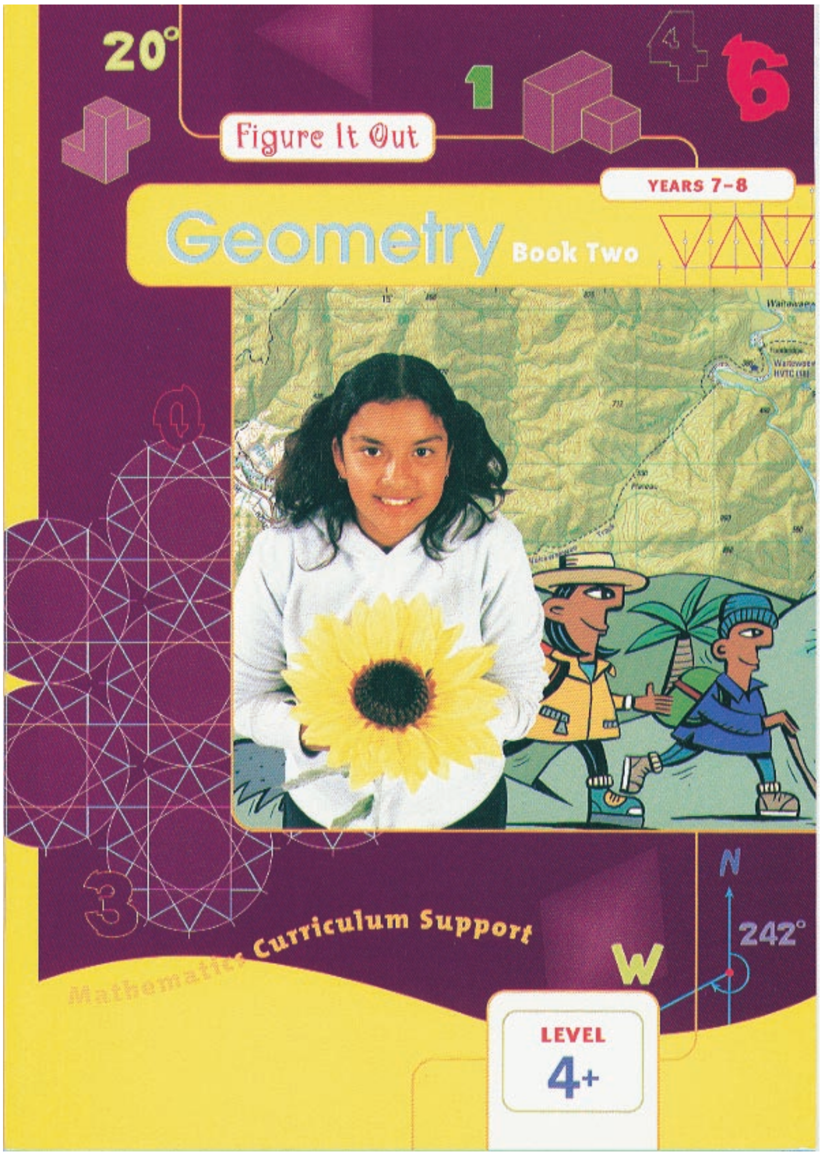
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

CONTENTS

The books for years 7–8 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand year 7–8 classrooms. The books have been developed and trialled by classroom teachers and mathematics educators and follow on from the successful series for levels 2–4 in primary schools.

Student books

The student books in the series are divided into three curriculum levels: levels 2–3 (linking material), level 4, and level 4+ (extension material). All the books are aimed at year 7–8 students in terms of context and presentation.

The following books are included in the series:
Number (two linking, three level 4, one level 4+, distributed in November 2002)
Number Sense (one linking, one level 4, distributed in April 2003)
Algebra (one linking, two level 4, one level 4+, distributed in August 2003)
Geometry (one level 4, one level 4+, distributed in term 1 2004)
Measurement (one level 4, one level 4+, distributed in term 1 2004)
Statistics (one level 4, one level 4+, distributed in term 1 2004)
Themes: *Disasters Strike!, Getting Around* (levels 4–4+, distributed in August 2003)

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and life experiences that are meaningful to students aged 11–13 years. The activities can be used as the focus for teacher-led lessons, as independent bookwork, or as the catalyst for problem solving in groups.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the student books includes full answers and explanatory notes. Students can use them for self-marking, or you can use them for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, comment on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/r/maths/curriculum/figure

Using Figure It Out in your classroom

Where applicable, each page starts with a list of equipment that the students will need to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum*, page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

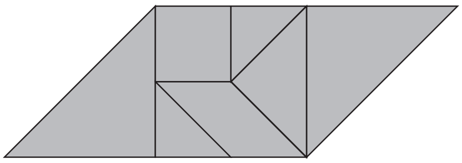
Answers Geometry: Book Two

Page 1 Tempting Tangrams

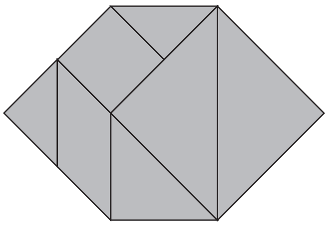
ACTIVITY

1. Practical activity
2. A and B: 4 square units; C, E, and G: 2 square units; D and F: 1 square unit
3. 16 square units
4. 4 units
5. They have the same base and height.
6. Practical activity. Here is one solution for each shape. Other solutions are possible.

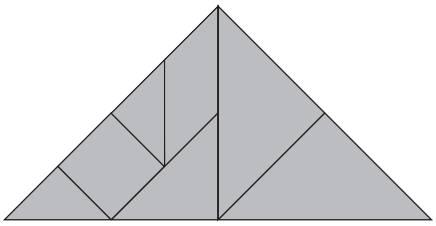
a.



b.



c.



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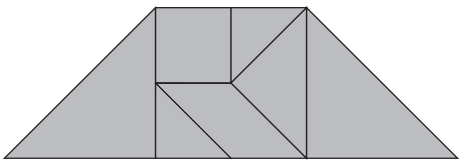
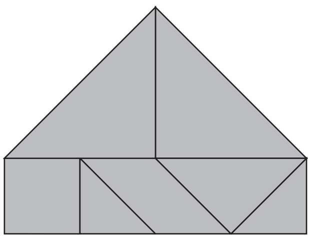


e.



Pages 2–3 Escher Envy

ACTIVITY ONE

Practical activity

ACTIVITY TWO

1.–2. Practical activities

Page 4 X-ray Vision

ACTIVITY

1. Practical activity. There are many buildings that will fit the given views. Three possibilities are:

2	1	3
1	2	1
1		1

2	2	3
2	2	
	1	

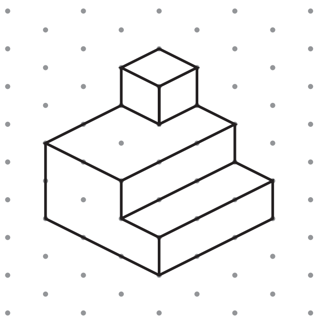
2	2	3
	2	
1	1	1

(The numbers give the height, in cubes, of each level.)

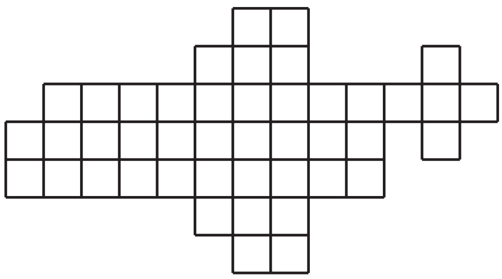
2. a. 16

2	2	3
2	2	2
1	1	1

b.

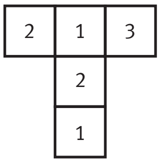


c. Here is one way of constructing the net. There are many other possibilities.

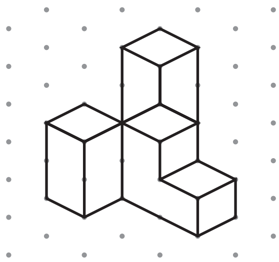


3. a. Practical activity

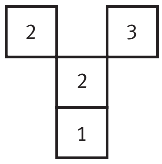
b. A bird's-eye view of the building would look like this:



c. A drawing on isometric dot paper would look like this:



4. Yes, an 8-cube building could be constructed by cutting out the single cube at the centre back. But this would leave three groups of cubes that are not physically connected to each other:



Page 5 Winning Ways

ACTIVITY

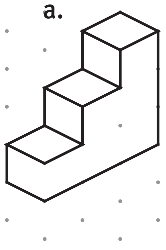
1. a. 6 cubes

b. 14 cubes

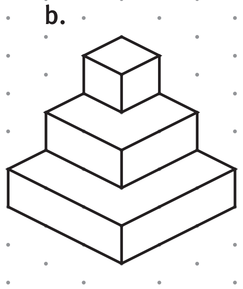
c. 10 cubes

2. Drawings should look like this:

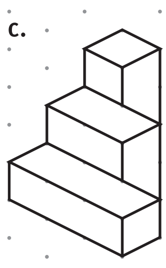
a.



b.



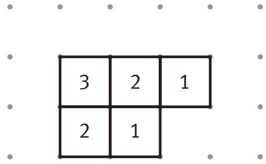
c.



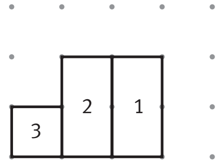
3. Practical activity. Results will vary.

4. Drawings and comments will vary. Three possible solutions, each using 9 cubes, are:

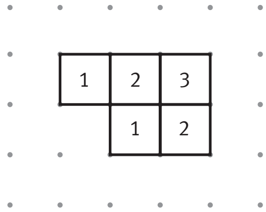
i.



ii.

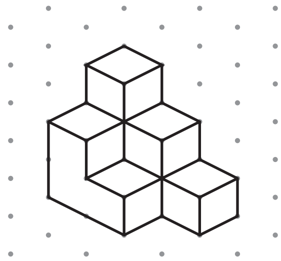


iii.

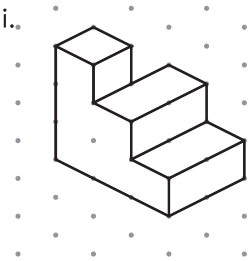


Isometric views of the three stands above are:

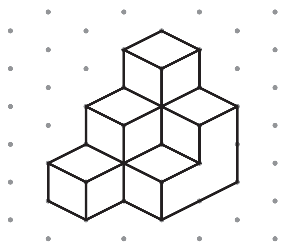
i.



ii.



iii.

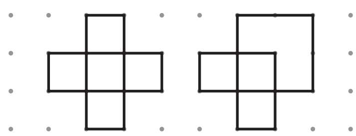


Designs i and iii are mirror images, so they are not really different. They provide a good solution because each person can get into position without anyone else blocking their way, the extra steps are hidden tidily behind the three winners' steps, and a minimum number of cubes is needed. Design ii is fine as long as the 2nd and 3rd place-getters stand back and allow the winner to get into position before moving forward.

Page 6 Missing Anything?

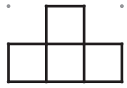
ACTIVITY

1. The two possible top views are:

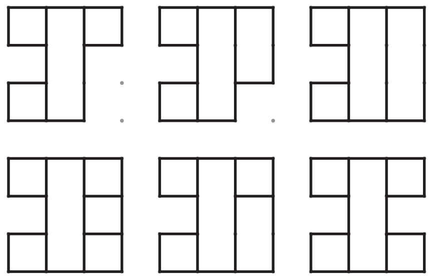


2. Possible front views are:

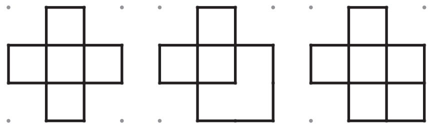
a.



b.

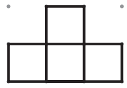


c.

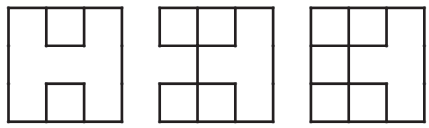


3. Possible left-side views are:

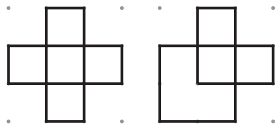
a.



b.

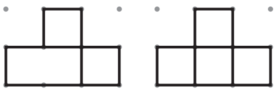


c.

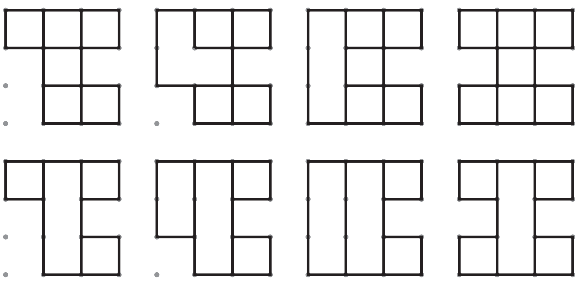


4. Possible rear views are:

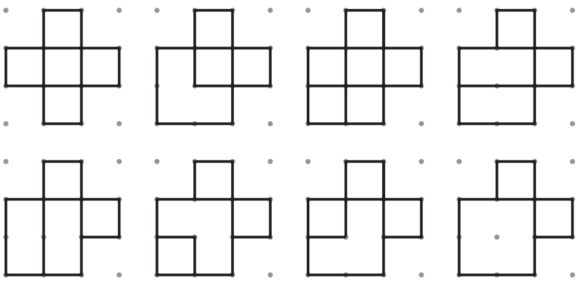
a.



b.



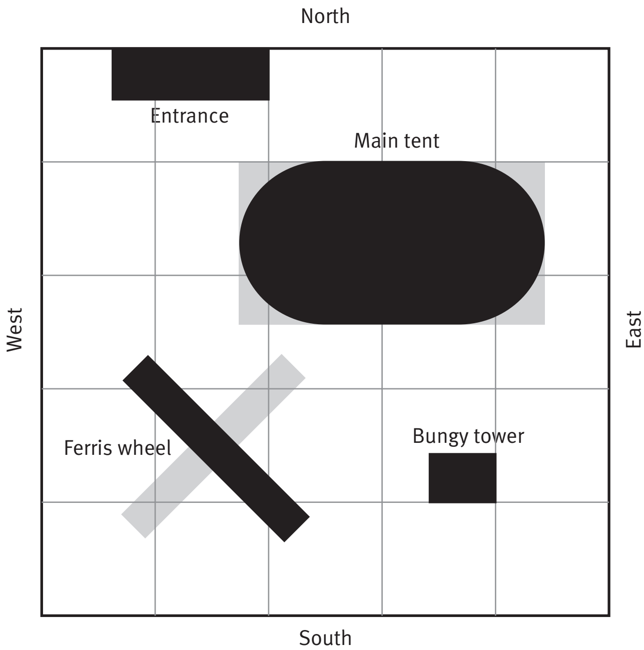
c.



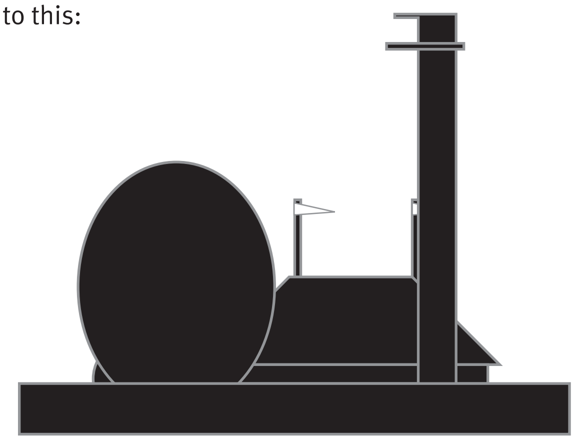
Page 7 Fair and Square

ACTIVITY

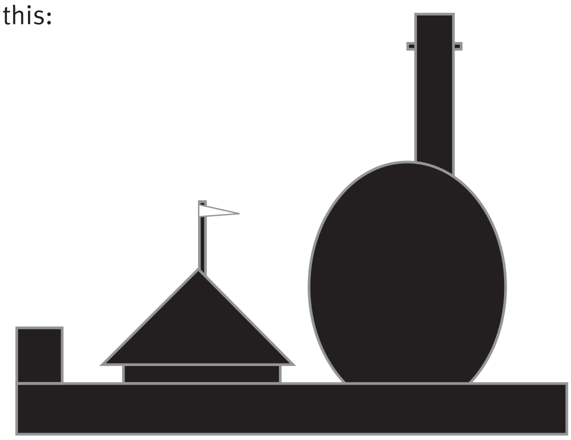
1. The top view should look similar to this (the tent can be rectangular and the ferris wheel can be in either marked position):



2. The view from South Street should look similar to this:



3. The view from West Avenue should look similar to this:

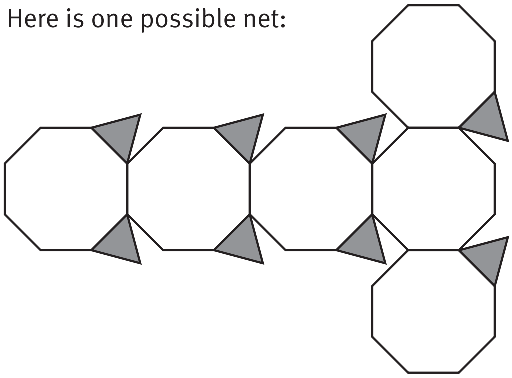


Pages 8–9 Tricky Truncations

ACTIVITY

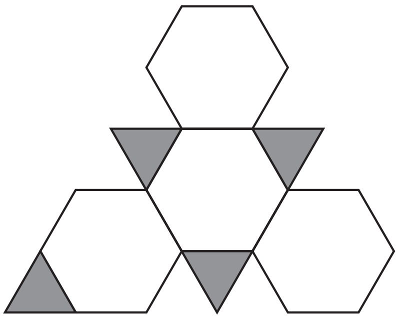
1. **a.** 6 octagons and 8 triangles

   b. Here is one possible net:



2. **a.** Practical activity

   b. Here is one possible net:

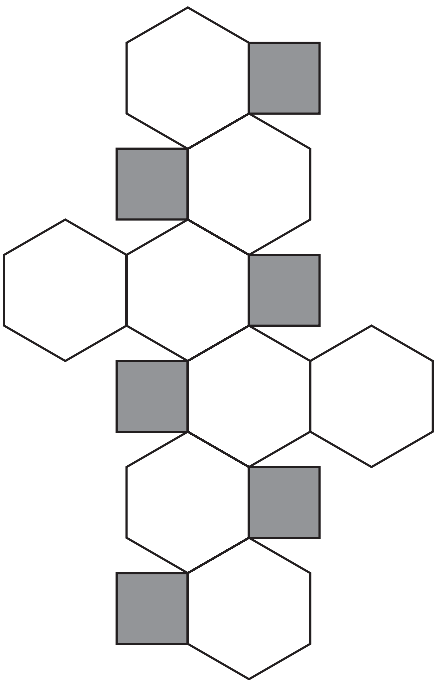


3. **a.** 8 hexagons and 6 squares

   b. 12 decagons and 20 triangles

   c. 20 hexagons and 12 pentagons

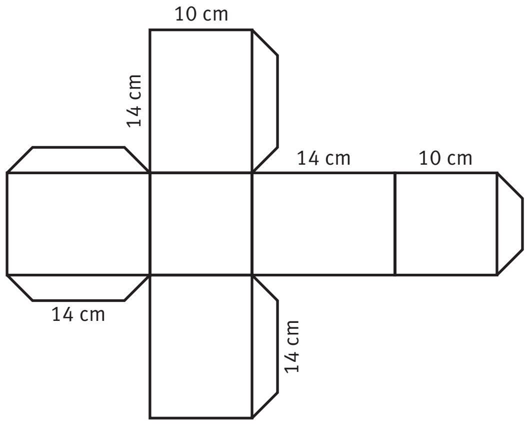
4. Here is one possible net:



Page 10 Perfect Packing

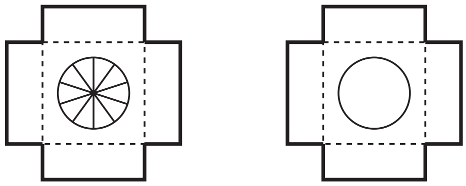
ACTIVITY

1. **a.** Here is one possible net:



   b. Practical activity

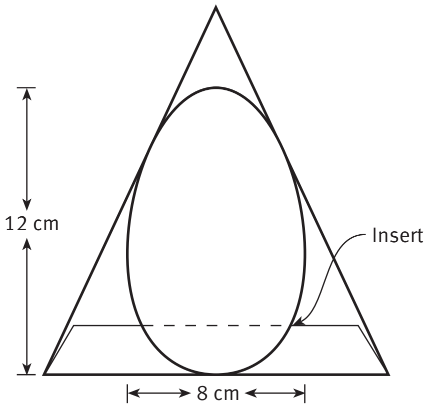
2. **a.** Here are two possible nets:



   Getting the diameter of the hole in the inserts just right is tricky. Possible methods include:
   - trial and improvement
   - using plastic putty to build a mould around the egg up to the quarter mark and measuring the required diameter from this
   - pressing the egg into packed, damp sand and measuring the diameter of the depression it has made
   - cutting "fingers" outward from the centre, bending these fingers at the required place, and using the position of the bends to determine the required diameter.

   b. Practical activity. The bottom insert may need to have a larger hole than the top insert because of the shape of the egg.

3. The dimensions of the base would need to be greater than the width of the egg to allow the pyramid to hold the widest part of the egg. Similarly, the height of the triangular faces would need to be greater than the length of the egg. There would need to be an insert in the base to stop the egg moving.



Although attractive and unusual, this shape may not be very practical for an emu egg container. It would be:

- either very tall or very wide compared with the egg (especially if a centimetre gap is left around the egg to protect it)
- difficult to pack for postage
- difficult to pack with others
- difficult to make
- tricky to open (presumably the egg is removed through the base).

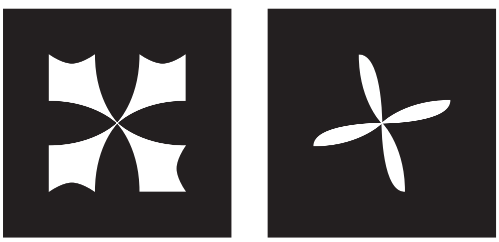
Page 11 Stencil Style

ACTIVITY

1. She can use the stencil below for the 1st colour, clean it, turn it 120°, use it for the 2nd colour, and repeat the process for the 3rd colour. She can do this because the design has rotational symmetry.

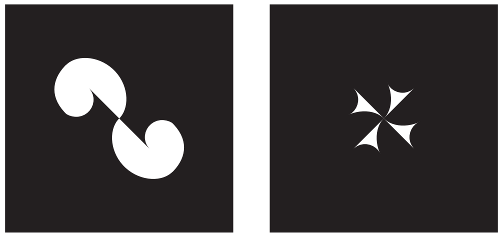
2. a.

b.



c.

d.

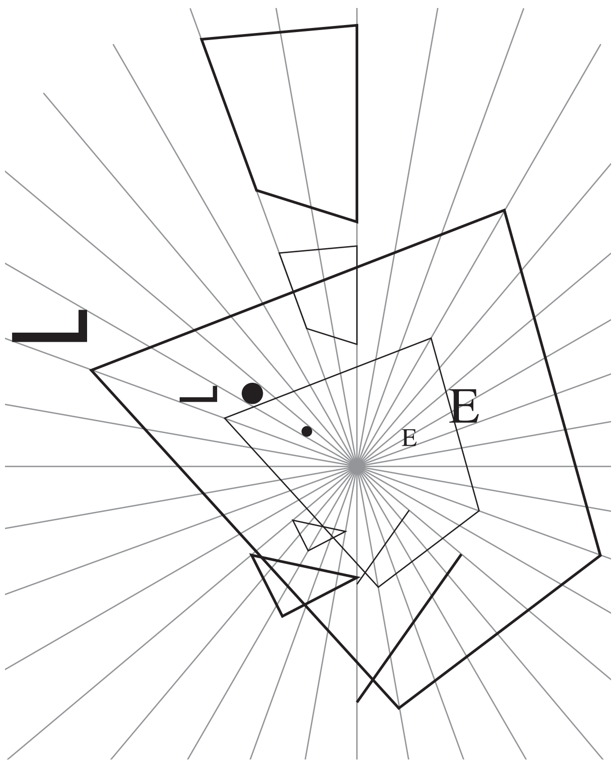


3. Designs and stencils will vary.

Page 12 Enlargement Explosion

ACTIVITY

Your copymaster should look similar to this:



Page 13 Design a Logo

ACTIVITY

1. Logos will vary. Check that you have used 2 transformations.
2. a. 20. (4 m = 400 cm. $^{400}/_{20}$ = 20)

   b. 3.2 m. (All lengths are increased by the scale factor of 20. The original width times 20 is 16 x 20 = 320 cm or 3.2 m.)
3. $^1/_4$. ($^5/_{20}$ = 0.25 or $^1/_4$)
4. Practical activity. The area will be $^1/_{16}$ of the original ($^1/_4$ x $^1/_4$).

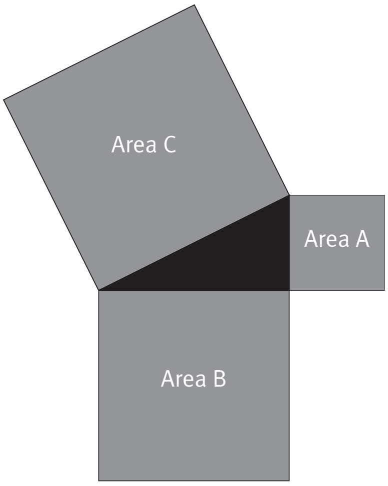
Pages 14–15 Pythagoras Power

ACTIVITY ONE

a. Practical activity

b. A and B together have the same area as C.

c. Diagrams should look like this:



d. Answers will vary, but the general idea is that, given a right-angled triangle, the combined area of the squares on the two shorter sides is the same as the area of the square on the longest side.

ACTIVITY TWO

1. a. Practical activity

   b. Results in the grey columns come from measurement, so they may vary slightly:

Triangle	Side a	Side b	Side h	$a^2 + b^2$	h^2	$a^2 + b^2 = h^2$?
i	5	16	16.8	25 + 256 = 281	281	Yes
ii	9	7	11.4	81 + 49 = 130	130	Yes
iii	5	6	7.8	25 + 36 = 61	61	Yes
iv	12	4	12.6	144 + 16 = 160	160	Yes
v	5	12	13.0	25 + 144 = 169	169	Yes

2. 30 cm.

   ($h^2 = a^2 + b^2$

   $= 15^2 + 26^2$

   $= 225 + 676$

   $= 901$.

   Using the √ key, h = 30 [or 30.0] cm)

INVESTIGATION

Triangle **v**. An infinite number of other triples can be found. Many of these are multiples of others (for example, [5, 12, 13], [10, 24, 26], [50, 120, 130], and so on). Here are the other "primitives" (those that are not multiples) that have a hypotenuse of less than 100: (3, 4, 5), (8, 15, 17), (7, 24, 25), (20, 21, 29), (12, 35, 37), (9, 40, 41), (28, 45, 53), (11, 60, 61), (33, 56, 65), (16, 63, 65), (48, 55, 73), (36, 77, 85), (13, 84, 85), (39, 80, 89), and (65, 72, 97).

Pages 16–17 Making Tracks

ACTIVITY ONE

1. Penn Creek Hut
2. (Because it is difficult to read the references exactly, the last digit in each group of 3 may vary by 1.)

   a. 045 393

   b. 983 352

   c. 999 309

   d. 012 365

   e. 019 258

ACTIVITY TWO

1. a. The bearing is 053° (measured clockwise from north, as always). The distance "as the crow flies" is approximately 7.4 km.

   b. The approximate actual length is 9 km.

   c. Answers will vary. Possible ideas include:
   - The track lies roughly in a north-east direction and is fairly straight.
   - After crossing a second footbridge, the track wanders across open, fairly level ground before heading up a valley.

- The track continues up the valley with little climbing until it comes to an old log hauler at about the 4 km point.
- After this, the track runs next to a creek and climbs steadily until it reaches a plateau at about the 6 km point.
- For the next kilometre, the track is fairly level at an altitude of about 500 m.
- The track then descends to the Otaki River for about 1 km.
- For the final kilometre, the track skirts around a bend in the Otaki River and across a small ridge before it reaches the hut.

2. a. About 130 m. (Mt Hector is 1 529 m, and Kime Hut is on the 1 400 m contour line.)

b. i. The bearing is 013°, and the distance is about 13.1 km.

ii. The bearing is 337°, and the distance is about 8.9 km.

Pages 18–19 Conic Cuts

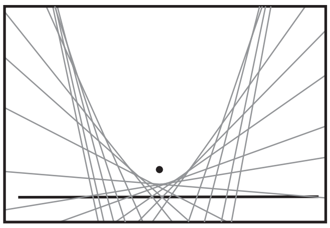
ACTIVITY ONE

a. The conic section is a circle only if the torch is held at right angles to the surface. If you alter this even slightly, the section becomes an ellipse.

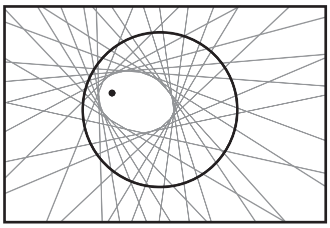
b. The conic section becomes a parabola when the outer edge of the beam is parallel to the surface.

ACTIVITY TWO

a. A parabola



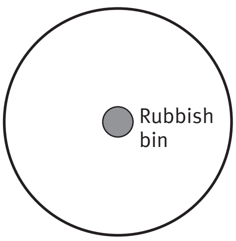
b. An ellipse



ACTIVITY THREE

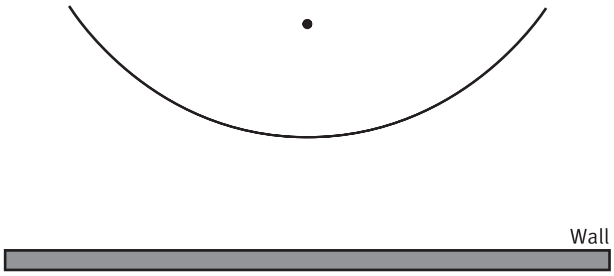
GAME ONE

The fairest way is to draw a circle around the rubbish bin. If the players shoot from any point on this circle, the throwing distance is the same for all of them.



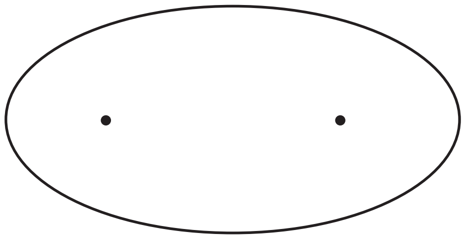
GAME TWO

Draw a parabola, as in the diagram. Every point on this curve is exactly the same distance from the cone and the wall, so X can be anywhere on it.



GAME THREE

The fairest way is to draw an ellipse, as in the diagram. If the two players start from anywhere on this curve, they will run the same distance.



Pages 20–21 Snazzy Snowflakes

ACTIVITY ONE

1. a. Practical activity

b. 4

c. They have rotational symmetry of order 4. (The object will fit its starting position 4 times in one complete turn.)

2. a. Practical activity

b. 6

c. They have rotational symmetry of order 6.

3. a. Practical activity. Use the method in **1** but fold an extra time before cutting. Start with very thin paper, or the cutting will be difficult.

b. Not by folding and cutting a square. Each fold doubles the number of axes, so the number of axes (except the first) is always even.

4. Practical activity. Results will vary.

5. a. This design has reflective symmetry along a vertical axis and a horizontal axis and rotational symmetry of order 2.

b. Practical activity. Designs will vary.

ACTIVITY TWO

1.

Number of folds	Number of layers	Number of axes of symmetry
1	2	1
2	4	2
3	8	4
4	16	8
5	32	16
10	1 024	512

2. 16 axes. This involves 5 folds and 32 layers of paper. Due to the size of the folded paper and its thickness, you are unlikely to be able to fold and cut it again.

INVESTIGATION

Answers will vary.

Page 22 Ancient Architecture

ACTIVITY

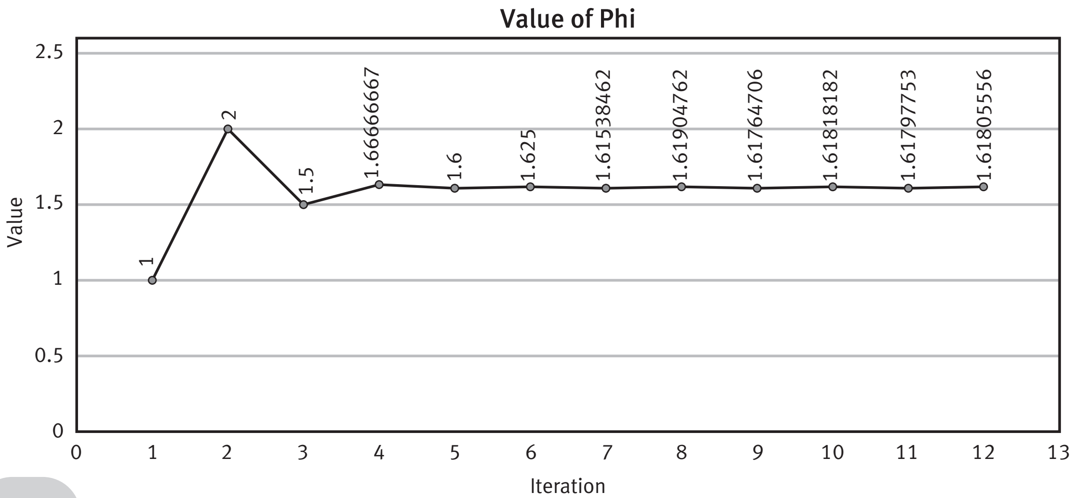
1.

Length	Height	Length ÷ height (as a decimal)
105 mm	64.5 mm	1.63

2. a.

Pattern	Approximate value of φ
1 ÷ 1	1
2 ÷ 1	2
3 ÷ 2	1.5
5 ÷ 3	1.66666667
8 ÷ 5	1.6
13 ÷ 8	1.625
21 ÷ 13	1.61538462
34 ÷ 21	1.61904762
55 ÷ 34	1.61764706
89 ÷ 55	1.61818182
144 ÷ 89	1.61797753
233 ÷ 144	1.61805556

b.



3. 297 ÷ 210 = 1.41. Although this number looks quite close to the golden ratio, the difference is enough to give a rectangle with very different proportions.

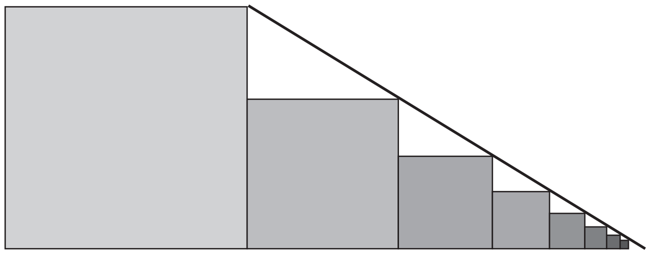
INVESTIGATION

Practical activity

Page 23 Golden Spirals

ACTIVITY

1. Practical activity
2. Practical activity. When lined up together, the squares will look like the following diagram. If you use a ruler, you will find that their top right-hand corners lie on a straight line.



INVESTIGATION

Practical activity

Page 24 Fantastic Fractals

ACTIVITY

1. Practical activity

2. a.

Step	Number of sides	Length of each side	Total perimeter	Area (in small triangles)
1	3	18	54	324
2	12	6	72	432
3	48	2	96	480

   b. With each step, the number of sides is multiplied by 4 while the length of the sides is divided by 3, so the total perimeter increases by a factor of $^4/_3$. This means that, with every step, the perimeter is $^1/_3$ greater than it was the time before.

   c. With each step, a triangle is added to each side. The number of sides is 4 times what it was before, and the size of the triangles is $^1/_9$ what it was before, so the increase in area is $4 \times {}^1/_9 = {}^4/_9$. This means that, with each step, the area is increasing, but by an ever-smaller amount.

   d. The perimeter is growing at an ever-increasing rate but always within an imaginary circular boundary passing through the vertices of the triangle you started with. The area is growing by an ever-smaller amount each time; the increase will soon be close to zero. (If you take this one step further, you will realise that the fractal flake has an infinite perimeter but a finite area!)

3. Practical activity

Teachers' Notes

Overview Geometry: Book Two

Page 1 **Tempting Tangrams**

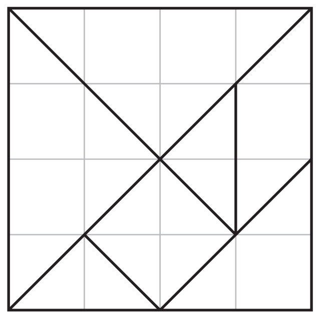
Achievement Objectives

- describe the features of 2-dimensional and 3-dimensional objects, using the language of geometry (Geometry, level 3)
- apply the symmetries of regular polygons (Geometry, level 4)
- find perimeters, areas, and volumes of everyday objects (including irregular and composite shapes), and state the precision (limits) of the answer (Measurement, level 5)
- classify objects, numbers, and ideas (Mathematical Processes, developing logic and reasoning, level 4)

ACTIVITY

The tangram is a deceptively simple geometrical puzzle. The pieces are easy to make using card and scissors but can be combined in ways that challenge both students and adults. This activity focuses on the areas of the different pieces relative to each other and to the completed square.

When making the pieces in question **1**, the students should use thin card so that they can fold it cleanly. If they later want to make a more durable set, they can draw a 4-by-4 grid of suitably sized squares on thicker card and then make the pieces by joining the appropriate intersections, as in this diagram:



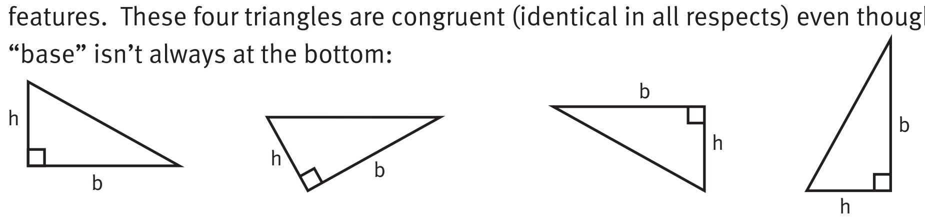
To answer questions **2** and **3**, the students need to compare the area of each piece with the area of the smallest right-angled triangle. They may need some help with the idea that a triangle can represent a square unit. If this is the case, you could get them to cut one of the small triangles in two and re-form the two halves as a square. Area does not have a shape!

In question **4**, the students need to give their answer in units (rather than length as measured by a ruler). They could work backwards from the area of the big square (16 square units) and use the fact that this area is found by squaring the length of the side, or they could make a square from one of the small triangles (as suggested above) and then use its side to measure the length of the long side of one of the big triangles. They should remember that, while area is expressed in square units, length is expressed in units that are not squared.

If your students have difficulty with questions **2–4**, you can simplify the maths for them by getting them to make a tangram set using a grid, as shown in the above diagram. If they leave the grid lines in place, they can easily compare the size of each piece by counting the squares. Conversely, you can make the maths more challenging by naming the area of another piece (one of the large triangles, for example) as 1 square unit and then getting the students to express the area of all other pieces in terms of this unit. To do this, they will have to use fractions.

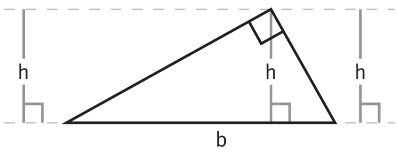
Question **5** is unlikely to be obvious or easy but creates a valuable teaching opportunity. The four pieces clearly do not have the same number of sides, the same shape, or the same area. The clue is the fact that these four pieces were cut from the same strip: the strip that was left when A, B, and C were removed. This means that they have the same height. They also have the same base length.

When students see the word "base", they often think that this means the bottom (horizontal) line of a shape. In everyday thinking, this is true, but in geometry, when we rotate a shape, we don't change its essential features. These four triangles are congruent (identical in all respects) even though the side labelled b for "base" isn't always at the bottom:

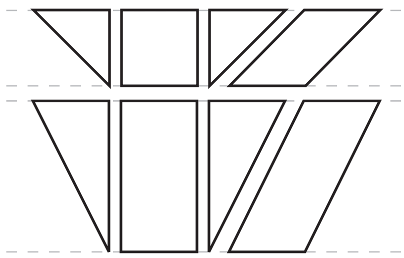


The teaching points here are that:

- "height" and "base" are nothing more than labels; we can call any side the base
- the height and base always form a right angle with each other
- sometimes, we need to draw a height line. This can be done at any convenient place:



As an extension, you could challenge your class to think about what happens to the areas of shapes when their heights change, as in this diagram:



Do the triangles still each have an area equal to half the rectangle? Does the area of the parallelogram still equal the area of the rectangle? If the height (the distance between the two parallel lines) is doubled, does the area of each shape also double? How could you show that this is, or is not, the case?

In question **6**, the students use their tangram pieces to make geometrical shapes. Some will find this difficult at first, but they will get better as they understand the relative sizes of the pieces and develop some strategies. One useful strategy is to try and place the two large triangles first; in most cases there are only a few positions that they can occupy. The two small triangles can be left until the end; their positions will become obvious once the larger pieces are in place.

You may like to discuss with your students the fact that the parallelogram occupies a differently-shaped space when it has been flipped over (turned reverse side up). This is because it does not have reflective symmetry. Some series of tangram puzzles require the parallelogram to be flipped; others do not.

If your students get the "tangram bug", they could explore the subject using the Internet. There are websites that will give them plenty of challenges. Some of these feature interactive tangram sets. To find them, try "interactive tangram" in a search engine. One good site is www.tangram.i-p.com

Pages 2–3 Escher Envy

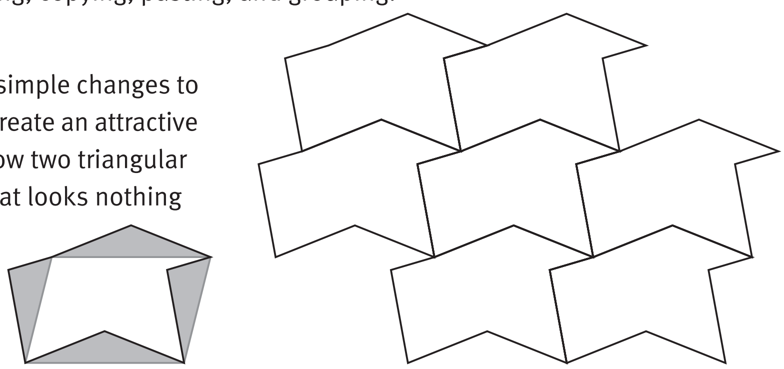
Achievement Objectives

- design and make a pattern which involves translation, reflection, or rotation (Geometry, level 3)
- describe patterns in terms of reflection and rotational symmetry, and translations (Geometry, level 3)
- apply the symmetries of regular polygons (Geometry, level 4)
- describe the reflection or rotational symmetry of a figure or object (Geometry, level 4)

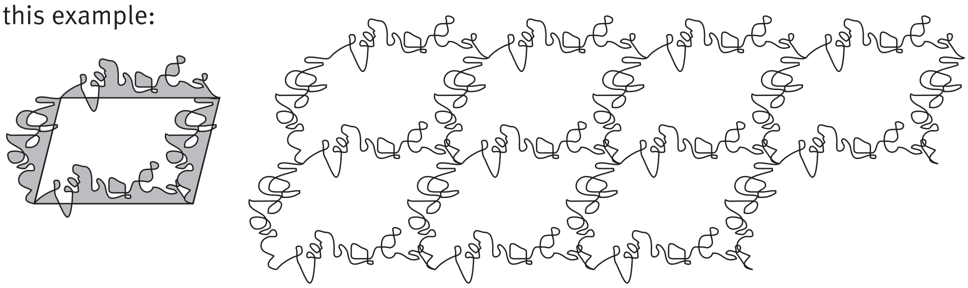
Although it is possible to create Escher-style tessellations without a computer (as Escher did), this activity assumes that your students will use one. A computer makes it possible for them to create attractive and complex patterns without the lengthy repetition that would otherwise be involved. If they are not sure how to use a drawing program, these two activities give them a suitable context in which to learn. The main functions they will need to use are drawing, copying, pasting, and grouping.

ACTIVITY ONE

If they wish, the students can make very simple changes to the sides of their parallelogram and still create an attractive tessellation. The next example shows how two triangular translations (shaded) can create a tile that looks nothing like the original parallelogram.



Alternatively, the students could make a less obviously geometrical tessellation by using the freehand drawing tool, as in this example:



ACTIVITY TWO

This time, the students base their tessellations on a hexagon. They use the same computer functions as in the first activity, with the addition of reflection and rotation. As they work, you should find them using the language of transformations to describe what they are doing.

Question **2** challenges the students to create a more complex design. They will find that it is surprisingly difficult to create a tessellation featuring a recognisable creature or object. If, however, they add eyes or a mouth, their shapes will start to look alive. Using a computer for this activity makes it easy to go back and add to or change the outline of the original tile to get a slightly different tessellation.

There are a number of websites devoted to tessellations and other sites that feature Escher's work. Totally Tessellated is an excellent site that includes demonstrations of how Escher's tessellations work and copymasters for students: http://library.thinkquest.org/16661 The official Escher website is www.mcescher.com

Page 4 X-ray Vision

Achievement Objectives

- make a model of a solid object from diagrams which show views from the top, front, side, and back (Geometry, level 4)
- design the net and make a simple polyhedron to specified dimensions (Geometry, level 4)
- draw diagrams of solid objects made from cubes (Geometry, level 4)
- make isometric drawings of 3-dimensional objects built out of blocks (Geometry, level 5)

ACTIVITY

No single view of a solid object will give us all the information we need in order to construct it. As the students will discover from this activity, even four views may not be enough to define a unique structure.

One approach to question **1** is to use trial and improvement. The students can use cubes to make the simplest possible model that fits the front view and then adjust the design until it also conforms with the other views.

Another approach is for them to see how much information they can deduce from the four views before they begin:

- The building is no more than 3 cubes wide, deep, or high.
- There is only 1 cube in the top layer, and it is in the back right-hand corner.
- The front of the building is 1 cube high, and the next row back is 2 cubes high at some point.

The Answers show three ways of making a model with 12 cubes, but there are other possibilities. The students could see how many they can find, using square grid paper to keep a record of their designs. They could then get a classmate to check their designs. Challenge them to do the checking without first seeing the made-up model.

Questions **2b** and **3c** involve isometric drawing. Students usually find it easy to learn this drawing technique, but if yours have not met it before, you should spend some time introducing it. Some practical tips include:

- Use the isometric dot paper the right way up. (The dots should form vertical lines.)

- Turn the object to be drawn so that the focus is on a vertical line (not a face).
- There are four possible views of an object sitting on its base (one for each of the vertical edges of a cube). Choose the view that best reveals the structure of the object.
- Only draw the edges that you can actually see from your viewing angle.
- Only draw the edges of cubes if 2 different faces meet at that edge.
- Only use pencil and have an eraser handy.
- Render (shade) the 3 planes (left, right, and top) differently. This will help the viewer to visualise the 3-D nature of the object.

When drawing the net for question **2c**, the students could begin with the 3-by-3 base, work outwards from there in 4 directions to get the sides, and then decide which side(s) they wish to attach the top parts to. As is usually the case with nets, there are many variations possible.

To build the model in question **3a**, the students can hold their earlier version at eye level and remove cubes one at a time, checking to see that they are not changing any of the four profiles.

In question **4**, it is possible to construct a model with 8 cubes, but if it were a real building, it would consist of three independent buildings touching at the corners, with no internal access linking them.

As a further task, you could challenge your students to construct another model, draw the 4 views (excluding the bird's-eye view), and get a classmate to make it with as many and as few cubes as possible.

Page 5 Winning Ways

Achievement Objectives

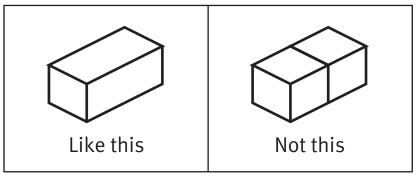
- make a model of a solid object from diagrams which show views from the top, front, side, and back (Geometry, level 4)
- draw diagrams of solid objects made from cubes (Geometry, level 4)
- make isometric drawings of 3-dimensional objects built out of blocks (Geometry, level 5)

ACTIVITY

Like the previous activity, this one asks the students to visualise 3-D objects from 2-D drawings and to create isometric drawings. This time, they work within a practical context.

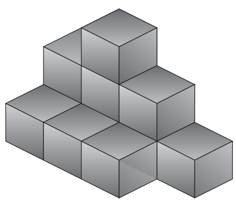
In question **1**, the numbers on the diagrams make it clear how many layers of cubes there are in each part of the structure.

When doing question **2**, the students should remember that where two or more adjoining cubes are part of the same unbroken plane (flat surface), the join in the blocks is not shown. See the previous activity for more guidance on isometric drawing.



Questions **3** and **4** pose a technology-type challenge, in which the students have to find the best outcome given the constraints of accessibility and resource minimisation and then justify their decision.

As an extension, the students could create some winners' stands (see the example to the right) using a 3-D drawing program. If your school doesn't have such a program, there are a number available on the Internet as downloadable freeware. Find them using keywords such as "3-D graphics freeware".



Once one cube has been created by constructing a square and clicking on an appropriate 3-D shape, it can be copied and dragged as many times as necessary to form the winners' stand. The students may find that they need to use the Control and/or Alt keys to position the cubes precisely.

Page 6

Missing Anything?

Achievement Objectives

- draw diagrams of solid objects made from cubes (Geometry, level 4)
- make isometric drawings of 3-dimensional objects built out of blocks (Geometry, level 5)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

ACTIVITY

This activity illustrates the fact that we often assume something is true based on incomplete information. It should be done after the two preceding activities, X-ray Vision and Winning Ways, because these activities explore the conventions of isometric drawing and give students practice at visualising 3-D objects. Students will find it much easier to answer the questions if they first make the objects using cubes.

In question **1a**, students may assume that there is no cube in the far (hidden) corner of the object because there is no cube in the three corners that they can see. There is, however, no reason why this should be so, and the Answers show the two possibilities.

In question **2**, the students consider the front views of each object. There is only 1 possible front view of object **a**, but object **b** may have from 1 to 3 cubes on the right wing, and if there are 2 cubes, these can be placed in 2 different configurations. So there are 4 possible views from this angle. In the case of **c**, the bottom right-hand (middle row) position is obscured. This means that there are 3 possible front views depending on whether there is a cube in the far right bottom row, and if there is, whether it is in the nearer or farther position. Similar thinking is needed to produce the different possible views in question **3**.

Question **4** is not easy, but the alternative views all require the same considerations that the students have met in questions **1–3**. For example, in **a**, is there is a cube in the far corner? One view will show a cube in that position; another won't. There are 8 possible views for **b** and 8 for **c**. You could challenge your students to find them all.

Page 7

Fair and Square

Achievement Objectives

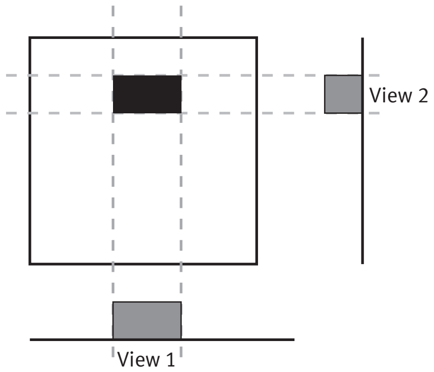
- draw pictures of simple 3-dimensional objects (Geometry, level 3)
- make a model of a solid object from diagrams which show views from the top, front, side, and back (Geometry, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

ACTIVITY

This activity builds on the previous three activities. Like them, it involves viewing objects from different directions. This time, the objects are more complex, and the supporting structure of dots and grids has been removed.

Although not obviously an exercise in scale drawing, the students will find that the best approach to question **1** is to take measurements from the illustration and use them to fix the location of the fairground attractions on the 5-by-5 grid. They can either use a scale that relates the measurements taken from the illustration to the size of the squares on their paper or draw up their own grid to match the side of the town square in the illustration.

The principle used to fix the location of the objects is illustrated in this diagram:



Note that this method only defines the minimum rectangular space occupied by the object; it doesn't define its shape. All the following shapes could occupy the same rectangular space and would have the same two views:

Measurement can also be used to fix the height of the objects. The heights are needed for questions **2** and **3**.

As an extension to this activity, the students could make scale sketches of a feature of their school or their town from 2 directions that are at right angles to each other. They could then have a classmate draw the 2 opposing views, based on those sketches.

Pages 8–9 Tricky Truncations

Achievement Objectives

- model and describe 3-dimensional objects illustrated by diagrams or pictures (Geometry, level 3)
- make a model of a solid object from diagrams which shows views from the top, front, side, and back (Geometry, level 4)
- design the net and make a simple polyhedron to specified dimensions (Geometry, level 4)

Of the five Platonic solids, three have equilateral triangles for their faces (the tetrahedron, octahedron, and icosohedron), one has squares (the cube), and one has pentagons (the dodecahedron).

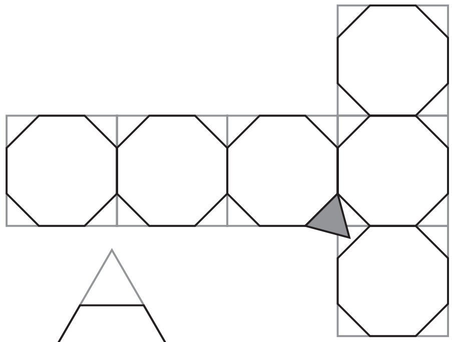
Students sometimes think that an octahedron is the same as 2 tetrahedrons joined base-to-base, but this is not the case. When 2 tetrahedrons are joined, the resulting solid has just 6 sides because 2 of the original 8 faces get "lost" in the joining process. An octahedron can, however, be thought of as two square-based pyramids joined base-to-base.

ACTIVITY

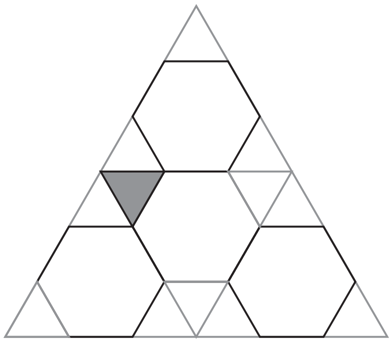
A cube (question **1a**) has 6 faces and 8 vertices. Each of the square faces loses a corner in the truncation process, turning the 6 squares into 6 octagons. The 8 corners become 8 equilateral triangles. The truncated part of each corner is a triangular pyramid: 3 isosceles right-angled triangles sitting on an equilateral triangle. These pyramids are identical to the stellations of the icosahedron in Fantastic Folding (page 9, *Geometry: Book One*, Figure It Out, Years 7–8).

When doing question **1b**, the students should begin by drawing the net of a cube and then replace each square with a regular octagon. They should then make an equilateral triangle that exactly matches the side of the octagon and put one of these at each truncated corner. (Only one of these has been placed in the following diagram.) The triangle can be joined to any of the three adjacent octagons.

If your students are able to use a computer drawing program for this task (and the following ones), they will be able to draw excellent nets using the polygon tool and the copying, rotating, grouping, and pasting functions. An alternative method is for them to cut patterns from cardboard and draw around these. Both methods remove much of the tedious repetition.



In question **2**, the students make a truncated tetrahedron from geoshapes or polydrons, if available, before drawing its net. When drawing the net, the students should start with the net of the untruncated tetrahedron and modify it. The diagram shows how this can be done. Only one (any one) of the three equilateral triangles at each point belongs to the net, but the other triangles could be used as gluing tabs.



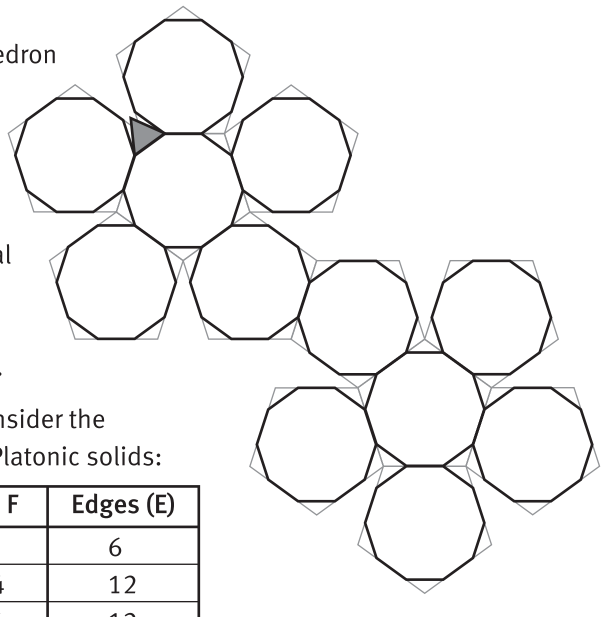
When thinking about the shapes they will get from the truncation in question **3**, the students should consider:

- The shape of the face of the untruncated solid and what this shape will become when each corner is removed. (The truncated face will always have double the number of sides.)
- The number of faces that meet at each vertex in the untruncated solid. (The vertex will be replaced by a regular polygon that has one side for each of the faces that meet at this point.)

When working out how many there are of each kind of face, the maths is similar for each of the three parts of question **3**. As an example, here is the reasoning behind the answers to question **3b**:

- A dodecahedron has 12 faces, each with 5 sides. When truncated, each of those faces will have 10 (twice as many) sides (that is, each face will be a decagon), but there will still be 12 faces.
- A dodecahedron has 20 vertices. (Each of the 12 faces has 5 vertices, which suggests that there should be 12 x 5 = 60 vertices in the solid. But because 3 faces share each vertex, each is being counted 3 times, so the actual number is 60 ÷ 3 = 20.)
- 3 faces meet at each vertex, so when the dodecahedron is truncated, each vertex will be replaced by an equilateral triangle. There are 20 vertices, so there will be 20 triangles.

This diagram shows the start of a possible net for a truncated dodecagon. Only one of the small equilateral triangles has been shown. The outlines of the original pentagon-shaped faces have been left in place. The students are likely to find it easiest if they do this, too.



As an extension to this activity, the students could consider the relationship of *vertices* and *faces* to *edges* in the five Platonic solids:

Solid	Vertices (V)	Faces (F)	V + F	Edges (E)
Tetrahedron	4	4	8	6
Cube	8	6	14	12
Octahedron	6	8	14	12
Dodecahedron	20	12	32	30
Icosahedron	12	20	32	30

From this, they may be able to generalise that V + F = E + 2, that is, the number of vertices plus the number of faces is always equal to the number of edges plus 2.

There are many websites devoted to the Platonic solids. To start with, the students could explore:
www.mathconsult.ch/showroom/unipoly/list.html
www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Plato.html

Perfect Packing

Achievement Objectives

- design the net and make a simple polyhedron to specified dimensions (Geometry, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

ACTIVITY

Question **1** is a straightforward task in which the students make a cuboid-shaped container that meets certain specifications. Encourage them to do the most professional job possible. The quality of the end product will depend on their:

- using pencil rather than pen for construction lines
- using a set square (or anything known to be square) when forming the right angles
- measuring accurately
- scoring lines before folding
- using suitable glue tabs
- using small quantities of PVA glue instead of sticky tape.

Like most nets, the net for the container can be drawn in more than one way.

The inserts in question **2** are important, not only for stopping the egg moving inside the container, but also to provide protection for the egg. Most students will have come across similar inserts inside Easter egg boxes. Chocolate eggs are much more fragile than emu eggs, yet the thin card of a well-made container does an excellent job of protecting them.

The students have to decide how big to make the diameter of the hole in the insert. Perhaps the best way of doing this is to trace the outline of the egg pictured in the book, measure its length, divide this by 4, and then determine the width of the egg at the points $^1/_4$ of the way from each end.

They should draw the circle using a compass centred on the point where the diagonals of the square insert cross. Instead of cutting out the hole, they could cut "fingers" radiating from the centre. This method allows for some error and small variations in egg size.

Question **2a** asks the students how they could get precise dimensions for a specific egg. Various possibilities are suggested in the Answers, but some students may realise that the decision to place the inserts $^1/_4$ of the way up each egg is probably an arbitrary one. If this is so, it may be easier to adjust the length of the four flaps on the insert than to keep playing around with the diameter of the hole.

When answering question **3**, the students could begin by drawing a sketch of the side view of the pyramid, starting with the emu egg itself. You could photocopy an egg shape about 12 centimetres by 8 centimetres and give this to them to work with. They will need to consider:

- using an insert near the bottom of the pyramid to hold the egg in place
- choosing a size for the base that won't make the pyramid too tall
- ensuring that the insert and the sloping sides fit neatly around the egg to prevent it from moving in transit
- giving the egg appropriate protection in a container of this shape.

Finally, the students should discuss the merits of the square-based pyramid compared with the cuboid as a suitable container for the egg. As part of this discussion, they could consider how a number of pyramid-shaped boxes would pack together.

As an extension, the students could investigate the design of containers used for hens' eggs, especially non-moulded containers.

Page 11

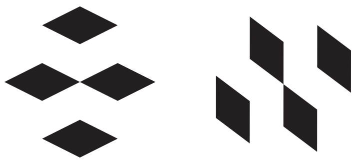
Stencil Style

Achievement Objectives

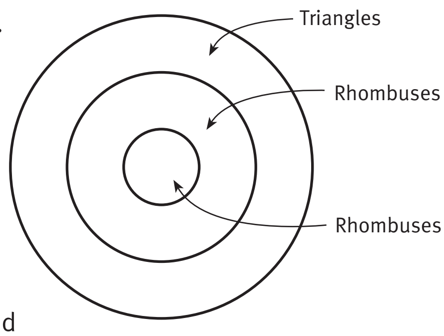
- describe the reflection or rotational symmetry of a figure or object (Geometry, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

ACTIVITY

Each of the designs in this activity could be done using a stencil for each of the individual repeated elements that make it up. (For example, a single rhombus stencil could be used 12 times to make the design on the T-shirt.) What is wanted, however, is a single stencil for each colour involved. In this way, the relationship between the individual elements is preserved as well as their shape. So, in question **1**, the stencil consists of 4 rhombuses and it is used 3 times. The first 2 positions are shown in the diagram:



The most difficult part of question **2** is **c**. This is because the elements of the stencil are not arranged symmetrically in relation to each other. Students can see that there are 3 colours, so they know they will be looking for a stencil that can be used 3 times. The finished design has 15 elements, so 5 of them must be on the stencil (3 rhombuses and 2 triangles). The students could imagine the design in terms of 3 concentric bands, with the 6 triangles on the outside, 6 rhombuses in the middle band, and 3 rhombuses in the inner band. They can then use deduction or trial and improvement to work out exactly which parts of the finished design are in each band on the stencil.



As the design has rotational symmetry and the stencil is used 3 times, it should be clear that the angle of rotation each time is 360 ÷ 3 = 120 degrees.

Experience with questions **1** and **2** should give the students ideas they can use when working on question **3**. When they have created their designs, they could show them to the rest of the class and explain their symmetry or invite others to explain how the symmetry works.

Page 12

Enlargement Explosion

Achievement Objectives

- enlarge, on grid paper, simple shapes to a specified scale (Geometry, level 3)
- enlarge and reduce a 2-dimensional shape and identify the invariant properties (Geometry, level 4)

ACTIVITY

Your students are likely to have encountered many examples of enlargement, even if they have not recognised them as such. The device may have been an overhead projector, movie projector, TV, photocopier, magnifying glass, telescope, microscope, binoculars, or the zoom lens of a camera. Model cars, trains, planes, and boats are made to scale, and so are enlargements.

Students will also have had experience of scale factors, particularly when using devices (such as a photocopier, a video game, or a video camera) that have a zoom feature.

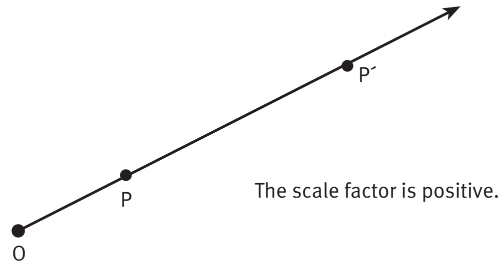
You could begin this activity by asking your students to think of and list as many examples of enlargement (including reduction) as they can. They can then go on to explore how enlargement works mathematically. This activity will help them to clarify the concepts and methods involved.

The three key concepts are:

- Enlargement affects every point in the plane except the centre, which is fixed.
- In an enlargement, all points move directly away from the centre in straight lines.
- Every enlargement has a scale factor that determines how far each point moves.

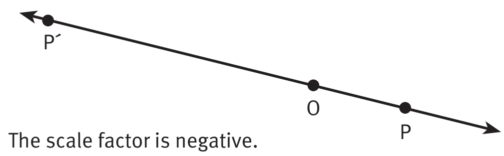
You may need to discuss the idea of there being a fixed centre. This is because in their experience of enlargement, the students will often not have been aware of any such centre. For example, when an overhead projector projects an image onto a screen, there is a centre (the bulb), though it is unlikely that the students have thought of it in these terms.

The diagram shows how a single point (P) moves in an enlargement where the centre is O and the scale factor is 3. P´ is the new location of P. (P´ is read as “P dash” or “the image of P”.) P´ is 3 times as far from O as P.



When the scale factor is:

- greater than 1, all points in the plane move further from the centre (and so shapes increase in size)
- greater than ⁻1 and less than 1, all points move closer to the centre (and so shapes reduce in size)
- 1, points stay where they are
- negative, points end up on the exact opposite side of the centre, as in the diagram on the right.



The students will usually think of enlarging a shape, not a point, but understanding how points behave will make it easy for them to enlarge a shape because a shape is nothing more than a selection of points. To enlarge a triangle, we only need to track the movement of 3 points (the vertices) to know where all the others have gone because shape does not change under enlargement.

The students could go on to draw their own simple geometrical shapes, specify a centre and scale factor for each, and then enlarge them. As above, the process involves drawing a line from the centre through each of the points that define its shape, measuring the distance of each point from the centre, and multiplying it by the scale factor to give the new position. Finally, they join the new positions of the key points.

The students could take one of their shapes and try using a new centre that is outside, inside, or on their shape and see what difference this makes. (The size, shape, and orientation of the enlarged figures will be identical, but they will be in different places on the paper.)

Page 13 Design a Logo

Achievement Objectives

- describe patterns in terms of reflection and rotational symmetry, and translations (Geometry, level 3)
- enlarge and reduce a 2-dimensional shape and identify the invariant properties (Geometry, level 4)

ACTIVITY

Like the previous activity, this one is about enlargement, but it also involves the other three transformations known to the students: translation, reflection, and rotation.

An A4 sheet of paper is ideal for question **1**. The students should begin by drawing a frame the correct size. They should take care to make this as accurate as possible, with corners that are right angles.

They will find it helpful to experiment on rough paper first as they try to come up with their design concept. They should look for ideas that have a maths theme, that incorporate at least 2 of the 3 transformations, and that they can work up into an attractive and tidy finished design. When the students are satisfied with their sketches, they can draw them more carefully as “finished art”. You could make a competition of the task and offer to make an overhead transparency of the three best designs.

When answering question **3**, the students need to compare the projected height of the logo (4 metres) with its actual height (20 centimetres). The question is: how many times greater than 20 centimetres is 4 metres? The answer, 20, is the scale factor of the enlargement. All dimensions, including the width, increase by this factor.

Question **3** involves the reverse process: a reduction. The printed height of 5 centimetres is to be reduced from 20 centimetres. Clearly, the scale factor is $^1/_4$ (or 0.25). At 4 centimetres, the width of the printed logo is also $^1/_4$ of the original width.

In questions **2** and **3**, the students have to find the scale factor by comparing 2 lengths. This method will always work provided they follow these two principles:

- They compare only *corresponding* lengths. This means that one is the enlarged version of the other. (They can’t find the scale factor by comparing the length of the first with the width of the second.)
- They divide the enlarged (new) length by the original (old) length. For example, in question **3**, we start with 20 (old) and end up with 5 (new). New divided by old gives $5 \div 20 = {^1/_4}$.

The answer to question **4** will come as a surprise to many students, but as long as they follow the instructions correctly, they will be convinced by it. The reason that the area of the printed logo is just $^1/_{16}$ of the area of the original logo is that *two* dimensions have been quartered, not just one. It has $^1/_4$ of the length *and* $^1/_4$ of the width, so it has $^1/_4 \times {^1/_4} = {^1/_{16}}$ of the area.

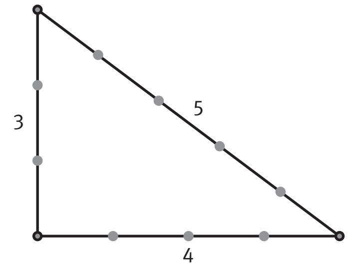
As an extension of question **4**, the students could explore the areas of different squares and rectangles when they are increased or decreased by a variety of scale factors. They could then try other shapes and see if they can work out what happens to their areas under various enlargements. From these experiences, they should be able to generalise that the area of *any* shape is always enlarged by the square of the scale factor. Reinforce the point that this is true of any 2-D shape, no matter how complex or irregular it may be.

Pages 14–15 Pythagoras Power

Achievement Objectives

- find an unknown side in a right-angled triangle using scale drawing, Pythagoras' theorem, or an appropriate trigonometric ratio (Geometry, level 5)
- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, level 4)

Pythagoras' theorem is the rule that enables us to find the length of a missing side in a right-angled triangle. The Egyptians knew of the rule and used it to square the corners of their fields, buildings, and the bases of their pyramids. They used a piece of rope with knots spaced at equal intervals. If they set the rope up around 3 pegs as in the diagram, they knew that the angle opposite the longest side was a right angle.



ACTIVITY ONE

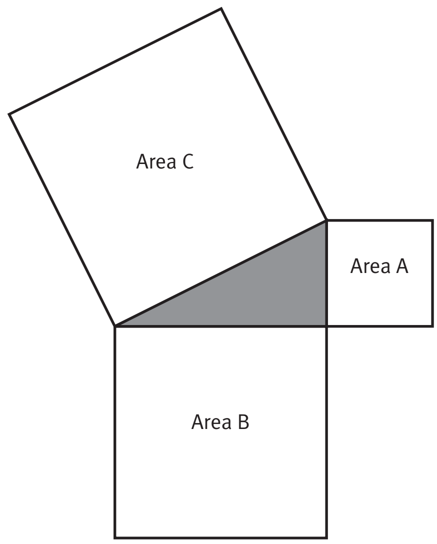
There are many ways of showing that Pythagoras' theorem is true, and this activity works through one of them. Students mark out, cut, and rearrange pieces from two squares of card to show that the square on the hypotenuse (area C) is equal to the sum of the squares on the other two sides (area A + area B).

For this demonstration to work, each student needs 2 squares of card the same size. They mark a point about a third of the way from the bottom left corner. There is nothing special about the third, but by dividing the side at this point, the students will get a triangle with sides that are clearly different lengths. This will make it easier for them to see what is happening. What is important is that the two lengths marked in diagram **i** are identical and that the 4 triangles in diagrams **ii** and **iii** are identical. When the students have finished the first three instructions, they should have the 4 triangles and a square. The students should note that the sides of the square have the same length as the longest side of the triangles.

In question **a v**, the students should see that they have 2 squares whose sides match the two shorter sides of the triangle.

In question **b**, the students need to compare the two arrangements, **iii** and **v**. The area of the background square is identical in each case, as is the area of the 4 triangles. So if you subtract the areas of the triangles from the areas of the squares, what remains in each case (the unshaded parts) must be the same. In other words, the sum of areas A and B must equal area C.

In question **c**, the students have to rearrange the squares around a triangle, as in the diagram, and then in question **d** describe what this arrangement appears to prove. They can use the term "longest side" instead of "hypotenuse" if they wish at this stage.



ACTIVITY TWO

The students will need to draw the triangles in question **1** with care. If they use grid paper with centimetre squares, the grid will give them the right angles and the measurements of the two shorter sides. They should measure the hypotenuse with a ruler, accurate to the nearest millimetre. They can complete the table with the help of a calculator. Make sure that they understand how to use the square and square root functions. Also make sure that they remember to round h^2 to the nearest whole number, or they may have trouble seeing that $a^2 + b^2$ does equal h^2 in each case.

To answer question **2**, the students use their understanding of the theorem (and a calculator) to work out that the hypotenuse must be very close to 30 centimetres. You could make up your own series of right-angled triangles and get the students to find the hypotenuse in each case. As an extension, you could challenge them to find a way of working out the length of a missing shorter side when the hypotenuse is known.

INVESTIGATION

The most famous of all Pythagorean triples is (3, 4, 5), and there is an infinite number of others that are multiples of this one, for example, (6, 8, 10) and (30, 40, 50). There is also an infinite number of other Pythagorean triples, excluding multiples, most of which consist of very large numbers. See this site for some: http://grail.cba.csuohio.edu/~somos/rtritab.txt

As an extension, the students could investigate some other demonstrations of Pythagoras' theorem. For 43 different proofs of the theorem, see www.cut-the-knot.org/pythagoras/index.shtml For information on Pythagoras himself, see www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Pythagoras.html

Pages 16–17 Making Tracks

Achievement Objectives

- draw and interpret simple scale maps (Geometry, level 3)
- specify location, using bearings or grid references (Geometry, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 4)

New Zealanders have a tradition of using the bush, mountains, and wilderness areas for recreation, tourism, and scientific purposes. Therefore, it is important for students to be able to locate features on a map and plan a route. This activity uses part of a sheet from the latest series of 1 : 50 000 topographical maps. This is the kind of map that would normally be used when tramping.

ACTIVITY ONE

Question **1** shows students how 6-digit grid references are used to specify a precise location. Note these points:

- The east–west reference is given before the north–south reference.
- The first 2 digits in each group of 3 are found on the map. (They are printed in blue and are not easy to see.)
- The third digit in each group of 3 has to be estimated or measured. To do this, the reader needs to imagine each cell of the grid divided into tenths in each direction.

In question **2**, the students use the reverse process. Various features are given, and the students have to determine the grid references for each of them.

Ideally, the students should practise their estimation skills when specifying the references because this is what people usually do in practice, but they could use a ruler if they find the estimation difficult. The grid lines form squares that are 20 millimetres by 20 millimetres. This means that every 2 millimetres represents $^{1}/_{10}$ of the width or height of a cell. Waitewaewae Hut is located on the (imaginary) vertical line that passes through the point 10 millimetres along from 04. 10 millimetres equates to $^{5}/_{10}$ of the distance, so the reference is 045. A similar process finds the north–south reference.

ACTIVITY TWO

This activity gives students experience at working out bearings and distance on a map.

To answer question **1a**, the students should:

- place the edge of a sheet of paper on the map so that it runs through the footbridge and the Waitewaewae Hut symbol

- place the centre of their protractor on Otaki Forks and rotate it until the 0 degrees line is oriented to the north (parallel to the vertical grid lines on the map)
- read the angle (measured clockwise) between north and the direction of the hut (represented by the edge of the sheet of paper) and write the angle as a 3-digit number in the format 053°
- use their ruler to measure the straight distance from Otaki Forks to the Hut. It is about 14.8 centimetres, so the distance is approximately 7.4 kilometres.

When doing question **1b**, students are told to use a piece of string, but they could use a narrow strip of card marked off in centimetres. This could easily be bent to follow the turns of the route. If these methods prove too fiddly, they may be able to use their rulers, measuring small sections of the track at a time to take account of the bends.

For question **1c**, the students could talk about:
- the *direction* of various sections of the track (for example, south-west to the first footbridge)
- the *features* along the way (for example, bridges, the old log hauler, valleys, streams, or a bend in the river)
- the *distances* to various features
- the *ups and downs* of the track (using information given by the contour lines).

Students need to use the fine brown contour lines when answering question **2a**. Each line represents a difference in height of 20 metres.

To answer question **2b**, the students use the same strategies as in question **1a**, but this time they centre their protractors on the symbol for Kime Hut. The bearing to the footbridge at Otaki Forks is about 337°. Those using a standard 180 degree protractor will have to make sure that they measure the correct angle and that they don't forget to add 180 degrees to their result.

As a possible extension, the students could investigate a topographical map that covers their own area. Because it would be a full sheet, it would cover a greater variety of terrain and give the students more features to describe, locate, and interpret. Local maps are usually available from local book stores and sports shops that specialise in tramping gear. A parent may be willing to loan a map.

Pages 18–19 Conic Cuts

Achievement Objectives

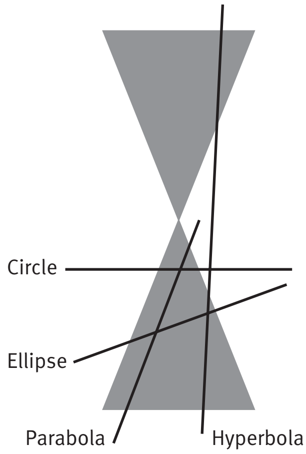
- describe the features of 2-dimensional and 3-dimensional objects, using the language of geometry (Geometry, level 3)
- find, and use with justification, a mathematical model as a problem-solving strategy (Mathematical Processes, problem solving, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)

These activities investigate three of the four conic sections, that is, the different cross-sections you can get by cutting a cone. The first of these, the circle, appears in the level 4 Measurement and Geometry achievement objectives. The others do not appear in the Algebra achievement objectives until level 6 and in the Geometry ones until level 8, but they are in the suggested learning experiences for levels 5 and 6. This illustrates the fact that students can enjoy exploring these curves and their applications without getting too far into the mathematical theory.

The cone is a familiar shape. Your students should be able to think of examples of how the cone shape is found in nature and how it is used in product design (for example, road cones, ice cream cones, funnels for pouring, or the nose cone on rockets). In each of these uses, the shape has been chosen for good, practical reasons. Can the students suggest what these might be?

The following diagram shows that, mathematically speaking, a cone actually has two parts that touch at a point. If it is cut:

- parallel to the base, we get a circle
- parallel to the sloping side, we get a parabola
- at an angle between the above two cases, we get an ellipse
- at an angle greater than the angle of the sloping side, we get an hyperbola.



Only the circle and the ellipse are closed curves. If a closed curve is not a circle, it's an ellipse. The parabola and hyperbola are open and extend into infinity. The hyperbola has two completely separate parts to it and is avoided in this activity because it is conceptually more difficult than the other three sections.

ACTIVITY ONE

This is a simple experiment in which light is used to define the shape of the three sections.

ACTIVITY TWO

In this activity, the students use paper folding to define the outline of a parabola and an ellipse. Your students should:

- use greaseproof paper or tracing paper so that they can easily see the marked point when they fold the top layer of paper onto it
- mark the point towards the edge of the long side of the sheet of paper
- make many creases so that the shape of the curve (which is formed by the edge of the unfolded part of the paper) is clearly defined
- use a pencil to draw a smooth line around the curve they have made.

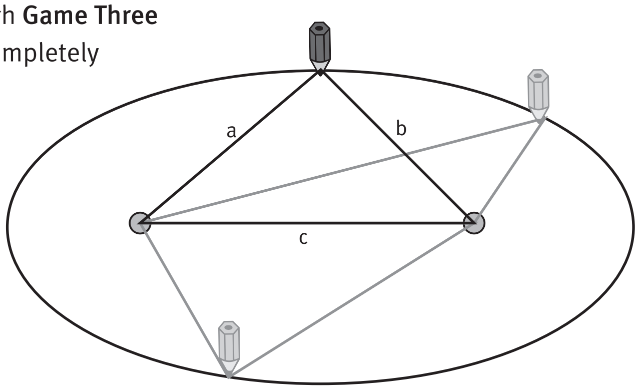
ACTIVITY THREE

Students should see that in **Game One**, to be fair, all players must stand the same distance from the rubbish bin. This means that they should stand on the circumference of a circle, with the bin in the centre.

Game Two is trickier. One approach is for the students to mark any point that is the same distance from the cone and the wall and then to mark other such points. These points will lie on the locus of the line that defines all suitable starting points. The students may need prompting to see that **Activity Two** question **a** is relevant here. The cone in the game is equivalent to the dot, the wall to the line, and the locus of points equidistant from both is a parabola. Folding the paper was a means of finding midpoints.

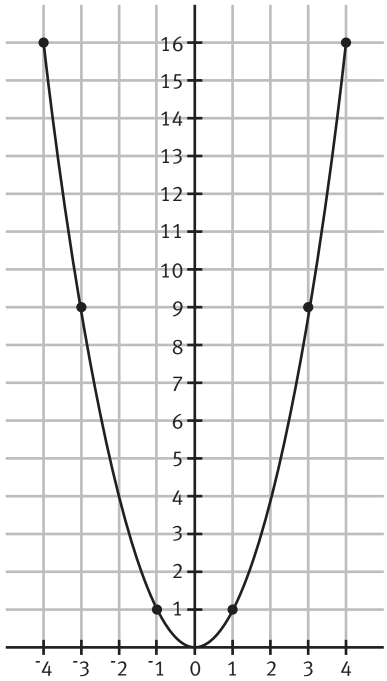
One approach to **Game Three** is to use a closed loop of string to represent the total distance travelled by each player and two drawing pins stuck into a piece of card to represent the cones. The player can be represented by a pencil. The pencil can move anywhere as long as it keeps the loop of string taut at all times. As it moves to all possible positions, the pencil draws an ellipse. If the students consider the 3 legs of the journey each player makes, they will see that, although lengths a and b change depending on the starting point, the total $a + b$ is constant (as is the total $a + b + c$). This means that the game will be fair as long as the two players start anywhere on the ellipse.

In **Activity Two**, question **b**, the students create an ellipse by finding points equidistant from a fixed point and a circle. Although **Game Three** also involves an ellipse, its shape is derived in a completely different way (using two fixed points).



As an extension to these activities, the students could create a parabola on square grid paper, using square numbers, as in the diagram, and joining the marked points with a smooth curve. (It's over to you whether you introduce the squares of negative numbers at this point.)

Another extension activity would be for the students to investigate the use of the parabola in technology, for example, in a torch or a solar cooker. They could also try to discover why the parabola is the most suitable shape for such purposes.



For a short history of the conic sections and a visual demonstration of them, see this University of Georgia site: http://jwilson.coe.uga.edu/EMT668/EMAT6680.F99/Erbas/emat6690/Insunit/conicsunit.html

Pages 20–21 Snazzy Snowflakes

Achievement Objectives

- describe the reflection or rotational symmetry of a figure or object (Geometry, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY ONE

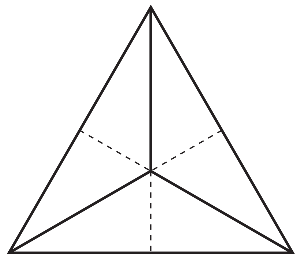
Question **1a** is a straightforward folding and cutting task, but if students want attractive and delicate results, they will have to cut away much of the paper, and this will require care.

These snowflakes have 4 lines (axes) of symmetry because they have 3 folds. The first fold gave 1 axis, the second, 2 axes; and the third, 4 axes. If the students turn their snowflake through one complete revolution about the centre point, they will see that it appears to occupy its original position 4 times. (The students can put a dot in a corner of their snowflake to help them follow the progress of the turn.) In mathematical terms, we say that the snowflakes have rotational symmetry of order 4.

The snowflakes in question **2** are a little more difficult. When the students come to make the third fold, they have to estimate its correct position. Whether they have succeeded becomes clear with the next fold. The outer edges should meet, and the folded shape should have a symmetrical pair of points that look a bit like foxes' ears. Cutting is more difficult: there are now 12 thicknesses of paper. This time, there are 6 fold lines and the snowflake fits its original position 6 times in a complete turn, so its order of reflective symmetry is 6 and its order of rotational symmetry is 6. The same is true of real snowflakes.

To make a snowflake with 8 axes of symmetry, as in question **3a**, students will need to use the same folds as for question **1** but make one further fold. They will need to start with a fairly large piece of thin paper if they are to manage the extra fold and the cutting. By now, they should be able to see that if they fold a square (through a common point), they will always get an even number of axes of symmetry. This gives them the answer to question **3b**.

If, however, they start with an equilateral triangle (they will need to use a compass to construct one accurately), they can make a snowflake with 3 axes of symmetry. They should fold the triangle in half along the 3 lines shown in the diagram. They should then re-fold it, making valleys of the dotted lines and bringing all three vertices together. They can then cut out parts to create a snowflake with 3 axes of symmetry and rotational symmetry of order 3.



The above folding idea could be pursued as part of question **4**, and there are many other possibilities, too. The important thing is that the students are thinking about what they are doing and trying to describe the results using geometrical language.

By folding and cutting as in question **5**, the students will get patterns that have two axes of symmetry and rotational symmetry of order 2. Additionally, one-half of each pattern can be translated to give the other. A pattern of this kind could be translated repeatedly to make an attractive frieze.

ACTIVITY TWO

Given their previous experiences, the students may be able to complete the table in question **1** with little extra thought. If not, they will be able to establish the pattern by folding further squares with the table in mind.

When answering question **2**, the students are likely to find that, at most, they can fold the square of paper 5 times. They will see from the table that 5 folds means 32 layers of paper. The folded paper is now too thick and too small (in its folded state) to allow for a further fold.

INVESTIGATION

Type "snowflake photos" in your Internet search engine to find sites that show actual snowflakes, greatly magnified.

For ideas on paper snowflakes, see sites such as these:
www3.ns.sympatico.ca/dstredulinsky/snow%20flakes/thumbs1.htm
www.montessoriworld.org/Handwork/foldingp/snowflak.html
http://snowflakes.lookandfeel.com/make-a-flake.swf

Page 22

Ancient Architecture

Achievement Objectives

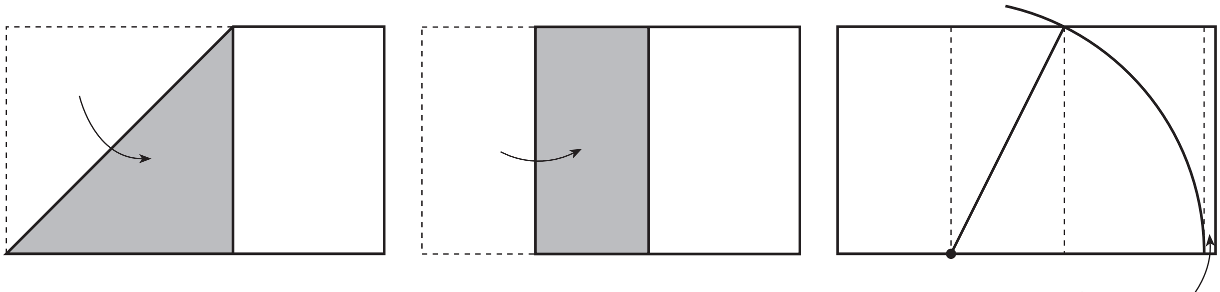
- enlarge and reduce a 2-dimensional shape and identify the invariant properties (Geometry, level 4)
- express one quantity as a percentage of another (Number, level 5)
- share quantities in given ratios (Number, level 5)
- use equipment appropriately when exploring mathematical ideas (Mathematical Processes, problem solving, level 4)

In ancient Greece, it was believed that, of all possible rectangles, the golden rectangle had the most pleasing proportions. Its proportions were often used in works of art and architecture. The golden rectangle is still used in art today, but authorities no longer agree that it has the perfection that was once claimed.

Mathematics in the New Zealand Curriculum suggests, as a level 4 development band activity, that students research and report on the use of geometrical ideas in art, "for example: the occurrences of the golden section in art and architecture".

ACTIVITY

In question **1**, the students measure the length and height of an illustration of the Parthenon. Alternatively, they could construct a golden rectangle and take measurements directly from this. The construction is quite straightforward, but the students will need to use a compass. The diagram shows how to start with any rectangle and, by folding, drawing, and cutting, to reduce it to the correct proportions.



Question **2a** asks the students to follow a pattern that will lead to an increasingly precise value for φ. Some may be intrigued about the nature of this number. Like π, φ is irrational. For a discussion on irrational numbers, see page 12 of the teachers' notes for *Measurement: Book One*, Figure It Out, Years 7–8.

When doing question **2b**, the students should enter the values from the second column of their table in a spreadsheet and then select the XY (scatter) option.

The students may be interested to see that the value of the golden ratio, correct to 50 places, is:
1.618 033 988 749 894 848 204 586 834 365 638 117 720 309 179 805 76.

The students can compare the proportions of an A4 sheet with those of the golden rectangle (question **3**) either by calculation or by following the steps in the diagram above. If they measure the A4 sheet, then divide length by breadth, they should get a result of 1.41, correct to 2 decimal places. This is well short of the value of φ. The 1.41 value is not chosen at random. It is $\sqrt{2}$ (the square root of 2, which is another irrational number). A rectangle whose sides are in the ratio of $\sqrt{2}:1$ can be halved to give 2 smaller rectangles whose sides are in exactly the same ratio as those of the original rectangle. That is why all sheets in the series A1, A2, A3, A4 … have the same shape.

INVESTIGATION

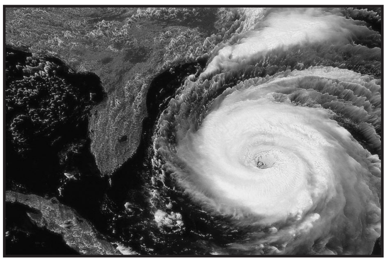
The students investigate the golden rectangle and the golden ratio, using the Internet. The following are good sites to begin with:
http://mathforum.org/dr.math/faq/faq.golden.ratio.html
www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fiblnArt.html

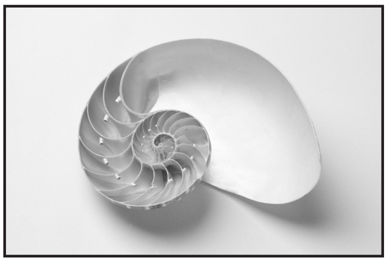
Page 23 Golden Spirals

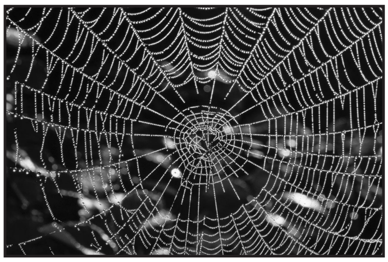
Achievement Objectives

- enlarge and reduce a 2-dimensional shape and identify the invariant properties (Geometry, level 4)
- report the results of mathematical explorations concisely and coherently (Mathematical Processes, communicating mathematical ideas, level 4)

Spirals appear in surprising places and range from the very small to the huge. The cochlea (chamber of the inner ear) is a spiral. So is the nautilus shell (and the shell of many other sea creatures), the shell of a snail, the webs of some spiders, the pattern of leaf distribution around the stems of some plants, whirlpools, tornadoes, and galaxies in space.


Cyclone


Nautilus


Spider's Web

ACTIVITY

To do the construction in question **1**, the students will need a sharp pencil, a good compass and ruler, and a set square (or other suitable right-angled object). Most students should find the diagrams self-explanatory, but they will need to take great care with their construction work. Even with care, they are unlikely to be able to complete more than 5 or 6 quarter arcs. The cumulative effect of small inaccuracies will mean that the sides of the ever-smaller rectangles are no longer in the correct ratio (φ).

As an alternative to pencil and paper, the students could use a computer drawing program. Starting with a large 90 degree arc, they can construct a continuous spiral by repeating this sequence:

- copy the previous arc
- rotate the new arc 90 degrees clockwise
- enlarge (scale) it by a factor of $^1/\varphi$ (to reduce its size by the correct amount)
- translate it so that it joins up with the previous arc.

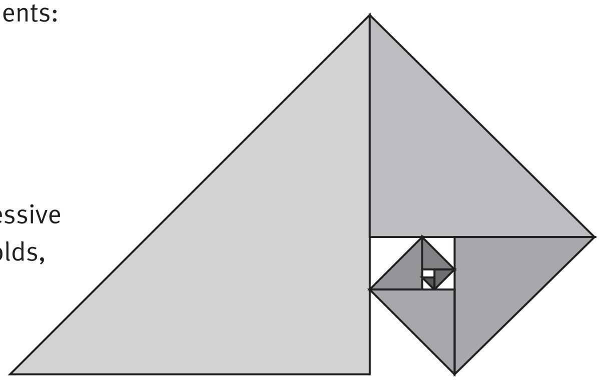
Students should see that each successive section of the spiral is a smaller version of the previous one. The spiral is therefore a fractal. See the final activity in this book for more on fractals.

In question **2**, the students investigate the squares that contain the 90 degree arcs of the spiral, not the arcs themselves. They will find that if they line them up as instructed, they can make a "ramp" down them, using a "plank" that rests on their top right-hand corners. In other words, the top right vertices of the squares are all in a straight line. The students may like to consider how many squares they could cut (or draw) before they become too small to be visible. Surprisingly few!

INVESTIGATION

These websites could be useful starting points for students:
www.spiral.co.nz/spirals/golden rectangle.asp
www.pithemovie.com/gold.html
www.arches.uga.edu/~iluvkids/Goldenspiral.html

As a further activity, students could make this fractal:
They start with a golden rectangle and fold down successive corners. Before making the fourth (and subsequent) folds, they will need to make a straight cut.



Fantastic Fractals

Achievement Objectives

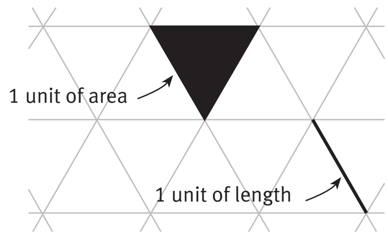
- enlarge and reduce a 2-dimensional shape and identify the invariant properties (Geometry, level 4)
- make conjectures in a mathematical context (Mathematical Processes, developing logic and reasoning, level 4)
- use equipment appropriately when exploring mathematical ideas (Mathematical Processes, problem solving, level 4)

A fractal is a curve or surface created by some process of successive subdivision. The result is a figure or design in which each part has the same characteristics as the whole. Fractals have become very important mathematically and have even been used to model urban growth.

ACTIVITY

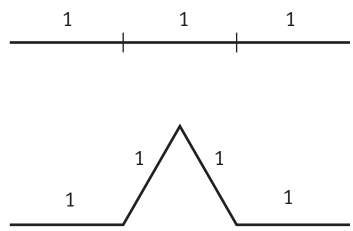
The diagrams on the right-hand side of the page should be self-explanatory for students doing question **1**. They will need to work in pencil so that they can remove parts of the outline as their flake develops.

By the time they get to step v, the students will find that they are working with very small lengths, and some may not want to complete the iteration (application of the process). They should still have done enough to be able to complete the table in question **2**.



In question **2**, the students should use as units the side of 1 small triangle on the isometric grid and the area of 1 small triangle (as in the diagram). This means, for example, that in step 1, the length of each side can be written simply as 18 units, the total perimeter as 54 units, and the total area as 324 units.

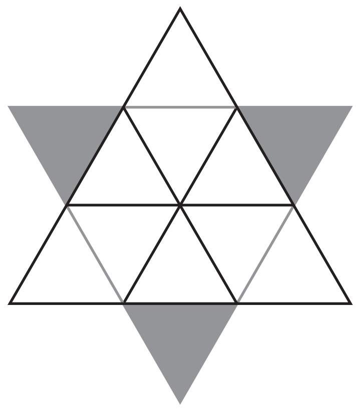
For question **2b**, the students can look at their chart, where they will see that the perimeter increases from 54 to 72, from 72 to 96, and from 96 to 128. If they use a calculator to compare each increase, they will discover that it is constant: 1.33. For example, 72 is 1.33 times greater than 54, and 96 is 1.33 times greater than 72.



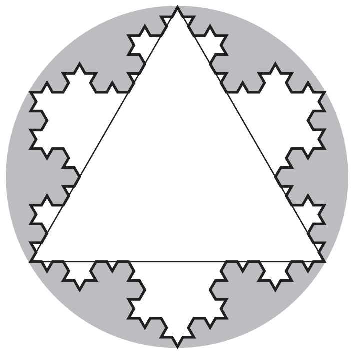
A less mathematical and more intuitive approach is to look at what happens to each line segment as we move from one iteration (application of the process) to the next. All that happens is that each line segment (no matter how small) is divided into 3 equal parts and the middle part is replaced by 2 parts, each the length of the part that was removed, as in the diagram. So where there were 3, there are now 4. The length has been increased by a third. The new length can be found by multiplying the previous length by $^4/_3$ (1.33).

Question **2c** is not easy. The explanation here amplifies that in the Answers:

- At each iteration, a small triangular-shaped area is added at the mid-point of each side, as in the diagram on the right:
- The area of each small triangle is $^1/_9$ the area of the previous triangle. (You can demonstrate this to your students using the diagram.)
- The area of the first, large triangle is 324 units (small triangles).
- At the first iteration, the area of each additional triangle will be $^1/_9$ x 324 = 36 units, and there are 3 of them (1 for each side), a total of 108. This is added to 324 to give a total area of 432.
- At the second iteration, the area of each additional triangle will be $^1/_9$ x 36 = 4 units, and there are 12 of them (1 for each side), a total of 48. This is added to 432 to give 480, and so on.



1st iteration



Question **2d** asks the students to predict what would happen if this process continued. The perimeter would keep on increasing: with every iteration, it grows by a third. The area, however, is finite: with every iteration, it grows by an ever-smaller amount. While the perimeter of the flake keeps growing forever, the area will never exceed that of a circle that could be drawn through the vertices of the original triangle.

As an extension to question **2d**, the students could use formulae in a spreadsheet, like the one below, to show what happens over, say, 10 or 20 iterations. Note that an extra column has been added (E). This column helps to make the formula needed in column F relatively simple. The formula for B3 is =4*B2, that for C3 is =C2/3, that for D3 is =B3*C3, and that for E3 is =E2/9. The formula for F3 is shown in the diagram:

Workbook 1

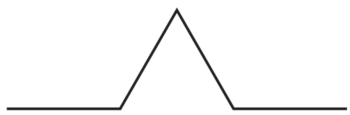
F3 =F2+B2*E3

	A	B	C	D	E	F
1	Step	Number of sides	Length of each side	Total perimeter	Area of 1 small triangle	Area (in small triangles)
2	1	3	18.0000	54	324.0000	324.0000
3	2	12	6.0000	72	36.0000	432.0000
4	3	48	2.0000	96	4.0000	480.0000
5	4	192	0.6667	128	0.4444	501.3333
6	5	768	0.2222	171	0.0494	510.8148
7	6	3072	0.0741	228	0.0055	515.0288

If the students go on to graph the data for the total perimeter, they will see that the curve quickly approaches the vertical. If they graph the data for area, they will find that the curve quickly approaches the horizontal. This confirms the explanations given in the Answers.

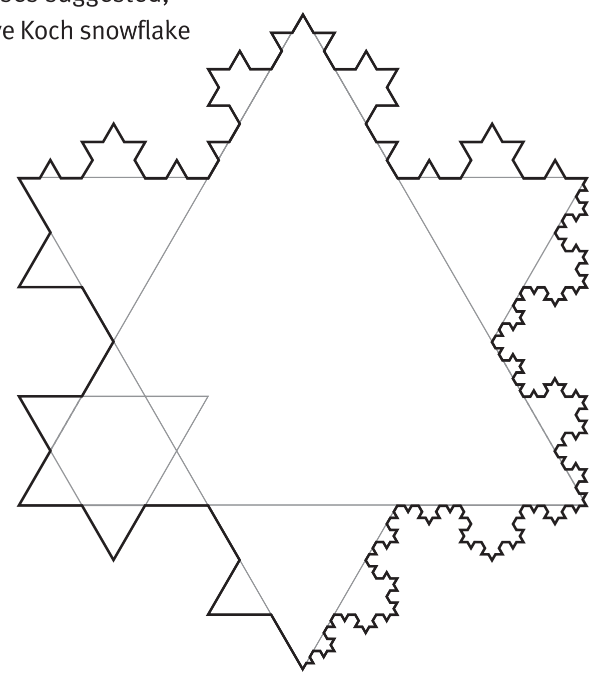
Question **3** asks the students to create a Koch snowflake using a computer. They will find this an interesting and achievable challenge that removes the painful repetition. At the same time, they will make use of all their knowledge of mathematical transformations.

They could start by drawing the biggest circle that will fit on the screen, then drawing the biggest equilateral triangle that will fit within the circle. This will ensure that their whole design fits on the screen.

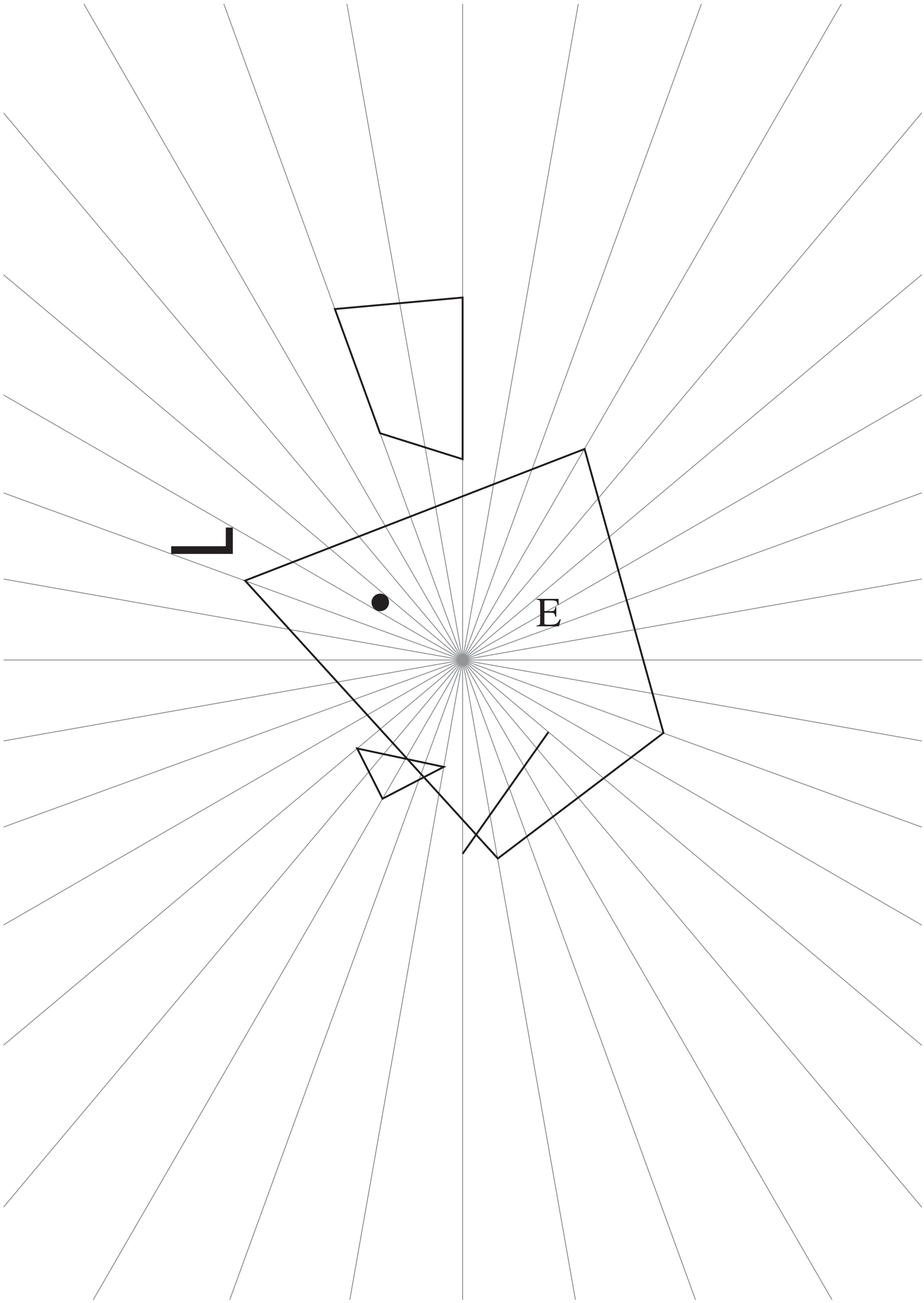


The finished snowflake is made entirely from a single 4-segment element like the one in this diagram. As long as the students construct it accurately the first time, they can use it repeatedly. The scale factor for the successive enlargements (reductions) is $^1/_3$ (0.33). The last diagram shows parts of the second, third, and fourth iterations built up around the original equilateral triangle. By following the diagrams and using the processes suggested, your students should be able to create and print an attractive Koch snowflake for themselves.

The students could also investigate fractals further using the Internet. Suitable sites include:
http://math.rice.edu/~lanius/frac
(for teaching ideas)
www.mbfractals.com
www.math.umass.edu/~mconnors/fractal/fractal.html



Enlargement Explosion



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