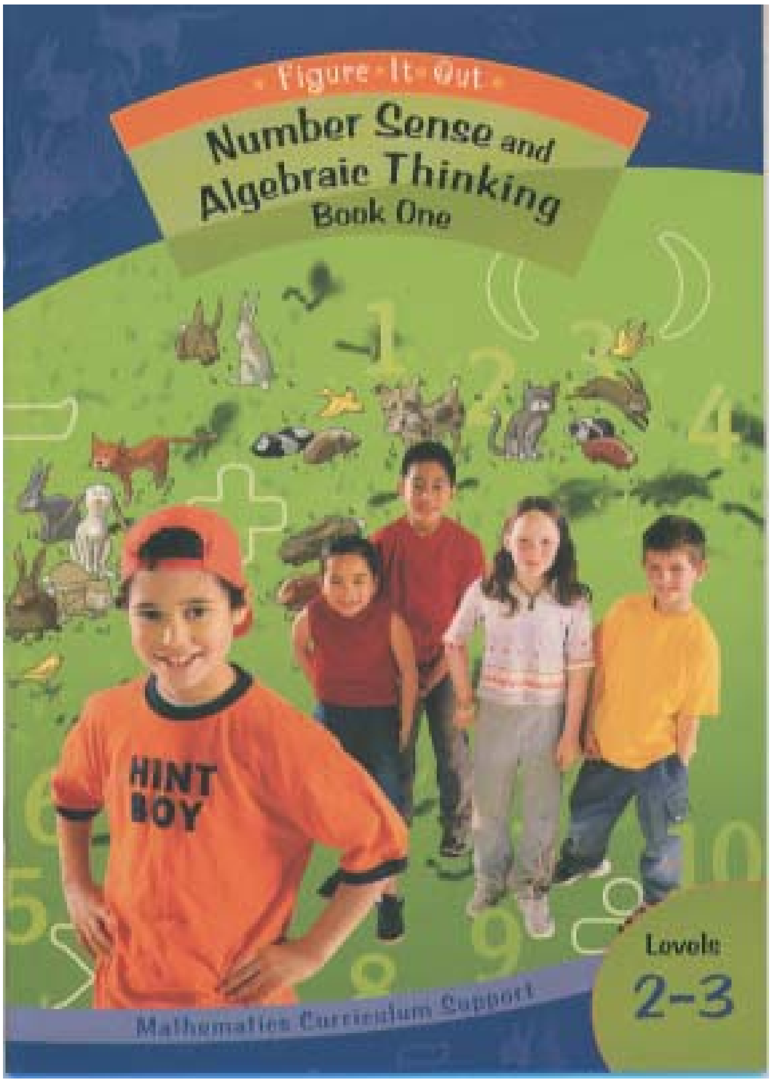
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Contents

Introduction

The books for levels 2–3 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand classrooms. These books are most suitable for students in year 4, but you should use your judgment as to whether to use the booklets with older or younger students who are also working at levels 2–3.

Student books
The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and the life experiences that are meaningful to students in year 4.

The activities can be used as the focus for teacher-led lessons, for students working in groups, or for independent activities. Also, the activities can be used to fill knowledge gaps (hot spots), to reinforce knowledge that has just been taught, to help students develop mental strategies, or to provide further opportunities for students moving between strategy stages of the Number Framework.

Answers and Teachers' Notes
The Answers section of the *Answers and Teachers' Notes* that accompany each of the *Number Sense and Algebraic Thinking* student books includes full answers and explanatory notes. Students can use this section for self-marking, or you can use it for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, Number Framework links, comments on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/r/maths/curriculum/figure

Using Figure It Out in the classroom
Where applicable, each page starts with a list of equipment that the students will need in order to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to the problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Figure It Out

Number Sense and Algebraic Thinking
Book One

Answers

Page 1: Wrapping up Wontons

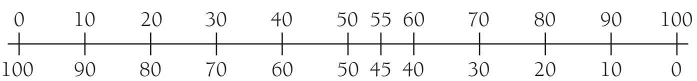
Activity One

1. Add piles **a** and **e**: 8 + 12 = 20
   Add piles **b** and **f**: 6 + 14 = 20
   Add piles **c** and **d**: 13 + 7 = 20

2. **a.** 23. (27 + 23 = 50)
   b. 31. (19 + 31 = 50)
   c. 12. (38 + 12 = 50)
   d. 44. (6 + 44 = 50)
   e. 29. (21 + 29 = 50)

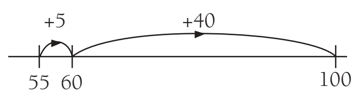
Activity Two

1. Methods will vary. She could use a double number line:

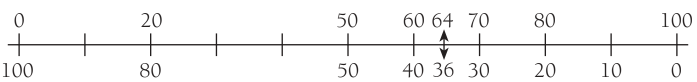


On this number line, An-Mei finds 55 on the top and works out that the complement on the bottom must be 45.

Another method is: 55 + 5 = 60 and 60 + 40 = 100, so An-Mei needs 5 + 40 = 45 items.



2. **a.** **i.** 36. (The double number line of complements to 100 shows 64 on top and 36 below:



Or: 64 + 6 = 70 and 70 + 30 = 100, so 6 + 30 = 36 items are needed.)

ii. 27. (73 – 3 = 70 and 70 + 30 = 100, so 30 – 3 = 27 items are needed.)

iii. 11. (89 + 1 = 90 and 90 + 10 = 100, so 1 + 10 = 11 items are needed.)

b. **i.** 68. (32 – 2 = 30 and 30 + 70 = 100, so 70 – 2 = 68 items were on the platter.)

ii. 59. (41 – 1 = 40 and 40 + 60 = 100, so 60 – 1 = 59 items were on the platter.)

iii. 63. (37 + 3 = 40 and 40 + 60 = 100, so 60 + 3 = 63 items were on the platter.)

3. Problems will vary.

Pages 2–3: Pet Boasting

Activity One

1. 21. (Using a "make tens" strategy, this is 8 + 2 + 6 + 4 + 1.)

8	2	6	4	1
10		10		1

2. 24. ($\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$, so $\frac{1}{4}$ are red. 6 are red, so 6 are yellow and 12 are blue. 6 + 6 + 12 = 24)

$\frac{1}{2}$		$\frac{1}{4}$	6 red
1 whole (all the birds)			
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

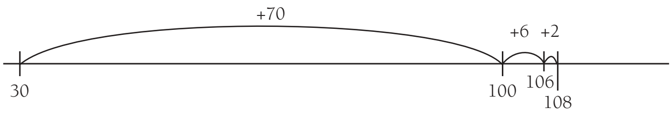
Activity Two

1. 108

2. Strategies may vary. A possible strategy is:
   Harry has 3 goats. Tanya has 3 x 3 or 3 + 3 + 3 = 9 pigs.
   Mari has 9 x 2 or 9 + 9 = 18 lambs.
   Losi has 18 ÷ 3 = 6 calves (3 x ☐ = 18).
   3 + 9 + 18 + 6 = 36. (You could use a tens strategy here:
   2 + 10 + 20 + 4 = 10 + 20 + 6
   = 36.)

Sonya has 2 x 36 or 36 + 36 = 72 rabbits (30 + 30 = 60, 6 + 6 = 12, 60 + 12 = 72).

The total number of pets is therefore 36 + 72 = 108. On a number line, the addition could be shown like this:



3. Problems and strategies will vary.

Page 4: 50 First

Activity

1. **a.** Practical activity. Discussion will vary. Te Rama tries to be the first one to get to 40. Megan's next number will take her to a total between 41 and 49, and then Te Rama can make the answer up to 50 in 1 turn. For example, 43 + 7 = 50.

b. Te Rama can win by making sure that, whatever Megan enters on any of her turns, he enters a digit that will take the total to the next 10. (But if he takes the first turn, Megan will win if she follows the same strategy.)

2. Te Rama could use the same make-10 strategy up to 30. On his next turn, he would need to make the total 37 so that Megan can't make 47. (If Megan's total is already more than 37, Te Rama would go straight to 47.)

3. Yes, Te Rama's strategy would work. You would simply aim to get to a decade (10) as quickly as you can.

4. You can win if you are the first to get to 200. To be certain of a win, you need to be the one to make 100 and then 200.

Page 5: Among the Dolphins

Activity

1. **a.** **i.** \$38.50. (31.50 + 7)

ii. \$38. (\$10 x 4 = \$40, 50c x 4 = \$2.00, then \$40 – \$2 = \$38; or \$9 x 4 = \$36, 50c x 4 = \$2, \$36 + \$2 = \$38)

iii. \$55. (1 family pass, 1 adult's ticket, and 2 children's tickets:
31.50 + 9.50 + 7 + 7 = 55.
Strategies for adding these include: 50c + 50c = \$1; then add \$9 to \$1 to make \$10; then add \$31 to \$10 to make \$41; 2 x \$7 = \$14, so add that to \$41 to make \$55. Two family tickets cost \$63, so that is not the best option.)

b. \$1.50. (2 x 9.50 = \$19; 2 x 7 = \$14; taking \$1 from the \$14 and adding it to \$19 makes \$20; together with the remaining \$13, this makes \$33, which is \$1.50 more than the family ticket.)

2. **a.** 3. (One way to work out 9 adults is:
[9 x \$10] – [9 x 50c]
= 90 – 4.50
= \$85.50
Then: 3 children = \$21.
\$85.50 + \$21 = \$106.50.
\$201 – \$106.50 = \$94.50, which is 3 x \$31.50.)

b. 24. (9 adults and 3 children, plus 3 x 2 adults and 3 x 2 children)

3. 24. (96 ÷ 4 = 24; or 100 ÷ 4 = 25. 25 – 1 = 24)

Pages 6–7: Keeping Score

Activity One

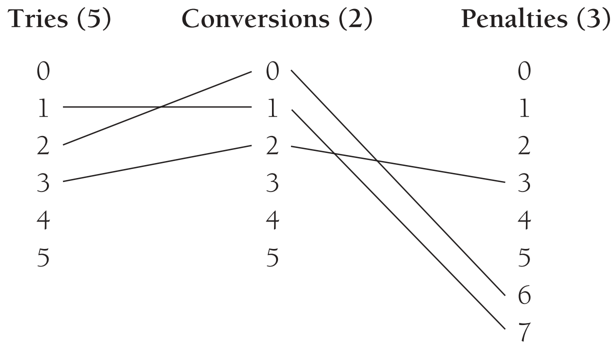
1. **a.** The final score was 27–18.
(5 + 5 + 7 + 7 + 3 = 27 or
5 x 4 + 2 x 2 + 3 = 27. 6 x 3 = 18)

b. The Rockers won by 9 points.

2. **a.** Tables can be set out in different ways. The table on the following page shows how the total score could be made up.

Rockers 28				Movers 17			
Tries (5 points)	Conversions (2 points)	Penalty and drop goals (3 points)	Total	Tries (5 points)	Conversions (2 points)	Penalty and drop goals (3 points)	Total
5 x 5 = 25	0 x 2 = 0	1 x 3 = 3	28	3 x 5 = 15	1 x 2 = 2	0 x 3 = 0	17
4 x 5 = 20	1 x 2 = 2	2 x 3 = 6	28	2 x 5 = 10	2 x 2 = 4	1 x 3 = 3	17
4 x 5 = 20	4 x 2 = 8	0 x 3 = 0	28	1 x 5 = 5	0 x 2 = 0	4 x 3 = 12	17
3 x 5 = 15	2 x 2 = 4	3 x 3 = 9	28				
2 x 5 = 10	2 x 0 = 0	6 x 3 = 18	28				
1 x 5 = 5	1 x 2 = 2	7 x 3 = 21	28				

b. Strategies will vary. The most useful strategy is to add up the points for a given number of tries and see if you can make up the difference with conversions and penalty or drop goals.
You might find it helpful to use a diagram like the one below. Start with 0 tries and see if you can score 28 from penalties only. Then use 1 try and see if you can score the remaining 23 points from a conversion and penalties. Trace a path for each option that works. For example:



c. There are three possibilities: Rockers 3 tries, 2 conversions, 3 penalty or drop goals (15 + 4 + 9) and Movers 1 try, 4 penalty or drop goals (5 + 12); Rockers 2 tries, 6 penalty or drop goals (10 + 18) and Movers 2 tries, 2 conversions, 1 penalty or drop goal (10 + 4 + 3); Rockers 1 try, 1 conversion, 7 penalty or drop goals (5 + 2 + 21) and Movers 3 tries and 1 conversion (15 + 2).

Activity Two

1. To make up the 6 tries and at least 2 more conversions, the Rockers could have scored 34 points through 3 tries, 2 conversions, and 5 penalty or drop goals. (15 + 4 + 15 = 34)

   Here is a possible strategy: If the Aliens scored 3 tries and converted 2 of them, that leaves 3 tries for the Rockers.

   The Rockers must have converted at least 2 of their tries because more than half of all the tries were converted. If we check out the possibilities, we find:
   - 3 converted tries is 7 x 3 = 21 points. 34 – 21 = 13. 13 is not exactly divisible by 3, so this isn't possible.
   - 2 converted tries and 1 unconverted try is 2 x 7 + 5 = 19, and 34 – 19 = 15. This is 5 penalty or drop goals, so this is possible.

2. a. Three other ways are:
   Rockers 4 converted tries and 2 penalty or drop goals (28 + 6) and Aliens 2 tries and 3 penalty or drop goals (10 + 9); Rockers 2 unconverted tries, 3 converted tries, and 1 penalty or drop goal (10 + 21 + 3) and Aliens 1 converted try and 4 penalty or drop goals (7 + 12); Rockers 2 converted tries, 1 unconverted try, and 5 penalty or drop goals (14 + 5 + 15) and Aliens 2 converted tries and 1 unconverted try (14 + 5).

   b. Strategies will vary. These could include the use of tables and trial and improvement. Most will include dividing by 3 the points remaining after tries and conversions.

Pages 8–9: Unusual Rulers

Activity

1. a. $\frac{1}{3}$ (because it takes 3 of Phoebe's foot to match 1 of Tim's arm). A ratio table might show it this way:

Tim's arm	6	5	4	3	2	1
Phoebe's foot	18	15	12	9	6	3

b. $\frac{1}{4}$ (because it takes 4 of Roy's hand span to match 1 of Tim's arm).
A ratio table might show it this way:

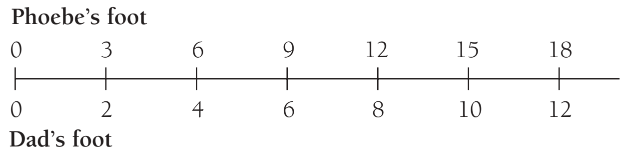
Tim's arm	6	5	4	3	2	1
Roy's hand span	24	20	16	12	8	4

2. a. 20 cm. (60 ÷ 3 = 20)

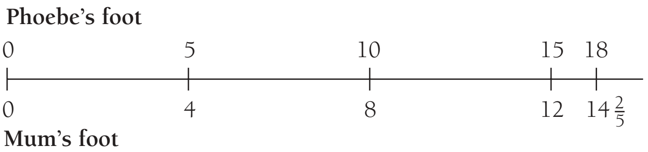
   b. 15 cm. (60 ÷ 4 = 15)

3. 360 cm or 3.6 m. You could calculate it with any or all of the children's units. Tim: 60 x 6; Phoebe: 20 x 18; Roy: 15 x 24

4. a. 12. A double number line could be used like this:



   b. $14\frac{2}{5}$ (rounded sensibly, this is 15 steps). A double number line could be used like this:



5. False (because if his steps were longer, he would not need as many to cross the field)

Investigation

1. Comments will vary, but they should include the idea that body part sizes vary from one person to another, and this variation would lead to misunderstandings. So standardised units are needed. When the measuring task requires accuracy to a millimetre or less, body parts are simply unable to be used efficiently.
Accurate measurement also requires carefully aligning the unit to the task, which is much more difficult with body parts than with a ruler.
Efficient measurement devices like rulers can align a number of units to the object at the same time, but if we used a body part, we would have to measure by repeating a single unit.
We now use mm, cm, m, and km to measure most lengths. We could choose to use a decimetre (10 cm = 1 dm) as well as a decametre (10 m = 1 dam) and a hectometre (100 m = 1 hm), although these are rarely used because the basic units do just as well.

2. Some historical Māori units are: matikara: a fingerspan; tuke: the cubit from elbow to fingertip; mārō: a full arm span from fingertip to fingertip; and the kete or basket (to measure capacity).

Pages 10–11: Worm Wipe-out

Activity

1. a. $1\frac{1}{2}$ tablets

   b. $2\frac{1}{2}$ tablets

   c. 2 tablets. (Rocky needs $1\frac{4}{5}$ tablets, so $1\frac{3}{4}$ won't be quite enough.)

2. a. \$3.50. (\$2 + $\frac{3}{4}$ of \$2)

   b. \$15.60. (2 x $1\frac{1}{2}$ + 2.80 x $2\frac{1}{2}$ + 2.80 x 2
   = 3 + 7 + 5.60
   = \$15.60)

3. More than 5 kg and less than or equal to $7\frac{1}{2}$kg. (It costs \$1.40 per $\frac{1}{2}$ tablet for 5 kg, and 70c per $\frac{1}{4}$ tablet for 2.5 kg.
\$1.40 + 70c = \$2.10)

4. a. Approximately 6 kg (cat) and 15 kg (dog)

   b. One strategy is to draw up a table and use trial and improvement. For example:

Cat		Dog		Total
Mass	Cost	Mass	Cost	
4 kg	\$2	10 kg	\$2.80	\$4.80
5 kg	\$2.50	15 kg	\$4.20	\$6.70
6 kg	\$3	20 kg	\$5.60	\$8.60
6 kg	\$3	15 kg	\$4.20	\$7.20

5. a. The most likely answer is: cat 7 kg and dog 12.5 kg.

   b. Strategies will vary. You might use a table, increasing the cost in both the cat and dog columns:

Cat		Dog	
Mass	Cost	Mass	Cost
4 kg	\$2	10 kg	\$2.80
5 kg	\$2.50	12.5 kg	\$3.50
6 kg	\$3		
7 kg	\$3.50		

Pages 12–13: DVD Decisions

Activity

1. a. $9

   b. i. $6.75. Methods will vary. Continuing Kinesha's thinking, you could say $3 is the same as $4 – (4 x 25c), so of the $3, each person pays $1 – 25c or 75c.
   $5 + $1 + 75c = $6.75

   ii. Strategies will vary. For example:
   $28 ÷ 4 = $7, and $1 ÷ 4 = 25c.
   $27 = $28 – $1, so $\frac{1}{4}$ of $27 is
   $7 – 0.25c, which is $6.75

   Another strategy is 8 + 8 + 8 + 3 = 28.
   $\frac{1}{4}$ of each of these bits is
   2 + 2 + 2 + 0.75 = 6.75

2. $27.50. One way to work this out is:
   30 + 25 = $55. 55 ÷ 2 = $27.50. Another way is to work out $\frac{1}{2}$ of 30 + $\frac{1}{2}$ of 25.
   15 + 12.50 = $27.50

3. $24. (32 ÷ 4 = 8. 32 – 8 = 24 or
   30 – 10 + 4 = 24)

4. a. $9. (39 – 30)

   b. Answers will vary. For example, she could pay for $\frac{1}{2}$ of *Making Masks* with 1 other.
   If Alana doesn't mind paying more than the others, she could pay $9 towards a video, and her friends could share the rest. For example, she could pay $9 towards *Smash Hits,* and 2 of the others could pay $8 each (9 + 8 + 8 = $25).

Investigation

Results will vary.

Page 14: Total Recall

Game

A game using basic facts

Page 15: Soccer Saturdays

Activity

1. a.

Week	Mon	Tue	Wed	Thu	Fri	Sat	Sun
1	W	W	W	W	O	O	O
2	O	W	W	W	W	O	O
3	O	O	W	W	W	W	O
4	O	O	O	W	W	W	W
5	O	O	O	O	W	W	W
6	W	O	O	O	O	W	W
7	W	W	O	O	O	O	W
8	W	W	W	O	O	O	O
9	W	W	W	W	O	O	O
10	O	W	W	W	W	O	O

   b. 6

2. There is a pattern of 4. From the first Saturday that he works, Tavai's dad has 4 Saturdays on, then 4 Saturdays off, then 4 Saturdays on, and so on.

   Here is one way of explaining why this happens: Tavai's dad has 4 days on and 4 days off, so his cycle of work and "weekends" has 8 days in it, which is 1 more than the 7 in a calendar week. This means that if he starts work this week on Tuesday, next week he will start on Wednesday (1 day later). Also, if he starts on Tuesday this week, he will have the next 4 Tuesdays off; if he starts work this Saturday, he will be able to watch Tavai for the next 4 Saturdays in a row.

3. 8 weeks

4. Neither works better because, over the course of the year, Tavai's dad gets half his Saturdays off in both cases. Tavai may prefer the 5 on 5 off arrangement because at most, there would be only 2 Saturdays in a row that his father couldn't watch him.

   A 5-day pattern would look like this:

Week	Mon	Tue	Wed	Thu	Fri	Sat	Sun
1	W	W	W	W	W	O	O
2	O	O	O	W	W	W	W
3	W	O	O	O	O	O	W
4	W	W	W	W	O	O	O
5	O	O	W	W	W	W	W
6	O	O	O	O	O	W	W
7	W	W	W	O	O	O	O
8	O	W	W	W	W	W	O
9	O	O	O	O	W	W	W
10	W	W	O	O	O	O	O

Page 16: Buying Batteries

Activity

1. **a.** \$3.75. ($\frac{1}{2}$ of \$7.00 is \$3.50, and $\frac{1}{2}$ of 50c is 25c. \$3.50 + 25c = \$3.75)

b. \$12.75. ($\frac{1}{2}$ of \$24.00 is \$12.00, $\frac{1}{2}$ of \$1 is 50c, and $\frac{1}{2}$ of 50c is 25c. \$12.00 + 50c + 25c = \$12.75)

2. **a.** Answers may vary. The price of each extra battery goes down by 30c as the pack gets bigger. (The first extra battery costs \$1.90, the second \$1.60, and so on.) Or: The price of *each* battery in a pack is 15c cheaper than the price of the batteries in the previous pack.

b. A 6-pack (\$8.70) + a 2-pack (\$4.10)

c. **i.** A 6-pack and a 4-pack. (\$15.70)

ii. Two 6-packs and a 3-pack. (\$23.10)

iii. Three 6-packs and a 2-pack. (\$30.20)

Page 17: Kapa Haka Practice

Activity

1.

Kapa Haka Practice		
	Number of students	Amount of equipment needed
Poi	30	60
Rākau	20	40
Waiata sheets	10	5

2. **a.–b.**

Kapa Haka Practice		
	Number of students	Amount of equipment needed
Poi	45	90
Rākau	30	60
Waiata sheets	15	8 (1 person doesn't have to share)

3.

Kapa Haka Practice		
	Number of students	Amount of equipment needed
Poi	102	204
Rākau	68	136
Waiata sheets	34	17

4. Strategies will vary. First you need to find the number of students in each group. You could use a box diagram to help you. This example uses the 60 students from question **1**:

60		
30	30	
20	20	20

Or you could divide 60 by 2 and 3. Once you have the numbers in each group, you can double or divide by 2 to find out how much equipment is needed. For the 90 students, you could use $1\frac{1}{2}$ times the 60-student numbers.

Pages 18–19: Clean Cars

Activity

1. Answers may vary. Possible ways to complete and explain each strategy are:

a. "... I added 50 and 30 to get \$80." Sam's strategy uses the fact that 10 + 6 = 16.

b. "... 8 tens or 80." Tanya's strategy uses the fact that 2 x 8 = 16.

c. "... 20 is 4 lots of 5. This gave me \$80." Ariana's strategy uses the fact that 20 – 4 = 16.

2. **a.**

	Sat (\$)	Sun (\$)	Total (\$)
Wash	42	36	78
Vacuum and wash	80	90	170
Vacuum, wash, and and polish	56	112	168
Vacuum, wash, polish, and deodorise	30	60	90

Note: The "Total (\$)" column is not asked for in the question, but it's useful for question **c**.

b. Strategies will vary and include those shown in question **1**. You could use your basic facts (8 x 7 = 56) for Saturday's vacuum, wash, and polish and double that amount for Sunday. Or you might add 50 + 50 + 12 = 112. The simplest way to find the total for the vacuum, wash, polish, and deodorise is to use your 10 times table.

c. \$506

3. a. Vacuum and wash

   b. Vacuum, wash, and polish

4. $44. (550 – 506)

Page 20: Emu Auctions

Activity

1. a. 42 emus. (16 x 6 = 96. 96 – 54 = 42)

   b. 7 paddocks. (54 ÷ 6 = 9. 16 – 9 = 7)

2. Paddock 1: 6 emus ($\frac{1}{10}$ x 60 = 6);
   paddock 2: 12 emus; paddock 3: 18 emus;
   paddock 4: 24 emus

3. Lot 1: 32 emus ($\frac{1}{2}$ x 64 = 32); lot 2: 16 emus;
   lot 3: 8 emus; lot 4: 8 emus

Page 21: Family Ties

Activity

1. a. Georgios is 1, Elana is 5, Theo is 10,
   Nikki is 15, Costas is 20, Mum is 40,
   Dad is 45, Nana is 75, and Grandad is 80.

   b. There are 4 possible solutions:
   Mum, Costas, Nikki (40 + 20 + 15 = 75);
   Dad, Nikki, Theo, Elana
   (45 + 15 + 10 + 5 = 75);
   Mum, Costas, Theo, Elana
   (40 + 20 + 10 + 5 = 75);
   Dad, Costas, Theo (45 + 20 + 10 = 75)

   c. There are 4 possible solutions:
   Mum, Costas, Nikki, Elana
   (40 + 20 + 15 + 5 = 80);
   Dad, Costas, Nikki (45 + 20 + 15 = 80);
   Nana, Elana (75 + 5 = 80);
   Dad, Costas, Theo, Elana,
   (45 + 20 + 10 + 5 = 80)

2. Georgios: 26 (1 + 25)
   Elana: 30 (5 + 25)
   Theo: 35 (10 + 25)
   Nikki: 40 (15 + 25)
   Costas: 45 (20 + 25)
   Mum: 65 (40 + 25)
   Dad: 70 (45 + 25)
   Grandad: 105 (80 + 25)
   The simplest strategy is to add 25 to each family member's current age.

3. Problems will vary.

Pages 22–23: The No Name Game

Game One

1. A possible set of rules is:
   - Each player has their own grid.
   - Take turns to throw the 3 dice.
   - Add up the total of your dice.
   - See if you can make 10 by adding the total of your dice to a number on your grid.
     If you can, cross out that number on your grid.
     If you can't, you miss your next turn.
   - The first player to cross out any 4 numbers is the winner.

2. Practical activity

Game Two

1. A game that involves adding numbers to make 100

2. Rules will vary.

Page 24: Cup Fever

Activity

1. a. Flag $4

   b. Cap $8

   c. T-shirt $16.
   ($84 ÷ 3 = $28. This is the cost of 1 flag, 1 cap, and 1 T-shirt. $4 + $4 x 2 + $4 x 4 = $28)

2. 8 caps, 1 flag, and 1 T-shirt.
   ($8 x 8 + $4 + $16 = $84)

3. There are many possibilities. For example:
   4 T-shirts + 5 flags
   2 T-shirts + 6 caps + 1 flag
   10 caps + 1 flag
   19 flags + 1 cap
   17 flags + 1 T-shirt.

Figure It Out

Number Sense and Algebraic Thinking

Teachers' Notes

Overview of Levels 2–3: Book One

Title	Content	Page in students' book	Page in teachers' notes
Wrapping up Wontons	Using compatible numbers	1	13
Pet Boasting	Using a variety of operations to solve problems	2–3	14
50 First	Adding compatible numbers	4	15
Among the Dolphins	Using a variety of strategies and operations to solve problems	5	17
Keeping Score	Using strategies to solve multiplication problems	6–7	18
Unusual Rulers	Looking at relationships between numbers to solve problems	8–9	19
Worm Wipe-out	Using a variety of operations to solve problems	10–11	21
DVD Decisions	Finding fractions of whole-number amounts	12–13	24
Total Recall	Using basic facts	14	25
Soccer Saturdays	Looking for number patterns	15	26
Buying Batteries	Finding fractions and adding money amounts	16	27
Kapa Haka Practice	Working with fractions and multiplication	17	28
Clean Cars	Using a variety of strategies to solve multiplication problems	18–19	29
Emu Auctions	Using fractions and different operations to solve problems	20	31
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About Number Sense and Algebraic Thinking

The *Number Sense and Algebraic Thinking* books in the Figure It Out series provide teachers with material to support them in developing these two key abilities with their students. The books are companion resources to Book 8 in the Numeracy Project series: *Teaching Number Sense and Algebraic Thinking*.

Number sense

Number sense involves the intelligent application of number knowledge and strategies to a broad range of contexts. Therefore, developing students' number sense is about helping them gain an understanding of numbers and operations and of how to apply them flexibly and appropriately in a range of situations. Number sense skills include estimating, using mental strategies, recognising the reasonableness of answers, and using benchmarks. Students with good number sense can choose the best strategy for solving a problem and communicate their strategies and solutions to others.

The teaching of number sense has become increasingly important worldwide. The need for this emphasis has been motivated by a number of factors. Firstly, traditional approaches to teaching number have focused on preparing students to be reliable human calculators. This has frequently resulted in their having the ability to calculate answers without gaining any real understanding of the concepts behind the calculations.

Secondly, technologies – particularly calculators and computers – have changed the face of calculation. Now that machines in society can calculate everything from supermarket change to bank balances, the emphasis on calculation has changed. In order to make the most of these technologies, students need to develop efficient mental strategies, understand which operations to use, and have good estimation skills that help them to recognise the appropriateness of answers.

Thirdly, students are being educated in an environment that is rich in information. Students need to develop number skills that will help them make sense of this information. Interpreting information in a range of representations is critical to making effective life decisions, from arranging mortgages to planning trips.

Algebraic thinking

Although some argue that algebra only begins when a set of symbols stands for an object or situation, there is a growing consensus that the ideas of algebra have a place at every level of the mathematics curriculum and that the foundations for symbolic algebra lie in students' understanding of arithmetic. Good understanding of arithmetic requires much more than the ability to get answers quickly and accurately, important as this is. Finding patterns in the process of arithmetic is as important as finding answers.

The term "algebraic thinking" refers to reasoning that involves making generalisations or finding patterns that apply to *all* examples of a given set of numbers and/or an arithmetic operation. For example, students might investigate adding, subtracting, and multiplying odd and even numbers. This activity would involve algebraic thinking at the point where students discover and describe patterns such as "If you add two odd numbers, the answer is always even." This pattern applies to *all* odd numbers, so it is a generalisation.

Students make these generalisations through the process of problem solving, which allows them to connect ideas and to apply number properties to other related problems. You can promote process-oriented learning by discussing the mental strategies that your students are using to solve problems. This discussion has two important functions: it gives you a window into your students' thinking, and it effectively changes the focus of problem solving from the outcome to the process.

Although the term "algebraic thinking" suggests that generalisations could be expressed using algebraic symbols, these Figure It Out *Number Sense and Algebraic Thinking* books (which are aimed at levels 2–3, 3, and 3–4) seldom use such symbols. Symbolic expression needs to be developed cautiously with students as a sequel to helping them to recognise patterns and to describe them in words. For example, students must first realise and be able to explain that moving objects from one set to another does not change the total number in the two sets before they can learn to write the generalisation $a + b = (a + n) + (b - n)$, where n is the number of objects that are moved. There is scope in the books to develop algebraic notation if you think your students are ready for it.

The Figure It Out *Number Sense and Algebraic Thinking* books

The learning experiences in these books attempt to capture the key principles of sense-making and generalisation. The contexts used vary from everyday situations to the imaginary and from problems that are exclusively number based to those that use geometry, measurement, and statistics as vehicles for number work. Teachers' notes are provided to help you to extend the ideas contained in the activities and to provide guidance to your students in developing their number sense and algebraic thinking.

There are six *Number Sense and Algebraic Thinking* books in this series:
Levels 2–3 (Book One)
Levels 2–3 (Book Two)
Level 3 (Book One)
Level 3 (Book Two)
Levels 3–4 (Book One)
Levels 3–4 (Book Two)

Page 1: Wrapping up Wontons

Achievement Objectives

- mentally perform calculations involving addition and subtraction (Number, level 2)
- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)

AC
EA
AA
AM
AP

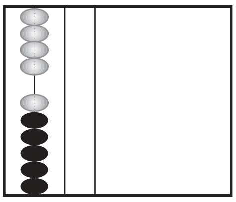
Number Framework Links

Use these activities to:

- encourage transition from advanced counting strategies (stage 4) to early additive strategies (stage 5)
- help the students who are beginning to use early additive strategies (stage 5) to become confident at this stage
- help your students to consolidate and apply their knowledge of groupings with 20, 50, and 100 (place value, stage 4).

Activity One

This activity encourages students to use compatible numbers to make groups of 20 and 50. With a guided teaching group, talk through the context and encourage the students to look for the key information. Check that they know what a wonton wrap is. Before the students move to making groups of 20 and 50, revise counting in 5s and 10s and addition facts to 20 (or to 10 first, if some students need that practice). Use the bead frame as a focus for some quick recall questions and skip-counting practice.



6 and how many more makes 10?

Encourage the students to move beyond materials to imaging piles larger than 10. You could ask them to image the extra wrappers (beads) on the piles as well as image the number and the action needed to make the pile of 20 or 50.

Activity Two

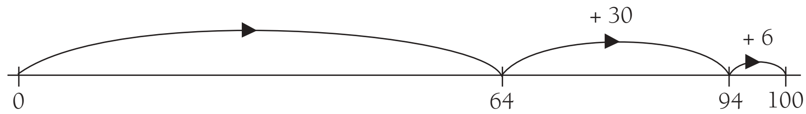
In this activity, the students solve addition and subtraction problems with a sum of 100. Likely compatible number strategies include attending to the tens that make 90 and then looking at the ones.

Make sure the students understand the change in context to a platter of food. A bead frame laid flat or a small hundreds board could be used to model a platter holding 100 items.

Avoid instructing the students in ways that lead them to simply memorise a set of actions. Working in groups will give the students the opportunity to "buy into" a strategy or choose to improve their thinking.

To encourage imaging, use an unnumbered hundreds board and mark only the position that shows the number of food items. For example, a counter on the 55 position shows how many items are left. The students could then image the amount needed to fill the board.

To reinforce the use of number properties, the students could use numbers and diagrams to record the oral language they use to explain their strategy. For example, for question **2a i:**



30 + 6 = 36

If you want to use these activities with independent groups, put the students in problem-solving groups of 3–4 students and have them discuss what each problem is about before they attempt some solutions. Encourage the groups to check that their solutions answer the questions.

Mixed-ability groups work well if the whole team has to be able to explain their solution strategies rather than leaving the best mathematician in the group to report back. Afterwards, have the groups come together to share their strategies and solutions. This will provide an environment for the students to see and hear a range of strategies and will generate an opportunity for those who are advanced counters to "buy into" a part–whole strategy.

You can help the students to clarify and reflect on each other's strategies and encourage part–whole thinking with interactions such as:
So you started at 27, then counted in tens to 97, then counted on 3 more. This helped you to see that you needed 70 and 3, which was a total of 73.
Did anyone work it out without having to do any counting?

Pages 2–3: Pet Boasting

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- solve problems of the type □ + *15* = *39* (Algebra, level 3)

Other mathematical ideas and processes

Students will also use letters to represent unknowns in a meaningful way.

AC
EA
AA
AM
AP

Number Framework Links

Use these activities to:

- encourage transition from advanced counting strategies (stage 4) to early additive strategies (stage 5)
- help the students who are beginning to use early additive strategies (stage 5) to become confident at this stage in all three operational domains (addition and subtraction, multiplication and division, and proportions and ratios)
- help your students to consolidate and apply their knowledge of basic multiplication facts.

Activities

These activities expose students to a range of strategies, from simple addition to multiplication and division with single-digit factors and multiplication using simple unit fractions.

To do these activities, the students need a basic understanding of multiplication and a knowledge of simple unit fractions.

Activity One

Question **2** challenges students to understand the relationship between halves, quarters, and the whole as a set.

If you are using materials and images, fraction pieces such as those in the Numeracy Project material master 4-19 (available at www.nzmaths.co.nz/numeracy/materialmasters.htm) or commercial fraction pieces will be useful. Ask the students: *How can we use this model or diagram to find the total number of birds?*

Extension

As an extension using number properties, have the students record equations showing how many of each type of bird there are, that is, $\frac{1}{2} \times 24 = 12$ blue birds and $\frac{1}{4} \times 24 = 6$ yellow.
Ask them: What happens to a set if you multiply it by a half or a quarter? The students should notice that multiplying by a fraction makes the number smaller. Try to generalise this observation by exploring multiplying by fractions less than 1 and then by fractions greater than 1. This will help the students gain the critical understanding that multiplying by 1 is the point at which the quantity remains constant.

Activity Two

If you are using this activity for guided teaching, encourage the students to first make sure that they understand the problem. They must be able to find the relationships that connect the clues presented by the characters. Use small groups (discussed in the notes for "Wrapping up Wontons") and have them present a report with diagrams that explain these connections. The students could then return to their groups to attempt the problems and report back on their strategies.

If you think materials will help the students, you could encourage the groups to act out the characters and use counters or beans to represent their pet. Animal strips like those in the Numeracy Project material master 5-2 could also be used or adapted to help solve the problems.

Page 4: 50 First

Achievement Objectives

- mentally perform calculations involving addition and subtraction (Number, level 2)
- continue a sequential pattern and describe a rule for this (Algebra, level 2)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- encourage transition from advanced counting strategies (stage 4) to early additive strategies (stage 5) and progression towards advanced additive strategies (stage 6)
- help your students to consolidate and apply their knowledge of groups of 5 and 10 up to 100 (place value, stage 4).

Activity

This activity is a variation on the traditional games of nim and murder 21. It has been structured so that students will use compatible numbers up to 10 (and later, up to 100) as a winning strategy, which will reinforce these number facts. Until the students become aware of this strategy, you will be able to focus on improving their mental calculation strategies using advanced counting or early additive part–whole thinking. It is also a great way for the students to have some fun with number. To do this activity, the students need to be able to sequence numbers to at least 50 for questions **1** to **3** and to 300 for question **4**.

To make the most of this game in a guided teaching group, make sure the students play it a number of times before they answer question **1**. During this initial phase, don't allow the students to enter their number into the calculator until they have explained the counting-on or part–whole strategies they used as they mentally added on their number. Use questions such as:
What number are you thinking of entering?
What total will that give you?
How did you get that total?

Question **1** brings out the number patterns that make the winning strategy. A good focusing question here is: *What "special number" do you need to reach to be sure that your next number will add up to the final winning number?*

When the students discover that they can control the outcome of the game if they're the player who adds a number that results in 40, ask them: *Is there a special number that you need to reach that lets you always make 40?* Work backwards repeating this procedure until all the special numbers are identified (10, 20, 30, and 40).

Question **2** challenges the students to find a winning strategy for a target number that is not a multiple of 10. The special numbers for 47 will be 37, 30, 20, and 10.

Question **3** provides a chance for students to use counting back or part–whole subtraction strategies. The winning strategy to hit 0 means that the first special number is 10. The number that controls 10 is 20, and so on back to 40.

Make sure the students look for the connection between the largest number they can add or subtract and the range in the special numbers. They should find that it is 1 more than the largest number. As 9 is the largest number, the first special number is 10 less than the target.

To generalise the full pattern, they will need to vary the range of numbers they can add or subtract. Stipulate that they can only add from 1 to 5 to make 50. Challenge them to find all the special numbers and test them out by playing some games with a classmate. They should find that the special numbers are found by subtracting 1 more than 5 from 50 and so on. They will be 44, 38, 32, 26, 20, 14, 8, and 2.

Subtracting this new range of numbers from 50 will provide many more opportunities to have the students explain and share their subtraction strategies. These may be counting-back strategies or early part–whole ones.

Question **4** extends the numbers used into the hundreds and changes the range of numbers that they can use to add. A target number of 300 with a range of numbers from 1 to 99 will mean that 200 and 100 are the special numbers for ensuring victory.

The students should note that in all these examples, the person who goes second can always win as long as they follow the strategy. Conversely, the first person can never win unless the other person makes a mistake.

Page 5: Among the Dolphins

Achievement Objectives

- mentally perform calculations involving addition and subtraction (Number, level 2)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- help the students who are beginning to use advanced additive strategies (stage 6) to become confident at this stage
- encourage advanced additive strategies (stage 6) for addition and multiplication, progressing to advanced multiplicative strategies (stage 7)
- help your students to consolidate and apply their knowledge of basic multiplication facts.

Activity

These problems use addition and multiplication, and provide a practical context to broaden and extend strategies.

Knowledge of basic facts is a key part of this activity. Some students may need to have access to a tables chart. The students also need to know the place value of 3-digit whole numbers.

These types of problems are ideal for small groups. The students share their understanding of what the problem is about and discuss ways to solve it. Then they report back to the other groups, sometimes during the process if they are having difficulties, and always after they have a solution.

Discuss the concept of family concessions. Many students will have experienced these at mini-golf courses, theme parks, or sports events. Have the students attempt to solve question **1** in their groups. Bring the groups together to discuss their strategies and to ensure that they understand how the group or family concession can reduce costs. Take this opportunity to check that the students understand that 50 cents is also "point 5 or a half of a dollar".

Question **2** increases the challenge. If the students follow the hint for question **2a**, they will find they have to account for $94.50 in family tickets. Focus on number sense by asking them:
Does this amount give you a clue as to how many family tickets were bought?

Question **3** gives a context for discussing ratio. You could explain the notation of 1:4 as a mathematical way to write this ratio. Use a double number line to show how it works as the numbers increase:

Adults	0	1	2	3	4	5	6	7	8
Children	0	4	8	12	16	20	24	28	32

Compare the ratio of adults to children with that of children to adults so that the students see and understand the inverse relationship of 1:4 and 4:1 and say these as "1 to 4" and "4 to 1".

Extend question **3** by asking the students what would happen if there was a group concession, for example, 4 adults and 16 children for $100. Use questions such as:
How many group concessions would the school need to buy? (6)
How much would the school pay altogether?
Have the students discuss this and record their thinking.

Students using this activity for independent practice or maintenance should be at least at the advanced multiplicative strategy stage.

Extension
The students could investigate what concessions are available at local attractions or on public transport. They could use this information to show how much money different groups would save by using the concession. They could also investigate why concessions are offered. This might involve a survey to establish "critical price points" over which people won't buy tickets.

Pages 6–7: Keeping Score

Achievement Objective

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)

AC
EA
AA
AM
AP

Number Framework Links

Use these activities to:

- encourage transition from early additive strategies (stage 5) to advanced additive strategies (stage 6) in the operational domain of multiplication and division
- help the students who are beginning to use advanced additive strategies (stage 6) to become confident at this stage in the operational domain of multiplication and division
- help your students to consolidate and apply their knowledge of basic facts.

Activities

The context of rugby scores allows students to develop early multiplicative strategies and use a combination of multiplication and addition in recording equations. The range of combinations that the students need to test to solve the problems encourages systematic recording and reinforces the value of persistence in problem solving.

To do these activities, the students need to be able to recall their basic facts unless you allow them access to a tables chart. They also need to understand the term "converted try" in rugby scoring. (After a team scores a try, they get an extra 2 points if their kicker successfully kicks a goal, that is, converts the try.)

Activity One

With a guided teaching group, use question **1** to evaluate the students' understanding of the scoring system used in rugby. (For example, some recent immigrants may not be familiar with the game.) Also evaluate their approach to organising the scoring data. Is it systematic? Do they need help in setting out the data? You could offer suggestions such as using tables, systematic listing, or writing equations.

In question **1**, a bead frame with its fives patterns is ideal for showing the score mounting as the students add the parts. The Rockers' score would be made by moving 4 lots of 5 beads and 2 lots of 2 beads and then adding on 1 lot of 3 beads. This shows that the final score is 27.

Question **2** provides a good model for a table. A possible table is shown in the Answers. Another variation is this matrix table:

Team	Tries	Conversions	Penalty goals	Drop goals	Total points
Rockers	5	0	1	0	28
Working	5 x 5 = 25		1 x 3 = 3		25 + 3 = 28
Movers	3	1	0	0	17
Working	3 x 5 = 15	1 x 2 = 2			15 + 2 = 17

Encourage the more able students to record all their working in one equation, for example, $5 \times 5 + 1 \times 3 = 28$. This will enhance their understanding of equations. You could also use this as an opportunity to discuss some basic order of operations rules (for example, multiplication before addition). Ask the students to explain a mental strategy for working out their equations.

Question **2a** will challenge most students to find an approach that is systematic. Direct the students to the table and ask them how they can use it to test different combinations in a sensible order. Have the students think about the total to see if it gives them some hints to start with. They may be able to use the divisibility patterns as clues. Encourage them to use statements such as: "A total that ends in 5 or 0 can be made with tries" or "An amount that can be divided by 3 can be made with penalty goals."

Question **2c** asks students to sift through the possibilities for both teams and find combinations that work. Because there are only three possibilities for the Movers, the students would find it easiest to work through these one at a time, seeing if they can match each possibility with a Rockers combination to give the desired result.

Activity Two

This activity extends the challenge by leaving the number of tries each team scored open but relative to each other. Encourage the students to try to improve their strategy to test all the possibilities. A number of reporting-back sessions may be necessary as the students try and examine ideas.

Extension

There are other sports that use a variety of ways to accumulate points. For example:
What combination of scoring shots are possible for a batter playing cricket to score 25 runs?
In what ways can a darts player score 18 points by throwing 3 darts?
In basketball, there are 1-point, 2-point (free throws), and 3-point shots. How might a player score 14 points?

Pages 8–9: Unusual Rulers

Achievement Objective

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)

Other mathematical ideas and processes

Students will also:

- use non-standard units for measuring
- find and use number relationships between these non-standard units as a basis for comparison. This has links to early algebraic thinking through the equivalence relationship.

Number Framework Links

AC
EA
AA
AM
AP

Use this activity and investigation to:

- encourage transition from advanced additive strategies (stage 6) to advanced multiplicative strategies (stage 7) in the operational domains of multiplication and division and proportions and ratios
- help your students to consolidate and apply their knowledge of co-ordinating numerators and denominators and help them to recognise equivalent fractions (fractional numbers, stages 6 and 7).

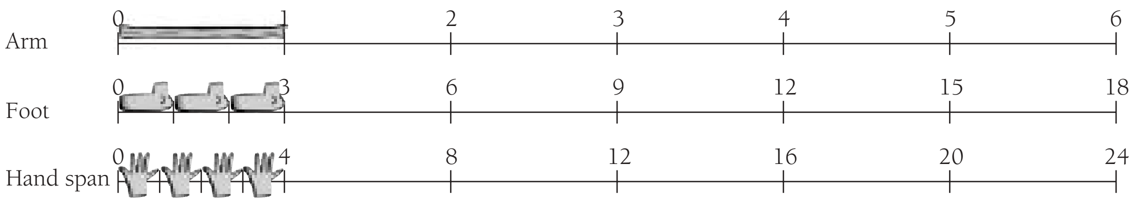
Activity

This activity integrates measurement and number by comparing some non-standard units of length as multiples and then as proportions. This can help students see the relationship between the units in different ways, depending on the strategy used in the comparison. A key idea is that the larger a unit is, proportionally fewer of it will fit in a given space in comparison with a smaller unit. For example, ten 10 centimetre lengths and twenty 5 centimetre lengths fit into the same space.

Before the students do this activity, remind them that there are 100 centimetres in a metre and have them do some quick conversions mentally:
How many metres in 200 centimetres? 300 centimetres? 400 centimetres?
and the reverse:
How many centimetres in 5 metres? 10 metres? 4.5 metres?

To introduce the problem, have the students compare the measurement strategies used by Tim, Phoebe, and Roy. Notice how Tim has transferred his arm unit to an instrument (a bamboo "ruler") to make his measuring easier. Phoebe and Roy are using repetitions of their actual feet or hand spans to count the units.

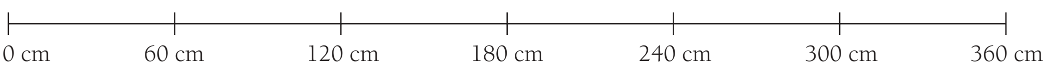
Use the graphic showing each person's measure of the room to compare the size of each unit. The students could transfer the graphic onto some number lines. This will help develop their concept of multiple scales. For example:



You could use suggestions such as:
Find where Phoebe's units line up with Tim's.
How many repetitions did she need to match 1 of Tim's units?
Find where all the units are at the same place. Use this to help you compare the size of each unit.
By following these suggestions, the students may see that Tim's unit is 3 lots of Phoebe's and 4 lots of Roy's.

In question **1**, a ratio table such as that shown in the Answers may be used to show the equivalent fractions that compare Tim's arm to Phoebe's foot and Roy's hand span.

Discuss ways to use Tim's, Phoebe's, and Roy's units to work out questions **2** and **3**. Strategies may include adding a number line to the diagram above to show counting in 60s, for example:

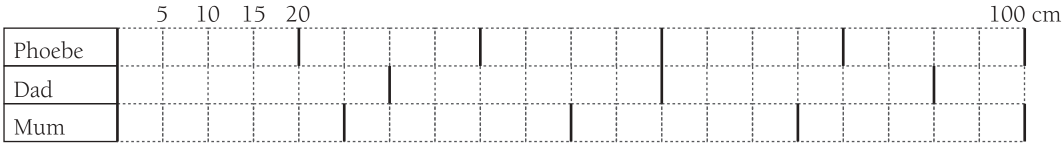


The students may use number properties to answer these questions. Those at strategy stage 6 may double 60 to make 120, then add 120 + 120 + 120 = 360 centimetres. Those at stage 7 may split 60 into 6 x 10, multiply 6 x 6 = 36, and then multiply 36 x 10 = 360. The factor 6 can be split into 2 x 3. So 60 x 2 = 120, then 120 x 3 = 360.

Question **4** is about the number of steps Phoebe and her father and mother will take to cross the room. The students could make a model of Phoebe's foot with multilink or other available cubes, using 1 cube to represent 5 centimetres (her foot is 20 centimetres long, so they will need 4 cubes), and similar models to represent Dad's foot and Mum's foot, 30 and 25 centimetres long respectively, and use each repeatedly to answer the question:
How many steps does it take until Dad's foot lines up with Phoebe's and Mum's foot lines up with Phoebe's?

Then ask: *So if 2 of Dad's steps match 3 of Phoebe's, how many of Dad's will match 18 of Phoebe's?*

Comparisons can be made using double number lines (as shown in the Answers), or alternatively, all the comparisons can be made using a single table like this:



The above table is particularly useful for question **4b**, where the answer involves a fraction:
So 4 of Mum's steps match 5 of Phoebe's. How many of Mum's will match 18 of Phoebe's?
Have the students share their strategies for solving this, for example: "18 ÷ 5 is 3 and a bit. That means that if Mum takes 3 lots of 4 steps, she still has to match Phoebe's 3 last steps." Using the table, the students should be able to see that $2\frac{2}{5}$ of Mum's steps are equal to 3 of Phoebe's steps. So Mum takes $12 + 2\frac{2}{5} = 14\frac{2}{5}$ steps, which has to be sensibly rounded to 15 steps.

A table like the above one can be used to make many other comparisons. You may also like to give your students rooms of different dimensions (in centimetres or metres) and ask them to say what the dimensions are using the foot lengths of Phoebe, Dad, and Mum.

Question **5** is a powerful question to see if students can explain the relationship between the size of the unit and the quantity used in the measurement task. To ensure that the students have generalised this relationship, ask them: *Is it always true that the bigger the unit, the lower the number needed to measure a length?*

Investigation

For the first investigation, the following prompts may be useful:
Two farmers are arguing over the position of their boundary. They both agree that it's 30 paces from the kauri tree, but they never agree where the paces end. Why not?
What is the easiest instrument to use to measure your height accurately, a metre ruler or your hand?
Why were rulers and tape measures developed?
In some jobs, centimetres are commonly used, while in others, millimetres are used. Why is this?

For the second investigation, the website www.nzetc.org/etexts/BesMaor/c6-1.html (Victoria University of Wellington) will provide access to an excellent description of traditional measures used by Māori.

Pages 10–11: Worm Wipe-out

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)

Number Framework Links

Use this activity to:

- help the students who are beginning to use advanced additive strategies (stage 6) to become confident at this stage in all three operational domains (addition and subtraction, multiplication and division, and proportions and ratios).
- encourage transition from advanced additive strategies (stage 6) to advanced multiplicative strategies (stage 7)

Activity

In this activity, the students can use a variety of strategies for simple ratio and proportion problems, ranging from advanced counting approaches to early proportional thinking. They will also consolidate their understanding of simple mixed fractions.

This activity should be introduced through guided teaching rather than as an independent activity unless students are confident at stage 7 or above on the Number Framework.

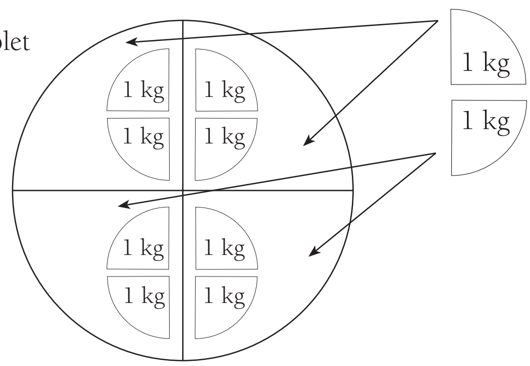
Begin with some mental exercises to check that the students understand how to find quarters and halves in mixed fractions and equivalences between halves and quarters. For example: *How many quarters are in* $\frac{3}{4}$? *How many halves are in* $1\frac{1}{2}$? *So how many quarters are in* $1\frac{1}{2}$? You should also check that the students understand that fractions can equally well be expressed as decimals. They may use both in this activity.

Introduce the context using the vet's instructions. The students could act out this initial situation using large paper circles folded or marked in quarters to model the tablets. Have them mark each quarter of the cat's tablet as 1 kilogram. This is a good opportunity to introduce the students to the concept of "rate" as a multiplicative relationship between two or more different measurement units. Here it is tablets per kilogram, but another common example is kilometres per hour, as in 60 km/h.

Send the students to their small groups to work out how many kilograms they need to link to each quarter of the dog's tablet. Have the students report back on their strategies. If they cannot solve this, give each group 10 rectangular labels of the same size labelled "1 kg of dog". Have them share these out onto the quarters of one of the dogs' tablets and insist that they assign all 10 equally to the quarter tablets.

Use the diagram below and ask: *How do we share these 2 kilograms among 4 quarters?*

Dog's tablet



Send the students back to their groups to attempt question **1** and then report back on their strategies. Encourage a number of strategies, for example: "I counted the kilograms on each quarter" or "I divided the total cat kilograms by 4 because 4 kilograms makes 1 tablet, and I divided the dog kilograms by 10 and then looked at the remainder." Rocky's mass doesn't fit neatly into the $\frac{1}{4}$ tablet rate, so his dosage has to be rounded up.

Question **2** extends the problem by adding the relationship of cost. This involves using two different rates simultaneously, the rate of mass to dose and the rate of tablets to cost. Send the students into their groups with the question: *What will you need to know to work out the cost of 1 dose?* When they realise that it depends on the number of tablets, they should attempt questions **2a** and **2b** and report back on their strategies.

For question **3**, ask the students: *What will you need to know to work out the mass of Brock the dog?* After the students have responded, sketch the complete set of relationships used in this problem (using multiples of $\frac{1}{4}$ of a tablet):

Mass	$\longleftrightarrow$	Tablets	$\longleftrightarrow$	Cost
2.5 kg		$\frac{1}{4}$		\$0.70

In question **4**, you may need to show the students how to record their trials systematically so that they can use each result as a hint until they find the best result.

Cat		**Dog**		**Total**
Mass	Cost	Mass	Cost	
4 kg	\$2.00	10 kg	\$2.80	\$4.80

Before the students attempt question **5**, have them record the cost of each fraction of a tablet for a dog and a cat, for example:

	Cat		**Dog**	
	Cost	Mass	Cost	Mass
Whole tablet	\$2.00	4 kg	\$2.80	10 kg
Half tablet	\$1.00	2 kg	\$1.40	5 kg
Quarter tablet	50c	1 kg	70c	2.5 kg

They can use this information to help them make a table (as in question **4**) or set up a double number line. For example:

Cat	Mass	1 kg	2 kg	3 kg	4 kg	5 kg	6 kg	7 kg	8 kg
	Cost	50c	\$1.00	\$1.50	\$2.00	\$2.50	\$3.00	**\$3.50**	\$4.00
Dog	Cost	70c	\$1.40	\$2.10	\$2.80	**\$3.50**	\$4.20		
	Mass	2.5 kg	5 kg	7.5 kg	10 kg	12.5 kg	15 kg		

Extension

The students may be able to see that the 35 in \$3.50 is the lowest common multiple of 7 x 5. You could also ask them why the answers for questions **3** and **4** are approximate. The reason is that the cat mass doesn't go up in 1 kilogram steps to match the $\frac{1}{4}$ tablets. A cat weighing 1.1 kilograms would also get $\frac{1}{4}$ tablet. This also introduces the concepts of limits of accuracy and rounding, which is also needed for question **1c**.

Pages 12–13: DVD Decisions

Achievement Objectives

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- solve problems of the type □ + *15* = *39* (Algebra, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity and investigation to:

- help the students who are beginning to use early additive strategies (stage 5) to become confident at this stage in all three operational domains (addition and subtraction, multiplication and division, and proportions and ratios)
- encourage transition from early additive strategies (stage 5) to advanced additive strategies (stage 6).

Activity

By combining the money context with different numbers of friends sharing costs, this activity allows a variety of strategies to be used with subtraction, division, and fractions.

The students at stage 6 (advanced additive) or above could attempt this activity independently in groups and report back.

Introduce this activity to a guided teaching group by pointing out that the DVD prices have been rounded up to a tidy dollar amount even though shops almost always price them at so many dollars and 95 cents. This rounding enables the students to calculate the shared costs better because we cannot split 5 cents in real money.

Introduce or revise $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of $1.00 as 25 cents, 50 cents, and 75 cents respectively. These facts will be useful in solving some problems. The students could record these with other facts that they need to memorise.

The students who need to use materials could act out the scenarios in each question with play money and a card representing each DVD. When they report back, discuss different ways to split the cost equally. Ideas may include sharing out dollars in units of $1, $5, or $10 notes, one at a time, and then sharing out coins in the same way, as needed, to meet the price.

For imaging, have the students pretend that their money is being carried by an adult and that they have to ask for the notes and coins they will use to buy each DVD. They receive the money after they have solved the sharing problem by imaging the money they need.

To encourage the students to use number properties, put them in small groups and ask them to find and share different ways of using and recording numbers and operations to solve the problems.

Highlight the connections between ideas. For example, question **1** can be solved by having the three friends sharing out $5, $5, and $5, then $2, $2, and $2, and finally, $2, $2, and $2 again while they keep a running total. The students who know □ x 3 = 27 should see that this is the same as sharing out $9, $9, and $9.

Question **1b** provides an opportunity to discuss and reinforce the connection between $27 ÷ 4 and finding $\frac{1}{4}$ of $27. Students don't automatically understand that dividing by a number and multiplying by its inverse (reciprocal) are just two different ways of viewing the same thing. (Dividing by 4 is accessible to early additive students if they use halving and halving.)

When students see that dividing by 4 has the same effect as multiplying by $\frac{1}{4}$ and vice versa, create a pattern by extending this to dividing by 2 and multiplying by $\frac{1}{2}$ and other pairs of reciprocals, such as $\frac{1}{3}$ and 3. Encourage the students to describe the pattern.

Investigation

Remind the students before they begin the investigation to round the cost of the DVDs to the nearest whole dollar.

Page 14: Total Recall

Achievement Objectives

- mentally perform calculations involving addition and subtraction (Number, level 2)
- recall the basic multiplication facts (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this game to:

- encourage transition from advanced counting strategies (stage 4) to early additive strategies (stage 5) and progression to advanced additive strategies with multiplication (stage 6)
- help the students who are beginning to use early additive strategies (stage 5) to become confident at this stage
- help your students to consolidate and apply their knowledge of basic multiplication facts.

Game

This game promotes the recall and use of basic facts. There will be many opportunities for you to use the "and how did you work that out?" question. You could also use this game to show how brackets can be used to control the order of operations.

If you are using this game in a guided teaching situation and you display and explain the rules, you will still need to revisit them as the students play their first few games. If you do want to show the students how to use brackets to control the order of operations, insist that they write the equation that shows how they combined their dice numbers. There will be many opportunities for them to use brackets to enable addition or subtraction to be done before multiplication, for example, $(3 + 5) \times 2$ will make 16, whereas $3 + 5 \times 2$ will make 13.

Review the variety of ways that equations can be made and highlight the increase in choice that the students have if they combine operations, mix around the digits, and use brackets. Encourage the students to manipulate expressions "around" a combination of 2 dice. For example, with a throw of 3, 2, and 5, you can use $3 \times 2 = 6$ as a start and make 1 and 11: $(3 \times 2 - 5)$ and $(3 \times 2 + 5)$.

When the students understand the game, it is an ideal maintenance or fun activity. Use it as a pairs activity where both students check each other's equations or as a small-group activity where a leader who understands the use of brackets can check the responses of the others.

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)

- Book 4: *Teaching Number Knowledge*
  Bowl a Fact, page 35 (a similar game with a different context).

Page 15: Soccer Saturdays

Achievement Objectives

- describe, in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- use words and symbols to describe and continue patterns (Mathematical Processes, developing logic and reasoning, levels 2–3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to help your students develop their knowledge of basic addition facts. Students can solve the calculations in these problems using advanced counting (stage 4) or early additive (stage 5) strategies.

Activity

This activity requires students to use logic and reasoning with a number pattern to solve a valid contextual problem.

The essence of this problem is the fact that the pattern repeats in a cycle of 8 days (4 on, 4 off) whereas there are only 7 days in a week. The "extra" day means that, each time, the cycle begins 1 day later than the last time. This appears on the grid as a diagonal pattern. Don't give the game away by explaining this to the students at the beginning because this would remove the problem-solving element and reduce the activity to a series of exercises.

Introduce the activity to a guided teaching group by focusing the students on Tavai's concern (how many Saturdays Dad can watch him play soccer) and then on Dad's work pattern of 4 days on and 4 days off. Give the students the table copymaster and send them into problem-solving groups to attempt question **1**. Then bring the groups back together to share their solutions.

Read through question **2** with the students and give them time to find and discuss any patterns.
Ask the students questions such as:
Are there always 4 Ws and then 4 Os in the sequence?
What pattern do you see when you look horizontally?
Diagonally?
Vertically?
What causes the pattern to "move" as it does?
What would the pattern look like if we had an 8-day week?
(Some students may need to make an 8-day chart to answer this question.)
Subsequent discussion should make the essence of the problem clear and explicit.

The students should fill in the table for question **4** for a minimum of 12 weeks. (It begins to repeat after 10 weeks, but the students need to go a little further to confirm that this is happening.) Because the cycle is now 5 + 5 = 10, it begins each time 3 days later than it did the time before (10 = 7 + 3). Encourage your students to find and describe the patterns within the completed table.

Extension
A similar problem would be to have the students explain why their birthday is on a different day of the week each year. Explore this for ordinary years and also for a leap year.
(365 = 52 x 7 + 1; 366 = 52 x 7 + 2; so the days next year are either 1 or 2 different, depending on whether the year is a leap year.)

Page 16: Buying Batteries

Achievement Objectives

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- help the students who are beginning to use advanced additive strategies (stage 6) to become confident at this stage in the operational domains of addition and subtraction and proportions and ratios
- encourage transition from advanced additive strategies (stage 6) to advanced multiplicative strategies (stage 7). (Use question **2** for this.)

Activity

This activity helps students to develop strategies for tackling simple fraction and proportion problems. Introduce it through a guided teaching group rather than as an independent activity unless your students are confident at stage 7 or above on the Number Framework.

Begin with some exercises to check that the students understand how to add using money notation. Then focus on why John chose to split $9.70 the way he did. If necessary, suggest that this made the problem easier for him to solve mentally.

When the students have completed question **1**, ask them to suggest a different way to split $9.70. Ideas may include $10 – 30c or $8 + $1 + 60c + 10c.

The students may find explaining the pattern in question **2** a challenge. They should be able to calculate the increase as the number of batteries grows:

Number of heavy-duty AA batteries	1	2	3	4	5	6
Cost	$2.20	$4.10	$5.70	$7.00	$8.00	$8.70
Increase		$1.90	$1.60	$1.30	$1.00	70c

Hints like *What is happening as John adds another battery?* may help them see that each additional battery is another 30 cents cheaper.

A more challenging way to look at the changes would be to have the students calculate the value of *each* battery in the pack as the number increases. This is a unit rate idea.

Number of heavy-duty AA batteries	1	2	3	4	5	6
Cost	$2.20	$4.10	$5.70	$7.00	$8.00	$8.70
Calculation	$2.20 ÷ 1	$4.10 ÷ 2	$5.70 ÷ 3	$7.00 ÷ 4	$8.00 ÷ 5	$8.70 ÷ 6
Cost of each battery	$2.20	$2.05	$1.90	$1.75	$1.60	$1.45

This pattern shows that the cost of each battery is reduced by 15 cents as one more battery is added.

The students who need to use materials to calculate each part of the pattern could draw large outlines of the batteries and use play money to distribute the price of each pack evenly among all the batteries.

To use number properties, the students will need to be able to divide or multiply. They could record their thinking as an equation, for example, $5.70 ÷ 3 = ☐ or ☐ x 3 = $5.70. You could make the problem accessible to advanced counting and early additive students by allowing them to use a calculator.

For questions **2b** and **2c**, the students will need to understand that John's biggest pack is a 6-pack, so combinations will need to be tried and tested. Encourage the students to notice that the solutions involve using as many 6-packs as possible and have them try to explain why this is true. (6-packs have the lowest unit cost.)

Page 17: Kapa Haka Practice

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- solve problems of the type ☐ + *15* = *39* (Algebra, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

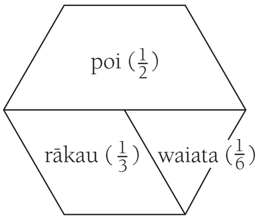
- help the students who are beginning to use early additive strategies (stage 5) to become confident at this stage in all three operational domains (addition and subtraction, multiplication and division, and proportions and ratios)
- encourage transition from early additive strategies (stage 5) to advanced additive strategies (stage 6)
- help your students to consolidate their knowledge of ordering unit fractions and simple equivalences (fractional numbers, stages 5 and 7).

Activity

This activity creates a context in which students work out fractions of a set. They can use a range of strategies in the process, including repeated halving, equal sharing, and using known addition and multiplication facts.

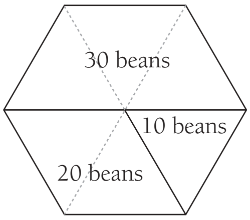
Introduce this activity through a guided teaching group rather than as an independent activity unless your students have strategies at stage 6 or above on the Number Framework. Begin with some mental exercises to practise finding $\frac{1}{2}$ and $\frac{1}{4}$ of a set, for example, $\frac{1}{2}$ of 10, 20, and 30 and $\frac{1}{4}$ of 8, 16, and 24.

The students who need to use materials can use the hexagon in a set of international pattern blocks to model the whole group. The trapezium will then represent the $\frac{1}{2}$ (the poi), whereas the rhombus will be the $\frac{1}{3}$ using rākau. Complete the hexagon with a triangle and ask the students: *How much of the whole hexagon is the triangle?* When they can see that it represents $\frac{1}{6}$ of the whole, make sure that they identify the $\frac{1}{6}$ with the rest of the students who will be in the group practising waiata.



When the students understand that $\frac{1}{6}$ of the group will practise waiata, check that they understand the connections between the numbers of students and the number of poi, rākau, and waiata sheets mentioned in the problem.

Send the students into small groups to attempt question **1**. They should report back before doing question **2**. Some students may need to sketch a large hexagon showing the division into $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$. They could use 60 beans or counters to represent the team members and model the solution by sharing out the beans over the hexagon. Encourage the students to use number properties to anticipate the result of equal sharing before they use materials.



The students who use number properties to find $\frac{1}{3}$ of the set will either divide by 3 or change the question to $3 \times \square = 60$ and ask themselves: "What is the value of the box?"

In question **2**, have the students look for an easy way to solve the problem. If necessary, prompt them to use question **1** to help them work out question **2**. Note that the number of waiata sheets should be rounded up to the nearest whole number to be sensible.

Question **3** will be more of a challenge. Remind the students that if 204 is too hard to work with, they can split it into easy parts. Send them into groups to find some good splits and report back on these. They may see that 204 = 60 + 60 + 60 + 24, so they only have to work out what share the 24 get because they know what 60 members get from question **1**. Similarly, 204 = 180 + 24, so they could use twice the answer for question **2** plus the shares for the extra 24.

Pages 18–19: Clean Cars

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- recall the basic multiplication facts (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Students using advanced counting (stage 4) strategies can solve these problems using materials and by skip counting, for example, in fives.

Use this activity to:

- help the students who are beginning to use early additive strategies (stage 5) to become confident at this stage in the operational domain of multiplication and division
- encourage transition from early additive strategies (stage 5) to advanced additive strategies (stage 6)
- help your students to consolidate their knowledge of basic facts.

Activity

This activity allows students to use strategies ranging from advanced counting to early multiplicative thinking. If necessary, help the students to link their counting or addition strategies to multiplication as a more efficient form of thinking for this type of situation. (The three strategies depicted all use multiplication.)

For this activity, a recall of basic multiplication facts is desirable. Students without this knowledge need to be able to skip-count in at least 5s and 10s and be able to derive an unknown multiplication fact from a known fact or use a basic fact reference chart.

Introduce the activity to a guided teaching group by discussing the differences in the grooming types and charges. Have the students convert the tally chart into numbers to ensure that they understand it.

Question **1** is the key to meeting the achievement objectives for this activity. To find out what strategies your students already have, ask them to attempt to solve the question individually or in groups before looking at the strategies suggested in the speech bubbles. After they report back, they can compare their strategies with those suggested in the speech bubbles and then complete those statements.

The students who are already confident at stage 5 (early additive) or above could begin by discussing and completing the speech bubble strategies.

For some students, you may need to clarify some of the strategies. Place value materials, counters, or cubes will be important in modelling these strategies. For question **1a**, ask the students: *How has Sam made it easier to work out 16 lots of 5?* The explanation should highlight the place value split:

16

10 + 6

so 16 x 5 is made of 10 x 5 and 6 x 5.

For question **1b**, repeat the question: *How has Tanya made it easier to work out 16 lots of 5?*
The students may need a chart like this to help them see the strategy:

Number of 5s	Number of 10s
2	1
4	2
6	3
...	...
16	8

If necessary, use a bead frame to model this pattern.

As an algebraic extension, ask the students to find a generalisation that describes *any number of 5s makes what number of 10*? The generalisation can be expressed as: "The number of 10s is half the number of 5s" or "There are twice as many 5s as 10s."

For question **1c**, again ask: *How has Ariana made it easier to work out 16 lots of 5?* In response, the students should identify a subtractive split.

16

20 – 4

so 16 x 5 is made of 20 x 5 minus 4 x 5.

Have the students work in small groups to attempt questions **2–4**. For the students who need to use materials, the bead frame would be an excellent tool to help them model the problem and for you to use as you re-explain some strategies.

For the students using imaging, the Numeracy Project material master 5-12, Number Line in Tens (available at www.nzmaths.co.nz/numeracy/materialmasters.htm), would help them to image the actions used on the bead frame and provide a link to recording with numbers.

The reporting back and sharing of strategies from each group will be the time to clarify and compare strategies. As the students report back, it can be helpful if you record the strategies they used on the whiteboard, making sure that the recording accurately reflects the report.

Page 20: Emu Auctions

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Students using advanced counting (stage 4) strategies can solve these problems using materials.

Use this activity to:

- help the students who are beginning to use early additive strategies (stage 5) to become confident at this stage in all three operational domains (addition and subtraction, multiplication and division, and proportions and ratios)
- encourage transition from early additive strategies (stage 5) to advanced additive strategies (stage 6)
- help your students to consolidate their knowledge of co-ordinating numerators and denominators in fractions and simple equivalences (fractional numbers, stages 6 and 7).

Activity

This activity encourages students to use diagrams of farm regions to help them find fractions of sets of emus. There is a wide range of strategy options available to students who have access to materials and diagrams.

Introduce this activity through guided teaching rather than as an independent activity unless your students are confident at stage 6 or above on the Number Framework and have a good understanding of equivalent fractions.

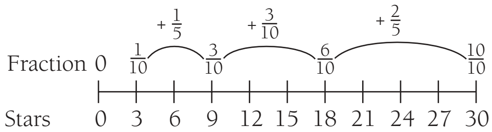
The students can clarify the problem by making a diagram of the farm in question **1**. Share a number of alternative arrangements for the farm paddocks, for example, paddocks arranged as grids in patterns of 4 x 4 or 8 x 2.

Ask the students using imaging:
How many paddocks of 6 emus do you think can be filled with 54 emus?
Now image the rest of the paddocks to work out the total needed to fill the farm.

Those students who find this too difficult could use materials to model all the emus on their diagrams and find out how many will fill the farm if 6 go in each paddock. If they use counting strategies, ask them to try and find the total again without counting. This will encourage part–whole thinking.

The question **2** scenario may need clarification. Relate the fractional size of the paddocks to the farmer's plan for equal space for each emu. In the Numeracy Project material master 7-8, Mystery Stars (available at www.nzmaths.co.nz/numeracy/materialmasters.htm), there is a circle divided into tenths with 3 stars in each tenth. Use this to explain the equivalent relationship between fifths and tenths and how if $\frac{1}{10}$ holds 3 stars, then $\frac{1}{5}$ holds 6 stars. You could also use it to show how $\frac{1}{10} + \frac{1}{5} + \frac{3}{10} + \frac{2}{5} = 1$.

On a double number line, compare the 30 stars on the whole circle of the material master with the fractions used in the problem:



Now ask the students to image 60 stars on the whole circle and find out how many are in each fraction. Connect this problem to the problem in question **2**. If necessary, the students could draw another number line using 60 emus instead of 30 stars.

Have the students use number properties to find the number of emus in each paddock. They could record their working as equations in a chart like the one below. A third column could show the equations using tenths. For example:

Paddock	Equation	Equation using tenths
1	$\frac{1}{10} \times 60 = 6$	$\frac{1}{10} \times 60 = 6$
2	$\frac{1}{5} \times 60 = 12$	$\frac{2}{10} \times 60 = 12$
3	$\frac{3}{10} \times 60 = 18$	$\frac{3}{10} \times 60 = 18$
4	$\frac{2}{5} \times 60 = 24$	$\frac{4}{10} \times 60 = 24$

In question **3**, explore the fraction that Maafi's father buys from each lot. Material master 7-7, Fraction Strips, or a fraction board will be an ideal tool for this. The students will need to use 4 strips that join up to make 1 whole. These strips need to match the conditions in the question. They should find $\frac{1}{2}$, $\frac{1}{4}$, and 2 lots of $\frac{1}{8}$ strips. Compare these to the box diagram so that the students connect these models to the problem.

Encourage the students who use imaging to image the number of emus on each part of the box diagram. If necessary, the students could use materials and model each emu. Discuss ways to work out the problem using numbers and have the students attempt to write equations, for example:

$\frac{1}{2} \times 64 = 32$

$\frac{1}{2} \times 32 = 16$

$16 \div 2 = 8$

Page 21: Family Ties

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- solve problems of the type $\square + 15 = 39$ (Algebra, level 3)
- use words and symbols to describe and continue patterns (Mathematical Processes, developing logic and reasoning, levels 2–3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to help the students who are using early additive strategies (stage 5) to become confident at this stage in the operational domains of addition and subtraction and multiplication and division. It is also suitable as an independent activity for students at the advanced additive stage (stage 6) or higher.

Activity

This activity explores relationships involving addition and multiplication. The family context encourages students to take the ideas out of the classroom and use them to explore similar relationships in their own lives.

Whether in a guided teaching group or in an independent group, the students need to discover how to use the one age they know (Georgios's) to work out the ages of the others. The key is to look for age statements that include Georgios's age. There are two of these. The students then work progressively from the known to the unknown.

Have the students move into small groups to find the ages. You could get them to number the statements in the bubble as 1–8 and to record the order in which they use the statements and the information they deduce from each.

Use question **2** as an opportunity for the students to see the pattern of adding 25 and use it as a generalisation for this question. Don't tell the students the "add 25" relationship. Let them solve a number of the ages for themselves. Encourage them to find the generalisation: *Is there a connection between the answers in question 1a and the ones you are now getting?*

In question **3**, have the students record their number clues in words and as equations.

Pages 22–23: The No Name Game

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- recall the basic addition and subtraction facts (Number, level 2)

Other mathematical ideas and processes

Students will also generalise patterns for finding compatible numbers (Algebra)

AC
EA
AA
AM
AP

Number Framework Links

Students can solve the calculations in these problems using advanced counting (stage 4) and early additive (stage 5) strategies.

Use these games to:

- develop the students' knowledge of counting in tens to 100 (place value, stage 4)
- develop the students' knowledge of addition to 10 and multiplying by 10 (basic facts)
- help the students who are beginning to use early additive strategies (stage 5) to become confident at this stage in the domain of addition and subtraction.

Games

These games build knowledge of compatible numbers to make the tidy number 10 and then the tidy number 100. Until the students build this knowledge, they will have to use counting methods to solve most problems. Once they have acquired this knowledge, they will be able to use it with part–whole strategies for addition and subtraction.

Game One

Introduce the first game through the interaction between Ariana and Iosua. After the students have played the game and worked out the rules, ask them to suggest a name that would suit.

Game Two

This game involves a significant extension to **Game One**. The students may wish to find a new name for this version of the game after playing it a few times. Note that 30 has been used twice because (long term) the score of 70 comes up more often than any other score. As an extension, ask: *Why is 90 not used on the scoring grid?* (The smallest total from the dice is 1 + 1 = 2 or, in this case, 10 + 10 = 20. 100 – 20 = 80, so 80 is the highest number that can be crossed off the grid.)

For question **2**, the students could vary the tidy number targets. For example, a target of 200 is possible, provided the numbers on the grid start from 20 rather than 10 and do not exceed 170. Whatever variations they come up with, the students need to check that all 9 numbers on their grid can be used.

Extension
You could extend **Game Two** by using only 2 dice and adding some special rules for dice throws:

Roll	What to do
Two 6s	Pass.
A 6 and a 5	Have an extra turn.
A 6 and a 4	Give the next player an extra turn.
Two 5s	Cross out any number on your grid.
Any other numbers?	Follow Ariana and Iosua's method above.
Can't cross out a number on your grid?	Pass.
First to cross out all your 9 numbers?	You're the winner!

After the students have played the game using these rules, ask: *Why are there special rules for 6 + 6, 6 + 5, 6 + 4, and 5 + 5?* (All of these scores give a total of 100 or greater, so there is no remainder when taken away from 100.)

The students could then try the game with a number such as 130 as the target and work out how this would simplify the dice throw rules for 2 dice.

Page 24: Cup Fever

Achievement Objective

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)

Other mathematical ideas and processes

Students will also:

- solve relationship problems
- use logic and reasoning through if/then statements.

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- encourage transition from early additive strategies (stage 5) to advanced additive strategies (stage 6)
- help the students who are beginning to use advanced additive strategies (stage 6) to become confident at this stage in the domains of addition and subtraction and multiplication and division.

Activity

The need to use the relationship between one item and another to work out the first item's value is a challenging but important step in pre-algebraic thinking. This activity is carefully scaffolded to encourage students to use trial-and-improvement strategies to solve this type of problem.

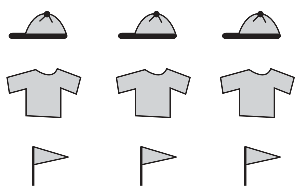
Only the students who are confident at stage 6 or above in numeracy strategies should attempt this activity independently.

Use the thought bubble to check that the students understand the multiplicative relationship between 3 caps, 3 T-shirts, and 3 flags and that by dividing by the common factor of 3, they get 1 of each. You could use materials to show this:

Caps

T-shirts

Flags



The students could use this chart arrangement for the souvenirs:

	Caps		T-shirts		Flags		Total
If	3	+	3	+	3	=	$84
Then	1	+	1	+	1	=	$28

One approach is to encourage your students to use the trial-and-improvement strategy suggested in the first speech bubble. They need to see that the incorrect answer to their trial can give them a clue as to how much to use in the next trial, rather than simply using random guesses in the rest of their trials. For example: "I tried 1 flag costing $2, so then 1 T-shirt cost $8 and 1 cap cost $4. The total would be $14, which is half of $28, so I should double the cost of the flags."

An alternative approach is to use the flag as a unit. For example:
A cap is worth 2 flags, and a T-shirt is worth 4 flags. So how many flags could you buy for the cost of 1 flag, 1 cap, and 1 T-shirt? (1 + 2 + 4 = 7 flags)
7 flags cost $28. $7 \times \square = 28$?
Once the cost of a flag is known ($4), this amount can be doubled and doubled again to get the cost of a cap and a T-shirt.

A trial-and-improvement strategy will work well in question **2**. Focus the students on the fact that there must be 8 of the same item. If they explore what 8 of each item will cost, they will find:

Item	Unit Cost	Total cost for 8
Cap	$8.00	$64.00
T-shirt	$16.00	$128.00
Flag	$4.00	$32.00

This chart shows that the only possible item to have a chance of costing $84 for 8 of that item plus 2 other items is the 8 caps.

Be sure to discuss a variety of strategies that can be used to work out each total. Some students will see the doubling pattern in the costs of flags, caps, and T-shirts as a quick way of working out the full costs for each item.

Question **3** asks the students to find other combinations that make (exactly) $84. There are many possible combinations. For example, the first bubble says that 21 flags make $84, but 2 flags can always be replaced by a cap and 4 flags by a T-shirt. You may wish to limit the number the students find by setting the challenge to be the first to make 10 different combinations that are correct.

Copymaster: Total Recall

6	26	24	1	30	2
19	45	48	17	9	21
12	35	16	28	13	15
10	4	32	20	25	27
6	3	5	22	8	36
23	11	0	14	31	18

Copymaster: Soccer Saturdays

Week	Mon	Tue	Wed	Thu	Fri	Sat	Sun
1	W	W	W	W	O	O	O
2	O						
3							
4							
5							
6							
7							
8							
9							
10							

Week	Mon	Tue	Wed	Thu	Fri	Sat	Sun
1	W	W	W	W	W	O	O
2	O	O	O				
3							
4							
5							
6							
7							
8							
9							
10							

Copymaster: The No Name Game

4	2	3
5	1	6

4	2	3
5	1	6

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