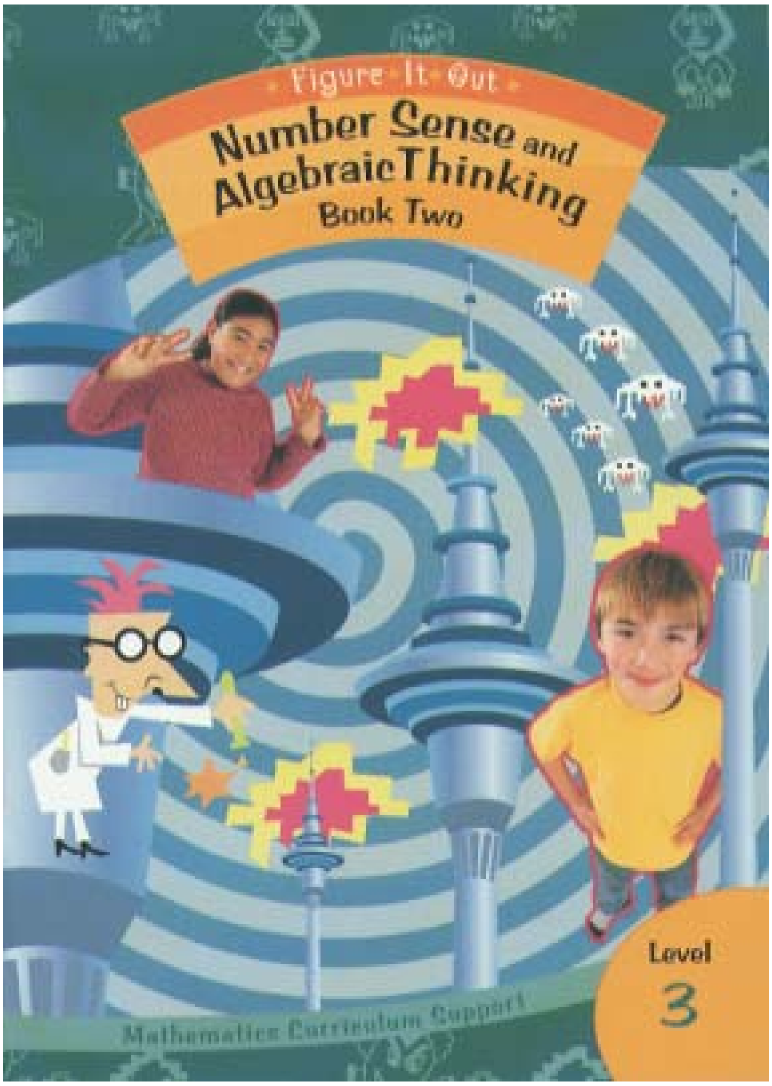
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Contents

Introduction

The books for level 3 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand classrooms. These books are most suitable for students in year 5, but you should use your judgment as to whether to use the books with older or younger students who are also working at level 3.

Student books

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and the life experiences that are meaningful to students in year 5.

The activities can be used as the focus for teacher-led lessons, for students working in groups, or for independent activities. Also, the activities can be used to fill knowledge gaps (hot spots), to reinforce knowledge that has just been taught, to help students develop mental strategies, or to provide further opportunities for students moving between strategy stages of the Number Framework.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the *Number Sense and Algebraic Thinking* student books includes full answers and explanatory notes. Students can use this section for self-marking, or you can use it for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, Number Framework links, comments on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/r/maths/curriculum/figure

Using Figure It Out in the classroom

Where applicable, each page starts with a list of equipment that the students will need in order to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to the problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Number Sense and Algebraic Thinking Level 3: Book Two

Answers

Page 1: Triple Trouble

Activity

1. **a.** i. $5\frac{2}{3}$ chocolate bars

b. v. 5, 6, and 6 people

c. iii. \$5.65, \$5.65, \$5.65, r5c. (\$5.65 each and 5c left over)

d. iv. 5.6666666 (on a calculator with 8 places)

e. ii. 5.7 cm

2. **a.** $5\frac{2}{3}$. ($5\frac{2}{3} \times 3 = 17$)

b. Answers will vary. The numbers differ in how close they are to exactly $5\frac{2}{3}$. They have been rounded differently because of the different contexts. For example, we don't use 1c and 2c coins any more, so the friend will have to round her amounts to \$5.65; most calculators display 8 digits; the number of pieces in a chocolate bar may be exactly divisible by 3.

3. **a.–b.**

Problems will vary. The story with a whole number answer should involve things that are not normally broken up into parts, such as people, animals, or cars. The story with an answer rounded to 1 or 2 decimal places might involve measuring weight, length, or volume.

Pages 2–4: Maths Detective

Activity

1. **a.** Phala

b. Wātene

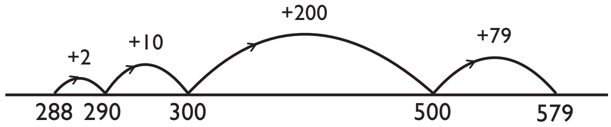
c. Kere

d. Maire

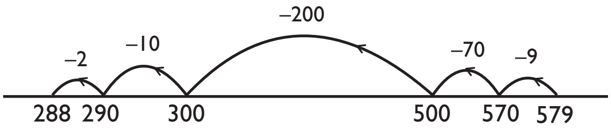
e. Ryan

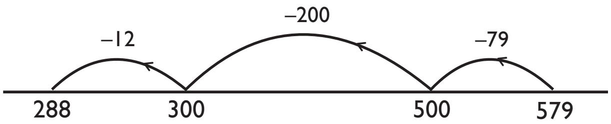
f. Tāne

2. Answers will vary. Some students may use the given strategies but add more interim steps, such as:



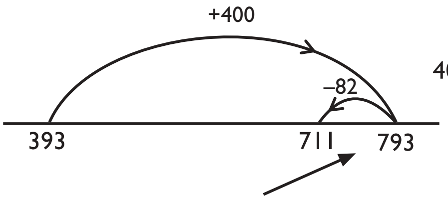
Ryan's strategy could also be varied by subtracting in jumps to tidy numbers:





3. Answers will vary. All the strategies worked well. For example, Ryan's adding-on strategy **(1e)** is straightforward for keeping track of in his head; Kere's equal adjustment strategy **(1c)** is concise and is a good use of number properties.

4. **a.** Phala (strategy **1a**):



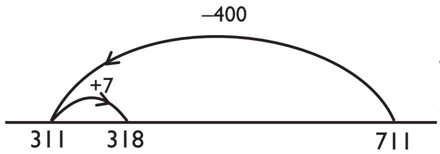
$400 - 82 = \boxed{318}$

This step (793 – ❑ = 711) is difficult in your head.

Wātene (strategy **1b**):
$711 - 300 = 411$
$411 - 90 = 321$
$321 - 3 = \boxed{318}$

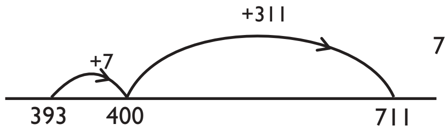
Kere (strategy **1c**):
$711 - 393$
$+7 \quad +7$
$718 - 400 = \boxed{318}$

Maire (strategy **1d**):



711 – 400 = 311
311 + 7 = $\boxed{318}$

Ryan (strategy **1e**):



7 + 311 = $\boxed{318}$

Tāne (strategy **1f**):
700 – 300 = 400
11 – 93 = $^{-}82$
400 – 82 = $\boxed{318}$

b. Answers will vary. Efficient strategies are Ryan's way of reversing and adding on (because it only takes two jumps), Maire's tidy numbers and compensating (because 393 is close to a tidy number), and Kere's equal adjustments strategy (again, because 393 is close to a tidy number). The other strategies work, but the mental calculations are trickier.

5. Problems will vary.

a. Ryan's adding on strategy will suit most pairs of numbers.

b. Phala's strategy of adding a tidy number and compensating will be straightforward if the starting number can be rounded up to a close tidy number. Maire's strategy of subtracting a tidy number and compensating will work well if the number being subtracted can be rounded up to a close tidy number.

c. Wātene's place value strategy will be most efficient for numbers where the corresponding digit in each place is smaller in the number being taken away, for example, in 584 – 362. Tāne's place value strategy works most efficiently when the starting number can be rounded up to a close tidy number.

d. Kere's equal adjustments strategy is simplest to use when the number being subtracted can be rounded up to a close tidy number.

6. Strategies and discussion will vary.

Page 5: Alien Estimates

Activity

1. Yes, Rongomai is correct. She rounded 82 145 down to 82 000 and 45 877 up to 46 000. Then she rounded 61 480 down to 61 000 and 67 952 up to 68 000. Ben had entered 1 instead of 7 on the calculator. The Martians' round 1 score should have been 129 432.

2. **a.** **i.** Round 2: Cyborgs 272 733, Martians 270 954

ii.–iii. Results will vary.

b. One possible strategy (Rongomai's) is to round the numbers to the nearest 1 000 and work out what 69 000 plus 204 000 is for the Cyborgs and what 176 000 plus 95 000 is for the Martians team.

3. Estimates will vary. A sensible estimate is 300 000 + 140 000 = 440 000. Any estimate should focus on the digits in the high value places and ignore those in the lower value places.

Pages 6–7: Pizza Split

Activity

1. Yes, they are both right. $\frac{2}{8}$ can be written as $\frac{1}{4}$. They are equivalent fractions.

2. **a.** Yes. 8 ÷ 2 does not give the same answer as 2 ÷ 8.

b. 8 ÷ 2 = 4; 2 ÷ 8 = $\frac{1}{4}$ (or $\frac{2}{8}$)

3. **a.** **i.** 12

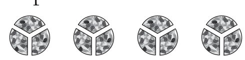
ii. 3. (12 ÷ 4)

iii. Practical activity.
A possible answer is:

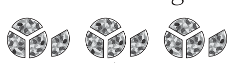
can be rearranged as

iv. 3 ÷ 4 = $\frac{3}{4}$

b. **i.** Practical activity.
A possible answer is:



can be rearranged as



ii. 4 ÷ 3 = $\frac{4}{3}$. (This can also be written as $1\frac{1}{3}$.)

4. Yes, the pattern does work. $\frac{5}{2}$ is 5 ÷ 2 and $\frac{2}{5}$ is 2 ÷ 5.

can be rearranged as:

5 pizzas ÷ 2 = 5 halves of a pizza each

and

can be rearranged as:

2 pizzas ÷ 5 = 2 fifths of a pizza each

$5 \div 2 = \frac{5}{2}$ (which can be written as $2\frac{1}{2}$)
$2 \div 5 = \frac{2}{5}$

The number on the bottom of the fraction (the denominator) tells you what sort of fraction it is. For example, 5 on the bottom means you have fifths, 4 means you have quarters, 3 stands for thirds, 2 stands for halves, and 1 stands for wholes. The number on the top of a fraction (the numerator) tells you how *many* you have: in $\frac{3}{4}$, you have 3 quarters; in $\frac{1}{4}$, you only have 1 quarter.

5. Yes, the answers do follow the pattern. $\frac{2}{8}$ is $\frac{1}{4}$ (one-quarter) and $\frac{4}{1}$ is 4 wholes. All whole numbers could be written as something over 1. For example, 15 could be written as $\frac{15}{1}$ or 345 as $\frac{345}{1}$, but we don't do this unless we need to. So for 8 pizzas divided between 2 people, each person gets 4 whole pizzas each ($\frac{4}{1}$). With 2 pizzas among 8 people, each person gets $\frac{2}{8}$ or $\frac{1}{4}$ each.

6. Yes. Problems will vary. The pattern will work for all of them, although you may have to change some of your fractions to their simplest equivalent form. For example, you may have worked out that $4 \div 12 = \frac{4}{12}$, which does not seem to match $12 \div 4 = 3$. However, $\frac{4}{12}$ is equivalent to $\frac{1}{3}$, and 3 is $\frac{3}{1}$.

Pages 8–10: Digit Shuffle

Activity One

1. The largest total possible is 1 827. This can be made with 973 + 854, 953 + 874, 954 + 873, or 974 + 853.

2. **a.** 1 170

   b. 1 395

   c. 4 ways: 753 + 642; 752 + 643; 742 + 653; 743 + 652

3. Answers will vary.

4. A possible set of instructions is: Put the 2 largest digits in the hundreds places of the 2 numbers. Put the 2 smallest digits in the ones places. Put the middle 2 digits in the tens places.

5. The instructions would be reversed. For example, put the 2 smallest digits in the hundreds places of the 2 numbers. Put the 2 largest digits in the ones places. Put the middle 2 digits in the tens places.

6. The largest total possible is 2 556. The smallest total possible is 774. There are lots of different ways of arranging the digits to get these totals, but the same basic rules for placing them apply. (See the answers for questions **4** and **5.**)

Activity Two

1. 531 is the largest answer possible:
   987 – 456 = 531

2. Answers will vary.

3. A possible set of instructions is: Choose the 3 largest digits and make the biggest number you can by putting the biggest digit in the hundreds place and the smallest of the 3 digits in the ones place. Take the 3 smallest digits and make the smallest number you can by putting the smallest digit in the hundreds place and the largest of the 3 digits in the ones place. Subtract the small number from the large number.

4. No. The smallest possible answer is 14, given by the subtraction 712 – 698.

Investigation

1. Yes. The smallest possible 3-digit numbers you could add together are 100 + 100 = 200.

2. 1 998. (999 + 999)

Page 11: Horsing Around

Activity

1. Answers may vary. Marama could adjust the numbers proportionally. She knows that 60 is 10 times more than the 6 in 6 x 20, so she could balance the equation by making the number in the box 10 times less than 20. (20 ÷ 10 = $\boxed{2}$.) It's the same idea as halving and doubling, but she is multiplying and dividing by 10 instead of by 2. So 6 x 20 = 60 x $\boxed{2}$.

2. She could multiply and divide by 6:
   10 x **6** = 60; 18 ÷ **6** = $\boxed{3}$.
   So 10 x 18 = 60 x $\boxed{3}$.

3. You could multiply and divide by 4:
   15 x **4** = 60; 16 ÷ **4** = $\boxed{4}$. (Or you could double and halve repeatedly: 15 x 16 = 30 x 8
   = 60 x $\boxed{4}$)

4. a. The horses will get 8 carrots each. (Multiply by 2 and divide by 2 [double and halve]:
   30 x 16 = 60 x ❑. 30 x **2** = 60.
   16 ÷ **2** = $\boxed{8}$. So 30 x 16 = 60 x $\boxed{8}$.)

   b. Answers will vary. You could work out 30 bags x 16 carrots using one of the strategies below to find out how many carrots there are altogether and then divide that by 60 horses to find out how many carrots each horse would get. (Note that with Marama's strategy, you don't have to work out how much food there is altogether, so her strategy is more efficient than those shown below.)

   To work out 30 x 16:
   - you could use doubling and halving repeatedly: 30 x 16 = 60 x 8
     = 120 x 4
     = 240 x 2
     = 480
   - you could use place value partitioning:
     30 x 16 = (30 x 10) + (30 x 6)
     = 300 + 180
     = 480
   - you could multiply by a tidy number and compensate:
     30 x 16 = (30 x 20) – (30 x 4)
     = 600 – 120
     = 480.

   To work out 480 ÷ 60:
   - you could subtract 60 repeatedly (remembering how many times you did it): 480 – 60 = 420,
     420 – 60 = 360 … and so on
   - you could use a fact you know:
     I know 48 ÷ 6 = 8, so 480 ÷ 60 is 80.

5. a.–b. Problems will vary.

Pages 12–13: Perfect Lasagne

Activity

1. a. Yes. The recipe is for 6 people. To make it a recipe for 30 people, each ingredient needs to be multiplied by 5 because 6 x 5 = 30.

   b.

	6 people	30 people
Pasta	240 g	1.2 kg (240 g x 5 = 1 200 g or 1.2 kg)
Mince	1.2 kg	6 kg (1.2 kg x 5 = 6 kg)

2. a. To find out how much pasta would be needed for 1 person. 240 g ÷ 6 = 40 g each.

   b. Liam needs to multiply the amount of pasta for 1 person by 30, that is,
   40 g x 30 people = 1 200 g.
   If Liam finds it difficult to multiply by 30, he could multiply by 3 and then by 10.

   c. 1.2 kg ÷ 6 people = 0.2 kg each.
   0.2 kg x 30 people = 6 kg.

3. Methods may vary. For example:

	Ara	Liam
Pasta	30 x 5 = 150 1 200 g x 5 = 6 000 g (6 kg) Or: 240 g x 25 = 60 x 100 = 6 000 g (6 kg)	240 g ÷ 6 = 40 g 40 g x 150 = 6 000 g (6 kg)
Mince	30 x 5 = 150 6 kg x 5 = 30 kg Or: 1 200 x 25 = 30 x 100 = 3 000 (30 kg)	1.2 kg ÷ 6 = 0.2 kg 0.2 kg x 150 = 30 kg

4. Answers will vary. Both strategies work, but Ara's avoids decimal fractions, so the mental maths is probably easier.

5. a. i. 1 050 g (1.05 kg) for 10 people

   ii. 6 300 g (6.3 kg) for 60 people

b. Liam's strategy would be the most straightforward for 10 people. He would find out how much tomato 1 person would need (630 g ÷ 6 = 105 g each) and then multiply this by 10.

Ara would need to work out that $6 \times 1\frac{2}{3} = 10$ people and then multiply 630 g by $1\frac{2}{3}$.

Ara's strategy would be the most straightforward for 60 people: she would multiply 630 g x 10 because 6 x 10 = 60. If you used Liam's strategy, it would still work, but you'd need to multiply 105 g x 60 people, which is more difficult than Ara's calculation.

Investigation

Recipes and adaptations will vary.

Pages 14–15: Token Gesture

Activity

1. a. $12.80. (32 x 40c = 1 280c, which is $12.80)
   b. $11.20. (28 x 40c = 1 120c, which is $11.20)
   c. $22.00. (55 x 40c = 2 200c, which is $22.00)
   d. $5.60. (14 x 40c = 560c, which is $5.60)
2. a. Mere could be dividing the number of tokens she has collected by 5 and working out the answer with a remainder if it has one, for example, for 19 tokens: 19 ÷ 5 = 3 r4. To work out the bonus, Mere must ignore the remainder, because those 4 tokens are not enough to get her another bonus dollar. So for 19 tokens, she would get a $3 bonus.

   Mere could work out whether the number of tokens is a multiple of 5. If it is, the bonus can be worked out by dividing the number of tokens by 5, for example, 20 ÷ 5 = $4 bonus. If it's not a multiple of 5, Mere could find the multiple of 5 that is just before the number of tokens and divide that by 5 instead. For example, for 19 tokens: 19 isn't a multiple of 5, but the multiple of 5 just before 19 is 15. 15 ÷ 5 = $3 bonus.

   b. i. $4. (23 ÷ 5 = 4 r3, so the bonus is still based on 20 tokens.)
   ii. $7. (37 ÷ 5 = 7 r2, so the bonus is $7, based on 35 tokens.)
   iii. $9. (48 ÷ 5 = 9 r3, so the bonus is $9, based on 45 tokens.)
3.

Total number of tokens collected	Money I make (including bonuses)
1	$0.40 (1 x 40c)
5	$3.00 (5 x 40c + $1)
9	$4.60 (9 x 40c + $1)
10	$6.00 (10 x 40c + $2)
12	$6.80 (12 x 40c + $2)
15	$9.00 (15 x 40c + $3)
20	$12.00 (20 x 40c + $4)
23	$13.20 (23 x 40c + $4)
25	$15.00 (25 x 40c + $5)
100	$60.00 (100 x 40c + $20)

4. a. Answers may vary. Mere will lose out on possible bonuses each time she sends in tokens that aren't multiples of 5. As long as she sends in multiples of 5, she won't lose out on any bonuses, regardless of when in March she sends them in. If she waits until the end of the month, she will only miss out on 1 bonus at the most (if she has a set of less than 5 left over).
   b. Answers may vary. One method is:
   153 x 0.40 = $61.20 (or 153 x 40c = 6 120c, which is $61.20) plus the bonus.
   153 ÷ 5 = 30 r3, so the bonus is $30.
   61.20 + 30 = $91.20 raised
5. a. 115 tokens
   b. Strategies will vary. One strategy is working from known quantities. You could perhaps start by estimating that it will be a bit more than 100 tokens because this would earn $60, according to the table in question **3**. The table also tells you that 15 tokens would earn $9. $60 + $9 = $69, and so the number of tokens must be 100 + 15 = 115. Another strategy is to start from the fact that Rewi needs 5 tokens for every $3 he earns. There are 23 lots of $3 in $69 (69 ÷ 3 = 23), so there would be 23 lots of 5 tokens;
   23 x 5 = 115.

6. Explanations will vary. The Serious Cereals tokens are a better deal if you can complete lots of 10 because for every 10 tokens collected, they pay \$6.50 (that is, 10 x 25c + \$4 bonus), while Breakfast Bonanza pays only \$6 (that is, 10 x 40c + \$2 bonus). However, it is not until you collect over 50 tokens that the Serious Cereals tokens will always give you a better deal. If you have collected 1–9, 14–19, 25–29, 35–39, or 47–49 tokens, Breakfast Bonanza would give you more money than Serious Cereals.

Pages 16–17: Digital Dilemmas

Activity

1. a. They need to find a solution for 3 000 ÷ 125 = ❑. One way to solve this is to reverse the problem, making it multiplication, and then perhaps use a doubling strategy: ❑ x 125 = 3 000. 2 x 125 = 250, 4 x 125 = 500, 8 x 125 = 1 000. Then they could triple both sides to make (3 x 8) x 125 = 3 x 1 000 or 24 x 125 = 3 000. 3 000 ÷ 125 = 24, which means that 24 standard photos could be stored on the camera.

   b. They need to find a solution for 3 000 ÷ 375 = ❑. One way to solve this is to reverse the problem to make it a multiplication: ❑ x 375 = 3 000. Tidy numbers might help them estimate, and if they realise that 8 x 400 = 3 200, they might try 8 as the solution. 8 x 375 = 3 000. Another way is to double repeatedly: 2 x 375 = 750; 4 x 375 = 1 500. 1 500 + 1 500 = 3 000, so 8 x 375 = 3 000, which means that 8 extra-wide photos would fit on the camera. Or they may realise that 375 is 3 x 125 kb. So $\frac{1}{3}$ of the 24 standard photos is 24 ÷ 3 = 8.

2. Explanations will vary. The basic equation is 2 625 ÷ 125 = ❑. Melanie may have known that every extra-wide photo taken uses up the space needed by 3 standard photos (125 x 3 = 375) and that 24 standard photos could fit altogether if there were no extra-wide ones, so 24 – 3 = 21.

3. a. i.

Extra-wide photos (375 kb)		
Number of photos	Memory used (kb)	Memory remaining (kb)
0	0	3 000
1	375	2 625
2	750	2 250
3	1 125	1 875
4	1 500	1 500
5	1 875	1 125
6	2 250	750
7	2 625	375
8	3 000	0

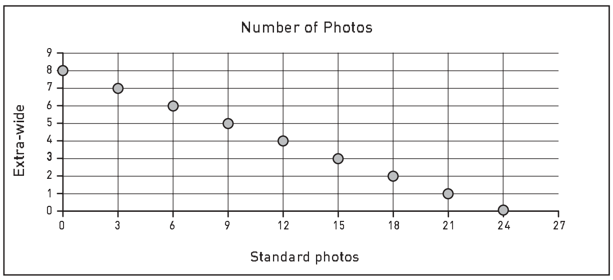
ii.

Standard photos (125 kb)		
Number of photos	Memory used (kb)	Memory remaining (kb)
24	3 000	0
21	2 625	375
18	2 250	750
15	1 875	1 125
12	1 500	1 500
9	1 125	1 875
6	750	2 250
3	375	2 625
0	0	3 000

   b. Discussion will vary. Note the reverse order of the second table and the way each row links to its matching row in the other table.

   c. 12 photos (6 of each size)

4. a.



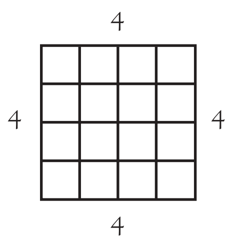
b. Every time Jake or Melanie take an extra-wide photo, the number of standard photos they can take decreases by 3.

Pages 18–19: Fenced In

Activity

1. Sam's run has a bigger area for the puppy to run in: his run is 15 m^2 (5 x 3) compared with Dad's 12 m^2 (6 x 2).

2. **a.** Practical activity. The largest run possible has an area of 16 m^2:



b. The largest rectangle is actually a square (all the sides are the same length).

c. A square. It would have sides of 3 m, and its area would be 9 m^2.

3. **a.** He might have gone 40 ÷ 4 = 10 and 10 x 10 = 100.

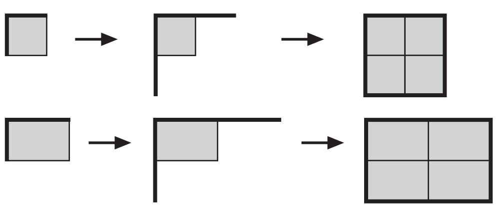
b. Yes.

4. **a.** The area gets 4 times bigger. (8 x 8 = 64; 64 = 16 x 4)

b. Yes, it always happens. A possible table is:

Original run			Bigger run		
Length (m)	Width (m)	Area (m^2)	Length (m)	Width (m)	Area (m^2)
4	4	16	8	8	64
3	5	15	6	10	60
2	6	12	4	12	48
3	6	18	6	12	72
4	3	12	8	6	48

c. When the sides of a rectangle are made twice as long, you can fit 4 of the original rectangles in the new rectangle. Possible diagrams include:



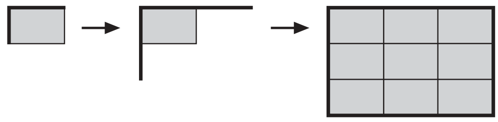
5. **a.** The area would get 4 times smaller.

b. Practical activity. Lengths used will vary. Some examples are:

Original run			Bigger run		
Length (m)	Width (m)	Area (m^2)	Length (m)	Width (m)	Area (m^2)
4	4	16	2	2	4
6	2	12	3	1	3
10	4	40	5	2	10

Investigation

a. It gets 9 times bigger (see diagram).



b. It gets 9 times smaller.

Page 20: Slide Shows

Activity

1.

Number of photos in the slide show	Time taken (seconds)
1	8
2	8 + 2 + 8 = 18
3	8 + 2 + 8 + 2 + 8 = 28
4	8 + 2 + 8 + 2 + 8 + 2 + 8 = 38
5	8 + 2 + 8 + 2 + 8 + 2 + 8 + 2 + 8 = 48

2. **a.** It would take 88 s. Possible short cuts include:
(9 x 8) + (8 x 2), or (8 x 10) + 8,
or (9 x 10) – 2.

b. 998 s, which is 16 min and 38 s. This could be worked out using one of the short cuts from **2a**:
(100 x 8) + (99 x 2) = 800 + 198
= 998
(99 x 10) + 8 = 990 + 8
= 998
(100 x 10) – 2 = 1 000 – 2
= 998

c. 8 photos. One way to work this out is:
1 min 18 s = 78 s. Each photo takes 8 + 2 = 10 s to show with its transition, apart from the last one, which only needs 8 s. 78 is made up of 7 groups of 10 and 1 group of 8, so there must be 8 photos. Another way is: 78 + 2 = 80; 80 ÷ 10 = 8.

3. a. Generalisations will vary, for example:
   - Multiply the number of photos by 15 and subtract 3.
   - Work out what 1 less than the number of photos is, multiply this number by 15, and then add on another 12.
   - Multiply the number of photos by 12, multiply 1 less than the number of photos by 3, and then add these two answers together.

   b. 20 photos. One way to work this out is: Each photo takes 12 + 3 = 15 s to show with its transition, apart from the last one, which only needs 12 s. 297 is made up of 19 lots of 15 and 1 lot of 12, so there must be 19 + 1 = 20 photos. (Use the fact that 2 x 15 = 30 to work out that 20 x 15 = 300 and adjust from there.) Another way is: 297 + 3 = 300; 300 ÷ 15 = 20.

Page 21: Enough Rice?

Activity

1. a. 12 days. (There are now double the people, so the sack of rice will last half as long. 24 ÷ 2 = 12)

   b. 48 days. (There are now half the original number of people to feed, so the sack will last twice as long. 24 x 2 = 48)

   c. 96 days. (You could double the amount of time it will last 2 people: 2 people for 48 days, 1 person for 96 days. Or you could notice that the sack will last 4 times longer for 1 person than for 4 people. 24 x 4 = 96)

2. a.–b.

How many whole days will a full sack of rice last?								
Number of people	1	2	3	4	5	6	7	8
Number of days	96	48	32	24	19	16	13	12

Methods will vary. One strategy is to work from the facts you know and adjust the numbers proportionally. For example, if you know that the sack will last 2 people 48 days, you could multiply the number of people by 3 and divide the number of days by 3 to work out that it would last 6 people for 16 days (2 x 3 = 6, 48 ÷ 3 = 16). Once you know that the sack feeds 6 people for 16 days, you could halve and double to work out that it would last 3 people for 32 days (6 ÷ 2 = 3, 16 x 2 = 32).

Working out how many days the sack will last for 5 and 7 people is harder because 5 and 7 don't relate easily to any of the facts you already know. You could work from knowing that the rice will last 1 person for 96 days and multiply by 5 and divide by 5 to work out how long it would last for 5 people: 1 x 5 = 5, 96 ÷ 5 = 19.2, which is 19 whole days, with a bit left over. Then multiply by 7 and divide by 7 to work out how long it would last for 7 people: 1 x 7 = 7, 96 ÷ 7 = 13.714285 (using a calculator), so the rice would last 7 people for 13 whole days, and there would be some left over, but not quite enough for the 14th day.

   c. 4.8 days. Methods will vary. If you worked from knowing that the rice will last 1 person for 96 days, you could multiply by 20 and divide by 20 to work out how long it would last 20 people. 1 x 20 = 20, 96 ÷ 20 = 4.8, so it would last 4 whole days, and there would be some left over but not quite enough for the 5th day. You could also work from knowing that it will last 4 people for 24 days and multiply by 5 and divide by 5 to work out how long it would last for 20 people. 4 x 5 = 20, 24 ÷ 5 = 4.8, that is, 4 whole days with some left over.

3. About 96 people. (If the sack lasts 1 person for 96 days, this means that each day they eat 1 ninety-sixth [$\frac{1}{96}$] of the sack. To use a whole sack in 1 day, 96 people would need to eat 96 ninety-sixths [$\frac{96}{96}$] in 1 day.)

4. a.–b. Problems will vary.

Pages 22–23: On Top of the World

Activity

1. a. $\frac{1}{12}$

   b. $\frac{1}{48}$

2. a. 8

   b. $\frac{1}{4}$

3. a.

Kere's table	
Aoraki	$1\frac{1}{2}$ or $\frac{3}{2}$
Taranaki	1
Sky Tower	$\frac{1}{8}$
The Beehive	$\frac{1}{32}$

b.

Manu's table	
Aoraki	48
Taranaki	32
Sky Tower	4
The Beehive	1

4. They are all correct because the numbers are comparing the height of Taranaki with that of different landmarks. The other landmarks are all different heights, so the comparisons are different fractional numbers (32 and 8 are also fractional numbers: in this case, for Taranaki in comparison to the other landmarks, 32 as a fraction is $\frac{32}{48}$ and 8 is $\frac{8}{12}$. A fractional number is a number used to compare a subset and its set or a part to the whole.)

Page 24: Decimal Fraction Buddies

Game

A game for finding decimal fraction addition pairs

Activity

1. Games will vary.
2. Answers will vary. Strategies could be similar to these:
   - I know that there are 5 tenths in 0.5, so I found pairs of tenths that add up to 5 tenths, such as 2 tenths + 3 tenths = 5 tenths (0.2 + 0.3 = 0.5).
   - I know that 5 tenths is equivalent to 50 hundredths, so I found pairs of hundredths that add up to 50 hundredths, such as 43 hundredths + 7 hundredths = 50 hundredths (0.43 + 0.07 = 0.5).
   - I know that 5 tenths is equivalent to 500 thousandths, so I found pairs of thousandths that add up to 500 thousandths, such as 485 thousandths + 15 thousandths = 500 thousandths (0.485 + 0.15 = 0.5).

Figure It Out

Number Sense and Algebraic Thinking Teachers' Notes

Overview of: Level 3: Book Two

Title	Content	Page in students' book	Page in teachers' book
Triple Trouble	Exploring the impact of context on answers to division	1	15
Maths Detective	Comparing addition and subtraction strategies	2–4	16
Alien Estimates	Making estimations in addition	5	20
Pizza Split	Investigating number properties in division	6–7	21
Digit Shuffle	Exploring the effects of operations and place value	8–10	23
Horsing Around	Finding and using number patterns in multiplication	11	24
Perfect Lasagne	Making decisions about operations and strategies	12–13	26
Token Gesture	Finding and using generalisations to predict number patterns	14–15	27
Digital Dilemmas	Describing patterns and relationships	16–17	30
Fenced In	Investigating relationships between length and area	18–19	32
Slide Shows	Finding and using generalisations for number patterns	20	34
Enough Rice?	Describing patterns and relationships	21	36
On Top of the World	Exploring relative values	22–23	38
Decimal Fraction Buddies	Finding decimal fraction addition pairs	24	39

Introduction to Number Sense and Algebraic Thinking

The *Number Sense and Algebraic Thinking* books in the Figure It Out series provide teachers with material to support them in developing these two key abilities with their students. The books are companion resources to Book 8 in the Numeracy Project series: *Teaching Number Sense and Algebraic Thinking*.

Number sense

Number sense involves the intelligent application of number knowledge and strategies to a broad range of contexts. Therefore, developing students' number sense is about helping them gain an understanding of numbers and operations and of how to apply them flexibly and appropriately in a range of situations. Number sense skills include estimating, using mental strategies, recognising the reasonableness of answers, and using benchmarks. Students with good number sense can choose the best strategy for solving a problem and communicate their strategies and solutions to others.

The teaching of number sense has become increasingly important worldwide. This emphasis has been motivated by a number of factors. Firstly, traditional approaches to teaching number have focused on preparing students to be reliable human calculators. This has frequently resulted in their having the ability to calculate answers without gaining any real understanding of the concepts behind the calculations.

Secondly, technologies – particularly calculators and computers – have changed the face of calculation. Now that machines in society can calculate everything from supermarket change to bank balances, the emphasis on calculation has changed. In order to make the most of these technologies, students need to develop efficient mental strategies, understand which operations to use, and have good estimation skills that help them to recognise the appropriateness of answers.

Thirdly, students are being educated in an environment that is rich in information. Students need to develop number skills that will help them make sense of this information. Interpreting information in a range of representations is critical to making effective decisions throughout one's life, from arranging mortgages to planning trips.

Algebraic thinking

Although some argue that algebra only begins when a set of symbols stands for an object or situation, there is a growing consensus that the ideas of algebra have a place at every level of the mathematics curriculum and that the foundations for symbolic algebra lie in students' understanding of arithmetic. Good understanding of arithmetic requires much more than the ability to get answers quickly and accurately, important as this is. Finding patterns in the process of arithmetic is as important as finding answers.

The term "algebraic thinking" refers to reasoning that involves making generalisations or finding patterns that apply to *all* examples of a given set of numbers and/or an arithmetic operation. For example, students might investigate adding, subtracting, and multiplying odd and even numbers. This activity would involve algebraic thinking at the point where students discover and describe patterns such as "If you add two odd numbers, the answer is always even." This pattern applies to *all* odd numbers, so it is a generalisation.

Students make these generalisations through the process of problem solving, which allows them to connect ideas and to apply number properties to other related problems. You can promote process-oriented learning by discussing the mental strategies that your students are using to solve problems. This discussion has two important functions: it gives you a window into your students' thinking, and it effectively changes the focus of problem solving from the outcome to the process.

Although the term "algebraic thinking" suggests that generalisations could be expressed using algebraic symbols, these Figure It Out *Number Sense and Algebraic Thinking* books (which are aimed at levels 2–3, 3, and 3–4) seldom use such symbols. Symbolic expression needs to be developed cautiously with students as a sequel to helping them recognise patterns and describe them in words. For example, students must first realise and be able to explain that moving objects from one set to another does not change the total number in the two sets before they can learn to write the generalisation $a + b = (a + n) + (b - n)$, where n is the number of objects that are moved. There is scope in the books to develop algebraic notation if you think your students are ready for it.

The Figure It Out *Number Sense and Algebraic Thinking* books

The learning experiences in these books attempt to capture the key principles of sense-making and generalisation. The contexts used vary from everyday situations to the imaginary and from problems that are exclusively number based to those that use geometry, measurement, and statistics as vehicles for number work. Teachers' notes are provided to help you to extend the ideas contained in the activities and to provide guidance to your students in developing their number sense and algebraic thinking.

There are six *Number Sense and Algebraic Thinking* books in this series:
Levels 2–3 (Book One)
Levels 2–3 (Book Two)
Level 3 (Book One)
Level 3 (Book Two)
Levels 3–4 (Book One)
Levels 3–4 (Book Two)

Page 1: Triple Trouble

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- state the general rule for a set of similar practical problems (Algebra, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to help your students to extend their advanced multiplicative part–whole strategies (stage 7) and advanced proportional strategies (stage 8) in the domain of multiplication and division.

Activity

This activity encourages students to think about how the remainder in a division problem can be treated differently according to the context. Students need to be at least advanced multiplicative thinkers (stage 7) to do this activity independently. However, advanced additive students can learn more about division by attempting the activity. They need to be familiar with fractions and decimal fractions, including how to round a decimal fraction to 1 or 2 decimal places.

You may need to define the term "decimal fraction", more commonly called a decimal. The term refers to a numeral that uses base 10 place values, such as tenths and hundredths, and a decimal point to name a fraction.

With a guided teaching group, you could present question **1** as a practical activity, particularly if the students are not yet confident about using stage 7 strategies in division. Give each pair in the group one of the problems to solve using equipment:

- For **a**: 17 "chocolate bars", each made of 3 (or 6) multilink cubes
- For **b**: 17 counters to represent the people and 3 pieces of paper to represent the tables. (This pair could also do problem **d**, which will not take long.)
- For **c**: $17 in toy money. (You will also need to have coins available for them to make exchanges.)
- For **d**: a calculator
- For **e**: a 17-centimetre piece of card to fold, a ruler, and some scissors.

Ask each pair to report back their findings and to demonstrate to the others what they did with their equipment to solve the problem. Record the different answers each pair came up with, for example, 5.7 centimetres or $5.65 with 5 cents left over.

Get the students to look at the box of answers in the student book. Ask the students questions such as:

Which answer in the list is best for the task that you did? Give reasons for your choice.
Are there any answers in the list that are suitable for more than one problem?

You can use question **2** to ask questions that promote algebraic thinking. All the answers in question **1** are ways of handling 17 ÷ 3, but whether they are "correct" depends on the context of the problem. For example, $5\frac{2}{3}$ is the result of dividing 17 by 3. When we attempt to write this as a decimal fraction, we get $5.66\dot{6}$. We can't work with an infinite string of 6s, so we approximate. The way we approximate always depends on the circumstances. We use only as many decimal places as we want to or can. Sometimes it isn't appropriate to use fractions or decimal fractions because the items (for example, people) cannot or should not be shared into parts. In such cases, we round the answer up or down to the nearest whole number.

Question **3** is a useful formative assessment point to see whether the students understand which contexts are suitable for whole number answers and which ones require 1 or 2 decimal places. Remind the students that the calculation could involve decimal fractions that have to be rounded, such as 13 ÷ 3 or 22 ÷ 7, or finite decimals, such as 19 ÷ 4 = 4.75. The students may need to use a calculator to help them find a problem that has an answer with many decimal places.

If the students are having difficulty thinking of contexts, ask the group to brainstorm things that shouldn't be broken up into pieces, such as people, animals, cars, or items of clothing, and things that you can measure to 1 or 2 decimal places, such as people's height in metres, distance travelled in kilometres, dimensions of a sheet of paper in centimetres, weight of food in kilograms, or liquid in litres.

Extension

Have the students investigate the Swedish rounding system used in supermarkets. They could also design a flow chart that explains the decisions involved in rounding a decimal fraction to 2 decimal places. Question starters for the flow chart could include:

Does your decimal fraction have more than 2 decimal places?

Is the number in the hundredths place greater than or equal to 5?

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)

- *Book 4: Teaching Number Knowledge*
  Swedish Rounding, page 28
  Sensible Rounding, page 28
  Dividing? Think about Multiplying First, page 37
- *Book 6: Teaching Multiplication and Division*
  Remainders (division problems with remainders), page 32.

Pages 2–4: Maths Detective

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- state the general rule for a set of similar practical problems (Algebra, level 3)
- record, in an organised way, and talk about the results of mathematical exploration (Mathematical Processes, communicating mathematical ideas, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to help the students to extend their advanced additive part–whole strategies (stage 6) in addition and subtraction.

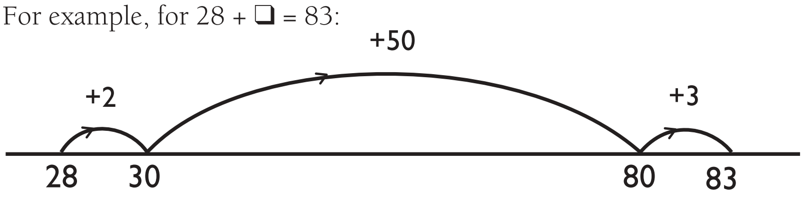
Activity

This activity is designed to encourage students to choose from a broad range of mental strategies by exploring how a subtraction problem can be solved in at least six different ways. It also gives students opportunities to compare the efficiency of strategies when they are applied to different problems, to interpret various recording techniques, and to be exposed to ways of describing solution paths.

This activity would be a useful follow-up for students who have just learned a range of additive strategies because it encourages them to explore the concept of "efficiency" and when each strategy might be useful. Note that for many calculations, several different strategies can be equally efficient.

Students need to be at least advanced additive part–whole thinkers (stage 6) to do this activity. They need to be familiar with the following part–whole strategies:

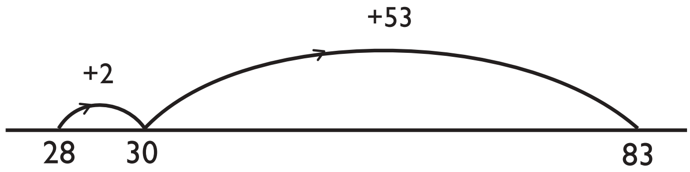
- Adding on by jumping to tidy numbers on a number line.
  For example, for 28 + ❑ = 83:



2 + 50 + 3 = $\boxed{55}$

28 + 2 = 30, 30 + 50 = 80, 80 + 3 = 83, 2 + 50 + 3 = 55

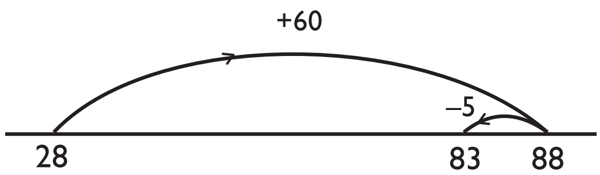
or in just two jumps:



2 + 53 = $\boxed{55}$

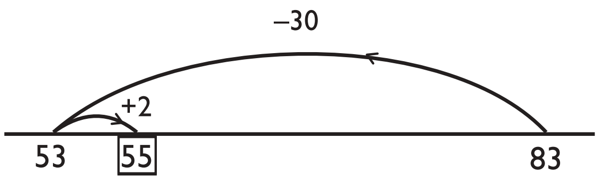
28 + 2 = 30, 30 + 53 = 83, 2 + 53 = 55

- Adding or subtracting tidy numbers and then compensating.
  For example, for 28 + ❑ = 83:



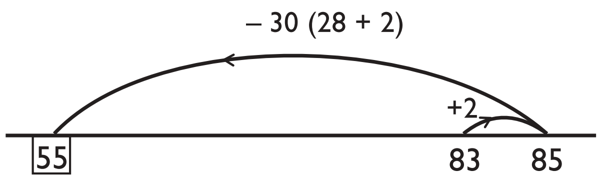
28 + 60 = 88, 88 – 5 = 83

or for 83 – 28 = ❑:



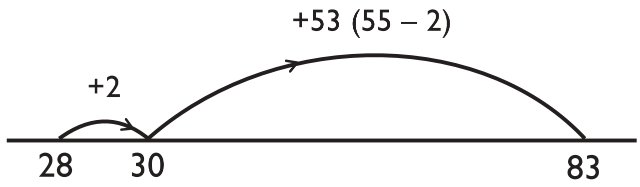
83 – 30 = 53, 53 + 2 = 55

- Making equal adjustments to give a tidy number that's easier to work with.
  For example, for 83 – 28 = ❑:



Add 2 to 83 and 28 to make 28 a tidy number: 85 – 30 = 55.

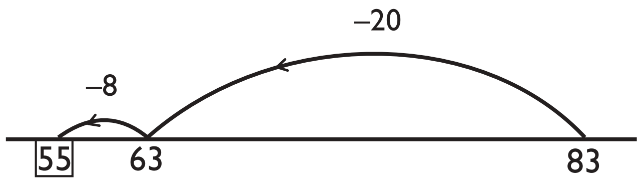
- Re-partitioning to make one number a tidy number.
  For example, for 28 + 55 = ❑:



28 + 55 = $\boxed{83}$

Take 2 from the 55 and add it to the 28 to make a tidy number → 30 + 53 = 83

- Using place value partitioning to break up the numbers.
  For example, for 83 – 28 = ❑:



83 – 20 = 63, 63 – 8 = 55 (or 83 – 20 = 63, 63 – 3 = 60, 60 – 5 = 55)

Another possibility is 80 – 20 = 60, 3 – 8 = $^{-}5$, 60 – 5 = 55. Note the significance here of going "back through a 10" to solve 63 – 8.

- Reversing a problem to change it from subtraction to addition or vice versa.
  For example: 83 – 28 = ❑ → 28 + ❑ = 83.

Students working independently could discuss parts of this activity in pairs or groups of 3–4. Before they start the activity, you could ask:
Why is it a good idea to know lots of different strategies for solving problems? (Although one strategy might work for lots of problems, other strategies will sometimes be better. If you only use one strategy all the time, you might be taking longer than you need to or finding the mental calculations harder than you should. Also, it might be useful to use a different strategy to check an answer.)
When a strategy is "efficient", what does that mean? (It has fewer steps than other strategies and you don't waste your time doing extra working out.)

You may need to define these terms:
- Adjust: change a number by adding or subtracting a bit.
- Compensate: make a further change to cancel the effect of an earlier one.
- Place value: the value of the place each digit occupies. (For example, the 5 in 57 is in the tens place.)
- Tidy number: a number that makes the calculation easy and is close to a number in a problem. Usually, in addition and subtraction, we use numbers like 50, 90, 300, and 4 000.

With a guided teaching group, ask the students to solve Mrs Coey's problem (579 – 288 = ❑) and to record how they solved it using number sentences or an empty number line. It's a good idea to have place value materials accessible in case a particular strategy needs clarification or validation.

Encourage comparisons by asking questions such as:
What clues did you use to match the students to their strategies?
Which ones were the easiest to match? Which ones were the most difficult? Why?
Did the students in Mrs Coey's class come up with any strategies that were different from ours?
Did we use any strategies that they didn't?

Focus the students on recording methods:
When you're recording your solution strategy on a number line, how can you make it easy for someone else to understand? Use your own ideas or ideas you saw from someone in your group or from the students in the book.

This activity gives students lots of opportunities to develop appropriate language to talk about their problem-solving strategies and to communicate their thinking clearly. It also helps them to come to a shared understanding of terminology such as "adding on", "using place value", "using a tidy number and compensating", or "making equal adjustments".

The students could make a "strategy toolbox" display by writing the name of each strategy onto a picture of a different tool. The metaphor of a toolbox may remind the students that particular strategies are likely to be suited to particular problems, even though they might be able to solve many problems with just one strategy. (You could probably use any of the tools in a toolbox to bang in a nail, but a hammer will be most efficient.) Keep the "toolbox" as a reference on the wall and use it to encourage the students to choose their strategy before they solve a problem and to justify that choice.

Question **6** is a useful formative assessment point to judge whether the students can explain why they've chosen particular numbers to suit certain strategies.

Some students are reluctant to use a variety of strategies and prefer to use one trusted method, even if it is less efficient. Try to give the students more practice in trying different strategies. Use phrases such as: *Let's all use Kere's method to solve this next problem.*

One way to encourage them to diversify their strategies is to deliberately choose numbers for problems that would be cumbersome for their preferred method but very quick using a different strategy. For example, 803 – 497 is easily solved using tidy numbers (803 – 500 + 3) or reversing (497 + 3 + 303).

Extension
In pairs, get the students to make a Wanted poster or to write a newspaper Situations Vacant ad that describes the characteristics of problems that can be solved efficiently using a particular strategy. Starters might include:
- Wanted! Problems that can be solved by making equal adjustments!
- We are looking for …
- These numbers can be recognised by …

These can be displayed for students to use as a reference to remind them to choose their strategies wisely.

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)
- *Book 5: Teaching Addition, Subtraction, and Place Value*
  Jumping the Number Line (adding on to tidy numbers), page 33
  Don't Subtract – Add! (reversing), page 34
  Problems like 67 – ❑ = 34 (reversing), page 42
  Problems like 23 + ❑ = 71 (adding a tidy number and compensating), page 35
  Problems like 73 – 19 = ❑ (subtracting a tidy number and compensating), page 38
  When One Number Is Near a Hundred (equal adjustments in addition), page 37
  Equal Additions (equal adjustments in subtraction), page 38
- *Book 8: Teaching Number Sense and Algebraic Thinking*
  Reversing Addition, page 7
  Subtraction to Subtraction, page 8
  When Subtraction Becomes Addition, page 8
  6 Minus 8 Does Work! (subtracting using negative numbers), page 31.

Page 5: Alien Estimates

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 3)
- explain the meaning of the digits in any whole number (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- encourage transition from early additive part–whole strategies (stage 5) to advanced additive part–whole strategies (stage 6)
- help the students to extend their advanced additive part–whole strategies (stage 6) in the domain of addition and subtraction.

Activity

This activity encourages students to use place value knowledge and additive part–whole strategies to estimate the results of calculations.

Students need to know numbers up to the hundreds of thousands to complete this activity. They need to be familiar with rounding numbers to the closest tidy number, for example, rounding 82 145 to the nearest tidy number in the thousands would give 82 000.

Some students may have problems with place value if, for example, they don't know that for 82 000 + 45 000, they can calculate 82 + 45 in the thousands place. Once they realise they can calculate in this way, you will need to ensure that they don't treat as ones units the numbers in the hundreds, thousands, ten thousands places, and so on. Place value houses are a useful way of clarifying any misconceptions that the students may have. (See The Power of Powers, pages 14–15, in *Number Sense and Algebraic Thinking: Book One*, Figure It Out, Level 3.)

This activity would be useful as an independent follow-up for a group that has just focused on rounding whole numbers to the closest tidy number in the tens, hundreds, or thousands.

With a guided teaching group, set the scene by asking about the students' favourite computer or games-machine programmes and whether they've ever played a multi-person computer game.

Some students may be reluctant to estimate the answers in question **1** and may want to work out the calculation exactly. Encourage them to focus on what Rongomai has explained in her speech bubble rather than starting with the boxes of scores. Ask questions such as:
Where has Rongomai got the numbers 82 and 46 from? (The 82 is really 82 000, which is Ben's score of 82 145 rounded down to the nearest tidy number in the thousands. The 46 refers to 46 000, which is Eseta's score of 45 877 rounded up to the nearest tidy number in the thousands.)
Why do you think Rongomai decided to round the scores to the nearest thousand? (She wanted to make them into numbers she could add easily in her head, such as 82 000 + 46 000, but still be pretty accurate. If she'd rounded to the nearest tens of thousands, her estimation wouldn't have been as close to the actual total [80 000 + 50 000 = 130 000].)
What strategy might she have used to work out 82 + 46? How would you do it? (Rongomai could have used place value partitioning [80 + 40 = 120, 2 + 6 = 8, then 120 + 8 = 128] or added a tidy number and compensated [82 + 50 = 132, 132 – 4 = 128] or made equal adjustments [82 + 46 = 80 + 48 = 128].)

For question **2**, before the students attempt to calculate the totals in round 2, ask: *How would you round each person's score if you were using Rongomai's strategy?*

For more estimating practice, the students could make up their own scores for round 3, follow the "deliberate mistake" step in question **2a ii**, and swap with a classmate.

You could use questions **2b** and **3** for formative assessment if the students are able to articulate the decisions they make when estimating.

Students who have difficulty rounding numbers to the nearest thousand may benefit from using an empty number line to help them visualise the distance of a number from its tidy number neighbours.

For a number like 69 034, tell the students that they're trying to find out which tidy number in the thousands it is closest to. Start your number line at 60 000 and get the students to count by saying the number words with you as you mark 61 000, 62 000, 63 000, … up to 70 000 on it.

Then ask: *Which two tidy numbers on the number line will the number 69 034 be between?* (Circle 69 000 and 70 000.) *These are its thousands tidy number neighbours. So which tidy number is 69 034 closest to on our number line?*

Encourage the students to use imaging with questions such as:
Which thousands tidy numbers are going to be our number's neighbours?
Which of these two tidy number neighbours is our number closest to?

The students using number properties may start talking in terms of the number of hundreds and how that will tell them that the number is closer to the next 1 000 up or to the 1 000 before.

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)

- *Book 4: Teaching Number Knowledge*
  Place Value Houses (identifying multi-digit numbers up to trillions), page 5
  Swedish Rounding (rounding to the nearest 5 cents), page 28
  Sensible Rounding (according to the context of the problem), page 28
- *Book 8: Teaching Number Sense and Algebraic Thinking*
  Checking Addition and Subtraction by Estimation, page 9
  Whole Number Rounding (rounding to the nearest 1, 10, 100, and 1000), page 19
  Rounding Decimals, page 21.

Pages 6–7: Pizza Split

Achievement Objectives

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- state the general rule for a set of similar practical problems (Algebra, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to help the students extend their part–whole strategies in division and proportions to advanced multiplicative strategies (stage 7).

Activity

Many students attempt to solve problems like 2 ÷ 8 by changing the order of the numbers, as they would in multiplication or addition, and calculating 8 ÷ 2 instead. This activity prompts the students to discover that division is not commutative, that is, they explore examples using a fraction kit and diagrams to confirm that 8 ÷ 2 and 2 ÷ 8 in fact give the inverse of each other ($8 \div 2 = \frac{4}{1}$ and $2 \div 8 = \frac{1}{4}$).

To do this activity independently, the students need to be able to:

- write symbols for fractional amounts, for example, three-quarters as $\frac{3}{4}$ and four-thirds as $\frac{4}{3}$
- make simple equivalent fractions, such as $\frac{1}{4}$ and $\frac{2}{8}$

- divide whole numbers into fractional parts, using materials such as a fraction kit
- change mixed fractions (such as $1\frac{1}{2}$) into top-heavy fractions (such as $\frac{3}{2}$) and vice versa.

With a guided teaching group, give each pair in the group a fraction kit (foam or plastic circles cut into various-sized fractions) so that they can explore the problems using materials that they can manipulate. If you don't have commercial ones available, the students can make their own using the Fraction Pieces material master 4-19 (available at www.nzmaths.co.nz/numeracy/materialmasters.htm). The students can choose the circles they need to solve each problem and then either cut them up to share out or shade each person's share.

Look out for students who interpret Mena's diagram in question **1** as $2 \div 8 = \frac{2}{16}$ because they see 16 pieces of pizza in the diagram with each person getting 2 of them. A fraction is a part of a whole amount, and you need to define what the whole is to make the size of the fraction meaningful. It would be correct to say "2 sixteenths of 2 pizzas", but also ask: *How much of 1 pizza is that?* It's important that the students understand that, in the context, 1 whole refers to the number 1, not to the whole group of pizzas. It is critical that the students understand this re-unitising if they are to progress in fractions. Emphasise this by asking questions such as:
If you ate 1 piece of pizza in Mena's diagram, what fraction of 1 whole pizza would you have eaten? ($\frac{1}{8}$)
If you ate 2 pieces of pizza, what fraction of 1 whole pizza would you have eaten? ($\frac{2}{8}$)
Would you still have eaten $\frac{2}{8}$ *of a pizza if you ate 1 piece from 1 pizza and 1 from the other? Show me.* (If I take $\frac{1}{8}$ from each pizza, it's the same size as if I take $\frac{2}{8}$ from 1 pizza.)
Can you show me what $\frac{1}{16}$ *of 1 whole pizza would look like?* (Half the size of one of the eighth pieces)
What about $\frac{2}{16}$ *of 1 whole pizza?* (The same size as one of the eighth pieces)

You may need to help some students decide which fractions in their kit they should use to solve the problems. Ask questions such as:
When Mena shows 3 pizzas divided among 4 people, her diagram shows the pizzas cut into quarters. How did Mena know to cut her pizzas into quarters? (If you're sharing between 4 people, each person will get a quarter.)
How did Zac know to cut his pizzas into thirds when he showed 4 pizzas divided among 3 people? (If you're sharing between 3 people, each person will get a third.)
What fraction pieces would be useful to choose to show 5 ÷ 2? 2 ÷ 5? (5 ÷ 2: halves because 5 is shared between 2 people; 2 ÷ 5: fifths because 2 is shared between 5 people)

The students will find it difficult to see the inverse pattern unless they write their fractions as common fractions rather than mixed ones. If they have written $4 \div 3 = 1\frac{1}{3}$, ask them to write it as a common fraction as well, that is, $\frac{4}{3}$.

In question **6**, if the students can't make the pattern work, encourage them to write their fractions in their simplest form. For example, in question **1**, Mena works out $2 \div 8$ as $\frac{2}{8}$, but in order to see it as an inverse of $8 \div 2 = \frac{4}{1}$, she would need to write $\frac{2}{8}$ as the equivalent fraction of $\frac{1}{4}$, which she realises in question **5**.

To encourage reflective discussion and generalisation, ask:
Were there any questions in this activity whose answer surprised you? How?
What patterns did you notice? How did you use them to predict the answers? What would you predict as the answer to 5 ÷ 15? How do you know?

A generalised solution to this is to create fifteenths, that is, cut each pizza into fifteenths. Each share will be $\frac{5}{15}$ because $\frac{1}{15}$ will come from each pizza. This generalises the link that $5 \div 15 = \frac{5}{15}$. Note that $5 \div 15$ is an operation or process, while $\frac{5}{15}$ is a number.

Extension

The students could investigate whether the order of the numbers matters in addition, subtraction, and multiplication. They could explain and summarise their findings for all four operations, using a chart or a computer slide-show presentation.

Some students could explore further the ideas that a fraction is a part of a whole amount and that the size of the fractional piece depends on the size of the whole. Ask:
Which is bigger, $\frac{2}{8}$ *of 1 pizza or* $\frac{2}{16}$ *of 2 pizzas?*
Is $\frac{1}{8}$ *of a small pizza the same as* $\frac{1}{8}$ *of a large pizza?*
Can a quarter ever be bigger than a half?

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)

- *Book 4: Teaching Number Knowledge*
  Little Halves and Big Quarters, page 19
  The Same but Different (equivalent fractions), page 31
- *Book 7: Teaching Fractions, Decimals, and Percentages*
  Trains (splitting whole numbers up into fractional parts), page 19
- *Book 8: Teaching Number Sense and Algebraic Thinking*
  To Turn or Not to Turn (changing order in division), page 12
  Equivalent Fractions, page 16
  Fractions Greater Than 1 (converting mixed fractions into common fractions and vice versa), page 17
  Fraction Number Lines (mixed and common fractions), page 18.

Pages 8–10: Digit Shuffle

Achievement Objectives

- explain the meaning of the digits in any whole number (Number, level 3)
- state the general rule for a set of similar practical problems (Algebra, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use these activities to help the students to develop place value knowledge to complement advanced additive part–whole strategies (stage 6).

Activities One and Two

These activities are intended to reinforce the principle of place value: that the value of a digit in a number changes according to the position it is in. Students explore what effect changing the order of the digits has on the solution in addition and subtraction equations.

The students who are not confident using a variety of mental strategies to quickly add and subtract 3-digit numbers could use a calculator so that they can concentrate on the place value. They may also be more willing to experiment with moving their numbers around to create new calculations.

However, they should use the calculator as a backstop rather than automatically reaching for it every time. Encourage them to look first at the numbers in the problem and decide whether they could solve it easily in their heads. For example, **Activity One**, question **2**, asks them to add 623 + 547, which is relatively straightforward when using a place value partitioning strategy such as 600 + 500 = 1 100, 20 + 40 = 60, 3 + 7 = 10, 1 100 + 60 + 10 = 1 170.

In **Activity Two**, question **4**, the numbers should be close enough together for the students to be able to use an adding-on strategy. Even when the calculator is used, the students should use mental strategies to estimate whether the answer shown on the calculator is correct or whether the numbers have been entered incorrectly; just because it's on the calculator screen doesn't guarantee it's right!

The questions that ask the students to write a set of instructions are ideal for promoting algebraic thinking. They are also useful for formative assessment.

Investigation

If the students have difficulties with Ese's investigation at the end of the activity, encourage them to experiment by adding a range of 3-digit numbers together. Ask:
What's the smallest 3-digit number possible?
What's the largest 3-digit number possible?

Some students might be interested in posing their own follow-up investigation question, such as predicting the number of digits in the answer when adding two 4-digit numbers, subtracting a 3-digit number from a 4-digit number, and so on.

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)
- *Book 4: Teaching Number Knowledge*
  Place Value Houses (identifying multi-digit numbers), page 5
  Number Hangman (identifying ones, tens, hundreds, thousands), page 5.

Page 11: Horsing Around

Achievement Objectives
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- state the general rule for a set of similar practical problems (Algebra, level 3)

AC
EA
AA
AM
AP

Number Framework Links
Use this activity to help the students to develop advanced multiplicative part–whole strategies (stage 7) in multiplication of whole numbers.

Activity

In this activity, students are encouraged to apply proportional strategies to multiplication problems to make them easier to calculate.

This would be a useful follow-up activity after you have introduced your students to proportional strategies such as doubling and halving because it shows them that they can multiply and divide by any number that will make the calculation easier. The students will need reasonable recall of their basic multiplication facts to be able to identify patterns readily.

With a guided teaching group, introduce the problem by emphasising that Marama is trying to find out how many items of food should go in each lunch bag without calculating how much food there is altogether. Say: *Marama has discovered a clever trick so that she doesn't have to work out how much food there is altogether, and we want to find out about this quick trick, even though we could solve the problem in other ways.*

For question **1**, help the students to connect the story problem to Marama's equation
6 x 20 = 60 x ❑ by asking questions such as:
Where does the 6 come from in the story problem?
Where does the 20 come from?
Where has the 60 come from?
When we've worked out the number that goes in the box, what will that number tell us?

Encourage the students to identify patterns in the equations by recording the two parts of the equation one above the other:
6 x 20
60 x 2
and ask questions such as:
What do you have to do to the 6 to make it into 60? (Multiply by 10)
What do you have to do to the 20 to make it into 2? (Divide by 10)
Do not accept "add zero" or "take away zero". If necessary, point out that 6 + 0 = 6 and 6 – 0 = 6.

Have the students record their answers on their diagrams:

$$\begin{tikzcd} 6 \times 20 \arrow[d, bend right=60, "\times 10"'] \arrow[d, bend left=60, "\div 10"] \\ 60 \times 2 \end{tikzcd}$$

Note that this uses proportional adjustment, like doubling and halving.

For questions **2–4**, facilitate the students' identification of patterns by continuing to record the equations in diagrammatic form and encouraging the students to maintain the "balance" by using an inverse operation, that is, if you *multiply* one number by 6, you'd *divide* the other number by 6.

$$\begin{tikzcd} 10 \times 18 \arrow[d, bend right=60, "\times 6"'] \arrow[d, bend left=60, "\div 6"] \\ 60 \times 3 \end{tikzcd} \qquad \begin{tikzcd} 15 \times 16 \arrow[d, bend right=60, "\times 4"'] \arrow[d, bend left=60, "\div 4"] \\ 60 \times 4 \end{tikzcd} \qquad \begin{tikzcd} 30 \times 16 \arrow[d, bend right=60, "\times 2"'] \arrow[d, bend left=60, "\div 2"] \\ 60 \times 8 \end{tikzcd}$$

To find out whether the students have generalised the ideas so far and are ready to apply the strategy of proportional adjustments to other equations, ask them to explain in their own words to a classmate, and then to the group, how they'd work out what would go in the box in this equation: 25 x 18 = 75 x ❑. The strategy that Marama is developing is considerably easier than the alternative of calculating 25 x 18 and then 450 ÷ 75.

Question **5** asks the students to write their own problems. Writing one that works is useful for formative assessment. It also tends to be more difficult than solving a problem that is provided. The students could easily write a problem using any numbers, but it is much more difficult to write a problem that results in a whole-number answer that suits the strategy.

The students need to recognise that for this strategy to work, one of the numbers on one side of the equation needs to be a multiple of the number on the other side. For example, in the equation 18 x 50 = ❑ x 100, 100 is a multiple of 50, and in the equation 12 x 33 = ❑ x 99, 99 is a multiple of 33.

Record the equations from the activity on the board to give the students an overview of problems that work: 6 x 20 = 60 x 2; 10 x 18 = 60 x 3; 15 x 16 = 60 x 4; 30 x 16 = 60 x 8; 25 x 18 = 75 x 6. Note that in this activity, all the adjustments happen to be based on the first term in each expression, but it could equally well be the second term that is the focus for adjustment. Proportional adjustment can be made to either number on either side. Putting the unknown in different places helps to strengthen the students' understanding of the equals sign. For example, 6 x 20 = 60 x ❑ could also be adjusted as 6 x 20 = ❑ x 2.

Ask:
What patterns and relationships can you see that will help you to choose numbers that will suit Marama's strategy? (The first number on the left-hand side of the equals sign is a factor of the first number on the right-hand side, for example in 6 x 20 = 60 x 2, 6 is a factor of 60 because 6 x 10 = 60.)
If you chose 8 as the first number on the left-hand side of the equals sign, what might the first number on the right-hand side be? (A multiple of 8, such as 16, 24, 32, 40, and so on)
If you chose 8 as the first number on the left-hand side of the equals sign and 24 as the first number on the right-hand side, what could the ▲ and ❑ numbers be? That is, 8 x ▲ = 24 x ❑. (The ▲ would have to be something that is a multiple of 3 because 8 x 3 = 24. It could be 12 or 45 or 333 …)

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)

- *Book 6: Teaching Multiplication and Division*
  Cut and Paste (doubling and halving, trebling and thirding), page 25
- *Book 8: Teaching Number Sense and Algebraic Thinking*
  Doubling and Halving (extended), page 14
  Multiplying by 25, page 14.

Pages 12–13: Perfect Lasagne

Achievement Objective

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to help the students to extend their advanced multiplicative part–whole strategies (stage 7) in the domains of multiplication and division and proportions and ratios.

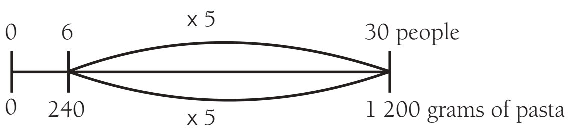
Activity

This activity encourages students to try two different strategies for increasing the amount of ingredients in a recipe and then to compare the efficiency of the strategies in different circumstances. The problems involve rates where two quantities measured with different units are compared, for example, grams of pasta with number of people.

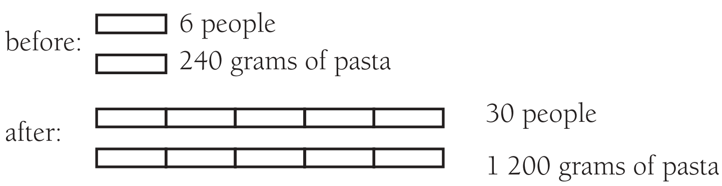
For this activity, the students will need to be able to choose flexibly from a range of multiplicative strategies. They will need to be able to multiply and divide decimal fractions by a whole number, for example, 1.2 ÷ 6, or 0.2 x 30, and to convert grams into kilograms and vice versa, for example, 1.2 kg = 1 200 g.

With a guided teaching group, discuss Ara's proportional strategy in question **1**. She works out that the recipe for 30 people would need to be 5 times bigger than the recipe for 6 people, so she multiplies each ingredient by 5. This strategy is very useful when the proportions can be easily identified and calculated. Ara's strategy is quick and efficient in this case. It's less straightforward if the proportion cannot be expressed as easily, such as later in the activity when she is making the recipe for 10 people and she has to multiply each ingredient by $1\frac{2}{3}$.

Ara could also use a proportional strategy to calculate 240 g x 5 by working out 240 x 10 = 2 400 and then halving it: 2 400 ÷ 2 = 1 200. Ara's strategy could be shown as a double number line:



or as a strip diagram:



In question **2**, Liam uses a different proportional strategy. He works out how much of each ingredient would be needed for just 1 person by dividing the ingredients in the recipe by 6. He then multiplies that amount by the number of people he needs to feed. This "unit rate" strategy will work in all situations, but it involves an unnecessary step when the number to be fed is a multiple of the number of people for whom the recipe was originally written.

Question **3** is useful for formative assessment. For this question, Ara could make the original recipe 25 times bigger (6 x 25 = 150) or multiply her ingredients for 30 people by 5 (from 30 x 5 = 150). If the students have difficulty getting started, ask questions such as: *How much bigger would the recipe for 6 people have to be to feed 150 people? What about 30 people? How might the facts in Ara's speech bubble help?*

Possible strategies for multiplying by 25 include:

- making proportional adjustments, for example:
  25 x 240 = 100 x 60 (quadruple and quarter)
  = 6 000
- using place value partitioning (25 x 200) + (25 x 40).

Liam will need to multiply his ingredients for 1 person by 150 people. Possible strategies for multiplying by 150 include:

- using place value partitioning, for example: 150 x 40 = (100 x 40) + (50 x 40)
  = 4 000 + 2 000
  = 6 000
- multiplying by 100 and then adding on half the answer, for example,
  100 x 40 = 4 000. Half of 4 000 = 2 000; 4 000 + 2 000 = 6 000.

Question **5b** promotes algebraic thinking because it requires the students to generalise the types of problems for which each strategy is most appropriate. Ara's strategy works where the measurements of one object are multiples of one another, for example: 6 people → 30 people.

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)

- *Book 6: Teaching Multiplication and Division*
  A Little Bit More/Less (using tidy numbers and compensation on multiplication problems), page 15
  Cut and Paste (making proportional adjustments in multiplication), page 25
  Multiplication Smorgasbord (choosing from a variety of multiplicative strategies), page 27
- *Book 8: Teaching Number Sense and Algebraic Thinking*
  Doubling and Halving (extended), page 14
  Multiplying by 25 (making proportional adjustments), page 14
  Whole Numbers Times Fractions, page 22.

Pages 14–15: Token Gesture

Achievement Objectives

- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- state the general rule for a set of similar practical problems (Algebra, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to help the students to consolidate and apply advanced multiplicative part–whole strategies in the domain of multiplication and division (stage 7).

Activity

This activity requires students to multiply a decimal by a whole number. It also involves a simple rate in dollars per token that makes the problem more demanding. In this activity, students use multiplicative strategies to calculate the amount earned by collecting tokens and bonuses and then identify patterns and relationships that could be used to predict earnings for any number of tokens collected.

To solve the calculations in this activity mentally, students need to be at least advanced multiplicative thinkers (stage 7) because they have to solve multiplication problems such as 28 x 40 and 55 x 40. Calculations such as those in question **1** require the stage 8 skills of multiplying decimal fractions (for example, 32 x $0.40), but this can also be done at stage 7 using whole numbers and then converting to dollars if necessary (for example, 32 x 40 cents = 1 280 cents or $12.80).

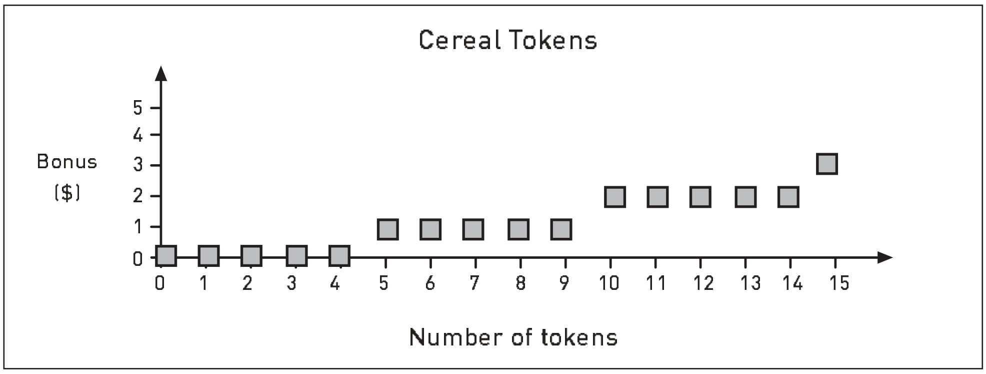
Set the scene with a guided teaching group by talking about promotions involving collecting tokens, receipts, or points that the students may have taken part in. Ask them what motivated them to collect the tokens, what tactics they used to get lots of them, and about any prizes they earned.

The calculations in question **1** give the students opportunities to apply and compare different multiplicative strategies. Get the students to discuss each question with a classmate or group of 3–4 first before sharing with the whole group. Possible strategies include:

- place value partitioning:
  14 x $0.40 = (10 x 0.4) + (4 x 0.4)
  = 4 + 1.6
  = $5.60
  14 x 40 cents = (10 x 40) + (4 x 40)
  = 400 + 160
  = 560 cents or $5.60
- using tidy numbers and compensating:
  28 x $0.40 = (30 x 0.4) – (2 x 0.4)
  = 12 – 0.8
  = $11.20
  28 x 40 cents = (30 x 40) – (2 x 40)
  = 1 200 – 80
  = 1 120 cents or $11.20
- doubling and halving repeatedly:
  32 x 40 cents = 16 x 80
  = 8 x 160
  = 4 x 320
  = 2 x 640
  = 1 280 cents or $12.80

For question **2**, get the students to discuss in their small groups how Mere could work out her bonus. Have the students share back with the whole group how they would apply their group's strategies. They need to recognise that it is the multiples of 5 that will help Mere to work out her bonus because it's not until she reaches each multiple of 5 that she gets another dollar. You could demonstrate this on a hundreds board by getting the students to flip or circle every fifth number as they count up, keeping a tally at the same time of how many bonus dollars Mere would get as she collects each extra group of 5. As the students count up towards the next multiple of 5, you could ask: *How much will Mere's bonus be now that she's got 18 tokens? 19 tokens? 20 tokens?*

This relationship could be plotted on a graph:



For obvious reasons, this is known as a step function.

You could use questions **3** and **4** to make formative assessments of the students' algebraic thinking. Listen as they work through the questions with their classmate or group to identify which students are merely using recursive (sequential) rules by adding another 40 cents or bonus repeatedly and which students have generalised by identifying the functional relationship, that is, the number of tokens x 0.4.

Some students may need to use a calculator when they are working out the money earned from 153 tokens so that they can concentrate on the algebraic aspects. Encourage them to evaluate for themselves when they need to use a calculator. Remind the students that number sense is very important when using a calculator because they need to check by "backwards estimation" that the answer they get is sensible and not the result of an incorrect entry.

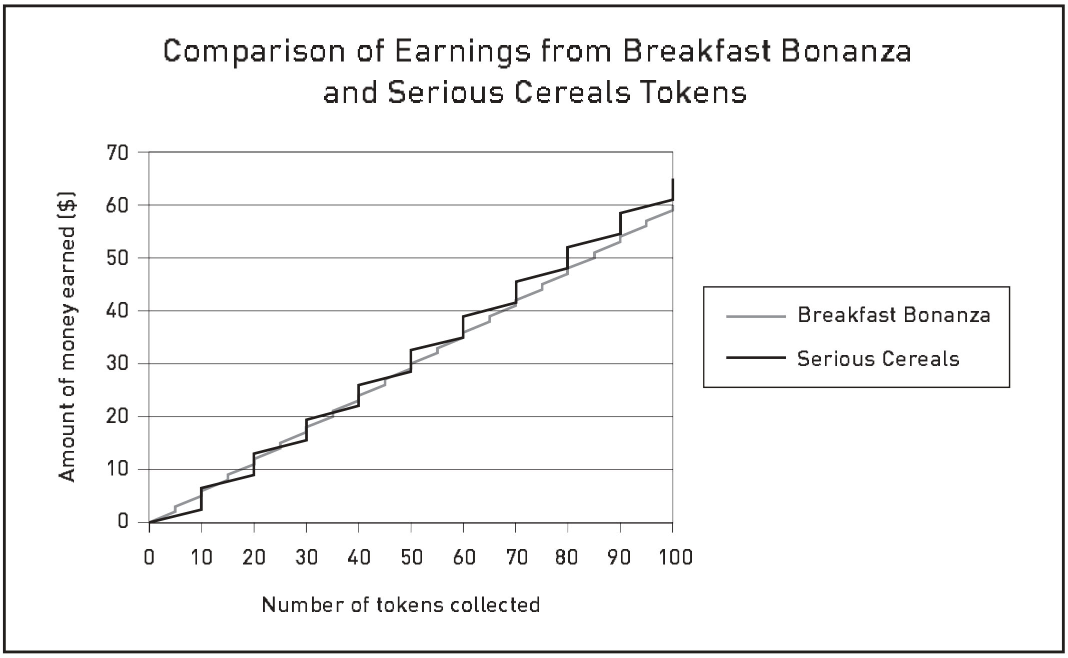
The students who are able to solve the reversed problem in question **5** will show a deep understanding of the way the patterns work. A simple approach might be to look at the table to find the number of tokens that earns close to $69 (such as 100 tokens earns $60) and use a trial-and-improvement strategy to hit the correct number. An alternative approach is to recognise that every 5 tokens collected represent total earnings of $3 (from $0.40 x 5 tokens + $1 bonus). This $3 is earned for every group of 5 tokens in a multiple of 5, for example, 20 tokens has 4 lots of 5 tokens, so Rewi earns 4 lots of $3. Using this approach, the students would see the $69 as being made up of 23 lots of $3 or 23 lots of 5 tokens (23 x 5 = 115).

As a challenge, you could extend the problem in question **5**: *Mere's school wants to raise $3,000 for new sports equipment. How many tokens will the students have to collect?*

- Every $3 earned needs 5 tokens, and there are 1 000 lots of $3 in $3,000, so they would need 1 000 lots of 5 tokens: 1 000 x 5 = 5 000 tokens.
- 10 tokens earns you $6, and 100 tokens earns you $60. 1 000 tokens would earn $600, and you'd need 5 times that to get $3,000. 5 x 1 000 tokens = 5 000 tokens.
  Working backwards to check:
  (5 000 tokens x $0.40) + (bonus of 5 000 tokens ÷ 5 x $1) = $2,000 + $1,000
  = $3,000.

For question **6**, you might need to suggest that the students use a systematic method of comparison, such as making a table or drawing a graph, because the comparison is not consistently better for one company or the other until you collect over 50 tokens. The Breakfast Bonanza company pays bonuses on multiples of 5, and Serious Cereals pays out on multiples of 10, so another worthwhile strategy is to compare how much money would be earned from every 10 tokens collected ($6 for Breakfast Bonanza tokens compared to $6.50 for Serious Cereals tokens, which means that, in the long term, Serious Cereals tokens would earn you more).

The students could use a computer graphing program to compare the two kinds of tokens. If they do, they will need to enter the data for each number of tokens (1, 2, 3, and so on up to at least 60) because the graphs are stepped. The steps can be copied in as "blocks" to make the data entry easier. The program will show the rise of each step to be steeply sloped, but it should be vertical. Most graph-drawing programs have this limitation. The graph shown below is not ideal, but it's similar to the graphs you and your students will be able to produce. The inaccuracy of the steps doesn't detract from the main point, which is that, over time, the two lines are getting further apart. The graph should clearly demonstrate that the return on Serious Cereals tokens is always greater once the number of tokens is greater than 50.



Extension

The students could write their own token and bonus problem for another group member to solve.

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)

- *Book 6: Teaching Multiplication and Division*
  Remainders (doing division problems with remainders), page 32
  Cut and Paste (doubling and halving, trebling and thirding), page 25
  Multiplication Smorgasbord (using a variety of strategies), page 27
  Royal Cooking Lesson (making proportional adjustments in division), page 30
  Cross Products (multiplying multi-digit whole numbers), page 37
- *Book 8: Teaching Number Sense and Algebraic Thinking*
  Checking Multiplication and Division by Estimation, page 11.

Pages 16–17: Digital Dilemmas

Achievement Objectives

- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- use graphs to represent number, or informal, relations (Algebra, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

AC EA AA AM AP

Number Framework Links

Use this activity to help the students to apply advanced additive and multiplicative part–whole strategies (stages 6–7) in the operational domains of addition and subtraction and multiplication and division.

Activity

In this activity, there are lots of opportunities for students to apply advanced multiplicative strategies as they explore the relationship between two amounts of memory allocated to photos of different formats on a digital camera.

Students need to be at least advanced multiplicative thinkers (stage 7) to solve all the calculations in this activity mentally. Students who are at stage 6 could do the activity using a calculator where necessary.

You may need to explain that kilobyte (kb) and megabyte (Mb) are units that measure how much information can be stored on a computer. A kilobyte is approximately 1 000 bytes, and a megabyte is approximately 1 000 000 bytes.

You could get the students interested in this activity by letting them take some photos on a digital camera, experimenting with the resolution setting, and comparing the results. Talk about how the photos are stored on a memory card (some cameras use memory sticks or disks) and how the higher the resolution, the greater the memory used by each photo. Extra-wide photos automatically have a higher resolution so that they can be printed bigger without looking fuzzy, but this means that they take up more memory.

Promote algebraic thinking in this activity by focusing discussion on how the numbers relate to each other and by getting the students to use patterns to predict answers.

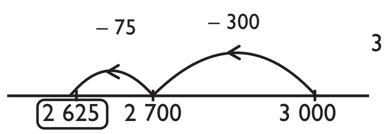
Many of the problems in this activity look daunting but can be solved quite easily using number sense strategies. Identify key number facts that may prompt the students to use their number sense strategies by asking:
Question **1a**: *What's double 125? Double 250? Double 500? Can you use the doubling we've just done to help you work out what 1 000 ÷ 125 is?* "There are 2 lots of 125 in 250, so there are 4 lots of 125 in 500 and 8 lots of 125 in 1 000. 1 000 ÷ 125 = 8." Write the facts on the board as the students identify them so that they can refer back to them.
Can you use the fact that 1 000 ÷ 125 = 8 to work out 3 000 ÷ 125? "If there are 3 thousands and each one has 8 lots of 125, there must be 3 x 8 = 24 lots of 125 altogether, so 3 000 ÷ 125 = 24."

Question **1b**: *What's double 375? Double 750? Double 1 500? Can you use the doubling we've just done to help you work out what 3 000 ÷ 375 is?* "There are 2 lots of 375 in 750, so there are 4 lots of 375 in 1 500 and 8 lots of 375 in 3 000. 3 000 ÷ 375 = 8"

Question **2**: *How many times does 125 go into 375?* (3 x 125 = 375) *If you know that 3 x 125 = 375 and that 2 x 375 = 750, how many lots of 125 are there in 750?* "There must be 6 x 125 because every 375 has 3 lots of 125 and there are 2 lots of 375.
❑ x 125 = 2 x 375
= 2 x (3 x 125)
= 6 x 125"

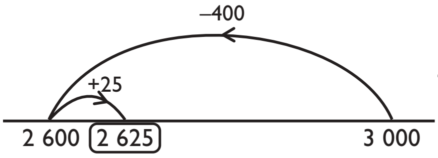
If you take 1 lot of 375 away from 3 000, how much is left? Encourage the students to use an empty number line if they need to. Two possible strategies are:

- Using place value partitioning: 3 000 – 300 = 2 700, 2 700 – 75 = 2 625



3 000 – 375 = $\boxed{2\ 625}$

- Using tidy numbers and compensating: 3 000 – 400 = 2 600, 2 600 + 25 = 2 625



3 000 – 400 + 25 = $\boxed{2\ 625}$

You've worked out that 3 000 – 375 = 2 625. You know that 3 x 125 = 375. You also know that 3 000 ÷ 125 = 24. Can you use those facts to help you work out how many lots of 125 there are in 2 625? (375 [or 3 lots of 125] were taken away from 3 000 to get 2 625. If there are 24 lots of 125 in 3 000 altogether and you've already taken away 3 of them, there must be 21 lots of 125 left. [24 x 125] – [3 x 125] = 21 x 125)

In question **3**, encourage the students to complete the tables using strategies such as known facts, tidy numbers and compensating, place value partitioning, and doubling. Remind them of key facts (such as 8 x 125 = 1 000, 24 x 125 = 3 000, 8 x 375 = 3 000, and 3 x 125 = 375) that they could use to help them. Encourage the students to compare the two tables and to make links between the numbers of extra-wide and standard photos and the memory used.

Some students may need to use a calculator at times during this activity so that they can concentrate on the algebraic aspects. Encourage them to evaluate for themselves when they need to use a calculator by asking them to look carefully at the problem to see whether they know a strategy they could try first. Remind them that number sense is very important when using a calculator because they need to check by "backwards estimation" that the answer they get is sensible and not the result of an incorrect entry.

Extension

The students may like to explore the relationship between resolution and memory using the school's digital camera.

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)

- *Book 5: Teaching Addition, Subtraction and Place Value*
  Problems like 37 + ❑ = 79 (using place value partitioning), page 36
  Problems like 73 – 19 = ❑ (using tidy numbers and compensating), page 38
- *Book 6: Teaching Multiplication and Division*
  Cut and Paste (doubling and halving, trebling and thirding), page 25
  Multiplication Smorgasbord (using a variety of strategies), page 27
  Cross Products (multiplying multi-digit whole numbers), page 37.

Pages 18–19: Fenced In

Achievement Objectives

- state the general rule for a set of similar practical problems (Algebra, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)

AC
EA
AA
AM
AP

Number Framework Links

For this activity, students should be using advanced additive strategies (stage 6) in the domain of multiplication and division.

Activity

In this activity, students explore how changes in the lengths of a rectangle's sides affect its area. Encourage them to use multiplicative reasoning.

You may need to define these terms:

- Area: the size of a surface contained by a boundary, measured in squares.
- Perimeter: the length of a line that encloses a shape, in this case, the length of the fence around the outside of the puppy run.
- Rectangle: a 4-sided shape with 4 right-angled corners. (Note that this definition includes squares. All squares are rectangles, but not all rectangles are squares.)

In the activity, the students create rectangles on geoboards and find multiplicative strategies to count the squares, so they don't need to know the formula for finding the area of a rectangle. If you don't have access to geoboards and rubber bands, supply square grid paper that the students can either cut up or draw rectangles on. Geoboards are useful because the students can show clearly what happens when the sides of a rectangle are lengthened by stretching out a different-coloured rubber band over the top of the first one.

The aim of question **2** is for the students to recognise that the largest possible rectangular area that can be made with any particular perimeter will always have 4 equal sides. (Note that for this activity, the students are working only with multiples of 4, so there are no fractions to deal with.) Some students think that squares and rectangles are two different shapes, and so they won't try a square on their geoboard because the question asks them to find the largest rectangle possible. If you notice this happening, encourage them to recognise that a square is actually a subset of the rectangle classification (a "regular" rectangle) by asking questions such as:
What special features does a rectangle have that make it a rectangle? (The students may find it helpful to look in the dictionary to add to their definition). As the students define each feature, make a list on the board. A possible definition is: A rectangle is a flat shape with 4 straight sides and 4 right-angled corners and 2 pairs of sides that are parallel (the lines go in the same direction). Discuss which of these features stem from other features. For example, *If the angles are right angles, is it possible for the pairs of sides to not be parallel?* (No!) Try to arrive at the most important criterion, that is, having a 90° angle at each corner. Then get the students to apply the criterion to a square to see if it's a kind of rectangle.
Does a square fit these criteria as well? Go down the list and tick each criterion off as the students agree that it does match. Tell the students that a square is a special sort of rectangle, just like an equilateral triangle is a special sort of triangle. It has all the same features as a rectangle plus one additional feature: all 4 sides are the same length.

In question **4**, the students explore what happens to the area of a rectangle when the lengths of the sides are doubled. Encourage the students to show the comparison on their geoboard or paper by using different colours for the original rectangle and the enlarged one and displaying them both at once. They can make large rectangles by pushing a few geoboards together and then using several rubber bands stretched thin (or lengths of wool) to make each side. With a guided teaching group, you could ask each student to make a different-sized rectangle. Record the information about each student's rectangle in a table as they share with the group and then compare the results.

The students should be able to identify a short cut for finding the area of a rectangle by looking for patterns, using the data generated in the table. To prompt them to identify the relationship between the length and width and the area, ask:
If you count 5 squares along the bottom row of a rectangle and 4 squares up the side, what's a quick way of working out how many squares are in the whole rectangle?
"I saw that there were 4 rows, so I worked out 4 lots of 5 squares. 4 x 5 = 20 squares."
Look at the numbers on your table in the length and width columns. How could you use the numbers 4 and 4 to make 16? How could you use the numbers 3 and 5 to make 15?
Does this work with the other rectangles on your table?
If you know that the length of a rectangle is 10 and its width is 5, how many squares would you predict there would be in its area?

Make sure that you connect the students' multiplicative expressions with the origin of the factors, that is, the columns and rows in the rectangles.

Questions **4c** and **5** are useful for formative assessment to evaluate whether the students can explain the relationship between the sides and the area when the sides are doubled or halved.

Investigation

Some students may like to extend the investigation by exploring what happens when the lengths of the sides are multiplied by 4 or divided by 4, multiplied by 5 or divided by 5, and so on. Ask: *Can you find a way to use square numbers ($2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $5^2 = 25$, and so on) to predict what will happen to the area of a rectangle if the sides are multiplied by 10 or divided by 10?*

Page 20: Slide Shows

Achievement Objectives

- state the general rule for a set of similar practical problems (Algebra, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to help the students to apply additive and multiplicative part–whole strategies (stages 5–7).

Activity

This activity encourages students to think systematically about the structure of a number pattern and to anticipate the result of calculations using identified patterns and rules.

Students need to be at least early additive part–whole thinkers (stage 5) in the domain of multiplication and division to do this activity. They need to understand that multiplication is a quick way of calculating repeated addition, for example, 8 + 8 + 8 + 8 = 4 x 8.

This activity has useful cross-curricular links to the Visual Language: Presenting strand of the English curriculum and would be a useful accompaniment to practical sessions on computers in which students are creating their own slide shows. Talk about computer slide shows that the students have seen or made. If possible, watch one on a computer. Show the students how you can choose the length of time each picture shows for and experiment with some of the transitions. Talk about how some of the transitions are instant and others, such as "dissolve" or "wipe down", take a couple of seconds, depending on the speed you select. In terms of visual language, you need to discuss the message that might be conveyed by quick breaks and long breaks, such as "dissolve".

The diagram and the equation in the table in question **1** are designed to help the students identify patterns in the numbers. For each new photo, 10 seconds are needed (made up of 8 seconds to view the photo and 2 seconds' transition), apart from the last photo (where no transition follows). Some students may need to draw or model the pattern physically (building from 1 photo to 2 photos and a transition, to 3 photos and 2 transitions, and so on) to help them visualise the structure of the pattern.

For question **2**, encourage the students to discuss their strategies in small groups and then report back to the whole group. Use this time to listen to and support the thinking of individuals as they interact with their peers.

The students should find it straightforward to add on 1 more transition and photo at a time to sequentially work out the time needed for 9 photos, but it is important that they look beyond recursive (sequential) rules such as "add 10 seconds" because these rules are only helpful if you know how long the slide show was before you added the extra picture. A generalisation enables students to work out how long any number of slides would take to show, and therefore it is much more powerful. To encourage the students to look towards a generalisation, ask questions such as:
What short cuts could Tama use to speed up the process of adding on 8 + 2 seconds?
Multiplication is a fast way of adding. What could you do instead of adding 8 + 8 + 8 + 8 + 8?
Can you see any groups that Tama could multiply to make a slide show with 5 photos?
"5 groups of 8 and 4 groups of 2: (5 x 8) + (4 x 2)"
"4 groups of 8 + 2 = 10 plus another 8 on the end: (4 x 10) + 8"
"I could imagine another 2 on the end and work out 5 groups of 8 + 2 = 10, then take the 2 off: (5 x 10) – 2."

Watch for students who assume that if they know a slide show with 5 photos takes 48 seconds, a slide show with double the photos would take double the time, that is, they think a slide show with 10 photos would take 96 seconds, but it would actually take 98 seconds. Challenge their thinking:
Convince me you're right.
Could you test your strategy on the 2-photo and 4-photo slide shows in the table?

For question **2c**, students show a deep understanding of the way the patterns work if they are able to solve the reversed problem using algebraic reasoning rather than simply adding on until they reach 78 by trial and improvement. Ask the students who solve the problem by adding on if they can also find another faster way to solve the problem. Ask students who need support:
How many seconds are there in 1 minute and 18 seconds?
What groups did you find in the table to use in your short cut?
How could you break 78 up into those groups?
Once you've broken the 78 seconds up into the same sort of groups you saw in the table, can you use that information to work out the number of photos?
Some possible ideas are:
"I saw groups of 10 made up of 8 + 2 in the table, but the last group didn't have its 2."
"I could add that missing 2 onto 78, which would give me 80, and then I could divide by 10 to work out how many groups there would have been: (78 + 2) ÷ 10 = 8 photos."
"I also saw groups of 10 made up of 8 + 2 in the table but with an extra group of 8 on the end. I could take the 8 off 78, which leaves 70, which could then be broken up into 7 groups of 10. So there'd be 7 photos plus another one in the last 8 seconds, and that's 8 photos altogether."

Question **3** is useful for making formative assessment observations as to whether the students can apply the ideas they have been explaining to create rules for similar problems using different numbers.

Extension
Get the students to write their own slide show problems for another group member to solve. Challenge them to write two types of problem: one where they have to work out how long the slideshow would take if they had ❑ pictures and another where they have to work out how many photos are shown in a slideshow lasting ❑ seconds. This is another valuable opportunity for formative assessment.

Challenge the students to write a set of instructions for working out the time needed for any number of photos where the times for the photo to show and the transition are not specified. Here is one possible set of instructions:
- Work out how long it takes to show a photo as well as change over to the next slide.
- Multiply this number by the number of photos you have.
- Subtract one transition time (because you don't need a transition at the end).

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)

- *Book 5: Teaching Addition, Subtraction, and Place Value*
  Make Ten (combining numbers to make calculations easier), page 26
- *Book 6: Teaching Multiplication and Division*
  Twos, Fives, and Tens (turning repeated addition into multiplication), page 10.

Page 21: Enough Rice?

Achievement Objectives

- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- help the students to apply and extend their advanced multiplicative part–whole strategies (stage 7) for proportional adjustment
- help the students to extend their ideas in the operational domain of proportions and ratios (stages 7–8)
- encourage transition to advanced proportional thinking (stage 8).

Activity

In this activity, students explore the relationship between the amount of time a sack of rice would last and the number of people eating. The relationship is an inversely proportional one: when one quantity is increased (for example, rice), the other is decreased (time). By contrast, in a *directly* proportional relationship, when one quantity is increased, so is the other. (For example, on a trip to the cinema, if the number of people increases, so does the total cost of admission.) The rice consumption is an example of a rate where different measures are related multiplicatively, that is, 1 sack : 4 people : 24 days.

This activity would be useful as a follow-up after the students have explored the strategy of doubling and halving and have recognised that it can be extended to adjusting by any inversely proportional amount, such as trebling and thirding, for example, $6 \times 9 = 3 \times 18 = 2 \times 27$, or multiplying and dividing by 4 or by 10.

The students in independent groups will need to break into pairs or groups of 3 to complete parts of this activity. Remind the students about the strategy of doubling and halving and suggest that they look for opportunities to use it in this activity.

Set the scene with a guided teaching group by asking the students about the foods their family might buy in big packets at the supermarket, such as cereals, potatoes, rice, or non-food items, such as washing powder or shampoo. Show them a big box of cereal and ask them to estimate how many days they think it might last their family. Ask:

What would happen if you had relatives come to stay and so you had twice as many people in your home who all liked that same cereal? How long would you expect the cereal to last then?

What about if some of your family went away on holiday and so you had only half the number of people eating the cereal? How long would you expect it to last then?

Divide the group into smaller groups. When they share back with the whole group, listen reflectively and summarise what they say by asking *So you … and then you … Is that what you did?,* at the same time recording number sentences to show what they did on the board. This ensures that you understand what the student has said, gives other group members another chance to hear the strategy, and provides a model of effective communicative language and recording.

For questions **1** and **2**, encourage the students to think about how they could apply the strategy of doubling and halving to work out how long the rice would last for 8 people, 2 people, and then 1 person. Promote the identification of patterns by recording summaries of all the problems as they are solved:
The rice lasts:
8 people for 12 days
4 people for 24 days
2 people for 48 days
1 person for 96 days. (This is called the unit rate. It is one way to solve any rate problem, although it is not always the most efficient.)
Ask the students:
What other patterns besides doubling and halving can you see in the numbers in these statements?
Can you tell me how the numbers relate to each other?

If the students need help to complete the table in question **2b**, ask general questions to begin with and then become increasingly focused as needed, for example:
What do you have to work out?
What do you know so far that might help you?
"You could start by filling in the easier cells first and working from there. For example, we've been told 4 people and 24 days, so we can easily do 2 and 48, 8 and 12, and 1 and 96."

A physical or diagrammatic model will help here:

<table>
<tr><td colspan="6">48</td><td colspan="6">48</td><td>2 people share</td></tr>
<tr><td colspan="3">24</td><td colspan="3">24</td><td colspan="3">24</td><td colspan="3">24</td><td>4 people share</td></tr>
<tr><td colspan="4">?</td><td colspan="4">?</td><td colspan="4">?</td><td>3 people share</td></tr>
</table>

"You could use trebling and thirding to work out how long the rice would last for 6 people based on the information for 2 people, and then you could halve and double that to work out how long it would last for 3 people."

The students will need to accept closeness in estimating the length of time needed for 5, 7, and 20 people because these calculations must approximate whole numbers of days.

These problems can be used to evaluate whether the students have generalised sufficiently to be able to apply the inverse proportional adjustment strategy even when the numbers are not easy multiples and factors. In fact, most students will tend to think additively, for example:

People	4	6	8
Days	24	16	12

Support the students' thinking if necessary by asking:
Are there any numbers of people on the table that you can double and halve or treble and third to make 5 or 7?
So you can't double and halve or treble and third for these ones, but could you find another relationship you could use, such as × 4 and ÷ 4, or × 5 and ÷ 5, or use fractions?
"I could use 'the rice lasts 1 person for 96 days' and × 5 and ÷ 5 to work out for 5 people and × 7 and ÷ 7 to work out for 7 people. So 96 ÷ 5 = 19.2 days for 5 people, and 96 ÷ 7 = 13.7 days for 7 people."

"I could use 'the rice lasts 4 people for 24 days' and x 5 and ÷ 5 to work out how long it lasts for 20 people because I know 4 x 5 = 20, so 24 ÷ 5 = 4.8 or 4 days."
Again, the use of diagrams will be very important.

For question **3**, support the students' thinking if necessary by asking increasingly focused questions, such as:
You know that 1 person takes 96 days to eat the rice. How could knowing that help you?
What fraction of the sack of rice would that 1 person have eaten after 1 day? (1 ninety-sixth, $\frac{1}{96}$)
How many ninety-sixths are there in a whole sack of rice?
How could knowing that help you?
If lots of people ate $\frac{1}{96}$ *each of the rice on the same day, how many people could you feed out of the sack?*

Encourage reflective discussion by asking, in a think-pair-share situation:
Does 42 x 67 = 42 x 29 x 67 ÷ 29? Explain your answer. (Yes, it does. Multiplying and dividing by the same number balances out, so 42 x 67 is unchanged.)

Extension
You could get the students to brainstorm examples of direct and inversely proportional relationships, for example, a person's mass on a see-saw to the distance from the fulcrum or the number of apples in a kilogram versus the mass of each apple.

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)

- *Book 6: Teaching Multiplication and Division*
  Cut and Paste (doubling and halving, trebling and thirding), page 25
  Proportional Packets (making proportional adjustments in division), page 28
  The Royal Cooking Lesson (making proportional adjustments in division), page 30
- *Book 8: Teaching Number Sense and Algebraic Thinking*
  Fractions in a Whole (finding fractions that add up to a whole), page 5
  Doubling and Halving (extending applications of doubling and halving), page 14
  Multiplying by 25 (making inverse proportional adjustments in multiplication), page 14.

Pages 22–23: On Top of the World

Achievement Objective

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)

AC
EA
AA
AM
AP

Other mathematical ideas and processes
Students will also explore patterns and relationships.

Number Framework Links
Use this activity to help the students to extend their ideas in the operational domain of proportions and ratios (stages 7–8).

Activity

In this activity, students explore the idea that a fraction can be used to compare one thing with another without knowing the exact height of either. This helps to develop ideas of "openness" and reunitising, which are critical in algebra. The students discover that the same object can have a completely different fraction when it is compared with a series of different objects.

For this activity, the students need to be able to make comparisons in fractional terms, for example, 8 is $\frac{2}{3}$ of 12 or 12 is $1\frac{1}{2}$ times bigger than 8. The diagram in the activity will help them do this. The students also need to recognise that numbers less than 1 (such as $\frac{2}{3}$) and numbers greater than 1 (such as $\frac{5}{4}$, $1\frac{1}{2}$, and $\frac{3}{1}$ or 3) are all fractional numbers.

Set the scene with a guided teaching group by talking about local landmarks and asking the students what they think tourists would find interesting in your local area. Ask the students whether they have ever visited the landmarks in the diagram and, if so, what they were like.

Encourage the students to refer back to the diagram for each new question and establish which landmark is now defined as the "benchmark" (the landmark against which all others will be compared). Encourage them to predict whether the other landmarks will be described using a fractional number larger or smaller than 1 in each situation. (If the landmarks are being compared with Taranaki, the height of Aoraki will be a fractional number larger than 1 because it's taller than Taranaki, but the Sky Tower will be a fractional number less than 1 because it's shorter than Taranaki.)

Question **4** is a useful point for making formative assessment observations to determine whether the students can explain that fractions are comparative numbers and comment on the importance of one whole in this comparison. Another useful question to promote algebraic thinking is: *Can a quarter ever be bigger than a half?* (Yes. $\frac{1}{4}$ of 40 is 10, and $\frac{1}{2}$ of 6 is 3. But you can't compare the physical sizes of quarters and halves unless they are both fractions of the same whole amount.)

For your information, the actual heights of the landmarks are: Aoraki: 3 754 metres, Taranaki: 2 518 metres, Sky Tower: 328 metres, and the Beehive: 49 metres. These heights have been used very approximately in the student activity.

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)

- *Book 4: Teaching Number Knowledge*
  Little Halves and Big Quarters, page 19. (This extends the idea in question **4**.)

Page 24: Decimal Fraction Buddies

Achievement Objective

- explain the meaning of the digits in decimal numbers with up to 3 decimal places (Number, level 3)

AC
EA
AA
AM
AP

Other mathematical ideas and processes

Students will also explore patterns and relationships.

Number Framework Links

Use this game and activity to help the students to consolidate their knowledge of part–whole strategies in addition and subtraction of decimal fractions. To do this activity, students need to be at least in transition from stage 6 to stage 7.

Game and activity

In this game and activity, students identify pairs of decimal fractions that add up to 1 and 0.5. (The copymasters for this game are at the end of these notes.) The students need to already be familiar with decimal fractions to 3 decimal places and should be able to add numbers such as 0.23 + 0.56 mentally, perhaps using a place value strategy (2 tenths + 5 tenths = 7 tenths, and 3 hundredths + 6 hundredths = 9 hundredths; 7 tenths and 9 hundredths can be written as 0.79).

With a group that is going to work independently, go over the instructions for the game and then, as a check, ask the students to find one pair of numbers on the board that add up to 1.

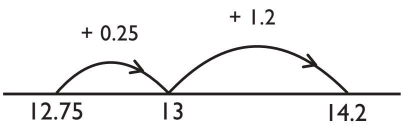
With a guided teaching group, write the following numbers on the board and ask the students to talk with a classmate to decide what decimal fraction each number needs to add up to 1 whole: 0.9, 0.7, 0.5, 0.99, 0.75, 0.08

Ask them to share their strategies for solving the problems and any patterns they notice that helped them. For example:
"For 0.9, I know that you need 10 tenths to make a whole, and those 10 tenths could be made up of 9 tenths and 1 tenth."
"To work out what makes 1 with 0.75, I can see there's already 7 tenths and 5 hundredths, so I'll need 2 more tenths and 5 more hundredths, and that's 0.25"
"I know that 0.75 is 75 hundredths. I think to myself '75 hundredths and how many more hundredths make 100 hundredths?' because I know 100 hundredths is the same as 1 whole. I can work out how many more hundredths are needed by adding on: 75 hundredths + 5 hundredths is 80 hundredths and another 20 hundredths makes 100 hundredths. So it's 25 hundredths, which is written as 0.25"

Encourage the students to describe decimal fractions using language that differentiates the fractions from whole numbers or from other uses of the decimal point, such as in money: *Say 14.56 as "14 point five six" or "14 plus 5 tenths plus 6 hundredths".* Physical models, such as deci-pipes or decimats, are also useful.

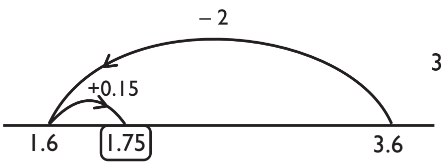
Ask: *Why might it be useful to be able to work out very quickly (or to know off by heart) pairs of numbers that add up to 1? Give me an example of a problem where you might find it useful.*
Two possible examples:

- Say you were adding decimal fractions and you wanted to use the strategy of adding to build up to tidy numbers. Instead of building up to numbers that end in 0 as you do with whole numbers, you could build to whole numbers, for example, for 12.75 + ❑ = 14.2: 12.75 + 0.25 =13, 13 + 1.2 = 14.2, 0.25 + 1.2 = 1.45



0.25 + 1.2 = $\boxed{1.45}$

- If you want to add or subtract a tidy number and then compensate, knowing pairs that make 1 whole will help you to know how much to compensate. For example, for 3.6 – 1.85 = ❑: 3.6 – 2 = 1.6, 1.6 + 0.15 = 1.75



3.6 – 2 + 0.15 = $\boxed{1.75}$

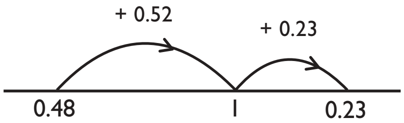
Get the students to look at the game board and see if they can each find two numbers that add up to 1. Then explain the rules of the game and get them to play it with a classmate.

If the students need support to find pairs of numbers that add to 0.5, ask questions such as:
How big is 0.5 compared to 1? (It's half the size.)
How many tenths are there in 0.5? (5)
If one of your numbers was 2 tenths, what would its partner have to be to add up to 5 tenths? (3 tenths, written as 0.3)
What other pairs can you find using tenths?
What's the fraction that's equivalent to 5 tenths using hundredths in the denominator? (50 hundredths, written as 0.50, which is the same as 0.5)

If one of your numbers was 43 hundredths, what would its partner have to be to add up to 50 hundredths? (7 hundredths, written as 0.07)
What's the fraction that's equivalent to 5 tenths using thousandths in the denominator? (500 thousandths, written as 0.500, which is the same as 0.5)
If one of your numbers was 485 thousandths, what would its partner have to be to add up to 500 thousandths? (15 thousandths, written as 0.015)

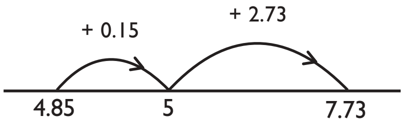
Extension
Challenge the students to apply what they have learned in the game and in the follow-up activity by asking them the following questions as think-pair-share discussion starters:
Make a list of the pairs of numbers that add up to 1 that you now know off by heart.
Can you use a pair that adds up to 1 to help you solve these problems?
Try using a strategy in which you add to build up to tidy numbers or you add or subtract a tidy number and then compensate.

- For 0.48 + ❑ = 1.23:
  0.48 + 0.52 = 1, 1 + 0.23 = 1.23, 0.52 + 0.23 = 0.75



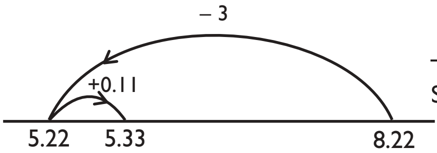
0.52 + 0.23 = $\boxed{0.75}$

- For 4.85 + ❑ = 7.73:
  4.85 + 0.15 = 5, 5 + 2.73 = 7.73, 0.15 + 2.73 = 2.88



0.15 + 2.73 = $\boxed{2.88}$

- For 8.22 – 2.89 = ❑:
  8.22 – 3 = 5.22, 5.22 + 0.11 = 5.33



– 3 + 0.11 = 2.89. 5.22 + 0.11 = 5.33
So, 8.22 – 2.89 = $\boxed{5.33}$

Make up a problem of your own in which it would be useful to know a pair that adds up to 1 to help solve the problem. Get a classmate to solve it.

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)
- *Book 4: Teaching Number Knowledge*
  Reading Decimal Fractions, page 8
  More Reading of Decimal Fractions, page 9
  Arrow Cards (using decimal place value), page 13
  Who Wins? (ordering decimal numbers), page 20
  Zap! (using decimal place value), page 26
- *Book 7: Teaching Fractions, Decimals, and Percentages*
  Pipe Music with Decimals (adding and subtracting decimals), page 22
  Candy Bars (adding and subtracting decimals), page 27.

Note that the behaviour of decimals can be disconcerting if the students are thinking in terms of "whole numbers with a dot". For example, with 1.375 + 0.625 = 2, the students may ask "Where's the point?". See *Book 5: Teaching Addition, Subtraction, and Place Value,* page 47.

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Extra-wide photos 375 KB		
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1	375	
2		
3		
4		
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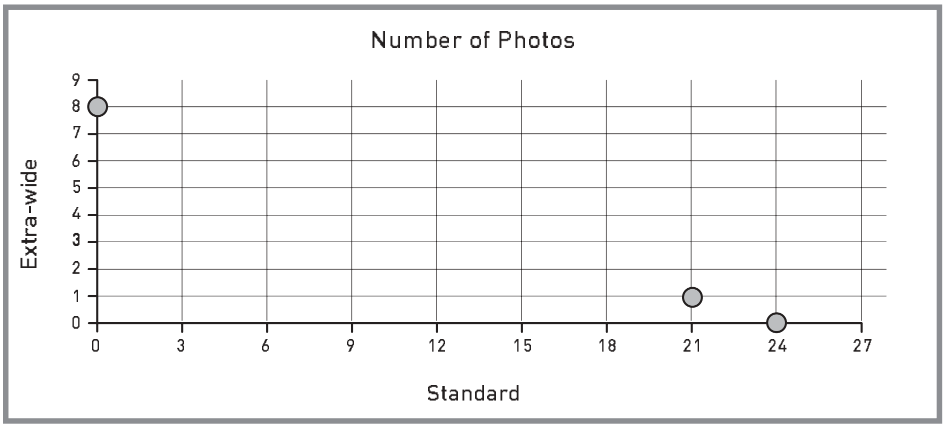

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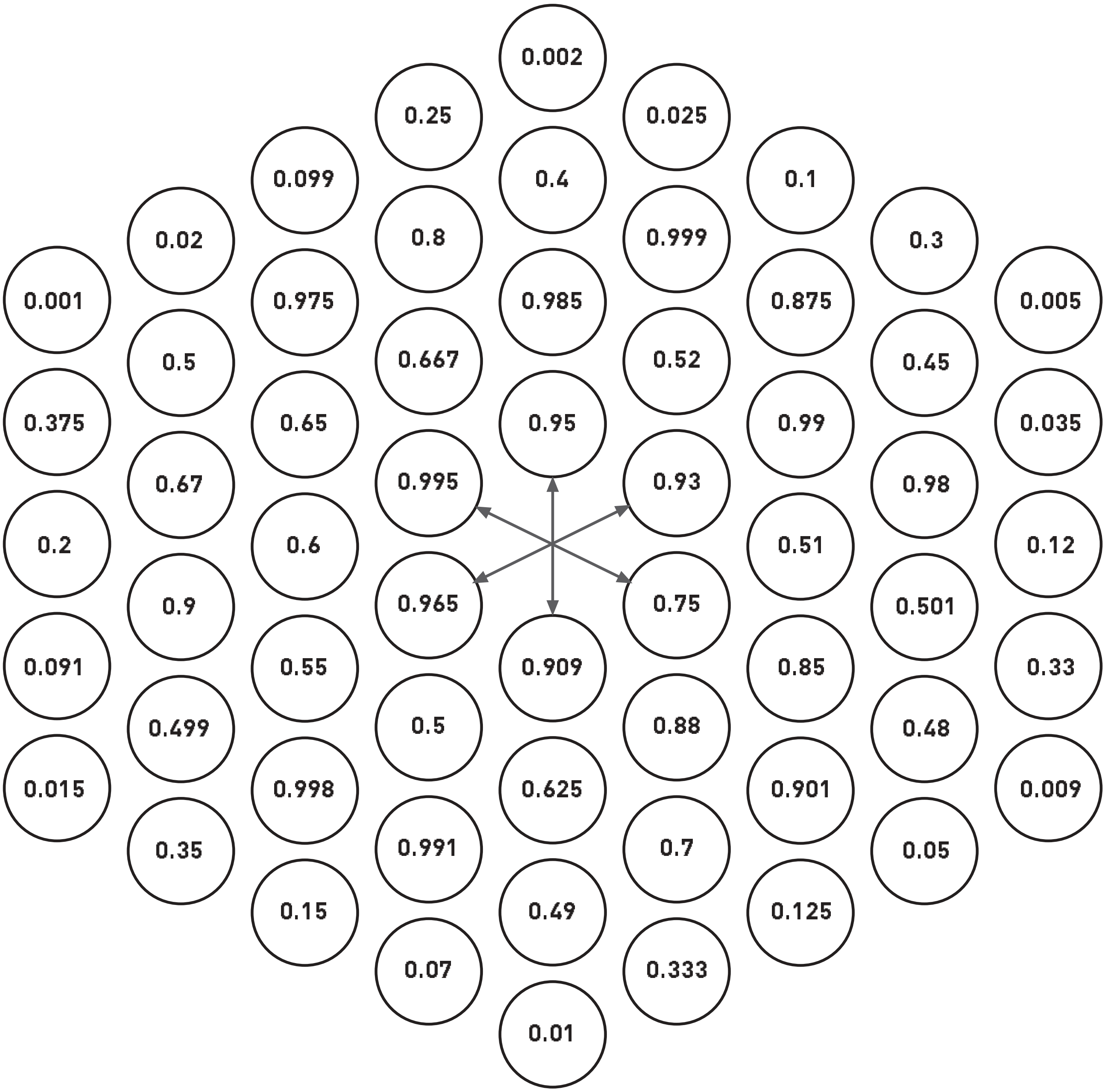

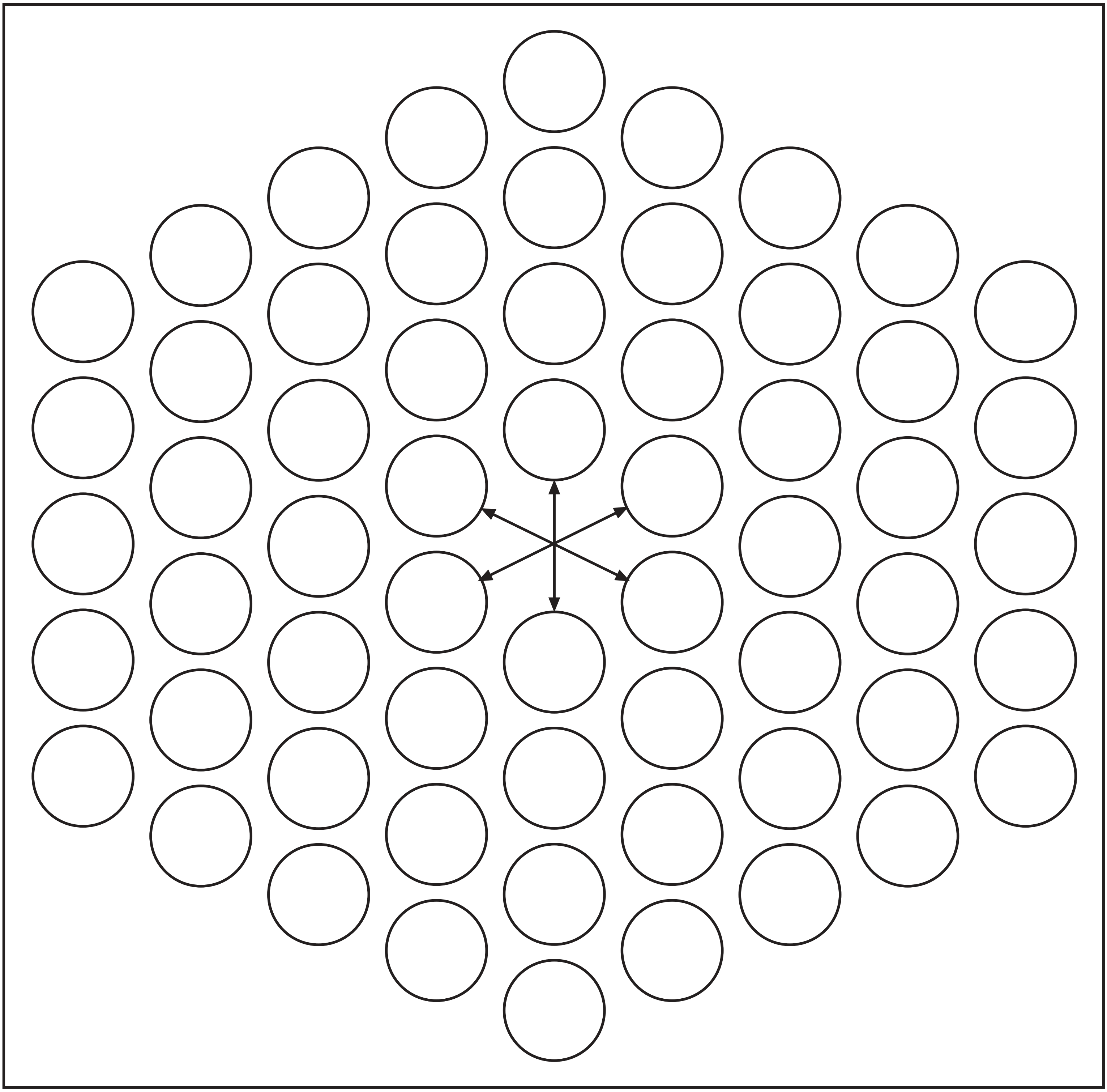

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