had risen another 5 cm by 3.20. From 3.20 until 3.40, the water level remained the same, but over the next 5 min, it went down 5 cm. The water level rose again over the next 5 min, to bring the level back to 35 cm. The water level remained the same for nearly 20 min and then reduced by 5 cm. 8 min later, the pool was emptied.

3. Practical activity

Page 24: Nimble Numbers

Game

A game using estimation and number strategies to solve problems

Figure It Out

Number Sense and Algebraic Thinking

Teachers' Notes

Overview of Levels 3–4: Book Two

Introduction to Number Sense and Algebraic Thinking

The *Number Sense and Algebraic Thinking* books in the Figure It Out series provide teachers with material to support them in developing these two key abilities with their students. The books are companion resources to Book 8 in the Numeracy Project series: *Teaching Number Sense and Algebraic Thinking.*

Number Sense

Number sense involves the intelligent application of number knowledge and strategies to a broad range of contexts. Therefore, developing students' number sense is about helping them gain an understanding of numbers and operations and of how to apply them flexibly and appropriately in a range of situations. Number sense skills include estimating, using mental strategies, recognising the reasonableness of answers, and using benchmarks. Students with good number sense can choose the best strategy for solving a problem and communicate their strategies and solutions to others.

The teaching of number sense has become increasingly important worldwide. This emphasis has been motivated by a number of factors. Firstly, traditional approaches to teaching number have focused on preparing students to be reliable human calculators. This has frequently resulted in their having the ability to calculate answers without gaining any real understanding of the concepts behind the calculations.

Secondly, technologies – particularly calculators and computers – have changed the face of calculation. Now that machines in society can calculate everything from supermarket change to bank balances, the emphasis on calculation has changed. In order to make the most of these technologies, students need to develop efficient mental strategies, understand which operations to use, and have good estimation skills that help them to recognise the appropriateness of answers.

Thirdly, students are being educated in an environment that is rich in information. Students need to develop number skills that will help them make sense of this information. Interpreting information in a range of representations is critical to making effective decisions throughout one's life, from arranging mortgages to planning trips.

Algebraic Thinking

Although some argue that algebra only begins when a set of symbols stands for an object or situation, there is a growing consensus that the ideas of algebra have a place at every level of the mathematics curriculum and that the foundations for symbolic algebra lie in students' understanding of arithmetic. Good understanding of arithmetic requires much more than the ability to get answers quickly and accurately, important as this is. Finding patterns in the process of arithmetic is as important as finding answers.

The term "algebraic thinking" refers to reasoning that involves making generalisations or finding patterns that apply to *all* examples of a given set of numbers and/or an arithmetic operation. For example, students might investigate adding, subtracting, and multiplying odd and even numbers. This activity would involve algebraic thinking at the point where students discover and describe patterns such as "If you add two odd numbers, the answer is always even." This pattern applies to *all* odd numbers, so it is a generalisation.

Students make these generalisations through the process of problem solving, which allows them to connect ideas and to apply number properties to other related problems. You can promote process-oriented learning by discussing the mental strategies that your students are using to solve problems. This discussion has two important functions: it gives you a window into your students' thinking, and it effectively changes the focus of problem solving from the outcome to the process.

Although the term "algebraic thinking" suggests that generalisations could be expressed using algebraic symbols, these Figure It Out *Number Sense and Algebraic Thinking* books (which are aimed at levels 2–3, 3, and 3–4) seldom use such symbols. Symbolic expression needs to be developed cautiously with students as a sequel to helping them recognise patterns and describe them in words.

For example, students must first realise and be able to explain that moving objects from one set to another does not change the total number in the two sets before they can learn to write the generalisation $a + b = (a + n) + (b - n)$, where n is the number of objects that are moved. There is scope in the books to develop algebraic notation if you think your students are ready for it.

The Figure It Out *Number Sense and Algebraic Thinking* books

The learning experiences in these books attempt to capture the key principles of sense-making and generalisation. The contexts used vary from everyday situations to the imaginary and from problems that are exclusively number based to those that use geometry, measurement, and statistics as vehicles for number work. Teachers' notes are provided to help you to extend the ideas contained in the activities and to provide guidance to your students in developing their number sense and algebraic thinking.

There are six *Number Sense and Algebraic Thinking* books in this series:
Levels 2–3 (Book One)
Levels 2–3 (Book Two)
Level 3 (Book One)
Level 3 (Book Two)
Levels 3–4 (Book One)
Levels 3–4 (Book Two)

Estimation and Rounding

Although students don't need to be bothered with the distinction at this stage, teachers need to understand that estimation and rounding, though closely related activities, are not the same thing.

Estimation is the process of coming up with a "ball park" outcome for a calculation. Estimation means simplifying the numbers to the point where we can work with them. Different people can come up with slightly different estimates for the same calculation. Estimation can be used:

- to predict a result before calculating it (out of interest or as a check against incorrect use of a calculator)
- to check that the result of a calculator calculation is reasonable (likely to be correct)
- to decide which of a number of possible outcomes is likely to be correct
- to get a rough idea of an outcome when we don't need to know the exact result
- as successive approximations that might lead to the exact answer, depending on how many steps are taken.

It's also helpful to know how to estimate if you don't have a calculator or paper handy.

Good estimating skills are extremely valuable. They are part of number sense (especially understanding of place value) and, where used in this student book, require a range of advanced multiplicative strategies (stage 7).

Rounding is the process of looking at the result of a calculation and making a deliberate decision about how exact it should be so that it:

- is sensible, given the context
- meets the purpose for which the calculation was done
- allows for "easy" interpretation.

Whole numbers may be rounded to the nearest 10, 100, 1 000, and so on. Decimal fractions may be rounded to 1, 2, 3, or more decimal places. The most useful kind of rounding is rounding to 1, 2, 3, or more significant figures, but students are not introduced to this concept until level 4 or later. For an explanation of significant figures, see *Answers and Teachers' Notes: Measurement: Book Two*, Figure It Out, Years 7–8, page 20.

When a number has been rounded, we usually say how it has been rounded (for example, "to the nearest whole number").

Page 1: Factor Patterns

Achievement Objectives

- recall the basic multiplication facts (Number, level 3)
- make up and use a rule to create a sequential pattern (Algebra, level 3)
- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem-solving, levels 3–4)

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Number Framework Links

Use this activity to develop your students' knowledge of basic multiplication facts and their ability to identify factors. This leads to divisibility rules, which students need in order to become proportional thinkers. The activity is most suitable for students who are in transition from advanced additive (stage 6) to advanced multiplicative (stage 7) or higher.

Activity

This activity explores patterns in the factors of the numbers 1 to 20. It builds on students' recall of multiplication facts and helps them to develop their understanding of square numbers and square roots.

Question **1** asks the students to make a list of all the factors for the numbers from 1 to 20. They could draw this up as a table, or you could show them how to record the factors for each number as a set, using set brackets, a comma to separate items, and listing each factor only once. For example, the number sentences for 16 are 1 x 16 = 16, 2 x 8 = 16, and 4 x 4 = 16, and the factors for 16 are {1, 2, 4, 8, 16}. Once the students have listed the factors, encourage them to look for patterns. Ask: *What is the name for numbers that have only 1 and the number itself as factors?* (Prime numbers have only two factors, 1 and the number itself. 1 is not considered to be a prime number because it only has one factor.)

Question **2** focuses on the pattern in numbers whose factors are half even numbers and half odd. (Students at the advanced additive stage will be able to describe the pattern [of half even, half odd factors] by adding on 4s or a combination of 4s and doubles and could still answer question **2d** using this method.) For question **2b**, the students should identify that, starting with 2, every fourth number fits the pattern. For question **2d**, ask:
How can you use this information to find out if 102 is in the pattern? (Take 2 off 102 and divide the answer by 4. 102 – 2 = 100. 100 ÷ 4 = 25. So 102 is an "every fourth number" and must fit the pattern.)
Can you find a generalisation to see if any number fits the pattern? (Take 2 off the number and divide the answer by 4. If the result is a whole number [that is, it has no remainder], the original number will follow the pattern.)

The students may not realise that the pattern is linked to prime factorisation. They are not asked to explain this, but you may wish to explore this a bit with them. Drawing up a list of the prime factors of numbers that work and those that don't work will help the students to recognise a pattern. For example:

Numbers that work:	18	22	26	30	34 …
Their prime factors:	2 3 3	2 11	2 13	2 3 5	2 17

Numbers that don't work:	12	15	16	20	24 …
Their prime factors:	2 2 3	3 5	2 2 2 2	2 2 5	2 2 2 3

The factors of numbers that work are 2 plus one or more odd primes. Numbers that don't work have two 2s and an odd prime, two odd primes, three 2s and an odd prime, four 2s, and so on.

The pattern in question **3** is based on exponents of 2. The students should see that the pattern involves doubling, and they may be able to obtain the tenth number in the pattern by continuing the pattern (question **3d**). Students who know about exponents should be able to describe the third number in the pattern, 8, as 2 x 2 x 2 or 2^3. Explain to them that the first number in a number sequence (or pattern) is known as the first term or term 1. This avoids over-using the word "number". Then say: *The second term in the pattern is 4 or 2 squared* (2^2)*, and the third term is 8 or 2 cubed* (2^3)*. Is there a short cut that you can use to find the value of any term in the pattern?* ($2^{\text{place in pattern}}$) The students can use this information to find that the tenth number in the pattern or 2^{10} is 1 024.

For question **4**, the students will have already noticed that not all even numbers have an even number of factors. Even numbers that are also square numbers will have an odd number of factors because the square factor is only listed once. For example, the factors for 16 are 1, 2, 4, 8, and 16 because 4 x 4 contributes just one factor. The students could also represent square numbers using an array.

The students could investigate further square patterns, continue sequences, and make generalisations using the starters below:

$1^2 = 1 \times 1 = 1$
$2^2 = 2 \times 2 = 4 = 1 + 3$
$3^2 = 3 \times 3 = 9 = 1 + 3 + 5$
and so on.

$10^2 - 9^2 = 100 - 81 = 19 \longrightarrow 10 + 9 = 19$
$9^2 - 8^2 = 81 - 64 = 17 \longrightarrow 9 + 8 = 17$
$8^2 - 7^2 = 64 - 49 = 15 \longrightarrow 8 + 7 = 15$
and so on.

A geometric explanation for these starters and another activity on the use of squares and exponents (powers) can be found in All Square, *Algebra*, Figure It Out, Levels 3–4, page 2. A spreadsheet activity on factors is The Great Factor Hunt, pages 2–3 of *Number Sense and Algebraic Thinking: Book One*, Figure It Out, Levels 3–4.

Pages 2–4: Mobilised

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- make up and use a rule to create a sequential pattern (Algebra, level 3)
- use graphs to represent number, or informal, relations (Algebra, level 3)

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Number Framework Links

Use this activity to:

- encourage transition from advanced additive strategies (stage 6) to advanced multiplicative strategies (stage 7) for the multiplication of decimals
- help the students to consolidate advanced multiplicative strategies (stage 7).

Activity

This activity involves the comparison of three cellphone plans with different rating schedules for peak-time calls, off-peak calls, text messaging, and monthly fees. The activity is suitable for students who are using strategies based on multiplication and division to solve problems with decimals and who can already order decimals up to three places.

Koleti calculates options according to the various payment plans and her pattern of use. These calculations are clearly explained in the Answers.

For the Earbasher off-peak call costs explored in question **1**, ask: *Why is 1 minute of time per month calculated as $10.49 when the off-peak calls are only 49 cents per minute?* (This plan has a $10 basic charge per month plus call time. It would cost $10 per month even if Koleti didn't call anyone.) Get the students to test Koleti's short cut against the values listed in the table on page **2** and then have them use the short cut to calculate the cost for 20 off-peak minutes for question **1a**. They could work in pairs to find a suitable generalisation.

The values for question **4** need to be entered into a spreadsheet so that the students can generate a scatterplot graph. The students could work in pairs to complete the spreadsheet and the graph before they discuss the best call plan for Koleti. They need to be aware of how many free texts each call plan allows because this will affect their calculations. Encourage the students to justify their ideas and the limitations of each option.

When the students are making a scatterplot in question **4b**, check that they understand the features of presentation, including a title (*What does the graph show?*), the labels for the horizontal axis (number of texts) and the vertical axis (cost in dollars), and the inclusion of a key to identify the company represented by each symbol. To get the computer to create the graph, the students need to use the XY (scatter) option. (For general information on the use of spreadsheets, see *Answers and Teachers' Notes: Algebra,* Figure It Out, Level 3, pages 26–27.)

For question **5**, the students need to understand the decimal relationship between dollars and cents (1 cent = $\$\frac{1}{100}$). Koleti's prepaid short cut can be expressed as:
Cost in cents = number of texts x 20 cents
Cost in dollars = number of texts x 0.2 dollars
Cost in dollars = number of texts x $\frac{2}{10}$ dollars (or $\frac{1}{5}$ dollars)
Cost in dollars = number of texts ÷ 5.

Some students may find this confusing because multiplication usually makes things bigger. Point out that "multiplied by" can be thought of as "of", for example, 20 x 0.2 is 20 lots of 0.2.

Extension

You could extend this activity by asking the students to vary the scenarios and recalculate the costs or by getting them to investigate real-life cellphone deals.

Challenge them to create a cellphone package that is ideal for students of their age.

Page 5: Missing Digits

Achievement Objectives

- solve problems of the type ☐ + *15* = *39* (Algebra, level 3)
- use a rule to make predictions (Algebra, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, levels 3–4)

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Number Framework Links

Use this activity with students at the advanced multiplicative stage who have a strong recall of addition and multiplicative basic facts and who are learning to solve number problems that have one or more "unknowns".

Activity

In this activity, the students use their knowledge of basic facts and number patterns to find missing digits in numbers. Most of the questions could be done using trial and improvement, but the students are likely to find it more efficient (and more satisfying) to first narrow the range of options as much as possible by studying the problem and seeing what they can deduce. You could suggest that they are learning to be number detectives.

You could introduce the activity to the whole class or a group using examples of your own. In this way, you leave all the problems in the book for the students to do themselves. Below are possible examples for group teaching or discussion.

Example 1: ☐☐3 + 3☐ = ☐44

- The last digit of the first number is 3; 3 + ☐ gives the final digit of the sum, 4.
- This means the missing digit must be 1 (because 3 + 1 = 4). Checking: 113 + 31 = 144.

Example 2: 1☐ x 3☐ = ☐2☐

- When you multiply this missing digit by itself, the answer (product) ends with that same digit.
- There are only three digits for which this is true: 1 (because 1 x 1 = <u>1</u>), 5 (because 5 x 5 = 2<u>5</u>), and 6 (because 6 x 6 = 3<u>6</u>).
- Try 1: 11 x 31 = 341. So it's not 1. Try 5. If 5 doesn't work, you would know it must be 6. In this case, 5 does work: 15 x 35 = 525.

Example 3: ☐6 x ☐ = 5☐

- The missing digit is even (because 6 is an even number, and whenever you multiply a whole number by an even number you get an even number).
- The digit must be 2, 4, 6, or 8 (the only even digits apart from 0, which wouldn't work because anything multiplied by 0 is 0).
- 4, 6, or 8 wouldn't do because the first number would have to be 46, 66, or 86, and if you multiplied these numbers by 4, 6, or 8, you'd get a number that was a lot greater than 50 something.
- This leaves 2 as the only possible even number. Checking: 26 x 2 = 52.

The dialogue alongside question **1** models how the students should be thinking. Question **2** asks them to write down the steps they used to solve each problem. For example, they could do this using numbered or bulleted points or in a table.

In question **3**, the students need to think about patterns (or strategies) they may have used, such as:

- odd + odd = even (for example: 9 + 5 = 14)
- odd – odd = even (for example: 9 – 5 = 4)
- even + even = even
- even + odd = odd
- even – odd = odd
- odd – even = odd
- odd x odd = odd
- even x even = even
- odd x even = even.

One approach to question **4** is for the students to make up some simple number sentences in the format "number +/–/x/÷ number = number" and then to look for a repeated digit that they could remove and replace with a box, thus creating a missing digit problem for a classmate to solve. For example:

- Make up a subtraction sentence and solve it: 253 – 152 = 101
- Now create a missing digit problem by replacing the digit 2 with a box: □53 – 15□ = 101; or the digit 5 with a box: 2□3 – 1□2 = 101; or the digit 1 with a box: 253 – □52 = □0□ .
- Check these possibilities and see what is involved in solving them. Some will need to be discarded as too simple. Not all will have a unique solution (for example, 2□3 – 1□2 = 101 works for any digit).

Some trial and improvement may be needed before each student has a small group of problems suitable for giving to a classmate.

Pages 6–9: Picking Patterns

Achievement Objectives

- use a rule to make predictions (Algebra, level 4)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- demonstrate knowledge of the conventions for order of operations (Number, level 4)
- solve simple linear equations such as *2□ + 4 = 16* (Algebra, level 4)

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AM
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Number Framework Links

Use these activities to encourage transition from advanced additive strategies (stage 6) to advanced multiplicative strategies (stage 7). You will need to provide some guidance.

Students who are doing the activity independently should have a good recall of their basic multiplication facts and be able to use strategies of commutativity and reversibility for multiplication and division.

Activity One

This activity gives students step-by-step instructions on how to use formulae in spreadsheets. The formulae generate simple patterns using basic multiplication and addition. Most spreadsheet programs allow the cursor to be clicked in the bottom right corner of a cell with a formula in it and then dragged through all the cells that the formula is to apply to.

Question **1** asks the students to look at the spreadsheet page and state the "3 times" relationship between the row numbers and the numbers in column A. Check that the students know how to identify a cell using column and row references: the letter (vertical component) identifies the column, and the number (horizontal component) identifies the row. So cell A3 can be found in the first column, third row down.

In question **1b i**, the numbers in the column are the row numbers multiplied by 3.
In question **1b iii**, the students need to solve either of the number sentences $3 \times \square = 39$ or $39 \div 3 = \square$.

In question **2**, the students learn how to use a spreadsheet formula and have three opportunities to practise the Fill Down method, putting the counting numbers from 1 to 15 in column A, multiplying those numbers by 3 in column B, and then subsequently changing the formula so that it multiplies by 4. In step 2, the students type =3*A1 into the B1 cell, then click and drag. You may need to direct them to the speech bubble that says that the asterisk is the spreadsheet symbol for multiplication.

In question **2b**, the students would need to change the formula in cell B1 to =4*A1 and use the Fill Down method again. Ask them to predict what change they should make and why they think this change should work.

In question **3a i**, the students need to recognise that the formula shown in cell B1 will multiply the numbers in column A by 8. In question **3b**, they change the formula in cell B1 to =3*A1+1 (algebraically, this could be written as $3n + 1$) and use it to create another pattern.

Activity Two

This is an excellent activity for encouraging the students' interest in patterns and the generalisations that go with them. In the activity, the students play a game that uses their understanding of spreadsheet patterns and formulae. Each student has a turn at changing the formula in cell B1 and having their classmate predict what the formula is from the number pattern on the spreadsheet. Initially, you may like to limit the scope of the changes to combinations of multiplication and addition or subtraction. In question **3**, the students can try also using division. You may need to direct them to the speech bubble that says that the spreadsheet symbol for division is the slash (/).

Discuss with the students what happens when there is a change in the multiplier and in the constant. For example, with =4*A1+2

Constant:
gives a fixed adjustment

Multiplier:
gives the increase for each
increase of 1 in column A

So changing the multiplier gives increases or decreases in the rate of change.

Pages 10–11: Growing Avocados

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 3)
- recall basic multiplication facts (Number, level 3)
- solve practical problems that require finding fractions of whole number or decimal amounts (Number, level 3)
- find a given fraction or percentage of a quantity (Number, level 3)

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Number Framework Links

These activities are suitable for students who use advanced multiplicative strategies (stage 7). They support the development of mental strategies to solve problems with fractions, decimals, and proportions, including the application of equivalent ratios and rates.

Activity One

While question **1** suggests a doubling strategy, there are a number of methods the students can use to obtain a solution. Get them to solve the problem independently and then share their strategies with a classmate. This will allow them to clarify and justify their methods and to see that other strategies are possible. You could then ask the students to solve a similar problem using the same strategies. For example, if the students solved question **1a** by distributing the 12 as 10 + 2, ask them to use the same method to solve a problem involving 14 trees:

Avocados = 14 x 150
= (10 + 4) x 150
= (10 x 150) + (4 x 150)
= 1 500 + 600
= 2 100

In question **1b**, the students could use doubling strategies to find out how many avocados Dan and Rochelle should expect from each tree: 12 x 150 = 1 800, so 24 x 150 = 1 800 + 1 800, which is 3 600, so 48 x 150 = 3 600 + 3 600, which is 7 200.

For question **1c**, encourage the students to make and justify an estimate before they calculate the answer. From the previous questions, they will know that 24 trees are too few and 48 trees are too many. Ask them to explain the strategy they used as well as their answer. For example, working from results they already have:
12 x 150 = 1 800
8 x 150 = 1 200
20 x 150 = 3 000.
So 40 trees would produce about 6 000 avocados in a good season.

In question **2**, the students need to understand that finding $\frac{1}{3}$ of the amount is the same as dividing by 3. If production per tree drops to $\frac{1}{3}$, the number of trees must be trebled to obtain the same yield. This is an example of proportional adjustment:
40 trees x 150 avocados = 6 000 avocados in a good season
120 trees x 50 avocados = 6 000 avocados in a bad season.

In question **3**, the students need to find the average yearly yield, given a good season and a bad season:

$$\text{Average avocados on a tree per season} = \frac{\text{good season + bad season}}{\text{two seasons}} = \frac{150 + 50}{2} = 100$$

Therefore Dan and Rochelle's average expected crop would be number of trees x 100.

Activity Two

In question **1a**, the students need to calculate how much each size of avocado is worth, rounded to the nearest 5 cents. You may need to remind them how this is done. (See also the discussion on estimation and rounding at the start of these notes.) For this question, $1.20 ÷ 6 = 20 cents for a small avocado; $2.20 ÷ 5 = 44 cents for a medium avocado (rounded to 45 cents because we no longer have 1- or 2-cent coins); and $2.50 ÷ 4 = 62.5 cents for a large avocado (rounded to 65 cents).

Question **1b** involves whole numbers only (as there is no indication that Brett will split the bags) and can be solved in a variety of ways, including counting on and doubling.

In question **1c**, a customer wants to spend up to $30 on avocados in a mixture of sizes. The students are asked to spend at least $28 of the $30. Expect them to offer more than one solution, writing down both the total cost and the change from $30. Encourage them to use a table here. Then they can use a pencil to work out combinations that meet the criteria, for example:

Bags	Small	Medium	Large
1	1.20	2.20	2.50
2	2.40	4.40	5.00
3	3.60	6.60	7.50
4	4.80	8.80	10.00
5	6.00	11.00	12.50
6	7.20	13.20	15.00
7	8.40	15.40	17.50

In question **2**, the students use their understanding of fractions to find out how much money Brett will get from his harvest. As part of the problem-solving process, they need to calculate (using a calculator if necessary):

- how many of each kind of avocado there are:
  $\frac{1}{5}$ x 3 000 = 600 small avocados
  $\frac{1}{2}$ x 3 000 = 1 500 medium avocados
  3 000 – (1 500 + 600) = 900 large avocados
- how many bags there are:
  600 ÷ 6 = 100 bags of small avocados
  1 500 ÷ 5 = 300 bags of medium avocados
  900 ÷ 4 = 225 bags of large avocados
- the total money Brett will get from the sale of each kind of bag: 100 x 1.20 = $120.00; 300 x 2.20 = $660.00; 225 x 2.50 = $562.50. 120 + 660 + 562.50 = $1,342.50

The students could set out this information more economically as a table:

	Small	Medium	Large
Number	3 000 ÷ 5 = 600	3 000 ÷ 2 = 1 500	3 000 – 600 – 1 500 = 900
Bags	600 ÷ 6 = 100	1 500 ÷ 5 = 300	900 ÷ 4 = 225
Price	100 x 1.2 = $120	300 x 2.2 = $660	225 x 2.50 = $562.50
Total sales: 120 + 660 + 562.50 = $1 342.50			

Activity Three

In this activity, the students are finding proportional relationships between measures. Encourage them to set out the information in a table and to discuss the links between the quantities.

$2\frac{1}{2}$ avocados	$\frac{1}{2}$ lemon	4 children
5 avocados	1 lemon	8 children
15 avocados	3 lemons	24 children
20 avocados	4 lemons	32 children

Pages 12–13: Non-stop Ninths

Achievement Objectives

- express a fraction as a decimal, and vice versa (Number, level 4)
- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, levels 3–4)

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EA
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Number Framework Links

Use this activity to encourage transition from advanced multiplicative strategies (stage 7) to advanced proportional strategies (stage 8).

Activity

In this activity, the students investigate non-terminating decimals. There are two kinds of non-terminating decimals:

- recurring, non-terminating decimals, in which digits repeat, such as $\frac{1}{3}$ = 0.333333… or $\frac{1}{11}$ = 0.090909…, and which can be renamed as common fractions;
- non-recurring, non-terminating decimals, in which the digits do not repeat, such as $\sqrt{2}$ = 1.4142135… and π = 3.14159… These numbers cannot be renamed as common fractions and are known as irrational numbers.

Recurring decimals can be shown by a dot over the repeating digit, for example, $0.\dot{3}$ (which is 0.333…) or $0.1\dot{6}$ (which is 0.1666…) or over the first and last of the repeating digits, for example, $0.\dot{0}\dot{9}$ (which is 0.090909…) or $4.53\dot{2}10\dot{6}$ (which is 4.53210621062…). Sometimes a line is used to show the recurring pattern, for example, 0.090909… can be shown as $0.\overline{09}$. Usually, a recurring decimal must be rounded before it can be used in a calculation. This is shown by indicating the number of decimal places that the number was rounded to. For example:

$\frac{2}{3}$ = 0.666…
= 0.667 (3 d.p.)
= 0.7 (1 d.p.)

Students usually first encounter recurring decimals when they are dividing by 3 or by multiples of 3. In the first part of this activity, the students gain practical experience in recurring decimals involving ninths. They will probably have used decimats when exploring the meaning of fractions. (See decimat copymaster at the end of these notes.) In those instances, a convenient cut or two would probably have been enough to divide the whole into its fractional parts. In question **1** of this activity, the decimal system means there are no convenient cuts that can be made to model the division of a whole (decimat) into 9 equal parts. Instead, the students divide the whole into 10 equal parts and model the division by sharing out what can be shared, dividing the remainder, and then repeating the process.

If your students are new to the concept of recurring decimals, use question **1** as a class or whole-group activity. This may reduce the possibility that the students could get confused with lots of little bits of paper. Alternatively, the students could work on the problem in pairs or small groups. If you use small groups, expect everyone in the group to be able to explain clearly the meaning of the cutting they have just done. Ask questions such as:

What is the value of this (uncut) mat? (1)
Now that it has been cut into 10, what is the value of each piece? ($\frac{1}{10}$ or 0.1)
How can I start sharing the 10 pieces equally between 9 people? (Give 1 bit to each person, with 1 piece left over.)
How can I now share the leftover piece equally between the same 9 people? (Cut it into 10 equal bits and give 1 bit to each person, with 1 piece left over.)

Now that the leftover piece has been cut into 10 pieces, what is the value of each little piece? ($\frac{1}{10}$ of $\frac{1}{10}$, which is $\frac{1}{100}$ or 0.01)
What is the total value of the shares that the 9 people now each have? (0.1 + 0.01 = 0.11)
Does each person now have exactly $\frac{1}{9}$ *of a whole (decimat)?* (No.)

Keep the focus on the size of each of the 9 shares as each little bit of decimat is added to it because the size of this share is the "answer" to the question *What does* $\frac{1}{9}$ *look like as a decimal?* Hopefully, the students will be able to see that although it is not practicable to cut a decimat into ever-smaller pieces, the process that it models can be continued indefinitely. With each application of the process, each of the 9 shares gets closer to exactly $\frac{1}{9}$ of the whole, but a share will never equal $\frac{1}{9}$. This is a difficult but fascinating idea.

When students have completed this activity, they could work in groups to create posters that demonstrate their cut decimats' idea and explain what they have learned.

In questions **3a** and **3b**, the students consider rounding 0.1111 to 2 decimal places. To do this, they need to make judgments based on the number line in the students' book and their knowledge of standard rounding procedures. (See the discussion on rounding at the start of these notes.) Question **4** also requires the students to round. Note that they can see the new prices as $\frac{1}{9}$ off the old price or $\frac{8}{9}$ of the old price. If necessary, they can use the number line in question **2** as a model. The number lines in the Answers have their money values underneath the line, but either way is acceptable.

Extension
The students could use a calculator to discover which unit fractions ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and so on) are finite and which are recurring and see if they can discover a way of predicting which is which. (The key to the prediction is the factors of the denominator. If the only factors are 2 and/or 5, the decimal will be finite. If there are any other factors in the denominator, the decimal will be recurring. So $\frac{1}{20} = \frac{1}{2 \times 2 \times 5}$ [finite] and $\frac{1}{24} = \frac{1}{2 \times 2 \times 2 \times 3}$ [recurring].)

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)
- *Book 8: Teaching Number Sense and Algebraic Thinking*
  Recurring and Terminating Decimal Fractions, page 38

Pages 14–15: Animal Antics

Achievement Objectives
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 3–4)
- find and justify a word formula which represents a given practical situation (Algebra, level 4)

AC
EA
AA
AM
AP

Number Framework Links
Use this activity to encourage transition from advanced multiplicative strategies (stage 7) to advanced proportional strategies (stage 8).

Activity

In this activity, students solve problems that involve proportional relationships, using equivalent ratios.

The students are likely to find it very hard to know where to begin with question **1**. The reason is that there are no obvious units to work with. But once they discover (or are shown) a way of dealing with this, they should have little difficulty comparing the relative strengths of the teams in each case. You may like to begin by letting your students spend some time in groups trying to find a way forward and then get them to report back on the problems they have or their successes. Alternatively, you may prefer to give them some scaffolding before they move into groups.

The simplest approach is to use either box diagrams or materials. (Visually, these amount to the same thing.) Materials could be strips of card that the students cut and label themselves. The strips need to be cut to fit the information in the two introductory illustrations. The yaks will be the shortest strips because they have the least pulling power. It doesn't matter what actual length the students choose for the yaks, but they need to cut and label three strips, all the same length. They then need to cut another strip that is exactly the length of three yak strips end to end. If they fold this strip in half and cut it on the fold, they will have two rhino strips. They can use all these strips to model the second set of information in the top panel:

<table>
<tr><td colspan="3">Rhino</td><td colspan="3">Rhino</td></tr>
<tr><td colspan="2">Yak</td><td colspan="2">Yak</td><td colspan="2">Yak</td></tr>
</table>

2 rhinos = 3 yaks

They should then cut a new strip that has the length of 1 rhino strip and 3 yak strips joined end to end. This new strip should be labelled Elephant. The students can now use their strips to model the first set of information in the top panel.

<table>
<tr><td colspan="9">Elephant</td></tr>
<tr><td colspan="3">Rhino</td><td colspan="2">Yak</td><td colspan="2">Yak</td><td colspan="2">Yak</td></tr>
</table>

1 elephant = 1 rhino + 3 yaks

It is very important that they understand that the length of the strip represents the strength of the animal: the longer the strip, the greater the pulling power.

By cutting additional strips for extra animals as needed, the students should now be able to use them to model each of the situations in question **1** and to solve the problems:

Question **1a**:

<table>
<tr><td colspan="9">Elephant</td></tr>
<tr><td colspan="3">Rhino</td><td colspan="3">Rhino</td></tr>
</table>

1 elephant should win against 2 rhinos.

Question **1b**:

<table>
<tr><td colspan="2">Yak</td><td colspan="2">Yak</td><td colspan="2">Yak</td><td colspan="2">Yak</td><td colspan="2">Yak</td></tr>
<tr><td colspan="3">Rhino</td><td colspan="3">Rhino</td><td colspan="3">Rhino</td></tr>
</table>

5 yaks should win against 3 rhinos.

Question **1c**:

<table>
<tr><td colspan="9">Elephant</td><td colspan="2">Yak</td><td colspan="2">Yak</td><td colspan="2">Yak</td></tr>
<tr><td colspan="3">Rhino</td><td colspan="3">Rhino</td><td colspan="3">Rhino</td><td colspan="3">Rhino</td></tr>
</table>

1 elephant and 3 yaks should win against 4 rhinos.

Using the strips, the students should find it easy to make up and answer their own tug-of-war questions involving any combinations of the three animals.

As a further activity, you could challenge your students to work out the relative strengths of the three animals as a ratio. To do this, they will need to use a notional "unit of strength". If they have trouble with this, suggest that they assign 2 units of strength to a yak. The elephant:rhino:yak ratio in its simplest form is 9:3:2. The box diagram illustrates this fact:

<table>
<tr><td colspan="9">Elephant</td><td>1 elephant = 9 units of strength</td></tr>
<tr><td colspan="3">Rhino</td><td colspan="3">Rhino</td><td colspan="3">Rhino</td><td>1 rhino = 3 units of strength</td></tr>
<tr><td colspan="2">Yak</td><td colspan="2">Yak</td><td colspan="2">Yak</td><td colspan="2">Yak</td><td>$\frac{\text{Yak}}{2}$</td><td>1 yak = 2 units of strength</td></tr>
</table>

Question **2a** requires the students to make proportional adjustments: they know how many bananas and apples are needed for each chimpanzee or monkey, so how many are needed for □ chimpanzees and □ monkeys? In question **2b**, the students have to work in reverse. This is not too difficult for part **i**. In this case, it is clear that the number of animals involved must be very small, so trial and improvement should quickly find the right combination. The students might note that it takes 4 bananas and 5 apples to feed a monkey and a chimpanzee. 8 bananas and 10 apples is twice that, so there must be 2 of each animal.

The students will probably find question **2b ii** more difficult because there appear to be so many possible combinations of chimpanzees and monkeys to consider. One method is to use the "one of each animal" formula to get close to the desired numbers, then adjust. 4 bananas and 5 apples feed 1 monkey and 1 chimpanzee, so 6 monkeys and 6 chimpanzees would need 24 bananas and 30 apples. This total is close to 29 bananas and 31 apples. Adjusting the 6-chimpanzee-and-6-monkey formula gives:

	Chimpanzees	**Monkeys**	**Bananas**	**Apples**
	6	6	24	30
Add chimpanzee and take away monkey	7 (+1)	5 (−1)	26 (+2)	29 (−1)
Add chimpanzee	8 (+1)	5	29 (+3)	31 (+2)
			(29)	(31)

The alternative to trial and improvement is to find a way of systematically sifting the possibilities. Here is a suggested way of doing this using a table:

Number of animals	Number of bananas (Chimps)	(Monkeys)	Number of apples (Chimps)	(Monkeys)
1	3	1	2	3
2	6	2	4	6
3	9	3	6	9
4	12	4	8	12
5	15	5	10	15
6	18	6	12	18
7	21	7	14	21
8	24	8	16	24
9	27	9	18	27
10	30	10	20	30
11	33	11	22	33
12	36	12	24	36
	29		31	

Once the numbers have been entered in the table, the students can use a pencil to circle combinations in the Bananas columns that add up to 29 and combinations of numbers in the Apples columns that add up to 31, as shown above. They should then list these combinations in another table, but with the focus on *numbers of animals*, not number of pieces of fruit as in the first table. There aren't many combinations that add up to 29 and 31, and it should be immediately clear that there is only one combination of animals that exists for both types of fruit:

(29 bananas)	
Chimps	**Monkeys**
6	11
7	8
8	5
9	2

(31 apples)	
Chimps	**Monkeys**
2	9
5	7
8	5
11	3

This kind of strategy works well in many different situations where we need to find one combination from a range of many possible combinations. Once the table has been drawn up, the students will quickly see that they can discard a lot of combinations for reasons such as "The sum is odd, so I only need to check combinations of even + odd."

For questions **3a** and **3b**, 2 salad rolls + 2 drinks = $12, and 3 salad rolls + 1 drink = $14. The students need to approach this logically and intuitively. For example:

1. If 2 salad rolls and 2 drinks cost $6, 1 roll and 1 drink cost $6.
2. 3 rolls and 1 drink cost $14, so the 2 extra rolls cost 14 – 6 = $8, or $4 for 1 roll.
3. Given that 1 roll and 1 drink cost $6, if 1 roll costs $4, the drink costs $2.

The students can use this information to create the generalisation for calculating the cost of lunch in question **3c**: number of drinks x $2 + number of rolls x $4 = cost.

In question **3d**, the students could calculate the cost of lunch as 3 x $4 rolls + 3 x $2 drinks = $18, so the amount spent on muffins is 28.50 – 18 = $10.50 for 3 muffins. 10.50 ÷ 3 = $3.50 for each muffin. Alternatively, the students could start with 1 roll + 1 muffin + 1 drink = $\frac{1}{3}$ x $28.50 and then use their answers to questions **3a** and **3b** to work out the cost of 1 muffin.

Pages 16–17: Using Mates

Achievement Objectives

- recall the basic multiplication facts (Number, level 3)
- write and solve problems involving decimal multiplication and division (Number, level 4)
- express a fraction as a decimal, and vice versa (Number, level 4)

AC
EA
AA
AM
AP

Number Framework Links

This activity, which involves multiplication, is suitable for students using advanced multiplicative strategies (stage 7) or higher.

Activity

This activity focuses on solving problems with fractions and decimals, using compatible numbers and the associative rule.

When you are teaching this activity (and Compatible Multiples, page **21**), it's very important to challenge the idea that multiplication always involves increasing the size of something (and division always involves reducing the size of something). Because this is always the case when working with *whole* numbers, some students don't realise that it isn't true for *all* numbers. The key idea is that multiplying any (positive) number by a number greater than 0 and less than 1 decreases it. (For example, 5 x 0.6 = 3; 3 is less than the 5 we started with.) See the Numeracy Project resource list below.

This activity does not involve negative numbers, but similar issues come up there: doubling a negative number gives a number that is less, not more, than the starting number.

In question **1**, Wiha realises that, instead of multiplying by 0.5, she can use a doubling and halving strategy to solve the problems quickly and obtain the weight of the packets in kilograms. Some students will find this confusing if they focus on the fact that multiplication usually makes things bigger. In this case, the answer is half of the multiplier. Discuss why this occurs. Point out that "multiplied by" can be thought of as "of", for example, 6 x 0.5 is 6 lots of 0.5.

Question **2** challenges the students to find compatible numbers to make multiplying decimals easier. The students need to be able to justify their choice of combinations, for example:
0.5: 0.5 x 2 = 1 or 0.5 x 10 = 5
2.5: 2.5 x 2 = 5 or 2.5 x 4 = 10.

In question **3**, the students apply their understanding of compatible decimal numbers and derived multiplication facts to solve problems that involve three numbers and explain their strategies. In doing this, they need to use the principles of commutativity, distributivity, and associativity:

- Commutativity is the principle of reversability that allows us to use whatever order suits us when adding or multiplying, for example, 8 x 0.5 = 0.5 x 8
- Distributivity is the principle that allows us to group numbers or split them for purposes of multiplication, for example, 12 x 14 = 12 x (10 + 4)
  = (12 x 10) + (12 x 4)
  = 120 + 48
  = 168
  This is the principle underlying part–whole thinking.
- Associativity is the principle that allows us to add or multiply more than two numbers by grouping them in whatever pairs suit us or the problem, for example, 2 + 3 + 4 = (2 + 3) + 4 or 2 + (3 + 4) or (2 + 4) + 3. In each case in this example, two of the numbers are added together and the result is added to the third number. (Addition and multiplication are binary operations: they can only be done with two numbers, not three or more. A computer adding or multiplying a million numbers would still pair them up at each stage to do so.)

With questions **3–6**, you could insist that the students use brackets each time they work with the associative principle. This will help them to understand what they are doing and also makes it clear to an onlooker how they are grouping pairs of numbers.
For example:
"20 x 806 x 0.5 = (20 x 0.5) x 806
= 10 x 806
= 8 060
0.5 is a half, so 20 can be halved to 10, which makes the multiplication of 806 easy."

"0.25 x 193 x 8 = (0.25 x 8) x 193
= 2 x 193
= 2 x 200 – 14
= 386
A quarter of 8 is 2, and 193 is nearly 200, so I just need to double 200 and take off 14 (that is, 7 x 2)."

"4 x 326 x 5 = (4 x 5) x 326
= 20 x 326
= 2 x 326 x 10
= 652 x 10
= 6 520.
4 times 5 is 20, which is 2 x 10, so I just need to double 326 and then multiply it by 10."

Question **4** offers more practice in using the strategies to solve number problems. Encourage the students to explain their methods and to try other students' strategies or similar examples.

Question **5** uses a more complex multiplication "mates" strategy. The students need to be able to recombine the factors to show they understand how this strategy works. For example:
8 x 23 x 25 = 2 x (4 x 25) x 23
= 2 x (100 x 23)
= 2 x 2 300
= 4 600
(The factors are reorganised to make 100, then doubled.)

Interesting extension points you can raise with the students include:
Does the grouping make any difference to the answer? For example, (8 x 23) x 25 = 8 x (23 x 25).
How are 3 factors linked to applications, particularly volume?
If multiplying a number by a decimal between 0 and 1 makes the answer smaller, what does division do?

In question **6**, the emphasis needs to be on the students' explanations of their strategies. Pairs could present their questions to the group, and then the students could discuss whether they had used the same strategy or a different one to solve each question and which strategy they found best.

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)

- Book 8: *Teaching Number Sense and Algebraic Thinking*
  Whole Numbers Times Fractions, page 22
  Fractions Times Whole Numbers, page 23
  A Fraction Times a Fraction, page 24
  When Big Gets Smaller, page 24
  When Small Gets Bigger, page 24
  Estimation in Decimal Multiplication and Division Problems, page 25

Pages 18–20: Lunchtime Mardi Gras

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- find and justify a word formula which represents a given practical situation (Algebra, level 4)

AC
EA
AA
AM
AP

Number Framework Links

Use these activities to encourage transition from advanced additive strategies (stage 6) to advanced multiplicative strategies (stage 7). **Activity Three** requires advanced multiplicative strategies or higher.

Activities

In these activities, students write and solve problems involving whole numbers and decimals. They may use double number lines and ratio tables to find proportional relationships between measures and to apply equivalent ratios.

Activity One

In question **1**, the students are given two visual methods to help them to solve a ratio problem: the double number line and the ratio table. These are used to compare the ratio of money to hoops. Eru's group has a ratio of $2 : 5 hoops, so you would pay $6 for 15 hoops (tripling the ratio terms).

In question **2**, the students can use the ratio table from question **1** and create other double number lines or ratio tables to answer the questions. Alternatively, they can apply the ratios directly to the problems and use appropriate strategies to solve them. Ask the students to explain their reasoning. For example:
"If $2 buys 5 hoops, $20 will buy 50 hoops and $60 will buy 3 x 50 = 150 hoops."
"If $2 buys 5 hoops, $10 will buy 5 times as many, that's 25 hoops, so $60 will buy 6 x 25 = 150 hoops."
"If $3 buys 9 hoops, $15 will buy 45 hoops and $30 will buy 90 hoops. 45 + 90 = 135, so the cost of 135 hoops must be 15 + 30 = $45."

To reinforce the mathematics, you could get each student to make up another table to show dollars and hoops, put new figures in the last two rows, and get a classmate to complete it.

The students need to summarise their findings. They should find that Mere's group made the most money. Get them to clarify their ideas with a classmate so that they can say why they think Mere's group made the most money. A possible explanation is: "For $1, Mere gave 2 hoops, Simon gave 3, and Eru gave $2\frac{1}{2}$, so Simon's stall gave you the best chance of winning. However, Mere's basic charge was only $1 compared with Simon's basic charge of $3, so Mere's stall may have appeared cheaper or appealed more to students who wanted to try the hoopla activity without spending too much money."

Activity Two

This activity involves the use of a dartboard. Make sure the students understand how the doubles and triples are scored. If possible, show this on a real dartboard.

In question **2a**, the students could use the idea of ratio tables from **Activity One** to find out how many darts a player can get for $3.

$	0.50	1	1.50	2.00	2.50	3.00
Darts	3	6	9	12	15	18

(The table could also be set out vertically.)

This ratio table could help the students establish a rule that the total number of darts = 6 x number of dollars. So if $20 were paid (question **2c**), the total number of darts would be 6 x 20 or 120 darts, and if 306 darts were thrown (question **2d**), the cost in dollars would be 306 ÷ 6 = $51.

In question **3**, the students have to justify the strategies they use to add up a combination of scores. These part–whole strategies could be based on tidy or compatible numbers for addition and on derived facts for multiplication.

The students need to be able to explain their strategy and why they chose it. Ask them to give another combination of three numbers for which a particular strategy would also be appropriate.

Extension
If possible, give the students practical experience in the game itself on a real dartboard, which is much more complex than the simplified one shown in the students' book. This would allow the students to combine larger numbers using doubles and trebles than they can with the simplified model in the students' book.

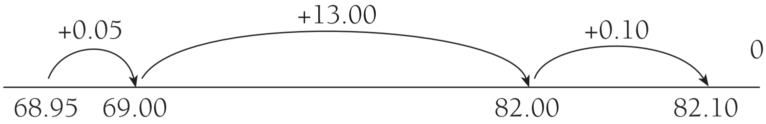
Activity Three

This activity requires the students to calculate how much money was made on the cake stall every 10 minutes. (This is known as a rate because money and time are measured in different units.) The students could display their work as a table.

Time (minutes)	Amount made ($)	Total ($)
10	12.50	12.50
20	22.15	34.65
30	34.30	68.95
40	13.15	82.10
50	20.70	102.80
60	18.55	121.35

Again, the question asks the students to explain the process they used to get their answers. Place value or tidy number strategies will work in either case. In difference problems like this, adding on (reversing) is usually an efficient mental strategy. For example, between 30 and 40 minutes, the difference is $82.10 – $68.95.

This can be calculated as:

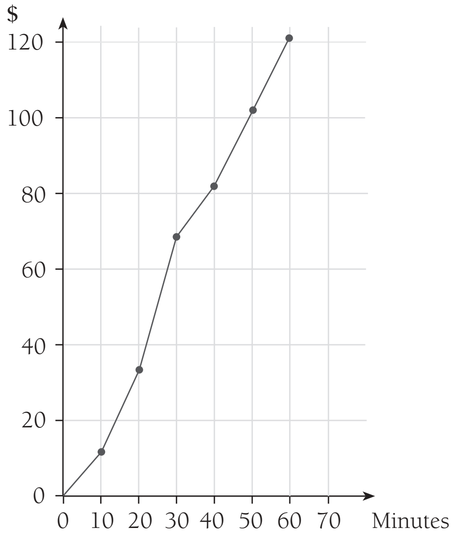


0.05 + 13.00 + 0.10 = $13.15

Extension

The students could represent the money total on a time-series graph, showing the time in minutes on the horizontal axis and the total amount of money (cumulative total) made on the vertical axis.

Cake Stall Earnings



The students could then use the visual representation to discuss the rate of sales in terms of the slope of the line at 10-minute intervals, that is, the steeper the slope, the more sales; and also whether the sales slowed down and for what possible reason. They could use this information to plan for next year's mardi gras, for example, *When would it be best to have most helpers at the cake stall?*

Activity Four

This is another activity about combining tidy numbers in addition. The students will have various answers and may combine strategies, such as:

- add the cents: (35 + 15 + 50) + (35 + 65) = $2 (to make whole dollars),
- 198 + 2 = $200 (make a hundred),
- 121 + 121 = $242 (doubles),
- (160 + 40) + (8 +2) = $210 (make 100, make 10),
- then combine the subtotals and $51 to make $703.

See the Answers for another possible combination of strategies.

Page 21: Compatible Multiples

Achievement Objectives

- write and solve problems involving decimal multiplication and division (Number, level 4)
- express a fraction as a decimal, and vice versa (Number, level 4)
- recall the basic multiplication facts (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

For this activity, students need to be advanced multiplicative (stage 7) or higher.

Activity

This activity uses a variety of strategies involving decimal and fraction compatibles to solve and write problems. Students also need good recall of multiplication facts to 10 x 10 and the corresponding division facts. Note that some students may be confused about multiplication answers being smaller than the multiplier, when the reverse is usually the case. As in earlier activities, a reminder that "multiplied by" can be thought of as "of" will help. (See the notes for pages **16–17**, Using Mates.)

This activity emphasises the development of efficient strategies to solve problems involving decimals and fractions. Encourage the students to share and justify the strategies they used to solve each problem. The activity assumes that students know the commutative property of multiplication (that the order of the factors does not affect the product). For example, for question **1a**, 5 x 7 x 0.2, the students may know that 0.2 is the same as $\frac{1}{5}$, so 5 x $\frac{1}{5}$ = 1, 1 x 7 = 7. Alternatively, they may know 5 x 0.2 = 1.0. Some strategies may not be based on compatible numbers but may be derived from known facts such as 5 x 7 = 35, so $\frac{1}{5}$ of 35 is 7.

Question **2** requires the students to use both fractions and decimals when coming up with their compatible numbers. Make sure they do so, as reinforcement and as good preparation for question **3**.

In question **3**, the students create four of their own problems based on compatible numbers, which they then ask a classmate to solve. You need to reinforce the compatible number patterns from question **1** by asking why particular combinations in that question were considered "compatible". Often it is because they multiply to 1 or another whole number such as 2, 3, 5, or 10. The students may come up with other compatible number combinations.

In question **4**, the students are asked to write a decimal multiplication and a fraction multiplication that use compatible number combinations and have a product of 20. For example, 0.2 x 5 x 20 = 20 and 10 x $\frac{1}{2}$ x 4 = 20. Get the students to explore several solutions and record these as a poster, Compatible Number Names for 20. The students can present their poster to the group and justify their number sentences by explaining which combinations are considered compatible numbers and why.

Pages 22–23: Even Pictures Talk

Achievement Objectives

- use graphs to represent number, or informal, relations (Algebra, level 3)
- find a given fraction or percentage of a quantity (Number, level 4)
- use their own language to talk about the distinctive features, such as outliers and clusters, in their own and others' data displays (Statistics, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to encourage transition from advanced additive strategies (stage 6) to advanced multiplicative strategies (stage 7).

Activity

This activity involves students describing, summarising, and comparing data, as well as identifying relationships within the data and communicating these ideas. It also requires students to use problem-solving strategies by making links between measurement and number.

Give the students time to explore and discuss interpretations of the data with a classmate and time to clarify and elaborate their ideas before and after studying the information presented in each graph. Each graph can be used as a model for an extension activity that involves the students collecting their own data to present in graph form to the class.

Different information is presented on five different kinds of graph, featuring either discrete or continuous information: a bar graph (question **1**), a pie or circle graph (question **2a**), a scatter graph (question **2b**), a stem-and-leaf plot (question **2c**), and a line graph (question **2d**). Some students may be familiar with certain styles of graphs and not with others, so the amount of questioning needed to develop ideas and to explore (and prepare) similar problems will vary.

In question **1**, the students are able to use the bar graph to find the total number of students in Room 1 by adding the frequencies of the flavour of chocolate liked by students, that is, 1 + 2 + 3 + 4 + 6 + 8 = 24. Each category can be represented as a fraction of the total number of students and/or converted to a percentage and compared to statements about the graph:
$\frac{2}{24}$ ($\frac{1}{12}$) like dark chocolate.
$\frac{3}{24}$ ($\frac{1}{8}$) like white chocolate.
$\frac{4}{24}$ ($\frac{1}{6}$) like peanut chocolate.
$\frac{6}{24}$ ($\frac{1}{4}$) like milk chocolate.
$\frac{8}{24}$ ($\frac{1}{3}$) like caramel chocolate.
$\frac{23}{24}$ like chocolate.

Therefore, all of the statements made are true: $\frac{1}{3}$ of the students in Room 1 prefer caramel chocolate, twice as many students prefer peanut to dark chocolate, $\frac{1}{8}$ of the students prefer white chocolate, and more than 95 percent of the students like chocolate.

In question **2a**, the data is represented as a whole on a pie graph and is divided into parts of that whole. The students can make general statements about the data on the pie graph using their observations of the relative size of each sector, for example, more students were born in Asia than Europe, half of the students were born in the Pacific, and the other half of the students were born in Europe, Asia, or Africa. Encourage the students to discuss more specific number information by measuring each sector in degrees, comparing it to the proportion of the whole circle, and recording this as a fraction:
Pacific: $\frac{180}{360} = \frac{1}{2}$ or 50%
Asia: $\frac{90}{360} = \frac{1}{4}$ or 25%
Europe: $\frac{60}{360} = \frac{1}{6}$ or $16\frac{2}{3}$%
Africa: $\frac{30}{360} = \frac{1}{12}$ or $8\frac{1}{3}$%.

For the pie graph, the number of students is not given. You could give the students different-sized classes and ask them to interpret the information and hence solve practical problems that require finding the fractions of whole numbers. For example: If there were 36 students in the class, how many would be born in Asia? If there were 24 …?

In question **2b**, the scatter graph data of hand spans and heights show a relationship (correlation) where the points could be grouped around a central line with a positive slope (gradient) from left to right. The graph shows that hand spans increase with the students' heights. The number of plots allows the students in Room 6 to be counted (18 students). Statements can be made about the range of data for two values: the hand span and the height. The students in Room 6 have hand spans between 12 and 21 centimetres and are between 130 and 170 centimetres tall. Your students can also make general statements about the trend in the relationship between the hand span and the height of a Room 6 student, for example, as the height of a student increases, the hand span increases.

Your students can be more specific about the relationship between the height of the student and the hand span by reading (estimated) values from the graph, for example,

$\frac{\text{hand span}}{\text{height}} \approx \frac{13 \text{ cm}}{130 \text{ cm}}$, which is $\frac{1}{10}$ or 10 percent.

They can compare all or several values so that they can make a general statement about this relationship, for example, "A student's hand span is about 10 percent of their height" and use this information to make predictions, for example, in response to "*If a new student in Room 6 had a hand span of 15 centimetres, about how tall might he be?*" The students could also rule a line through the scatter dots using a slope of "best fit", which in this case would leave 9 dots above the line and 9 dots below.

In question **2c**, the data is presented as a stem-and-leaf plot. This graph quickly shows the range of data and how it is distributed over that range through several decades. The students can find the number of students in Room 2 by counting the number of leaves on the graph (19 students). The students should be able to make statements about the range of hours of television watched by Room 2 students from 0 to 41 hours, the mode or value that occurs most frequently (13 hours), and the median (13 hours). The students could also make statements about the distribution of the data:

"Most students in Room 2 watch between 10 and 20 hours of TV in a week."

"$\frac{10}{19}$ or just over half watch between 10 to 19 hours of TV in a week."

"$\frac{5}{19}$ or slightly more than 25% of students watch less than 10 hours of TV in a week."

You could also encourage the students to compare the number of hours of TV watched to the numbers of hours in a week to see if the data portrayed is realistic, for example, 168 hours per week (24 hours per day x 7 days) – 35 hours (at or travelling to school) – 56 hours (sleeping at 8 hours per night) – 41 hours (watching TV) = 36 hours for everything else.

In question **2d**, the line graph shows the relationship between the water level of a paddling pool and the time of day. Encourage the students to be specific in their discussion about the information presented in the graph.

As part of their stories, the students could suggest why the water level rose and fell at different rates, such as the hose filling the pool was turned off at 3.15, but no one got in until 5 minutes later. The students could also suggest reasons for the maximum and minimum levels of water and why they occurred at particular times. (Perhaps there was some energetic splashing around 3.40!) The child or children in the pool may have got out at 4.10, and 8 minutes later, the pool was emptied.

Page 24: Nimble Numbers

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- make sensible estimates and check the reasonableness of answers (Number, levels 3–4)

AC
EA
AA
AM
AP

Number Framework Links

This game is suitable for students using advanced multiplicative (stage 7) strategies. It can easily be adapted for those using advanced additive strategies (stage 6).

Game

In this game, students use estimation and number strategies to work out hidden whole numbers. The game will support the development of strategies based on multiplication and division and gives students excellent practice with basic facts and factors. It can also provide practice with estimation.

You will need to check that the students know how to type formulae into a cell. They also need to know that the * (asterisk) is the spreadsheet symbol for multiplication and that the / (slash) is the spreadsheet symbol for division.

You can easily adjust this game to suit particular groups of students. Here are some suggestions:

- For students at the advanced additive stage, limit the size of the numbers they can use to 20. This will still allow over two dozen pairs of numbers that meet the criteria without using 1 as a factor: (20, 10), (20, 5), (20, 4), (20, 2), (18, 9), (18, 6), (18, 3), (18, 2), (16, 8), and so on. These numbers will give the students a lot of practice with basic facts.
- For students at the advanced additive stage who are struggling to make the next stage, change the fourth formula to read =A50/10. This will mean that they can always find the first number simply by multiplying the result they see in the fourth cell by 10. The second number can then be easily found using any of the other three clues.
- For students at the advanced multiplicative stage, increase the size of the numbers they can use to 100. This means that, to be a factor, the maximum size of the second number can be no more than 50. Some of the many possible pairs are: (100, 50), (96, 16), (55, 11), (84, 12) … This will increase the pool of potential pairs of numbers and will require the students to make use of a wider range of mental strategies as they add, subtract, multiply, and divide.
- To greatly increase the pool of possible number pairs, delete the fourth formula (=A50/A51). The three clues are still sufficient for the numbers to be found. Possible number pairs now include: (23, 17), (85, 9), (14, 13), (48, 15) … The rules state that the first number is always the bigger number, so students won't have to meet negative numbers.
- To make things more difficult, delete the second or third formula. This will leave just two clues, which is still enough for a solution to be found but will mean that the students have to make very good use of those two clues. If you decide to do this, try limiting the numbers that the students can use to ones that are 20 or less until they have gained some confidence.
- Widen the range of numbers that can be used to include numbers with one or two decimal places. The students will need to use estimation more.
- Vary the scoring rules or the total number of points needed to win a game. Able students could be given a higher score to aim for in order to win, or as soon as one person has a lead of 3 points, they are declared the winner.

Copymaster: Non-stop Ninths

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