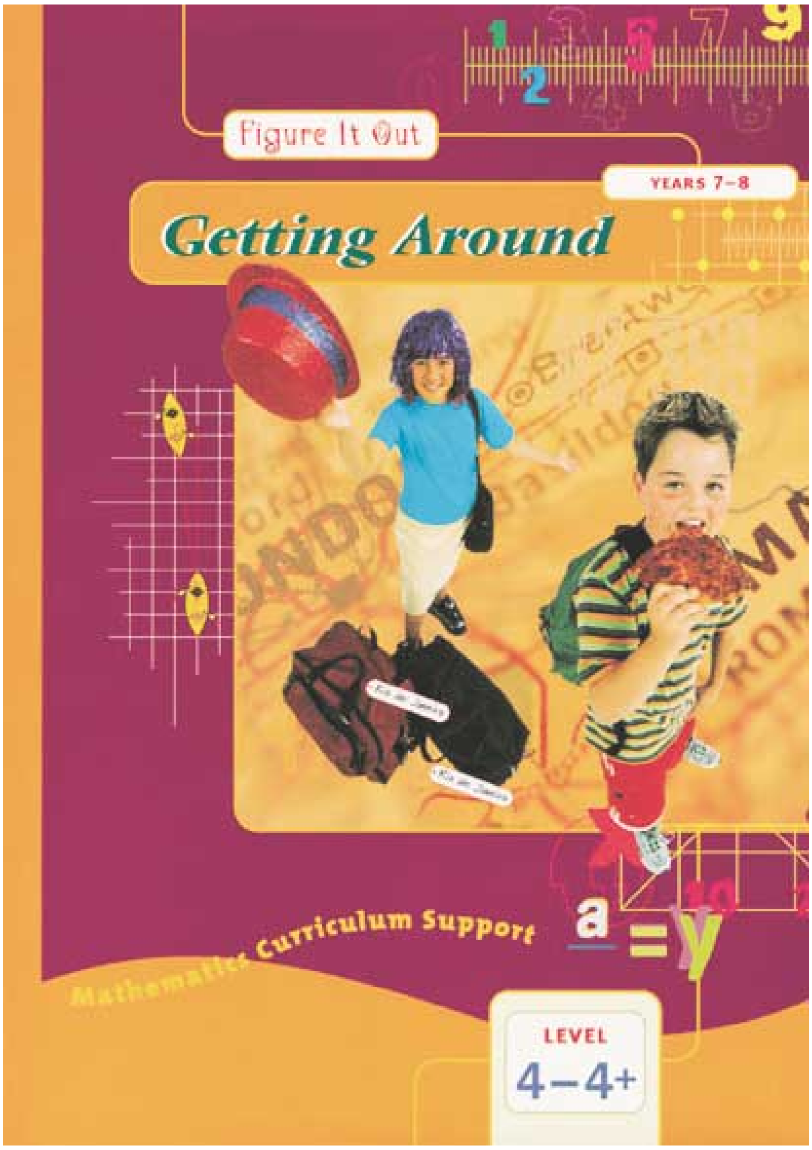
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

CONTENTS

The books for years 7–8 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand year 7–8 classrooms. The books have been developed and trialled by classroom teachers and mathematics educators and follow on from the successful series for levels 2–4 in primary schools.

Student books

The student books in the series are divided into three curriculum levels: levels 2–3 (linking material), level 4, and level 4+ (extension material). All the books are aimed at year 7–8 students in terms of context and presentation.

The following books are included in the series:
Number (two linking, three level 4, one level 4+, distributed in November 2002)
Number Sense (one linking, one level 4, distributed in April 2003)
Algebra (one linking, two level 4, one level 4+, distributed in August 2003)
Geometry (one level 4, one level 4+, distributed in term 1 2004)
Measurement (one level 4, one level 4+, distributed in term 1 2004)
Statistics (one level 4, one level 4+ distributed in term 1 2004)
Themes: *Disasters Strike!, Getting Around,* (level 4, distributed in August 2003)

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and life experiences that are meaningful to students aged 11–13 years. The activities can be used as the focus for teacher-led lessons, as independent bookwork, or as the catalyst for problem solving in groups.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the student books includes full answers and explanatory notes. Students can use them for self-marking, or you can use them for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, comment on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/r/maths/curriculum/figure

Using Figure It Out in your classroom

Where applicable, each page starts with a list of equipment that the students will need to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum*, page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Answers

Theme: Getting Around

Page 1 Up and Off

ACTIVITY

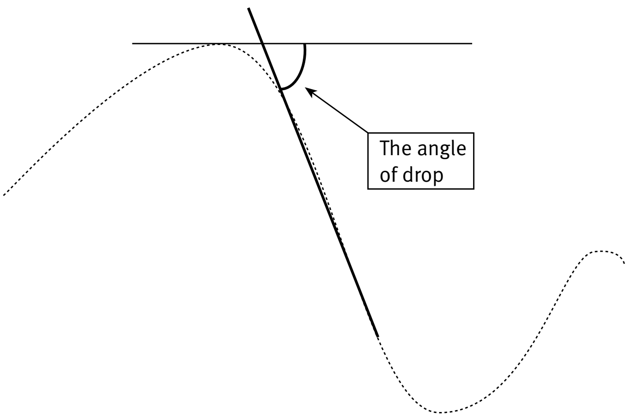
	ATR	Saab	737	767	Beechcraft
1.	200 km	176 km	306 km	328 km	196 km
2.	1 hr 17 min	1 hr 27 min	50 min	47 min	1 hr 18 min
3.	500 km/h	440 km/h	765 km/h	820 km/h	490 km/h

(Times have been rounded to the nearest min, distances to the nearest km, and speeds to the nearest km/h. Given information is shaded.)

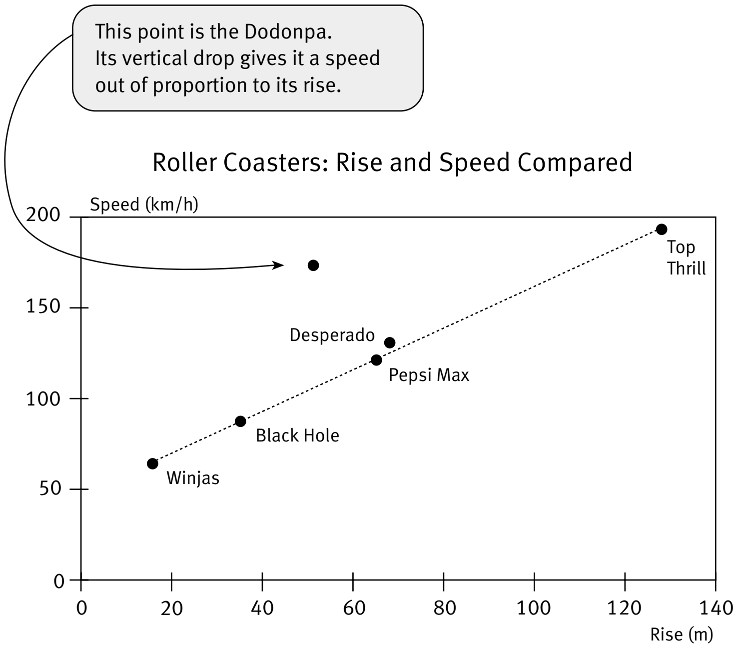
Pages 2–3 Roller Coasting

ACTIVITY

1. Your diagram should look something like this:

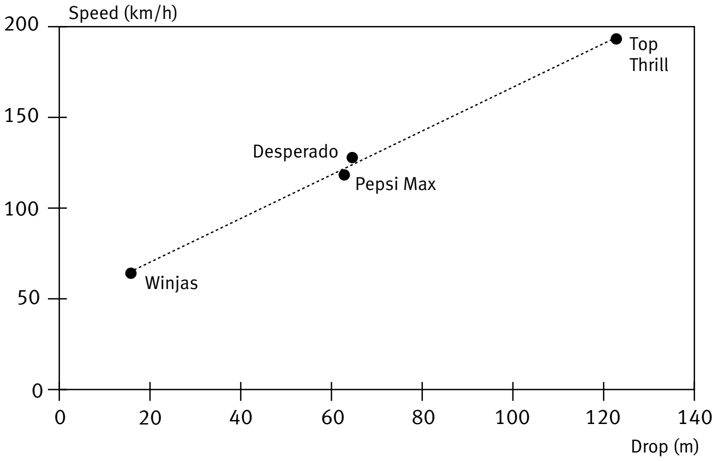


2. Rise, length of drop, and angle of drop all affect the speed, so Jim and Wiremu are both right. The number of people in the train does not appear to have much effect.
   If you put the figures into a spreadsheet and make graphs from them, they would show that there is a definite connection between rise and speed (see the graph in the next column), but that the angle of drop (for example, for the Dodonpa) can increase the speed still further.



The graph at the top of the next page shows that there is a very strong connection between the length of the drop and the speed.

Roller Coasters: Length of Drop and Speed Compared



3. a. Coca Cola reaches a top speed of 72 km/h. Estimates will vary.

   b. Estimates will vary, but most should be about 35 m. (Lethal Weapon actually drops 29 m.)

   c. Explanations will vary. The speed of the Coca Cola roller coaster can be estimated from the first graph by drawing a vertical line through the point on the horizontal scale that represents 27.7, noting where it cuts the graph, then reading across to the vertical scale from that point.

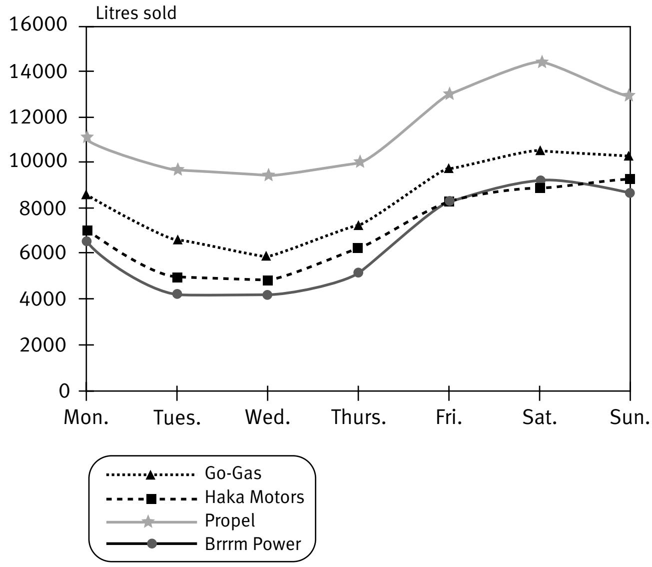
   The drop of the Lethal Weapon roller coaster can similarly be read from the second graph. Draw a horizontal line through the vertical axis at the point representing a speed of 88.5 km/h. Note where it cuts the graph, and from this point draw a line vertically until it cuts the horizontal axis. The point at which it cuts this axis is a reasonable estimate for the drop.

Pages 4–5 Have Your Fill

ACTIVITY

1. a. The graph from your spreadsheet should look similar to this one:

Petrol Sales



   b. The sales for each of the service stations follow a similar pattern through the week. Propel has the best sales record, followed by Go-Gas and Haka Motors, with Brrrm Power in fourth place. But on Saturday, Brrrm Power outsells Haka Motors. Sales dip at all four outlets during the week, rising on Friday and peaking on Saturday.

2.

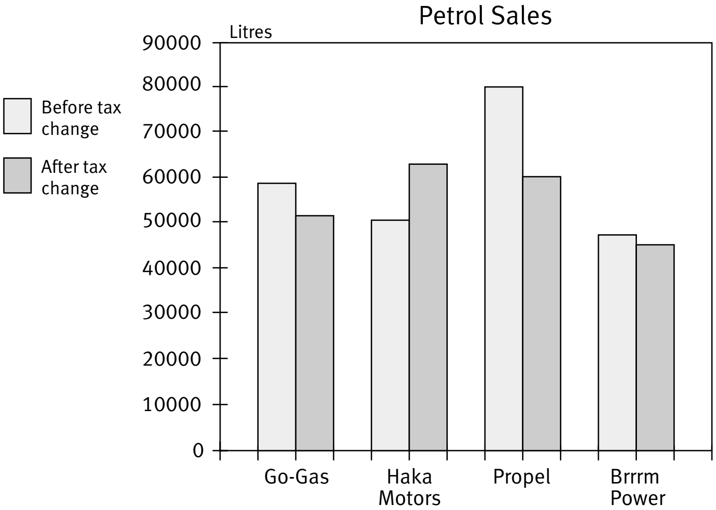
Petrol Sales 1

	Go-Gas	Haka Motors	Propel	Brrrm Power
Monday	8 575	6 949	11 238	6 511
Tuesday	6 659	5 466	9 757	4 379
Wednesday	5 934	5 126	9 479	4 290
Thursday	7 120	6 124	10 004	5 332
Friday	9 781	8 129	13 025	8 127
Saturday	10 362	8 943	14 567	9 203
Sunday	10 188	9 240	12 999	8 661
Total litres	58 619	49 977	81 069	46 503
a. Revenue @ $1 per litre	$58,619	$49,977	$81,069	$46,503
b. 20% of total	$11,724	$9,995	$16,214	$9,301

3. a. Go-Gas and Brrrm Power still get 20c for each litre sold:
110c – 80c (costs as before) – 10c (new tax) = 20c.
Haka Motors now gets 10c per litre:
100c – 80c – 10c = 10c.
Propel gets 25c per litre:
115c – 80c – 10c = 25c.

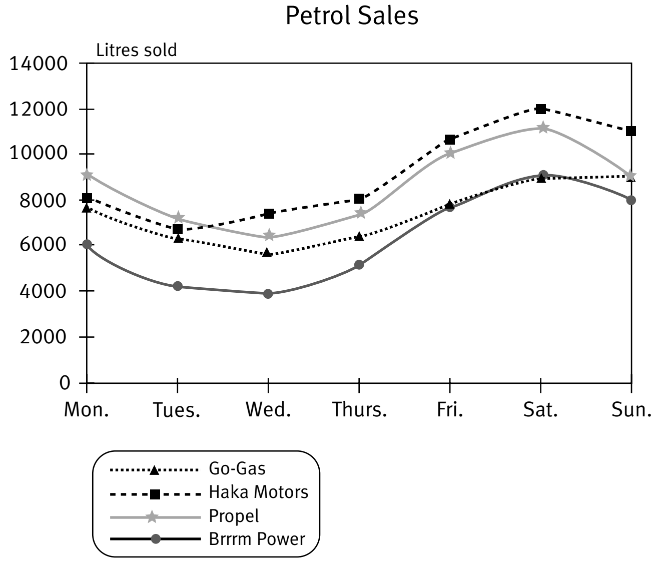
b. Petrol Sales 2

	Go-Gas	Haka Motors	Propel	Brrrm Power
Monday	7 893	8 134	9 236	6 001
Tuesday	6 356	6 987	7 405	4 113
Wednesday	5 453	7 132	6 223	3 998
Thursday	6 264	8 003	7 435	5 148
Friday	7 853	10 678	9 998	7 866
Saturday	8 790	11 987	10 961	8 785
Sunday	9 112	11 055	9 046	8 056
Total litres	51 721	63 976	60 304	43 967



c. See graph above. Go-Gas and Brrrm Power sold slightly less petrol (88.2% and 94.5% of former sales respectively). Propel sold a lot less petrol (74.3% of former sales). Haka Motors sold a lot more petrol (28% more).

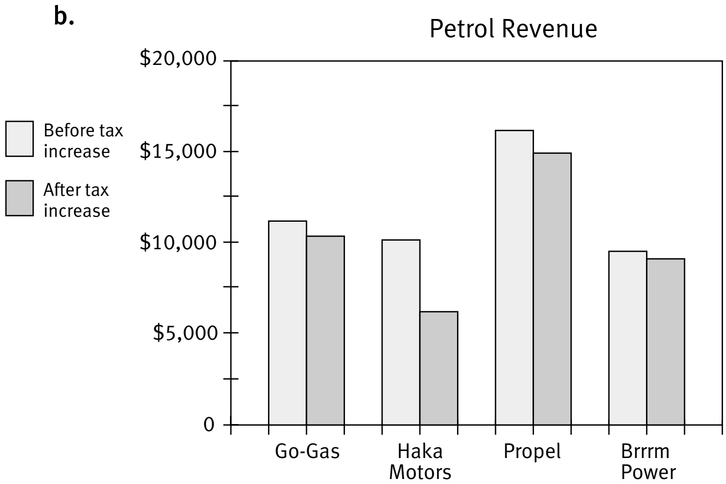
4. a.–b. See graph below. The weekly trend continues of sales dipping during the week and then rising to a peak on Saturday.



5. a. Service Stations' Share of Money from Sales: Before and After

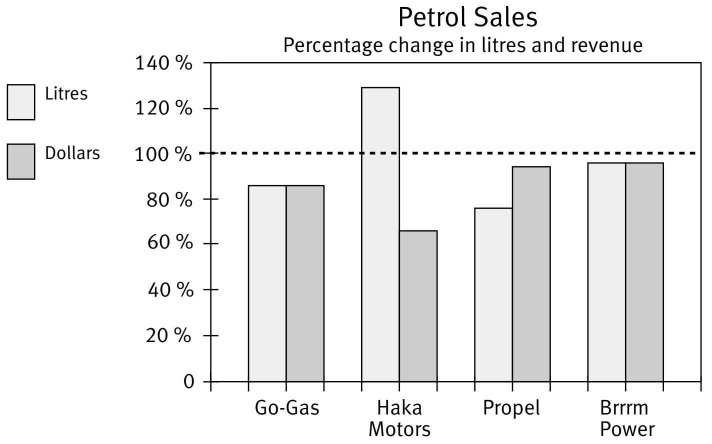
	Go-Gas	Haka Motors	Propel	Brrrm Power
Before 10c tax				
Litres	58 619	49 977	81 069	46 503
Service station's share	$11, 724	$9, 995	$16,214	$9 301
After 10c tax				
Litres	51 721	63 976	60 304	43 967
Service station's share	$10, 344	$6,398	$15,076	$8,793

b.



The before-and-after graph above shows revenue after taxes and oil company charges have been paid. We know that Haka Motors have increased the number of litres sold (see 3a), but it is clear that they have achieved this only by taking a massive dip in revenue after tax.

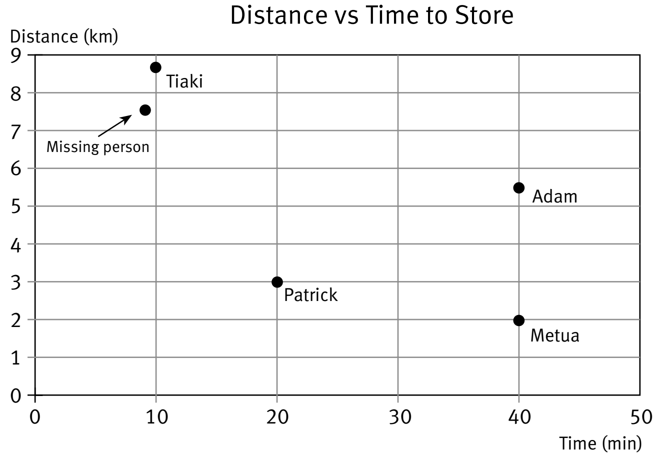
The extra graph below shows percentage change in litres sold and revenue earned (after charges and tax). It shows that Haka Motors has increased sales greatly but lost a lot of revenue in the process. The other three service stations have all sold fewer litres. In the case of Go-Gas and Brrrm Power, revenue has dropped in line with a drop in litres sold. Propel has managed to keep its revenue up while selling fewer litres, so its profit per litre has improved. Propel has therefore done better out of the changed price structure than the other service stations.



Pages 6–7 Ways to the Store

ACTIVITY

1. a.



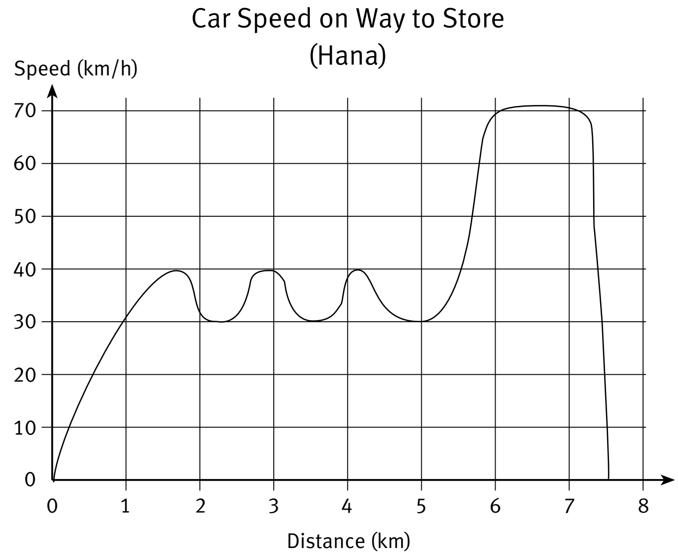
   b. Hana. Her distance from the village store (7.5 km) is not accounted for on the graph. Her distance is 1 km less than that of Tiaki, who also travels by car, so she would probably travel at a similar speed and take about 9 minutes.

   c. See the graph for **1a**.

2. Patrick biked. Compared with Adam, his campsite is about half the distance from the store. Patrick took half the time that Adam took to get there from his site, so he travelled at the same speed as Adam and it is likely that he biked too.

3. Hana would take about 50 minutes by bike. (Patrick took 20 min to cycle 3 km. Hana has to cycle 2.5 times as far [2.5 x 3 = 7.5 km], so she is likely to take about 2.5 times as long: 2.5 x 20 = 50 min.)

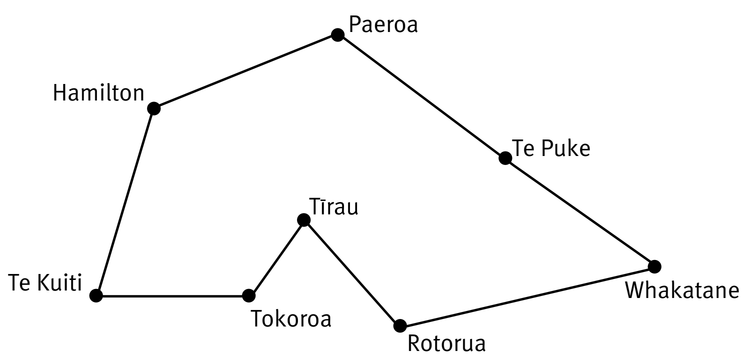
4. Your graph should look something like this:



Pages 8–9 Holiday Drop-offs

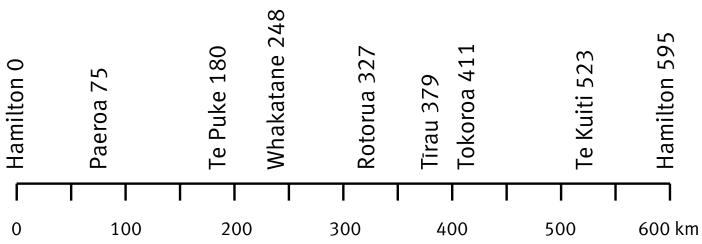
ACTIVITY ONE

1. a. Answers may vary. The shortest route (starting at Hamilton and heading for either Paeroa or Te Kuiti) is:



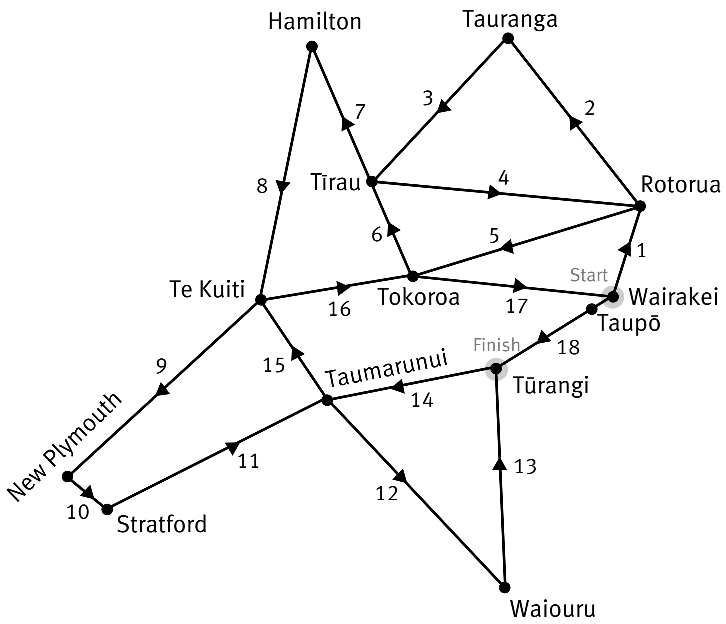
2. The total distance for the route shown above is 595 km.

3. Answers will vary. For the route shown above, with Paeroa the first stop, the distance model would be similar to this:



ACTIVITY TWO

1. a. Yes. They will go through some towns twice and finish at a different town from where they start.



   b. Tūrangi

2. Answers will vary.

Pages 10–11 Coast to Coast

ACTIVITY

1. 9.69 km/h
2. a. 2 hrs 14 min 56 s
   b. 26 km/h (rounded from 25.79)
3. a. 86.8 km
   b. 5 hrs 26 min 50 s
   c. 5 km/h (rounded from 4.77)
   d. He was running over uneven terrain with a lot of uphill and downhill work. Also, this was the final stage in a gruelling day.
   e. 10.9 km/h
4. 8:04:37 a.m.
5. 5 hrs 35 min
6. 1:39:37 p.m.
7. a. 2 hrs 58 min 50 s
   b. 24 km/h (rounded from 23.5)
   c. 238.8 km
   d. 17 hr 7 min 33 s

INVESTIGATION

Answers will vary.

Pages 12–15 Day Trippers

ACTIVITY ONE

1. a. Catch: the 8.40 a.m. bus to town
   the 9.15 a.m. ferry to Devonport
   the 3.45 p.m. ferry back to town
   the 4.15 or the 4.30 p.m. bus home.
2. 6 hrs 10 min
3. The 3.15 p.m. ferry (to connect with the 4.00 p.m. bus)

ACTIVITY TWO

1. 10 a.m.
2. 1 hr 30 min
3. They arrived at 2 p.m. and left at 4 p.m.
4. a. Answers will vary. They stopped for 10–12 minutes, which could have been to go to a dairy, chat to friends, adjust a bike part, or any similar activity.
   b. Answers will vary. Given the time of day and the length of the stop, they probably stopped for lunch.
5. a. Between 4 and 5 p.m.
   b. 15 km/h
6. a.–b. Answers will vary.

ACTIVITY THREE

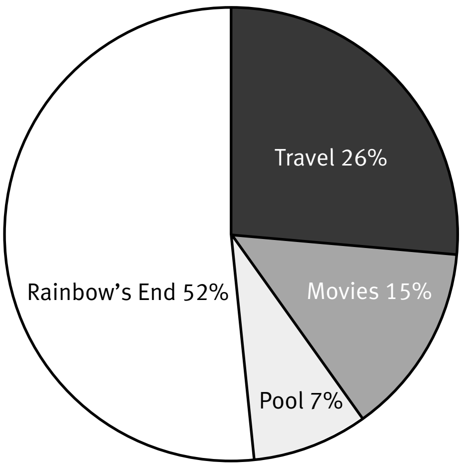
1. a. 6.22 m/s (2 d.p.)
   b. 22.39 km/h (2 d.p.)
   c. 3.40 km (2 d.p.)
2. a. 10.86 m/s (2 d.p.)
   b. 39.10 km/h (2 d.p.)

ACTIVITY FOUR

1. Measurements may vary slightly, but they should be between 4.5 and 5 km.
2. Answers will vary. A possible route is from the Marina, south of Browns Island, round Musick Point, and down to Cockle Bay. This is about 15 km on the map.

ACTIVITY FIVE

1. a. Fishing $16.80
   Movies/pool $21.00
   Rainbow's End $50.00
   Kayaking $7.90
   b. $95.70
   c. 25.81% (2 d.p.)
2. The pie graph should be similar to this one:



The percentages (to 2 d.p.) for each expense are:
travel 25.81%
movies 14.63%
pool 7.31%
Rainbow's End 52.25%

The angles (rounded to nearest degree) are:
travel: 25.81% of 360° = 93°
movies: 14.62% of 360° = 53°
pool: 7.31% of 360° = 26°
Rainbow's End: 52.25% of 360° = 188°

Pages 16–17 Time Zones

ACTIVITY

As noted in the activity, daylight saving is excluded from these answers.

1. a. Flight TG 992: Thailand is 5 hrs behind New Zealand. Departure time is 1430 – 5 hrs = 0930 Thai time, so flight time is 2230 – 0930 = 13:00 (13 hrs).

   b. Flight QF 25: Los Angeles is 20 hrs behind New Zealand. Departure time is 1940 – 20 hrs = 2340 USA time, so flight time is 1030 – 2340 = 10:50 (10 hrs 50 min).

   c. Flight PH 732: Apia is 23 hrs behind New Zealand. Departure time is 0445 – 23 hrs = 0545 Sāmoan time, so flight time is 0935 – 0545 = 03:50 (3 hrs 50 min).

   d. Flight NZ 121: Melbourne is 2 hrs behind New Zealand. Departure time is 0645 – 2 hrs = 0445 Australian time, so flight time is 0845 – 0445 = 04:00 (4 hrs).

   e. Flight JL 099: Tokyo is 3 hrs behind New Zealand. Departure time is 0930 – 3 hrs = 0630 Japanese time, so flight time is 1625 – 0630 = 09:55 (9 hrs 55 min).

   f. Flight BA 0268: London is 12 hrs behind New Zealand. Departure time is 1940 – 12 hrs = 0740 United Kingdom time. 1515 – 0740 = 07:35, so flight time is 07:34 + 24:00 = 31:35. Flights to the United Kingdom take much longer than 7 or 8 hrs, so it must be 31 hrs 35 min (including a stopover).

2. a. The flight from Sydney to Hong Kong left Sydney at 1430 – 9 hrs = 0530 Hong Kong time. (Hong Kong is 2 hrs behind Australia, so the flight left Sydney at 0530 + 2 hrs = 0730 Australian time.)

   b. The flight from New York to Madrid left at 1645 – 7hrs = 0945 Madrid time. (Madrid is 6 hrs ahead of New York, so the flight left New York at 0945 – 6 hrs = 0345 New York time.)

   c. The flight from Los Angeles to Rio de Janiero left Los Angeles at 1100 – 16 hrs = 0500 on the previous day, or 1900 Rio time. (Rio is 5 hrs ahead of Los Angeles, so the plane left Los Angeles at 1900 – 5 hrs = 1400 Los Angeles time.)

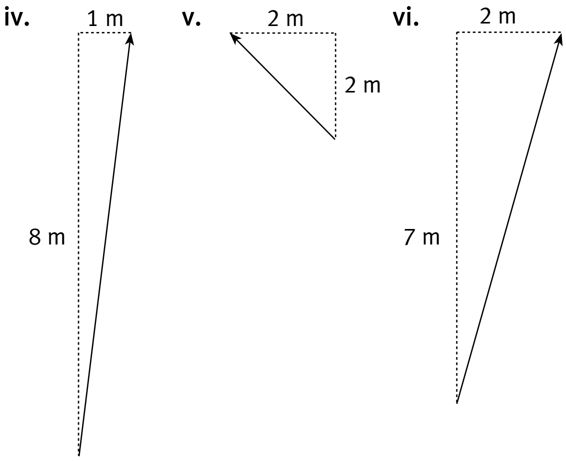
   d. The flight from Paris to Rome left Paris at 0745 – 2 hrs = 0545 Rome time. (Rome and Paris share the same time.)

3. Answers will vary.

Pages 18–20 Making Waves on the Whanganui

ACTIVITY

1.



(Explanations for each diagram are:

iv. Maasi paddles 1 stroke on the left, and Dave paddles 1 stroke on the right.

v. Dave paddles 1 stroke on the right, and Maasi drags her paddle on the left.

vi. Both Maasi and Dave paddle 1 stroke on the left side of the canoe.)

2. Answers will vary. There are many possibilities. A diagram for a route that works is shown on the following page.

3. Answers will vary. There are many possibilities.

4. Square each of the two parts of the vector and add them together. The bigger the sum of the squares, the faster the current. For example, the current with the vector $\binom{-2}{5}$ is faster than the current with the vector $\binom{3}{4}$ because $(-2)^2 + 5^2 = 29$, which is greater than $3^2 + 4^2 = 25$.

Pages 21 The Right Money

ACTIVITY

1. Australia: $710.64

Indonesia: 3749064 rupiah

Fiji: $859.36

Tonga: 970.48 pa‘anga

USA: $455.92

Britain: £284.32

France: €394.80

South Africa: 3271.20 rand

2. Exchange rates could vary, depending on when they exchanged their money. Based on the exchange rates shown in the table above, they would be:

a. Australia: $85.28 (0.12 x $710.64)

Indonesia: 674831.52 rupiah (0.18 x 3749064)

Fiji: $154.68 (0.18 x 859.36)

Tonga: 174.69 pa‘anga (0.18 x 969.68)

USA: $22.80 (0.05 x 455.92)

Britain: £14.22 (0.05 x 284.32)

France: €47.38 (0.12 x 394.80)

South Africa: 588.82 rand (0.18 x 3271.20)

b. Australia: $92.93 (85.28 ÷ 0.9177)

Indonesia: $129.00 (674831.52 ÷ 5231.42)

Fiji: $134.45 (154.68 ÷ 1.1505)

Tonga: $126.42 (174.69 ÷ 1.3818)

USA: $38.53 (22.80 ÷ 0.5918)

Britain: $38.15 (14.22 ÷ 0.3727)

France: $91.88 (47.38 ÷ 0.5157)

South Africa: $133.78 (588.82 ÷ 4.4014)

3. 87.73%, based on the percentage left in question **2**. (Your working might look like this:

Total left = $785.14

8 x 800 = $6,400

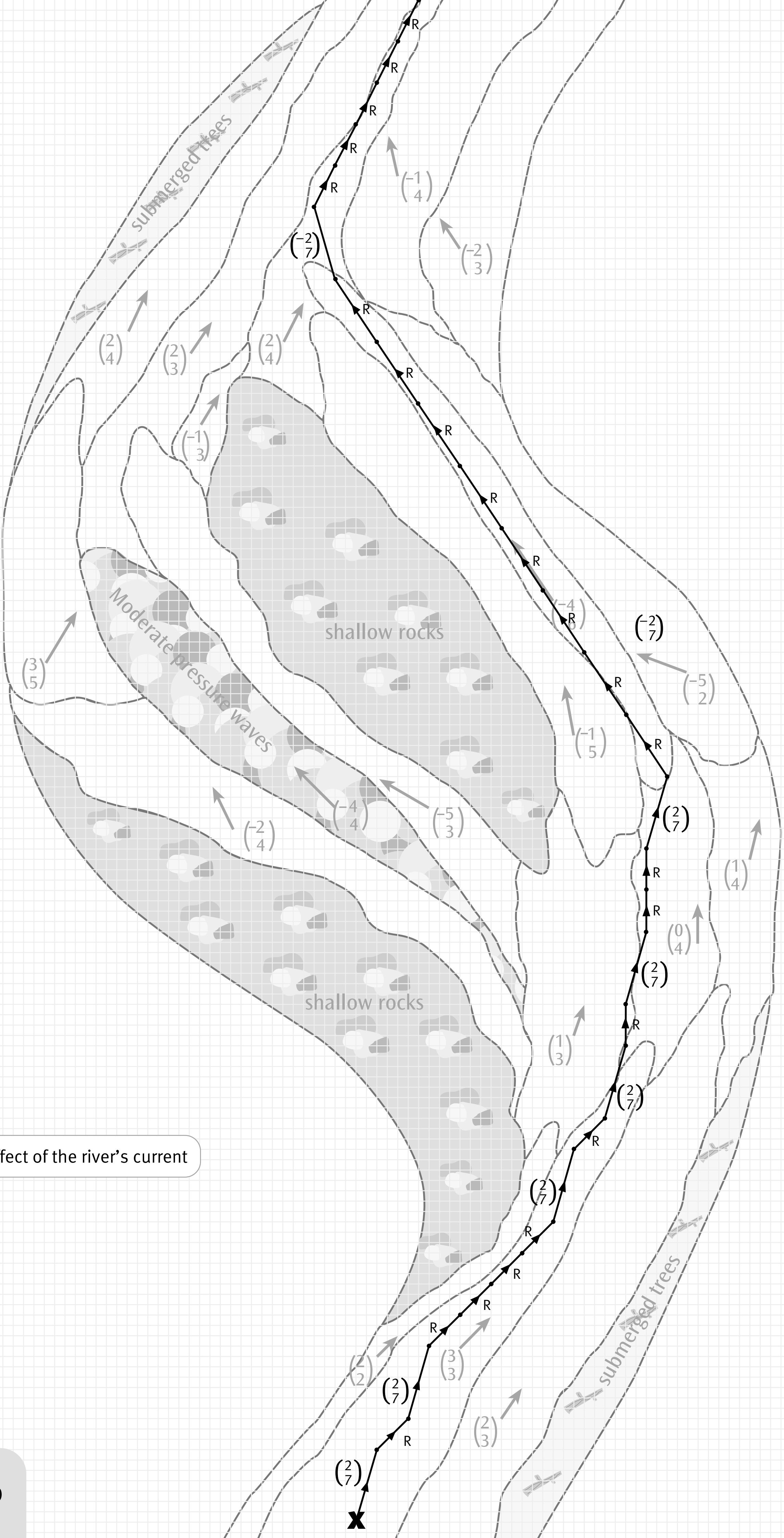
$$\text{\% left} = \frac{785.14}{6\,400} = 0.1227 = 12.27\%$$

100% – 12.27% = 87.73%)

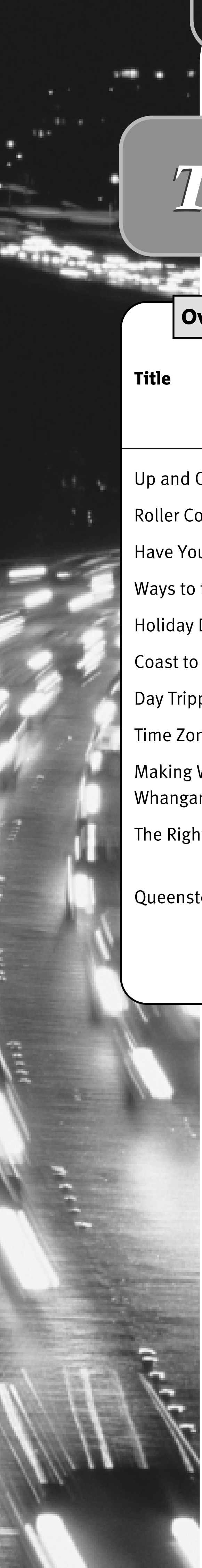
Pages 22–24 Queenstown Extravaganza

ACTIVITY

Answers will vary.



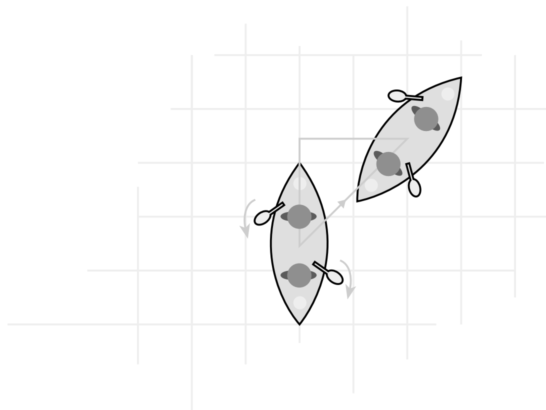
R is the effect of the river's current



Teacher's Notes

Overview **Theme: Getting Around**

Title	Content	Page in students' book	Page in teachers' book
Up and Off	Solving problems with ratio	1	12
Roller Coasting	Looking for relationships	2–3	15
Have Your Fill	Solving problems with percentages	4–5	16
Ways to the Store	Using and interpreting graphs of relationships	6–7	17
Holiday Drop-offs	Using networks to solve problems	8–9	18
Coast to Coast	Calculating speed, time, and rate	10–11	19
Day Trippers	Interpreting timetables, graphs, and maps	12–15	22
Time Zones	Working with 24 hour time and time zones	16–17	25
Making Waves on the Whanganui	Modelling situations with vectors	18–20	26
The Right Money	Using computation and percentages to solve money problems	21	28
Queenstown Extravaganza	Solving problems with time	22–24	29



Page 1

Up and Off

Achievement Objectives

- express quantities as fractions or percentages of a whole (Number, level 4)
- share quantities in given ratios (Number, level 5)
- interpret and use information about rates presented in a variety of ways, for example, graphically, numerically, or in tables (Measurement, level 5)

Other mathematical ideas and processes

Students will also:

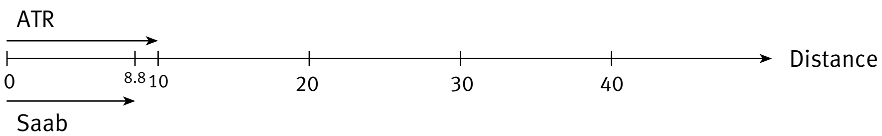
- use multiple number lines to solve problems
- increase and decrease a number by a given ratio.

ACTIVITY

For the ratios to have meaning, it is important that the students see the relationship between the speeds of the different aircraft. For example, if the ATR travels 10 kilometres in a given time, then the Saab will fly 8.8 kilometres in the same time. Expressed as a ratio, Saab : ATR is 8.8 : 10. The students will need to use this ratio to create an equation that calculates the Saab's distance in relation to the ATR's 200 kilometres. Ask the following questions to help the students to develop the numerical intuition they need to see that Saabs fly slower than ATRs:
"If an ATR flies 200 kilometres in a given time, will a Saab fly a longer or shorter distance in the same time?"
"Will it take a Saab more or less time than an ATR to fly 200 kilometres?"

Number lines are a useful graphical tool for helping students to visualise problems of this sort. The diagram below shows the graphical comparison of an ATR and Saab where the ATR has flown 10 kilometres.

ATR and Saab Comparison

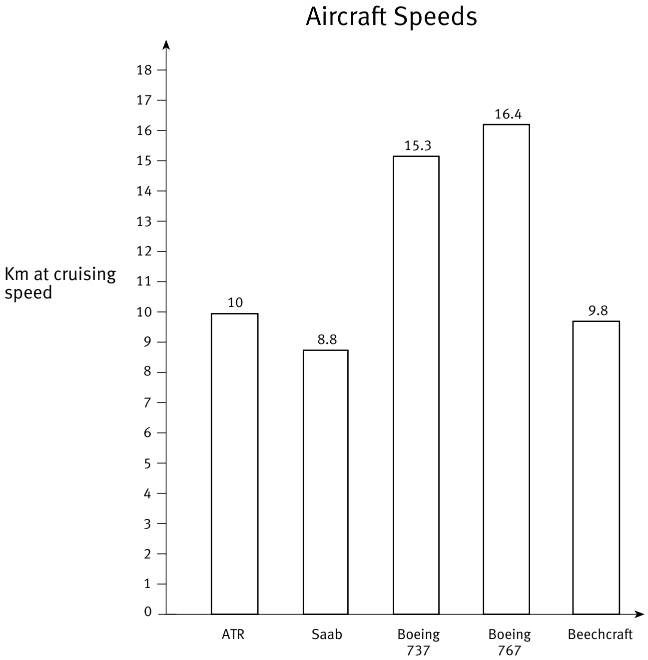


The students can then be asked to mark the position of the Saab when the ATR has flown 20 kilometres. The ATR has flown twice as far, so the Saab will also have flown twice as far, 17.6 kilometres. The students can then extend their number line so that they can plot the ATR at 200 kilometres. Ask "Where will the Saab be then?" The ATR has flown 10 times as far as its last position (20 kilometres), so the Saab will have flown 10 times its previous distance, which is 176 kilometres.

By graphing the comparative paths of the two aircraft, the students will see how the ratio linking them is maintained. They could check this link by comparing the ratios at the 10, 20, and 200 kilometre marks.
A table may help:

ATR	Saab	Ratio Saab : ATR
10	8.8	8.8 : 10
20	17.6	17.6 : 20 = 8.8 : 10
200	176	176 : 200 = 8.8 : 10

By converting this ratio into a fraction or a decimal, we can easily compare distances travelled by the ATR and the Saab in the same time period. For example, if the ATR has flown 350 kilometres, then the Saab has flown $^{8.8}/_{10}$ of that distance or 350 x 0.88 = 308 kilometres. Number lines can be drawn to show the relationship between the speeds of the ATR and the other aircraft, or better still, the relative positions of all the planes can be plotted on a single horizontal bar chart (a kind of multiple number line).
A possible bar chart is shown here:



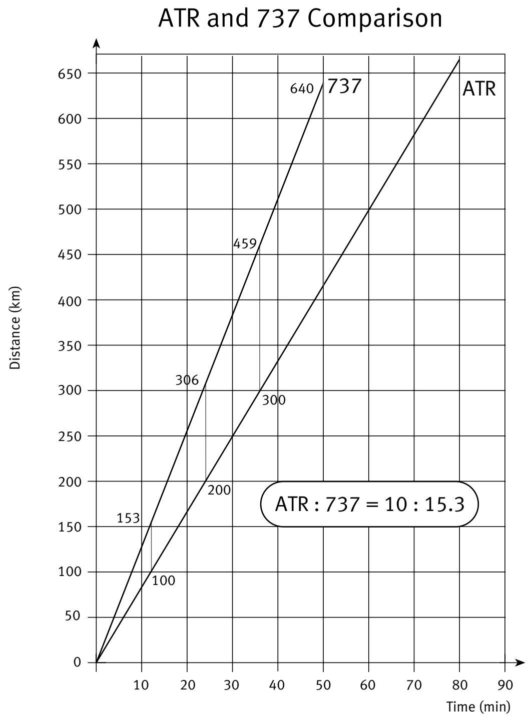
Question **2** raises the inverse relationship between speed and time. This can be a stumbling block for many students. You may need to help them gain the intuition to determine whether a quantity is being increased or decreased by a given ratio. They also need to know that multiplying by a number less than 1 *decreases* the original number and multiplying by a number more than 1 *increases* the original number.

The following questions may be helpful:

- Does a 737 fly faster than an ATR?
  (Yes, in the ratio 15.3 : 10.)
- In that case, will the ATR take more or less time to fly 640 kilometres than a 737? (More)
- Is the ratio 15.3 : 10 used to increase or decrease 50 minutes? (Increase)

So the time needed for an ATR to fly 640 kilometres is 50 x ($^{15.3}/_{10}$) minutes. Asking similar questions will show that a 767 will fly the distance in less time than the 737. So the 50 minutes is decreased by the ratio 15.3 : 16.4, or the 767 time = 50 x ($^{15.3}/_{16.4}$) minutes.

It may be helpful to illustrate this graphically by having the students plot a straight line graph comparing the flight times of two aircraft. This is shown below.



The line representing the 737 is drawn by linking the origin to the point (50,640). When the 737 is at 153 kilometres, the ATR will be at 100 kilometres. When the 737 is at 306 kilometres, the ATR will be at 200 kilometres. The line for the ATR can now be drawn by linking these two points with the origin and projecting the line past 640 kilometres. The time for this flight can be read off the horizontal axis. It should be approximately 77 = 50 x (15.3 ÷ 10) minutes.

The concept of increasing or decreasing by a given ratio is explored further in question **3**. The ratio of the speed of the Beechcraft to the speed of the ATR is 9.8 : 10. This comes from our original information. An ATR flies faster than a Beechcraft, so the speed of 490 km/h must be increased by the given ratio (multiplied by a factor of more than 1). The speed of the ATR is 490 x ($^{10}/_{9.8}$) = 500 km/h. Similarly, a Saab flies slower than a Beechcraft, so 490 kilometres will be decreased by the ratio 8.8 : 9.8. That is, the speed of the Saab is 490 x ($^{8.8}/_{9.8}$) = 440 km/h.

Another approach to this type of problem is as follows:
For every 9.8 kilometres the Beechcraft flies, the Saab will fly 8.8 kilometres. If the Beechcraft flies 490 kilometres in 1 hour, how many units of 9.8 kilometres has it flown? (490 ÷ 9.8 = 50) So, in 1 hour, the Saab will fly 50 units of 8.8 km, which is 50 x 8.8 = 440 kilometres.

CROSS-CURRICULAR LINKS

Social Studies

The students could plan a visit to a real or an imaginary friend in another part of New Zealand. They could research flight timetables to identify the quickest route to their destination. They then draw a diagram or picture of a possible means of transport to that place in the distant future and list possible positives and negatives for people travelling from one place to another so rapidly.

Achievement Objectives

Demonstrate knowledge and understandings of:

- causes and effects of events that have shaped the lives of a group of people (Time, Continuity, and Change, level 4)
- how and why people experience events in different ways (Time, Continuity, and Change, level 4)

Pages 2–3 Roller Coasting

Achievement Objectives

- sketch and interpret graphs on whole number grids which represent simple everyday situations (Algebra, level 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)

Other mathematical ideas and processes

- Students will learn to identify the relationships between variables when there are a number of variables involved.

ACTIVITY

In this activity, students identify factors that influence the speed of roller coasters.

Entering the data on a spreadsheet will enable the students to look for relationships between the data fields and the resultant speeds. For example, if an XY graph is created, with speed plotted on the vertical axis and height of rise on the horizontal axis, an interesting relationship is revealed. This is shown in the graph in the Answers. There is a clear relationship between the height of rise and the speed: the bigger the rise, the higher the speed. A similar relationship is found between the speed and the length of drop (see the graph in the Answers). There is no such relationship between the number of people and the speed (as an additional graph would show clearly). For example, the Dodonpa takes only 32 people but screams along at the second fastest speed.

The students might comment on the gaps in the data. If they do, discuss this with them and point out that, in most cases, the fact that data is missing does not invalidate conclusions drawn from the data that does exist.

Question **3** calls for interpolation, that is, estimating missing values based on known relationships. The Coca Cola ride has a height lying somewhere between Winjas and the Black Hole but closer to the Black Hole. The ratio of the Coca Cola ride's height to that of the Black Hole is 27.7 : 34.7. If we used this ratio, the speed of the Coca Cola ride would be 85 x (27.7 ÷ 34.7) = 67.9 km/h. The actual speed achieved by this coaster is 72 km/h, so our interpolation is reasonable.

To determine the drop length of the Lethal Weapon, a graph of the speed plotted against the length of drop as in the Answers provides an excellent guide. There is clearly a close linear relationship between the points. From this, we can extrapolate that the drop will be somewhere around 35 metres. In reality, however, the drop is 29 metres. This indicates that the angle of drop must be very steep.

CROSS-CURRICULAR LINKS

As a group, the students could identify a possible heritage site in the local area that could be developed as a tourist attraction. They then devise an impact report for the local council that identifies the key features of that place or environment, its historical value for the range of groups in the area, and the impact that the development would have on the local people's lifestyles and environments. The report should include the positives and negatives that the council needs to consider and conclude with a justified recommendation.

Achievement Objectives
Social Studies
Demonstrate knowledge and understandings of:

- how and why people view and use resources differently and the consequences of this (Resources and Economic Activities, level 4)
- the impact of the spread of new technology and ideas on culture and heritage (Culture and Heritage, level 4)

Health and Physical Education

- access and use information to make and action safe choices in a range of contexts (Personal Health and Physical Development, level 4)
- specify individual responsibilities and take collective action for the care and safety of other people in their school and in the wider community (Healthy Communities and Environments, level 4)

Pages 4–5 Have Your Fill

Achievement Objectives

- find a given fraction or percentage of a quantity (Number, level 4)
- sketch and interpret graphs which represent everyday situations (Algebra, level 5)
- interpret and use information about rates presented in a variety of ways, for example, graphically, numerically, or in tables (Measurement, level 5)
- report the results of mathematical explorations concisely and coherently (Mathematical Processes, communicating mathematical ideas, level 4)

Other mathematical ideas and processes

Students will also:

- make effective comparisons between two sets of data presented as spreadsheets
- model situations using spreadsheets.

ACTIVITY

Creating a computer spreadsheet is the most efficient way of analysing this situation, but it can be done on a calculator as long as the students write their figures down in columns. The graphing part of this exercise is made much simpler by using a spreadsheet. Note that the students should use formulae to determine totals and make other calculations. It makes no sense to create a spreadsheet of data, use a hand-held calculator to perform the calculations, and then enter the results into the cells as "dead" data. Keeping the spreadsheet "live" makes it possible to ask "what if?" questions of the data.

As can be seen in the answer to question **1**, weekends are the busiest time for petrol sales. Once the total number of litres sold by each service station is found, the total money taken, or gross sales, is easily found because 1 litre sells for 1 dollar. The revenue earned (after charges and taxes) is 20 cents per litre, which in this scenario is 20 percent. (Note that, in real terms, service stations make about 3 percent profit on the pump price.) The students should be reminded that 20 percent is a ratio, 20 : 100. So the profit of each service station is found by reducing the gross sales by this ratio. For example, Go-Gas's profit is $58 619 x 0.2 = $11,724. This could be expressed another way using the operator "of". To find 20 percent of $58 619, we calculate $^{20}/_{100}$ x $58,619, which is 0.2 x $58 619 = $11,724 (all values rounded to the nearest whole dollar).

Once the new table has been created for question **3** and the total litres sold found for each service station, the new gross income and net revenue can be determined. In the case of Go-Gas and Brrrm Power, the calculations are straightforward:
Gross sales = litres sold x $1.10. (This includes the new tax.)
Profit = litres sold x $1.00 x 0.2. (The new tax goes straight to the Council. The service station gets 20 percent of the price that the petrol was before the Council tax was imposed.)

Haka Motors chooses to absorb the new tax. So their gross income is still found by multiplying the litres sold by $1.00, but their percentage earnings are halved. Before, Haka Motors earned 20 cents out of every dollar it made. Now it must pay 10 cents to the Council from those earnings. So Haka Motors' earnings are only 10 cents in every dollar or 10 percent of gross income.

Propel has complicated things a little. Petrol at Propel now costs $1.15 per litre. So its gross income is total litres x $1.15. They now get the original profit of 20 cents out of every dollar plus the additional 5 cents that they have added on top of the levy. They therefore earn 25 cents on every litre of petrol sold, so their earnings equal litres sold x $0.25.

Propel has done better out of the changed pricing regime than the other service stations. In reality, this is an unlikely situation unless Propel is so far away from the other service stations that it is not worthwhile for people to buy their fuel elsewhere. Petrol is a highly sensitive commodity, and people will be influenced by a difference in cost, no matter how small.

Pages 6–7 Ways to the Store

Achievement Objectives

- sketch and interpret graphs on whole number grids which represent everyday situations (Algebra, level 4)
- read and construct a variety of scales, timetables, and charts (Measurement, level 4)
- collect and display time-series data (Statistics, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

Other mathematical ideas and processes

- Students will learn to use the gradient of a distance-time graph to determine the speed at which a given journey was completed.

ACTIVITY

In this activity, the students need to take careful measurements off the scale map provided and link these to the points on the graph. They will be able to do this most easily using a narrow strip of thin card marked in centimetres (1 cm represents 0.5 km) or a piece of string or thin wire.

Once they have the distances of each person from the store, the students can identify which point on the graph belongs with which holidaymaker. They will find that there is no point to represent Hana, who is 7.5 kilometres from the store.

Patrick travels 3 kilometres in 20 minutes, which is 9 km/h. This is obviously far too slow to be car travel, and it would be extremely fast walking. A comparison with Adam, who is known to use a bike, shows that he travels about twice as far as Patrick in twice the time. This means their speeds are similar, which makes it highly likely that Patrick also uses a bike for the trip.

Patrick covers 3 kilometres in 20 minutes on his bike, which is 1.5 kilometres in 10 minutes. Hana, who is the missing person, has 2.5 times as far to travel, so she could expect to take 2.5 times as long, which is 50 minutes. A graphical method of getting the same result would be to extend a line through the points representing the two known cyclists (Patrick and Adam) until it cuts the horizontal line representing 7.5 kilometres. The point where these two lines cut should be over the 50 minutes mark on the horizontal (time) scale.

Question **4** challenges the students to plot speed against distance. The horizontal axis of the graph needs to extend to 8 kilometres and the vertical axis to the maximum speed of 70 km/h. Using a strip of card or a piece of string, the students will need to mark the distances of each bend or corner measured from the camp store to Hana’s campsite. There are only three very sharp corners, each of which will be represented on the graph by a dip.

To get value out of the question, the students need to carefully plot the curve, not simply draw a wiggly line to indicate that the car is accelerating and decelerating as it goes around the curves. They may also like to consider whether the graph should slope away more steeply than it rises. This could lead to an interesting discussion of driving habits!

Pages 8–9 Holiday Drop-offs

Achievement Objectives

- effectively plan mathematical exploration (Mathematical Processes, problem solving, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

Other mathematical ideas and processes

Students will also:

- explore simple networks
- plot cumulative distances on a number line

ACTIVITY ONE

The first thing to notice in this problem is that none of the whānau lives in Taupō, so there is no reason to go there. To find the shortest route through all the other towns, starting at Hamilton and returning to Hamilton, we apply these principles:

- Always move forward. Don't enter loops that take you back to a town you have already visited.
- Where there is a choice of roads to the next town, choose the shortest.

Road	Distance	Cumulative distance
Hamilton – Te Kuiti	72 km	72 km
Te Kuiti – Tokoroa	112 km	184 km
Tokoroa – Tīrau	32 km	216 km
Tīrau – Rotorua	52 km	268 km
Rotorua – Whakatane	79 km	347 km
Whakatane – Te Puke	68 km	415 km
Te Puke – Paeroa	105 km	520 km
Paeroa – Hamilton	75 km	595 km

The distance model in question **3** is an effective way of demonstrating the relative distance each passenger has to travel to reach their destination. A follow-up activity could involve the students planning a tour through their own region, as in **Activity Two**, or around their own city, and then plotting cumulative distance on a number line, as in this question.

ACTIVITY TWO

For this activity, the students can try out different possible routes on their photocopy of the map (see copymaster). The activity involves Euler paths. (See the teachers' notes for All Fired Up in *Disasters Strike!* Figure It Out, Years 7–8.) There is an odd number of roads leading to and away from Wairakei. All other towns except Tūrangi have an even number of roads. So the end point for this journey is Tūrangi, and the journey is certainly possible, as shown in the Answers. For the students to make up a similar circuit of towns in their own region, the condition to be met is that a maximum of two towns have an odd number of roads radiating from them. Where all the towns have even numbers of roads radiating from them, a circuit can start anywhere and end up back at the beginning. If two towns have odd numbers of roads radiating from them, one must be the start of the circuit and the other the finish.

If this condition is not met by the towns and roads in your region, discuss if and where a road could be built to make a circuit possible.

A useful website about Euler and networks is www.mathforum.org/isaac/problems/bridges2.html

CROSS-CURRICULAR LINKS

Social Studies

You could ask the students to choose either Taupō or Rotorua as a location and to research a local legend associated with that place. The students then give a presentation that shows the impact of the place or environment on an individual or group of people at that time.

Achievement Objective

- demonstrate knowledge and understandings of how places reflect past interactions of people with the environment (Place and Environment, level 4)

Pages 10–11 Coast to Coast

Achievement Objectives

- read and interpret everyday statements involving time (Measurement, level 3)
- perform calculations with time, including 24-hour clock times (Measurement, level 4)
- express a fraction as a decimal, and vice versa (Number, level 4)

Other mathematical ideas and processes

- Students will also investigate the relationship between distance, speed, and time.

ACTIVITY

There are three key skills involved in this activity, all of which need to be specifically taught.
They are:

- counting elapsed time
- converting time expressed in minutes and seconds to time expressed as a decimal fraction
- performing calculations with distance, speed, and time.

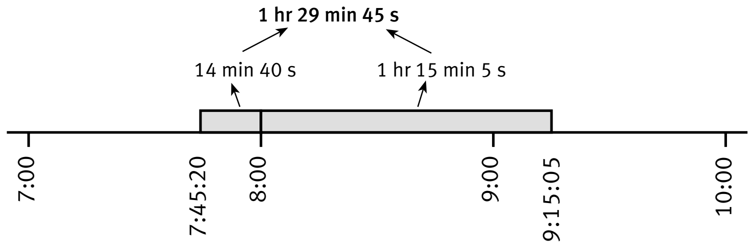
In the activity, time is expressed in the form 7:45:20. Check that your students understand this notation; it is unlikely to be a problem to those with digital watches.

Counting elapsed time

Although there are algorithmic methods of working out elapsed time by subtraction, the students are unlikely to appreciate what they are doing or why. A better method is to count on to the next whole hour, then on from that hour to the final time, and then to add the two times together to get total elapsed time.

Example: what is the elapsed time from 7:45:20 a.m. to 9:15:05 a.m.?

The time *to* 8 a.m. (the next whole hour) is 14 minutes and 40 seconds. The time *from* 8 a.m. is 1 hour, 15 minutes, and 5 seconds. Adding minutes to minutes and seconds to seconds, we get an elapsed time of 1 hour, 29 minutes, and 45 seconds. This can be represented on a time line:



The students should practise adding up amounts of time where the seconds come to more than 60 or the minutes come to more than 60. Most students will know that 72 minutes is 1 hour and 12 minutes, so they should have little trouble with the concept involved.

Converting minutes and seconds to a decimal fraction

A calculator is normally needed to convert time expressed in minutes and seconds to time expressed as a decimal fraction, and the process will require explicit teaching.

The students need to understand that 1 hr 29 min 5 s means 1 hour + 29 minutes + 5 seconds. They also need to understand that 5 seconds is $^{5}/_{60}$ of a minute and 29 minutes is $^{29}/_{60}$ of an hour:

- on a calculator, 5 seconds is $^{5}/_{60}$ = 0.0833 minutes
- 29 minutes 5 seconds is therefore 29 + 0.0833 = 29.0833 minutes
- 29.0833 minutes is $^{29.0833}/_{60}$ = 0.4847 hours
- 1 hour, 29 minutes, and 5 seconds is therefore 1 + 0.4847 = 1.4847 hours.

The sequence of operations on the calculator is 5 ÷ 60 [=] + 29 [=] ÷ 60 [=] + 1 =. (The operator in square brackets is not required on many calculators, but it may be safer to encourage the students to use it.)

Distance, speed, and time

All students will bring certain intuitive understandings to a discussion on distance, speed, and time. These should be the starting point for teaching this topic.

Typically, students will understand that:

- if they are travelling at 100 km/h, they will cover 100 kilometres in 1 hour, 200 kilometres in 2 hours, and so on (distance = speed x time);
- if they drive 150 kilometres in 2 hours, they will drive 75 kilometres in 1 hour, so their speed is 75 km/h (speed = distance ÷ time);
- if they have 180 kilometres to travel and they are driving at 90 km/h, it will take them 2 hours to reach their destination (time = distance ÷ speed).

The problem with distance, speed, and time is working out the relationships in a particular problem, and you should encourage the students to go back to first principles like the ones above, rather than apply a formula. Because the relationships are often inverse, you should expect the students to think about the kind of answer they are looking for. You could ask: "If distance is increased, what will this do to time? If speed is increased, what does this do to time?"

A final consideration is units. Time is measured in hours (and parts of hours), distance is usually measured in kilometres (and parts of kilometres), and speed is usually measured in kilometres per hour. The students need to take care to use the correct units in each situation and to abbreviate them correctly.

In this activity, the students need to solve applied problems involving distance/speed/time from a variety of angles. The students could work out the average speed in question **1** like this:

$$\text{Average speed} = \frac{2.8 \text{ km}}{17 \text{ min } 20 \text{ s}} \quad \text{or} \quad \frac{2.8 \text{ km}}{17.\dot{3} \text{ min}} \quad \text{or} \quad \frac{2.8 \text{ km}}{0.2\dot{8} \text{ hr}}$$

$$= 9.69 \text{ km/h (2 d.p.)}$$

Calculating Dad's speed on the first cycling leg (question **2b**), we see that he covered 58 kilometres in 2 hours, 14 minutes, and 56 seconds. The first step is to convert the time to hours expressed in decimal terms: 56 seconds is $^{56}/_{60}$ = 0.933 minutes, so altogether, we have 14 + 0.933 = 14.933 minutes. Converting this to hours, we have $^{14.933}/_{60}$ = 0.2488 hours, so altogether, the time taken is 2 + 0.2488 = 2.2488 hours. Now that we have the time as a decimal, we can use this to calculate average speed.
Speed = distance ÷ time, so speed = 58 ÷ 2.2488 = 25.79 km/h, which is 26 km/h rounded to the nearest kilometre.

In **3e**, the average speed for day 1 is found by dividing the total distance travelled by the total time taken. The total distance is 86.8 kilometres and the total time is 7:59:06 or 7.9850 hours. So Dad's average speed is 86.8 ÷ 7.985 = 10.87 km/h.

Note that this is *not* the same as finding the average of 9.69, 25.79 and 4.77 (Dad's speed on each of the three stages), which is 13.42 km/h. The reason we can't use this approach is that the stages are different lengths.

In question **4**, we have the reverse situation in which the resulting time is expressed as a decimal and we need to convert it to hours, minutes, and seconds. Dad covered 15 kilometres at an average speed of 26 km/h, so the time he took is 15 ÷ 26 = 0.5769 hours. This is a bit over half an hour, so we need to be working with minutes: 0.5769 x 60 = 34.62 minutes. This is 34 minutes plus a little over half a minute (0.62), so we now need to work with seconds: 0.62 x 60 = 37 seconds. So, Dad took 34 minutes and 37 seconds to cycle from Klondyke Corner to Mount White Bridge.

CROSS-CURRICULAR LINKS

Achievement Objectives

After they have done some research on the Coast to Coast race, you could put the students in pairs and ask them to create a crisis scenario that might occur during the race. They then write an account of the crisis, describing the scenario and what organisation was required by people (organisers and participants) to resolve it.

Social Studies

Demonstrate knowledge and understandings of:

- how people organise themselves in response to challenge and crisis (Social Organisation, level 4)
- how and why people exercise their rights and meet their responsibilities (Social Organisation, level 4)

Health and Physical Education

- specify individual responsibilities and take collective action for the care and safety of other people in their school and in the wider community (Healthy Communities and Environments, level 4)

Pages 12–15 Day Trippers

Achievement Objectives

- perform calculations with time, including 24-hour clock times (Measurement, level 4)
- express quantities as fractions or percentages of a whole (Number, level 4)
- draw and interpret simple scale maps (Geometry, level 3)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)
- read and construct a variety of scales, timetables, and charts (Measurement, level 4)

Other mathematical ideas and processes

Students will also:

- interpret key features of a distance–time graph to determine the speed at which the journey was undertaken
- convert speed expressed in metres per second to speed in kilometres per hour.

ACTIVITY ONE

This activity reinforces the work done on pages **10–11**. When Adam and Tiaki go to Devonport, the time calculations are straightforward and linear. Leaving home at 8.30 a.m., they arrive at the bus station at 8.30 a.m. + 4 min = 8.34 a.m. The next bus departing for town is at 8.40 a.m., which will have them in the city by 8.40 a.m. + 25 min = 9.05 a.m. Walking for 2 minutes to the ferry takes them to 9.07 a.m., and the next available ferry is at 9.15 a.m. A 20 minute ferry ride has them in Devonport at 9.15 a.m. + 20 min = 9.35 a.m.

The return trip includes the constraint to be home at 5.00 p.m. If they are to be home by 5.00 p.m., they must start walking from the bus stop at 5.00 p.m. – 4 min = 4.56 p.m. If the bus ride is 25 minutes, it must leave town no later than 4.56 p.m. – 25 min = 4.31 p.m. The latest bus satisfying this condition leaves at 4.30 p.m. (although the boys could also catch the 4.15 bus). Allowing for the 2 minute walk between the ferry terminal and the bus station, as well as 20 minutes for the ferry trip, the boys must leave Devonport no later than 4.30 p.m. – 2 min – 20 min = 4.08 p.m. So the latest ferry they can take is 3.45 p.m.

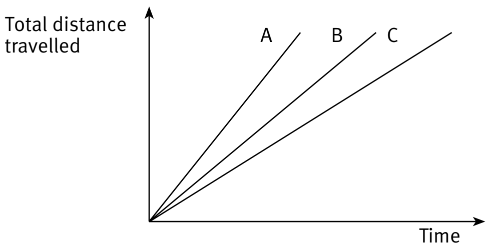
Using the same process in question **3**, the students can find ferry and bus times that will get the boys home by 4.30 p.m.

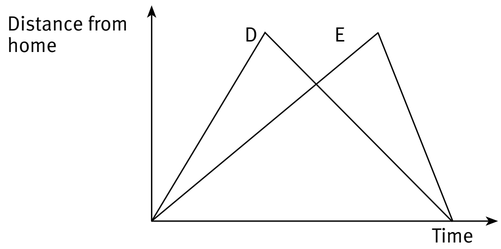
ACTIVITY TWO

Time-series graphs always tell a story. Exactly what story depends largely on the labelling of the vertical axis (the horizontal one always represents time). The students need experience in interpreting the different features of these graphs:

- A sloping line means that change is occurring. This might be change in distance, sales, profit, population, light, weight, savings, the number of people living in the home, electricity consumption, water flow, Internet usage, the number of pages in a newspaper, in fact, anything at all that changes over time.
- The steeper the slope, the faster the change is occurring.
- A horizontal line means that no change is taking place at that point in time.
- A vertical line means that change has occurred instantaneously.
- A slope can be positive (sloping upwards from left to right) or negative (sloping downwards from left to right). A positive slope always indicates an increase in whatever is being measured; a negative slope indicates a decrease.
- If a line is straight, the rate of change is constant (a steady speed, or whatever, is being maintained).
- A time-series graph is always continuous unless there is a gap due to missing data.
- Where a graph has clear scales, measurements can be taken and calculations made.

Simple graphs like these can be used as a focus for discussion:

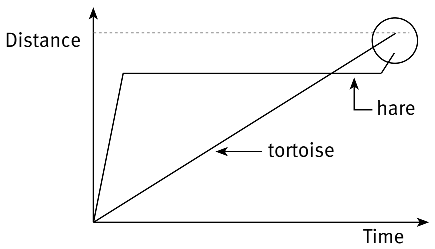




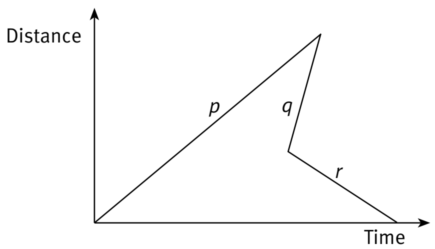
In the first graph, persons A, B, and C all travel exactly the same distance (all three lines stop the same distance up the vertical scale). But A travels fastest (shown by steepest slope on the graph) and gets there first (as measured on the horizontal time scale). B travels faster than C and gets there before C.

In the second graph, we are graphing distance *from home*, not total distance covered. Both D and E travel from home the same distance, but then the distance from home starts reducing until the distance is 0 (zero), which means they are back at their starting point (home). D gets to the destination before E (in half the time, actually, as measured against the horizontal scale), but D and E get home at the same time because D is slower on the return trip.

The following distance–time graph illustrates the progress of the hare and the tortoise on their famous race. Although not fully labelled, its principal features should be clear. The long horizontal segment represents the time the hare spent asleep (going nowhere). The horizontal line at the top indicates the end of the race. The circled gap shows that the hare gave up when he saw that the tortoise had reached the finish line ahead of him.



The last graph (below) cannot represent a time–series scenario. Why? Because the backwards sloping segment marked *q* implies that time can go backwards, which it can't. For this reason, every next point on a time–series graph must be to the right of the last point plotted.



ACTIVITY THREE

Back to roller coasters! In this activity, we move from the graphical representation of speed, distance, and time to calculations using different units (metres per second and kilometres per second).

Question **1a** asks for the speed of the roller coaster in metres per second.
Distance ÷ time = 485 metres ÷ 78 seconds, which is 6.22 m/s (2 d.p.). For question **b**, this leads to 22 392 m/h (6.22 x 60 x 60), which is 22.39 km/h. Another way to work out question **b** is:
485 metres = 485 ÷ 1 000 = 0.485 km. 78 seconds = 78 ÷ (60 x 60) = $0.0\dot{2}1\dot{6}$ hours. (There are 60 x 60 seconds in an hour.) The speed of the coaster is therefore 0.485 ÷ $0.0\dot{2}1\dot{6}$ = 22.38 km/h (or 22.39 if rounded earlier).

ACTIVITY FOUR

This is an exercise in reading scale maps. On the map given, 1 centimetre represents 1 kilometre, so the students should not find it difficult to select a course of between 15 and 20 kilometres in length because there are many options to choose from.

ACTIVITY FIVE

You will need to teach the students how to organise their data when solving audit problems. If a spreadsheet can be used for this task, use it because it enforces organisation of the data. If not, then a table will do nicely. The data can be arranged as follows:

Event	Cost each	Total cost
Bus to fishing	$4.40	$8.80
Ferry to fishing	$4.00	$8.00
Total trip 1	$8.40	$16.80
Movies	$7.00	$14.00
Wave pool	$3.50	$7.00
Total trip 2	$10.50	$21.00
Rainbow's End	$25.00	$50.00
Total trip 3	$25.00	$50.00
Kayaking bus 1	$2.20	$4.40
Kayaking bus 2	$1.75	$3.50
Total trip 4	$3.95	$7.90
Total	$47.85	$95.70

The total cost for each trip is then read from the table, and the cost of their transport for the week can be calculated: $8.80 + $8.00 + $4.40 + $3.50 = $24.70. So the percentage of their week's budget spent on transport is $24.70 ÷ $95.70 = 25.81%.

Drawing the pie graph to represent their expenditure requires the students to divide 360 degrees into appropriate sectors. We know that 25.81 percent of the budget was spent on transport. There are 360 degrees in a circle, so 25.81 percent of 360 degrees or 0.2581 x 360° = 93° (to 1 d.p.) is the size of the angle at the centre of the circle that represents transport. A table will prove useful when determining the size of the angles for the four different sectors.

Item	% of total	Size of angle
Travel	25.81%	0.2581 x 360° = 93°
Movies	14.63%	0.1463 x 360° = 53°
Pool	7.31%	0.0731 x 360° = 26°
Rainbow's End	52.25%	0.5225 x 360° = 188°

When doing tasks like this, the students need to understand that 25.81% = 0.2581, that 14.63% = 0.1463, and so on. It is also important that they do not truncate or round excessively part-way through their calculations because this will affect the accuracy of their result. In this case, the angle should be rounded at the end of the calculation to the nearest whole degree because protractors cannot be read to a greater degree of accuracy. A computer spreadsheet program would do a good job of producing the pie chart.

CROSS-CURRICULAR LINKS

You could present the students with the scenario that another class is to visit the students' class for the day to share knowledge and skills in relation to the topic they are currently studying. The students could develop ways that they might make the sharing of this knowledge and skills an enjoyable and valuable experience for everyone involved in the day.

Achievement Objectives

Social Studies

- demonstrate knowledge and understandings of how and why people exercise their rights and meet their responsibilities (Social Organisation, level 4)

Health and Physical Education

- describe and demonstrate a range of assertive communication skills and processes that enable them to interact appropriately with other people (Relationships with Other People, level 4)

Pages 16–17 Time Zones

Achievement Objectives

- perform calculations with time, including 24-hour clock times (Measurement, level 4)

Other mathematical ideas and processes

- Students will also learn to adjust times from different zones.

ACTIVITY

This task reinforces earlier work with 24 hour time but with the added complication of moving between different time zones. As the teacher, you should work through this activity before giving it to your students because it is very easy to make mistakes!

To see what information they have and what they don't have, the students may like to put the given information into a table, or tables, like the ones below. Here, the given information is in the shaded cells; what needs to be found is in the white cells.

Question	Flight	City of Departure	Destination	Departure time (New Zealand)	Arrival time (local)	Difference (hours)	Departure time at destination	Flight duration
1. a.	TG 992	Auckland	Bangkok	1430	2230	– 5	0930	13:00
b.	QF 25	Auckland	Los Angeles	1940	1030	– 20	2340	10:50
c.	PH 732	Auckland	Apia	0445	0935	– 23	0545	03:50
d.	NZ 121	Auckland	Melbourne	0645	0845	– 2	0445	04:00
e.	JL 099	Auckland	Tokyo	0930	1625	– 3	0630	09:55
f.	BA 0268	Auckland	London	1940	1515	– 12	0740	32:15

Question	Flight	City of Departure	Destination	Departure time (local)	Arrival time (local)	Difference (hours)	Departure time at destination	Flight duration
2. a.		Sydney	Hong Kong	0730	1430	– 2	0530	09:00
b.		New York	Madrid	0345	1645	+ 6	0945	07:00
c.		Los Angeles	Rio de Janiero	1400	1100	+ 5	1900	16:00
d.		Paris	Rome	0545	0745	0	0545	02:00

In each case, the time zone difference between departure point and arrival point needs to be determined. This information can be found at the front of the telephone directory, in the section headed International Country and Area Codes. It can also be found on a number of Internet sites, including www.timetableanddate.com/worldclock. If the telephone directory is used, daylight saving will need to be ignored (as in the instructions in the student book). If the Internet is being used, students will be able to take daylight saving into account.

In question **1**, a suitable approach is:

- Determine what the time is in the destination country at the point when the plane is flying out from the departure country.
- Calculate the difference between this time and the arrival time to get the travel time (flight duration).

Using **1e** as an example:

- The departure time in the destination country is 0930 – 3 hrs = 0630.
- From 0630 to 1625 is 05:30 + 04:25 = 09:55 (9 hours and 55 minutes).

Using **1f** as a further example:

- The departure time in the destination country is 1940 – 12 hrs = 0740.
- From 0740 to 1515 is 04:20 + 03:15 = 07:35 (7 hours and 35 minutes). But this distance can't be flown in such a short time, so there must be an additional 24 hours in there (including a stopover). The total time thus becomes 31:35 (31 hours and 35 minutes).

In question **2**, the task is very similar, except that the students are given arrival time and flight duration, and from these they have to work out the departure time.

It would be useful to have pairs of students comparing their answers for these activities. Where they get different results, they can determine who has made the error and what the error is. Working is given in the answers for each part of these two questions so that, if necessary, the students can get help with the process.

Pages 18–20 Making Waves on the Whanganui

Achievement Objectives

- draw and interpret simple scale maps (Geometry, level 3)
- solve practical problems which can be modelled, using vectors (Geometry, level 5)

Other mathematical ideas and processes

Students will also:

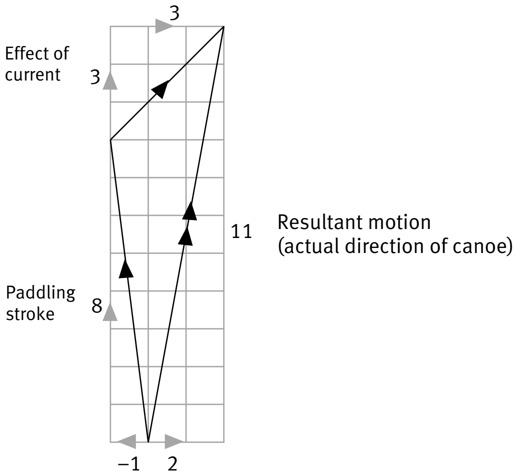
- learn how to use vectors to model speed and direction
- learn how to add vectors.

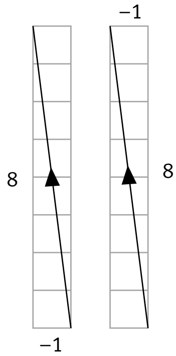
ACTIVITY

(For an introductory discussion of vectors, please refer to the teachers' notes for Oil Spills in *Disasters Strike!* Figure It Out, Years 7–8.)

Question **2** requires the students to add vectors together in order to find the resultant vector. In simple terms, if Maasi and Dave paddle the boat, they will travel along a line defined by their paddling vector. Suppose that the stroke they decide on is stroke **i**, Maasi right and Dave left. The vector defining their motion is $\binom{-1}{8}$. However, the flow of the river will influence the motion of the boat as well. If Maasi and Dave shipped their oars (and did not paddle at all), the river current would be the only influence upon the boat. This motion is defined by the vector $\binom{3}{3}$. Combining the force of the stroke and the current, we would have paddling + current = resultant motion. In vector terms, this would be $\binom{-1}{8} + \binom{3}{3} = \binom{-1+3}{8+3} = \binom{2}{11}$. So the movement of the boat can be defined by the single vector $\binom{2}{11}$.

Resultant Motion of the Boat





Note that both these vector triangles represent $\binom{-1}{8}$ and are identical in value and meaning.
(1 step to the left and 8 forwards gets you to the same place as 8 forwards and 1 to the left.)

To find a safe course through the river, the students need to start at point X, decide on a stroke Maasi and Dave should use, and then combine this with the current vector at that point to determine where the boat will be before applying the next stroke. For example, starting at X, if Maasi and Dave both paddle once on the left, the motion will be stroke + current $= \binom{2}{7} + \binom{3}{3} = \binom{2+3}{7+3} = \binom{5}{10}$. So, from X, the new position of the boat is found by moving 5 blocks to the right and then up 10. We notice that, at this point in the river, the vector describing the current is again $\binom{3}{3}$. Choosing the same stroke again, the movement of the boat is stroke + current $= \binom{2}{7} + \binom{3}{3}$, which is $\binom{5}{10}$. Plotting this movement, 5 across and 10 up, places us just 4 units away from the shallow rocks. We need to move further to the right with as little forward movement as possible and very quickly. Our best stroke for this is for Dave to left paddle and Maasi to right drag. The movement is then stroke + current $= \binom{2}{2} + \binom{3}{3}$, which is $\binom{5}{5}$. Continuing in this way, making corrections to the choice of stroke where obstacles are encountered, we will find a safe course through the river. There are, of course, many possible vector routes that will get Maasi and Dave safely through the rapids.

Question 4 challenges the students to work out how to use vectors to determine the strength of the current. To do this, they need to compare the length (magnitude) of the vectors. If two vectors have the same length, they represent the same speed. All these vectors have exactly the same magnitude:
$\binom{2}{3}, \binom{-2}{3}, \binom{2}{-3}, \binom{-2}{-3}, \binom{3}{2}, \binom{-3}{2}, \binom{3}{-2}, \binom{-3}{-2}$.

The standard method for finding the length of a vector is to use Pythagoras' theorem, which squares each component of a vector, adds these figures, and then finds the square root. However, the students can easily compare vectors by simply squaring and adding the two components (numbers) together. The bigger the result, the greater the magnitude.

Page 21 The Right Money

Achievement Objectives

- express a decimal as a percentage, and vice versa (Number, level 4)
- express quantities as fractions or percentages of a whole (Number, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)
- solve practical problems involving decimals and percentages (Number, level 5)
- interpret and use information about rates presented in a variety of ways, for example, graphically or in tables (Measurement, level 5)

Other mathematical ideas and processes

Students will learn about the principles of currency exchange as well as where to find current information about international exchange rates.

ACTIVITY

Current foreign exchange rates can be found at many websites, including www.westpactrust.co.nz and www.nationalbank.co.nz
An online currency converter can be found at www.bonz.co.nz/hmm/xchange.html

In question **2** of the activity, the students are asked to calculate what percentage remains of their funds. Many calculators have a percentage button, but it is best not used. The students should get used to 5 percent as 0.05, 10 percent as 0.1, 31 percent as 0.31, 152 percent as 1.52, and so on.

In question **3**, the students are asked what percentage of their original budget of has been spent. This is done after part **b** of question **2** is completed, using the method laid out for currency conversion at the start of the activity, and is shown in the Answers. If all the remaining New Zealand dollar values are added from each country, this amounts to $785.14 (using the exchange rates as shown in the students' book.)
The budget for the entire trip was $800 x 8 countries = $6, 400. This means they spent $6,400 – $785.14 = $5,614.86. The ratio of money spent : budget is 5,614.97 : 6 400 or, expressed as a percentage, $^{5\,614.86}/_{6\,400} = 0.8773$
$= 87.73\%$.

Another way of working this out is shown in the Answers.
This problem provides an ideal extension opportunity for students who understand spreadsheets. By setting up a table showing all the exchange rates with formulae to calculate the local currency equivalents, an interesting dynamic model of this scenario could be created.

CROSS-CURRICULAR LINKS

The students could use a range of tools, under close teacher supervision, to make contact with a peer in a different place or environment, in New Zealand or overseas. During this interaction, they should compare each other's needs, wants, and aspirations and help each other to identify the steps that each might need to take to achieve their goals for the future.

Achievement Objectives

Social Studies

- demonstrate knowledge and understandings of how and why people view and use resources differently and the consequences of this (Resources and Economic Activities, level 4)

Health and Physical Education

- describe and demonstrate a range of assertive communication skills and processes that enable them to interact appropriately with other people (Relationships with Other People, level 4)

Pages 22–24 Queenstown Extravaganza

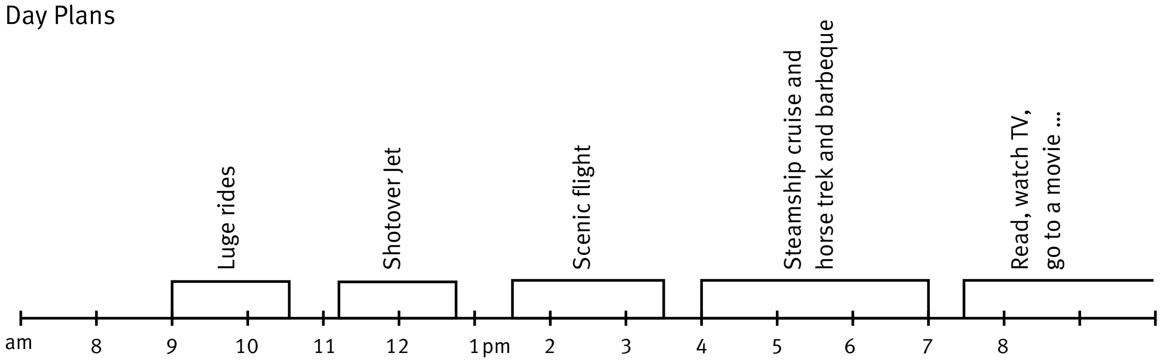
Achievement Objectives

- perform calculations with time, including 24-hour clock times (Measurement, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

ACTIVITY

The problem here is an "excess of riches": there is so much to do and so little time to do it in. This is another situation where a number line could prove useful. In fact, three number lines could be used, one for each day (see an example for 1 day below). Nothing, apart from the scenic flight, seems to start before 9.00 a.m. or run after 10.00 p.m. Some of the attractions have defined starting times while others are flexible. The solutions provided by the students will vary according to taste and preference. It may be interesting to hold a discussion about energy levels and realistic goals. Students who have been to the Gold Coast in Australia may be able to inform the class of their experiences when visiting 4 theme parks over 4 sequential days.

Day Plans



CROSS-CURRICULAR LINKS

Social Studies

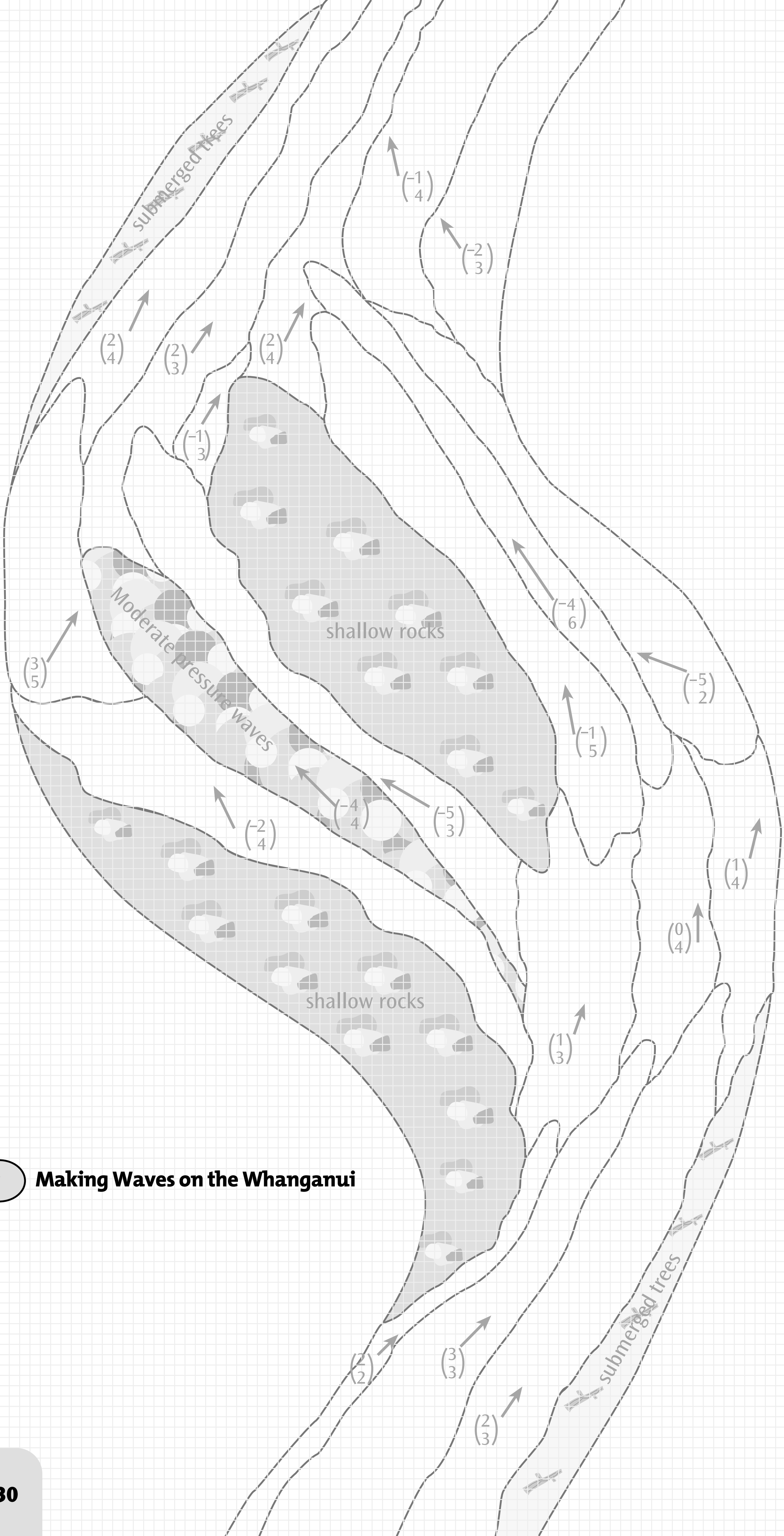
The students could, in groups, choose a place in New Zealand that is considered a tourist destination. They then identify the resources within that area that attract tourists, the ways people earn a living related to tourism, and the ways that tourists might be safeguarded. Each group could then prepare a visual display to show the relationships between the resources, the groups who utilise these resources to earn a living, and the safety strategies put in place to protect tourists. Encourage the students to use a range of information and communications technology (ICT) tools in their presentation.

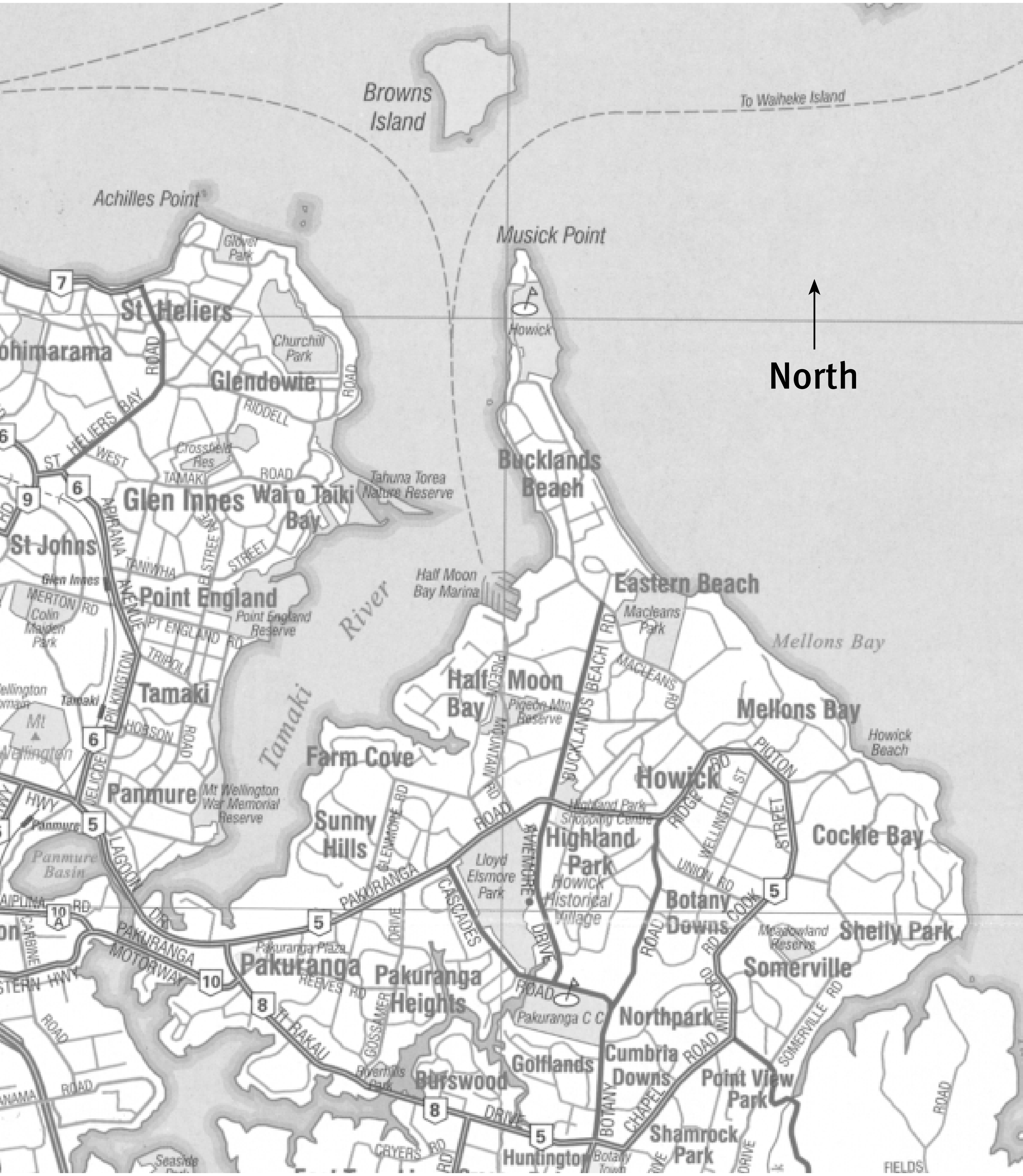
Achievement Objectives

Demonstrate knowledge and understandings of:

- how and why people view and use resources differently and the consequences of this (Resources and Economic Activities, level 4)
- how and why individuals and groups seek to safeguard the rights of consumers (Resources and Economic Activities, level 4)

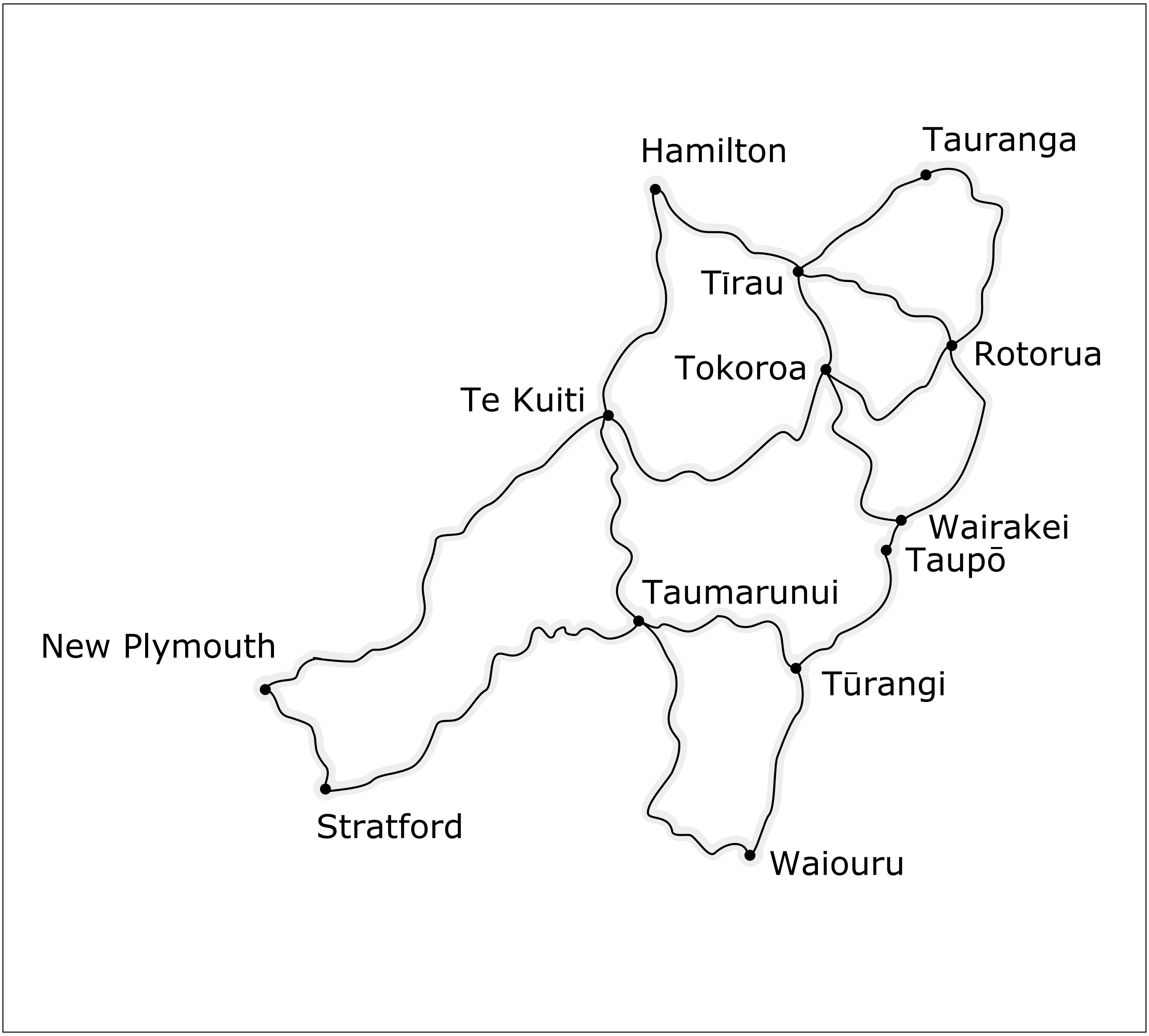
Copymaster **Making Waves on the Whanganui**





Scale 0 1 2 3 4 5

ACTIVITY TWO MAP



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