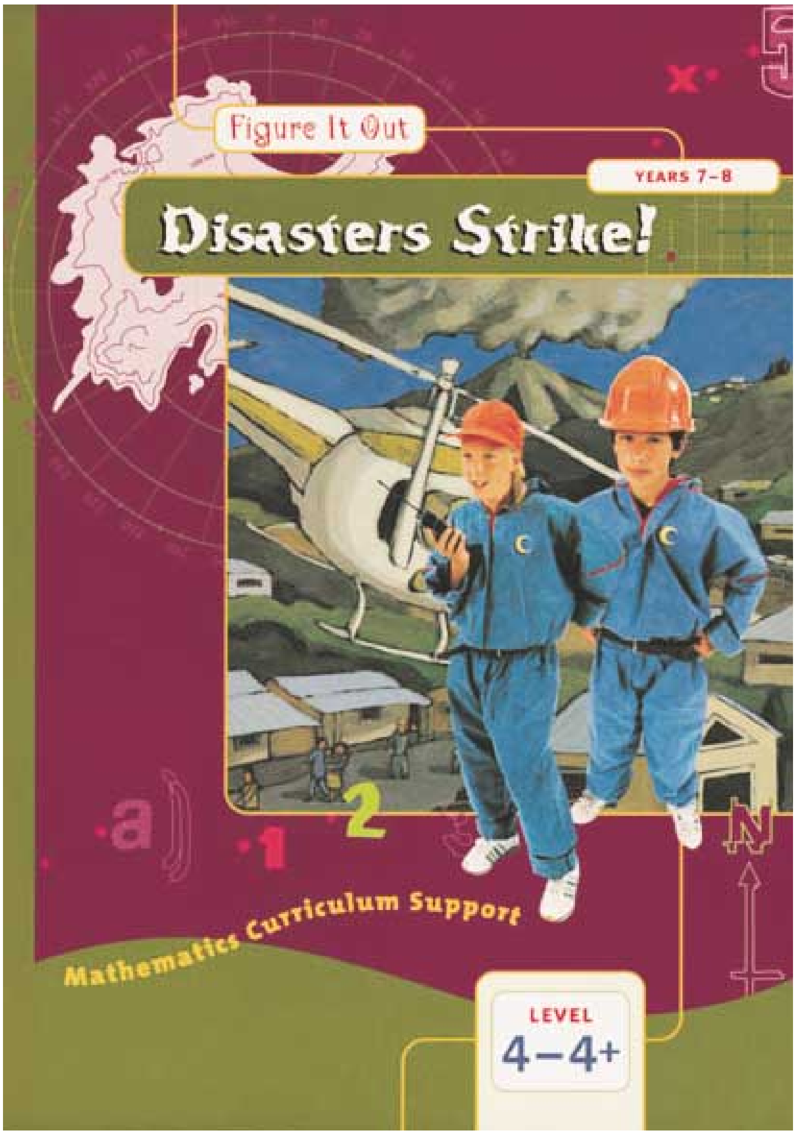
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

CONTENTS

The books for years 7–8 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand year 7–8 classrooms. The books have been developed and trialled by classroom teachers and mathematics educators and follow on from the successful series for levels 2–4 in primary schools.

Student books

The student books in the series are divided into three curriculum levels: levels 2–3 (linking material), level 4, and level 4+ (extension material). All the books are aimed at year 7–8 students in terms of context and presentation.

The following books are included in the series:
Number (two linking, three level 4, one level 4+, distributed in November 2002)
Number Sense (one linking, one level 4, distributed in April 2003)
Algebra (one linking, two level 4, one level 4+, distributed in August 2003)
Geometry (one level 4, one level 4+, distributed in term 1 2004)
Measurement (one level 4, one level 4+, distributed in term 1 2004)
Statistics (one level 4, one level 4+, distributed in term 1 2004)
Themes: *Disasters Strike!, Getting Around* (level 4, distributed in August 2003)

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and life experiences that are meaningful to students aged 11–13 years. The activities can be used as the focus for teacher-led lessons, as independent bookwork, or as the catalyst for problem solving in groups.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the student books includes full answers and explanatory notes. Students can use them for self-marking, or you can use them for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, comment on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/r/maths/curriculum/figure

Using Figure It Out in your classroom

Where applicable, each page starts with a list of equipment that the students will need to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Answers Theme: Disasters Strike!

Page 1 Counter Catastrophe

ACTIVITY

A possible answer is:
Chinda: fires; Hawke: aircraft emergencies;
Piri: earthquakes; Latu: shipwrecks; Alec: floods. You could also justify Chinda as floods and Alec as fires.

Pages 2–3 Oil Spill

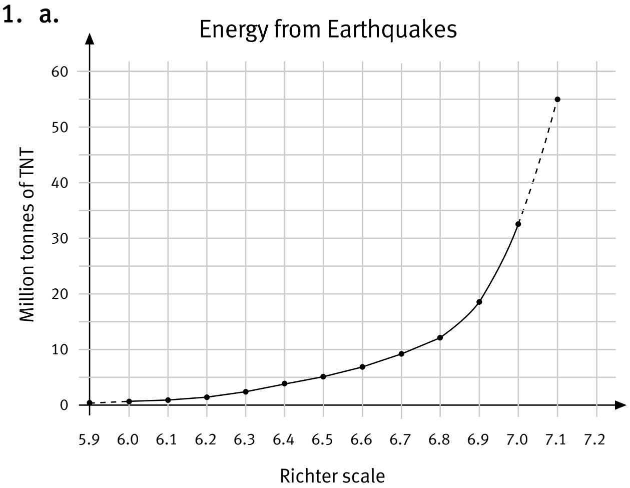
ACTIVITY

1. a. About 7.5 km from the gannet colony
   b. It should have been taken from buoy C.
2. a. See the map on the next page. Parts of the oil slicks near buoy E and F will reach the coast first.
   b. The effect of winds, tides, and ocean currents would cause the slick to move at different speeds in different places.
3. Yes, just. The oil slick will be close to the shrimp farm and other nearby parts of the coastline in 3 days, but it will not be right on the coastline, so the detergent treatment should stop it reaching the shrimp farm.

Pages 2–3 Moving and Shaking

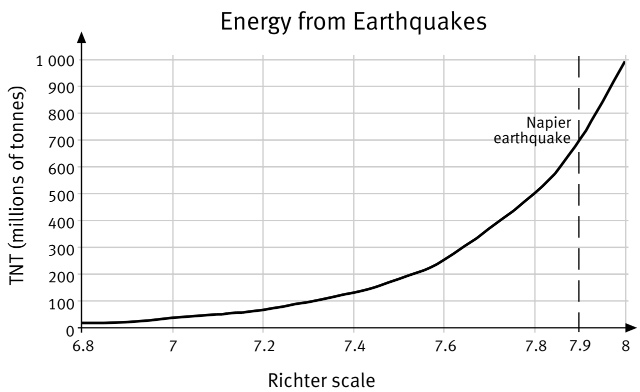
ACTIVITY

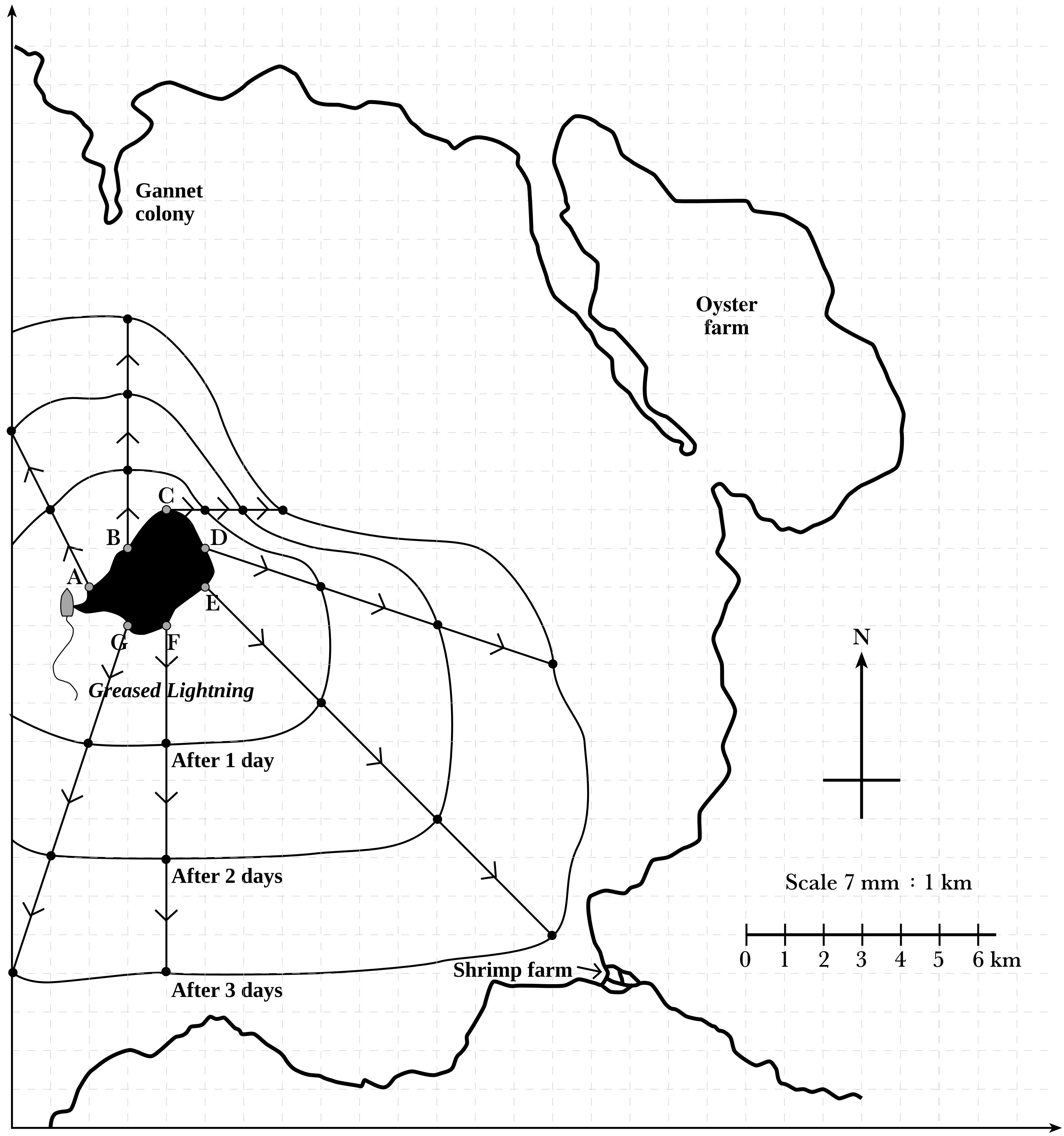
1. a.



Note: The dotted line shows how the line could be extended to show higher and lower values.

   b. Approximately 800 000 tonnes for 5.9 on the Richter scale. (This cannot be read off the graph, but it has to be less than 1 million (see 6.0 on the table in the student book). The differences between the figures in the table also point to approximately 800 000. Answers between 700 000 and 900 000 are acceptable.)
   Approximately 45 million tonnes for 7.1 on the Richter scale. (Answers between 40 and 50 million are acceptable.)
2. a. 160 000 000
   b. One billion is 1 000 000 000 or 10^9.
   One trillion is 1 000 000 000 000 or 10^{12}.
   (This is the United States version, which is uniformly accepted these days.)
3. Approximately 700 million tonnes of TNT. (Answers between 500 and 900 million tonnes are acceptable.) A graph that could be used to estimate this is:





Gannet colony
Oyster farm
C
B
D
A
E
G
F
Greased Lightning
After 1 day
After 2 days
After 3 days
Shrimp farm
N
Scale 7 mm : 1 km
0 1 2 3 4 5 6 km

Pages 6–7 A Near Miss

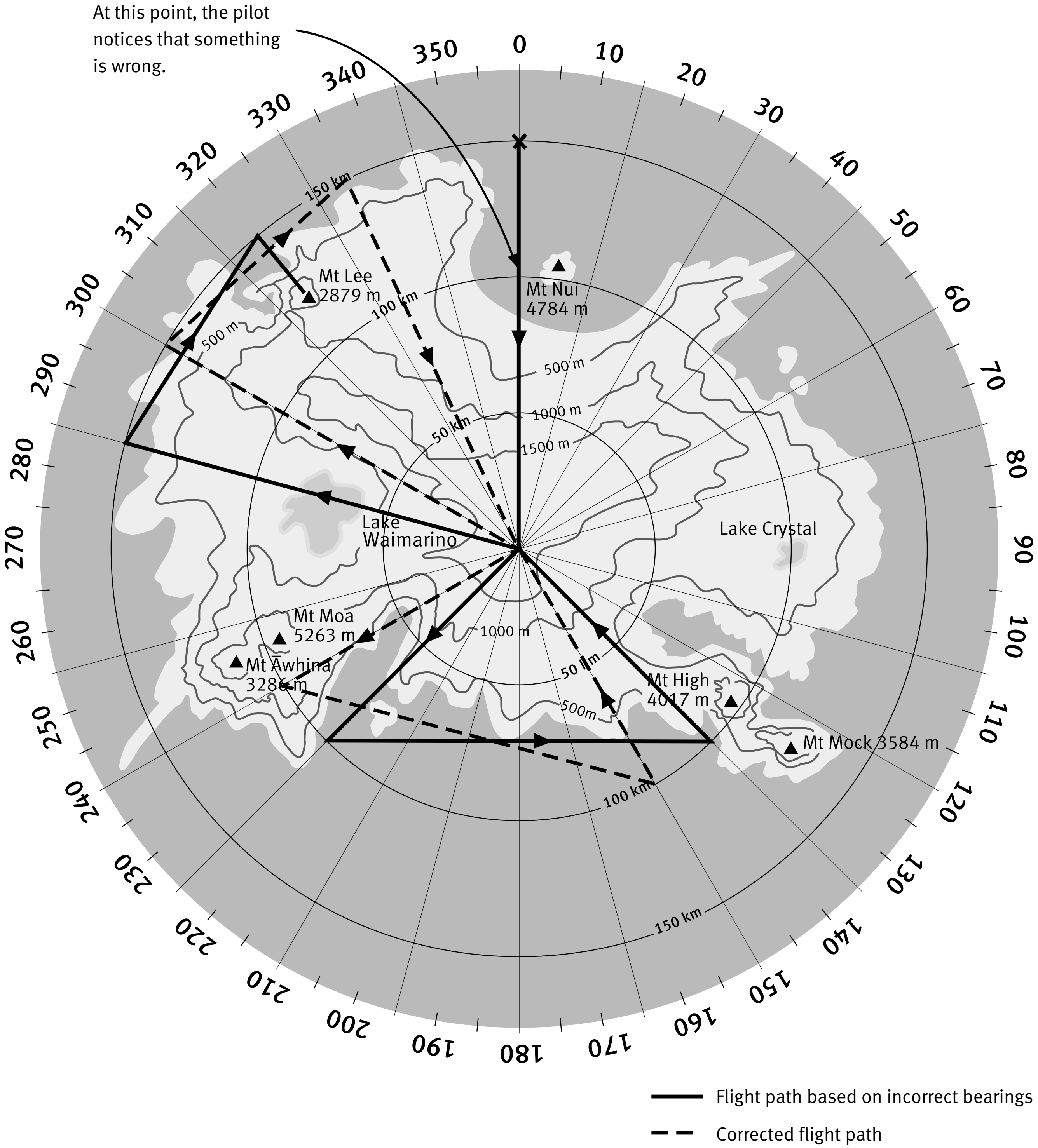
ACTIVITY

1. The flight path is drawn on the map below (see question 4).
2. a. He notices that Mount Nui is on port, not starboard.

   b. They would fly into Mt Lee.
3. The problem will be fixed if Hawke tells the pilot to continue to the origin and then correct the co-ordinate system on the computer by adding 15° to each of the bearings given in the flight instructions.
4. A possible flight path is drawn on the map below.

INVESTIGATION

A change of co-ordinates was given as a reason for the Erebus disaster.



Pages 8–10 High and Dry

ACTIVITY

1. It will slow down because the depth of the ocean is decreasing, and so the formula value will decrease. Tsunamis slow down but increase in height as the water gets shallower close to shore.
2. 2 746 s, which is 45 min 46 s
   Your distances and times may vary slightly, but your table should be similar to this:

	Average depth (m)	Speed of wave (m/s)	Distance (m)	Time (s)
a.	5 000	221.47	16 000	72
b.	3 000	171.55	23 000	134
c.	1 250	110.74	12 000	108
d.	300	54.25	24 500	452
e.	55	23.23	14 500	624
f.	5	7.00	9 500	1 356
			Total	2 746 (45 min 46 s)

3. **a.** A possible order for the minimum distance to safe land could be:

Brownlie family	3.5 km
Pye family	2.6 km
McNeil family	1.6 km
Delaney family	1.2 km
Rānui family	1.0 km
Fafeita family	0.2 km
Taunoa family	safe

   Note that these are only approximate distances. The families would try to reach further than the minimum distance if they had time.

   b. Discussion will vary. The Brownlie and Pye families are most likely to need helicopter assistance because they may not have enough time to get to safe ground after they get the tsunami warning. Other families in the area should be able to walk to safe ground in the time available.

4. Answers will vary. Many highly populated areas are below the 10 m contour. These include central Wellington, much of Napier and Gisborne, and Christchurch.

Page 11 Fire Alarm!

ACTIVITY

1. 7 088 ha (70.88 km^2, to 2 d.p.)
2. She should place a fire lookout at the highest point, 539 m, so that the fire watcher can see over the whole forest.
3. **a.** Designs will vary but should be at least 35 m tall so that there are clear sight lines above any mature trees.

   b. Scale models will vary.

   c. Diagrams will vary.

Pages 12–14 Ash in the Air

ACTIVITY

1. A strong westerly wind is blowing the ash eastwards, creating an elliptical shape.
2. The approximate areas are:

Hours	1	2	3	4
Area (km^2)	9	19	38	57

   Difference 10 19 19

   Looking at the differences, the growth appears to have stabilised at around 19 km^2 per hr, so 57 + 19 + 19 gives about 95 km^2 as a reasonable estimate of the area after 6 hrs.

3. **a.** About 1.7 m deep

   b. About 2 m deep

   c. About 1 m deep

4. **a.** Chalet, Slalom, and Titiromaunga need to be evacuated immediately.

   b. Reasonable estimates are:
   Rockville and Skipton: no ash;
   Chalet: up to 3.5 m of ash; Slalom: about 1 m of ash; Titiromaunga: up to 0.8 m of ash.

Page 15 Volcanic Volumes

ACTIVITY

1. a. 1 000 000 m^2

   b. 1 000 million m^3 (1 000 000 000 m^3 or 1 billion m^2)

   c. 10^9 m^3

2.

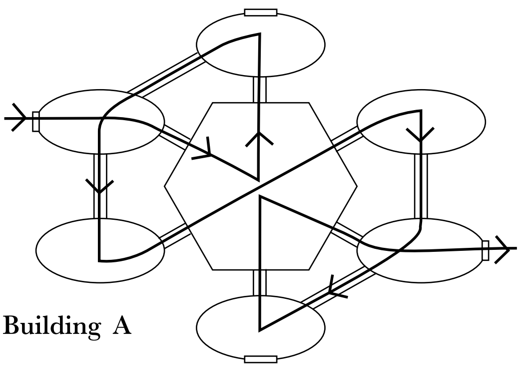
Volume of Erupted Material		
	km^3	m^3
Lake Pupuke	0.06	60 million
Rangitoto	2.45	2 450 million
Maungarei	0.21	210 million
Maungakiekie	0.35	350 million
Mt Māngere	0.19	190 million
Puke Kiwiriki	0.018	18 million
Te Tātua-o-Riukiuta	0.175	175 million
Pukekawa	0.0254	25.4 million
Ōwairaka	0.0387	38.7 million

3. Mt St Helens: 2
   Taupo: 40.82 (to 2 d.p.)

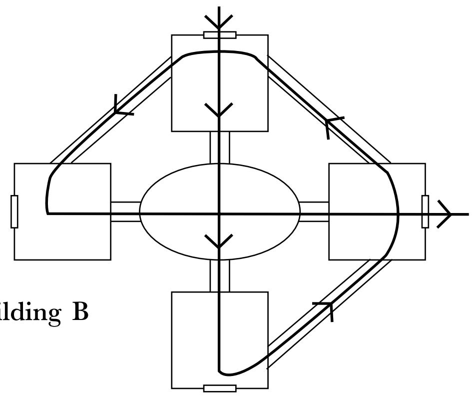
Page 16 All Fired Up

ACTIVITY

1. Possible routes are:



Building A



Building B

2. Drawings of buildings will vary.

Page 17 Virus Alert

ACTIVITY

1. a. 90

   b. 13 min 20 s

2. a. 108 000 computers

   b. 13 min 20 s

3. a. 1 111 computers (1 + 10 + 100 + 1 000)

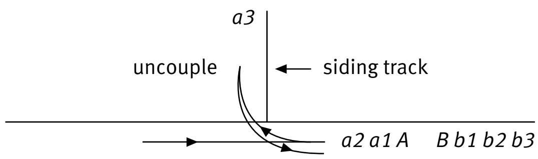
   b. Seven generations ($10^6 + 10^5 + 10^4$ …)

Pages 18–19 Multiple Mishaps

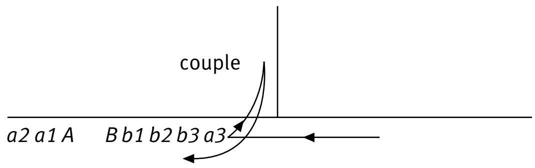
ACTIVITY

1. A possible solution that minimises coupling and uncoupling is shown below. In each step, the diagram shows the position of the trains and carriages when the step is complete.

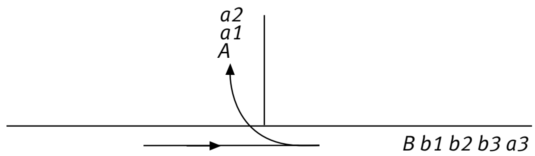
i *B* reverses sufficiently far along on the right to allow *A* to go completely past the siding. *A* reverses into the siding, uncouples *a3*, and then returns to a position in front of *B*.



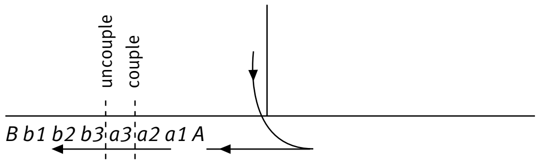
ii Both trains move independently until they occupy the left section of the track. *B* reverses into the siding, links up with *a3*, and pulls it back out onto the left section of the track.



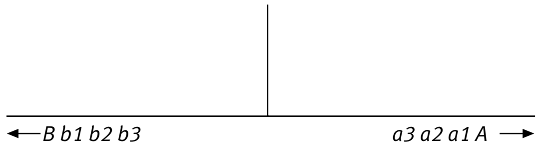
iii Both trains move independently until they occupy the right section of the track. *A* then reverses into the siding with *a1* and *a2*.



iv *B* immediately takes its 4 carriages back into the left section of track. *A* pulls *a1* and *a2* out of the siding onto the right section of the track, then reverses and couples with *a3*, which *B* then uncouples.



v Both trains head off in the desired directions with all their carriages in the original order.



2. One possible order is: i, c, d, k, a, h, f, j, g, e, b

3. One way to remove the break is to take out every second break each time. This limits the total number of dominoes that can fall by mistake at any one time. The arrows point to the breaks that are to be removed in each step.

Step i

100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100

Step ii

200 | 200 | 200 | 200 | 200 | 200 | 200 | 200

Step iii

400 | 400 | 400 | 400

Step iv

800 | 800

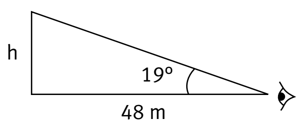
4. The most economical solution is to undo the links on the 2-link pieces and use these four separate pieces to join the other pieces.

5. The order is: 1, 6, 2, 10, 3, 7, 4, 9, 5, 8

Pages 20–21 Peanut in Trouble

ACTIVITY

1. a. The tree is 18 m tall.
This could be found by using the scale of the diagram, 1 cm : 3 m, and measuring accurately.
Alternatively, you could compare the height of the tree to the distance of 48 m.
You may be advanced enough to use trigonometry to solve the problem:



$^{h}/_{48}$ = tan 19°
$\Rightarrow$ h = 48 x tan 19°
$\Rightarrow$ h = 16.5 m

Adding the height from the ground to Patu's eyes gives 16.5 + 1.5 = 18 m.

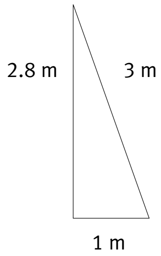
b. Using the scale 1 cm : 3 m gives 4.5 m above ground level. Alternatively, the lower branches are about one-quarter of the way up the trunk. $^{1}/_{4}$ of 18 is 4.5, so the branches are 4.5 m above ground level.

2. Discussion will vary.

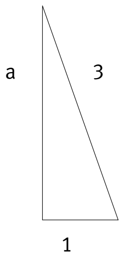
3. a. Patu places the base of the ladder one-third of the height of the ladder away from the tree, for example, $^{1}/_{3}$ x 3 m = 1 m and $^{1}/_{3}$ x 6 m = 2 m.

b. The angle remains unchanged (at about 70.5°) because, although the size of the triangle changes, the shape doesn't.

c. The 3 m ladder reaches about 2.8 m up the trunk. The 6 m ladder reaches about 5.7 m up the trunk. A scale diagram using a scale of 1 cm : 1 m will have these measurements:



If you used Pythagoras' theorem, your working could look like this:



$$
\begin{aligned}
a^2 + 1^2 &= 3^2 \\
a^2 &= 3^2 - 1^2 \\
a^2 &= 8 \\
\Rightarrow a &= \sqrt{8} \\
&= 2.8
\end{aligned}
$$

d. The 4 m ladder reaches about 3.8 m up the trunk, and the 5 m ladder reaches about 4.7 m up the trunk. The 5 m ladder is best.

Pages 22–23 A Disastrous Day

GAME

A game involving manipulating numbers

Page 24 A Helping Hand

ACTIVITY

1. a. It takes 3 years before the birds are adult and can lay their first eggs. After that, they lay every second year.

   b. 3 eggs are laid each time by every adult female, and all survive.

   c. 2 male and 1 female each time to each adult female.

2. a. After 17 years, there will be 767 birds (excluding any deaths).

   b. After only 11 years, there will be 486 birds (excluding any deaths).

YEARS 7–8

Teachers' Notes

Overview **Theme: Disasters Strike! (level 4–4+)**

Page 1 Counter Catastrophe

Achievement Objectives

- classify objects, numbers, and ideas (Mathematical Processes, developing logic and reasoning, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

Other mathematical ideas and processes

Students will also:

- develop analytical reading skills essential to the understanding of a problem
- interpret and analyse loosely formed statements to present an objective solution to a problem.

ACTIVITY

The ability to read a set of statements analytically is an essential skill. Logic problems like this require students to use their "reading with understanding" skills to sift through the information and sort the statements into a logical order from which they can draw conclusions.

Note that the answer is not obvious on a first reading. The information has to be read several times before all the links can be made. A discussion about how detectives solve crime may be helpful. The clues are examined over and over again before the links become clear.

The most difficult task is deciding where to start and what strategy to use. A table, as suggested in the activity and shown in full below, is often a good approach to logic problems because the process of completing the table requires the student to sift and organise the facts.

Once the table has been created, the student can examine the clues and record positive and negative links on the table, using ticks and crosses or question marks. Especially at the start of the process, it is just as important to know what a particular person definitely does not do as it is to know their speciality.

The clues:

1. Both Chinda and Alec work with water. For one, it is always the problem; for the other, it is usually part of the solution.
2. In built-up areas, Piri's emergencies can cause Chinda's emergencies.
3. Water is always a factor in Latu's emergencies; sometimes it is for Hawke, too.
4. Piri's emergencies can cause much more widespread destruction than Hawke's.

Clue 1

This clue tells us that Chinda and Alec deal with fires and floods. (Although it is true that water is a factor in other types of emergencies, such as shipwrecks, floods are the only emergency in which, by definition, water is *always* the problem.) We can summarise our deductions at this point with the following table:

	Shipwrecks	Floods	Fires	Aircraft	Earthquakes
Chinda	X	?	?	X	X
Hawke		X	X		
Piri		X	X		
Latu		X	X		
Alec	X	?	?	X	X

Clue 2
Piri can't be the shipwreck person because shipwrecks won't cause a problem in built-up areas. Also, we know from the first clue that Piri is not the fires or floods person, so the question is whether he is the aircraft or earthquakes person. Either type of emergency is more likely to cause a fire than a flood in a built-up area, so fires could be Chinda's specialty, which leaves floods to Alec:

	Shipwrecks	Floods	Fires	Aircraft	Earthquakes
Chinda	X	X	✓	X	X
Hawke		X	X		
Piri	X	X	X	?	?
Latu		X	X		
Alec	X	✓	X	X	X

However, earthquakes can also result in flooding (for example, if a dam bursts near a built-up area), so it could also be that Chinda is the floods person, leaving fires to Alec.

Clue 3
Given that Chinda and Alec are the fire and floods people, there is only one other type of emergency in which water is always a factor, and that is shipwrecks, so these must be Latu's responsibility. This leaves us with aircraft and earthquake emergencies to be allocated between Piri and Hawke:

	Shipwrecks	Floods	Fires	Aircraft	Earthquakes
Chinda	X	X	✓	X	X
Hawke	X	X	X	?	?
Piri	X	X	X	?	?
Latu	✓	X	X	X	X
Alec	X	✓	X	X	X

Clue 4
Earthquakes can cause more widespread destruction than aircraft emergencies, so Piri must be the earthquakes expert and Hawke the aircraft emergencies expert:

	Shipwrecks	Floods	Fires	Aircraft	Earthquakes
Chinda	X	X	✓	X	X
Hawke	X	X	X	✓	X
Piri	X	X	X	X	✓
Latu	✓	X	X	X	X
Alec	X	✓	X	X	X

Many students thrive on problems like these once they get a taste for them. Most bookshops sell inexpensive logic puzzle books that can be used as a classroom resource for improving analytical reading strategies. *Judy's Logic Problems* is a website with many good logic problems for different ages. It also gives advice on constructing and solving problems and has links to other similar sites. The address is http://pages.prodigy.net/spencejk

CROSS-CURRICULAR LINKS

The students could imagine that they are part of a team responding to a hypothetical disaster. In groups, they could create a plan of action that outlines how their team will respond and who is responsible for each role. Their plan would need to consider action from the time that they receive the information through to a debrief on how effective the team was. The role plays could be recorded or acted out for the other groups to view and evaluate.

Achievement Objectives

Social Studies

Demonstrate knowledge and understandings of:

- how people organise themselves in response to challenge and crisis (Social Organisation, level 4)
- how and why people exercise their rights and meet their responsibilities (Social Organisation, level 4)

Health and Physical Education

- describe and demonstrate a range of assertive communication skills and processes that enable them to interact appropriately with other people (Relationships with Other People, level 4)
- specify individual responsibilities and take collective action for the care and safety of other people in their school and in the wider community (Healthy Communities and Environments, level 4)

Pages 2–3 Oil Spill

Achievement Objectives

- solve practical problems which can be modelled, using vectors (Geometry, level 5)
- draw and interpret simple scale maps (Geometry, level 3)

Other mathematical ideas and processes

Students will also:

- draw vectors on a grid when given the components
- explore vector addition numerically and graphically
- extrapolate a trend when given reasonable historical data.

Before tackling this activity, it would be helpful for the students to gain some understanding of how oil slicks behave. Some of the websites that could be of great value within the classroom are:

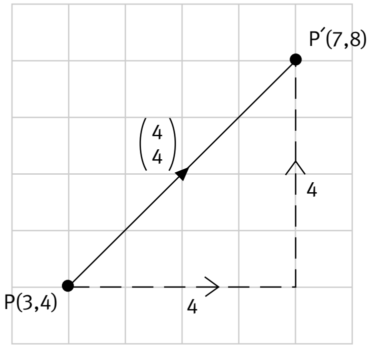
- New Zealand Maritime Safety Authority: Managing an oil spill www.msa.govt.nz/protection/managing.htm
- Brucie and Albert Caring for the Coast: www.brucecgull.com/html/oil_slick.htm
- US Environmental Protection Agency www.epa.gov/oilspill/eduhome.htm
- Environment Canada www.ec.gc.ca/ee-ue/pub/chocolate_e/toc.asp

The last two sites include ideas for classroom experiments that can be carried out to model the spread of an oil slick.

ACTIVITY

The buoys used to track the movement of the slick must be plotted on the map in order to understand the way the slick is moving. Each buoy moves a certain distance over a 24 hour period due to ocean currents and winds. The forces acting on the buoys will be acting on the oil slick as well. The change in position of each buoy during a day is called a *translation*. Vectors with two components (west-east and north-south) are the ideal mathematical tool for describing the movement of each buoy.

Vectors describe a translation:

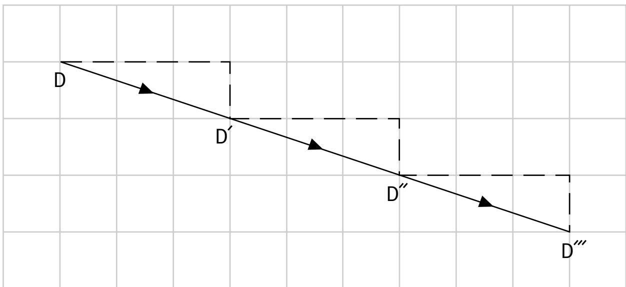


The buoy at position (3,4) is translated to position (7,8) when the vector $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$ has acted on it.

The top number in the vector is often known as the *x* component and describes horizontal movement east (+) or west (–). The bottom number is the *y* component and describes vertical movement north (+) or south (–). Whether the number is at the top or bottom of the vector and whether it is positive or negative are both critical details, and order does matter! The students need to learn that they should not put a dividing bar between the top and bottom components as they would in a fraction because the two numbers are separate.

A value of 4 in the top number means the buoy moves 4 kilometres towards the east. However, –4 means that the buoy is travelling 4 kilometres in the opposite direction to east, which is west. Similarly, if the bottom number of the vector is negative, the buoy is travelling south, not north. It would be a good idea for the students to examine a number line and refresh their memories as to how the positive and negative signs affect direction of movement along the number line.

In this activity, the vectors describe the movements of each buoy during 1 day. If we examine just one of these buoys, for example, D, over 3 days, we have the following:



So the movement of buoy D over a 3 day period is described by applying the vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ three times. Note that the start point for each translation is the end point of the previous translation. For the students to predict the movement of the slick, they will need to apply the given vectors to the buoys, each application representing the movement for 1 day. The best strategy is to draw the location of all buoys at the end of day 1 and then draw in the outline of the slick. Repeat this for days 2 and 3.

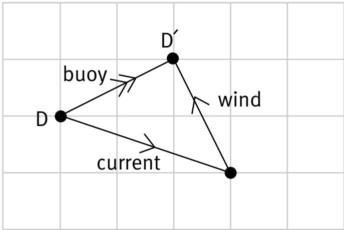
Notice that the translation of buoy D for the entire 3 day period can be described by the vector $\begin{pmatrix} 9 \\ -3 \end{pmatrix}$, which is the same as $\begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

This shows that vectors can be added by adding together the top numbers and then the bottom numbers of each vector. The result is known as the *resultant vector*.

As an extension activity, the students could consider the problem in a little more detail. It is unlikely that the ocean current and the wind will be travelling in exactly the same direction at the same speed. Suppose the movement of buoy D over 1 day due to the current is described by the vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and the wind by the vector $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$.

Where will the buoy move to in 1 day? Adding the vectors gives $\begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

The following diagram shows the combined effect of the wind and the current on buoy D:



$$\begin{pmatrix}3\\-1\end{pmatrix} + \begin{pmatrix}-1\\2\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix}$$

Note that, on a diagram, vectors are added by joining the "tail" of the second vector to the "head" of the first vector. When two vectors are added, the resultant vector is marked with two arrowheads and the three vectors form a triangle. Note also that whether you add wind to current or current to wind, you get the same outcome.

A further extension task would be to remodel this situation as follows:
Take the original vectors given in the activity as the daily translations operating on each buoy due to ocean currents. Then suppose a constant wind blows across the bay, described by the vector $\begin{pmatrix}2\\1\end{pmatrix}$. Which buoy will reach the coast first? Will Latu now be able to prevent disaster striking?

Hint: The daily resultant vector for buoy D would be $\begin{pmatrix}3\\-1\end{pmatrix} + \begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}5\\0\end{pmatrix}$. Find the resultant daily vectors for each of the remaining buoys and then re-plot the movement of the slick using these new vectors.

CROSS-CURRICULAR LINKS

Social Studies

The students could imagine that they are reporters who have arrived at the scene of an oil spill that has just occurred. They need to design questions to ask someone who is responsible for the supertanker to determine what their priorities are. They also need to design questions for an interview with a member of a conservation group who is concerned about the effects the oil spill will have on the environment. The interviews should consider what resources are available within New Zealand to minimise the disaster.

Achievement Objective

- demonstrate knowledge and understandings of how and why people view and use resources differently and the consequences of this (Resources and Economic Activities, level 4)

Science

The students could investigate the chemical changes that occur when oil slicks are treated with detergent. They could also examine the effects of oil slicks on the environment.

Achievement Objectives

- investigate and describe ways of producing permanent or temporary changes in some familiar materials (Material World, level 4)
- investigate the positive and negative effects of substances on people and on the environment (Material World, level 4)
- investigate a local environmental issue and explain the reasons for the community's involvement (Planet Earth and Beyond, level 4)

Pages 4–5 **Moving and Shaking**

Achievement Objectives

- sketch and interpret graphs on whole number grids which represent simple everyday situations (Algebra, level 4)
- explain the meaning and evaluate powers of whole numbers (Number, level 4)

Other mathematical ideas and processes

Students will also:

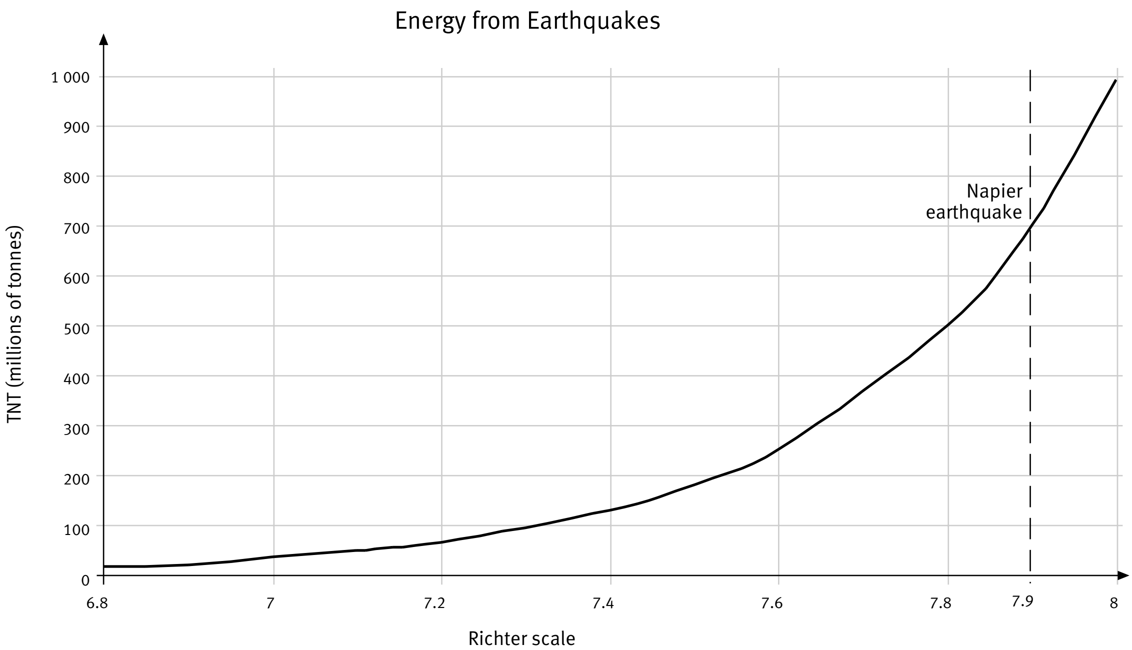
- make extrapolations and interpolations using a graph
- be introduced to the concept of exponential growth.

ACTIVITY

In this activity, students use the Richter scale and tonnes of TNT to investigate the energy released in earthquakes.

In question **1**, to find how many tonnes of TNT are required to simulate 5.9 and 7.1 on the Richter scale, we have to look beyond the boundaries of the given data. These values can be approximated by using the graph to *extrapolate* the required information. Another way of expressing this would be to "obtain an approximate reading by studying the trend of the graph beyond the given data". To do this, we extend the graph beyond its plotted boundaries, following the trend of the line or curve. An extrapolation from the Richter scale graph is shown in the Answers. The students could extend the line on their own graphs and use it to read an approximate value off the vertical axis for 7.1.

To find the TNT equivalent of the Napier earthquake, a process known as *interpolation* is applied to a graph of the data already provided or extrapolated. Simply put, interpolation is the process of estimating values that lie between given points on the graph. This is shown in the diagram below.



For 5.9, which is difficult to read off the graph, they need to keep in mind that it has to be less than 1 million, the value given for 6.0 on the student page. As suggested in the Answers, if the students calculate the differences between the tonnes of TNT, they should see a decreasing pattern (… 1.1, 0.8, 0.5, 0.3). So, the next lowest rating on the Richter scale, 5.9, should be about 0.2 less than the 1.0 million tonnes for 6.0, that is, about 800 000 tonnes.

An excellent website on New Zealand earthquakes is www.gns.cri.nz

For detailed information on the Napier earthquake, try
www.library.christchurch.org.nz/Childrens/NZDisasters/Napier.asp
For more information on the Richter scale, see www.matter-antimatter.com/earthquakes.htm

CROSS-CURRICULAR LINKS

Social Studies and Health and Education

The students could create a timeline of major disasters within New Zealand's history. Each student could identify a disaster for which they carry out detailed research. Their study needs to include the effects that the disaster had on the people at the time and ways in which it changed their lives. Using the underlying concept of hauora, from the health and physical education curriculum, the students could list the impacts under the four headings: physical, mental and emotional, social, and spiritual well-being.

Achievement Objectives

Social Studies

Demonstrate knowledge and understandings of:

- causes and effects of events that have shaped the lives of a group of people (Time, Continuity, and Change, level 4)
- how and why people experience events in different ways (Time, Continuity, and Change, level 4)

Health and Physical Education

- identify the effects of changing situations, roles, and responsibilities on relationships and describe appropriate responses (Relationships with Other People, level 4)

Science

The students could investigate how our landscape's behaviour in an earthquake is determined by its geological structure.

Achievement Objectives

- collect and use evidence from landforms, rocks, fossils, and library research to describe the geological history of the local area (Planet Earth and Beyond, level 4)
- investigate and describe processes which change the Earth's surface over time at local and global levels (Planet Earth and Beyond, level 5)

See also: Building Science Concepts Book 40.

Pages 6–7 A Near Miss

Achievement Objectives

- read and construct a variety of scales, timetables, and charts (Measurement, level 4)
- design and use models to solve measuring problems in practical contexts (Measurement, level 5)

Other mathematical ideas and processes

Students will also:

- take into account three measures: distance, direction, and altitude
- make decisions based upon a simultaneous assessment of these measures.

ACTIVITY

This activity will challenge even the more able students. It focuses on map reading and route finding, but it poses at least four difficulties you need to bear in mind when setting your students to work on it:

- Students may find themselves irresistibly drawn to the conclusion that they are looking at Antarctica and that the point in the centre of the diagram, the origin, is the South Pole. This confusion comes from the shape of the territory, the "pole" in the centre, the circular scale around the edge, and the explicit mention of Mount Erebus in the investigation. The problem with this is that compass bearings lose their meaning at the poles: at the South Pole, every direction is north.

- Some of the flight directions are given as polar co-ordinates (the direction you fly and the distance you travel), while others are given as destination points (you fly directly towards a point whose location you are given). These are two quite different systems and may confuse some students.
- If students are paying close attention to the landmarks listed in the chart, they will not be able to reconcile them with the other instructions. In the first leg, Mount Nui should be on starboard (on your right as you travel), but it is on port (left). The students are asked to identify this problem in question **2**.
- It can be very difficult for students to interpret and fix the error of 15 degrees if they are confused about the clockwise and anticlockwise movements involved.

Looking at the four issues in order:

If this were Antarctica, the gradations around the edge of the diagram would be lines of longitude, not bearings in the sense of clockwise direction from north. If they were lines of longitude, the scale would be labelled 0° to 180° and then back to 0° because longitude is measured west and east from the Greenwich meridian. For this reason, the diagram should be taken to represent some totally different, completely fictional country. The students can then think of the gradations on the scale as points on a compass: 0° is north, 90° is east, 180° is south, and 270° is west. The circles are then unrelated to circles of latitude and are simply circles drawn around a fixed reference point for the convenience of the pilot (in this case, the student).

The second issue should not be a major problem as long as the students are aware that there are two different systems here for specifying the legs of the journey. In the first system, the pilot is told how far to go and in what direction. In the second system, the pilot does not need to be told the distance; only the destination. Both are legitimate, though normally they would not be mixed like this.

Many students will ignore the landmarks when initially plotting the flight path, thinking they are extraneous details or only there for those who can't do without them. However, when the students get to question **2**, they will need to look closely at the first leg and match all the landmarks to their flight path. They should then notice that Mount Nui is on port, not starboard, and that the flight path leads directly to Mount Lee (and disaster).

The best way for the students to feel confident about the clock direction is to hold a pencil on the chart to represent the first leg of the journey (which should have Mount Nui on starboard) and to keep it fixed there while they rotate the chart into the position they were given (Mount Nui on port). This will confirm that the bearings have been rotated clockwise. To find the correct instructions, every compass direction on the list therefore needs to be rotated anticlockwise: just add 15 degrees to it.

Finally, the students need to pay attention to the contours of the land if they are to see what the danger is. The tsunami activity on pages **8–10** explores the meaning of contour lines in some detail.

For an excellent summary report of the Mt Erebus disaster, visit the website
www.library.christchurch.org.nz/Childrens/NZDisasters/Erebus.asp

CROSS-CURRICULAR LINKS

Using a variety of resources, you could introduce Robert Scott's ill-fated trip to reach the South Pole. The students could imagine that they are a member of Robert Scott's party on this trek and write or record a set of questions that investigate what motivates an individual to explore in remote, extreme climates. They could ask a classmate to answer the questions from their findings.

Achievement Objectives

Social Studies

Demonstrate knowledge and understandings of:

- causes and effects of events that have shaped the lives of a group of people (Time, Continuity, and

Change, level 4)

- how and why people experience events in different ways (Time, Continuity, and Change, level 4)

Health and Physical Education

- identify the effects of changing situations, roles, and responsibilities on relationships and describe appropriate responses (Relationships with Other People, level 4)

Pages 8–10 High and Dry

Achievement Objectives

- draw and interpret simple scale maps (Geometry, level 3)
- carry out measuring tasks involving reading scales to the nearest gradation (Measurement, level 4)

Other mathematical ideas and processes

Students will also:

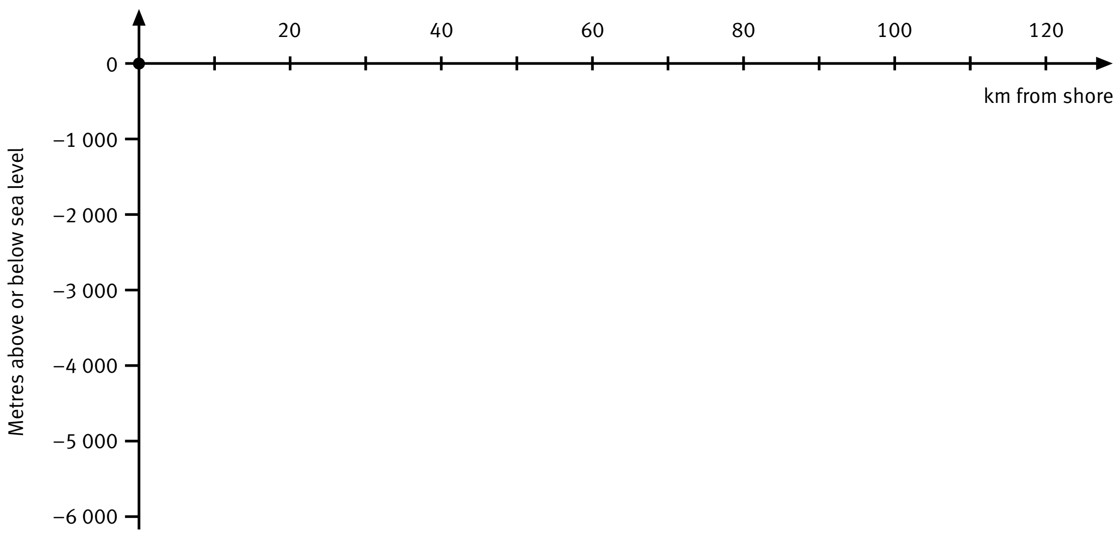
- read map contours
- investigate the relationship between distance, time, and speed
- use square roots in formulae.

ACTIVITY

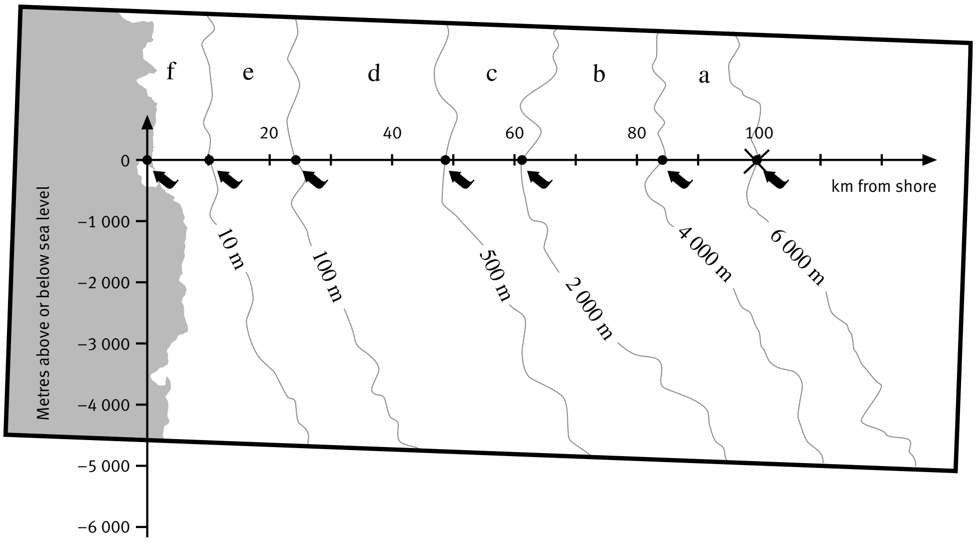
One of the challenges in this activity is to understand what is meant by "contours". Contours on a map are lines that show areas of the same height or, in the case of oceans, areas of the same depth. In the map on page **9** of the student book, the line marked 500 metres means that every bit of the ocean floor along that line is 500 metres below sea level. Likewise, the 2 000 metre line shows all the terrain that is 2 000 metres deep. However, even with this information, it can be difficult for the students to visualise the terrain by simply looking at a map and mentally processing the numbers. In this situation, the students can learn a lot by generating a profile of the ocean floor using the "cross section" method. To do this, they could follow these steps:

i. Draw a set of axes. Let the *x* axis represent horizontal distance and the *y* axis vertical elevation. The scale used for the axes must be the same as that used on the map, that is, 1 centimetre represents 10 kilometres on the horizontal. For the vertical scale, make 1 centimetre represent 1 000 metres.

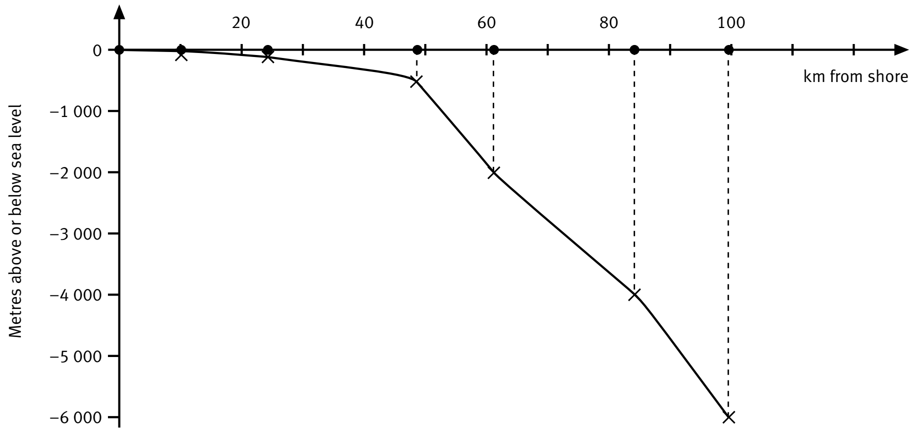
Ocean floor contours, axes only:



ii. Overlay your axes on the contour map so that the horizontal axis matches up exactly with the line of the transect (the line showing the direction in which the tsunami will move). Make a mark on the axis where each contour line crosses (the coastline crosses at 0).



iii. For each of the points you have marked along the horizontal axis, plot the depth. Join the points with a smooth curve, and you will have a scale approximation of the profile of the ocean floor.



Alternatively, the students could enter the distance and the depths into a spreadsheet and use the program to draw a similar graph (select XY Scatter) of the ocean floor profile. Note that the profile shown is exaggerated because of the scale used. To get an accurate representation, they would need to use the same scale on both axes.

As a reinforcement exercise, the students could be given photocopies of any contour map with a marked cross section. Their task is to generate the profile. This could lead to a discussion about how terrain affects travelling time, especially if journeying on foot.

Returning to the original problem, in question **3**, the students are asked to create an evacuation plan for the region. One strategy is:

i. For each of the seven places with people:
   - identify the nearest point that is above 10 metres
   - measure the distance to that point
   - decide who can get to that high ground without assistance in the time available.

ii. For those needing helicopter assistance:
   - Who should be evacuated first?
   - Where should they be taken (to the closest high ground)?

You may need to put a limit on the capacity of the helicopter.

For more background information and a stunning collection of photographs, visit The Pacific Tsunami Museum website at www.tsunami.org. Another site, www.geophys.washington.edu/tsunami, has an excellent computer-generated animation of a tsunami as it travels across the oceans.

CROSS-CURRICULAR LINKS

Social Studies and Health and Education

The students could investigate and then list the physical, cultural, and emotional reasons for living in a particular location. Ask them to critically analyse, using a continuum, which points they consider to be of greatest significance. They then could design the ideal place to live that meets the needs of a specified family unit.

Achievement Objectives

Social Studies

- demonstrate knowledge and understandings of why and how people find out about places and environments (Place and Environment, level 4)

Health and Physical Education

- access and use information to make and action safe choices in a range of contexts (Personal Health and Physical Development, level 4)

Science

The students could investigate the impact of tsunamis on land formation and erosion.

Achievement Objective

- investigate and describe processes which change the Earth's surface over time at local and global levels (Planet Earth and Beyond, level 5)

Page 11 Fire Alarm!

Achievement Objectives

- read and construct a variety of scales, timetables, and charts (Measurement, level 4)
- draw and interpret simple scale maps (Geometry, level 3)
- model and describe 3-dimensional objects illustrated by diagrams or pictures (Geometry, level 3)
- make a model of a solid object from diagrams which show views from the top, front, side, and back (Geometry, level 4)

Other mathematical ideas and processes

Students will also:

- convert units of area from one measure to another
- use isometric drawings to create 3-dimensional models.

ACTIVITY

In this activity, students convert scale distance and area into actual distance and area.

Before the students set out to calculate the area of the forest, review the units they will need to use. A hectare is an area equivalent to a square that is 100 metres by 100 metres, which is (10 x 10) x (10 x 10) = 10^4, or 10 000 square metres. A good learning experience for the students is to mark out a rectangle with an area of a hectare on the school grounds. As an extension, have them calculate the size of the school grounds in hectares. Then, to develop an appreciation of scale, have them draw their school grounds using a scale such as 1 : 1 000.

To prepare the students for the forest problem, discuss how to convert square metres to hectares. We've seen that 1 ha = 10 000 m^2. If possible, mark off an area that has both its length and breadth somewhere between 100 and 200 metres. Have the students calculate the area in square metres to begin with. For example, if the section is 140 metres by 120 metres, the area is 16 800 m^2. To find this area in square centimetres, there are two options from which to choose:

- convert the length and breadth to centimetres and then calculate the area;
- convert the square metres to square centimetres by dividing the area in square metres by 10 000 (1 square metre = 100 centimetres x 100 centimetres, or 10 000 cm^2).

The students would benefit from doing both calculations in order to appreciate the value of each approach. The students also need to understand the relationship between hectares and square kilometres. A hectare occupies an area 100 metres by 100 metres, and a square kilometre occupies an area 1 000 metres by 1 000 metres, so there are 100 (10 x 10) hectares in a square kilometre.

The students will need to decide what units they are going to use for their calculations. They can either:

- convert the scale lengths to actual lengths and then calculate the area (1 centimetre represents 125 000 centimetres, which is 1.25 kilometres);
- do all the calculations in scale units and convert the result to hectares at the end.

The first alternative is probably better. As a general principle, the students should be encouraged to work in units appropriate to the problem. For an area like this, the kilometre is the best choice because that is the unit we use for large-scale land measurements. If they choose to work in small units such as metres or centimetres, they will find themselves dealing unnecessarily with very large numbers and will greatly increase the risk of error.

The simplest calculation is probably this:

i. The rectangle is 7.2 by 6.3 centimetres.

ii. As 1 centimetre represents 1.25 kilometres, the actual dimensions are 9 kilometres by 7.875 kilometres, giving an area of 70.88 square kilometres (to 2 d.p.).

iii. As there are 100 hectares in a square kilometre, the area of the forest in hectares is 70.88 x 100 = 7 088 hectares.

The Lookout

The best point on which to build the lookout is obviously the highest point in the forest. The highest point according to the map is 539 metres. However, trees will be growing in this area as well, so the lookout must be higher than mature pine trees, which are approximately 32 metres in height.

Some design issues to take note of will be the size of the base, how the base will be secured to the ground, and the stability of the structure. Many students may be tempted to build the model first and then draw the plans. However, the desired learning outcome is that the students should be able to build according to scale. To achieve this, it is vital that they create a scale drawing first, using the given scale of 1:200. This means that if the lookout is 35 metres (3 500 centimetres) high, the height of the drawing should be $^{3\,500}/_{200}$ = 17.5 centimetres high, as should the model.

Some students have a great deal of difficulty with spatial perception and moving from a 2-dimensional drawing to a model. The opposite is also true. For these students, it may be worthwhile to have them plan their model using isometric block diagrams. The model they design will understandably be very "square". However, the learning goal here is for the students to gain confidence and familiarity with scale models and 3-dimensional drawings. Once this confidence has been gained, a more open approach to the planning and drawing can be adopted.

Cross-curricular links

Social Studies and Health and Education

The students could explore the importance of the forests to Māori in pre-European times. Included in this study could be the beliefs of Māori around conservation. The students could make comparisons, using a Venn diagram, between conservation as it was in the 19th century and conservation as it exists in the 21st century, with an emphasis on the effects that conservation has on the people. The students could draw up a plan outlining ways to ensure that our native trees are preserved. Where appropriate, encourage them to relate this plan to their school or local environment.

Achievement Objectives

Social Studies

Demonstrate knowledge and understandings of:

- why and how individuals and groups pass on and sustain their culture and heritage (Culture and Heritage, level 4)
- how and why people view and use resources differently and the consequences of this (Resources and Economic Activities, level 4)

Health and Physical Education

- specify individual responsibilities and take collective action for the care and safety of other people in their school and in the wider community (Healthy Communities and Environments, level 4)

Science

The students could investigate the impact of forest fires on the environment and how living things adapt.

Achievement Objectives

- investigate and describe special features of animals and plants which help survival into the next generation (Living World, level 4)
- investigate a national environmental issue and explain the need for responsible and co-operative guardianship of New Zealand's environment (Planet Earth and Beyond, level 5)

Pages 12–14 Ash in the Air

Achievement Objectives

- model and describe 3-dimensional objects illustrated by diagrams or pictures (Geometry, level 3)
- carry out measuring tasks involving reading scales to the nearest gradation (Measurement, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)

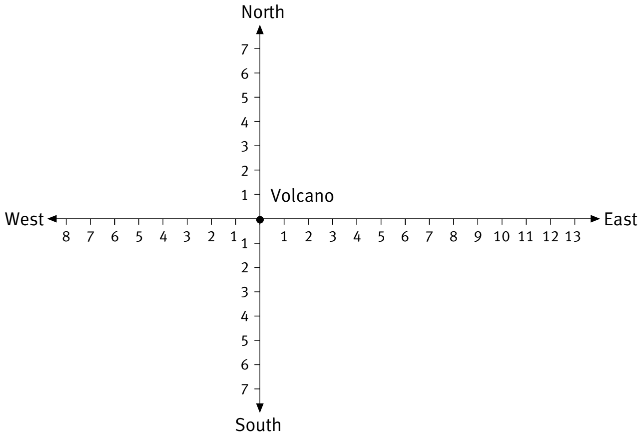
Other mathematical ideas and processes

Students will also:

- use simple formulae
- work with areas of circles and ellipses
- interpret graphs
- learn to use everyday objects to simulate complex situations and understand how these models can be linked to the real situation.

ACTIVITY

The following practical demonstration that models the situation will help the students to appreciate what this problem is about. On a sheet of graph paper, mark a pair of axes as shown in the figure below. Note that the origin is the centre of the volcano.



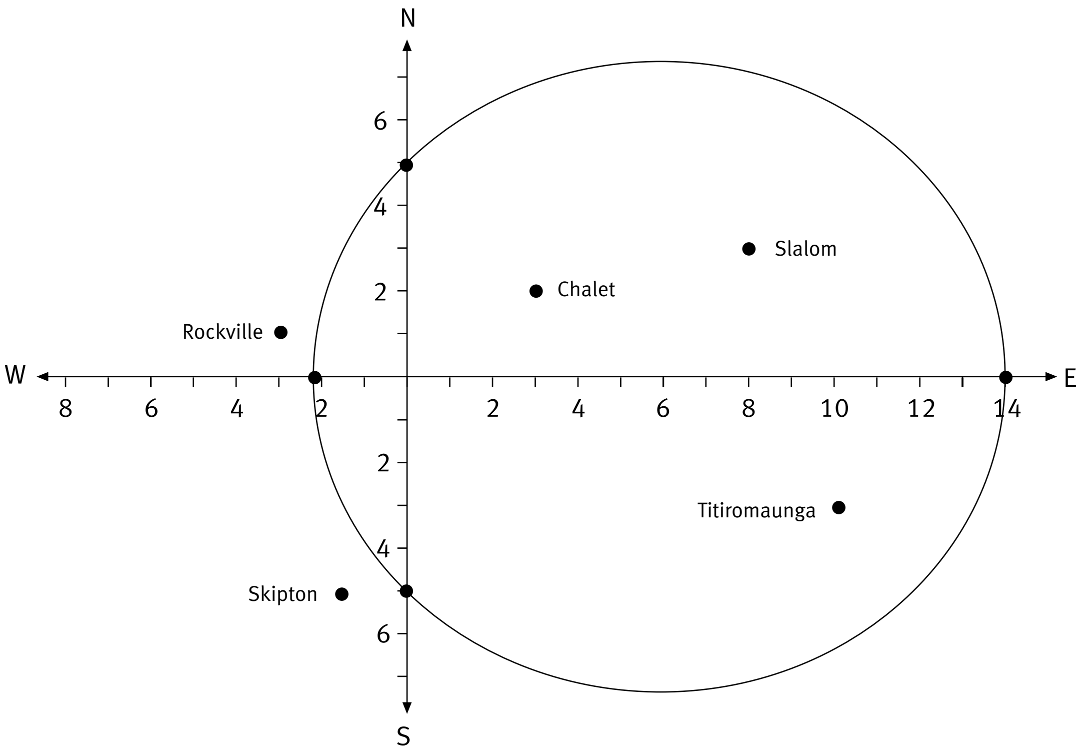
Salt is an ideal substance for simulating ash. Use a good salt cellar with a single free-running hole, or perhaps a dry beaker from which the salt can be poured. Hold the salt container directly over the origin of the set of axes. Pour the salt in an even stream for 1 minute and then carefully trace the outline of the salt pile onto the graph paper. Without disturbing the pile, pour the salt for a further 1 minute and trace around the pile again. Be sure that you pour the salt from the same height as before. Repeat the experiment for a further 3 or 4 minutes, tracing the outline of the pile each time.

Pour the salt back into the beaker and examine the outlines of the pile for each minute of salt pouring. Have the students calculate the area covered by the salt and record it in a table. The outline should be a circle, so the circle formula for area, $A = \pi r^2$, can be used.

This experiment models the distribution of the ash cloud in windless conditions. The example shown in the students' book allows for a wind blowing from west to east. This can be modelled using the same equipment as before with the addition of a small fan. Begin with a fresh piece of graph paper and draw in the axes, with the volcano at the origin. Tape the paper down so that it doesn't move. Now place a fan so that it blows across the paper as you pour the salt. Using the same method as before, draw around the "ash" for each minute of pouring. This time, the shapes will be elliptical and the area can be calculated using the formula $A = \pi ab$, where a is the shortest possible radius measured from the centre and b is the longest possible radius.

Explain to the students that what has been done with salt would normally be modelled mathematically, using a computer.

When working on question **4**, the students will find it helpful to sketch an approximation of the extent of the ash on the set of axes showing the towns. The required information comes from the two graphs. The west-east graph shows us that the spread starts at approximately 2.4 kilometres west of the volcano and stretches 14 kilometres to the east of the volcano. The centre of the ellipse is on the west-east axis at the point (5.8,0). The diagram below shows a bird's-eye of the area covered by the ash.



In making an estimate of the depth of ash, allowance must be made for the fact that the towns do not lie on the west-east axis. If we take a north-south line through Chalet, for example, the ash will be thickest where this line crosses the west-east axis. The ash will be slightly shallower at Chalet and definitely shallower at Slalom and Titiromaunga.

CROSS-CURRICULAR LINKS

Social Studies

The students could explore the advantages and disadvantages of new technology on the lives of people, for example, cellphones. They could debate a position on how these developments have impacted on their lives and the lives of others.

Achievement Objectives

Demonstrate knowledge and understandings of:

- the impact of the spread of new technology and ideas on culture and heritage (Culture and Heritage, level 4)
- how and why people view and use resources differently and the consequences of this (Resources and Economic Activities, level 4)

Science

The students could investigate how instruments are used to predict the effect of the ash cloud. They could also discuss the effect of the ash cloud on the environment and the influence of weather patterns on the direction and impact of the ash cloud.

Achievement Objectives

- investigate and offer explanations of how selected items of technology function and enhance everyday activities of people (Physical World, level 4)
- investigate the positive and negative effects of substances on people and on the environment (Material World, level 4)
- investigate major factors and patterns associated with weather, and use given data to predict weather (Planet Earth and Beyond, level 4)

See also: Building Science Concepts Book 12.

Volcanic Volumes

Achievement Objectives

- explain the meaning of the digits in decimal numbers with up to 3 decimal places (Number, level 3)
- explain the meaning and evaluate powers of whole numbers (Number, level 4)
- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (Measurement, level 3)

Other mathematical ideas and processes

Students will also:

- visualise very large numbers
- convert from one unit to another.

ACTIVITY

For successful learning to occur, the students need to visualise and understand what a cubic metre is and how to determine how many of them make up a cubic kilometre. Having students physically handle and measure 3-dimensional objects, as well as calculate the volume of the objects in different units, is a vital step in the learning process.

On first exposure, it will be difficult for the students to visualise a cubic kilometre, but one could be simulated using a scale model. A cubic metre could be constructed quite easily using wooden blocks or a wire frame structure. 1 metre = 1 000 millimetres, so a cubic metre is the same as:

$$\begin{aligned} 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m} &= 1\,000 \text{ mm} \times 1\,000 \text{ mm} \times 1\,000 \text{ mm} \\ &= 1\,000\,000\,000 \text{ mm}^3 \\ &= 10^9 \text{ mm}^3. \end{aligned}$$

In the same way, since 1 kilometre = 1 000 metres, 1 $km^3 = 10^9$ m^3.

Notice that 10^9 = 1 000 000 000, which is 1 000 x 1 000 000 or 1 000 million. This is a useful way of looking at the conversion of cubic kilometres to cubic metres. It means that 1 km^3 = 1 000 million cubic metres. So, 2.45 km^3 = 2.45 x 1 000 million m^3 or 2 450 million m^3.

A useful website about volcanoes and the Auckland volcanic field is www.arc.govt.nz/volcanic/field.htm

CROSS-CURRICULAR LINKS

Social Studies

The students could research an area in New Zealand or overseas where active or extinct volcanoes exist. They could investigate the history of the area, including the last eruption, and how that particular area has been populated over time. They then carry out a “plus, minus, interesting” exercise on the advantages and disadvantages of living in this area now. They could present their findings as a slideshow.

Achievement Objectives

Demonstrate knowledge and understandings of:

- how places reflect past interactions of people with the environment (Place and Environment, level 4)
- why and how people find out about places and environments (Place and Environment, level 4)

Science

The students could research the effect of ejected volcanic material on the landscape and the environment and its impact on land formation and erosion.

Achievement Objectives

- investigate the positive and negative effects of substances on people and on the environment (Material World, level 4)
- collect and use evidence from landforms, rocks, fossils, and library research to describe the geological history of the local area (Planet Earth and Beyond, level 4)

See also: Building Science Concepts Book 12.

Page 16 All Fired Up

Achievement Objective

- find, and use with justification, a mathematical model as a problem-solving strategy (Mathematical Processes, problem solving, level 4)

Other mathematical ideas and processes

Students will also:

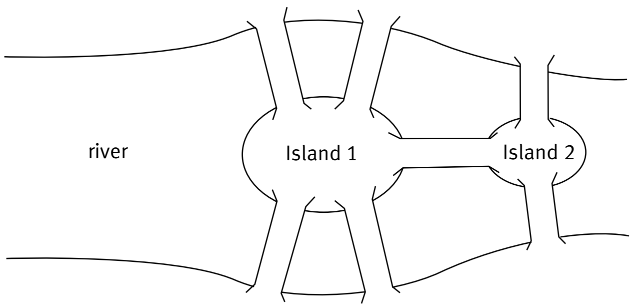
- use networks to model physical situations
- develop a strategy for finding Euler paths through a network.

ACTIVITY

Finding paths through networks in which each section is used once only is a task that fascinates many students. It has the appeal of a maze, although it is very different. Students can quickly learn that the start and finish points for travelling through the network can be found by inspection rather than by trial and error. Furthermore, a simple rule can determine whether or not the problem can be solved at all!

Leonhard Euler, pronounced "Oiler", (1707–1783) is the father of graph theory, a mathematical discipline that includes networks. He lived in Königsberg, Prussia, where the Sunday afternoon pastime was to walk over the seven bridges of the Pregel River.

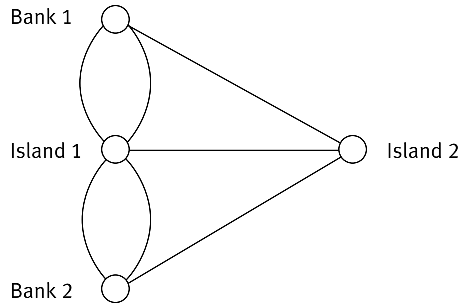
The diagram below models the Königsberg bridges network.



A challenge for walkers was to cross every bridge once and once only and to end up at their starting point. Euler proved that this was impossible and, in doing so, developed a branch of mathematics that is used for modelling all sorts of situations, such as the smoke-filled buildings in this activity.

In this activity, the students are trying to find a path that goes through every tunnel once only. This is known as an Euler path. (If you had to return to your starting point as well, you would be looking for an Euler circuit.)

From a teaching point of view, it would be good to let the students initially attack the problem using trial and error. They may find it helpful to strip the diagrams of all unnecessary details, re-drawing them as lines and nodes. Once Euler paths have been identified, ask the students to identify why these paths work and others don't.

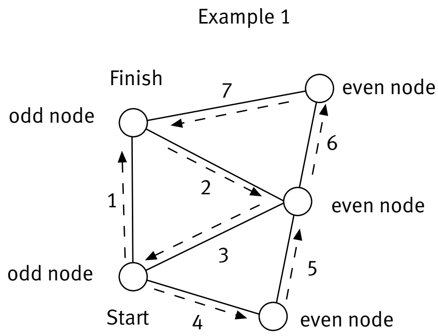


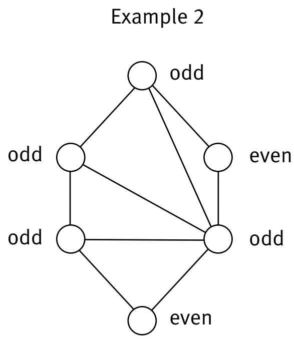
Other possible questions include:
"When a path is found, does the starting point (or node) of the network have an even number of tunnels connected to it or an odd number?"
"Does the ending point (or node) of the network have an even or odd number of tunnels connected to it?"
"Do all other nodes in the network have an even or an odd number of tunnels connected to them?"

The answers to the above questions give the basic rules for finding Euler paths and circuits through any network. These are:

- For a network to have an Euler path, the start and finish nodes must have an odd number of paths connected to them. All other nodes must have an even number of connecting paths.
- For a network to have an Euler circuit (a path that begins and ends at the same point), all the nodes must have an even number of paths.

Using these rules, we can look at any network and quickly determine whether or not it has an Euler path and, if so, where to start.

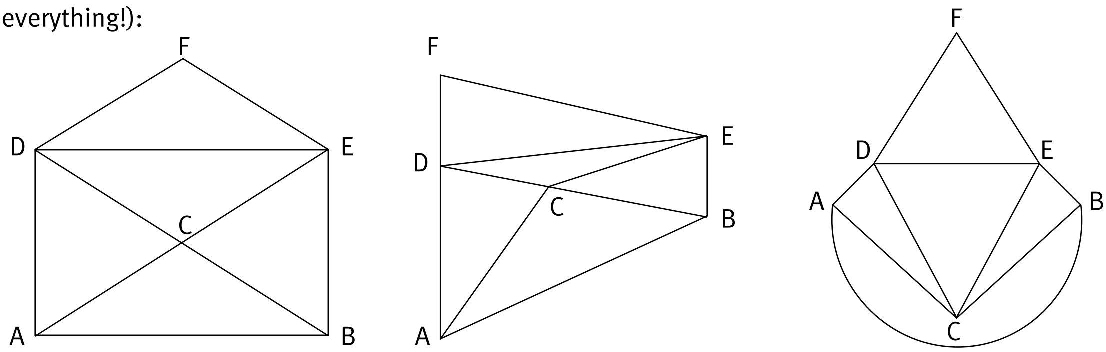




There is no solution in example 2 because there are too many odd nodes.

As an extension, the students can draw their own networks using these simple rules. You can also ask: "What could be done to the Königsberg bridge problem to make it an Euler path and then an Euler circuit?"

A further extension task is to challenge the students to create a series of networks that have an identical structure but that look very different, as in the following examples (note that shape means nothing; structure is everything!):



A useful website is www.mathforum.org/isaac/problems/bridges2.html

Page 17 **Virus Alert**

Achievement Objectives

- find a given fraction or percentage of a quantity (Number, level 4)
- perform calculations with time, including 24-hour clock times (Measurement, level 4)
- explain the meaning and evaluate powers of whole numbers (Number, level 4)

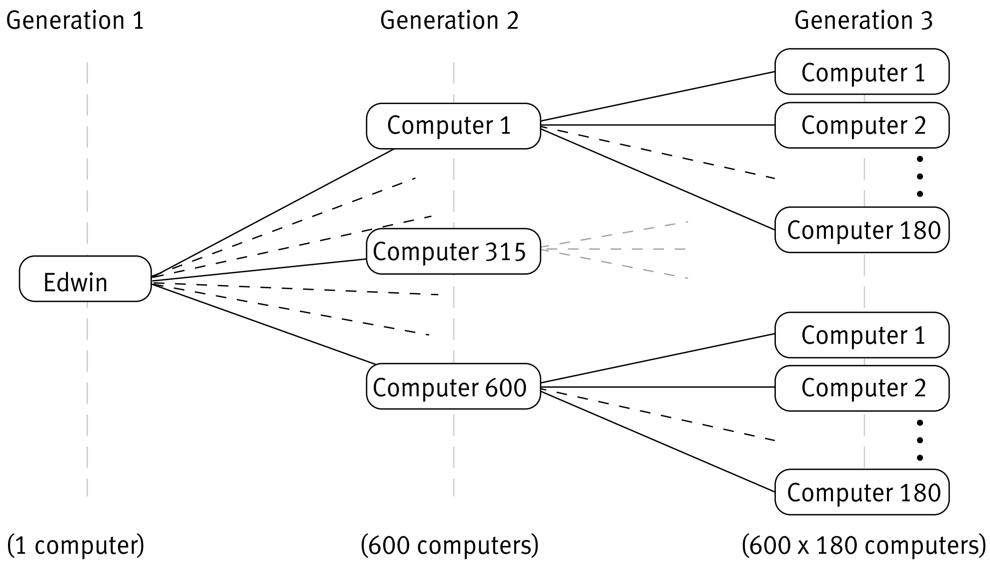
Other mathematical ideas and processes

Students will use simple models that simulate growth of a population or the spread of information through a community.

ACTIVITY

This activity provides the opportunity for some lively classroom discussion centred around the meaning of percentages. For example, 15 percent of Edwin's 600 addresses is 600 x 0.15 = 90. This is the number of computers infected every 2 minutes. The total time taken to infect all 600 computers is found by calculating how many units of 90 there are in 600: $^{600}/_{90} = 6.66\dot{6}$. Therefore, the time taken is $6.66\dot{6} \times 2 = 13.33\dot{3}$ minutes or 13 minutes and 20 seconds ($0.333\dot{3} \times 60 = 20$).

Now the problem gets interesting. If each of the 600 infected computers sends the virus to another 180 computers, the virus then infects 600 x 180 = 108 000 computers. A tree diagram will help the students see this.



When the virus gets to the 600 computers listed in Edwin's address list, it works on each of these computers at the same time. So we need only to find the time it takes to infect the 180-member address list of one of these computers.

We are told that it again takes 2 minutes to infect 15 percent of the address list. Because this is the same percentage as before, it will take exactly the same length of time to infect a list of 180 addresses as it did for Edwin's list of 600: 13 minutes and 20 seconds. Some students will be puzzled by this, so let's calculate it. 15 percent of 180 = 27 and $^{180}/_{27} = 6.66\dot{6}$. So the time taken to infect all 180 addresses on the list is $6.66\dot{6} \times 2 = 13.33\dot{3}$ minutes, or 13 minutes and 20 seconds. The point is that, in both cases, it takes 2 minutes to attack the same percentage (15 percent) of each list. It doesn't matter if the lists are of different lengths; the percentage is the same, so the overall outcome is the same.

In question **3**, we have a virus that spreads exponentially. Every infected computer infects another 10 computers. The first generation starts with 1 computer. It infects 10 computers. Each of those 10 computers infects another 10, which is 10 x 10 = 100. Each of the 100 computers infects 10 computers:

$10 \times 100 = 1\,000$. Adding all these up gives $1 + 10 + 100 + 1\,000 = 10^0 + 10^1 + 10^2 + 10^3$. The sum of computers infected after n generations is $10^0 + 10^1 + 10^2 \ldots + 10^{n-1}$. It takes 7 generations before Catdis has infected a million computers.

Pages 18–19 Multiple Mishaps

Achievement Objectives

- use equipment appropriately when exploring mathematical ideas (Mathematical Processes, problem solving, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- devise and follow a set of instructions to carry out a mathematical activity (Mathematical Processes, communicating mathematical ideas, level 4)

Other mathematical ideas and processes

Students will explore:

- co-operative strategies for finding the optimal solution for an entire system
- basic risk management techniques
- the use of number sequences in game strategies.

ACTIVITY

Ideally, the shunting problem should be solved using model railway track and the required rolling stock. If you don't have this equipment, 8 wooden blocks with a cup hook screwed into each end would serve just as well. Mark the carriage blocks *a1*, *a2*, *a3*, and so on, and the engine blocks *A* and *B*. Mark out a track on a piece of cardboard, being careful to ensure that the siding can take only 2 carriages and an engine.

There are different possible solutions to this problem, but they all require the drivers of the two engines to co-operate. Diagrams and a possible solution are included in the Answers.

As an extension activity, the students could be asked to come up with two different solutions and then explain why one of them is better than the other. The criteria for such a judgment could include the number of separate forwards and backwards movements involved, the number of couplings and uncouplings required, and whether the locomotives and their carriages end up in the original order. There could be a class-wide search for the simplest solution.

The students will be familiar with games that involve challenges similar to that posed in mini-disaster two. If desired, they can easily model the pile of steel sheets using paper or cardboard rectangles. They will discover that, although the starting sequence is clear, there are different possibilities for the later sheets.

The domino problem is a perfect example of risk management. By placing a break between every 100 dominoes, Makil is reducing the risk of disaster. He is not removing it altogether, but the worst that can happen is that 100 dominoes can tumble unexpectedly. The removal strategy is critical. If all the breaks were to be taken out in sequence, the risk of untimely collapse would increase by 100 dominoes each time. Just before the last break is removed, the whole lot except the last 100 dominoes could collapse! So, by removing every second break, Makil increases the risk in a controlled fashion.

An opportunity exists here for discussing risk management and mathematical strategies that contain risk. For instance, if a sweet factory makes bags of jelly beans, can every bag be checked for quality and correct contents? How could the risk of poor product going to the shops be managed realistically?

For the links problem, there is a simple solution. By opening the links of the two 2-link chains, Merania has 4 loose links, which is exactly the number of joining links required to connect the 4 remaining chains.

Furthermore, it is an economical solution since it takes a minimal amount of opening and closing to achieve the desired result.

In the card problem, Yvette's cards have to be placed in a strict sequence for the tricks to work. There are 10 cards in the pack, and every alternate card is placed down starting with the first, so we have the positions for 1, 2, 3, 4, and 5. Let the letters A, B, C, D, and E represent unknown cards for now. Then the sequence so far is 1, A, 2, B, 3, C, 4, D, 5, E. This means that 1 will be placed down and A moved to the back, 2 placed down and B moved to the back, and so on. After the 5 has been placed down and E moved to the back, the remaining cards will be in the order A, B, C, D, E. A will be the next card to place down, so it must be 6. B will go to the back of the pack, so C is 7 and E must be 8. The remaining order is B, D. E was placed down, so B goes to the back of pack. D is 9 and B is 10. Therefore, the original order of the cards was 1, 6, 2, 10, 3, 7, 4, 9, 5, 8.

Pages 20–21 Peanut in Trouble

Achievement Objective

- enlarge and reduce a 2-dimensional shape and identify the invariant properties (Geometry, level 4)

Other mathematical ideas and processes

Students will also:

- work with scale drawings
- use ratio to find unknown elements in similar triangles.

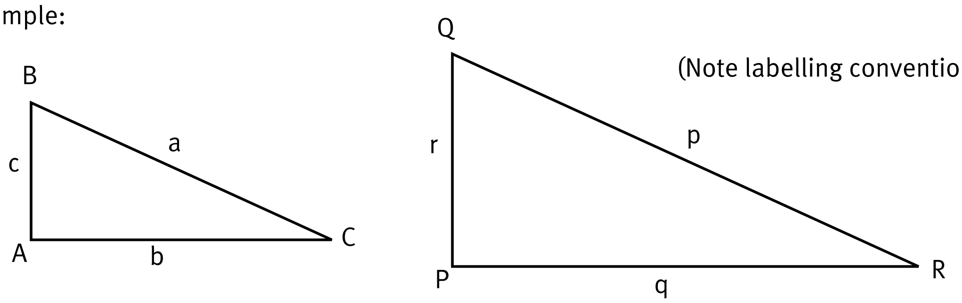
ACTIVITY

Although this problem can be solved using trigonometric ratios, these are not introduced until level 5 of the curriculum and are best avoided unless you have some very able students.

The best approach is to use the fact that the drawing supplied is to scale. The students will need to discover for themselves what the scale is by measuring and comparing the only length they are given (48 metres from Patu to the base of the tree) with its scale length, 16 centimetres. They will see that 1 centimetre on the diagram represents 3 metres in real life. It is a short step from here to determining that, if the height of the tree in the diagram is 6 centimetres, the height of the tree in real life must be 6 x 3 = 18 metres. The bottom branches in the diagram are 1.5 centimetres from the ground, or 1.5 x 3 = 4.5 metres in reality. The fact that the angle of inclination is 19 degrees plays no part in solving the problem using scale.

Questions **2** and **3** invite exploration of similar triangles (triangles that are the same shape but a different size) and the way in which pairs of corresponding sides have the same ratio.

For example:



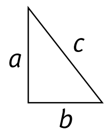
(Note labelling conventions.)

If A = P, B = Q, and C = R, then

$\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$ and $\frac{p}{a} = \frac{q}{b} = \frac{r}{c}$

$\frac{a}{b} = \frac{p}{q}$, $\frac{b}{c} = \frac{q}{r}$, $\frac{c}{a} = \frac{r}{p}$, and so on.

A scale diagram to help with questions **3c** and **d** is provided in the Answers. You could also use these questions to introduce more able students to Pythagoras' theorem:



$c^2 = a^2 + b^2$ or $a^2 = c^2 - b^2$

In this case, they need to find a, so for the 3 metre ladder:

$$\begin{aligned} a^2 &= 3^2 - 1^2 \\ &= 9 - 1 \\ &= 8 \\ \Rightarrow a &= \sqrt{8} \\ &= 2.8 \end{aligned}$$

Pages 22–23 A Disastrous Day

Achievement Objectives

- explain the meaning and evaluate powers of whole numbers (Number, level 4)
- demonstrate knowledge of the conventions for order of operations (Number, level 4)
- explain satisfactory algorithms for addition, subtraction, and multiplication (Number, level 4)
- express quantities as fractions or percentages of a whole (Number, level 4)

Other mathematical ideas and processes

Students will practise and improve their capacity for advanced mental calculations and adapt these techniques to simple problem-solving situations.

GAME

You can use this game as an opportunity to observe the numeracy skills of your class and to assess a student's ability or level of development. Observing the game could help you to group students into teams of similar numeracy skill or development. If you wish, you can easily write your own sets of cards emphasising different skills.

The game provides an incentive for students to improve their numeracy skills. Each throw of the dice requires them to perform mental calculations using the digits. Encourage them to do all possible calculations and not just settle for the addition option. By doing this, they will gain the maximum advantage in terms of outcome.

After some practice, the students will develop a "feel" for the numbers and start choosing the optimal calculation intuitively. Notice that because of the competitive nature of the game, players will be doing the calculations after every throw to check on their peers!

Over time, the students should notice that subtracting the square of the smaller number from the square of the larger number generally gives the greatest number but that there is an important class of exceptions to this.

More able students may enjoy analysing the results from the different methods more closely.

Algebraically, the approaches can be written as follows (a is the larger of the two numbers if a and b are different):

1. $a + b$
2. $a^2 - b^2 = (a + b)(a - b)$
3. $2(a - b)$

When the two numbers are the same, $(a - b) = 0$ and methods 2 and 3 will both give zero. So the best approach will be to add the two numbers.

When there is a difference of 1 between the numbers, $(a - b) = 1$, so methods 1 and 2 will both give the same result (which will be 3 or more). Method 3 will give the number 2. When $(a - b)$ is greater than 1, method 2 will give a bigger number than method 1.

Method 3 will never give the largest number. This is because it always gives a number less than or equal to that given by method 2. Both methods have a common factor of $(a - b)$, but $(a + b)$ is always greater than or equal to 2.

The Safe House and Lifesaver cards offer further opportunity and incentive to improve numeracy skills. Because some of the cards have a lot of information to be processed, some students may need to read them for themselves. Have another student hold the card up with the answer covered. A word of warning: most students have acute memories for correct answers when playing games, so adding to the collection of cards by using the same question structures and changing the numbers may be a useful ploy.

Page 24 A Helping Hand

Achievement Objectives

- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)
- find and justify a word formula which represents a given practical situation (Algebra, level 4)

Other mathematical ideas and processes

Students will develop their spreadsheet-modelling skills.

ACTIVITY

The most important task for any teacher in helping students to see patterns in number sequences is to show the students how to look at what's important to the problem. The students are asked to comment on how often the birds lay their eggs. Nothing has been said about the gender of the eggs, just how many. So all we need to study is the total column on Taki's spreadsheet, and there we see that the total bird population increases only every 2 years. Therefore, eggs hatch only every 2 years.

To determine how many eggs are laid every year, we need to look at who lays the eggs. Only the female birds can do this. So, when we had 1 female adult bird, the population increased by 3. On this basis, 2 female adults would have 6 chicks. Summarising this information, we have:

Number of adult female birds	Number of eggs
1	3
2	6
4	12
8	24

Looking at the gender of the chicks, we find that there is an obvious pattern in which two-thirds of the eggs are male and one-third is female.

Male chick	Female chick	Eggs
2	1	3
4	2	6
8	4	12
16	8	24

To predict the number of years before the bird population reaches 400 birds, we study the number pattern for the total. It can be summarised as follows:

Year	Total population	Increase
1	2	
3	5	3
5	11	6
7	23	12
9	47	24

We notice that the increase in population at each "egg hatch" is double the increase of the previous year. So we can complete the table as follows:

Year	Total population	Increase
11	95	48
13	191	96
15	383	192
17	767	384

If there were 4 chicks and half of them were of each gender, as in question **2b**, the spreadsheet would look like this:

Year	Adult Male	Adult Female	Young Male	Young Female	Total
1			1	1	2
2			1	1	2
3	1	1	2	2	6
4	1	1	2	2	6
5	3	3	6	6	18
6	3	3	6	6	18
7	9	9	18	18	54
8	9	9	18	18	54
9	27	27	54	54	162
10	27	27	54	54	162
11	81	81	162	162	486
12	81	81	162	162	486

Given this scenario, the number 400 is reached in 11 years.

While all the above reasoning has been done by extracting data from the table provided, the students would benefit from modelling the situation on a spreadsheet. Wherever possible, use the rules above and apply them to cell formulae when creating the spreadsheet rather than manually calculating and entering the numbers one by one. This will enable the spreadsheet to be manipulated by the students to see the effects of a change in birth patterns.

Below is a spreadsheet showing all the cell formulae:

	A	B	C	D	E	F
1	Year	Adult Male	Adult Female	Young Male	Young Female	Total
2	1			1	1	=SUM(B2:E2)
3	2			1	1	=SUM(B3:E3)
4	3	=B3+D3	=C3+E3	=C4*2	=C4*2	=SUM(B4:E4)
5	4	=B4	=C4	=D4	=E4	=SUM(B5:E5)
6	5	=B5+D5	=C5+E5	=C6*2	=C6*2	=SUM(B6:E6)
7	6	=B6	=C6	=D6	=E6	=SUM(B7:E7)
8	7	=B7+D7	=C7+E7	=C8*2	=C8*2	=SUM(B8:E8)
9	8	=B8	=C8	=D8	=E8	=SUM(B9:E9)
10	9	=B9+D9	=C9+E9	=C10*2	=C10*2	=SUM(B10:E10)
11	10	=B10	=C10	=D10	=E10	=SUM(B11:E11)
12	11	=B11+D11	=C11+E11	=C12*2	=C12*2	=SUM(B12:E12)

(Note that =SUM(B2:E2) is the same as =B2+C2+D2+E2.)

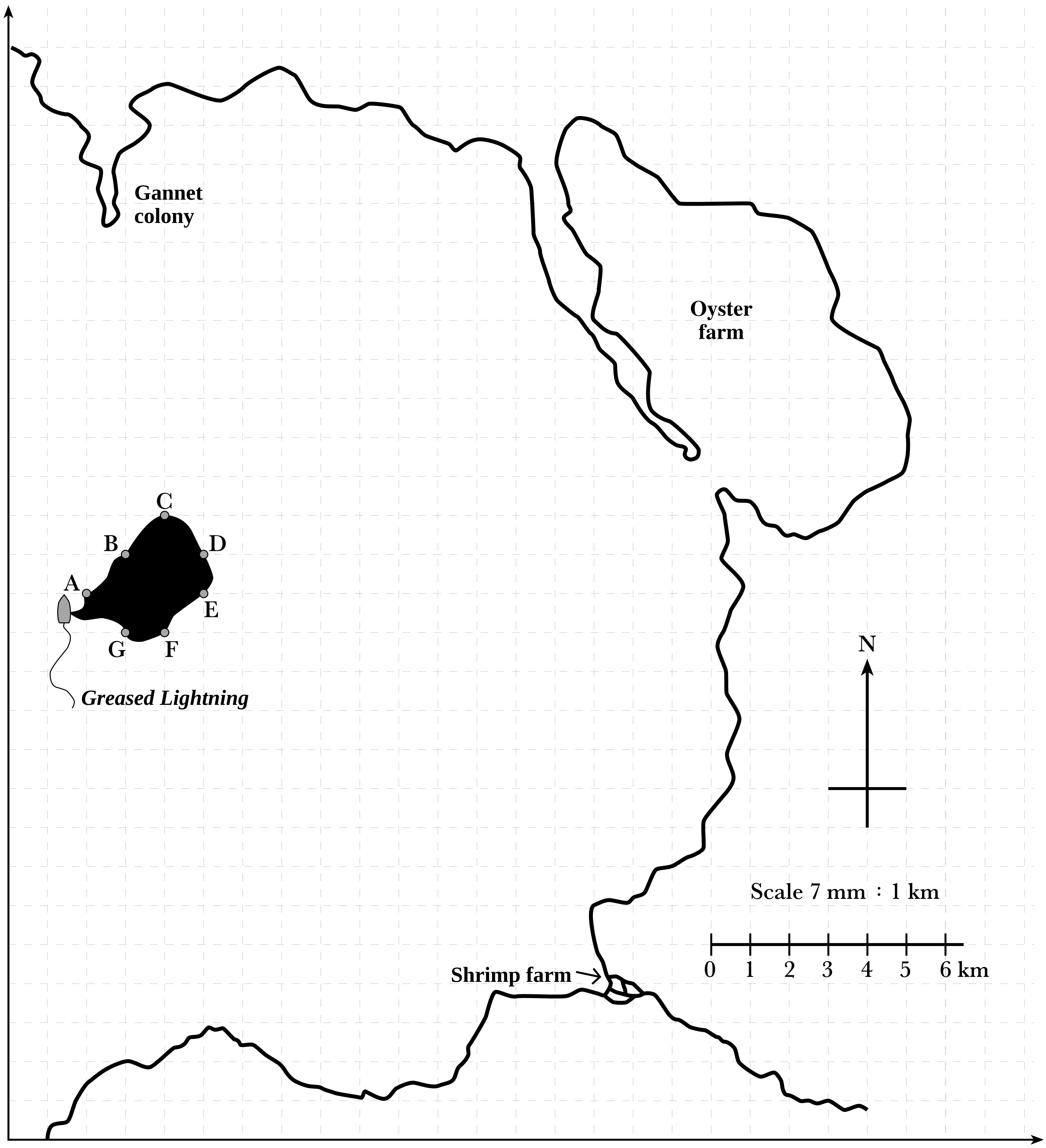
CROSS-CURRICULAR LINKS

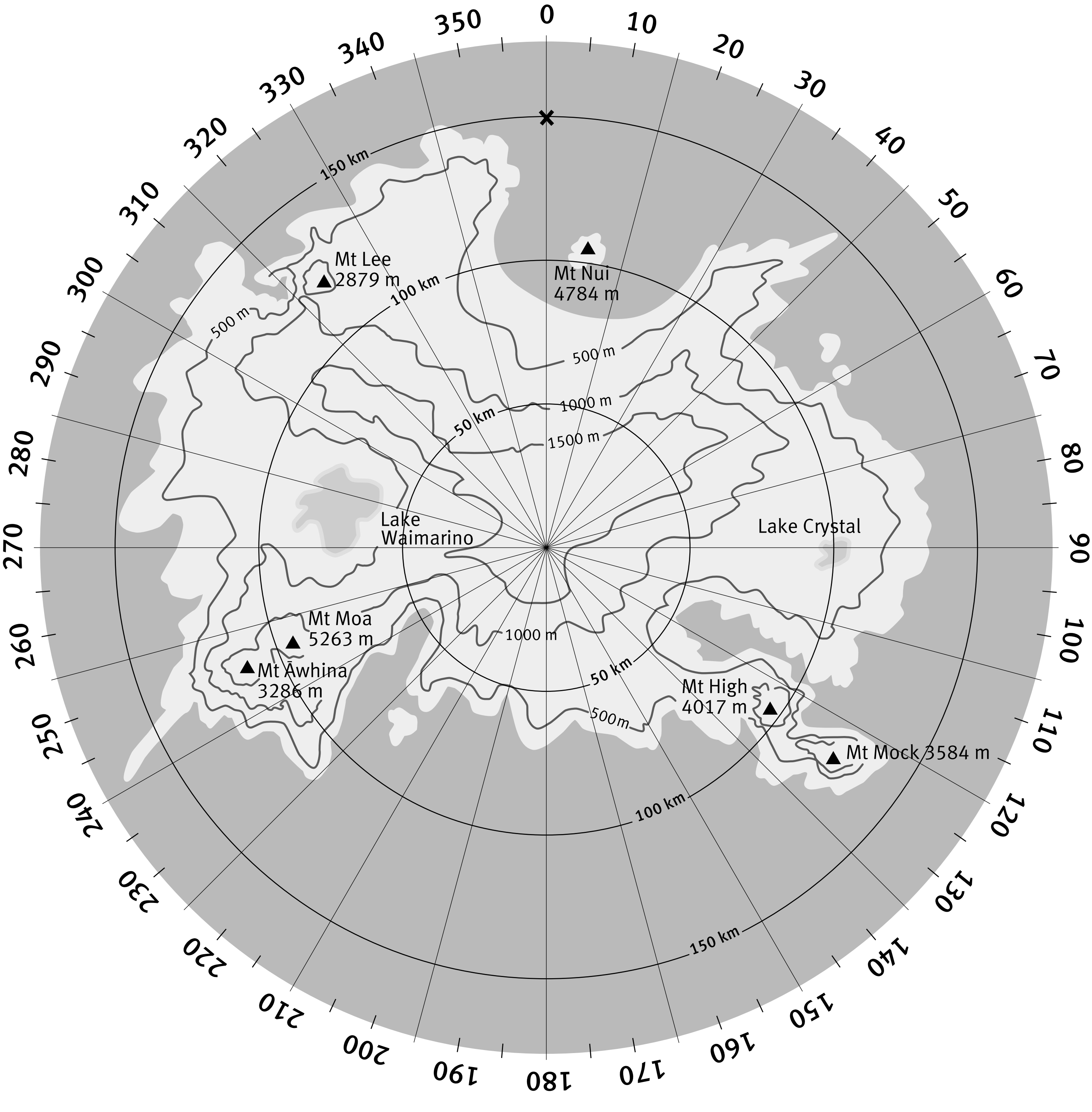
Science

Students who are interested in science generally, or in native fauna and endangered species in particular, will probably realise that this activity presents a simplified mathematical model for saving endangered species rather than an actual result. This could create useful classroom discussion or even a project. The students could investigate the way mathematics is used to find reasonable estimates and explore the variables and uncertainties that are involved in such estimates.

Achievement Objectives

- investigate and describe special features of animals or plants which help survival into the next generation (Living World, level 4)
- investigate a local environmental issue and explain the reasons for the community's involvement (Planet Earth and Beyond, level 4)


Gannet
colony
Oyster
farm
C
B
D
A
E
G
F
Greased Lightning
N
Scale 7 mm : 1 km
0 1 2 3 4 5 6 km
Shrimp farm



Scale 1 cm : 20 km

0 20 40 60 80 100 km

Safe House Cards

Safe House

What number am I?
- I'm a factor of 32.
- I'm a square number.
- I'm less than 10.

Answer: 4

Safe House

What is 6.54 p.m. in 24 hour time?

Answer: 18:54

Safe House

What number am I?
- I'm under 20.
- I'm odd.
- I'm a multiple of 5 and 3.

Answer: 15

Safe House

Dana nets 30 out of 50 attempts at goal.
What percentage is this?

Answer: 60%

Safe House

$^-5 \times 2 = \square$

Answer: $^-10$

Safe House

Andrew succeeds in 4 out of 5 shots at goal.
What percentage is this?

Answer: 80%

Safe House

$^-3 \times {}^-4 = \square$

Answer: 12

Safe House

How many vertices does a cuboid have?

Answer: 8

Safe House

$\sqrt{81} = \square$

Answer: 9 or $^-9$

Safe House

Simplify $2a \times 7$

Answer: $14a$

Safe House

$2^4 = \square$

Answer: 16

Safe House

Name a polygon with 6 sides.

Answer: hexagon

Safe House

How many minutes are there in $\frac{3}{4}$ of an hour?

Answer: 45

Safe House

How many hours are there in a week?

Answer: 168

Safe House

What is $\frac{4}{5}$ as a decimal?

Answer: 0.8

Safe House

$\frac{2}{3}$ of 12 = $\square$

Answer: 8

Safe House

15 – 3 x 4 = $\square$

Answer: 3

Safe House

How many millimetres is 1.4 metres?

Answer: 1 400 mm

Lifesaver Cards

Lifesaver

At 7 a.m., the temperature in Taupo was –3°C. At 2 p.m., it is 14°C. How many degrees has it risen?

Answer: 17°C

Lifesaver

How many minutes are there in 2 hours, 40 minutes?

Answer: 160

Lifesaver

A recipe uses $\frac{1}{4}$ of a 6 kilogram bag of flour. How much flour does it use?

Answer: $1\frac{1}{2}$ kg

Lifesaver

What is the chance of drawing a diamond from a pack of cards that has no jokers in it?

Answer: $\frac{1}{4}$

Lifesaver

Concrete is made of 6 parts builders' mix and 1 part cement. How much builders' mix is needed to make 28 kilograms of concrete?

Answer: 24 kg

Lifesaver

How many sides does a square-based pyramid have?

Answer: 5

Lifesaver

How much would you get paid for 5 hours work at $6.50 an hour?

Answer: $32.50

Lifesaver

$\frac{1}{4}$ x $\square$ = 8

Answer: 32

Lifesaver

If you need 35 minutes to roast every 500 grams of beef, how long will it take to roast 1.5 kilograms?

Answer: 105 min or 1 hr 45 min

Lifesaver

On Tuesdays, it costs 89c for a cheeseburger and $1.50 for a medium serving of chips. How much is it for 2 cheeseburgers and 2 servings of chips?

Answer: $4.78

Lifesaver

In a scale diagram, if 2 centimetres represents 50 metres, how much does 9 centimetres represent?

Answer: 225 m

Lifesaver

If $x = 3$ and $y = 5$, what does $4x + 2y$ equal?

Answer: 22

Lifesaver

If the area of a square is 64 cm^2, how long are the sides?

Answer: 8 cm

Lifesaver

How many minutes are there from 11.38 a.m. to 12.15 p.m.?

Answer: 37

Lifesaver

What is the volume of a cube with sides that are 3 centimetres long?

Answer: 27 cm^3

Lifesaver

$\sqrt{64} = \square$

Answer: 8 or $^-8$

Lifesaver

How many centimetres are there in 5.2 metres?

Answer: 520

Lifesaver

$10^4 = \square$

Answer: 10 000

Lifesaver

$20 - 6 \times 2 = \square$

Answer: 8

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