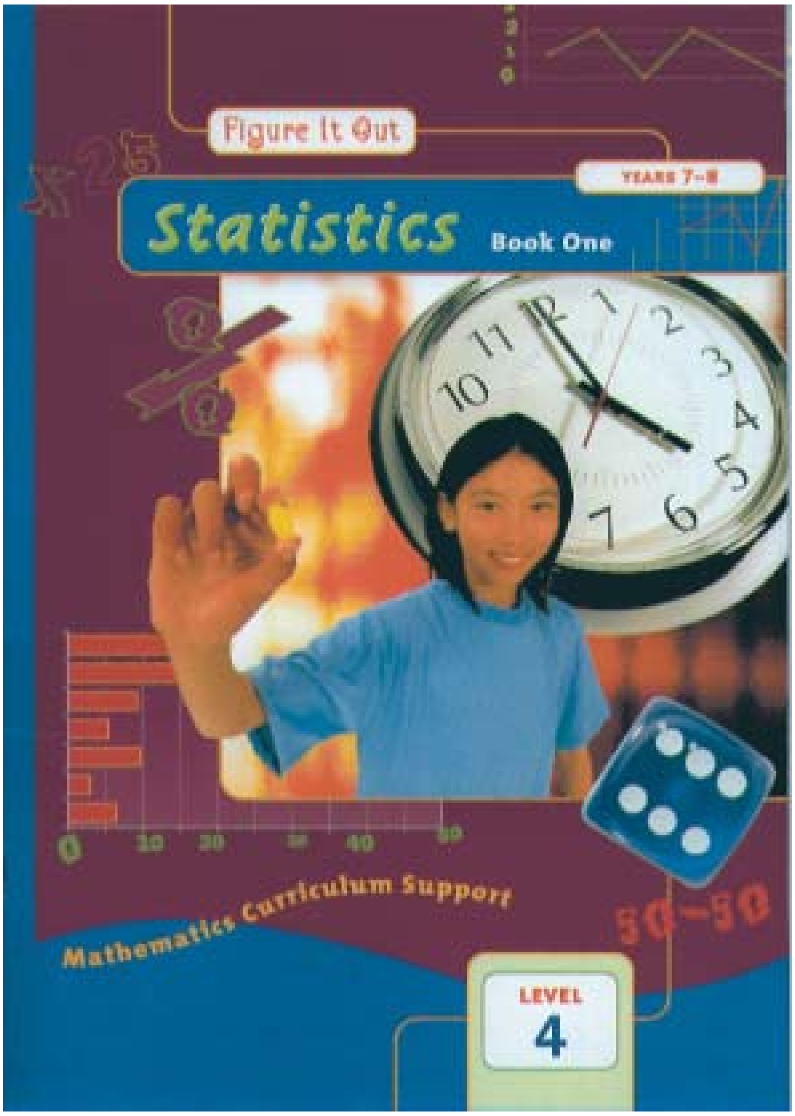
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

CONTENTS

The books for years 7–8 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand year 7–8 classrooms. The books have been developed and trialled by classroom teachers and mathematics educators and follow on from the successful series for levels 2–4 in primary schools.

Student books

The student books in the series are divided into three curriculum levels: levels 2–3 (linking material), level 4, and level 4+ (extension material). All the books are aimed at year 7–8 students in terms of context and presentation.

The following books are included in the series:
Number (two linking, three level 4, one level 4+, distributed in November 2002)
Number Sense (one linking, one level 4, distributed in April 2003)
Algebra (one linking, two level 4, one level 4+, distributed in August 2003)
Geometry (one level 4, one level 4+, distributed in term 1 2004)
Measurement (one level 4, one level 4+, distributed in term 1 2004)
Statistics (one level 4, one level 4+, distributed in term 1 2004)
Themes: *Disasters Strike!, Getting Around* (level 4, distributed in August 2003)

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and life experiences that are meaningful to students aged 11–13 years. The activities can be used as the focus for teacher-led lessons, as independent bookwork, or as the catalyst for problem solving in groups.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the student books includes full answers and explanatory notes. Students can use them for self-marking, or you can use them for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, comments on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/r/maths/curriculum/figure

Using Figure It Out in your classroom

Where applicable, each page starts with a list of equipment that the students will need to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Answers Statistics: Book One

Page 1 Monster Munch

ACTIVITY

1. It will not be enough because they can't be sure of an even distribution of nuts. For example, if one cookie has 16 nuts, another will be 1 short.
2. **a.–e.** Answers will vary.

   f. Not necessarily. A random distribution will never guarantee anything.
3. Answers will vary. Options include recommending that the company replaces the guarantee of 15 nuts with a statement that there is an average of 15 nuts in each cookie or changes the production method so that the nuts are machine-counted into the individual cookies.

Pages 2–3 Future Options

ACTIVITY

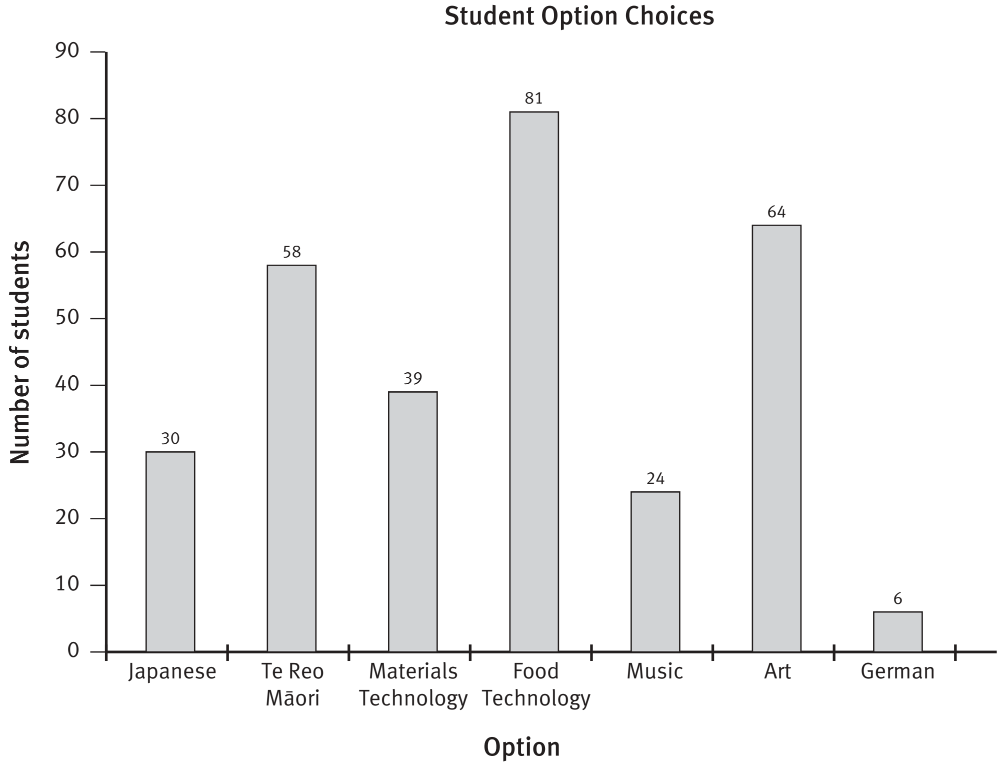
1. The bar graph is best. It shows how popular each subject is and the number of students taking it. The doughnut graph and the pie graphs do not give numbers. The two with the separate legends (keys) are hardest to understand. Even in the best pie graph (iii), it is hard to compare the size of the sectors without measuring them.
2. 101. (There are 202 option choices, and each student chose two options.)
3. **a.** Practical activity. One way to make a prediction is to look at what the other students have chosen and assume that those who were away on the day would make similar choices. (If $^1/_{10}$ of the students surveyed chose Japanese, it is reasonable to guess that $^1/_{10}$ of those absent would also choose Japanese.)

   b. Answers may vary. Using the method described in **a**, the projected numbers would be 30, 58, 39, 81, 24, 64, and 6, as in this table:

Option	Number of students	Estimating choice of missing students using ratios	Present + absent	Total
Japanese	20	$^{20}/_{202}$ x 100 =.10	20 +.10	30
Te Reo Māori	39	$^{39}/_{202}$ x 100 =.19	39 +.19	58
Materials Technology	26	$^{26}/_{202}$ x 100 =.13	26 +.13	39
Food Technology	54	$^{54}/_{202}$ x 100 =.27	54 +.27	81
Music	16	$^{16}/_{202}$ x 100 =.8	16 +.8	24
Art	43	$^{43}/_{202}$ x 100 =.21	43 +.21	64
German	4	$^{4}/_{202}$ x 100 =.2	4 +.2	6

   c. Practical activity. The above table shows how the numbers have been combined.

d. Here is one example of a suitable graph:



4. a. Probably not. 6 students is unlikely to be enough for a year 9 class.

b. 3. This would give 3 classes of 27, which is a reasonable size for a class.

5. a.–c. Practical activity. Results will vary.

Pages 4–5

What's the Question?

ACTIVITY

a. The question is of limited use because it has two possible meanings. Which should be banned, dogs or children? It is also a leading question because "vicious" is an emotive word. Avoid language of this kind.

b. The term "family" is too broad. The surveyor should use another term (like "household") or define "family". The surveyor should not suggest that the maximum size of a family is 7.

c. The question contains negatives like "not", "no", and "non". These should be avoided because they are confusing. Even worse is the use of two or more negatives.

d. Scientists can be found to support both sides of the argument. This question tells the person what answer they should be giving. The surveyor should present both sides to the listener.

e. Make this question more specific. The hearer will be unsure what sort of information the surveyor is looking for, and it will be difficult to classify the responses people give.

f. Specify time in intervals ("between 7 and 8 p.m.") rather than exact times.

g. Sharpen the time interval: "How many times in an average week?" or "How many times in the last 7 days?" or offer categories such as "About once a week" or "More than once a week".

Pages 6–7 Collect and Reflect

ACTIVITY

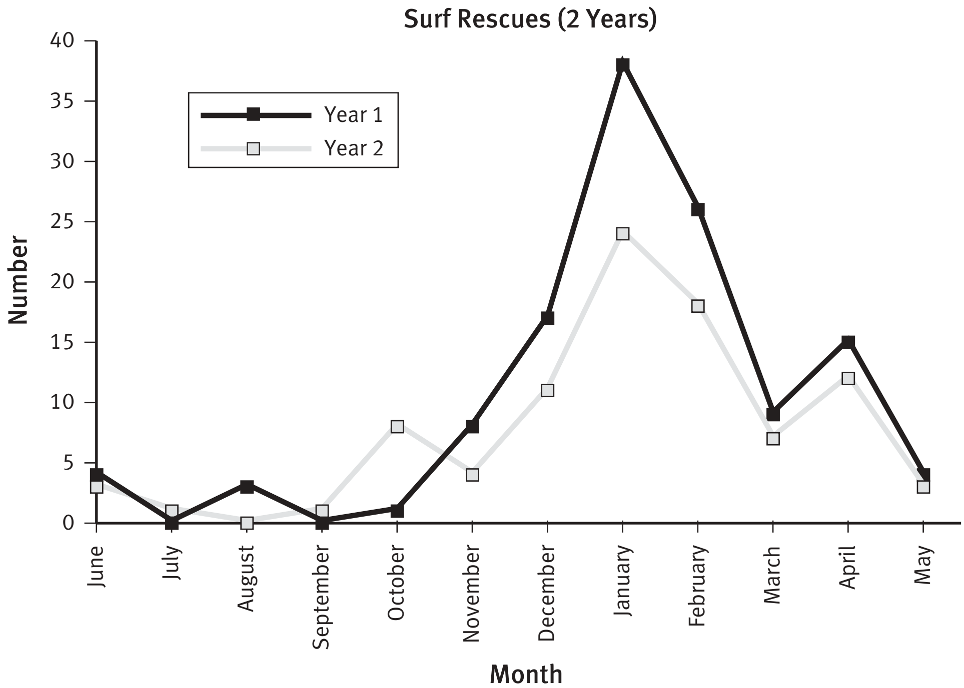
Answers will vary greatly depending on the choice of activity, but when you have finished, your experiment should have these features:

- a data collection containing at least 20 results for your experiment (but if it is easy to collect data for your experiment, you should collect more)
- your data collected and sorted into a table with suitable headings
- one or more graphs that enable a viewer to quickly see the important facts you have discovered
- mean, median, and spread (you could use a box-and-whisker graph to present the mean and spread visually)
- "The story in the picture": what you have discovered from your investigation or what you have proved.

Pages 8–9 Surf Stats

ACTIVITY

1. Your graph could be similar to this one or could show the 2 years as a single line:



2. a. January is the month with the most rescues in both years. January is the main summer holiday month, so more people are at the beach.

   b. There is a downward trend. If you compare the same months in the different years, the second year generally has a lower number of rescues. This could be the result of an effective water safety campaign or of poor weather meaning that fewer people were at the beach.

   c. October in the second year has an unexpectedly high number of rescues. Explanations will vary. The reason could be a one-off rescue involving a lot of people or a hot Labour Day meaning more people swam and were involved in water-based activities.

3. Paragraphs will vary but should include at least three points similar to the following:

   - The graph gives the number of rescues by geographical area.
   - It covers the period 2001–2002.
   - By far the most rescues (50%) were in the Northern Region.
   - 85% of rescues were in the North Island; 15% were in the South Island.

- Most of the rescues were in areas with warmer water.
- The Northern Region includes Auckland. It contains far more people than any other region, so it has more swimmers.
- The order of the bars runs from north to south.
- Canterbury had about 11% of the rescues.

4. a. 118.8%. Rescues sometimes use more than one piece of equipment.
   b. 93.6%. (100 – 6.4)
   c. The tube is the most obvious answer, but you could argue that the boys should concentrate on the lesser-used equipment because they will be least familiar with it.

Pages 10–11 Discipline Dilemmas

ACTIVITY

1. Temuera is correct. 33 people disagree, and 19 agree.

   Rachel is also correct. 57 people agree, and 43 disagree.
2. a. Rachel's scale requires people to have a definite opinion. Temuera's scale allows people to "opt out" and avoid making a decision.
   b. It depends what you're looking for. Temuera's results are useful in that they show that a large number of people don't have feelings one way or the other. Rachel's survey shows that when people are forced to make up their minds, most decide against smacking. But are forced opinions worth much? Both surveys show equally well that few people feel strongly about smacking.
3. "Undecided" people either may not have a definite opinion or may see both advantages and disadvantages in the proposition.
4. a. Results will vary.
   b. Children may be more strongly against smacking because they have had some recent experience of being smacked. Parents may feel that smacking is an important discipline tool.

Pages 12–13 Down the Plughole

ACTIVITY

1. Answers will vary depending on the number of occupants per household and whether they have a garden, pool, and so on. Usage is likely to be between 1 000 and 2 000 litres per person per week.
2. a. Answers will vary.
   b. Answers will vary. A bar graph is likely to be a good choice.
   c. Answers will vary but should include reference to total usage and the major uses of water.
3. Answers will vary.
4. Answers will vary but may include fitting a water-efficient shower rose, taking shorter showers, brushing teeth without the tap running continuously, not washing clothes after one use, using the dishwasher only for a full load, fitting a dual-flush toilet cistern, and fitting a timer to the garden sprinkler.
5. 28.8 m^3. (Minutes in a week: $7 \times 24 \times 60 = 10\,080$. Bucketfuls: $10\,080 \div 3.5 = 2\,880$. Total litres: $2\,880 \times 10 = 28\,800$. Volume in cubic metres: 28 800 L = 28.8 m^3.)

INVESTIGATION

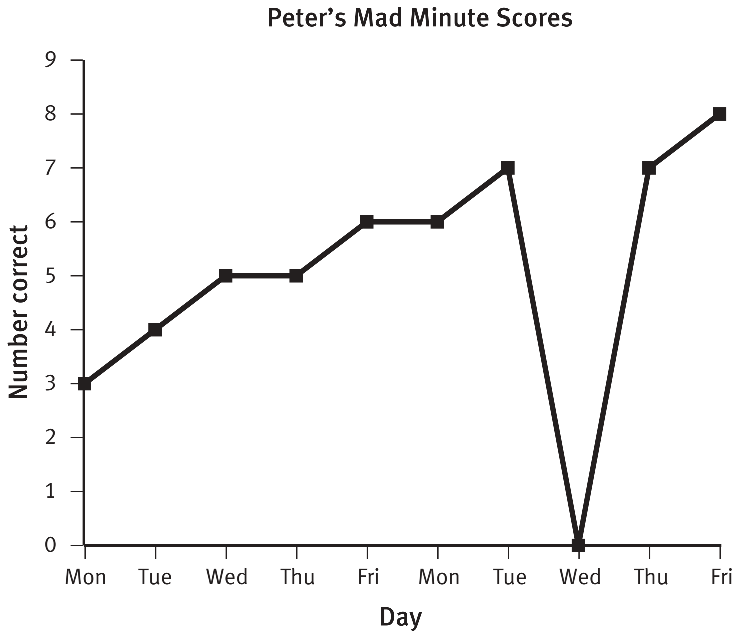
Answers will vary.

Pages 14–15 Mad Minute

ACTIVITY

1. a. Answers will vary. For example, the numbers on Monday were harder, she was tired after a busy weekend, she was out of practice, and so on.
   b. Yes. She has improved from getting only 2 correct on Monday and Tuesday of the first week to getting 5 correct on the Thursday and Friday of the following week.
2. a. Rawinia's worst score is Lome's best score. Rawinia's scores are generally improving, but Lome finishes the 2 weeks with the same score that he started with.
   b. His results are up and down and not very high. His worst score was 0, and he got this three times. His best score was 2. There is no evidence of improvement.

3. a.



b. Comments will vary. Peter consistently improved over the 2 weeks except for the second Wednesday. He may have been absent from school that day, or he may have added the first and second numbers incorrectly and got all the others wrong as a result.

4. a. Statements will vary. Rawinia has shown a steady improvement and is consistently scoring 8 by the end of the week.

b. Comments will vary. There is not enough evidence to show for certain whether she has peaked or has simply reached a plateau and is spending some time there before improving further.

5. It would take Rawinia longer to work out the answers, so she would probably get fewer right.

Page 16 Channel Surfing

ACTIVITY

1. Rose, but even her figure is probably too high. A large number of people are returning from work or eating dinner at this time. There are plenty of others who don't watch TV because they prefer doing other things. In spite of what Joanna says, it is not clear if she means all New Zealanders or all people watching TV at that time.

2. a. She may have interviewed mainly young people or just her household (as suggested in the bottom picture).

b. Yes. Viewing habits vary greatly between age groups. If she surveyed people in a retirement village, she would find that a high proportion watches the news in the evening, but if she surveyed teenagers, she would be likely to find that the opposite is true.

3. a.–d. Practical activity. Results will vary.

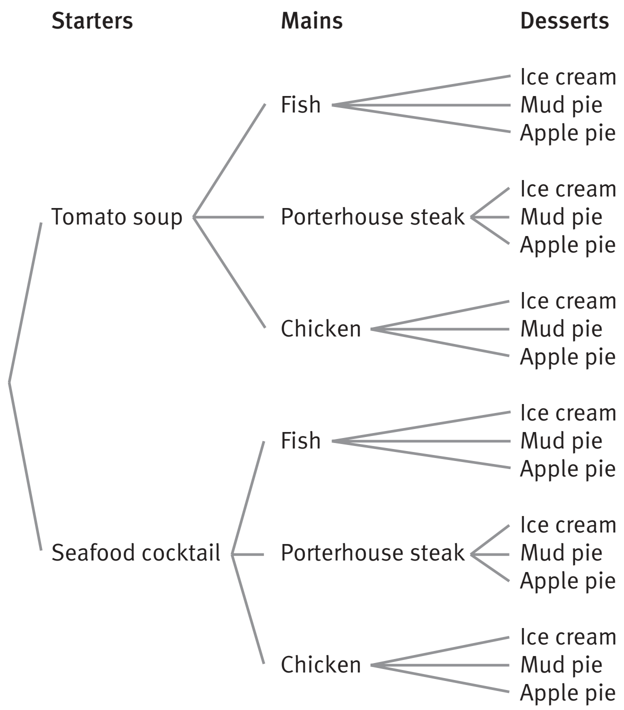
4. TV networks. Advertisers could use the data to decide when and where to place advertisements so they will reach certain audiences.

5. The sample group is too small. Generally, the larger the sample size, the more accurate the results. Also, data must be collected using systematic methods if the results are to be valid.

Page 17 Family Feast

ACTIVITY

1. Yes, Mum is right. A tree diagram will show this:



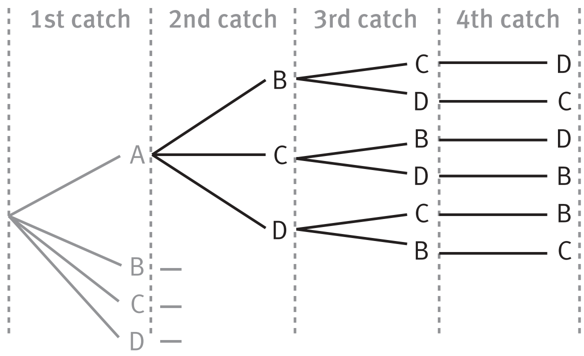
2. TFI, TFM, TFA, TPI, TPM, TPA, TCI, TCM, TCA, SFI, SFM, SFA, SPI, SPM, SPA, SCI, SCM, SCA. There are 18 different combinations of 3 courses.
3. 27
4. 36
5. 4 starters, 4 mains, and 3 desserts give 48 combinations. The order makes no difference (for example, 3 starters, 4 mains, and 4 desserts also give 4 combinations).

Page 18 Catch of the Match

ACTIVITY

1. 24. Assuming that A is the best catch, we get this set of possibilities:

   ABCD, ABDC, ACBD, ACDB, ADCB, ADBC.



   Similar lists and diagrams can be produced for each of B, C, and D as best catch. Each list or diagram represents 6 different combinations. 4 lists x 6 combinations = 24 entries.

2. If 4 catches give 24 possibilities, adding an extra catch means 24 x 5 =120 possibilities. This could also be shown on a tree diagram, but it would require a large sheet of paper and some very careful planning to ensure that everything fitted!
3. No. Other people may have the correct answers as well, in which case, the winner is drawn from all those who have sent in a correct entry.
4. No. He would have to complete 720 entry forms.

Page 19 Across the River

GAME

A game for investigating probability

ACTIVITY

1. a. 1 is impossible. 2 and 12 are very hard to get.

   b. 6 and 8 are quite easy. 7 is easiest of all.

2.

		Dice one					
		1	2	3	4	5	6
Dice two	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

The probability of obtaining each number is:

1: $^{0}/_{36}$; 2: $^{1}/_{36}$; 3: $^{2}/_{36}$; 4: $^{3}/_{36}$; 5: $^{4}/_{36}$; 6: $^{5}/_{36}$; 7: $^{6}/_{36}$; 8: $^{5}/_{36}$; 9: $^{4}/_{36}$; 10: $^{3}/_{36}$; 11: $^{2}/_{36}$; 12: $^{1}/_{36}$

3. The dice should have the numbers 0, 1, 2, 3, 4, 5 and 1, 1, 1, 7, 7, 7.

Pages 20–21 Wallowing Whales

GAME

A game for investigating probability

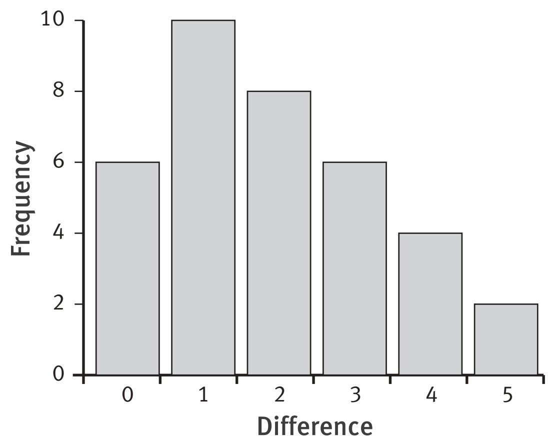
ACTIVITY

1. Answers will vary.

2. a.

		Dice one					
		1	**2**	**3**	**4**	**5**	**6**
Dice two	**1**	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2
	5	4	3	2	1	0	1
	6	5	4	3	2	1	0

b. **Difference of 2 Dice**



c.

Difference	0	1	2	3	4	5
Probability	$^{6}/_{36}$	$^{10}/_{36}$	$^{8}/_{36}$	$^{6}/_{36}$	$^{4}/_{36}$	$^{2}/_{36}$

3. a. Finn's strategy is unlikely to be effective. Although 1 has the highest probability, in the long run, he can expect to get it only 10 times out of 36. For the other 26 out of 36 times, his turn will count for nothing.

b. A more effective strategy is to spread his counters from 0–3, with proportionally more on 1. He then has a $^{30}/_{36}$ probability of getting a useful result each time he rolls the dice.

Page 22 Dodgy Dice

ACTIVITY

1. Explanations will vary. A dice could be picked because it has high numbers or no low numbers. (You need to be aware that the higher numbers are more likely to score the points in this competition.)

2. a.–b. Practical activity. Results will vary.

c.–d. Practical activity. Results will vary, but in the long run, it should become clear that each dice can beat another one and can be beaten by another one. (See the results for question **3** below.) So no dice is the "best dice" in all circumstances. Increasing the points should make no difference.

3. a.

		Red dice					
		0	**1**	**7**	**8**	**8**	**9**
Blue dice	**5**	B	B	R	R	R	R
	5	B	B	R	R	R	R
	6	B	B	R	R	R	R
	6	B	B	R	R	R	R
	7	B	B	=	R	R	R
	7	B	B	=	R	R	R

b. When it is blue versus red, you can expect that in the long run, red will win 22 out of 36 times, the two dice will draw 2 out of 36 times, and blue will win 12 out of 36 times.

c.

		Yellow dice					
		3	**4**	**4**	**5**	**11**	**12**
Blue dice	**5**	B	B	B	=	Y	Y
	5	B	B	B	=	Y	Y
	6	B	B	B	B	Y	Y
	6	B	B	B	B	Y	Y
	7	B	B	B	B	Y	Y
	7	B	B	B	B	Y	Y

		Red dice					
		0	1	7	8	8	9
Yellow dice	3	Y	Y	R	R	R	R
	4	Y	Y	R	R	R	R
	4	Y	Y	R	R	R	R
	5	Y	Y	R	R	R	R
	11	Y	Y	Y	Y	Y	Y
	12	Y	Y	Y	Y	Y	Y

d. In the long run, you would expect blue to win 22 out of 36 times, yellow to win 12 out of 36 times, and a draw 2 times out of 36.

e. In the long run, you would expect red to win 16 out of 36 times and yellow to win 20 out of 36 times.

4. a.–b. A dice labelled 1, 2, 3, 9, 10, and 11 *nearly* works: it will beat red, has a small advantage over yellow, and is the equal of blue. (See the tables below.)

		Red dice					
		0	1	7	8	8	9
New dice	1	N	=	R	R	R	R
	2	N	N	R	R	R	R
	3	N	N	R	R	R	R
	9	N	N	N	N	N	=
	10	N	N	N	N	N	N
	11	N	N	N	N	N	N

		Yellow dice					
		3	4	4	5	11	12
New dice	1	Y	Y	Y	Y	Y	Y
	2	Y	Y	Y	Y	Y	Y
	3	=	Y	Y	Y	Y	Y
	9	N	N	N	N	N	N
	10	N	N	N	N	N	N
	11	N	N	N	N	N	N

		Blue dice					
		5	5	6	6	7	7
New dice	1	B	B	B	B	B	B
	2	B	B	B	B	B	B
	3	B	B	B	B	B	B
	9	N	N	N	N	N	N
	10	N	N	N	N	N	N
	11	N	N	N	N	N	N

Page 23 What's the Chance?

ACTIVITY

1. Several are definite:

i. $^1/_2$

iv. $^1/_{26}$

v. 0. (India does not play netball at an international level.)

vi. 0. (The Prime Minister must be a New Zealand citizen.)

vii. 0, during your lifetime.

Other answers will vary greatly depending on personal circumstances and personal assessment.

2. Discussion will vary. In most cases, personal circumstances will determine that people come up with different outcomes, for example:

iii. Some will be keeping up frequent email correspondence with friends, and others won't. So probabilities could vary from close to 0 through to 1. Someone who does not have a computer will have 0 probability of getting email unless they access it through someone else's computer.

viii. Most will have a high probability of getting junk mail on any given day, perhaps 4 out of 5 (or $^4/_5$ or 0.8). If, however, they live in an out-of-the-way place, they may never get any, in which case the probability is 0.

ix. The assigning of probabilities to sports events is usually determined as much by personal loyalties and prejudices as anything!

x. Some mothers never buy lottery tickets, so they would have 0 probability of winning $1,000,000; others regularly buy tickets, in which case, the probability is marginally greater than 0!

xi. Some will bike every day (giving a probability of 1), when it is not wet (this probability may be $^9/_{10}$ or 0.9), and others never bike (in which case the probability is 0).

xii. If Grandma has already died before her 80th birthday, the probability of her reaching 80 is 0; if she is nearly 80 and in good health, the probability may be $^{19}/_{20}$ (or 0.95).

3. Events and placements will vary. Answers should be sensible.

Page 24 Game Show

ACTIVITY

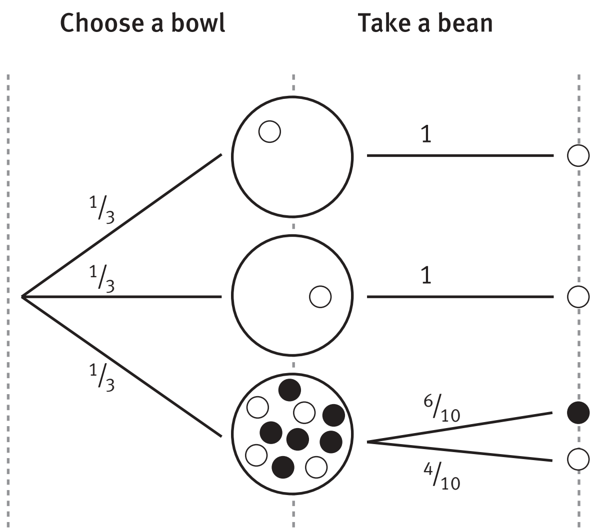
1. There is the same number of yellow beans as red ones.

2. Answers will vary. Two possibilities are 2Y 2R, 2Y 2R, 2Y 2R and 1Y 3R, 1Y 2R, 4Y 1R.

3. a. 1Y, 1Y, 4Y 6R

b. With the beans arranged in this way, Liang is certain to get a bowl with a yellow bean in it. If she chooses bowl A or B, she will get a yellow bean (because there is no other possibility). If she chooses bowl C, she has 4 chances in 10 of getting a yellow bean, so overall, her chances are 80%: $^1/_3 + {}^1/_3 + ({}^1/_3 \times {}^4/_{10})$ or $^5/_{15} + {}^5/_{15} + {}^2/_{15} = {}^{12}/_{15}$, which is $^4/_5$ or 80%.

Your tree diagram could look like this:



Teachers' Notes

Overview

Statistics: Book One

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Page 1

Monster Munch

Achievement Objectives

- collect appropriate data (Statistics, level 4)
- make statements about implications and possible actions consistent with the results of a statistical investigation (Statistics, level 4)
- find, and use with justification, a mathematical model as a problem-solving strategy (Mathematical Processes, problem solving, level 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)

Other mathematical ideas and processes

Students will also:

- use random numbers
- model a statistical procedure.

ACTIVITY

This activity introduces students to the concepts of randomness and mathematical modelling.

In statistics, "random" means that there is no pattern or reason behind a selection. In particular, it means that a person running an experiment or activity has no control over whom or what is selected. For example, Lotto numbers are selected randomly, using a machine built for the purpose and designed so that its operation is transparent. The balls can come out in any order, and participants usually have confidence that the process is fair and impartial. That is, they believe that every combination of numbers has the same chance of being selected.

Some students may think that making a random selection means choosing "a few from here and a few from there for no particular reason". If they use this approach, they can't be sure that their selection does not have an unintended bias. Randomness requires a system or method that *guarantees* that the selection is free from bias, including hidden bias.

Once the concept of randomness has been explored, the idea of random numbers follows naturally. Random numbers are numbers that occur in no particular order and with no pattern. No matter how carefully you study a sequence of random numbers, you cannot be sure what the next number in the sequence will be. Each number has the same chance of coming next.

Convenient sources of random numbers include:

- A calculator. Press the RAN# key. (Sometimes the shift key needs to be pressed first.) On a calculator, random numbers are normally displayed as decimals. If the students need a random number between 0 and 9, they should use the last digit; if they need a number between 0 and 99, they should use the last two digits.
- Car number plates. Use the last digit only for a random number between 0 and 9. Ignore personalised number plates.
- The phone book. Again, use the last digit for a random number between 0 and 9. The first three digits of a phone number will not be random because they usually denote an area.
- A computer spreadsheet program. Select Insert/Function/RAND, then use the cursor to drag the bottom right-hand corner of the active cell down the column to give a sequence of decimal numbers. Use the last digit in each cell as your random number.

You should discuss the concept of a mathematical model before the students attempt the activity.

The term "model" describes a piece of mathematics we use to imitate or replicate something that happens in the real world. A model can be a statistical experiment (like the one in the activity) or, for example, a table, an equation, or a graph.

In general, models simplify a real-life process by focusing on the important features only. For example, what happens when we drop a rock off a bridge? We can model its acceleration due to gravity using a formula that ignores the impact of wind resistance. In this activity, the students can explore how peanuts are distributed in a batch of cookies, without the ingredients, the facilities, or the mess.

Some students may find the instructions in question **2b** hard to follow, but the task itself is straightforward. Check that everyone understands that the random numbers relate to the cookie numbers, not to the number of peanuts.

It could be useful to get the students to record the results of their experiments on the board. They should notice very quickly that very few (if any) of the sets of results are identical. This can lead into a discussion of the fact that when sampling, each sample is likely to give a different result.

It is important that the students come to see that, when they are dependent on something that is random, they can never *guarantee* the outcome. Even if 1 000 peanuts were added to a batch of 10 monster brownies, this would not absolutely guarantee that there would be at least 15 peanuts in each!

Pages 2–3 Future Options

Achievement Objectives

- express quantities as fractions or percentages of a whole (Number, level 4)
- plan a statistical investigation arising from the consideration of an issue or an experiment of interest (Statistics, level 4)
- collect appropriate data (Statistics, level 4)
- choose and construct quality data displays (frequency tables, bar charts, and histograms) to communicate significant features in measurement data (Statistics, level 4)

Other mathematical ideas and processes

Students will also:

- read a table
- use a spreadsheet
- choose appropriate data displays
- calculate proportions.

ACTIVITY

This activity appears straightforward but may cause difficulties. In particular, the students need to understand fractions and be able to convert them to decimals and percentages if they are to mathematically predict option choices for the additional 50 students. They also need to know how to calculate what fraction of a number a quantity represents. This suggests that they should be at stage 7 (advanced multiplicative part–whole) of the Number Framework.

It is important to discuss with the students the characteristics of a quality data display. Essentially, it means that the key ideas can be seen at a glance, there is no undue loss of data, and the information is presented honestly (without distortion). A graph with axes must have:

- an explanatory title
- axes ruled at right angles

- suitable, honest scales, marked off in equal intervals
- labelled axes that identify the units used.

For other kinds of graph, the sectors must be labelled or a key provided.

Students using a computer graphing program should be strongly discouraged from choosing one of the numerous 3-D graph options. These look sophisticated, but they are hard to read, and they distort areas visually. A 2-D graph will almost always be a better choice.

Another useful concept that could be discussed with an able group is “data richness”. A data-rich graph is one that retains as much of the original information as possible (instead of merging or eliminating it) while displaying it with a minimum of ink (for printed graphs) or pixels (for on-screen graphs). A data-rich graph can be explored in detail from different angles and is able to tell a number of stories.

Your students could examine the types of graphs that are able to be drawn using a spreadsheet program or that are published in newspapers and magazines. Many will not meet the above criteria. They could make a wall display of such graphs, criticising their shortcomings.

Pages 4–5 What’s the Question?

Achievement Objectives

- plan a statistical investigation arising from the consideration of an issue or an experiment of interest (Statistics, level 4)
- collect appropriate data (Statistics, level 4)

ACTIVITY

This activity asks what makes a good survey question. Students could work on the task in groups. By pooling their different opinions, they will gain a better understanding of the issues involved in writing good questions. They are also more likely to be able to work out the meaning of the specialised language used in some of the examples.

When they have completed this activity, ask your students what they have learned. You may like to collate their ideas by writing on the board a list of things to remember when making up questions for a survey.

The list could include avoiding:

- excessively long questionnaires
- questions that need too much explaining
- long words, specialised language, and abbreviations that people may not understand
- emotive language
- unnecessary use of negatives, especially the use of double negatives
- statements that are vague or that can be understood in more than one way.

The students could look at the kinds of questions asked in surveys. Points for discussion could include:

- Do the questions require unstructured or structured responses? If unstructured, people can respond as they wish. If structured, people are given a number of choices and have to choose which one best fits with their situation or point of view. This kind of item will often also allow for an unplanned answer by including an option such as “other; please state …”. Structured responses are usually much easier to collate, write up, and display.
- What are the different ways in which responses can be structured? These include ticking the box, ranking on a scale, and putting a set of items in order of preference. What is each kind useful for? When thinking about this, the students should consider whether the survey is designed to gather facts or opinions.

- Are the questions intended for a small, well-defined group (such as the members of a church or a club), or are they intended for a much wider and more representative population?
- Do we need to trial surveys? The purpose of trialling is to check that:
  – the instructions make sense
  – the questions are in a logical order
  – the questions make sense
  – the categories for structured responses are sensible and cover the possibilities
  – the responses people make are useful and meet the purpose of the survey.

As an extension, the students could design and trial a questionnaire on a topic of their choice. This could be written up as a wall display, outlining the process used and the purpose of each of the questions.

Pages 6–7 Collect and Reflect

Achievement Objectives

- plan a statistical investigation arising from the consideration of an issue or an experiment of interest (Statistics, level 4)
- collect appropriate data (Statistics, level 4)
- choose and construct quality data displays (frequency tables, bar charts, and histograms) to communicate significant features in measurement data (Statistics, level 4)
- report the distinctive features (outliers, clusters, and shape of data distribution) of data displays (Statistics, level 4)
- make statements about implications and possible actions consistent with the results of a statistical investigation (Statistics, level 4)
- find, and authenticate by reference to appropriate displays, data measures such as mean, median, mode, inter-quartile range, and range (Statistics, level 5)

Other mathematical ideas and processes

Students will also use long-run relative frequency.

ACTIVITY

In this activity, students start with a question and then try to answer it by collecting and analysing appropriate data. Once they have chosen a question to investigate, they should closely follow the 4-step statistical process that is set out at the beginning of the activity. The process is the principal teaching point.

The activity illustrates the wide variety of questions that can be made the subject of a statistical investigation. Because each investigation is so different, each group will learn something different. For this reason, it will be useful for each group to report to the class on what they did and give their findings and conclusions.

The activity bridges curriculum levels 4 and 5. Before the students begin, you may need to revise earlier statistical work and make sure that they understand key vocabulary (for example, the words data, experiment, numeric, and survey).

Encourage the students to think about which kind of graph is the most suitable for their data. Frequency tables, bar charts, histograms, and time-series graphs are specifically mentioned in the curriculum's achievement objectives; pie graphs are also mentioned in the suggested learning experiences. Frequency polygons (line graphs, where the line forms a polygon with the *x* and *y* axes) are not mentioned in level 4, but an introduction to these will help the students to develop the skills they need for level 5.

Here are some questions you could ask your students:

- What sort of data will you get in this investigation?
  - Category data is data collected by category (for example, the colour of passing cars).
  - Discrete data is numeric, whole-number data (data that is obtained by counting, not measuring).
  - Continuous data is numeric data that comes from measurement and is limited only by the accuracy of the measuring device used.
  - Time-series data is numeric data that is collected at regular intervals over a period of time. Time-series data may be either discrete or continuous.

  Discrete numeric data is introduced as a concept in level 3; continuous numeric data is introduced in level 4.

- What is the best way to record data as you collect it?
  - A list is appropriate where the range of possible results is great, such as the number of pages in a book, or for time-series data, where the order of the data needs to be retained.
  - A stem-and-leaf plot is useful if the data consists of discrete numbers drawn from a limited range, for example, scores out of 50 in a maths test or measurements of arm span.
  - A tally chart is useful if the data has few categories, for example, the number obtained when throwing a dice.

- Do you need to group your data before displaying it?

  Continuous data always needs to be grouped, and discrete data needs to be grouped unless the data range is very limited or there are only a few categories. Intervals of 5 centimetres may be appropriate for arm span, intervals of 50 pages for a book, and intervals of 5 for a maths test scored out of 50. Encourage the students to use between 5 and 10 intervals of equal size, where possible, when grouping their data. This will usually allow trends to be seen clearly without sacrificing too much detail.

  The students may like to try grouping the data in several different ways (with different-sized intervals or intervals that start at different points) to see how the display changes. Sometimes the choice of intervals can obscure or reveal a message. This activity could be a valuable whole-class learning experience and may illustrate the fact that data can be manipulated to fit with a particular agenda.

- What sort of data display is appropriate to the data you have collected?
  - A bar chart is used for discrete data or data collected in categories. It shows the frequency of each category. The bars do not touch each other.
  - A histogram is used for continuous data. It shows the frequency of data within each interval. The bars touch each other.
  - A frequency polygon is used for either discrete or continuous data, and it shows trends.
  - A pie graph is used for discrete, continuous, or category data. It shows what fraction of the whole is occupied by each category.
  - A time-series graph is used for data that has been gathered over time. It shows how something changes as time goes by.
  - A stem-and-leaf plot is used for discrete or continuous data. It is similar to a bar graph, but it is on its side. It is useful for working out the median and quartiles and as a preparation for drawing other graphs.

- What has the graph told you?

  Look at the *distribution* (shape) of your set of results. Consider outliers (one-off, unusual results) and clusters (groups) within the distribution. When dealing with line graphs or time-series graphs, consider any seasonal and long-term trends.

The students should be able to make several comments about what their graphs show. For example:

- "Most of the data lies between 5 and 8."
- "Red is the most common colour."
- "One result is out on its own. The cause could be …"
- "Nobody scored over 15."
- "The boys did better than the girls."

- What statistics would be useful here?

  Although suggested learning experiences for level 4 include finding the mode and estimating the mean and median, statistical calculations are not officially introduced until level 5 of the curriculum. Calculations could therefore be used as an extension activity.

Comments specific to the student investigations

i. This investigation is based on the concept of long-run relative frequency (the outcome that is expected over time but that may not be obvious in the short term). For the results to be meaningful, that is, to show any trend, at least 50 trials (throws) will be needed.

ii. For each individual student, 2 or 3 timings could be taken and then averaged.

iii. The best approach is for the students to measure a number of paces and then average the results.

iv. This activity should involve the students surveying a range of people. The results may be hard to display, especially if some of the numbers come from very long streets or if the students live in rural areas.

v. Lower-ability students could find the measurement needed here difficult. Once they have made the measurements, they need to be able to convert them from millilitres to centimetres cubed. The curriculum suggests that this kind of conversion is a level 5 learning experience. The students should understand the meaning of such conversions, especially if they are giving feedback to the class later on.

vi. Remind your students not to strip trees bare in their quest for leaves! Also, encourage them to measure whole, full-grown leaves rather than new, damaged, or eaten ones.

vii. If the books are arranged alphabetically by author, the students could find the number of pages of, say, every fifth and tenth book in the A author section, the B author section, and so on.

viii. This is another task that requires an averaging strategy.

Other suggested sources of data for investigations:

- test scores
- the number of people in a household
- the size of hand spans or arm spans
- heights
- time spent viewing television or playing computer games
- temperature
- barometric pressure.

Pages 8–9 Surf Stats

Achievement Objectives

- collect and display time-series data (Statistics, level 4)
- report the distinctive features (outliers, clusters, and shape of data distribution) of data displays (Statistics, level 4)

Other mathematical ideas and processes

Students will also read and interpret data displays.

ACTIVITY

If possible, the students should create the line graph asked for in question 1 using a computer spreadsheet and graphing program. They could enter the data in a single column and produce a single line graph for the 2 years, but if they enter the data in 2 columns (as below), they can create a graph that shows the 2 years as separate lines. This will make their comparisons easier.

	A	B	C
1	Month	Year 1	Year 2
2	June	4	3
3	July	0	1
4	August	3	0
5	September	0	1
6	October	1	8
7	November	8	4
8	December	17	11
9	January	38	24
10	February	26	18
11	March	9	7
12	April	15	12
13	May	4	3

Highlight (select) the cells that contain the data and choose the scatter graph option. This will give you a double line graph with the data points between the tick marks on the horizontal axis. You can correct this by following these steps:

- Right-click on any of the labels on the horizontal axis (on a Macintosh, hold down the control key and click).
- Choose the Format option.
- Click on the Scale tab.
- Remove the tick next to "Value (Y) axis crosses between categories".

If you want help with computer spreadsheets and graphs, see the introduction to the teachers' notes for *Statistics: Book Two*, Figure It Out, Years 7–8.

When they have completed questions **1** and **2**, it may be appropriate to get the students to design their own investigation involving the collection of time-series data. This could mean recording the temperature at every hour throughout the day, taking the evening NZSX 50 index, or recording the number of cars travelling past a particular point during every tenth minute. Some data collected over a short period of time (for example, the temperature) will show a trend. Other data (for example, the NZSX 50 index) will show a trend only if studied over a long period of time.

The graphs given in questions **3** and **4** are useful ones for the students to read and comment on. You could increase the difficulty of the maths by asking your students to calculate fractions and/or percentages and to make comments based on these. You could also ask more demanding questions, such as "How many of the rescues in the Northern Region are likely to have used an inflatable rescue boat?"

Question **3** asks the students to describe what the graph shows. It is important that they do not make unsubstantiated guesses about causes and treat them as facts. When they describe what the graph shows, encourage them to restrict their comments to statements of fact that can be read or calculated from the graph.

Pages 10–11 Discipline Dilemmas

Achievement Objectives

- plan a statistical investigation arising from the consideration of an issue or an experiment of interest (Statistics, level 4)
- collect appropriate data (Statistics, level 4)
- evaluate others' interpretations of data displays (Statistics, level 4)

ACTIVITY

This activity further develops students' understanding of the process of designing a survey and builds on the idea that the way a question is asked can influence the outcome. In this case, the categories that are offered control the outcome.

The subject matter of this survey requires sensitive treatment. The parents of your students may have strongly held views at either end of the continuum, including the view that any and all smacking is child abuse. Some of the students may have experienced beatings in the home and assume that this is what advocates of the "right to smack" think is acceptable.

An important lesson that could be drawn from this activity is that when views are being canvassed on emotive issues, the issues need to be clearly defined or the results of the survey will have very little statistical value. A key issue in this activity is the meaning of the term "smacking". Because it means different things to different people, it is not at all clear what definition any surveyed individual is responding to. This means that the data gathered will not contribute much to the general debate surrounding this subject.

Whenever results from poorly designed surveys are used to "prove" anything, statistics have been misused. Encourage your students to ask questions whenever they encounter statistics, such as:

- "Where is this data from?"
- "How large was the sample?"
- "How was the data collected?"
- "What was the intention of the person collecting the data?"

Pages 12–13 Down the Plughole

Achievement Objectives

- plan a statistical investigation arising from the consideration of an issue or an experiment of interest (Statistics, level 4)
- collect appropriate data (Statistics, level 4)
- choose and construct quality data displays (frequency tables, bar charts, and histograms) to communicate significant features in measurement data (Statistics, level 4)
- report the distinctive features (outliers, clusters, and shape of data distribution) of data displays (Statistics, level 4)
- make statements about implications and possible actions consistent with the results of a statistical investigation (Statistics, level 4)

Other mathematical ideas and processes

Students will also:

- complete a data table
- make metric conversions
- work with fractions, decimals, and percentages
- work with time and do time conversions.

ACTIVITY

This activity integrates work from a number of curriculum strands. Your students will need to have good multiplication skills in order to attempt it and will need to be able to work with time, fractions, decimals, and percentages. This means they should at least be at stage 7 (advanced multiplicative part–whole) of the Number Framework. They should also have the skills to draw a range of quality data displays and be able to identify which graph type is appropriate to the data. For an explanation of what is meant by quality graphing, see the notes for Future Options (pages **2–3** of the students' book), and for more on choosing appropriate graph types, see the notes for Collect and Reflect (pages **6–7** of the students' book).

The table in the activity has a large number of blank cells, and the students should only fill out those that help them with their calculations. For example, "average times per day" is meaningless when it comes to "topping up pool". This is a water use that may happen once every 1 or 2 weeks in summer and never in winter.

Point out to your students that, when they are writing their paragraph on "Where does the water go?", they are to base their comments on the information from the table and graph.

There are a number of very useful websites related to water usage and water conservation, and local authorities usually have printed information that they are happy to give students who are doing a related project.

The investigation challenges students to work out the water and heating costs of the family's daily showers. Working out the cost of heating the water will be a major challenge if done experimentally. An alternative approach is to use averaged data from a media source, for example, *Consumer* magazine (June 2003), and apply this information to an individual household.

Pages 14–15 Mad Minute

Achievement Objectives

- collect and display time-series data (Statistics, level 4)
- make statements about implications and possible actions consistent with the results of a statistical investigation (Statistics, level 4)
- make statements about time-related variation as a result of a statistical investigation (Statistics, level 5)

Other mathematical ideas and processes

Students may also investigate the Fibonacci sequence.

ACTIVITY

Before setting this task, see the comment in the notes for Surf Stats (pages **8–9** of the students' book) on the graphing of time-series data.

It may be worth playing Mad Minute with your class several times so that they understand the concept and can collect some local data for analysis.

This activity will be most effective when followed by a discussion of the answers that your students write for the questions. The goal is to develop a common language for interpreting and describing time-series data. Students working at levels 4 and 5 of the curriculum should be understanding and using terms such as trend, plateau, and peak.

As an extension, you could get your students to try Mad Minute starting with the numbers 1, 1 and then ask them if they recognise the sequence they have just written down. This sequence, 1, 1, 2, 3, 5, 8, 13, 25 ..., is known as the Fibonacci sequence. Interested students could research this fascinating sequence using the Internet. They could then report their findings to the whole class.

Page 16 Channel Surfing

Achievement Objectives

- plan a statistical investigation arising from the consideration of an issue or an experiment of interest (Statistics, level 4)
- collect appropriate data (Statistics, level 4)
- choose and construct quality data displays (frequency tables, bar charts, and histograms) to communicate significant features in measurement data (Statistics, level 4)
- report the distinctive features (outliers, clusters, and shape of data distribution) of data displays (Statistics, level 4)
- make statements about implications and possible actions consistent with the results of a statistical investigation (Statistics, level 4)

Other mathematical ideas and processes

Students will also use percentages.

ACTIVITY

This activity can be used to build on the work done in What's the Question? and Discipline Dilemmas (pages **4–5** and **10–11** of the students' book). If you have not already attempted these activities, review the discussion on them in these notes.

This activity introduces the students to the concepts of a *population* (an entire group) and a *sample* (part of the population). Both are very important statistical concepts.

If a population is small, it may be realistic to survey everyone. Occasionally, everyone in a large population is surveyed (as in a census, general election, or a referendum), but the costs of doing this are so great that such occasions are rare. Generally, statisticians use the less complex and less costly alternative of surveying a sample. As long as the sample is selected so that it is genuinely representative of the population, surveying those in the sample should give a result that is very close to the result that would be obtained by surveying everyone.

The challenge is to ensure that those in the sample have the same characteristics, in the same proportion, as those in the population. This will normally mean that characteristics such as the gender mix and age mix of the sample closely match those of the population.

In theory, when a sample is being selected, the method used should give every member of the chosen population an equal chance of being chosen. This ensures that the views of the sample accurately reflect those of the population. If surveying a local town or suburb, one strategy would be to survey every house with a street number that is a multiple of 10. This is called a systematic sample. By contrast, a survey in which people are stopped on the street uses an informal sample; the only people surveyed are those who are easy to locate. Statistically, this is a haphazard way of finding out the views of the population because people who are in the same place at the same time often have a common characteristic that causes them to be there or means that they are able to be there.

A further consideration is the "method of contact", that is, the way those selected for the sample will be contacted. Face-to-face surveys are more likely to get a response than faxes or emails. Response rates are very important for the validity of the statistics collected. Those who do not answer a survey are known as non-respondents. If a lot of people in the sample do not respond to the survey, any conclusions reached are likely to be invalid because they will not genuinely reflect the views of the population as a whole. People who do not respond tend to fall into categories (for example, busy businesspeople). As a result, the survey fails to uncover the views of a whole segment of the sample, and therefore of the population. In practice, it is very hard to get a high level of response to a survey, and this is one reason why the results of any survey have a "margin of error".

These important ideas can be introduced to students studying at this level, but their treatment should be kept simple. The goals at this stage are to get the students to recognise the difficulties involved in conducting surveys and to develop in them a healthy scepticism for the results of any survey. They will revisit these concepts if they continue to study statistics in senior high school.

When developing their surveys, your students need to note that the more questions they ask, the more work there will be in collating, sorting, displaying, and interpreting their data. A group of able students may take 2 weeks to complete question **3**. Students working alone will take longer because the data-collection phase is time consuming. Students working in groups are likely to get much more out of the task because they will debate the issues while sharing the workload.

Page 17 Family Feast

Achievement Objective

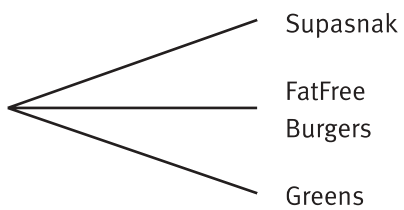
- find all possible outcomes for a sequence of events, using tree diagrams (Statistics, level 4)

ACTIVITY

This is a straightforward activity that students could do simply by listing all the possible outcomes. However, it is important that they learn to be systematic so they do not miss any of the options. One of the best ways is to use a tree diagram. Of the three activities in this book suited to the use of tree diagrams, this is the simplest, so it is best tackled before Catch of the Match and Game Show (pages **18** and **24** of the students' book).

Although tree diagrams are a useful tool, students often do not understand how to draw them or interpret them. The following notes explain a number of important points.

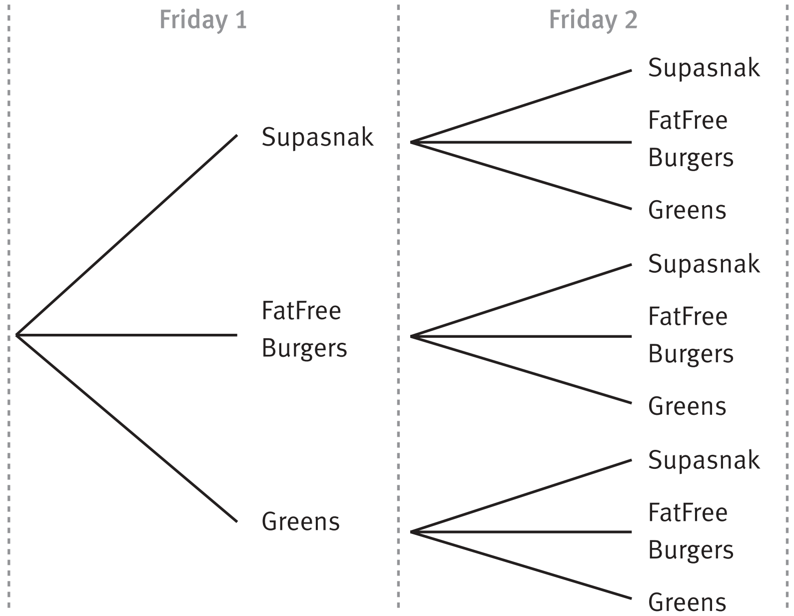
Branches begin at a common point on the left-hand side of the page. The number of branches depends on the number of options. In the following scenario, a family is deciding which of the three local restaurants to go to for a meal:



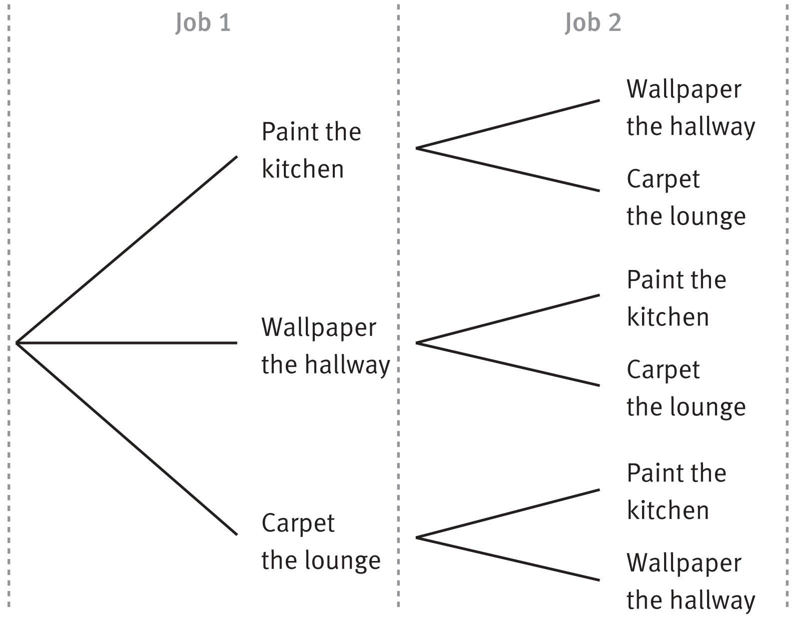
The various outcomes should be lined up vertically. (The vertical lines in the examples below, and in some of the diagrams in the Answers, are only an aid to tidy construction.)

If there is more than one decision to be made, there will be another set of branches to the right of the first.

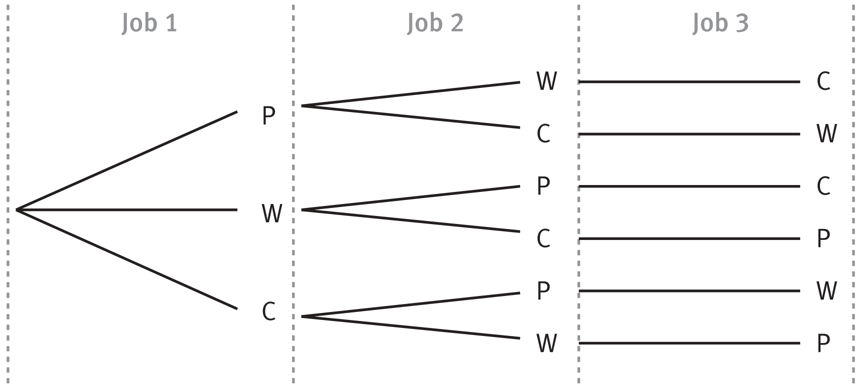
For example, this family decides that they can afford to eat out two Friday nights in a row. Their options can be set out in this way:



Often the second option eliminates whatever choice was made the first time around. In the following scenario, a couple thinks about the order in which they should do major jobs around their home. Clearly, once they have painted the kitchen (for example) they can tick that off their list:



To reduce the clutter in diagrams, it is common to abbreviate the names of the options or outcomes. So the above scenario, including job 3, becomes:



Note that it is much easier to construct a tree diagram using a computer drawing program than it is to draw one tidily by hand. Diagrams usually involve a lot of repetition, and copying, pasting, and grouping reduce the work involved. Also, using a computer, it is very easy to space branches evenly and to line them up vertically.

If they look *down* the last column of any tree diagram, the students can see how many branches there are by the number of ends or terminations. If they read *along* each branch from left to right, they can see what each branch represents. For example, the branch that reads WCP represents the outcome in which wallpapering is followed by carpeting, then painting.

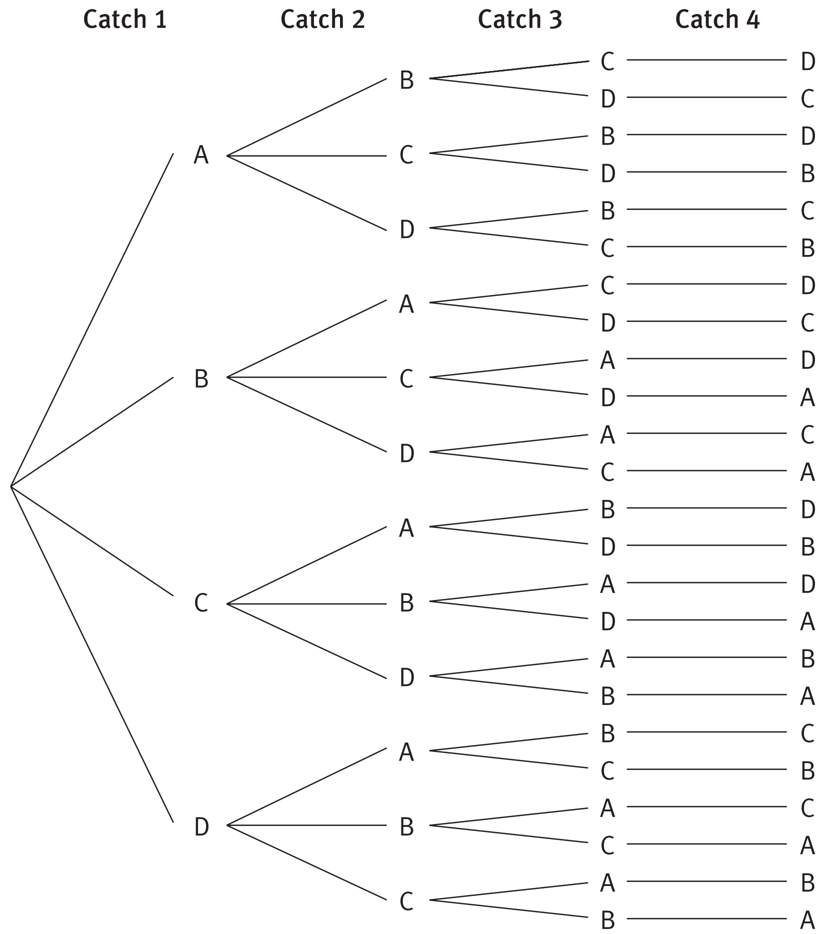
Page 18 Catch of the Match

Achievement Objective

- find all the possible outcomes for a sequence of events, using tree diagrams (Statistics, level 4)

ACTIVITY

Give the students the opportunity to sort this activity out for themselves. Most will choose to list all the possible outcomes for the scenario in question **1**, but if they have already met tree diagrams, they may want to use this method as an alternative. Here is a tree diagram that shows the number of ways in which 4 catches can be ordered:



For a detailed discussion on drawing tree diagrams, see the notes for Family Feast (page **17** of the students' book). See also the tree diagrams in the Answers.

Questions **2** and **4** require the students to find out how the number of permutations (arrangements in which order matters) can be calculated. The numbers involved quickly become too great for a list or a tree diagram to be practicable.

If there are 4 things (such as objects, actions, or people) to be put in order, the number of possible arrangements is 4 x 3 x 2 x 1 = 36. If there are 5 things, the number of possible arrangements becomes 5 x 4 x 3 x 2 x 1 = 120. The multiplier decreases by 1 each time because there is 1 fewer thing available for selection (since it has already been "taken").

Note that the number of permutations of 5 things can be written: 5! = 120. Likewise, 6! = 720. 6! is read "6 factorial". Some students may have noticed that scientific calculators have a factorial key. If they experiment with this key, they will discover that the number of permutations increases rapidly every time a further choice is added. In fact, they will discover that the number of permutations quickly exhausts the capacity of their calculator.

Page 19 Across the River

Achievement Objectives

- use a systematic approach to count a set of possible outcomes (Statistics, level 3)
- find all possible outcomes for a sequence of events, using tree diagrams (Statistics, level 4)
- determine the theoretical probabilities of the outcomes of an event such as the rolling of a die or drawing a card from a deck (Statistics, level 5)

Other mathematical ideas and processes

Students will also put fractions (with a common denominator) in order of size.

Game and Activity

Students can play this game even if they have little understanding of probability. If observant, they will discover that the different totals that can be obtained from throwing two dice are not equally likely. You could discuss why this might be the case. The students can then use either the table suggested in question **2** or a tree diagram as suggested by the curriculum to provide an answer to the question.

Even students who understand some probability can have trouble seeing that there are 36 different ways to get the totals 2 to 12. Discuss this, along with the idea that there is more than one way to get a total such as 7. You could point out that if you get a 1 on the first roll, you can get the desired total (7) by getting a 6 on the second roll. If you get a 2 on the first roll, it could be followed by a 5 on the second. A 3 could be followed by a 4, or a 4 by a 3. In fact, no matter what you get on the first roll, you have a chance of getting a total of 7 from the two rolls.

Once these facts have been established, you can ask, "What is the probability of getting a total of 7?" The answer can be recorded as a statement (6 out of 36) or as a fraction ($^6/_{36}$ or $^1/_6$).

In question **3**, the students will first need to realise that they must get each total (1–12) the same number of times. Because there are 36 ways to get a total and there are 12 different totals, each one will have to occur 3 times in a table. Even when they have realised this, they are unlikely to find a solution simply by trial and improvement.

The notes for Dodgy Dice (page **22** of the students' book) explain that the probability exemplars give a different view of what students can be expected to understand about probability compared with the curriculum document or the NCEA level 1 achievement standards.

Pages 20–21 Wallowing Whales

Achievement Objectives

- use a systematic approach to count a set of possible outcomes (Statistics, level 3)
- predict the outcome of a simple probability experiment, test it, and explain the results (Statistics, level 5)
- determine the theoretical probabilities of the outcomes of an event such as the rolling of a die or drawing a card from a deck (Statistics, level 5)

Other mathematical ideas and processes

Students will also:

- learn about long-run relative frequency
- use a table to list all possible outcomes for a sequence of two events
- consider simple games that use dice.

GAME

Students can play this game with little initial support. They will soon notice that certain numbers come up with greater frequency than others and will alter their strategy to reflect this. When they have played the game for 20–30 minutes, stop the play and discuss with your students what they have discovered.

Once they are aware that there may be better strategies, they can examine the theoretical probabilities behind the game.

ACTIVITY

Note that this activity should be set before Dodgy Dice (page **22** of the students' book), which requires a more formal understanding of the same ideas.

Question **2a** asks the students to complete a difference table. Along with tree diagrams, tables are a useful tool for helping them to see that, in certain situations, some outcomes are more likely than others. Tables allow students to see at a glance how many different ways there are of getting each outcome. The more ways there are, the greater the probability is. Because tables are 2-D, they have the limitation that they can only be used for 2-step events (for example, a dice rolled twice).

Question **2b** asks for a bar graph showing the frequency of each of the differences. If your students are using computers for this task, they may have difficulty getting the computer to label the horizontal axis correctly. If this is the case, they should follow these steps:

- Choose Chart from the menu bar.
- Select Source Data.
- Click on the Series tab.
- Click the cursor in the panel that says "Category (X) axis labels".
- Go to the spreadsheet and highlight the cells with the correct labels in them (0–5).
- Click on OK.

Question **2c** asks the students to turn the frequencies they have found in question **2b** into probabilities. Suggest that they think in terms of "chances out of 36" and then write their answers as fractions. For more discussion on this, see the notes for What's the Chance? (page **23** of the students' book).

Question **3b** is another good opportunity for developing the concept of long-run relative frequency. Discuss with your students how they might "prove" their strategy is better. Some will want to prove it by playing the game according to that strategy. In this case, ask them, "What will it prove if you win a game? Does it mean you will also win the next game? Will you win every game you play using this strategy? If not, how often are you likely to win? How could you work this out?" Students may want to play a lot of games to see what happens. At some point you can ask, "Is there a quicker way of working out how often you should win?" This may encourage students to think about the underlying probabilities.

Page 22 Dodgy Dice

Achievement Objectives

- determine probabilities of events based on observations of long-run relative frequency (Statistics, level 5)
- determine the theoretical probabilities of the outcomes of an event such as the rolling of a die or drawing a card from a deck (Statistics, level 5)
- predict the outcome of a simple probability experiment, test it, and explain the results (Statistics, level 5)

Other mathematical ideas and processes

Students will also learn to express probability as a fraction.

ACTIVITY

This activity reflects level 4 of the mathematics exemplars rather than the curriculum document. In the exemplars, students not only use fractions to record probabilities but deal with more sophisticated investigations by systematically counting all outcomes or using tree diagrams and then assigning numerical probabilities on the basis of this count. Entering the results into a table is a simple and logical development of these level 4 skills. See www.tki.org.nz/r/assessment/exemplars/maths/stats_probability/sp_4a_e.php

Unfortunately, many students do not have these skills and are especially weak on fraction concepts. If this is true of your students, you will need to work on these skills before assigning this activity. You should also revise the language of probability.

Dodgy Dice is best used following Wallowing Whales (pages **20–21** of the students' book), which introduces the idea of long-run relative frequency. This useful concept means that if you do not know the probability of something happening, you can work it out (experimentally) by completing a large number of trials and then using this formula:

$$\text{Experimental probability of an event} = \frac{\text{number of times the event happens}}{\text{number of trials}}.$$

Note that an *event* is something you want to happen (like getting an odd number when throwing a dice or getting blue to win). A *trial* is an attempt to get the desired outcome. You will need to explain these words if the formula is to make sense to the students.

At first, when the students play with the dodgy dice, one colour will win, then the other, with no discernible pattern. But as the number of trials increases, a trend will emerge in which one colour is dominant. After 50 trials, the students should have a tentative experimental probability, and after 100 trials, they should have a result that is unlikely to change much with further trialling. If a group of students is playing simultaneous games, recording the results on a master table will quickly give enough trials to provide a very accurate result for the experimental probability.

If the students have already met probabilities as fractions, they can make use of the following formula, which looks very similar to the one above but uses numbers that can be precisely determined (rather than obtained from trials):

$$\text{Theoretical probability of an event} = \frac{\text{number of ways the event can happen}}{\text{sample space}}.$$

The *sample space* is the complete set of all possible outcomes.

The tables in the Answers are the basis on which the theoretical probability of success for each dice is calculated.

Page 23 What's the Chance?

Achievement Objectives

- estimate the relative frequencies of events and mark them on a scale (Statistics, level 4)
- determine the theoretical probabilities of the outcomes of an event such as the rolling of a die or drawing a card from a deck (Statistics, level 5)

Other mathematical ideas and processes

Students will also:

- express probability as a number between 0 and 1
- place fractions, decimals, and percentages on a number line
- convert fractions into their decimal and percentage forms
- use ratios to express probability.

ACTIVITY

This activity establishes the key idea that probability can always be expressed as a number between 0 and 1. It also provides a useful context for learning how to place fractions and decimals on a number line.

The mathematical context of this activity is appropriate for students who are at stage 7 (advanced multiplicative part–whole) or 8 (advanced proportional part–whole) of the Number Framework. It provides a good follow-up to an introductory discussion on the concepts and language of probability.

An introductory discussion could begin with a question such as "What is the chance that someone will one day walk on the Sun?" The purpose of the question is to explore ways of describing a situation that has no chance of eventuating.

Hopefully, someone will suggest "zero" or "nil"; if not, you may need to prompt them. A follow-up question could be "Is it possible to find an event with a smaller chance (or probability) of happening than this?" Once you have established that if something is impossible, it has a probability of zero, the label "0" can be attached to the far left end of a line on the board or a length of rope on the floor. The line or rope represents a continuum. If you wish, the standard notation can be introduced at this point:

P (someone will walk on the Sun) = 0.

This is read as "The probability that someone will walk on the Sun is zero."

The second challenge is to establish that if something is inevitable, it has a probability of 1. A suitable starter question could be "What is the chance that the Sun will come up tomorrow?" This should lead to the label 1 being placed at the far right end of the continuum represented by the line or rope.

Once the extremes of 0 and 1 are established, explore the meaning of the space between them. At first, you could get your students to judge whether an event is impossible, possible, or certain. Each event can be given a label and placed either at the appropriate extreme or in-between.

"What is the chance you will eat potatoes tonight?" Apart from opening up a discussion on family and cultural differences, a question such as this will generate responses like "not very high" that can then be quantified. A family may eat this vegetable 3 times a week, so the chance of potatoes on any particular night becomes "3 out of 7" (unless the family has a rigid weekly menu, in which case the probability on any particular night may be 0 or 1!).

It is also important to discuss events where the probability is evenly balanced or nearly so. A suitable question could be "What is the chance that the next child born in our town will be a girl?"

This example could lead to the idea that there may be ways of calculating probability, at least in some circumstances. It could also be used to introduce the idea that a probability can be written as a fraction, leading to the understanding that "one out of two" can also be written as $^1/_2$.

Note that, because this is a different conceptual use for fractions, it is important not to assume that your students will automatically see that words like "3 out of 7" can be converted to a fraction, or that they can interpret a fraction written in the form $^3/_7$ as a probability. Explicit teaching is needed.

Bear in mind that research shows that students often have difficulty using number lines. While they are an essential tool, do not assume that a student can read them or place numbers on them. This is especially the case when working with decimals or fractions.

Page 24 Game Show

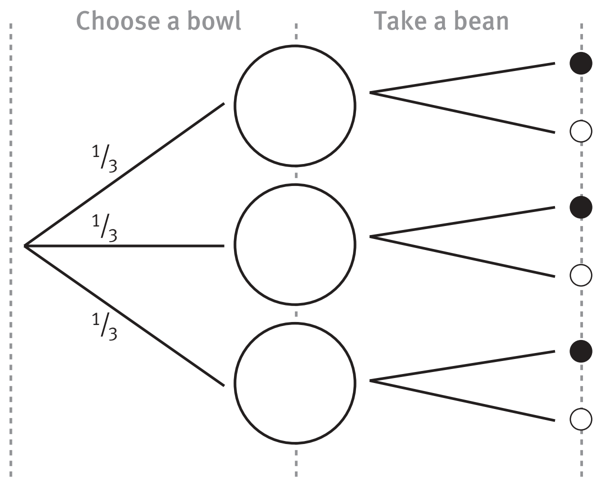
Achievement Objectives

- find all possible outcomes for a sequence of events, using tree diagrams (Statistics, level 4)
- determine the theoretical probabilities of the outcomes of an event such as the rolling of a die or drawing a card from a deck (Statistics, level 5)

ACTIVITY

All students should be able to attempt this activity, especially if they work in pairs. The instructions are brief and use no technical words. For a bit of drama, you may like to introduce it from the front of the classroom, using actual bowls and beans.

By now, students will have had practice with tree diagrams, but they may have trouble working out how to use a tree in this rather different context. The key is for them to recognise that, once they have sorted the beans into bowls, they have two choices (first the bowl, then a bean), so they will need a tree that has 2 sets of branches. The basic shape will be like this:



Question **3b** involves both the multiplication and addition of fractions and moves the problem on to a different level of understanding and skill.

Another game show problem worth exploring is the "Monty Hall paradox", which asks whether contestants can improve their chances of winning a game show by changing their choice at the last moment. This problem caused major arguments among mathematicians when it was first discussed, along with some very red faces. An Internet-based research project could be set, with students acting out the game show and explaining what they have discovered. Simulations for the problem are also available. Students could begin by typing Monty Hall into an Internet search engine. There are a number of good sites to choose from, including some that are accessible for an interested student.

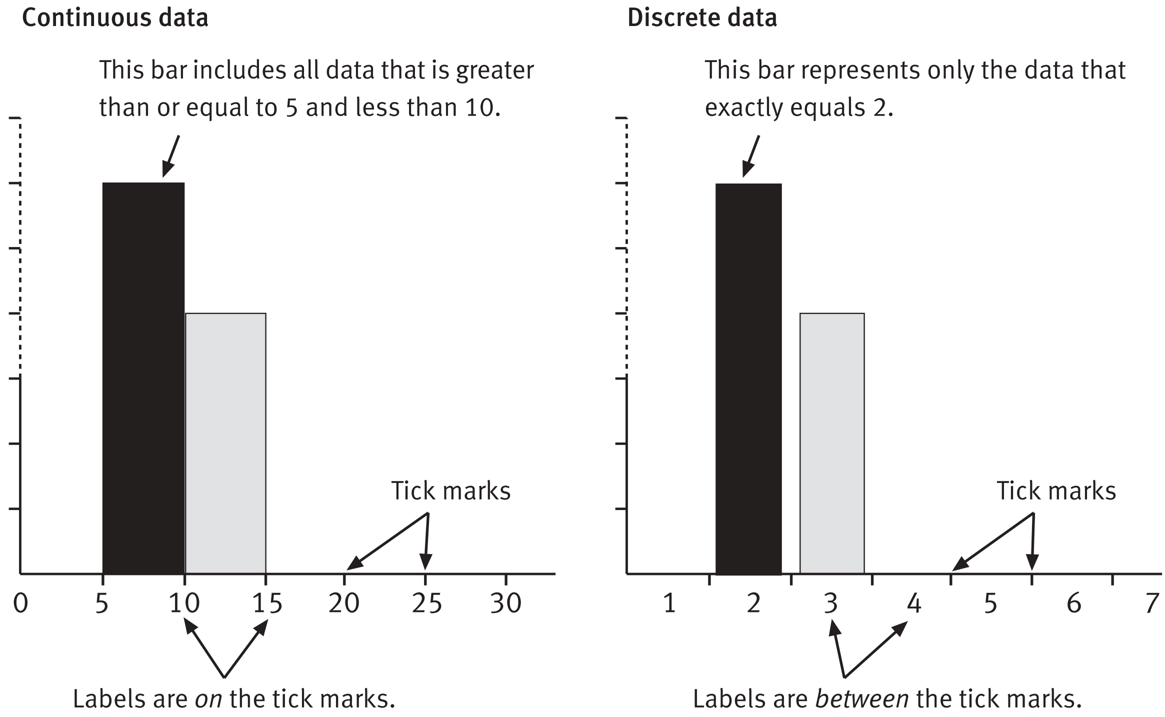
Appendix Should axes be labelled on the lines or between them?

Many students are thoroughly confused about where to write the labels on the horizontal axis of a graph. Do they put them on the lines (tick marks) or between the lines?

Some of this confusion comes from the different models students experience when they are learning number, but the protocols for different types of graph also vary. As far as students in years 7–8 are concerned, it will be sufficient if they can distinguish between discrete and continuous data and can remember this guideline:

- When data is in categories or discrete, the labels are placed centrally beneath the appropriate bar of a histogram, *between* the tick marks.
- When data is continuous, the labels on the axes are placed *on* the tick marks.

This diagram summarises these key ideas:



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