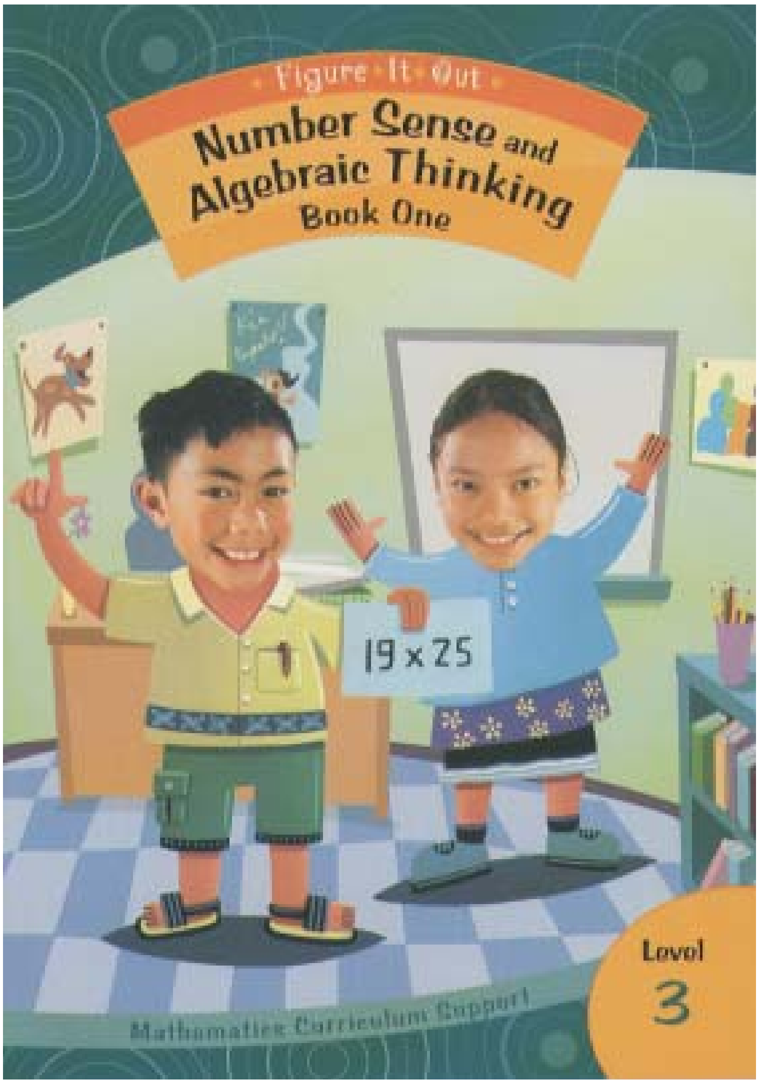
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Contents

Introduction

The books for level 3 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand classrooms. These books are most suitable for students in year 5, but you should use your judgment as to whether to use the books with older or younger students who are also working at level 3.

Student books

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and the life experiences that are meaningful to students in year 5.

The activities can be used as the focus for teacher-led lessons, for students working in groups, or for independent activities. Also, the activities can be used to fill knowledge gaps (hot spots), to reinforce knowledge that has just been taught, to help students develop mental strategies, or to provide further opportunities for students moving between strategy stages of the Number Framework.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the *Number Sense and Algebraic Thinking* student books includes full answers and explanatory notes. Students can use this section for self-marking, or you can use it for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, Number Framework links, comments on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/r/maths/curriculum/figure

Using Figure It Out in the classroom

Where applicable, each page starts with a list of equipment that the students will need in order to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to the problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Figure It Out

Number Sense and Algebraic Thinking
Level 3: Book One

Answers

Page 1: Taking Time

Activity

1. a. i. Days
   ii. Hours
   iii. Minutes
   iv. Seconds

   b. i. $11\frac{1}{2}$ yrs old. 4 200 ÷ 365 = 11.5
   ii. 11 yrs old. 96 360 ÷ 365 ÷ 24 = 11
   iii. $10\frac{1}{2}$ yrs old.
   5 523 840 ÷ 365 ÷ 24 ÷ 60 = 10.5
   iv. 10 yrs old. 315 532 800 ÷ 365 ÷ 24 ÷ 60 ÷ 60 = 10

2. Answers will vary depending on your age. For example, if you are $10\frac{1}{2}$, your answers will be:
   a. $10\frac{1}{2}$ yrs
   b. 3 833 days
   c. 91 980 hrs
   d. 5 518 800 min
   e. 331 128 000 s

3. a. Centuries. A century is 100 yrs.
   0.1 is $\frac{1}{10}$. $\frac{1}{10}$ of 100 is 10.
   b. 0.01. (10 is $\frac{1}{100}$ of 1 000)

4. Answers will vary. To find out your age in centuries, you need to divide your age in yrs by 100. To find out your age in millennia, divide your age in yrs by 1 000. For example:
   - 9 yrs old is 0.09 centuries or 0.009 millennia
   - $9\frac{1}{2}$ yrs old is 0.095 centuries or 0.0095 millennia
   - 10 yrs old is the same as James (0.1 centuries or 0.01 millennia)
   - $10\frac{1}{2}$ yrs old is 0.105 centuries or 0.0105 millennia
   - 11 yrs old is 0.11 centuries or 0.011 millennia.

Investigation

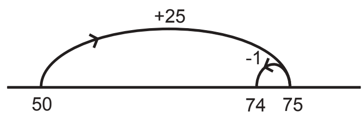
The unit of a year comes from how long the earth takes to circle the sun (approximately 365.25 days). We have leap years because each year is close to 365.25 days long, and we need to catch up on the 0.25 days every 4 years.

Pages 2–4: Tidying Up

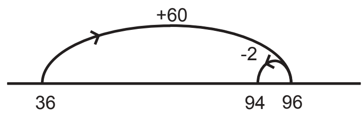
Activity One

1. Tidy number strategies will vary. One possible strategy and its matching number line is shown for each answer.

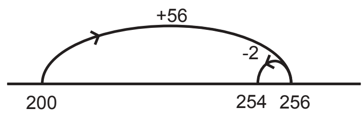
   a. 74. (50 + 25 = 75. 75 – 1 = 74)



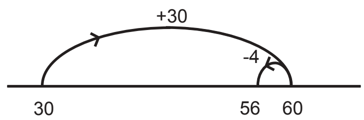
   b. 94. (36 + 60 = 96. 96 – 2 = 94)



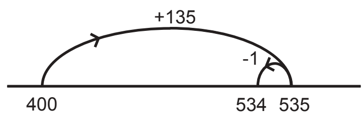
   c. 254. (200 + 56 = 256. 256 – 2 = 254)



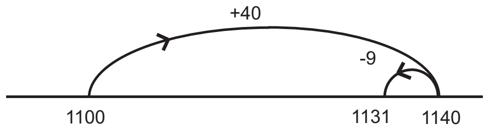
   d. 56. (30 + 30 = 60. 60 – 4 = 56)



   e. 534. (400 + 135 = 535. 535 – 1 = 534)



f. 1 131. (1 100 + 40 = 1 140.
1 140 – 9 = 1 131)



2. Strategies may vary. A possible strategy for each answer is given below.

a. 149. Using place value: 20 + 120 = 140, 3 + 6 = 9. 140 + 9 = 149

b. 145. Using tidy numbers: 100 + 46 = 146. 146 – 1 = 145

c. 3 333. Using tidy numbers: 3 100 + 234 = 3 334. 3 334 – 1 = 3 333

d. 339. Using place value: 100 + 200 = 300, 5 + 34 = 39. 300 + 39 = 339

e. 109. Using place value: 40 + 60 = 100, 5 + 4 = 9. 100 + 9 = 109

f. 164. Using tidy numbers: 130 + 36 = 166. 166 – 2 = 164

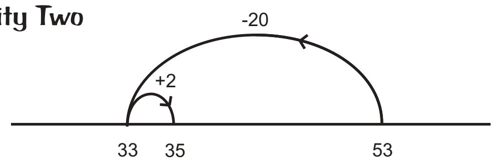
g. 642. Using tidy numbers: 300 + 345 = 645. 645 – 3 = 642

h. 1 577. Using place value: 400 + 100 = 500, 50 + 20 = 70, 5 + 2 = 7.
1 000 + 500 + 70 + 7 = 1 577

3. The tidy number strategy is always useful when one number, such as 21 or 198, is close to a tidy number (20 or 200). Usually, the number being "tidied" will end in 1, 2, or 3 or in 7, 8, or 9.

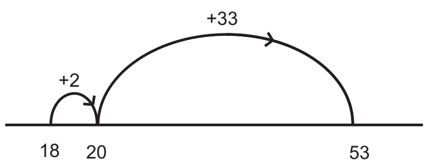
Activity Two

1.



2. Possible strategies include:

- Reverse the problem (18 + ❑ = 53) and add on. 18 + 2 = 20. 20 + 33 = 53.
  2 + 33 = 35



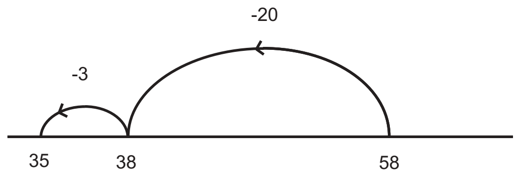
- Use place value strategy. Take off the tens and then the ones in steps: 53 – 10 = 43. 43 – 3 = 40. 40 – 5 = 35
- Make equal adjustments. Add 2 to each number: 55 – 20 = 35

3. Opinions will vary.

- Alex's strategy makes it easier to subtract mentally.
- Reversing and adding on is also a good ` method if you jump off tidy numbers. It's easy to add 2 to 3 at the end.
- Place value strategy may not be as useful in this problem because you have to do 43 – 8, which involves breaking up a 10.
- Taking away 20 is easier than taking away 18.

Activity Three

1.



2. Solutions and possible thinking are:

a. 42
Tidy number: 71 – 30 = 41. 41 + 1 = 42
Place value: 71 – 20 = 51. 51 – 9 = 42

b. 12
Tidy number: 85 – 70 = 15. 15 – 3 = 12
Place value: 80 – 70 = 10. 5 – 3 = 2.
10 + 2 = 12

c. 45
Tidy number: 97 – 50 = 47. 47 – 2 = 45
Place value: 90 – 50 = 40. 7 – 2 = 5.
40 + 5 = 45

d. 34
Tidy number: 62 – 30 = 32. 32 + 2 = 34
Place value: 62 – 20 = 42. 42 – 8 = 34

3. **a.** A tidy number strategy is most useful when the number that you are taking away is close to a tidy number.

b. Place value strategy is easier when the digits in each corresponding place are smaller in the number you are taking away so that you don't have to break any tens or rename. For example, for 85 – 73, it's easy to subtract 80 – 70 and then 5 – 3.

4. Strategies will vary. Efficient strategies include:

a. 51. Place value: 70 – 20 = 50, 4 – 3 = 1.
50 + 1 = 51
or 74 – 20 = 54. 54 – 3 = 51

b. 24. Tidy numbers: 53 – 30 = 23.
23 + 1 = 24

Adding on: 29 + ❑ = 53. 29 + 1 = 30.
30 + 23 = 53. 1 + 23 = 24

c. 60. Place value:
13 tens – 7 tens = 6 tens or 60
Adding on: 70 + ❑ = 130. 70 + 30 = 100.
100 + 30 = 130. 30 + 30 = 60

d. 24. Place value: 90 – 70 = 20, 8 – 4 = 4.
20 + 4 = 24
Adding on: 74 + ❑ = 98. 74 + 6 = 80.
80 + 18 = 98. 6 + 18 = 24

e. 4 701. Tidy numbers: 5 000 – 300 = 4 700.
4 700 + 1 = 4 701
Adding on: 299 + ❑ = 5 000.
299 + 1 = 300. 300 + 700 = 1 000.
1 000 + 4 000 = 5 000.
1 + 700 + 4 000 = 4 701

f. 63. Tidy numbers: 161 – 100 = 61.
61 + 2 = 63
Adding on: 98 + ❑ = 161. 98 + 2 = 100.
100 + 61 = 161. 2 + 61 = 63

g. 122. Place value: 200 – 100 = 100,
40 – 20 = 20, 5 – 3 = 2.
100 + 20 + 2 = 122

h. 642. Place value: 800 – 200 = 600,
90 – 50 = 40, 6 – 4 = 2.
600 + 40 + 2 = 642

Page 5: Megabytes of Memory

Activity

1. The cards' memory size doubles (for example, 2 x 32 = 64; 2 x 256 = 512), but the prices do not double as the memory size doubles. The cards go up by $30 or $40 with every increase in memory size.

2. **i.** 160 photographs (5 x 32)

ii. 320 photographs (5 x 64)

iii. 640 photographs (5 x 128)

iv. 1 280 photographs (5 x 256)

v. 2 560 photographs (5 x 512)

3. **a.** Answers will vary. Beth can't buy any 256 MB or 512 MB cards because then she wouldn't have enough money left for 2 more cards of any size. A possible table to show the options (including the number of photographs for question **b**) is:

MB	**32**	**64**	**128**	**256**	**512**		
Cost	$50	$80	$120	$160	$190	Total ($)	Number of photos
	3					150	480
	2	1				180	640
	1	2				210	800
	2		1			220	960
	1	1	1			250	1 120
		3				240	960

b. Beth's best option is 1 x 32 MB, 1 x 64 MB, and 1 x 128 MB cards, which would give her 1 120 stored photographs.

4. Answers will vary depending on the information found. (High resolution photos take between 1 and 5 MB [1 000–5 000 kB], depending on the resolution.)

Pages 6–7: What's Best?

Activity One

1. Discussion will vary. Possible comments include:

Strategy	Advantages	Disadvantages
Adding on	It's useful when you don't know multiplication facts.	It can be time-consuming.
Double and double again	It's useful if you can double easily. It can make the problem easier to work out mentally.	It only works where one of the numbers is a small even number (such as 4, 6, or 8) or a small even number multiplied by a tens number (such as 20).
Tidy numbers	It's great for problems where a number ends in 1, 2, or 3 or in 7, 8, or 9. Tidy numbers can simplify a problem and make it easier to solve mentally.	It can be confusing if you try to tidy numbers ending in 4, 5, or 6.
Place value	It breaks up the problem. It's sometimes easier to work things out in steps.	Sometimes there can be too much information to remember when you're trying to add all the bits together.
Doubling and halving	You can simplify a problem by making large numbers smaller.	You need a good basic facts knowledge. It doesn't always make the problem easier to calculate.
Known facts	You can start with what you know and build on that knowledge.	If the problem gets big, there can be too much for you to remember when you're trying to add all the bits together.

2. Opinions will vary. If you know 8 x 9, doubling and halving is an efficient strategy. The tidy number strategy is also a smart way of simplifying the problem because 18 is close to 20, so it can be easily solved in your head. Place value strategy is also efficient for this problem if you know 4 x 8 = 32.

3. Discussion will vary.

   Adding on: This would take a long time because you'd have to add 9 lots of 24 (or 24 lots of 9). It would be easy to lose track of how many you'd added on so far.

   Double and double again: This is not a sensible strategy to use because 9 is an odd number.

   Tidy numbers: This is an efficient strategy because 9 is close to 10, a tidy number:
   9 x 24 = (10 x 24) – (1 x 24)
   = 240 – 24
   = 216

   You could also use the tidy number 25, which is close to 24, if you're good at multiplying by 25:
   9 x 24 = (9 x 25) – 9
   = 225 – 9
   = 216

   Place value: This is an efficient strategy because there aren't too many steps to remember in your head: 9 x 20 = 180; 9 x 4 = 36; 180 + 36 = 216

   Doubling and halving: This strategy doesn't give you an easy fact that you'd probably know straight away, but it might help you make the problem easier to calculate using another strategy. You'd have to double and halve a few times to help: 9 x 24 = 18 x 12 = 36 x 6 = 72 x 3, or you could third and treble: 9 x 24 = 3 x 72

   Using known facts: Whether this strategy would be efficient for you would depend on which facts you knew. If you knew 9 x 12 = 108, you might double that to get 9 x 24. Or you could work out (9 x 10) + (9 x 10) + (9 x 4).

4. **a.** **i.** Efficient strategies include:

Doubling repeatedly: 2 x 26 = 52;
2 x 52 = 104, 2 x 104 = 208

Double and halve repeatedly:
8 x 26 = 4 x 52
= 2 x 104
= 208

Place value: (8 x 20) + (8 x 6)
= 160 + 48
= 208

Using known facts: If you know that 4 x 25 = 100, then 8 x 25 = 200. Add one more lot of 8 so there are 26 lots of 8: 200 + 8 = 208

ii. Efficient strategies include:

Tidy numbers: (50 x 7) – 7 = 350 – 7
= 343

Place value: (40 x 7) + (9 x 7)
= 280 + 63
= 343

b. Reasons will vary.

5. **a.** Efficient strategies include:

i. Double and halve: 1 000 x 4 = 4 000
Place value: (5 x 8) x 100 = 40 x 100
= 4 000

ii. Tidy numbers (because 19 is close to a tidy number): 20 x 25 = 500.
500 – 25 = 475

iii. Divide by 3 and treble (which turns the problem into a known fact): 9 x 9 = 81

b. Answers will vary.

Activity Two

1. Problems will vary, but each one should have at least 1 factor that is close to a tidy number, for example: 3 x 198, 29 x 5, 15 x 28
2. Problems will vary. For doubling repeatedly, one of the factors should be 4, 8, or 16, for example: 4 x 46, 23 x 8. For halving and doubling, the problem should turn into a known fact when halved and doubled, for example: 4 x 16 can be solved as 8 x 8, or 12 x 25 can be solved as 6 x 50.
3. Problems will vary. Place value strategy is especially useful when there is no renaming, for example: 8 x 12 = 8 x 10 + 8 x 2.

4. **a.** One of the numbers in the problem ends in 1, 2, or 3 or in 7, 8, or 9.

b. One number is even and can easily be halved. The other number is easy to double.

5. Practical activity

Pages 8–9: Just Right!

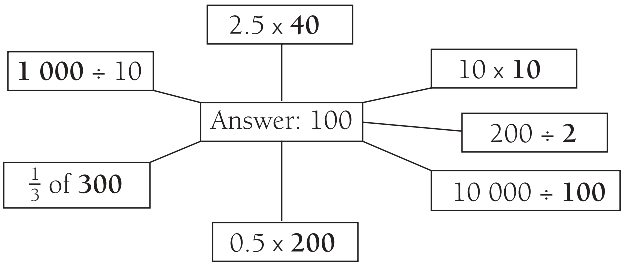
Activity

1. 2.25 ÷ 200 ml = 2 250 ÷ 200
= 11 cups.
(Ignore or drink the remainder.)
2. 75 ÷ 4 = 18 remainder 3. 19 cars are needed. (Ignore the empty seats.)
3. 170 ÷ 38 = 4 remainder 18. Harata thinks that each person will eat 4 or 5 drumsticks. (You know that 160 ÷ 40 = 4, so 170 ÷ 38 is more than 4 each, so there would be 4 or 5 pieces each.)
4. He averages 75 s (or $1\frac{1}{4}$ min) per length. (15 min is 15 x 60 = 900 s. 900 ÷ 12 = 75 s. There is no remainder.)
5. 129 ÷ 11 = 11 remainder 8. The coach can make 11 full teams and have the remaining players as reserves. There will be 8 reserves.
6. 8 ÷ 6 = $1\frac{2}{6}$ or $1\frac{1}{3}$. If the pizzas are cut into sixths, that will be 8 pieces or $1\frac{1}{3}$ pizzas each. There will be no remainder.
7. Between 0 and 8. (Each group of 9 doughnuts is 1 per person.)
8. Problems will vary.

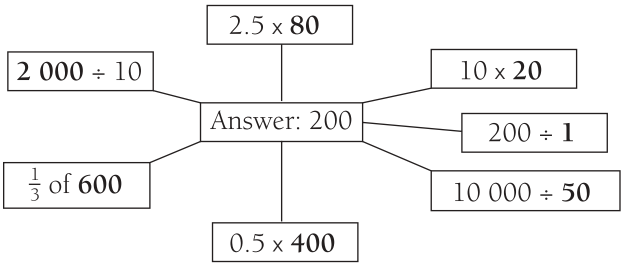
Pages 10–11: Branching Out

Activity One

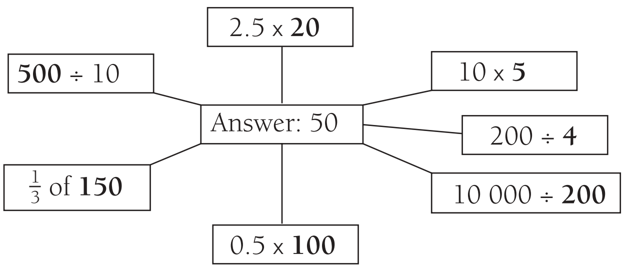
1. Note: The ❑ answers are in bold.



2.



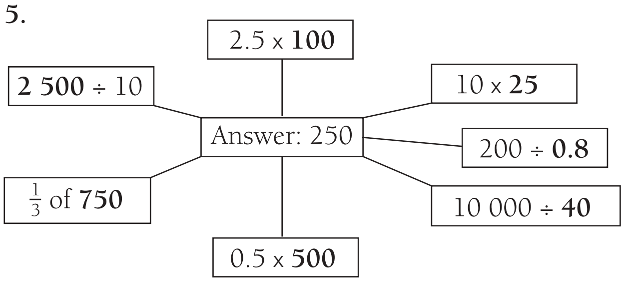
3. You could change your answers from question **1** by halving the ❑ amount when it's a multiplication expression or when it's ❑ ÷ 10 and doubling the ❑ amount in the other division expressions (where the ❑ is the divisor, not the dividend, as in 100 ÷ ❑).
The web would look like this:



4. Answers may vary. The multiplication ❑ numbers and the ❑ ÷ 10 numbers in the 50 web are $\frac{1}{2}$ those in the 100 web, and those in the 200 web are double those in the 100 web.

   The other division ❑ numbers in the 50 web are double those in the 100 web, and those in the 200 web are halved.

5.



Activity Two

1. For the 2 branches that begin with 4 x 5: One factor is halved and the other is doubled each time. For the branch that begins 10 + 10, 9.9 + 10.1: 0.1 (or $\frac{1}{10}$) is taken from one addend and added to the other each time. For the 2 branches that begin with 20 ÷ 1: The dividend (the number being divided) and the divisor (the number doing the dividing) are halved or doubled each time. So for all the branches, what happens to each number in the expression happens to all the numbers in the same position in that branch.

2. The 4 x 5 branches can be extended to 0.25 x 80 and 32 x 0.625 (or 32 x $\frac{5}{8}$).
   The 10 + 10 branch can be extended to 9.8 + 10.2, 9.7 + 10.3, and so on.
   The 20 ÷ 1 branches can be extended to 2.5 ÷ 0.125 and 160 ÷ 8.

   Answers will vary for the 2 branches with $\boxed{?}$ in them. Check yours on a calculator.

Pages 12–13: Run like the Wind

Activity

1. Plans will vary. Iain has 6 x 120 min = 720 min available for training each week. That's 720 ÷ 3 = 240 min for each part of the event. By the end of Tuesday, he has spent a total of 60 + 45 = 105 min swimming, 45 min running, and 90 min cycling.

   For the other 4 days (Sunday is a rest day), he must spend 240 – 105 = 135 min swimming, 240 – 45 = 195 min running, and 150 min cycling.

One possible plan is:

	Swim	Run	Cycle
Mon.	60		60
Tue.	45	45	30
Wed.	30	50	40
Thu.	35	50	35
Fri.	30	50	40
Sat.	40	45	35

On your plan, check that the numbers across each row for each day add up to 120 min, which is how much time Iain spends training each day (except Sunday), and that the numbers down each column add up to 240 min, which is how much time he trains in each sport each week.

2. a. Many different schedules are possible. Here is one:

	Swim	Run	Cycle
Mon.	45	45	30
Tue.	45	45	30
Wed.	60	0	60
Thu.	60	0	60
Fri.	45	45	30
Sat.	65	25	30

If Iain trains for 720 min in total each week and he keeps the cycling the same as before (240 min), he will need to spend 320 min swimming and 160 min running (because 320 is double 160).

On your plan, check that the numbers across each row for each day add up to 120 min, which is how much time Iain spends training each day (except Sunday), and that the swimming column adds up to 320 min, the running column adds up to 160 min, and the cycling column adds up to 240 min.

b. Iain is spending $\frac{4}{9}$ of his time swimming, $\frac{2}{9}$ of his time running, and $\frac{1}{3}$ or $\frac{3}{9}$ of his time cycling.

3. Answers will vary, but here is one possible division of Iain's total training time:

280 min Swim	160 min Run	280 min Cycle

4. a. $\frac{2}{12}$ or $\frac{1}{6}$

b. Just over 6 wks ($1\frac{1}{2}$ mths)

c. No. $\frac{1}{4}$ of a yr = 3 mths, $\frac{1}{3}$ of a yr = 4 mths, $\frac{1}{8}$ of a yr = $1\frac{1}{2}$ mths.
$3 + 4 + 1\frac{1}{2} + 2$ (tennis) = $10\frac{1}{2}$, which is $1\frac{1}{2}$ mths short of a yr.

5. If they are fractions, add them. They will add up to 1 or to an equivalent fraction such as $\frac{5}{5}$, $\frac{10}{10}$, or $\frac{12}{12}$. If they are percentages, they will all add up to 100 percent. (You could use a fraction kit and see if all the fraction pieces will join together to make a whole pie, or you could shade in a squared diagram and see if all the fractions and percentages fill up the whole box.)

Pages 14–15: The Power of Powers

Activity One

1.

Power of 10	10^5	10^4	10^3	10^2
Value	100 000	10 000	1 000	100

2. a. i. 100 000 000

ii. 10

iii. 1

b. Answers will vary, but the basic idea is that every time you multiply by 10, the 1 moves along one place to the left. As the power gets bigger, you multiply by more lots of 10 and move the 1 along more places.
When you are reducing the power by 1, for example, from 10^3 to 10^2, you are dividing by 10. 10^2 is 100, so 10^1 is $100 \div 10 = 10$. $10 \div 10 = 1$, so 10^0 is 1.

Activity Two

1. **a.** On your place value houses, you should have these powers of 10:

	Term	Trillions			Billions			Millions			Thousands			Ones		
		h	t	o	h	t	o	h	t	o	h	t	o	h	t	o
i.	10^6									1	0	0	0	0	0	0
ii.	10^{11}				1	0	0	0	0	0	0	0	0	0	0	0
iii.	10^9						1	0	0	0	0	0	0	0	0	0
iv.	10^0															1

b. **i.** 10^6: one million

ii. 10^{11}: one hundred billion

iii. 10^9: one billion

iv. 10^0: one

2. **a.** **i.** 1 000 000 (100 x 100 x 100)

ii. 1 000 000 (1 000 x 1 000)

iii. 100 000 000 (10 000 x 10 000)

iv. 100 000 000 (100 x 100 x 100 x 100)

b. 100 can be written as 10 x 10, so 100^3 can be written as (10 x 10) x (10 x 10) x (10 x 10), which is 10^6 or 1 000 000.
1 000 can be written as 10 x 10 x 10, so $1\,000^2$ can be written as (10 x 10 x 10) x (10 x 10 x 10), which is also 10^6 or 1 000 000.
A similar process can be used to show that $10\,000^2 = 100^4$.

c. **i.** A billion (1 000 000 000)

ii. A trillion (1 000 000 000 000)

iii. A hundred million (100 000 000)

3. **a.** 100 tens

b. 100 000 tens

c. 1 ten

d. 1 000 tens

e. 1 000 tens

f. 100 000 tens

4. **a.** 10 hundreds

b. 100 hundreds

c. 10 000 hundreds

d. 10 000 hundreds

e. 10 000 hundreds

Pages 16–17: Crafty Combinations

Activity

1. 55. (1 + 10 = 11, 2 + 9 = 11, 3 + 8 = 11, 4 + 7 = 11, 5 + 6 = 11. 5 x 11 = 55)

2. Yes. (12 + 10 = 22, 11 + 11 = 22, 13 + 9 = 22, 14 + 8 = 22. 4 x 22 = 88)

3. $179. (21 + 29 = 50, 26 + 24 = 50, 27 + 27 = 54. 25 + 50 + 50 + 54 = 179)

4. 2 000. 800 + 200 = 1 000, 14 + 986 = 1 000, 555 + 445 = 1 000, and – 999 – 1 = – 1 000. 1 000 + 1 000 + 1 000 – 1 000 = 2 000

5. **a.** 14. (One way is: 4 lots of $\frac{1}{2}$ = 2. 1 + 1 = 2, 2 + 2 = 4, 3 + 3 = 6. 2 + 2 + 4 + 6 = 14. Another way is to combine pairs of numbers to make 4: $\frac{1}{2} + 3\frac{1}{2} = 4$, 1 + 3 = 4, $1\frac{1}{2} + 2\frac{1}{2} = 4$. 3 x 4 + 2 = 14)

b. 8. (Look for decimals that add up to 2. 0.1 + 1.9 = 2, 1.2 + 0.8 = 2, 0.5 + 1.5 = 2, 1.6 + 0.4 = 2. 2 + 2 + 2 + 2 = 8)

c. 6. Look for fractions that add up to 1.
$\frac{3}{4} + \frac{1}{4} = 1$, $\frac{2}{5} + \frac{3}{5} = 1$,
$\frac{2}{3} + \frac{1}{3} = 1$, $\frac{4}{7} + \frac{3}{7} = 1$.
$1 + 1 + 1 + 1 = 4$. $10 - 4 = 6$.

6. a. They all contain numbers that combine to give whole numbers that are easy to add or subtract.

b. Not always. Sometimes the numbers will not add up to whole numbers that are easy to add or subtract.

Pages 18–20: Fraction Tagging

Activity

1. a. i. $\frac{4}{8}$
ii. $\frac{1}{2}$
iii. $\frac{2}{4}$
iv. $\frac{3}{6}$
b. i. $\frac{4}{6}$
ii. $\frac{3}{4}$
iii. $\frac{8}{10}$
iv. $\frac{5}{10}$

2. a. 5. ($\frac{1}{2}$ of 10)
b. 5. ($\frac{1}{2}$ or $\frac{2}{4}$ of 10)
c. 2. ($\frac{1}{5}$ of 10)
d. 7. ($\frac{7}{10}$ of 10)

3. a. 12. ($\frac{1}{2}$ of 24)
b. 16. ($\frac{2}{3}$ of 24. $\frac{1}{3}$ is 8, so $\frac{2}{3}$ is 16.)
c. 12. ($\frac{2}{4}$ or $\frac{1}{2}$ of 24)
d. 20. ($\frac{5}{6}$ of 24. $\frac{1}{6}$ is 4, so $\frac{5}{6}$ is 20.)
e. 9. ($\frac{3}{8}$ of 24. $\frac{1}{8}$ is 3, so $\frac{3}{8}$ is 9.)
f. 2.4. ($\frac{1}{10}$ of 24)

4. a. $7\frac{1}{2}$ or 7.5. The row is divided into 2 equal parts, so, **a** is half of 15.

b. 5. The row is divided into 3 equal parts, so **b** is $\frac{1}{3}$ of 15.

c. 9. The shaded area is equal to 3 out of 5 parts ($\frac{3}{5}$). $\frac{1}{5}$ of 15 is 3, so $\frac{3}{5}$ is 9.

d. 4.5 or $4\frac{1}{2}$. The shaded area is equal to 3 out of 10 parts ($\frac{3}{10}$).
$\frac{1}{10}$ of 15 is 1.5 or $1\frac{1}{2}$, so $\frac{3}{10}$ is 4.5 or $4\frac{1}{2}$.

5. Problems will vary.

Page 21: Calculation Chains

Activity

1. a. 1
b. 909 090
c. 5 050 505
d. 3 939 393
e. 1 012

2. a. 11
b. 9
c. 5
d. 100
e. 0.5

3. a. 5
b. 50
c. 80
d. 4
e. 0.5

4. Problems will vary.

Pages 22–23: Taking Turns

Activity

1. Room 1:
2 hrs. 12 x 20 = 240 min. 240 ÷ 60 = 4.
4 hrs ÷ 2 computers = 2 hrs per computer.
Or: 12 students at 2 computers is 6 students per computer.
6 x 20 min = 120 min or 2 hrs per computer.
Room 2:
4 hrs. 6 pairs x 40 min = 240 min, which is 4 hrs.
Room 3:
4 hrs. 24 x 20 = 480 min, which is 8 hrs.
8 hrs ÷ 2 computers = 4 hrs per computer.
Or: 24 students at 2 computers is 12 students per computer. 12 x 20 min = 240 min or 4 hrs per computer.

Room 4:

2 hrs. 24 x 20 = 480 min, which is 8 hrs. 8 hrs ÷ 4 computers = 2 hrs per computer. Or: 24 students at 4 computers is 6 students per computer. 6 x 20 min = 120 min or 2 hrs per computer.

Room 5:

1 hour 40 min. 15 pairs x 20 = 300 min. 300 min ÷ 3 computers = 100 min per computer, which is 1 hr 40 min. Or: 30 students at 3 computers in pairs is 5 pairs per computer. 5 x 20 min = 100 min, which is 1 hr 40 min.

2. Strategies will vary. Note that the school day is 6 hrs (9 a.m.–3.00 p.m.)

Room 1:

1 hr per day per student
Possible working: 12 students at 2 computers is 6 students per computer. 6 hrs shared between 6 students is 1 hr each.

Room 2:

1 hr per day per pair (shared time)
Possible working: 12 students share 1 computer in pairs, so there are 6 pairs. 6 hrs shared between 6 pairs is 1 hr per pair.

Room 3:

30 min ($\frac{1}{2}$ hr) per day per student
Possible working: 24 students at 2 computers is 12 students per computer. 6 hrs shared between 12 students is 30 min each.

Room 4:

1 hr per day per student
Possible working: 24 students at 4 computers is 6 students per computer. 6 hrs shared between 6 students per computer is 1 hr each.

Room 5:

1 hr 12 min per day per pair (shared time)
Possible working: 30 students in pairs is 15 pairs. 15 pairs at 3 computers is 5 pairs per computer.
6 hrs shared between 5 pairs is 72 min or 1 hr 12 min each.

3. Answers may vary. Room 5 students could get the most time, but it is in pairs. Rooms 1 and 4 students could get the most time by themselves on the computer. You could argue that Room 5's time is less hands-on time per student because Room 5 students work in pairs, so they could only interact with the computer half of their time, or 36 min.

4. Answers will vary. There are 12 computers and 102 students. There should be 1 computer for every 8.5 students (1:8.5), but it is not possible to share them evenly because of the class numbers. If all the students used the computer in pairs or if they all used them individually, it would be fairer if Rooms 1 and 2 both had 1 computer each (1:12), Rooms 3 and 4 both had 3 computers each (1:8), and Room 5 had 4 computers (1:7.5). It would be even fairer if the school bought an extra computer for Rooms 1 and 2 to share because then they would have a ratio of 1 computer for every 8 students (1:8 instead of 1:12). Another alternative is for Rooms 1 and 2 to share 3 computers (1:8), Rooms 3 and 4 to have 3 computers each (1:8), and Room 5 to have 3 computers (1:10).

Investigation

Results will vary.

Page 24: Planning Paths

Activity

1. a.

Number of flowers	Number of hexagonal tiles	Number of triangular tiles
1	4 + 2 = 6	6
2	4 + 4 + 2 = 10	6 + 6 = 12
3	4 + 4 + 4 + 2 = 14	6 + 6 + 6 = 18
4	4 + 4 + 4 + 4 + 2 = 18	6 + 6 + 6 + 6 = 24
5	4 + 4 + 4 + 4 + 4 + 2 = 22	6 + 6 + 6 + 6 + 6 = 30
12	4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 2 = 50	6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 = 72
20	4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 2 = 82	6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 = 120

b.

	Number of flowers	Number of hexagonal tiles	Number of triangular tiles
i.	12	(12 x 4) + 2	12 x 6
ii.	20	(20 x 4) + 2	20 x 6

c. Number of hexagonal tiles = (the number of flowers x 4) + 2
Number of triangular tiles = the number of flowers x 6

2. a. (8 x 4) + 2 = 34 hexagonal tiles and 8 x 6 = 48 triangular tiles

b. (16 x 4) + 2 = 66 hexagonal tiles and 16 x 6 = 96 triangular tiles

c. (100 x 4) + 2 = 402 hexagonal tiles and 100 x 6 = 600 triangular tiles

Investigation

1.–2. Practical activity

Figure It Out

Number Sense and Algebraic Thinking

Teachers' Notes

Overview of Level 3: Book One

Title	Content	Page in students' book	Page in teachers' notes
Taking Time	Exploring units of time	1	17
Tidying Up	Comparing addition and subtraction part–whole strategies	2–4	20
Megabytes of Memory	Exploring patterns and relationships	5	22
What's Best?	Using different strategies to solve multiplication problems	6–7	24
Just Right!	Solving division problems with remainders	8–9	27
Branching Out	Using operations and exploring relationships	10–11	29
Run like the Wind	Working with fractions and ratios	12–13	32
The Power of Powers	Linking place value to powers of 10	14–15	33
Crafty Combinations	Using combining strategies	16–17	35
Fraction Tagging	Finding fractions of sets	18–20	37
Calculation Chains	Using place value in operations	21	39
Taking Turns	Using relationships to calculate amounts	22–23	40
Planning Paths	Finding and using generalisations for patterns in geometric designs	24	41

Introduction to Number Sense and Algebraic Thinking

The *Number Sense and Algebraic Thinking* books in the Figure It Out series provide teachers with material to support them in developing these two key abilities with their students. The books are companion resources to Book 8 in the Numeracy Project series: *Teaching Number Sense and Algebraic Thinking*.

Number sense

Number sense involves the intelligent application of number knowledge and strategies to a broad range of contexts. Therefore, developing students' number sense is about helping them gain an understanding of numbers and operations and of how to apply them flexibly and appropriately in a range of situations. Number sense skills include estimating, using mental strategies, recognising the reasonableness of answers, and using benchmarks. Students with good number sense can choose the best strategy for solving a problem and communicate their strategies and solutions to others.

The teaching of number sense has become increasingly important worldwide. This emphasis has been motivated by a number of factors. Firstly, traditional approaches to teaching number have focused on preparing students to be reliable human calculators. This has frequently resulted in their having the ability to calculate answers without gaining any real understanding of the concepts behind the calculations.

Secondly, technologies – particularly calculators and computers – have changed the face of calculation. Now that machines in society can calculate everything from supermarket change to bank balances, the emphasis on calculation has changed. In order to make the most of these technologies, students need to develop efficient mental strategies, understand which operations to use, and have good estimation skills that help them to recognise the appropriateness of answers.

Thirdly, students are being educated in an environment that is rich in information. Students need to develop number skills that will help them make sense of this information. Interpreting information in a range of representations is critical to making effective decisions throughout one's life, from arranging mortgages to planning trips.

Algebraic thinking

Although some argue that algebra only begins when a set of symbols stands for an object or situation, there is a growing consensus that the ideas of algebra have a place at every level of the mathematics curriculum and that the foundations for symbolic algebra lie in students' understanding of arithmetic. Good understanding of arithmetic requires much more than the ability to get answers quickly and accurately, important as this is. Finding patterns in the process of arithmetic is as important as finding answers.

The term "algebraic thinking" refers to reasoning that involves making generalisations or finding patterns that apply to *all* examples of a given set of numbers and/or an arithmetic operation. For example, students might investigate adding, subtracting, and multiplying odd and even numbers. This activity would involve algebraic thinking at the point where students discover and describe patterns such as "If you add two odd numbers, the answer is always even." This pattern applies to *all* odd numbers, so it is a generalisation.

Students make these generalisations through the process of problem solving, which allows them to connect ideas and to apply number properties to other related problems. You can promote process-oriented learning by discussing the mental strategies that your students are using to solve problems. This discussion has two important functions: it gives you a window into your students' thinking, and it effectively changes the focus of problem solving from the outcome to the process.

Although the term "algebraic thinking" suggests that generalisations could be expressed using algebraic symbols, these Figure It Out *Number Sense and Algebraic Thinking* books (which are aimed at levels 2–3, 3, and 3–4) seldom use such symbols. Symbolic expression needs to be developed cautiously with students as a sequel to helping them recognise patterns and describe them in words. For example, students must first realise and be able to explain that moving objects from one set to another does not change the total number in the two sets before they can learn to write the generalisation $a + b = (a + n) + (b - n)$, where n is the number of objects that are moved. There is scope in the books to develop algebraic notation if you think your students are ready for it.

The Figure It Out *Number Sense and Algebraic Thinking* books
The learning experiences in these books attempt to capture the key principles of sense-making and generalisation. The contexts used vary from everyday situations to the imaginary and from problems that are exclusively number based to those that use geometry, measurement, and statistics as vehicles for number work. Teachers' notes are provided to help you to extend the ideas contained in the activities and to provide guidance to your students in developing their number sense and algebraic thinking.

There are six *Number Sense and Algebraic Thinking* books in this series:
Levels 2–3 (Book One)
Levels 2–3 (Book Two)
Level 3 (Book One)
Level 3 (Book Two)
Levels 3–4 (Book One)
Levels 3–4 (Book Two)

Teaching strategies

Many of the activities in the student book lend themselves to the use of the following teaching strategies:

- Get the students to discuss each question (or a particular question) in small groups first before sharing with the whole group. One way to promote effective listening is to have a student share another person's idea from their small-group discussion. Tell the students that you expect each member of the group to be able to explain the group's solution and that any one of them could be asked to be the reporter.
- To encourage reflective discussion at the end of an activity, you could write some sentence starters on the board and ask your students to complete at least one sentence orally or in writing. For example:
  - A strategy I tried was …
  - I noticed that …
  - I found a short cut for …
  - Something I want to remember is …
  - Today I learnt …
  - A pattern I used was …
  - It was hard, but I managed to …
  - It was satisfying when …

Ask:

Did you see or hear someone else use a good strategy today? What was it?
What useful tips could you give a friend in another group before they did this activity?

Page 1: Taking Time

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 3)
- read and interpret everyday statements involving time (Measurement, level 3)

Other mathematical ideas and processes

Students will also explore patterns and relationships (Algebra, level 3).

Number Framework Links

AC
EA
AA
AM
AP

Use this activity to help your students to consolidate and apply advanced multiplicative part–whole strategies (stage 7) and knowledge in the domain of multiplication and division.

Activity

In this activity, students use different units of time to measure age. They explore the relationships between units such as seconds, minutes, hours, years, centuries, and millennia. The calculations required in the activity involve large numbers, so the use of calculators is appropriate. Recognising and working with the relationships between the units involves multiplicative thinking that may be quite difficult for some students, so they may need extra help from you.

In their calculations, the students will need to use their knowledge of the number of seconds in a minute, the number of minutes in an hour, the number of hours in a day, and the number of days in a year.

You could introduce an independent group to the activity by saying: *I can measure time in days or hours. What other units can I use?* (Centuries, years, months, weeks, minutes, seconds, milliseconds)

You may need to define these terms:

- Unit: any standard amount that we use for measuring how much or how little we have of something. (Illustrate this with some examples.) For more on units, see the Measurement section of Book 9 in the Numeracy Project series.
- Leap year: a year in which 29 February is an extra day, so there are 366 days instead of 365. (Note that, for this activity, the students are instructed to ignore leap years in their calculations, although the investigation challenges them to find out why leap years are necessary.)
- Millennium: a thousand years.

Divide a guided teaching group into smaller groups of 3–4 students and say: *Brainstorm as many units of time as you can with your group. Write each one onto a small piece of paper. Now put your pieces of paper into order from the biggest unit of time to the smallest.* (Possible units are: nanosecond [1 thousand millionth of a second], millisecond [1 thousandth of a second], second, minute, hour, day, week, month, year, decade, century, and millennium.)

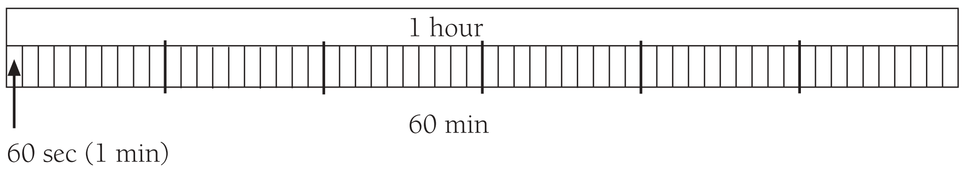
While the students are in their small groups, ask the questions in the activity orally, referring to the pictures and speech bubbles in the students' books.

The calculator has an important role in this activity because it allows the students to focus on the relationships between the units of time rather than on the actual calculations. Estimation and number sense is still vital when using a calculator because it is very easy to make errors entering the numbers. Encourage the students to "estimate in reverse" by looking at the result they have on the calculator and then judging whether or not it is a sensible answer to the problem they are solving.

In order to develop estimation skills, students need to develop some benchmarks to judge their estimates against. Useful benchmarks for question **1** are the numbers of days, hours, minutes, and seconds there are in 1 year. For example, the students can work out that there are 365 x 24 = 8 760 hours in a year, which is about 9 000. If they use 9 000 as a benchmark for the number of hours in a year, they are better able to estimate how many hours old a 10-year-old would be, as well as to identify which of the students in the activity might be measuring their age using hours.

The students will also need to understand the proportional nature of units. The smaller the units used to measure a given amount, the greater the number required:

10 years	1	1	1	1	1	1	1	1	1	1
520 weeks	52	52	52	52	52	52	52	52	52	52



After the students have completed the activity, you can promote algebraic thinking by posing questions that focus on how the units are related. The group can then talk about these in a think-pair-share discussion. Possible questions and student ideas (using calculators) are:

If you know how many hours old someone is, what would you do to work out how many seconds old they are?

"I'd multiply the number of hours by 60 (to get minutes) and then by 60 again (to get seconds)."

"I'd multiply by 3 600 because that's how many seconds there are in an hour. I know there's 3 600 seconds in an hour because I worked out 60 lots of 60 seconds."

If you know how many hours old a person is, how would you work out how many centuries old they are?
"I'd divide the number of hours by the number of hours in a year (8 760) to find out how old they are in years and then divide that by 100 (the number of years in a century). I know there's 8 760 hours in a year because I worked out 24 hours in a day x 365 days."

If the students are having difficulty, help them build an understanding of the relationships between the units by solving simpler problems first. Diagrams such as those above should help. Possible questions and student ideas (using calculators) are:
How would you work out how many seconds there are in an hour?
"I know there are 60 seconds in 1 minute and 60 minutes in 1 hour, so I would work out 60 lots of 60 seconds. 60 x 60 = 3 600. There are 3 600 seconds in an hour."
How could you use that number (3 600) to help you work out how many seconds there are in a week?
"I know there are 24 hours in a day, so there are 24 lots of 3 600 seconds in a day.
24 x 3 600 = 86 400. There are 7 days in a week, so that's 7 lots of 86 400 seconds.
7 x 86 400 = 604 800 seconds."
Can you work out how many hours there are in a year?
"I know there are 365 days in 1 year. So I multiplied the number of days in the whole year (365) by the number of hours in 1 day (24). 365 lots of 24 hours = 365 x 24
= 8 760 hours."
How could you use that number (8 760) to help you work out how many minutes and seconds there are in a year?
"I'd multiply the number of hours in a whole year (8 760) by the number of minutes in 1 hour (60). 8 760 lots of 60 minutes is 8 760 x 60 = 525 600 minutes."
"I'd multiply the number of minutes in a year (525 600) by the number of seconds in 1 minute (60). 525 600 lots of 60 seconds is 525 600 x 60 = 31 536 000 seconds."
If you know how many hours there are in 1 year, how could you work out how many hours old a 5-year-old would be?
"I'd work out 5 lots of the number of hours in a year (8 760). 5 x 8760 = 43 800 hours."
If you know how many hours old a 5-year-old is, how could you use that number to work out how many seconds old he or she is?
"I'd work out 3 600 lots of 43 800, which is the number of seconds in 1 hour multiplied by the 5-year-old's age in hours. (I know the number of seconds in 1 hour because I went 60 minutes x 60 seconds.) 3 600 x 43 800 = 157 680 000 seconds"
"I'd multiply the number of hours (43 800) by 60 (minutes in each hour) and multiply it again by 60 (seconds in each minute), which would make the 5-year-old 157 680 000 seconds old."

Extension
Possible investigations include:
- How many seconds do we spend at school each year?
- Find out the average life expectancy of a person like yourself living in New Zealand (with the same birth year, gender, and ethnicity). Based on your daily routine now, how long will you spend over your entire life doing activities such as sleeping, watching television, reading, travelling, talking on the phone, playing sport, doing chores, and doing other activities?
- How do scientists measure very short or very long periods of time?

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)
Some extension ideas for this activity can be found in:

- *Book 8: Teaching Number Sense and Algebraic Thinking*
  Leap Years, page 35
  Turn of the Century, page 35
  The Lunar Year (lunar-based calendars), page 36
- *Book 9: Teaching Number through Measurement, Geometry, Algebra, and Statistics*
  Measurement section, pages 3–18.

Pages 2–4: Tidying Up

Achievement Objectives

- use equipment appropriately when exploring mathematical ideas (Mathematical Processes, problem solving, level 3)
- record, in an organised way, and talk about the results of mathematical exploration (Mathematical Processes, communicating mathematical ideas, level 3)

Other mathematical ideas and processes

Students will also:

- explore patterns and relationships
- explore computation and estimation.

Number Framework Links

AC
EA
AA
AM
AP

Use these activities to:

- promote transition from early additive strategies (stage 5) to advanced additive strategies (stage 6) by supporting the students' use of a range of addition and subtraction strategies
- help the students to extend and consolidate advanced additive part–whole strategies (stage 6) in addition and subtraction.

Numeracy Professional Development Project learning experiences that will help to develop students' understanding of these strategies are listed at the end of the notes for this activity. (Many of the notes in this book contain similar references.)

Activities One, Two, and Three

In these activities, students use and compare strategies for addition and subtraction, focusing on tidy numbers and compensation and on place value partitioning strategies.

These activities would be useful as a follow-up to exploring tidy number and place value strategies. Here is a useful summary of these strategies:

- In the tidy number and compensation strategy, students add or subtract a close tidy number, usually a multiple of 10 or 100, then adjust the answer to compensate. For example, $53 - 18 = (53 - 20) + 2$.
- In the place value partitioning strategy, students break the numbers up into their place value components and then add or subtract them one part at a time. For example, to solve $58 - 23$, the 23 could be partitioned into $(20 + 3)$ and taken off one part at a time. $58 - 20 = 38$, then $38 - 3 = 35$. Another way to use this strategy is to break up both the numbers so $58 - 23$ becomes $(50 + 8) - (20 + 3)$. This can be solved by grouping place value numbers: $50 - 20 = 30$ and $8 - 3 = 5$. $30 + 5$ is 35.

Students will need to work in pairs or small groups to complete parts of these activities.
Before they start, ensure that they understand what Alex has done in **Activity One** by asking them to

solve 49 + 25 using his strategy. For example: $49 + 25 = (50 + 25) - 1$

$$= 75 - 1$$
$$= 74$$

Ask:

Why is it a good idea to know how to solve a problem using lots of different strategies?
(Different numbers suit different strategies, and the best strategy is the one that you find quickest and easiest in that situation. By using different strategies, we learn more about how numbers behave when we use the operations.)
When a strategy is "efficient", what does that mean? (It gets you the answer quickly and easily, with as few steps as possible. This usually means less mental effort is required.)

You may need to define these terms:

- Tidy number: multiples of 10, 100, and 1 000.
- Equation: a number sentence that contains an equals sign.

These activities are also suitable for use with a guided teaching group. The initial parts of **Activity One** and **Activity Three** lend themselves to the Numeracy Project strategy teaching model that uses materials, images, and number properties. That is, the students move from materials to symbols to words. (See *Book 3: Getting Started*.)

Students who need to use materials could solve Alex's problem (or Raukura's in **Activity Three**) using place value equipment such as canisters and beans, bundled sticks, or tens money ($100, $10, $1). Record what the students do on an empty number line when they share their solution paths with the group.

When the students are able to solve the problems successfully, promote imaging by setting up the beans and canisters behind a screen for each problem and asking the students to describe what is there and what they would do to manipulate the materials to solve the problem. Alternatively, you could ask them to describe the hops they would make on an empty number line. If necessary, let the students remove the screen and move the materials or draw the hops on a number line as they share their solution paths with the group.

Using number properties is the next step. Students who are using imaging successfully to solve problems may be able to move on to using just the numbers. They need to understand the number properties involved before going on to compare strategies or to classify the types of problem suited to a particular strategy.

Some students are reluctant to use a variety of strategies and prefer to use one trusted method, even if it is less efficient. Some ways to give your students more practice in trying different strategies are:

- Suggest: *Let's all use Alex's method to solve this next problem.*
- Have group discussions about why being able to use a variety of strategies is useful.
- When groups are sharing their different strategies, record them all on one empty number line, using different colours for each strategy. Ask the group which strategies were most efficient (that is, that were streamlined and fast, needing fewer hops on the number line).
- Sometimes, deliberately choose numbers for problems that would be cumbersome for the student's preferred method but very quick using a different strategy.
- Emphasise the importance of efficiency by asking questions such as: *Now that you've solved this problem, what would you do differently next time with a similar problem?* or *Is there a faster way of solving that problem?*

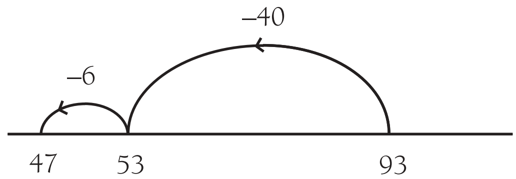
Many of the problems in these activities could also be solved efficiently by adding on. For example, for 64 – 38: "I started at 38 and hopped up 2 to 40, then 24 more to 64. 2 + 24 = 26." The intention of this activity is to compare the efficiency of different strategies, so welcome any alternative efficient strategies that are offered for discussion.

Key questions to promote algebraic thinking include:
What is it about the numbers in that problem that made you choose that strategy?
Can you make up another problem that would be good for the tidy number strategy or the place value strategy?

For **Activity One**, question **3**, and **Activity Three**, question **3**, ask the students when tidy number or place value strategies would be most useful. You can use this to observe whether the students are able to articulate any generalisations they may have made.

Tidy number strategies are always useful when one number in an addition problem is close to a tidy number (that is, it ends in 1, 2, 3 or in 7, 8, 9) or in subtraction problems when the number you are taking away is close to a tidy number (such as 65 – 29).

Place value strategies are most useful in addition problems where there is no renaming (for example, 42 + 53) or in subtraction problems where the digits in the number you are taking away are smaller than the ones in the same place in the larger number so that you don't have to do any renaming (for example, 76 – 32). Place value strategies are always useful, but they are not always convenient. For example, 93 – 46 involves renaming, but it works if "back through 10" is known:



Some suggestions for reflective discussion include:
Is there a strategy that you've seen or used today that you'd like to use more often?
What kinds of numbers or problems will you be looking out for to try with this strategy? Give me an example.

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)

- *Book 5: Teaching Addition, Subtraction, and Place Value:*
  Don't Subtract – Add! (using addition to solve subtraction problems), page 34
  Problems like 23 + ❑ = 71 (adding tidy numbers and compensating), page 35
  Problems like 73 – 19 = ❑ (subtracting tidy numbers and compensating), page 38
  Problems like 37 + ❑ = 79 (using place value partitioning), page 36.

Page 5: Megabytes of Memory

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)

AC
EA
AA
AM
AP

Number Framework Links

This activity gives students an opportunity to apply advanced multiplicative part–whole (stage 7) strategies in multiplication.

Activity

In this activity, students explore the relationship between the price and the size of various memory cards for digital cameras. For the investigation in question **4**, the students will need to have access to a digital camera, its manual, digital camera brochures, or the Internet.

Introduce an independent group to this activity by discussing how digital photos are stored on a memory card (some cameras use memory sticks or disks), the difference between high and low resolution (the higher the resolution setting, the more memory is taken up by each photo), and how you can buy different amounts of memory. Memory is measured in megabytes (MB).

You may need to define these terms:

- Low resolution: the dots (pixels) that make up the picture are not as close together as they are in a high-resolution picture. A low-resolution picture takes up less memory than a high-resolution picture, but it isn't as sharp or clear.
- Megabyte (MB): a unit that measures how much information can be stored on a computer.

With a guided teaching group, you could motivate the students' interest in this activity by letting them take some photos on a digital camera, experiment with the resolution setting, and compare the results. Discuss storage, resolution, and memory as above.

For question **1**, encourage the students to identify patterns in the different sizes and prices of the memory cards by suggesting that they organise the information in the picture into a table, with a column for the sizes and another for the prices. Ask: *What patterns can you see in the memory card sizes?* (They double in size each time.)
Does this same pattern happen with the prices? (No. They increase by $30 or $40 each time.)

For question **2**, encourage the students to work out mentally, in groups of 3–4, how many photos will fit on each card and then to share their answers and strategies with the whole group. This will encourage higher levels of participation and give you opportunities to listen, to support thinking, and to make formative assessments of individuals.

Some students may work out that 5 x 32 MB is 160 photos and then double this number successively for each of the other cards, for example, 160 x 2 = 320 photos will fit on the 64 MB card. Others may work out, for each card, "5 lots of ❑ MB" or "5 x ❑, where ❑ is the number of MB on a card". Strategies for multiplying by 5 include:

- place value partitioning: 5 x 32 = (5 x 30) + (5 x 2)
- doubling and halving: 5 x 32 = 10 x 16

For question **3**, again ask the students to investigate this in small groups before sharing their ideas with the whole group.

Encourage the students to work systematically by asking questions such as:
How could you make sure that you have recorded all the different options?
Could you show your information in a table? (See the table given in the Answers.)
Are there any options where Beth can buy 3 cards without having to spend the whole $250?
Are there any card sizes that Beth can't afford? (Remember, she needs to buy cards for 3 digital cameras.)

Extension
If Beth had $300 to buy the 3 memory cards instead of just $250, how might this change her spending decisions?

This activity could also lead into an exploration of the power of 2:

$4 = 2 \times 2$
$\quad = 2^2$
$8 = 2 \times 2 \times 2$
$\quad = 2^3$
$16 = 2 \times 2 \times 2 \times 2$
$\quad = 2^4$

Ask:
Can you use this pattern to work out the power of 2 that equals 32, 64, and the capacity of the other memory cards?
Can you use the pattern to work out what 2^1 and 2^0 equal? ($2^1 = 2$, $2^0 = 1$. For an explanation of 10^0 (and therefore 2^0), see the answers and notes for the Power of Powers activity in this book.)

Pages 6–7: What's Best?

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- state the general rule for a set of similar practical problems (Algebra, level 3)

AC
EA
AA
AM
AP

Number Framework Links

These activities will help students to consolidate advanced multiplicative part–whole thinking (stage 7) by comparing a range of strategies for multiplying whole numbers.

Activities

Students need to be able to choose appropriately from a broad range of mental strategies when solving problems. These activities are designed to help them develop that ability by having them compare and identify the types of problems and numbers that would best suit each multiplication strategy.

Students need to be at least progressing towards being advanced multiplicative part–whole thinkers (stage 7) to complete these activities. They need to understand how to use each of the strategies outlined below and have good recall of their multiplication basic facts.

These activities give students lots of opportunities to come to a shared understanding of terminology such as "adding on", "using place value", and "doubling and halving", which they need when talking about their problem-solving strategies.

Activity One

Before the students look at the activity, pose the problem: *There are 4 teams of 18 players going to the rugby championships. How many players are going altogether?*

Record 4 x 18 = ❑ on the board or in the class modelling book (a scrapbook for recording outcomes and strategies). Encourage the students to describe their solution path with a classmate or small group before the range of strategies is shared in the main group. The students may need to use materials to help them understand each strategy.

Record all the different ways of calculating the answer on the board as they are described and reflect back what each student said, using phrases such as: *So you went … and then you … Is that how you did it?* This gives the listeners visual support as well as another chance to hear each explanation.

Now ask the students to read the strategies used by Ashleigh's group in **Activity One**.
Then ask:
Which strategies are the same as ours?
Which ones are different? (Record any new ones on the board.)
Can you model these strategies with the place value materials?

The students can approach the questions in the activity orally as think-pair-share or group discussion starters. They should record their thinking so that they can report back. The chart in the Answers provides a suitable structure for recording their responses to question **1**.

Activity Two

Before starting this activity, draw a table with three columns on the board:

Tidy number strategies	Doubling strategies	Place value strategies

Then ask:
What are some examples of problems from ***Activity One*** *that can be solved quickly and easily with these strategies?*

Tidy number strategies	Doubling strategies	Place value strategies
49 x 7	4 x 18	4 x 18
8 x 26	500 x 8	9 x 24
19 x 25		

(Some problems may be suited to more than one strategy. Note that 27 x 3 best suits a thirding and trebling strategy.)

What do you notice about the numbers in the tidy number column? (One number in each problem is close to a tidy number, for example, 49 and 19.)
What do you notice about the numbers in the doubling strategies column? (One number is 4 or 8; both numbers are even.)

This activity provides opportunities for promoting algebraic thinking. Question **4** in **Activity Two** asks students to identify similarities between problems that are best solved using tidy number strategies or doubling and halving. This takes them away from specific examples to properties that are always true for all numbers.

At the end of the activity, the students make up their own problems, and you could use these to assess who has understood and generalised the characteristics of the problems that suit each strategy. Students should be able to explain why they chose particular numbers. Note that doubling and halving is one of a family of strategies that includes thirding and trebling, for example, 18 x 33 = 6 x 99. Doubling repeatedly is one example of a family that uses factors of a factor, for example, 6 x ❑ = 3 x 2 x ❑ or 2 x 3 x ❑. Place value works on any problem, but renaming sometimes makes it difficult. Student reasons could include:

- For a tidy number strategy:
  "I want one of the numbers in the problem to be close to a tidy number so that it's easy to round in my head. If I wanted it to be a 2-digit number, I could choose 1, 2, 3, 7, 8, or 9 for the ones digit, for example, 41 or 49 are close to 40 and 50. If I wanted it to be a 3-digit number, I could make it ❑ hundred and ninety ❑, for example, 693 or 697, which are both close to 700. So I could end up with problems like 6 x 49 = 6 x 50 – 6 or 3 x 697 = 3 x 700 – 9."
- For doubling and halving:
  "I want to turn the problem into an easier problem or a fact I know. I want at least one of the numbers in the problem to be even so that it's easy to halve, for example, 6, 8, 10, 12, 14, 16, 18, or 20. (I need to know my multiplication facts to use these.) I want the other number to end up being a times table I know when it's doubled, for example, if I chose 4, I could use my 8 times tables. I know how to multiply by tens or hundreds, so I could make the second number half of a tens or hundreds number, for example, 25, 35, 50, or 200. So I'd end up with problems like 16 x 4 = 8 x 8, 18 x 50 = 9 x 100, or 14 x 35 = 7 x 70."
- For doubling repeatedly:
  "I want one of the factors to be 2, 4, or 8 so that I can double for 2, double and double again for 4, or double and double and double for 8. The doubling would be easier if I made the digits in the other number small, for example, 13. So I could end up with problems like 8 x 13 = 4 x 26 = 2 x 52 or 4 x 32 = 2 x 64."

- For a place value strategy:
  It's easier to use place value partitioning if the digits in the numbers are small, such as 3 x 213, so that there is no renaming. It's also easier if one number has only one digit or is a multiple of 10 or 100, so that there are only two or three parts to add together, such as
  5 x 423 = (5 x 400) + (5 x 20) + (5 x 3), or 50 x 423 = (50 x 400) + (50 x 20) + (50 x 3),
  or 500 x 423 = (500 x 400) + (500 x 20) + (500 x 3).

Some students are reluctant to use a variety of strategies and prefer to use one trusted method. You could use the following strategies to support those who are having difficulties:
- Try giving them more practice in trying different strategies: *Let's all use Sarah's method to solve this next problem.*
- Have group discussions about why being able to use a variety of strategies may be useful.
- Emphasise the importance of efficiency by asking questions such as: *Now that you've solved this problem, what would you do differently next time if you were doing a similar problem? or Is there a faster way of solving this problem?*

After the students have completed the activity, pose questions for the group to talk about in a think-pair-share discussion:
Which multiplication strategy do you use most often? Why?
Now that you have compared some strategies, is there a strategy that you will try to use more often? What kinds of problems will you use it for?
Which "special numbers" will you keep an eye out for in multiplication problems that will tell you to try a certain strategy?
"Numbers close to tidy numbers, such as 41 or 199, will tell me to try a tidy number strategy."
"If there is a 4 or an 8 in the question, I can try doubling repeatedly. I can look out for numbers that can be halved and doubled to make a multiplication fact I know."

Extension
Have the students, in pairs, make a Wanted poster or write a newspaper Situations Vacant advertisement that describes the characteristics of problems that can be solved efficiently using a particular strategy. Sentence starters might include:
- Wanted! Problems that can be solved using tidy numbers!
- We are looking for …
- Successful applicants will be/have …
- These numbers can be recognised by …

Display the posters or advertisements as a reference to remind the students to choose their strategies wisely.

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)
- *Book 6: Teaching Multiplication and Division:*
  Multiplying Tens (tidy numbers), page 14
  A Little Bit More/A Little Bit Less (compensation), page 15
  Cut and Paste (doubling and halving), page 25
  Multiplication Smorgasbord (tidy numbers, place value partitioning, doubling, proportional adjustment), page 27
- *Book 8: Teaching Number Sense and Algebraic Thinking*
  Doubling and Halving, page 14.

Pages 8–9: Just Right!

Achievement Objective

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity:

- to encourage transition from advanced additive strategies (stage 6) to advanced multiplicative strategies (stage 7)
- to help your students extend their advanced multiplicative part–whole strategies (stage 7) in division.

Activity

This activity encourages students to think about what to do with remainders in a division calculation. They have to decide whether to round up or down or whether the remainder should be left as a remainder, according to the context of the problem. In all cases, the students need to understand that a remainder means that the amount remaining is not enough to make another whole group (whatever that is, in the context of the problem).

Students need to be at least advanced multiplicative part–whole thinkers (stage 7) to do this activity independently because they need to use a range of mental strategies to solve division problems. For these remainder problems, place value strategies are the most useful.

Students in transition to stage 7 should be able to complete the activity with teacher support. Encourage them to choose from the multiplicative strategies that they can use, derive answers from known facts, and use appropriate place value equipment if they need to model a problem.

Students working independently will need to work in pairs or groups of 3 to complete parts of this activity. Those who have difficulty solving the division problems mentally could use part–whole diagrams or equipment such as tens money ($100, $10, $1), beans and canisters, fraction kits, or squares of paper that they can fold and cut.

With a guided teaching group, set the scene by asking:
You have 15 chairs and you need to put them into groups of 4 chairs. How many groups of 4 will there be? (There will be 3 groups, and 3 chairs will be left over; $15 \div 4 = 3$ remainder 3.)
What about if you cooked 15 toasted sandwiches and 4 people each ate an equal share of them? How many toasted sandwiches would each person get? ($3\frac{3}{4}$)

This activity gives rise to some key questions that you can ask to promote algebraic thinking. Question **7** is designed to find out whether students recognise that, when a number is divided by 9, there could be a remainder of 0, 1, 2, 3, 4, 5, 6, 7, or 8. If there were more than 8 doughnuts remaining, another whole group of 9 could be made. If the students have difficulty understanding this, get them to try systematically dividing some numbers by 9 and work out the reminder each time. One pair might try all the numbers in the 30s and another all the numbers in the 40s. Compare the results and ask: *Why is the remainder never bigger than 8?*

To see if your students have generalised this idea to all numbers, ask questions such as:
If you were sharing the doughnuts between 5 people, how many doughnuts could be left over? (0, 1, 2, 3, or 4)

For question **8**, challenge the students to write a division problem where the remainder should be ignored (as in question **1**) or the remainder is a whole number (as in questions **2** and **5**) or where there is no remainder (as in questions **4** and **6**).

Get the students to write their problem on a large piece of paper so that the whole group can read it. Before they start to solve each other's problems, ask them: *Can you predict just by reading the words of these problems whether you should ignore (or not ignore) any remainder?*

Students who do not have many multiplicative strategies may need to use equipment to solve the problems in the activity:

- For question **1** (2 250 millilitres ÷ 200 millilitres): equipment that can be used to show thousands, such as tens money or place value blocks
- For question **2** (75 cars ÷ 4 children): beans and canisters or bundles of sticks
- For question **3** (170 chicken drumsticks ÷ 38 people): beans and canisters or bundles of sticks
- For question **4** (15 minutes ÷ 12 lengths): place value equipment such as tens money (900 seconds ÷ 12) or 15 squares of paper to represent each minute, folded and cut into fractions (quarters) as required. You could also have a linear model, using strips of adding machine tape.
- For question **5** (129 players ÷ 11 in a team): beans and canisters, tens money, or bundles of sticks
- For question **6** (8 pizzas ÷ 6 boys): circles or squares of paper to fold and cut.

Ask: *What important things do you want to remember about remainders?* (Remainders can be treated in different ways: they can be left as a remainder in whole numbers or divided up into fractions or decimal fractions. It's important to look at the context of the problem carefully so you know if the answer needs to be a whole number and the remainder can be ignored or if the reminder should be divided further to make a fraction, or a decimal fraction, and how many decimal places it would be sensible to round the decimal number to.)

Extension

The following extension suggestions illustrate the principles of remainders: that is, the context tells you how to treat the remainder. Parts of ones are defined as fractions, but if you are forming sets, you must make a complete set, leave the remainder alone, or round up. Ask:
How many different (correct) answers can you come up with for 29 ÷ 3? (9 and ignore the remainder; 9 remainder 2; $9\frac{2}{3}$; $9.\dot{6}$; 9.6666667; 9.67; 9.7; 10, 10, and 9; $9.65, $9.65, and $9.70)
Choose two of these answers and make up word problems that would suit each answer.
The students will need to keep the following in mind:

- For 9 remainder 2, problems should involve things that can't be subdivided, such as people, cars, clothing, animals, and so on.
- For $9\frac{2}{3}$, problems should involve things that are typically cut into pieces, such as pizza or chocolate, or that are often referred to in fractional terms, such as "I've read $9\frac{2}{3}$ books" or "I've done $9\frac{2}{3}$ laps of the field."
- For 9.67, problems should involve measurement in metres, kilograms, or kilometres, where you might round to 2 decimal places because it's not necessary to be any more accurate with your measuring, or items paid for by EFTPOS to the nearest cent.
- For 10, 10, and 9, problems should involve things that can't be subdivided, such as people, animals, or cars.
- For $9.65, $9.65, and $9.70, problems should involve cash, where the amounts are rounded to the nearest 5 cents.

Go back to the "setting the scene" chair questions suggested for the guided teaching group and ask: *In both these questions, you had to work out 15 ÷ 4. How is it that both answers, 3 remainder 3 and $3\frac{3}{4}$ are correct?* (Whether you rename the remainder or leave it left over depends on whether the remainder is something you can split.)

Note that when division involves making sets, remainders as fractions are a bit more complicated. For example: *You have 67 sweets altogether and you want to put them in packets of 4. How many packets can you make?* ($16\frac{3}{4}$ packets or 16 packets and 3 sweets left over)

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)

- *Book 4: Teaching Number Knowledge*
  Sensible Rounding, page 28
  Dividing? Think about Multiplying First, page 37
- *Book 6: Teaching Multiplication and Division*
  Long Jumps (using repeated addition and known multiplication facts to solve division problems), page 19
  Goesintas (using multiplication to solve division problems), page 20
  Proportional Packets (proportional adjustment in division), page 28
  The Royal Cooking Lessons (proportional adjustment in division), page 30
  Remainders (division problems with remainders), page 32
  Little Bites at Big Multiplications and Divisions (using factors of factors), page 38
- *Book 8: Teaching Number Sense and Algebraic Thinking*
  Finding Remainders (using a calculator), page 35

Pages 10-11: Branching Out

Achievement Objectives

- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- make up and use a rule to create a sequential pattern (Algebra, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use these activities to:

- help your students to consolidate and apply advanced additive and advanced multiplicative part–whole strategies (stages 6 and 7) in multiplication and division of whole numbers
- extend the students who are beginning to use proportional part–whole strategies so that they become confident at this type of thinking.

Activities

In these activities, students identify patterns and relationships between equations. They explore the effects of proportionally adjusting number expressions.

Students need to be at least advanced additive part–whole thinkers (stage 6) to do this activity independently. They should be familiar with fractions and decimal fractions and multiplying and dividing by multiples of 10 and 100.

Students working independently will need to work in pairs or groups of 3 to complete parts of this activity and will need photocopies of the web copymasters to work on.

You may need to define these terms:

- Expression: symbols connected by an operator (+, –, x, ÷) with no equals signs, for example, 40 x 6 or 69 + 8.
- Relationship: how any two things (in this case, expressions) are connected, how changing one changes the other, and what patterns you can see.

Activity One

Encourage the students to change the expressions around to make them easier to solve. For example: *If you know "something divided by 10 is 100", how could you change this into a multiplication problem that could help you?* (❑ ÷ 10 = 100 has the same answer as 10 x 100 = ❑.)

Students who don't have a strong grasp of multiplicative strategies may need to confirm their predictions on a calculator after they make each one because it is impossible to identify useful patterns based on inaccurate answers. Encourage them to build on this instant feedback by looking for patterns and using them to predict related answers.

Asking your students questions about the relationships between the webs will help them to focus on identifying patterns they can use to predict answers:
What connections can you see between the 100 and the 200 webs?
What do you have to do to that number to make it into this new number?
Is it the same for the other spokes on the web?
Which ones are different?

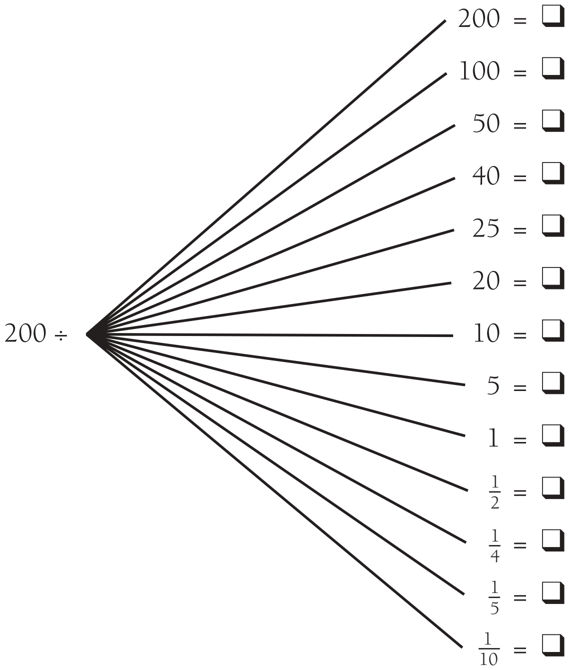
Note that the relationships between the expressions are not as simple as they first appear. For multiplication expressions, when the answer is doubled, all the boxes in the multiplication expressions are doubled. In the division expressions, the action depends on the position of the ❑. Where the ❑ is the divisor, for example, 200 ÷ ❑, doubling the answer is matched by halving the box number.

There are three key issues here:

- Division is not commutative (reversible). 8 ÷ 4 is not the same as 4 ÷ 8.
- Increasing the variable (❑) in ❑ ÷ 10 increases the result by the factor of the increase (because $\frac{1}{10}$ of something big is always greater than $\frac{1}{10}$ of something small).
- Increasing the variable in 10 ÷ ❑ decreases the result proportionally, for example, 10 ÷ $\boxed{2}$ is twice 10 ÷ $\boxed{4}$ because the more people there are sharing the 10 items, the less they each get.

These ideas can be easily demonstrated using examples.

Question **5** is a useful formative assessment checkpoint for algebraic thinking. Do the students use the multiplicative relationships they have discovered, or are they still needing to guess and check the answers? Students who understand the patterns and relationships they have been exploring in questions **1–4** are likely to see that they can solve most of the questions by multiplying their answer to question **1** by 2.5. However, most will have trouble with 200 ÷ ❑ = 250 and may wonder "How is it that a number can become bigger by dividing?" One approach you could take to this is to ask your students to complete the pattern in the following:



Once they realise that the number they are looking for in 200 ÷ ❑ = 250 is less than 1, they can search for it using trial and improvement. Another approach is to see 200 ÷ $\frac{1}{2}$ as *How many halves do you get in 200?* This explains the answer of 400 (halves). The unit of the answer is the real issue: it's not ones, it's halves.

Activity Two

In question **1**, students are asked to identify patterns in each branch of the web so that they can continue the same pattern. This familiarises them with the various ways that expressions can be adjusted to make them easier to solve without changing the answer. This can be helpful when solving problems mentally. The branch $\boxed{20 \div 1} \rightarrow \boxed{10 \div 0.5}$ continues to explore division by numbers less than 1 (but greater than 0).

Challenge the students to use fractions and decimal fractions for the $\boxed{?}$ branches of the big web. Get them to check each other's webs by using a calculator to do the equation on the end of each branch.

To encourage reflective discussion, ask the following questions in a think-pair-share discussion:
Did you use any short cuts today that helped you predict the answers?
What patterns did you notice that would be useful to remember?
Did any patterns surprise you?
Why might it be useful to change expressions so that they have different numbers that still give the same answer (like you did on your big webs)?

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)

- *Book 7: Teaching Fractions, Decimals and Percentages*
  Candy Bars (adding and subtracting decimal fractions), page 27
- *Book 8: Teaching Number Sense and Algebraic Thinking*
  Reversals with Multiplication and Division, page 10
  Doubling and Halving (extending applications of doubling and halving, trebling and thirding), page 14
  Whole Numbers Times Fractions and Fractions Times Whole Numbers, pages 22–23
  When Big Gets Smaller (multiplying by decimal fractions less than 1), page 24
  When Small Gets Bigger (dividing by positive numbers less than 1), page 24.

Pages 12–13: Run like the Wind

Achievement Objective

- solve practical problems that require finding fractions of whole number and decimal amounts (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- help the students to apply advanced additive part–whole strategies (stage 6) in the operational domains of addition and subtraction and proportions and ratios
- help the students to apply advanced multiplicative part–whole strategies (stage 7) when solving problems involving fractions.

Activity

In this activity, students apply their strategies in whole-number addition and identify fractions of sets that make up training schedules for a triathlete. They need to be able to use a range of addition strategies (stage 6) and also be able to describe parts of a group in fractional terms, for example, to say that 160 minutes out of 720 is $\frac{2}{9}$ (stage 7). This requires them to see common factors in the numbers, for example, 160 and 220 have common factors of 2, 5, 10, and so on.

If possible, set the scene and motivate a guided teaching group by inviting a triathlete to talk about their training schedule.

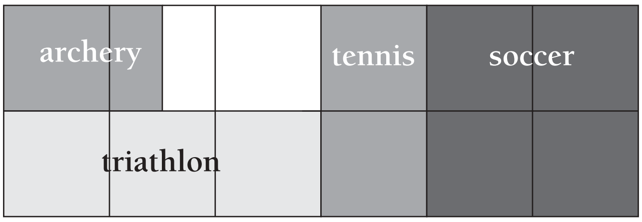
If the students need support to fill in the table in question **1**, encourage them to work systematically by asking think-pair-share questions such as:
If you add the numbers across the table on each day, what will they add up to?
How many hours will Iain train for in each week? How many minutes is this?
Iain wants to train for the same amount of time in each activity by the end of the week. How much time should he spend swimming each week?
How could this information help you with your table?

Supportive questions for question **2** could include:
Before his new coach stepped in, Iain was training for the same amount of time in each activity. What fraction did he spend doing each activity? ($\frac{1}{3}$)
Iain kept his cycling time the same (240 minutes). How many minutes are left for the other two sports each week? (480 minutes)
How could Iain divide up his 480 minutes so that he spends twice as much time swimming as running?
(Strategies could include: "I guessed and checked, for example, 100 + 200 = 300, 150 + 300 = 450, 160 + 320 = 480." "I worked out that $\frac{1}{3}$ of 480 = 160 minutes for running and so $\frac{2}{3}$ of 480 = 320 minutes for swimming because $\frac{2}{3}$ is twice as much as $\frac{1}{3}$.")

Have the students explore question **2b** in small groups and then share their strategies and answers with the whole group. Some students might solve the problem by working out what fraction of the whole 720 minutes each sport is allocated, for example, 160 minutes running out of a total of 720 minutes is $\frac{2}{9}$.

If students need support, ask them to look at all the numbers involved: 160, 240, 320, and 720. Ask:
What's the biggest number you can find that will divide into each of these numbers without any remainders? (80)
What fraction of 720 is 80? ($\frac{1}{9}$)
How can you use this information to help you answer question ***2b****?*

When doing question **4**, some students may like to draw or make a diagram. If they need support, suggest that they make a rectangle of 12 squares, where one square represents each month.



Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)

- *Book 7: Teaching Fractions, Decimals, and Percentages*
  Birthday Cakes (using multiplication to find a fraction of a set), page 14
  Fractional Blocks (using patterns to find fractions of shapes and sets), page 15
- *Book 8: Teaching Number Sense and Algebraic Thinking*
  Equivalent Fractions, page 16

Pages 14–15: The Power of Powers

Achievement Objectives

- explain the meaning of the digits in any whole number (Number, level 3)
- explain the meaning and evaluate powers of whole numbers (Number, level 4)

AC
EA
AA
AM
AP

Number Framework Links

Use these activities to help your students develop knowledge of place value and powers of 10 to support multiplicative thinking.

Activities

Introduce these activities to an independent group by reading what Aarif and his teacher say about 10^3. Reinforce that 10 to the power of 3 means 10 x 10 x 10, not 10 x 3 or 3 tens.

Activity One

Encourage the students in a guided teaching group to say "I moved the 1 along 8 places" rather than "I added 8 zeroes" when they are predicting what 10^8 would be. The rule "add a zero" to multiply by 10 is conceptually inaccurate. Mathematically, adding 0 to a number doesn't change it: 12 + 0 = 12. This "rule" also sets students up for problems later when they are multiplying decimal fractions because 5.6 x 10 does not equal 5.60. It's better for students to learn to understand mathematical principles that are always true than to learn lots of rules that need to be continually changed in different situations.

The generalisation in this case is that when we multiply by 10, the numbers shift along 1 place to the left; it is the digits that move, not the decimal point. You can demonstrate this by writing the number 43 on a strip of paper under the place value houses and moving the strip along 1 place to the left to multiply by 10. This creates a space under the ones column, where a 0 has to be written as a "place-holder" because if we didn't have the house labels above the numbers, we wouldn't know whether the number should be 43 or 430. It is important to use place value materials to support this generalisation.

Typically, students will jump to the conclusion that 10^0 is 0, but it isn't! The easiest way to demonstrate this is to use a table or place value houses and get the students to follow the pattern from the left to the right, starting at 10^5 and dividing by 10 each time to reduce the power. Make sure that when they get to 10^2, they see it as 100. So 10^1 is 100 ÷ 10 = 10, and 10^0 is 10 ÷ 10 = 1.

	Thousands			Ones			
	h	t	o	h	t	o	
	10^5	10^4	10^3	10^2	10^1	10^0	(reduce power by 1)
	100 000	10 000	1 000	100	10	1	(divide by 10)

For your own information, note that anything to the power of 0 (apart from 0 itself) has the value of 1. For example, in the extension activity for page **5**, Megabytes of Memory, the pattern leads to 2^0, which is 2 ÷ 2 = 1.

Activity Two

Place value houses are an effective way of helping students to say large numbers. Encourage them to say the number in each house and then the name of the house. For example,
483 065 000 would be said "four hundred and eighty-three million, sixty-five thousand".

In question **2**, students work with powers of numbers that are themselves multiples of 10. They are likely to find it helpful to rewrite each number using only 10s. In this way, they will see that

$$\begin{aligned}100^3 &= 100 \times 100 \times 100\\ &= (10 \times 10) \times (10 \times 10) \times (10 \times 10)\\ &= 10 \times 10 \times 10 \times 10 \times 10 \times 10\\ &= 10^6\end{aligned}$$

A key idea is that the grouping of factors does not affect the value:

$$\begin{aligned}100^3 &= (10 \times 10) \times (10 \times 10) \times (10 \times 10)\\ &= (10 \times 10 \times 10) \times (10 \times 10 \times 10)\\ &= (10 \times 10) \times (10 \times 10 \times 10 \times 10)\\ &= 10 \times 10 \times 10 \times 10 \times 10 \times 10\end{aligned}$$

For questions **3** and **4**, some students might find it helpful to manipulate or image tens money ($100, $10, $1) and work out how many tens and hundreds are in numbers by making exchanges. For example, they may work out that 10^3 is $1,000, which is the same as ten $100 notes or one hundred $10 notes. Other students may be able to manipulate the numbers in question **3** to work out how many tens are in a number by partitioning one lot of 10 out of the factors. 10^3 can be written as (10 x 10) x 10, which is the same as 100 x 10, so there are 100 lots of 10 in 10^3.

To work out how many hundreds are in the numbers in question **4**, some students may be able to partition 100, or 10 x 10, out of the list of factors. For example, 100^2 can be written as

$$\begin{aligned}(10 \times 10) \times (10 \times 10) &= 10 \times (10 \times 10 \times 10)\\ &= 10 \times 1\,000 \text{ (ten thousand)}\end{aligned}$$

or

$$\begin{aligned}(10 \times 10) \times (10 \times 10) &= 100 \times 100\\ &= 10\,000.\end{aligned}$$

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)

- *Book 4: Teaching Number Knowledge*
  Place Value Houses (identifying multi-digit numbers), page 5
  Zap (finding the meaning of digits in different places), page 26
  Digits on the Move (multiplying by 10, 100, and 1 000), page 29

Pages 16–17: Crafty Combinations

Achievement Objectives

- state the general rule for a set of similar practical problems (Algebra, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- help students extend and consolidate advanced additive part–whole strategies (stage 6) in addition and subtraction
- encourage students to move to advanced multiplicative part–whole thinking (stage 7) by applying the combining strategy to addition of fractions and decimal fractions.

Activity

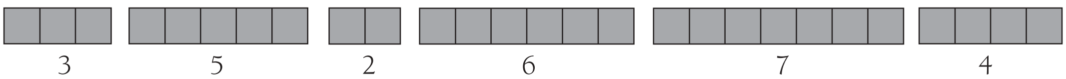
This activity encourages students to look for clues that could help them select a strategy (in this case, combining numbers) to solve a problem more easily. Students are often encouraged to look for combinations of numbers that add to 10 or multiples of 10 when they first use combining strategies, but this activity helps students to recognise that the strategy can be used with other numbers as well. The strategy is applied to addition and subtraction of whole numbers and then to fractions and decimal fractions.

For questions **1–4** and **6**, students need to be advanced additive thinkers (stage 6) or ready to make the transition to this stage. Students who are not familiar with fractions and decimal fractions may find question **5** an interesting introduction to them.

Before an independent group starts work on the activity, say: *Sometimes when you need to add several numbers together, you can combine some of the numbers first and then add the answers together. Why might you do this?* Then work through the first example together. Some students might like to record by drawing curved lines to connect the pairs of numbers and writing the answer along the line.

Note that combining involves the idea of average or "middle". For example, in question **3**, the numbers are around a middle of 26: 21, 24, 25, 26, 27, 27, 29.

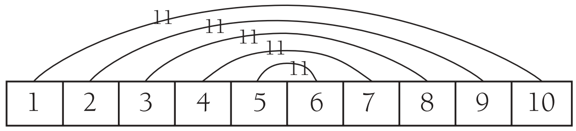
Use the same introductory question with a guided teaching group. Then work through the introduction and question **1** as a practical activity. Get the students in the group to make the following numbers by linking cubes together:



Write 3 + 5 + 2 + 6 + 7 + 4 on the board and then say: *You might notice that some of these numbers combine to make exact 9s. Which ones are they?* Get the students to join the cubes physically to make 3 groups of 9.

Show the students a hundreds board. Say: *I want to work out what the first row of the hundreds board adds up to, but I want to find a quicker way than adding each one in order. I can see that 1 + 10 and 2 + 9 both add up to 11. How might that be useful?*

After the students have found the other groups of 11, reinforce the idea of combining with this diagram:



Give each student a photocopy of the combinations copymaster so that they can do their own recording of their linking for question **1** and the rest of the questions.

With question **5**, the students might find it useful to make the fractional numbers with a foam fraction kit.

Question **6** is designed to promote algebraic thinking. It asks students to think about when combining would be a useful strategy. Encourage the students to look carefully at the numbers in problems before they rush into solving them so that they have time to notice the clues that might help them to solve the problem with minimal difficulty and can choose a strategy that suits the problem. (One clue is the clustering of numbers around a middle number, as described above for question **3**.)
To encourage this, ask questions such as:
Look at the problem carefully before you start to solve it. Which strategies might you use? Which do you think would be best? Why?
Now that you've done the problem, were you happy with your choice of strategy? If you were doing it again, what would you do differently?

Extension
Have the students investigate the use of combining strategies to add consecutive numbers for:
- the numbers 1 to 20
- all the numbers on a hundreds board
- multiples of 5 up to 100 (5, 10, 15, 20 …)
- any sequence of consecutive numbers, such as 23 + 24 + 25 + 26 + 27 + 28, or 20 + 22 + 24 + 26.

Ask:
Which numbers will you combine to help you add all these numbers quickly?
Does it make a difference if there is an odd or an even number of numbers to add?
What instructions would you give someone who wanted to quickly add a sequence of consecutive numbers?

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)
- *Book 5: Teaching Addition, Subtraction, and Place Value:*
  Make Ten, page 26
  Compatible Numbers, page 26
  Three or More at a Time, page 42
- *Book 8: Teaching Number Sense and Algebraic Thinking*
  Adding Sequences, page 34

Another useful resource is the lesson Multiple Ways to Add and Subtract, found in the Additional Lessons section of the NZ Maths website: www.nzmaths.co.nz/numeracy/Other%20material/Lessons.htm

Pages 18–20: Fraction Tagging

Achievement Objective

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity with students who are early additive part–whole thinkers (stage 5) through to advanced multiplicative part–whole thinkers (stage 7) to encourage them to apply part–whole strategies to finding fractions of numbers.

Activity

In this activity, students use a fraction wall to compare equivalent fractions and then find fractions of numbers. The 1 whole is assigned different values on different walls, which develops the concept that fractions are measured relative to 1 whole and the value of the fraction depends on the size of the whole. This is known as re-unitising and is an important feature of proportional reasoning.

This is useful as a follow-up activity for students who have been introduced to finding fractions of numbers using:

- early additive part–whole strategies (stage 5), such as deriving from known addition facts, for example: $\frac{1}{3}$ of 15 = 5 because 5 + 5 + 5 = 15
- advanced additive part–whole strategies (stage 6), such as using repeated halving or known multiplication and division facts, for example:
  $\frac{1}{4}$ of 48: half of 48 is 24; half again is 12
  $\frac{1}{4}$ of 48: I know 4 x 10 = 40 and 4 x 2 = 8, so 4 x 12 = 48 and $\frac{1}{4}$ of 48 is 12
- advanced multiplicative part–whole strategies (stage 7), for example:
  finding $\frac{4}{5}$ of 30 by calculating $\frac{1}{5}$ of 30 = 6, then 4 x 6 = 24 or 30 – 6 = 24.

You might like to have the students in a guided teaching group construct their own fraction wall before they start question **1**. Give each student 8 strips of paper and challenge them to fold them so that they are divided up into halves, thirds, quarters, fifths, sixths, eighths, and tenths. If the strips are 24 centimetres long, students at stages 6–7 could work out how big each section should be in centimetres by working out $\frac{1}{2}$ of 24, $\frac{1}{3}$ of 24, $\frac{1}{4}$ of 24, and so on. (They may need a calculator to work out the fifths and the tenths.) The students can then use their own fraction wall to complete question **1**.

Ensure independent groups understand how the walls work before they do the activity. A good place to start is the wall in question **2**. Ask:
*What fraction of the top row could **2b** be?*
*How many bricks would there be along the bottom row of this wall if you could see all the bricks in **2d**? How do you know?*
The students may say:
"The 3 bricks are nearly the same size as 1 brick in the thirds row. That's 3 x 3 = 9, and 1 more would fit, so that's 10."
"I could see 3, and I estimated how many more would fit along the row."
"It's the same size as the bottom row in the next fraction wall, and there are 10 in that row."
*So what fraction of the top brick is **2d**? How do you know?* ($\frac{7}{10}$. The other part is $\frac{3}{10}$, and $\frac{3}{10} + \frac{7}{10} = \frac{10}{10}$, which is 1 whole.)

For the guided teaching group, use the questions above to help the students to use the walls for questions **2–4**. Students working at stage 5 may need counters to share into equal groups on the fraction wall if they don't know a relevant addition fact. Length models constructed from 10, 15, and 24 multilink cubes would be very helpful.

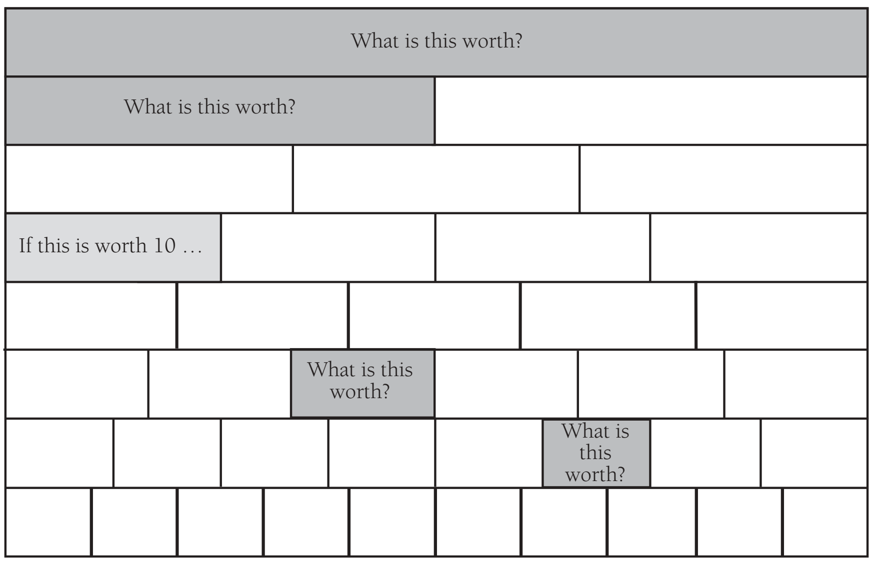
Encourage the students at stage 6 to look for facts they know that could help them, for example:
You're trying to find $\frac{2}{3}$ *of 24. What do you know about the 3 times table and 24 that could help?*
You're trying to find $\frac{3}{5}$ *of 15. What do you know about the 5 times table and 15 that could help?*

Encourage students at stage 7 to use multiplicative strategies, such as finding one part first and multiplying that by the number of parts required. Ask:
You're trying to find $\frac{3}{5}$ *of 15. Could you find out what* $\frac{1}{5}$ *of 15 is first?*
How could you use that to find $\frac{3}{5}$ *of 15?*

Students need to understand iteration of a fraction unit, for example, $\frac{3}{4}$ is $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ and $\frac{4}{5}$ is $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$.

For question **5**, make sure that the students choose numbers for the tops of their walls that are possible to solve using the strategies they know. (Finding $\frac{1}{6}$ of 17 is very difficult to do!) Possible questions and student ideas are:
When you were working out the 24 wall, which fractions were easy to find? Which were harder?
"The fractions with numbers that divide evenly into 24 were easiest, like halves (÷ 2), thirds (÷ 3), and sixths (÷ 6). The fractions that didn't go evenly into 24 were harder, such as the tenths, because we had to use decimal fractions." (This connects to factors. That is, 2, 3, 4, 6, 8, and 12 are factors of 24.)
Would it have been easy to find $\frac{1}{6}$ *of 10? Why or why not?*
"No. It's really hard to divide 10 by 6 in your head."
Why do you think the numbers 10, 24, and 15 were chosen for the walls in the activity?
"Lots of other numbers will divide into them, especially the 24."
What other numbers could you choose for the top of your wall so that your classmate could work out the answers in their head?
"I could choose numbers like 12, 18, 20, 30, 32, and 36."
*The **a** brick on each wall is always half. So why was it worth 5 on wall 2, 12 on wall 3, and* $7\frac{1}{2}$ *on wall 4?*
(This question promotes algebraic thinking.)
"It depends on how much the whole is worth; the half bricks are worth different amounts because the whole brick on the top of each wall is worth different amounts on different walls."
Can $\frac{1}{4}$ *of a number ever be bigger than* $\frac{1}{2}$ *of another number? Give me an example.*
"Yes. $\frac{1}{4}$ of 40 = 10, but $\frac{1}{2}$ of 12 is only 6."

Extension
Students could make up and solve fraction walls where the clue number is on a different brick from the "1 whole" brick, for example:



Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)

- *Book 4: Teaching Number Knowledge*
  Little Halves and Big Quarters (how a half can be smaller than a quarter), page 19
- *Book 7: Teaching Fractions, Decimals, and Percentages*
  Animals (using sharing and repeated addition to find fractions of a set), page 7
  Hungry Birds (using addition and subtraction to find fractions of a set), page 11
  Birthday Cakes (using multiplication to find fractions of a set), page 14

Page 21: Calculation Chains

Achievement Objective

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

Other mathematical ideas and processes

Students will also explore patterns and relationships.

Number Framework Links

AC
EA
AA
AM
AP

Use this activity to:

- help students apply advanced additive part–whole strategies (stage 6) in addition and subtraction of large numbers
- help students apply advanced multiplicative part–whole strategies (stage 7) in multiplication and division.

Activity

In this activity, students apply number sense and their knowledge of place value to work out how a number has been transformed on a calculator. Number sense principles that may be explored through this activity include whether a number gets bigger or smaller when it's multiplied or divided by a whole number or by a decimal number.

Question **1** is useful for students who are working at stage 6. To be able to apply appropriate number sense to questions **2** and **3**, students should be making the transition to stage 7 or be already working at this level, although it is possible to benefit from the activity through trial and improvement with a calculator.

Some students may need encouragement to try a strategy beyond guessing and checking on the calculator. Possible strategies to solve 111 x ❑ = 1 221 include:

- Estimation: "I know 111 x 10 is 1 110, so it has to be a little more than that."
- Opposite operation or working backwards: "I'm going to use the opposite operation and work out 1 221 ÷ 111."
- Looking at the ones place: "The product in 111 x ❑ = 1221 suggests only a 1 is possible in the ones place of the box number."

Encouragethe use of these strategies by asking questions such as:
Try estimating what would go in the box before you try it on the calculator. Do you think the answer you get on your calculator is going to be more or less than your estimate? Why?
What happens to a number when you multiply it by 10? 100? 1 000?
What happens to that number when it is divided by 10? 100? 1 000?
What facts do you already know that might help you estimate the answer?
Could you use a tidy number strategy to help you estimate the answer?
What's the opposite of adding, multiplying, or dividing? How might using the opposite operation help?
Could you use the answer and work backwards?

Extension

In question **3e**, some students will be surprised that you can divide 12.5 by something and get a bigger number (25). This could start an interesting investigation:
When you multiply a number by another number, does the first number always get bigger?
When you divide a number by another number, does the first number always get smaller?
Note that Branching Out (pages **10–11** of the student book) raises the same issues.

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)

- *Book 8: Teaching Number Sense and Algebraic Thinking*
  When Big Gets Smaller, page 24
  When Small Gets Bigger, page 24

Pages 22–23: Taking Turns

Achievement Objective

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

Other mathematical ideas and processes

Students will also explore patterns and relationships.

Number Framework Links

AC
EA
AA
AM
AP

Use this activity to help students apply advanced multiplicative part–whole (stage 7) strategies in the operational domains of multiplication and division and proportions and ratios.

Activity

In this activity, students compare computer access times across classes that have different numbers of students and computers, using relationships in a table of information to make calculations and comparisons. For this activity, students need to be multiplicative part–whole thinkers, preferably advanced (stage 7). Time has a base of 60, not 10, so be alert for potential problems if the students do not take this into account.

Some students in a guided teaching group may need assistance to use the information in the table. Ask supporting questions for question **1**, such as:
How many students are in Room 1?
If each of those students uses a computer for 20 minutes, how much computer time is that? How did you work that out?
There are 2 computers in Room 1. If the computers are used for a total of 4 hours, how long is each computer used for on average?
What maths did you use to work this out? Can you write what you have done as number sentences?
(12 students x 20 minutes ÷ 2 computers = 120 minutes (2 hours) for each computer.
12 x 20 ÷ 2 = 120)
Can you use this same process for the other classrooms?
If students need assistance for question **2**, ask supporting questions such as:
What clues can you find in Te Rama and Tina's speech bubbles?
How many hours could the computers be used for?
Room 1 has 2 computers, so how many hours of computer time are available?
If there is 12 hours of computer time available, how much could each student in Room 1 get?
An alternative strategy is outlined in the Answers.

Investigation

To encourage reflective discussion, discuss findings from the investigation and any recommendations the students have for improving your school's or classroom's computer sharing system.

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)

- *Book 6: Teaching Multiplication and Division*
  Multiplication Smorgasbord (solving multiplication problems using a variety of strategies), page 27
  Proportional Packets (solving division problems by making proportional adjustments), page 28

Page 24: Planning Paths

Achievement Objectives

- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- make up and use a rule to create a sequential pattern (Algebra, level 3)

AC
EA
AA
AM
AP

Number Framework Links

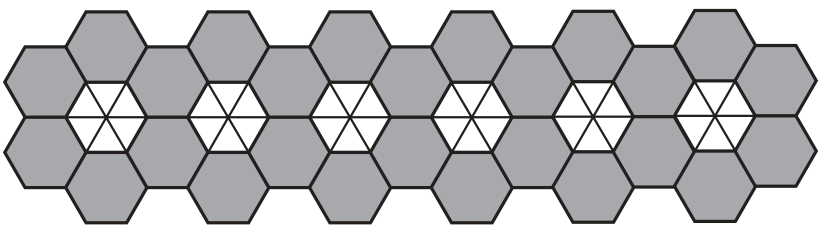
Use this activity with students who are early additive part–whole thinkers (stage 5) through to advanced multiplicative part–whole thinkers (stage 7) to encourage them to apply known part–whole strategies to repeated addition or multiplication calculations.

Activity

This activity encourages students to think systematically about the structure of a pattern and to predict patterns through the use of generalisations or equations. Students need to be at least early additive part–whole thinkers (stage 5) to do this activity mentally, although students working at earlier stages may be able to do so with the support of materials like pattern blocks.

You could set the scene with a guided teaching group by talking about "makeover" shows on TV: *What would you have done to your house and garden if you were on one of these shows?*

Get the students to use foam pattern blocks to make a path using triangles and hexagons, as pictured.



Ask: *If we wanted to add another flower to the path, how many more triangles and hexagons would we need?*

In question **1**, the diagram and the table (which uses numbers to represent the tiles) are designed to help the students see how the pattern is formed. For each new flower, 4 hexagons and 6 triangles are needed, but for any path, there are always 2 extra hexagons needed to complete the last flower (or to start the first). Some students may need to make paths with 1 flower, 2 flowers, 3 flowers, and so on to help them visualise these relationships.

Students should find it straightforward to add on 1 more flower at a time, but it is important that they look beyond iterative (sequential) rules such as "add 4 hexagons and 6 triangles" because these rules are only helpful if you know how many of each type of tile were used in a path with 1 fewer flower. A general pattern or generalisation enables students to work out how many tiles are needed for any length of path and is therefore much more powerful. To encourage the students to look towards the generalisation, ask questions such as:

What short cuts could Kalila use to make it faster than adding on 4 hexagons and 6 triangles every time?

Multiplication is a fast way of adding. What could you work out instead of 4 + 4 + 4 + 4 + 4?

Can you see any groups that Kalila could multiply to make a path with 12 flowers? (12 groups of 4 hexagons [12 x 4] and 12 groups of 6 triangles [12 x 6], then add the extra 2 hexagons)

If you make a 12-flower path and you multiply 12 x 4 hexagons and 12 x 6 triangles, would this give you all the tiles you need? How do you know?

If the students think that 48 hexagons and 72 triangles would be enough, encourage them to test this with a smaller number, such as the 4-flower pattern drawn or one they've made, or to use the patterns in the table to see that they'd need to add another 2 hexagons on.

Could you use the same short cut for 20 flowers? How could you test to check?

In question **2**, watch for students who assume that the 16-flower path will just have double the tiles needed in the 8-flower path. Challenge their thinking by saying:

Convince me that you're right.

How could you test that?

Does that strategy work for patterns that you've made?

Could you test your strategy on the 2-flower and 4-flower path in the table?

Question **1c** requires students to apply their patterns and short cuts to writing a generalisation for any length of path. To promote algebraic thinking, say to the students:

Describe what you would do to work out how many tiles you'd need for a really long path.

Write a list of instructions for someone else to follow your short cuts. (To work out the number of hexagons, multiply the number of flowers in the path by 4 and then add 2 more to finish the last flower. To work out the number of triangles for the centres of the flowers, multiply the number of flowers in the path by 6.)

Investigation

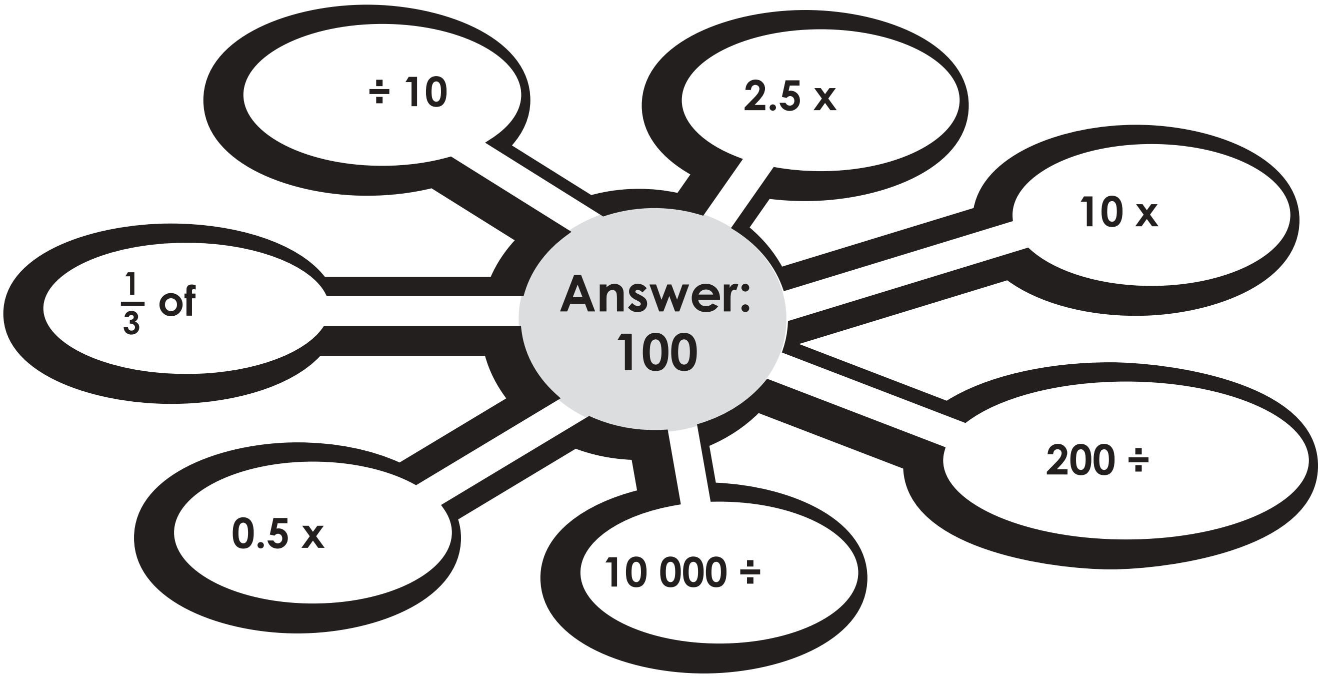
Encourage the students to lay their pattern out in a way that makes a short cut clear, like the diagram of the flower path in the student book.

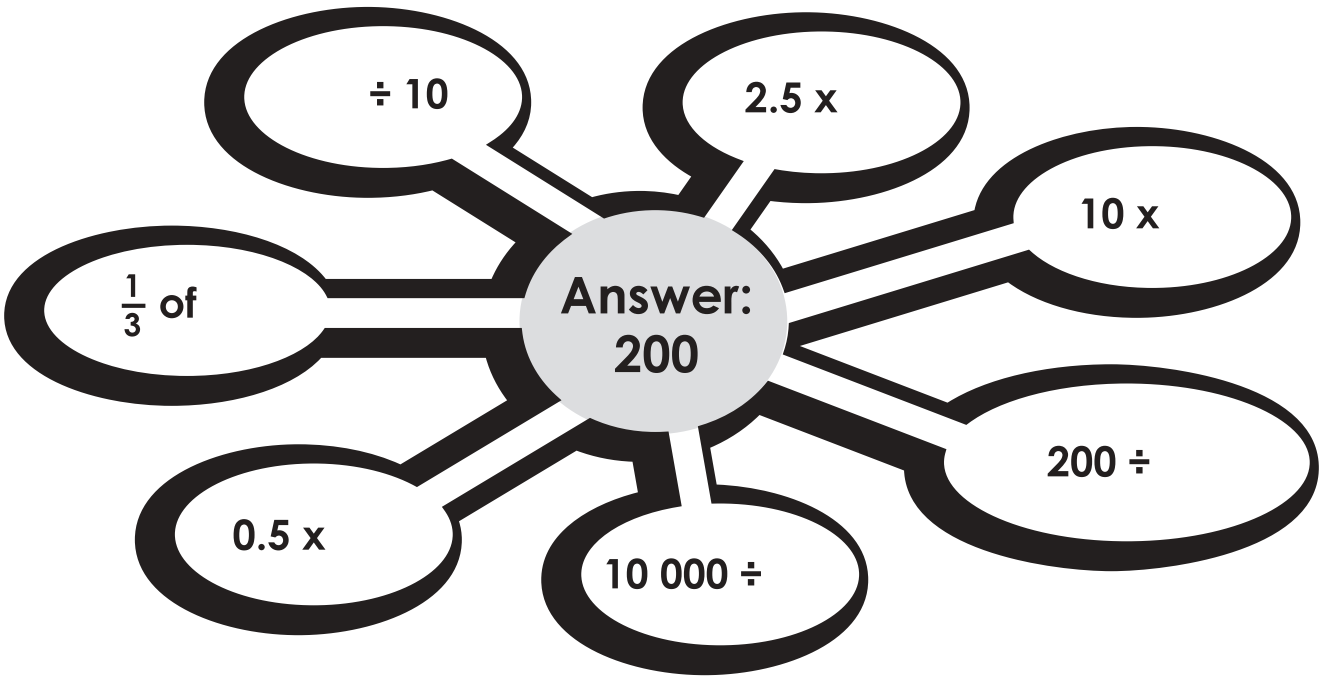
Extension

Have the students work backwards: *Kalila used 86 hexagonal tiles. How many flowers did she make? How many triangular tiles did she need?*

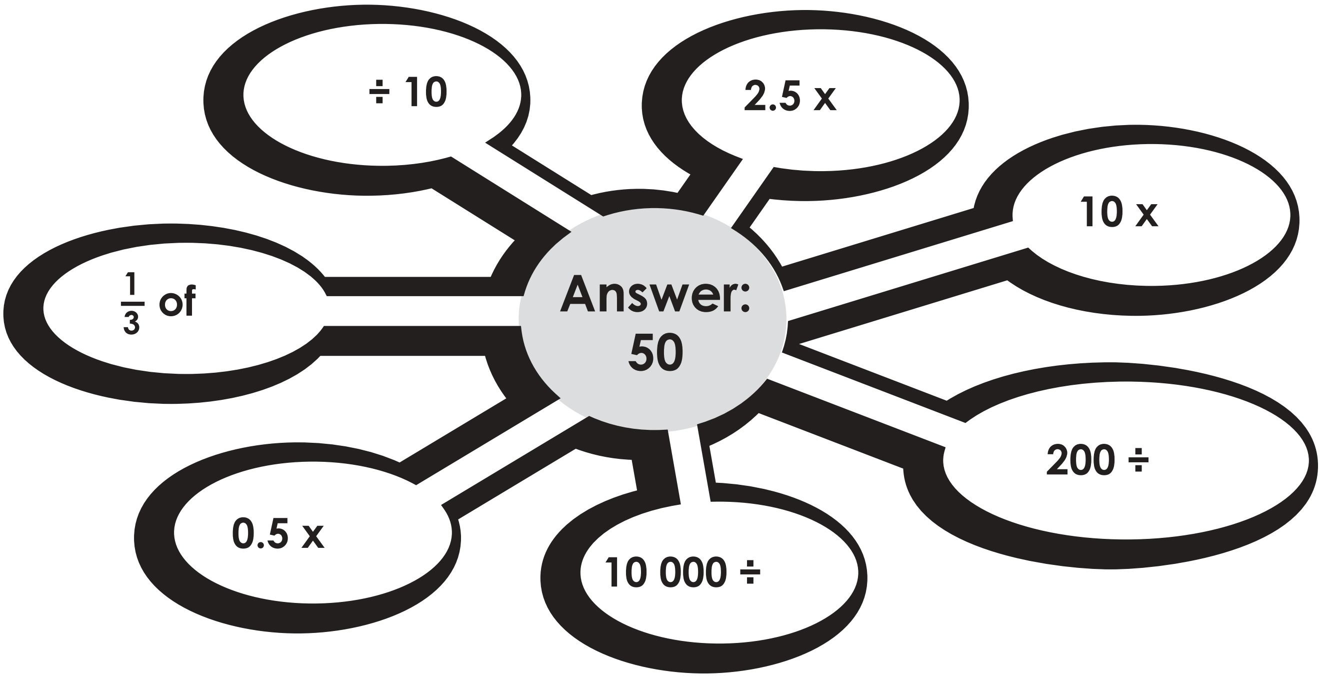
The students could also consider the equivalence of different rules for the same pattern. For example, *Mike says that for 12 flowers, it will take (12 x 6) – (11 x 2) hexagons. Where do his numbers come from?*

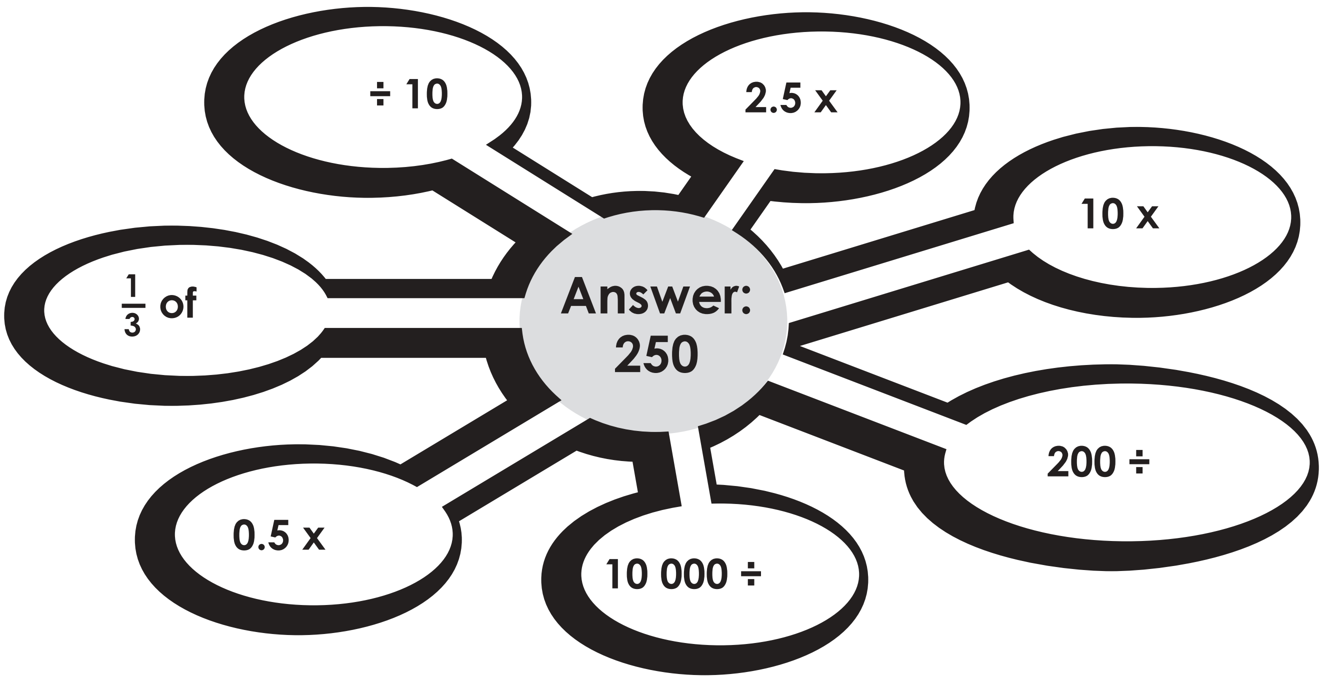
Copymaster: Branching Out



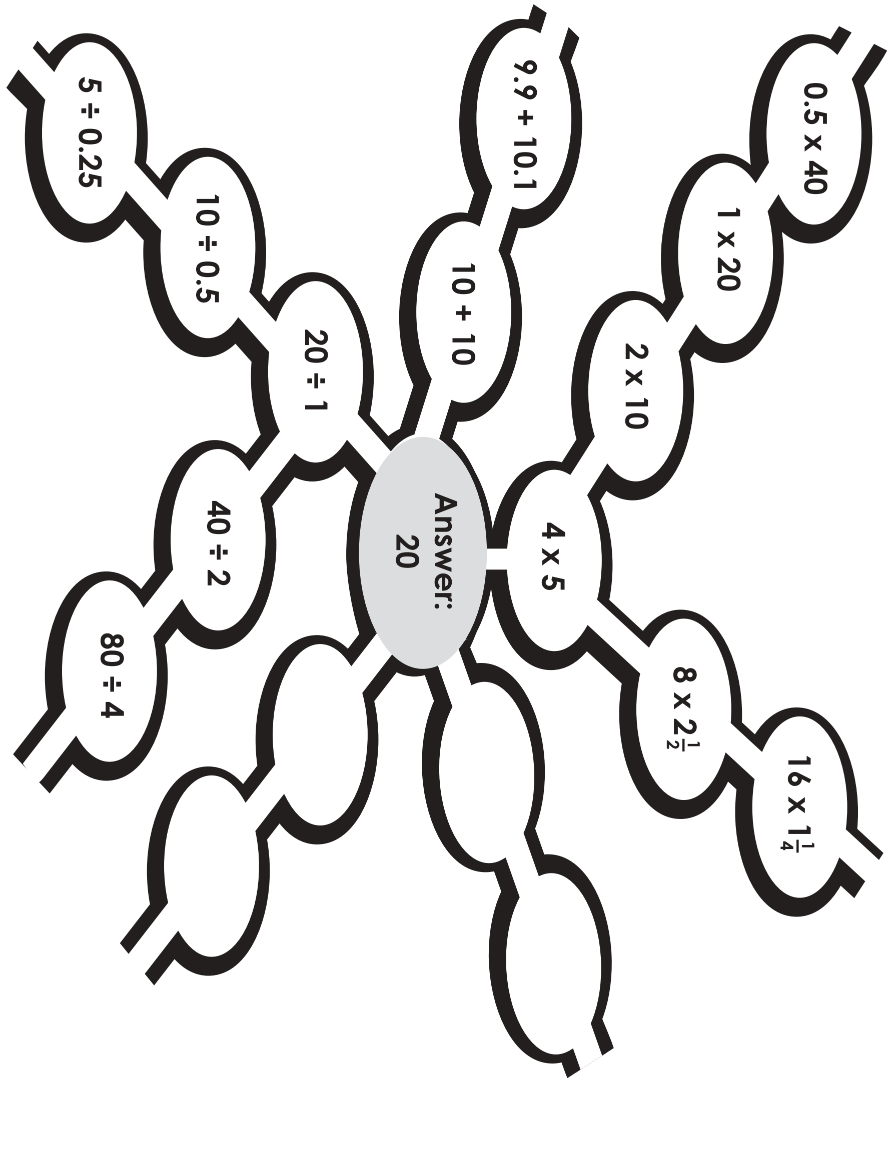


Copymaster: Branching Out





Copymaster: Branching Out



Copymaster: The Power of Powers

Trillions			Billions			Millions			Thousands			Ones		
h	t	o	h	t	o	h	t	o	h	t	o	h	t	o

Trillions			Billions			Millions			Thousands			Ones		
h	t	o	h	t	o	h	t	o	h	t	o	h	t	o

Trillions			Billions			Millions			Thousands			Ones		
h	t	o	h	t	o	h	t	o	h	t	o	h	t	o

Copymaster: Crafty Combinations

Question 1:

1	2	3	4	5	6	7	8	9	10

Question 2:

12	11	10	13	11	14	9	8
Bus A	Bus B	Bus C	Bus D	Bus E	Bus F	Bus G	Bus H

Question 3: \$27 \$26 \$24 \$25 \$27 \$21 \$29

Question 4:

$$14 + 800 - 1 + 555 + 986 - 999 + 445 + 200 = \square$$

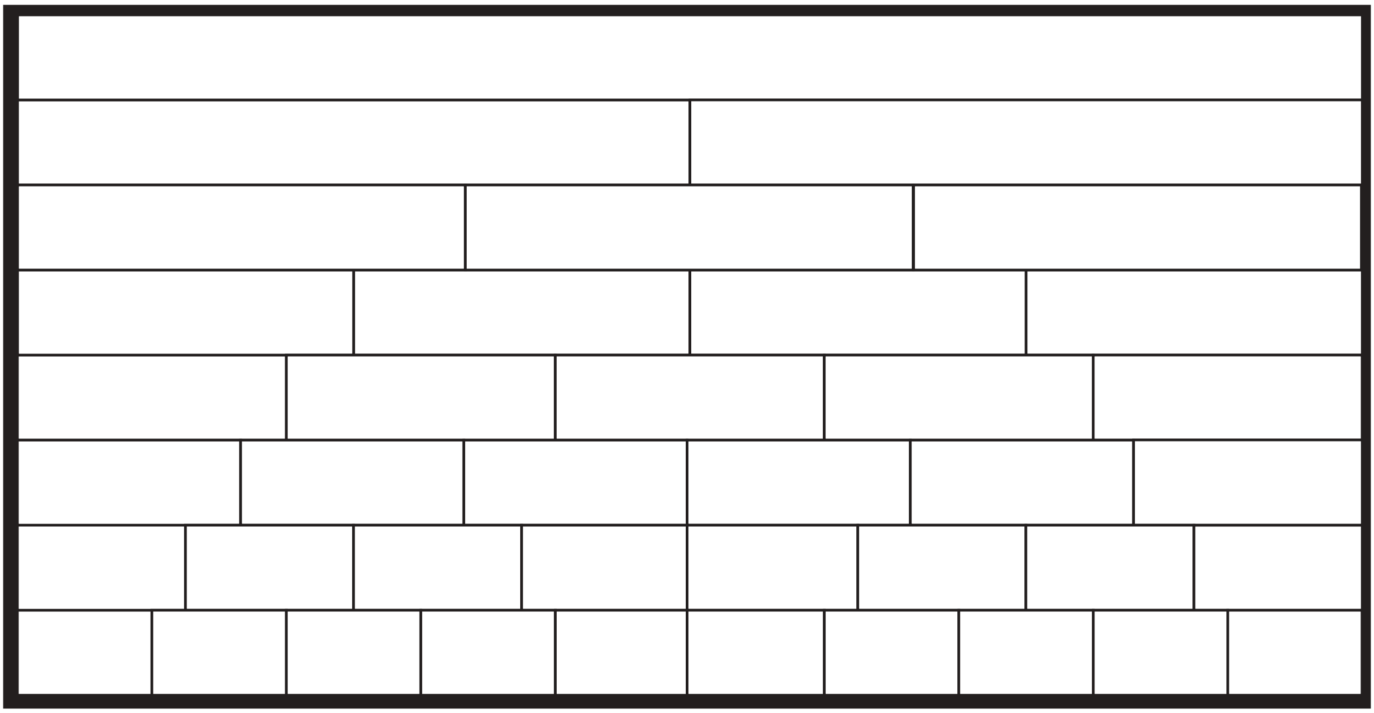
Question 5:

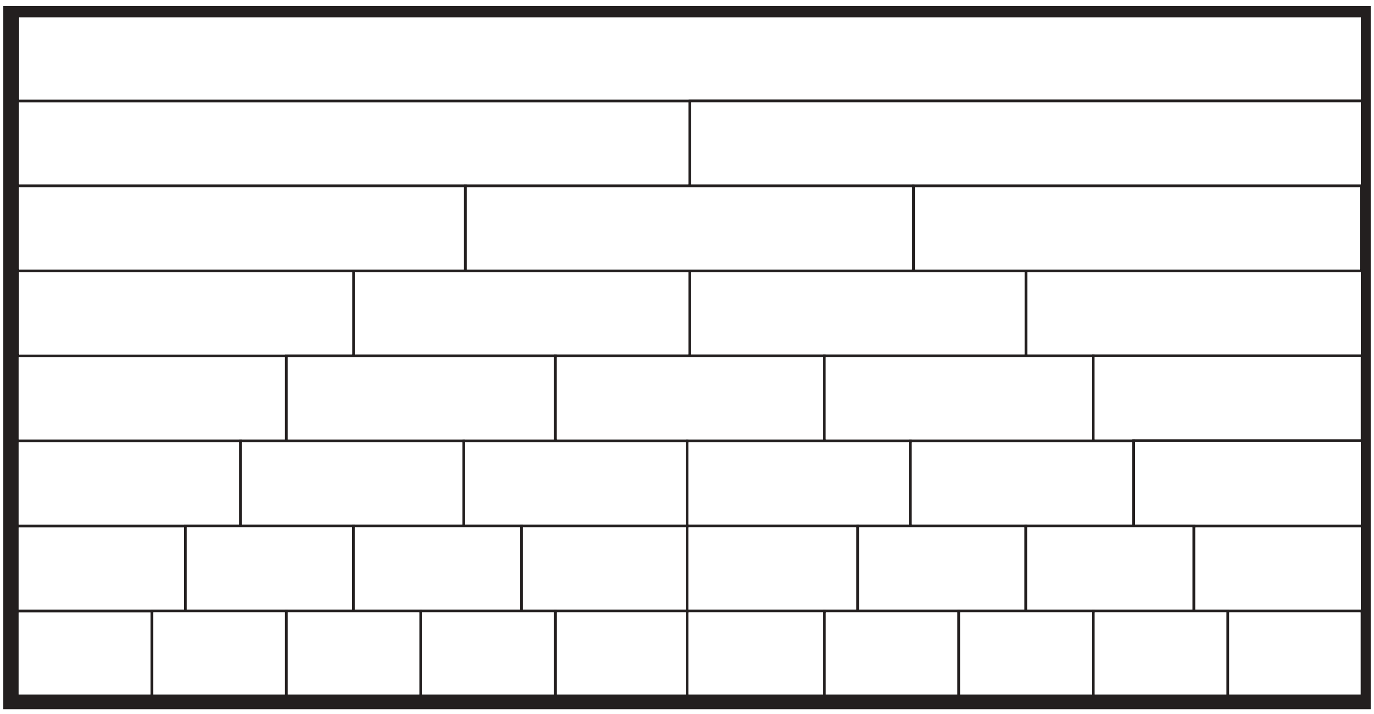
a. $\frac{1}{2} + 1 + 1\frac{1}{2} + 2 + 2\frac{1}{2} + 3 + 3\frac{1}{2} = \square$

b. $0.1 + 1.2 + 0.8 + 0.5 + 1.6 + 1.5 + 0.4 + 1.9 = \square$

c. $10 - \frac{3}{4} - \frac{2}{5} - \frac{2}{3} - \frac{4}{7} - \frac{3}{5} - \frac{1}{4} - \frac{1}{3} - \frac{3}{7} = \square$

Copymaster: Fraction Tagging





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