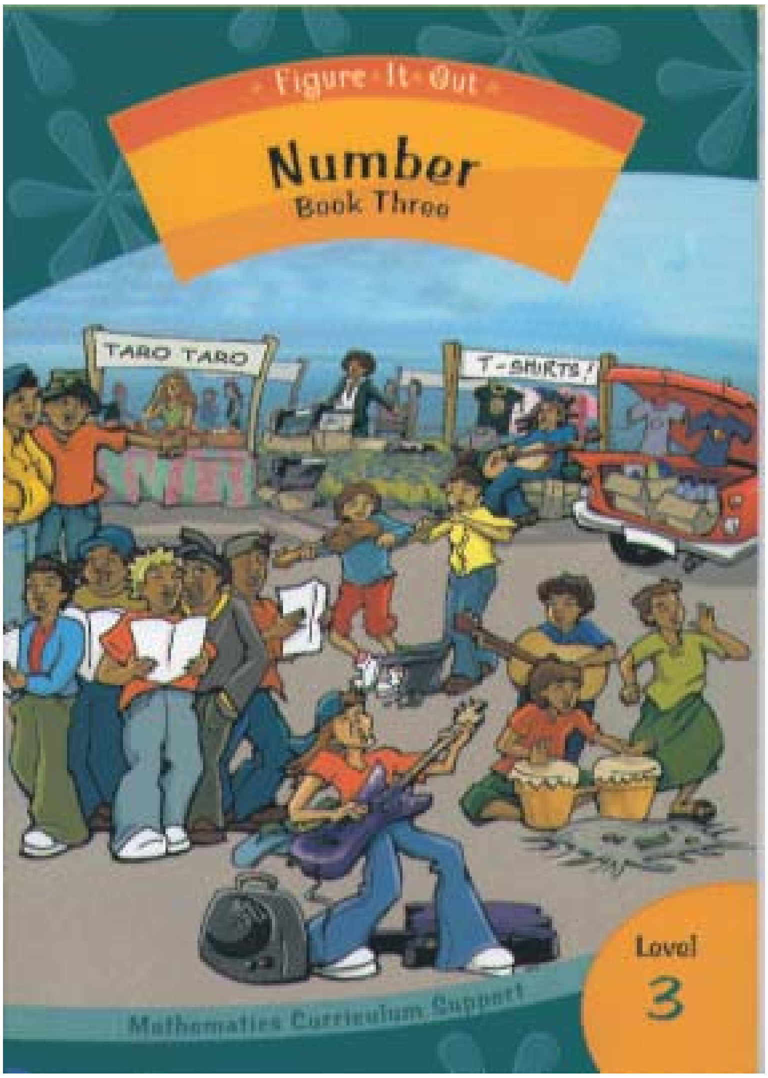
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Contents

Introduction

The books for level 3 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand classrooms. *Number: Book Two* and *Number: Book Three* have been developed to support teachers involved in the Numeracy Project. These books are most suitable for students in year 5, but you should use your judgment as to whether to use the books with older or younger students who are also working at level 3.

Student books

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and the life experiences that are meaningful to students in year 5.

The activities can be used as the focus for teacher-led lessons, for students working in groups, or for independent activities. Also, the activities can be used to fill knowledge gaps (hot spots), to reinforce knowledge that has just been taught, to help students develop mental strategies, or to provide further opportunities for students moving between strategy stages of the Number Framework.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the student books includes full answers and explanatory notes. Students can use this section for self-marking, or you can use it for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, comments on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/r/maths/curriculum/figure

Using Figure It Out in the classroom

Where applicable, each page starts with a list of equipment that the students will need in order to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to the problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Figure It Out

Number: Book Three

Answers

Page 1: Crazy Compatibles

Activity

1. 10 of the following pairs:
25 + 75 = 100
75 + 25 = 100
72 + 28 = 100
28 + 72 = 100
85 + 15 = 100
15 + 85 = 100
71 + 29 = 100
29 + 71 = 100
67 + 33 = 100
33 + 67 = 100
10 + 90 = 100
90 + 10 = 100
32 + 68 = 100
68 + 32 = 100
60 + 40 = 100
40 + 60 = 100
31 + 69 = 100
69 + 31 = 100
30 + 70 = 100
70 + 30 = 100
55 + 45 = 100
45 + 55 = 100
35 + 65 = 100
65 + 35 = 100
18 + 82 = 100
82 + 18 = 100
17 + 83 = 100
83 + 17 = 100
41 + 59 = 100
59 + 41 = 100
51 + 49 = 100
49 + 51 = 100

2. a. 103. (23 + 2 + 77 + 1 = 103)
   b. 103. (45 + 2 + 55 + 1 = 103)

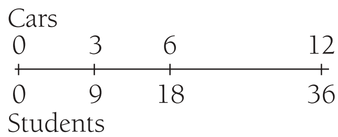
3. a. 101
   b. 104
   c. 102
   d. 104
   e. 103
   f. 104
   g. 102
   h. 111
   i. 105
   j. 660. (340 + 60 = 400. 400 + 600 = 1 000. 60 + 600 = 660)
   k. 703. (297 + 3 = 300. 300 + 700 = 1 000. 3 + 700 = 703)
   l. 130. (870 + 30 = 900. 900 + 100 = 1 000. 30 + 100 = 130)

4. Practical activity

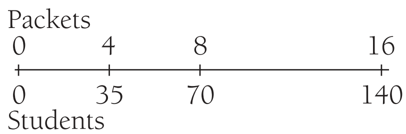
Page 2: Numbers on the Line

Activity

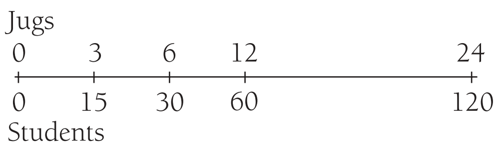
1. 3 cars. The double number line could look like this:



2. 4 packets of biscuits. A possible double number line is:



3. 3 jugs. A possible double number line is:



4. a. 54 students. (You could use halving and doubling instead of the double number line below. So 18 x 3 becomes 9 x 6 = 54.)

b. 30 potatoes. A possible double number line, useful for both answers, is:

Potatoes
1 9 18 30
3 27 54 90
Students

Page 3: Banking Issues

Activity

1. a. 19 $10 notes and 7 $1 coins
   b. 46 $10 notes and 5 $1 coins
   c. 120 $10 notes and 6 $1 coins
2. a. i. $270
   ii. $310
   iii. $540
   b. $1,120
   c. $60
3. a. 6 $100 notes, 1 $50 note, 7 $1 coins
   b. 30 $100 notes, 1 $50 note, 15 $1 coins
   c. 124 $100 notes, 0 $50 notes, 21 $1 coins

Pages 4-5: Skimming Stones

Activity

1. Possible strategies include:
   - looking for compatible numbers, as Lagi did
   - adding the numbers in groups of 2 (4 + 6 = 10, 8 + 3 = 11, 7 + 8 = 15, 10 + 11 + 15 = 36)
   - using basic facts (4 + 6 = 10, 10 + 8 = 18, 18 + 3 = 21, 21 + 7 = 28, 28 + 8 = 36)
   - averaging the numbers and multiplying by 6 (the 6 and 8 in Jordan's scores can be seen as two 7s, and 6 x 7 = 42).

   The total number of bounces for each person is: Te Rama: 36; Penny: 36; Harina: 32; Jordon: 42; Lagi: 36; and Ford: 42.
2. Answers will vary. For example, you could take the 4 best skims of the two leading contenders (Jordan and Ford have the same total for 6 skims), you could find the mean of each person's skims, or you might decide that Harina is the new champion because she was the only one to have a skim with 9 bounces.
3. Ford. (38, compared to Jordan's 36)
4. Jordan. 4 of his 6 skims had 7 bounces, and the other 2 skims had close to 7.

Game

A game for using mental strategies for addition

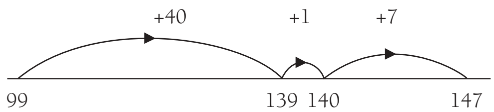
Pages 6-7: The Strategy Strut

Activity One

1. The strategies should be similar to the ones below.

   a. 99 + 48 = 147

   Sui:



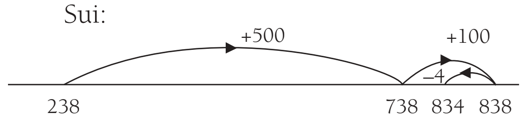
   Hamish: Round 99 up 1 to 100.
   Round 48 up 2 to 50.
   100 + 50 = 150
   Take off 3 = 147.

   Taufa: 90 + 40 = 130
   9 + 8 is 9 + 9 – 1 = 17
   130 + 17= 147

   Erin: 99 + 1 = 100
   48 – 1 = 47
   100 + 47 = 147

   b. 238 + 596 = 834. The strategies should be similar to the ones below.

   Sui:



   Hamish: Round 238 up 2 to 240.
   Round 596 up 4 to 600.
   240 + 600 = 840
   840 – 6 = 834

Taufa: 200 + 500 = 700
30 + 90 = 120
8 + 8 – 2 = 14
700 + 120 + 14 = 834

Erin: Take 4 from 238 to make
596 + 4 = 600.
234 + 600 = 834

2. Answers will vary. The best strategy for each problem will depend on the numbers involved in the problem and which strategy or strategies you feel most comfortable using.

3. Preferred strategies will vary (see comment in **2** above). The answers are:

   a. 73

   b. 93

   c. 84

   d. 176

   e. 180

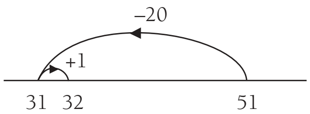
   f. 54

Activity Two

1. **a.** **i.** 51 – 19 = 32. The strategies should be similar to the ones below.

Hamish: 51 – 10 = 41
41 – 1 = 40
40 – 8 = 32

Sui:



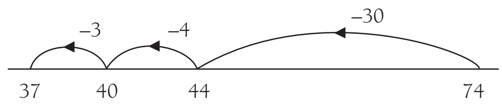
Erin: 51 – 20 = 31
31 + 1 = 32

Taufa: 19 + □ = 51
19 + 1 = 20
20 + 30 = 50
50 + 1 = 51
1 + 30 + 1 = 32

ii. 74 – 37 = 37. The strategies should be similar to the following:

Hamish: 74 – 30 = 44
44 – 4 = 40
40 – 3 = 37

Sui:



Erin: 74 – 40 = 34
34 + 3 = 37

Taufa: 37 + □ = 74
37 + 3 = 40
40 + 30 = 70
70 + 4 = 74
3 + 30 + 4 = 37

b. Answers will vary. (See comment for **Activity One**, question **2**.)

2. Preferred strategies will vary. The answers are:

   a. 25

   b. 22

   c. 37

   d. 45

   e. 37

   f. 38

   g. 38

   h. 64

   i. 106

   j. 135

   k. 75

   l. 68

Page 8: Slippery Slope

Activity

1. **a.** 19 m to the top.
54 + 6 = 60, 60 + 13 = 73, 6 + 13 = 19 m

   b. 27 m from the bottom.
52 – 2 = 50, 50 – 20 = 30, 30 – 3 = 27 m

   c. He slipped 35 m.
63 – 3 = 60, 60 – 30 = 30, 30 – 2 = 28,
3 + 30 + 2 = 35 m or
28 + 2 = 30, 30 + 33 = 63, 2 + 33 = 35 m

   d. 46 m up the slope.
27 + □ = 73, 27 + 3 = 30, 30 + 40 = 70,
70 + 3 = 73, 3 + 40 + 3 = 46 m

2. Practical activity

Page 9: Calendar Capers

Activity One

1. Answers will vary, but when you add 3 consecutive horizontal, vertical, or diagonal numbers on a calendar grid (or any grid array) and divide by 3, your answer is the middle number. For example, 11 + 12 + 13 = 36. 36 ÷ 3 = 12. This works for any 3 such numbers, so it will work for any month.
2. Answers may vary. We are finding the mean (average) of 3 consecutive horizontal, vertical, or diagonal numbers. The mean is found by "levelling" numbers to make them the same. So to find the mean of 8, 9, and 10, we take off 1 from the 10 and put it on the 8 so that all the numbers become 9. (The mean of 8 + 9 + 10 is 27 ÷ 3 = 9.)

Activity Two

1. a. Explanations may vary. One explanation is: Sam took the 48, divided it by 4, and then took off 4 (because the mean of the numbers in a 4-grid array is always 4 more than the first number). This gave him 8, the smallest number. The next number has to be 1 more than 8 (9), the third 7 more than 8 (15), and the fourth 8 more than 8 (16). So the 4 numbers are 8, 9, 15, and 16.

   Another way of explaining this is:
   The 4 numbers add up to 48. In a 4-grid array, a total of 16 is added to 4 times the first number. 48 – 16 = 32. 32 ÷ 4 = 8, which gives you the smallest number in the square. If n = the number given (48) and s = the smallest number, then $(n \div 4) - 4 = s$. $s + 1$, $s + 7$, and $s + 8$ will give you the other three numbers.

s	$s + 1$
$s + 7$	$s + 8$

8	9
15	16

   b. Numbers will vary. Sam's method will work for any set of 4 numbers in a 4-grid array.

2. Yes. Explanations will vary. A possible explanation is:
   Take any set of 4 numbers: 17 18
   24 25
   When we look at the numbers, we see:
   17 17 + 1
   17 + 7 17 + 8
   If we add all these together, we get
   4 x 17 + (1 + 7 + 8) = 68 + 16.

   As a generalisation: Let the first number be s.
   Then the next will be $s + 1$,
   the third will be $s + 7$,
   and the fourth will be $s + 8$.
   The total will be $4 \times s + 16$.

Page 10: Worth the Work

Activity

1. $445.00
2. $246.00
3. $216.05
4. a. $298.85

   b. $146.15

Page 11: Busking Blues

Activity

1. a. i. $16.20
   ii. $14.05
   iii. $18.90
   iv. $21.70

   b. i. $4.05
   ii. $^1/_4$. (Each of the 4 buskers gets $12.15. This is $^3/_4$ of the $16.20 that the 3 buskers each get, so they get $^1/_4$ less.)
2. a. $509.60. ($661.50 – $151.90)

   b. $19.60 less. ($661.50 ÷ 5 = $132.30. The single busker gets $151.90. $151.90 – $132.30 = $19.60)

Pages 12–13: What a View!

Activity One

Strategies will vary. The answers are:

a.	221	**b.**	84	**c.**	114
d.	64	**e.**	414	**f.**	57
g.	135	**h.**	119	**i.**	196
j.	144				

Activity Two

1. **a.** 48 windows. Methods will vary.
Possible methods include:

i. 6 x 10 – 2 x 3 – 2 x 3 = 60 – 6 – 6
= 48

ii. 4 x 10 – 6 = 34
2 x 10 – 6 = 14
34 + 14 = 48

iii. 2 x 4 + 2 x 3 + 2 x 10 + 2 x 7
= 8 + 6 + 20 + 14
= 48

b. $302.40. Methods will vary.
Two possible methods are:

i. $6 x 50 = $300
$0.30 x 50 = $15.00
$6.30 x 2 = $12.60
$315 – $12.60 = $302.40

ii. $6 x 40 = $240
$6 x 8 = $48
$0.30 x 40 = $12.00
$0.30 x 8 = $2.40
240 + 48 + 12 + 2.40 = $302.40

2. **a.** 384 windows. Methods will vary.
Possible methods include:

i. 10 x 24 = 240. 12 x 6 = 72,
72 + 72 = 144. 144 + 240 = 384

ii. 22 x 24 = 528. 12 x 12 = 144.
528 – 144 = 384

iii. 22 x 6 = 132. 2 x 132 = 264.
10 x 12 = 120. 264 + 120 = 384

b. $2,419.20

Pages 14–15: Easy Nines

Activity One

1.

	Using my 10 times table	Down a decade and digits adding up to 9	Using my 3 times table	Answer
6 x 9 =	6 x 10 = 60 One group of 6 less: 60 – 6 = 54	It will be in the 50s. 5 + □ = 9 5 + 4 = 9, so it's 54.	6 x 3 = 18. Double 18 is 36. Add 18 and 36 to get 54.	54
9 x 9 =	9 x 10 = 90 90 – 9 = 81	It will be in the 80s. 8 + 1 = 9, so it's 81.	9 x 3 = 27. Double 27 is 54. Add 54 and 27 to get 81.	81
3 x 9 =	3 x 10 = 30 30 – 3 = 27	It will be in the 20s. 2 + 7 = 9, so it's 27.	3 x 3 = 9. Double 9 is 18. Add 18 and 9 to get 27.	27
5 x 9 =	5 x 10 = 50 50 – 5 = 45	It will be in the 40s. 4 + 5 = 9, so it's 45.	5 x 3 = 15. Double 15 is 30. Add 30 and 15 to get 45.	45
2 x 9 =	2 x 10 = 20 20 – 2 = 18	It will be in the 10s. 1 + 8 = 9, so it's 18.	2 x 3 = 6. Double 6 is 12. Add 12 and 6 to get 18.	18
8 x 9 =	8 x 10 = 80 80 – 8 = 72	It will be in the 70s. 7 + 2 = 9, so it's 72.	8 x 3 = 24. Double 24 is 48. Add 48 and 24 to get 72.	72

2. Answers will vary. One possible strategy is to use doubling. For example, for 7 x 9, double the 7 three times and then add 1 more group of 7:
7 x 2 = 14
14 x 2 = 28
28 x 2 = 56
56 + 7 = 63
(7 x 9 = 63)

Activity Two

1. A game for practising the 9 times strategies

2. a. 36

b. 63

c. 90

Answers for **d–i** and some possible strategies using the 9 times table are:

d. 95. (19 x 5 is the same as 9 x 5 plus 10 x 5.)

e. 392. (49 x 8 is the same as 40 x 8 plus 9 x 8.)

f. 174. (29 x 6 is the same as 20 x 6 plus 9 x 6.)

g. 9. (81 ÷ 9 = □ is the same as 9 x □ = 81.)

h. 7. (63 ÷ 9 = □ is the same as □ x 9 = 63.)

i. 54. (□ ÷ 9 = 6 is the same as 6 x 9 = □.)

Pages 16–17: Jumping Practice

Activity

1. a. Estimates will vary, but they should be close to those given below.

i. Vaitoa:
hop = 1 800 mm
step = 2 400 mm
jump = 3 300 mm

ii. Hira:
hop = 1 500 mm
step = 2 250 mm
jump = 3 250 mm

iii. Grant:
hop = 1 600 mm
step = 2 400 mm
jump = 3 800 mm

b. Discussion will vary. There are 10 mm in a cm, so each mm distance can be divided by 10 to get cm (for example, 1 500 mm ÷ 10 = 150 cm).

2. a.

Distances in centimetres				
	Sheena	Vaitoa	Hira	Grant
Hop	165	180	150	160
Step	250	240	225	240
Jump	315	330	325	380
Total distance	730	750	700	780

b. Discussion will vary. Sheena needs to improve on her hop and jump to beat Vaitoa. She is already better than Hira overall, but Hira's jump is better than hers. Grant's jump is about 65 cm more than Sheena's, which is a lot to make up, so she may also need to improve on her hop and step if she wants to have an overall distance of more than 780 cm.

Game

A game for practising converting units of measurement

Page 18: Arcade Adventure

Activity

1. a. 2 310 tokens

b. 240 tokens

c. 2 870 tokens

d. Explanations will vary, but they should be based on multiplying by 10. (You should avoid saying "add a 0": multiplying by 10 involves place value, with each digit moving 1 place to the left. For example, if you add a 0 to 24, you still get 24. If you multiply 24 by 10, you get 240.)

2. a. 56 tokens

b. 160 tokens

c. 120 tokens

3. 106 games

4. a. 6 games

b. 12 games

c. 24 games

5. 18 games (twice as many)
6. Practical activity

Page 19: Wheels Galore

Activity

1. 78 wheels
2. 9 tandems (and 5 mountain bikes)
3. 4 tandems (and 2 unicycles)
4. 17 wheels (includes trailer [2 wheels] and car [4 wheels]). (Light trailers only have 2 wheels, but if you thought that the trailer had 4 wheels, then your answer would be 19 wheels.)
5. Problems will vary.

Page 20: Stacking Up

Activity

1. a. i. $3/8$. (3 eighths)
   ii. $1/3$. (1 third)
   iii. $5/8$. (5 eighths)
   iv. $3/5$. (3 fifths)
   v. $2/7$. (2 sevenths)
   vi. $1/12$. (1 twelfth)

   b. i. $1/8$. (1 eighth)
   ii. $2/3$. (2 thirds)
   iii. $2/8$ (2 eighths) or $1/4$ (1 quarter)
   iv. $2/5$. (2 fifths)
   v. $4/7$. (4 sevenths)
   vi. $8/12$ (8 twelfths) or $3/4$ (3 quarters)

   c. i. Yellow + Blue + Pink + Green
   3 eighths + 1 eighth + 3 eighths + 1 eighth = 8 eighths
   = 1 whole

   ii. Yellow + Blue
   1 third + 2 thirds = 3 thirds
   = 1 whole

   iii. Yellow + Blue + Green
   5 eighths + 2 eighths + 1 eighth = 8 eighths
   = 1 whole

   iv. Yellow + Blue
   3 fifths + 2 fifths = 5 fifths
   = 1 whole

   v. Yellow + Blue + Green
   2 sevenths + 4 sevenths + 1 seventh = 7 sevenths
   = 1 whole

   vi. Yellow + Blue + Pink + Green
   1 twelfth + 8 twelfths + 1 twelfth + 2 twelfths
   = 12 twelfths
   = 1 whole

2. i. $3/8 + 1/8 + 3/8 + 1/8 = 8/8$
   $= 1$
   ii. $1/3 + 2/3 = 3/3$
   $= 1$
   iii. $5/8 + 2/8 + 1/8 = 8/8$
   $= 1$
   iv. $3/5 + 2/5 = 5/5$
   $= 1$
   v. $2/7 + 4/7 + 1/7 = 7/7$
   $= 1$
   vi. $1/12 + 8/12 + 1/12 + 2/12 = 12/12$
   $= 1$

Page 21: Marble Marvels

Activity

1. He should choose the second jar. The first jar would give him 52 marbles, and the second jar would give him 58 marbles. Methods will vary. For example:
   78 ÷ 3 = 26. 26 x 2 = 52
   232 ÷ 4 = 58
2. Answers will vary. Other possible methods include:
   "Half of 232 is 116. Half of 116 is 58, so a quarter of 232 is 58."
   "A third of 78 is 26. 26 + 26 is 52, so two-thirds of 78 is 52."
   "A quarter of 200 is 50. A quarter of 32 is 8, so a quarter of 232 is 50 + 8 = 58."
   "90 is 12 more than 78. Two-thirds of 90 is 60, and two-thirds of 12 is 8, so two-thirds of 78 is 60 – 8 = 52."

3. a. The second jar. (It contains 44, and the first one contains 42.)
   b. The second jar. (It contains 24, and the first one contains 23.)
   c. The second jar. (It contains 45, and the first one contains 34.)
   d. Either jar. (Both jars contain 60 marbles.)
   e. The first jar. (It contains 78, and the second one contains 72.)
4. Answers will vary. In their simplest forms, possible solutions include:

63-marble jar	42-marble jar	Number of marbles
$^1/_9$	$^1/_6$	7
$^2/_9$	$^1/_3$	14
$^1/_3$	$^1/_2$	21
$^4/_9$	$^2/_3$	28
$^5/_9$	$^5/_6$	35
$^6/_9$	$^1/_1$	42

Pages 22–23: Fraction Frenzy

Activity

1. Practical activity. Your columns should have equivalent fractions in them. (Equivalent fractions are different ways of expressing the same fraction.) Karyn has arranged her first row from the smallest to the largest fraction.

$^1/_5$	$^1/_4$	$^1/_3$	$^1/_2$	$^3/_5$	$^2/_3$	$^3/_4$	$^2/_2$
$^2/_{10}$	$^2/_8$	$^2/_6$	$^6/_{12}$	$^9/_{15}$	$^6/_9$	$^9/_{12}$	$^{10}/_{10}$
		$^4/_{12}$	$^2/_4$				$^4/_4$
			$^7/_{14}$				$^7/_7$

2. a. in the $^1/_4$ column
   b. in the $^1/_3$ column
   c. in the $^1/_3$ column
   d. in the $^3/_5$ column
   e. in the $^1/_5$ column
   f. in the $^1/_2$ column
   g. in the $^1/_4$ column
   h. in the $^2/_2$ column
3. a. $^2/_5 = {}^4/_{10}$ or $^6/_{15}$
   b. $^3/_7 = {}^6/_{14}$
   c. $^2/_3 = {}^4/_6$, $^8/_{12}$, or $^{10}/_{15}$
4. Answers will vary. For example, $^2/_6$ and $^4/_{12}$ both simplify to $^1/_3$. They all take up the same length on the fraction wall. The top number (the numerator) and the bottom number (the denominator) can both be divided by 2 (and in the case of $^4/_{12}$, by 2 again) to get $^1/_3$. So, equivalent fractions simplify to the same simplest form. Another example is $^4/_{10}$ and $^8/_{20}$, which both simplify to $^2/_5$, so they are equivalent fractions.
5. a. $^6/_9 = {}^2/_3$
   b. $^6/_9 = {}^{66}/_{99}$
   c. $^6/_9 = {}^8/_{12}$
6. 1 000

Page 24: Surf's Up!

Activity

1. i. a. The Surf Shop. ($42 vs $43.20)
      b. $1.20
   ii. a. Surf the Waves Company. ($76.80 vs $84)
      b. $7.20
   iii. a. The Surf Shop. ($33.60 vs $37.40)
      b. $3.80
   iv. a. Surf the Waves Company. ($48 vs $60.20)
      b. $12.20
2. a. $200.40
   b. $224.80
   c. $24.40
3. Problems will vary.

Figure It Out

Number: Book Three
Teachers' Notes

Overview of Number: Book Three

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Numbers on the Line	Using double number lines	2	14
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Skimming Stones	Using mental strategies for addition	4–5	17
The Strategy Strut	Exploring addition and subtraction strategies	6–7	18
Slippery Slope	Using the up-and-back-through-tens strategy	8	20
Calendar Capers	Using patterns, addition, and subtraction to solve calendar problems	9	21
Worth the Work	Using addition and subtraction to solve problems	10	22
Busking Blues	Solving problems with division and subtraction	11	23
What a View!	Using multiplication arrays	12–13	24
Easy Nines	Deriving multiplication and division facts	14–15	25
Jumping Practice	Estimating and converting units of measurement	16–17	26
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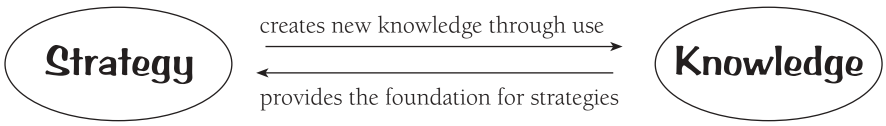
Introduction to Number

There is a remarkable commonality in the way many countries around the world are now teaching arithmetic. Changes in the approaches reflect the evolving demands of everyday life, a greater volume of classroom-based research about how students learn, and a desire to improve general levels of numeracy.

In the past, arithmetic teaching has focused on preparing students to be reliable human calculators. The prevalence in society of machines that calculate everything from supermarket bills to bank balances has meant that students now require a wider range of skills so that they can solve other problems flexibly and creatively.

The Figure It Out series aims to reflect these trends in modern mathematics education. A range of books is provided at different levels to develop both number skills and number sense. The Number books are aimed at developing students' understanding of the number system and their ability to apply efficient methods of calculation.

The development of the Figure It Out series has occurred against the backdrop of a strong drive for improved standards of numeracy among primary-aged and intermediate students. A key element of this drive has been the creation of the Number Framework as part of the Numeracy Strategy. The framework highlights this significant connection between students' ability to apply mental strategies to solving number problems and the knowledge they acquire.



Learning activities in the series are aimed at both the development of efficient and effective mental strategies and increasing the students' knowledge base.

Links to the Number Framework

There are strong links to the stages of development in the Number Framework in *Number*, Books Two and Three for level 3 and levels 3–4. The knowledge- and strategy-based activities in the level 3 books are at the early additive part–whole to advanced additive/early multiplicative stages of the Number Framework. Those in the level 3–4 books also link to the advanced multiplicative (early proportional) part–whole stage of the Number Framework.

Information about the Number Framework and the Numeracy Project is available on the NZMaths website (www.nzmaths.co.nz/Numeracy/Index.htm). The introduction includes a summary of the eight stages of the Number Framework. Some of the Numeracy Project material masters are relevant to activities in the Figure It Out student books and can be downloaded from www.nzmaths.co.nz/Numeracy/materialmasters.htm
Books for the Numeracy Development Project can be downloaded from www.nzmaths.co.nz/Numeracy/2004numPDFs/pdfs.htm

Terms

Teachers who are not familiar with the Numeracy Project may find the following explanations of terms useful.

The Number Framework: This is a framework showing the way students acquire concepts about number. It comprises eight stages of strategy and knowledge development.

Knowledge: These are the key items of knowledge that students need to learn. Knowledge is divided into five categories: number identification, number sequence and order, grouping and place value, basic facts, and written recording.

Strategies: Strategies are the mental processes that students use to estimate answers and solve operational problems with numbers. The strategies are identified in the eight stages of the Number Framework.

Counting strategies: Students using counting strategies will solve problems by counting. They may count in ones, or they may skip-count in other units such as fives or tens. They may count forwards or backwards.

Part–whole thinking or part–whole strategies: Part–whole thinking is thinking of numbers as abstract units that can be treated as wholes or can be partitioned and recombined. Part–whole strategies are mental strategies that use this thinking.

Partitioning: Partitioning is dividing a number into parts to make calculation easier. For example, 43 can be partitioned into 40 and 3, or 19 can be partitioned into 10 and 9 or thought of as 20 minus 1.

Relevant stages of the Number Framework for students using the level 3 and 3–4 *Number* books:
Stage five: early additive part–whole: At this stage, students have begun to recognise that numbers are abstract units that can be treated simultaneously as wholes or can be partitioned and recombined. This is called part–whole thinking.

A characteristic of this stage is the derivation of results from related known facts, such as finding addition answers by using doubles or "teen" numbers. For example, students at this stage might solve 7 + 8 by recalling that 7 + 7 = 14, so 7 + 8 = 15. They might solve 9 + 6 by knowing that 10 + 6 = 16, so 9 + 6 = 15. They might solve 43 + 35 as (40 + 30) + (3 + 5), which is 70 + 8 = 78.

Stage six: advanced additive/early multiplicative part–whole: At this stage, students are learning to choose appropriately from a repertoire of part–whole strategies to estimate answers and solve addition and subtraction problems.

Addition and subtraction strategies used by students at this stage include:
- standard place value with compensation (63 – 29 as 63 – 30 + 1)
- reversibility (53 – 26 = □ as 26 + □ = 53)
- doubling (3 x 4 = 12 so 6 x 4 = 12 + 12 = 24)
- compensation (5 x 3 = 15 so 6 x 3 = 18 [3 more])

Students at this stage are also able to derive multiplication answers from known facts and can solve fraction problems using a combination of multiplication and addition-based reasoning. For example, 6 x 6 as (5 x 6) + 6; or $^3/_4$ of 24 as $^1/_4$ of 20 is 5 because 4 x 5 = 20, so $^3/_4$ of 20 is 15, so $^3/_4$ of 24 is 18 because $^3/_4$ of the extra 4 is 3.

Stage seven: advanced multiplicative part–whole: Students who are at this stage are learning to choose appropriately from a range of part–whole strategies to estimate answers and solve problems involving multiplication and division. For example, they may use halving and doubling (16 x 4 can be seen as 8 x 8) and trebling and dividing by 3 (3 x 27 = 9 x 9).

Students at this stage also apply mental strategies based on multiplication and division to solve problems involving fractions, decimals, proportions, ratios, and percentages. Many of these strategies involve using equivalent fractions.

Page 1: Crazy Compatibles

Achievement Objectives

- explain the meaning of the digits in any whole number (Number, level 3)
- recall the basic addition and subtraction facts (Number, level 2)
- mentally perform calculations involving addition and subtraction (Number, level 2)

Activity

To do this activity, students need to know the basic addition and subtraction facts. Encourage your students to use the facts they know to derive the facts they don't know. For example, 55 + 49 can be seen as 55 + 45 + 4 and 49 + 49 can be seen as 50 + 50 – 2.

Compatible numbers add together to make rounded or tidy numbers, such as 3 + 7 = 10, 30 + 70 = 100, and 300 + 400 = 700. Use examples such as these to ensure that the students see the connections between hundreds, tens, and ones. (See *Number: Book One*, Figure It Out, Level 2, page 6 for an activity that focuses on these connections.) The students can use these connections and tidy numbers to solve problems such as 25 + 76 = ☐ and 297 + ☐ = 1 000 in question **3**.

After the students have done question **1**, you could ask them: "What other type of problems can compatible numbers help you to solve mentally?" (Any that include numbers that have a tidy number total close by. For example, 47 is close to 50 and 31 is close to 30. So 47 – 31 could be seen as 50 – 30 – 4.)

Some students may find question **2** difficult because they cannot see the connections between the known and unknown facts. You could prompt them by drawing their attention to the example given in the boy's speech bubble, which builds on one of the compatibles in question **1**.

Question **3** builds on this understanding and extends it to 3-digit numbers. It also draws on the students' understanding of place value.

In question **4**, the students have to make their own compatible numbers grid, using numbers that add up to a total other than 100. The students' grids will need to have an even number of squares, such as 3 by 4 or 4 by 4.

Further discussion and investigation

You could extend the idea of compatible numbers to fractions and decimals, provided that the students understand these concepts.

Page 2: Numbers on the Line

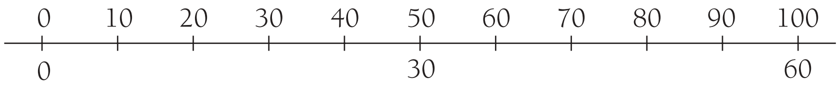
Achievement Objectives

- recall the basic multiplication facts (Number, level 3)
- mentally perform calculations involving addition and subtraction (Number, level 2)
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- write and solve problems which involve numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

Activity

For this activity, students need to use their basic addition, subtraction, multiplication, and division facts. They also need to know how to use a number line to solve proportion and ratio problems.

A double number line has a set of data placed above the line and a second set placed below. For example:



The explanation in the students' book shows how Kaylee uses a double number line to compare the time it takes her and her father to tramp 16 kilometres with the time it takes a group to tramp 4 kilometres. Kaylee starts with

and uses a halving strategy to work out the in-between hours and kilometres.

In question **1**, the basic number line is given, and the students have to use a strategy, possibly halving, to find the number of cars for 9 students. In this case, it is the bottom line, the number of students, that needs to be halved (18) and halved again (9) so that the corresponding halves on the top line (6 and 3) can be used to answer the question.

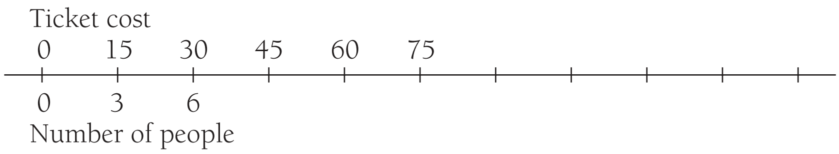
In questions **2**, **3**, and **4**, the students have to make their own number lines and put the appropriate marks in the correct places to solve the problems.

Further discussion and investigation

You can extend this activity by creating further problems for the students to solve by imaging the number lines and marks to solve the problems mentally.

Alternatively, you could introduce problems that require the number lines to be extended so that numbers can be doubled rather than halved as they are placed along the line. For example:

"If tickets to the circus cost $15 for 3 people, how much would they cost for 24 people?"



Have the students make up some problems of their own that can be solved using a double number line. For example, they could find the amounts of ingredients needed if a recipe is made 4 or 6 times larger.

There are many class-based activities that use various ratios, such as the ratio of parents to students or buses to pupils. Use these as realistic problems for the students to solve using number lines.

Page 3: Banking Issues

Achievement Objectives

- represent a sum of money by two or more different combinations of notes and coins (Measurement, level 2)
- explain the meaning of the digits in any whole number (Number, level 3)
- recall the basic multiplication facts (Number, level 3)

Activity

To do this activity, students need to be able to recognise money denominations. They also need to be able to count in tens, hundreds, and thousands and to multiply by tens and hundreds.

Most students will be able to recognise the notes and coins, but they may not know the number of tens, hundreds, and so on in the various amounts. To help them understand this, use a place value chart so that the students can read the values of the notes and amounts.

Tens *Millions*	**Ones** *Millions*	**Hundreds** *Thousands*	**Tens** *Thousands*	**Ones** *Thousands*	**Hundreds** *Ones*	**Tens** *Ones*	**Ones** *Ones*	**Tenths**	**Hundredths**
				$1,000	$100	$10	$1	10c	1c

To do the questions, the students need to be able to recognise the number of ones, tens, and hundreds in a dollar value such as $178. (Note that 1 hundred, 7 tens, and 8 ones can also be seen as 17 tens and 8 ones or 178 ones.) Some students may have difficulty with this concept. It is not necessary to introduce traditional names such as "face, place, and total value" because the activity only requires the students to understand the total number of tens, hundreds, thousands, and so on.

Encourage the students to use toy money to model the amounts until they see the pattern in the numbers. For example, $197 has 19 tens (and 7 ones).

Show the students how the numbers move on the place value chart when they multiply and divide by 10. For example, when they are multiplying 24 by 10, the 24 moves one place to the left to become 240.

Many students are confused about multiplying and dividing by 10 and think that it's simply a case of "adding a 0 or taking a 0 off". Challenge this language by asking them to add 0 to 24. The answer, of course, is 24. Similarly, if they take 0 from 240, the answer is still 240. The digits remain where they are. The students who understand this concept will say "Because the digits have moved 1 place to the left, we put a 0 in the vacant ones place."

This effect can be shown powerfully on a calculator when the students enter 10 x 24 and watch the display as they press the equals button. They can then use the constant function to multiply by 10 repeatedly by pressing $\boxed{=}$ a number of times and watch the digits move one place to the left each time. (On some calculators, the students will need to enter 10 $\boxed{\times}$ $\boxed{\times}$ 24 before they can use the constant function.)

You may wish to extend this concept to multiplying by a factor of 100 or 1 000.

Question **1** asks the students to find the numbers of notes and coins in a given total. In question **2**, the students multiply by 10 and then add and subtract the totals, and in question **3**, they divide the values into hundreds, fifties, and ones.

Further discussion and investigation

Have the students relate the place-value concepts to other metric measures, such as the number of grams in a kilogram.

In pairs or small groups, the students could find large numbers mentioned in newspaper and magazine articles or advertisements and discuss the number of tens, hundreds, and so on in each number.

The students could investigate how many ones, tens, and so on there are in 1 million.

An interesting investigation could be to research how money developed. The students could also find out why New Zealand changed from the British system of pounds, shillings, and pence (£, s, and d) to decimal currency. They could discuss what benefits and/or disadvantages the introduction of decimal currency has brought.

Pages 4–5: Skimming Stones

Achievement Objectives

- mentally perform calculations involving addition and subtraction (Number, level 2)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 3)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 3)
- make sensible estimates and check the reasonableness of answers (Number, level 3)

Activity

To do this activity, students need to know what a "mean" is (a reminder is given on the page). They will use it in the game, but they may also decide to use the mean as a way of finding the new champion in the activity. For the game, students also need to understand rounding.

Two major concepts are encouraged on the page: finding various mental strategies to add a series of numbers and developing an understanding of what an average (mean) is. The activity shows how the strategy of compatible numbers and making sets of 10 can be used and asks the students to use other strategies to find the total. These strategies could include using basic facts (up to and including adding 9), using 5s instead of 10s, or "averaging the numbers" and multiplying by 6. Jordan's skim scores, 7, 6, 7, 8, 7, and 7, are very suitable for averaging the numbers, as shown in the Answers, but this strategy could also be applied to Penny's (3 x 5 + 3 x 7) and Ford's (6 x 7, with the extra 1 in each of the 8s being added to the 4).

Various answers and strategies are possible for question **2**. After the students have finished the activity, encourage them to share their strategies with a group of classmates. Some suggestions are given in the Answers.

Some students may already have suggested the strategies in questions **3** and **4** as a way of determining the new champion, but they still need to apply these strategies and, in the case of question **4**, to justify their answer.

Game

A copymaster for the game is provided at the end of these notes. To find their score for each turn, the students need to calculate the mean. They could add the numbers and divide by the number of counters, but if they find division difficult, they could use the strategy of averaging out the numbers and multiplying by the number of skims, as suggested for question **1**. For example, for skim scores of 9, 7, 7, and 4, take 2 from the 9 and add it to the 4. This now makes 7, 7, 7, and 6, so a realistic mean is 7. The students can use a calculator to check the mean. At times, the calculator check will display decimal fraction numbers, so you may need to discuss how to round numbers to the nearest whole number. The game requires the students to "flick" counters, so it would also be appropriate to discuss the correct behaviour for playing the game.

Further discussion and investigation

The students could practise using other students' mental strategies for adding a set of numbers. Ask them: "Would you need to change the strategy if the numbers included tens and/or hundreds? How would you do this?"

Discuss the various ways of finding the "champion" and see if the group or class can come to a consensus as to the fairest way.

The students could investigate sports in which the worst score is eliminated, for example, diving. (See also **Number**: *Book Three*, Figure It Out, Levels 3–4, pages 2–3, for an activity in which lowest scores are discarded.)

Pages 6–7: The Strategy Strut

Achievement Objectives

- mentally perform calculations involving addition and subtraction (Number, level 2)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 3)

Activities One and Two

For these activities, students are exploring strategies for adding and subtracting 2- and 3-digit numbers.

The activities encourage the students to use a variety of mental strategies to solve addition and subtraction problems and to think about which is the best strategy for solving a particular problem.

The best strategy for any problem depends on:

1. The numbers involved. Some numbers lend themselves to certain strategies more readily than others. For example, for question **1a**, because 99 is so close to 100, Hamish's rounding and Erin's tidy number strategies seem much easier to work with than Sui's number line (100 + 50 or 100 + 47 seems easier or quicker than 99 + 1 + 40 + 8). In **Activity Two**, using Taufa's strategy of changing subtraction to addition also depends on the numbers involved. For example, for 103 – 99 = ☐, it's easy to go 99 + ☐ = 100, 99 + 1 = 100, 100 + 3 = 103, so 99 + (1 + 3) = 103 and 99 + 4 = 103. So 103 – 99 = 4. But for 103 – 28 = ☐, changing to addition isn't so efficient.
2. The student's preferences. Students tend to use the strategy or strategies they feel comfortable with, as the students in the students' book have. But it's very important that you don't just accept any strategy that the students use. You should encourage the students to look for and use the *most efficient strategy for the problem*.

The purpose of these activities is to give students experience with different strategies and see what's most useful for a particular problem. As a student's number sense develops, they will be able to more quickly see which are the most efficient strategies to use. Students need to be flexible enough to change the strategy they use according to the problem.

In rounding and compensation, numbers are rounded to the nearest tidy number like 10, 20, 100, or 150 and then added. The answer is adjusted according to whether the numbers were rounded up or down.

In number lines, jumps are made along a number line to a tidy number (for example, 30) and then to other tidy numbers, such as multiples of 10, and then a small jump is made to finish if necessary. The jumps are then added (or subtracted) to find the answer. After the students get used to using number lines on paper, they should be able to visualise (or image) them to solve problems.

In the place value strategy, numbers are broken into place value parts (tens and ones), each part is then added, and adjustments (renaming) through part–whole methods (for example, 7 is seen as 8 – 1) are made.

Tidy numbers relate to part–whole thinking, in which numbers are broken into parts so that tidy numbers are made.

Activity One gives the students four number strategies and asks them to use each strategy to solve the two problems in question **1**. Encourage the students to record each strategy as they use it because many may just slip back to using the strategy they are most comfortable with.

Before the students look at the strategies given in the activity, you could give them the problem that the students on the page are working on and have them brainstorm various strategies. Ask the students to name the strategy they suggest. The students could then go on to look at the strategies given on the page in the students' book.

After using the strategies, the students are asked to decide which strategy they found the "best". The students' choices of the best strategy will vary depending on their knowledge of strategies and their recall of basic facts and part–whole thinking. Be aware that some students may revert to using written forms that are not asked for.

There are many different contexts that you can use for this type of question to give your students the opportunity to practise choosing and using various strategies.

Activity Two approaches subtraction in a similar way. Strategies offered in the examples include number lines, tidy numbers, place value, part–whole, and a new strategy: converting subtraction to addition.

Many students may not have realised that subtraction problems can be solved by using the opposite operation (addition). This may be their first introduction to this idea, and some students may have difficulty grasping the concept. Work through Taufa's strategy with them and help them apply it to one of the problems in question **1a**.

Further discussion and investigation

As an extension, the students could use the same strategies to apply to 4- and 5-digit addition. For example, using a rounding and compensation strategy:

2 643 + 1 298 = $\square$
2 643 + 7 = 2 650
1 298 + 2 = 1 300
2 650 + 1 300 = 3 950
3 950 – 7 – 2 = 3 941.

Have them make a table of all the different strategies that can be used for addition and subtraction and then see if some strategies are better for some types of numbers. For example, numbers close to a 10, such as 19 and 32, might be best suited to a tidy number strategy, large numbers might be suited to a rounding and compensation strategy, and all numbers might be suited to a place value strategy.

Page 8: Slippery Slope

Achievement Objectives

- recall the basic addition and subtraction facts (Number, level 2)
- mentally perform calculations involving addition and subtraction (Number, level 2)
- pose questions for mathematical exploration (Mathematical Processes, problem solving, level 3)

Activity

This activity builds on The Strategy Strut by showing students another strategy they could use for addition and subtraction. For this activity, students need to be able to count forwards and backwards in tens and use mental strategies for addition and subtraction. They also need to know their addition and subtraction basic facts.

You may need to discuss "up and back through tens" with your students to make sure that they understand what it means. It's actually "up and back through *a multiple* of 10", but that is a bit long-winded. This strategy is similar to the tidy numbers or rounding and compensating strategies in The Strategy Strut.

This may be the first time the students have been introduced to solving problems by adding and subtracting using a multiple of 10. Encourage them to use and then visualise a number line so that they can see how they can add (or subtract) to a 10. (The notes for The Strategy Strut explain how to use number lines.) If the students have difficulty visualising a number line, get them to draw it and mark in the jumps to tens. If they always jump to the next multiple of 10, encourage them to make larger jumps. For example, for 38 + ☐ = 73, instead of going 38 + 2 = 40 and then jumping to 50, 60, 70, and + 3 (that is, 2 + 10 + 10 + 10 + 3 = 35), encourage them to go 38 + 2 = 40 and then straight to 70 + 3 (that is, 2 + 30 + 3 = 35).

Discuss with the students whether visualising a number line helps them solve the problems mentally and have them explore other mental strategies they could use to solve the problems.

The students may misinterpret the language of the problems and add instead of subtract or vice versa. For example, in question **1d**, the students may focus on "how far from the top" instead of "how far up the slope".

Encourage the students to practise and extend the strategies they use to 3-digit numbers.

Page 9: Calendar Capers

Achievement Objectives

- write and solve problems which involve numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- recall the basic addition and subtraction facts (Number, level 2)
- mentally perform calculations involving addition and subtraction (Number, level 2)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 3)

Activities One and Two

For these activities, students need to know their addition and subtraction basic facts, be able to divide numbers (perhaps using a calculator), find an average (mean) of a set of numbers by manipulating numbers rather than by dividing, and understand the meaning of "vertical", "horizontal", and "diagonal".

The students should find this an interesting activity because there are many patterns to be found in a number array such as a calendar, and calendars are commonly used around the school and at home.

Activity One requires the students to add 3 numbers and then divide the total by 3. They should notice that the result is always the middle number of the sequence. Instead of dividing, the students may wish to "level" the numbers. For 3 numbers in sequence, this means taking an amount off the largest number and giving it to the smallest number so that all the numbers end up the same. For example, using 15, 23, and 31, take 8 from the 31 and add it to the 15 so that all the numbers become 23.

This is a great way to introduce students to the idea of a "mean", that is, adding a set of numbers and then dividing by the number in the set: 15 + 23 + 31 = 69; 69 ÷ 3 = 23. (See also pages **4–5** of the students' book.) The students should notice that if the numbers from the calendar are in a horizontal row, the difference between them is 1; if they are vertical, it is 7 (because there are 7 days in a week); and if they are on the top left and bottom right diagonally, the difference is 8, that is, 1 week and 1 day. If they are on the top right and bottom left diagonally, the difference is 6 (1 week less 1 day).

Activity Two is an extension of **Activity One**, but the students now have to discover the relationship between 4 numbers. Sam already knows the relationship and has worked it out in his head, but the students have to find out what he did.

Two possible methods that Sam could have used are explained in the Answers.

Further discussion and investigation
You could ask the students:
"Does this levelling of the mean work for 5 numbers in a row, column, or diagonal? Why or why not?"
"Are there any other sets of numbers (such as a 3 by 2 or 3 by 3 grid) that work in a similar way?"
"Do these patterns work on a hundreds board?"
"How and why is a hundreds board different from a calendar?"

Cross-curricular link
Science
The students could investigate how the calendar was developed and its relationship to the Moon and the Sun.

Page 10: Worth the Work

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)

Activity

In this activity, students solve money problems involving addition and subtraction, use estimation skills, and extract information from a table.

Some students may need to discuss the numbers in the table because they may not, at first, agree with the amounts in the Pay to date column if they forget to take into account the fact that a job may be done more than once a week. For example, Rachel does the dishes 3 times a week at $2 a time, which is $6 x 10 weeks = $60.

Question **1** should be a mental addition activity. Strategies could include basic facts knowledge and finding tens and hundreds: 60 + 40, 150 + 55.

In question **2**, discourage the students from using a calculator and possibly even a pencil and paper. The mental strategy for taking $199 from $445 could be to round $199 to $200, take the $200 away from $445 to make $245, and then add $1 more to make $246.

Question **3** initially looks difficult, but it can be done by taking away $30 and then adding 5 cents. The students could try doing this mentally.

For question **4**, again encourage the students to look for a way they can solve the problem in their heads. Before they start, make sure everyone understands that they need to consider the cost of the skateboard and helmet and the pads as well as the amounts mentioned in question **4**. One strategy is: $19.90 is close to $20; 20 + 50 = 70, and $70 – 10 cents = $69.90. The students then need to add the cost of the skateboard and helmet and the pads.

Ask the students:
"Is it quicker to find the answers by working mentally, using a pencil and paper, or using a calculator?" "What must you know if you are to 'believe' the calculator's answers?"

The students will hopefully see that working mentally will often be quicker because pencil and paper methods can be slower and more tedious. For the calculator to be an effective tool, the students must be able to estimate the answer so that they know if they have pushed the correct buttons. Estimation skills require quick recall of all basic facts, number sense, knowledge of place value, and multiplying and dividing by 10, 100, and so on.

After the students have completed the page using mental strategies, they could check their own work with a calculator and share their strategies with a classmate.

Further discussion and investigation
The students could look at some current advertising material and work out some more problems based on buying items listed in the advertisements with a set amount of money.

If appropriate, have the students make a table of the money they have received from jobs done around the home and work out how much they could save in 10 weeks. If they had a special item they wanted to buy for themselves, how long would they have to work in order to afford it?

Page 11: Busking Blues

Achievement Objectives

- recall the basic multiplication facts (Number, level 3)
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)

Activity

In this activity, the students will use operations with dollars and cents and work with fractions. They should know their multiplication and division basic facts.

Encourage the students to solve the division problems in their heads rather than using a pencil and paper or their calculators. Some students may need to revise mental strategies for division of whole numbers and decimals. A common strategy is to "break up the amounts" to make the divisions easier. If we take $48.60 ÷ 3, we can split the 48.60 up into 30, 18, and 0.6 (for easy dividing by 3). For $94.50 ÷ 5, we can split the dollar amount into 90 and 4.50 (for easy dividing by 5). The students can check their answers by multiplying the shared amount by the number of people.

Question **1b** requires the students to divide $48.60 by 4 and then to subtract this amount from the amount found in **1a i**. The second part asks them to find what fraction the amount found for **1b i** is of the amount found for **1a i**. If the students have difficulty with this, remind them that they are looking for how much less the buskers earn as a fraction and discuss the relationship between the two amounts, $4.05 and $16.20. You could do this by separating the dollars and cents. 16 is 4 x 4, and 20 is 4 x 0.05. So $4.05 is 1/4 of $16.20.

For question **2**, a table is a useful way of keeping track of the amounts for each group.
A possible table is:

Number of buskers	Amount earned after 1 day	Amount earned after 7 market days	Amount earned per person after 7 market days
3 people	$48.60		
2 people	$28.10	$196.70	$98.35
5 people	$94.50		
1 person	$21.70	$151.90	$151.90
4 people	$48.60		

The students need to multiply the 1-day amounts by 7 and then divide this amount by the number of buskers. Wherever possible, encourage the students to work out the problems mentally and check their answers using written methods or a calculator.

You could have an interesting discussion with your students about how much each member of the group that earns the most each day receives in comparison with the other buskers. The students need to consider the pros and cons of busking alone or with others: an individual busker possibly earns more, but buskers performing together can support each other.

Pages 12–13: What a View!

Achievement Objectives

- recall the basic multiplication facts (Number, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

Activities One and Two

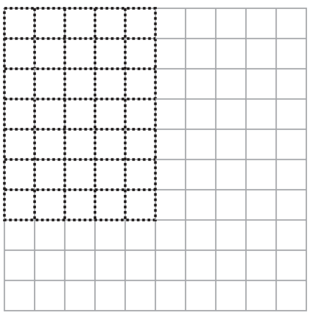
These activities are similar to those on pages **6–7** of the students' book in that they present a variety of strategies for students to experiment with and use, including multiplication strategies and the use of arrays.

Arrays are used to help develop the students' understanding of what they are doing when they are multiplying and to show them how two factors can be broken into parts that are easier to work with (just as addends can be in addition).

As explained in the notes for pages **6–7**, the best strategies for a particular problem depend on the numbers involved and what strategies the students feel most comfortable using. Encourage your students to be flexible in their use of strategies and to look for the most efficient way of solving problems.

Before attempting these activities, the students should have had experience using an array to find and show multiplication facts. Happy Hundreds, Numeracy Project material master 6-5 (available at www.nzmaths.co.nz/numeracy/materialmasters.htm), is a good resource for this, as is The Field of 100 Sheep in *Basic Facts*, Figure It Out, Level 3, pages 16–17.

The use of additional 10 by 10 squared grids, plus the pieces of coloured overhead transparency suggested in the students' book for use in this activity, can help the students to understand the mental strategies they can use to find the factors.



This shows 7 x 5, or 35 squares.

Introduce the students to these problems by giving them a similar problem before assigning the page and show them how to use the pieces of coloured overhead transparency. Encourage the students to develop a mental image for each one.

The diagram on the student's page shows an 18 by 13 array. The piece of coloured overhead transparency allows the students to see a group of 10 windows and a group of 3 windows across the top, as well as a group of 10 and a group of 8 down the side.

Activity One describes four strategies for solving the problem:

- rounding and compensation
- partial products
- halving and doubling
- doubles and partial products.

Note that the numbers in questions **a** to **j** have been selected to encourage the students to use a variety of mental strategies. 19 x 6 is close to 20 x 6 (then take off 6), and 7 x 28 is close to 7 x 30 (then take off 14). For numbers like 6 x 24, the students could make 6 x 20 and then 6 x 4, use doubling and halving (12 x 12), or use rounding and compensation. Encourage them to use known multiplication facts to derive what they need to know.

If the students still seem unsure of the strategies, have them make up further examples and solve them using shaded grids or coloured overhead transparency sheets.

In part **b** of each question in **Activity Two**, the students have to work out the price if Sparkle charges $6.30 per window. You may wish to encourage the students to multiply the money amounts mentally. 48 times $6.30 could be solved by adding 50 times $6 and 50 times $0.30 and then subtracting $12.60.

Further discussion and investigation
The students could use a mental strategy to work out the cost of cleaning the windows of the classroom, the whole school, or a nearby building.

Pages 14–15: Easy Nines

Achievement Objectives

- recall the basic multiplication facts (Number, level 3)
- make sensible estimates and check the reasonableness of answers (Number, level 3)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)

Activities One and Two

In these activities, students use their addition skills, explore the relationship between multiplication and division, use known facts to derive unknown ones, and show their understanding of multiplication as repeated addition.

These activities explore three methods for solving problems involving multiplying by 9. The 9 times table is often hard for students because it is usually the last to be introduced. Although the students have learned all the facts in the 9 times table in earlier tables (apart from 9 x 9, which they may have met through an exploration of square numbers), they often do not make these connections (for example, that 3 x 9 is the same as 9 x 3). Moreover, not enough time may have been allocated to memorising and maintaining this last times table.

Each of these strategies needs to be explored so that the students can call on one of them when they "forget" a basic fact or when they need to extend to larger numbers, such as 18 x 9.

Activity One, question **1** asks the students to complete their copy of the Easy Nines table (see the copymaster at the end of these notes), using the three methods explained on the page. Some students may need assistance in understanding each of the methods, especially if their recall of tables is not strong.

Question **2** asks the students if they can think of another strategy to solve the problems. The students may come up with several strategies. You may wish to explore these with the students as a group or class and then compare the effectiveness of each of the strategies. One strategy is to divide a number by 3 and then by 3 again to find the answer to a 9 times table fact.
81 ÷ 3 ÷ 3 = 9, and so 9 x 9 = 81.

Activity Two starts with a game using the 9 times table. To be efficient, the students should have a good recall of the 9 times table. If they don't, you could allow them to write out the 9 times table up to 9 times 10 and refer to it as they play the game. This will enable them to practise and familiarise themselves with the 9 times table in a fun environment.

In question **2**, the students need to use their knowledge and strategies to solve various problems. At first, the students may not see how they can use the 9 times table to help them with some of the problems, and they may revert to other methods. Some possible strategies are given in the Answers. The students may also come up with other possible strategies that may not necessarily involve using the 9 times table. For example, 50 x 8 – 8 is a more efficient way to solve 49 x 8 than 40 x 8 + 9 x 8 or 40 x 9 + 9 x 9 – 49.

Pages 16–17: Jumping Practice

Achievement Objectives

- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (Measurement, level 3)
- explain the meaning of the digits in any whole number (Number, level 3)
- explain the meaning of the digits in decimal numbers with up to 3 decimal places (Number, level 3)
- make sensible estimates and check the reasonableness of answers (Number, level 3)

Activity

In this activity, students add 3- and 4-digit numbers, add whole numbers and decimal numbers, read and draw number lines, and multiply and divide by 10, 100, 1 000, and so on.

In question **1**, the students are asked to estimate. Estimation is about closeness, and the student responses should not be marked wrong unless they are too far from the actual answer. The more practice we give students in estimating, the closer their estimates will become. Many students believe that a correct estimate is the same as the actual answer, and they don't like having incorrect answers, so they measure and then make the estimate the same as the measurement. Students need to be encouraged to take a few risks in their learning. Estimation is a safe way of taking a risk because the estimated answer does not need to be exact.

If the students have not been introduced to rounding, this may be an appropriate time to do it because rounding can assist with estimation. The general rule is that 5, 6, 7, 8, and 9 round up and 1, 2, 3, and 4 round down.

The estimation in this activity differs from other, more common, estimation in that it's not estimating the answer to an operation. It's an estimated reading from a number line that does not have enough information on it to give an accurate reading. The students will probably base their estimates on how far along each section they estimate a particular mark to be. They will also need to use strategies such as tidy numbers to make estimates of jumps that fall either side of a marked number. The students could check their estimates by adding them together to see if the total of their estimates is close to the total of the jump.

To help the students convert millimetres to centimetres and metres, revise the place value chart by having the students put the units on a chart like the one below.

Tens *Thousands*	Ones *Thousands*	Hundreds *Ones*	Tens *Ones*	Ones *Ones*	Tenths	Hundredths	Thousandths
	kilometre			metre		centimetre	millimetre
				3	0	0	0

Make sure the students realise that the numbers move to the left when multiplied by 10 and move to the right when divided by 10. Note that the decimal point does not move.

In question **2**, the students complete a chart, using their estimated distances from question **1**, and then add those distances to find the total distances. The chart is labelled Distances in centimetres, and the students need to use the correct form of measurement (that is, centimetres) in the table to make a valid comparison.

Game

When the dice is thrown, the students will need to be able to convert the dice number to the correct millimetre number so that it can be recorded on the playing board. This conversion should be a mental activity in which the students multiply and divide by multiples of 10.

Further discussion and investigation

On the place value chart, 1 centimetre is one-hundredth of a metre, which means that a centimetre unit needs to be placed in the hundredths column. In New Zealand, we don't usually use decimetres (dm) for 10 centimetres. Discuss with the students whether it would be helpful to use this measurement so that we could have names for each place value column.

Page 18: Arcade Adventure

Achievement Objectives

- explain the meaning of the digits in any whole number (Number, level 3)
- recall the basic multiplication facts (Number, level 3)
- make sensible estimates and check the reasonableness of answers (Number, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

Activity

In order to do this activity, students should be able to solve simple division and multiplication problems, work with simple ratios, and be able to multiply and divide by 10, 4, and 8.

Before the students attempt this activity, remind them about place value columns and relationships. The students can also use toy money to find the number of tens in 400 or hundreds in 2 000. Questions **1a**, **b**, and **c** require the students to multiply each number by 10. Encourage them to see that multiplying by 10 is a matter of shifting the digits one place to the left (that is, it is not a matter of shifting the decimal place). In question **1d**, they are asked to explain their method(s). Ask them to share their methods with a classmate and then write down what they did.

In questions **2a**, **b**, and **c**, the students need to multiply by 8. They should do this mentally rather than by using written algorithms. They can calculate 7 x 8 using their basic facts, think of 20 x 8 as 2 x 8 x 10, and think of 15 x 8 as 5 x 8 plus 10 x 8.

Question **3** requires division of a 3-digit figure by 8. Encourage the students to estimate the answer by rounding the number of tokens used in a game up to 10. Thus, 848 ÷ 10 = 84.8. This should show them that their actual answer will be more than 84. One way of working out the answer in their heads is to go 848 = 800 + 48. 800 ÷ 8 = 100, and 48 ÷ 8 = 6, so 848 ÷ 8 = 106.

For question **3**, the students need to realise that they must work with division because the problem is the reverse of the previous questions. Some students may just carry on multiplying, not understanding the change of focus.

Question **5** compares two games. The ratio for Freedom Buster is 4 tokens per game (4:1), while the ratio for Stunt Cycle is 8 tokens per game (8:1). The students need to be able see the relationship between the ratios (4 is half of 8, so half the number of tokens is needed for 1 game of Freedom Buster) and then see how many games of Freedom Buster can be played for the same cost as 9 games of Stunt Cycle (twice as many, which is 18).

In question **6**, the students are given the opportunity to make up their own problems. Before they share their problems with a classmate, they need to be able to work out the answers themselves. (This helps prevent the students from creating very large or complicated operations that are too difficult for others to solve.)

Further discussion and investigation
The students could undertake further exploration of ratios, such as doubling recipe quantities or investigating parent to student ratios for class trips.

Page 19: Wheels Galore

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- pose questions for mathematical exploration (Mathematical Processes, problem solving, level 3)

Activity

This activity involves the use of multiplication and division skills and doubling and halving.

The students need to extract from the sentences the mathematics required to solve the problems. Many students have difficulty with this because the skills needed for reading and understanding informational text are different from those needed for normal reading. Encourage the students to ask themselves questions about the information, such as: "What does this tell me?" "What do I have to find?" "How should I solve this?" "Is there enough, not enough, or too much information?" "Can I draw a picture of the information given?"

The activity gives information about various types of cycles, and from the information given, the students have to extract the correct number of cycles and/or riders. These types of problems are usually solved through a trial-and-improvement strategy. So that the students don't lose sight of the information provided from question to question, ask them to lay it all out in a table. For example, in question **2**, the students are asked how many tandems there are if there are 23 riders and a total of 14 mountain bikes and tandems in a group. The trial-and-improvement table on the following page could help.

Mountain bikes	Tandems	Total number of racers
6	8	6 x 1 + 8 x 2 = 22
7	7	7 x 1 + 7 x 2 = 21
5	9	5 x 1 + 9 x 2 = 23

By listing the possible combinations in the table, the students can see that there are 9 tandems in the race. Students who are having difficulty could use materials such as different-coloured cubes or counters to model each situation.

For question **3**, you may need to explain what "half as many" means. Again, the students can use a table to help with this.

In question **4**, the students may forget to add the wheels of the trailer and the wheels of the car to find the total.

When the students are making up their own problems in question **5**, have them put the data into a table, check their answers, and then extract the information from the table to write the problem. For example, the table could be:

Racers	Wheels	Unicycles	Mountain bikes	Tandems
45	65	5	20	10

The related problem might be: "Quinten saw that there were 45 people, including unicycle, mountain bike, and tandem racers, in the round-the-lake race. He counted 65 wheels. How many of each type of cycle were in the race?"

The students can improve their problem-solving skills by revisiting the activities and solving them using different strategies, such as working backwards. The students will probably benefit more from solving a problem 5 different ways than solving 5 different problems the same way.

Page 20: Stacking Up

Achievement Objectives

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- write and solve story problems which involve halves, quarters, thirds, and fifths (Number, level 2)

Activity

For this activity, the students need to be able to read and write fractions and understand that fractions are part of a whole and part of a set.

Before the students begin this activity, they should have had experience with finding fractions of a stack of objects, such as multilink cubes. If they have not, you could introduce fractions to them in the following way.

The students need to understand that a fraction is a part of all the units in the stack. If you take a stack of 6 cubes, made up of 5 red cubes and 1 white cube, $^5/_6$ of the stack is red and $^1/_6$ of the stack is white. You could then change the number of colours but keep 6 cubes in the stack. "If I have 3 blue cubes and they make up $^3/_6$ of the stack, how much of the stack is left?" Now change the number of cubes in the stack so that the students are introduced to the idea that they have to know what the "whole" is before they can work out the proportion of the parts of the whole.

You will need to encourage the students to make connections between the fractions written in words and those written numerically. Often the students can tell you about parts of a whole but cannot match the concept with the written words or numerical symbols.

This activity introduces the students to stacks made up of 3 or more colours, and they have to find what fraction each colour is of the whole. Encourage the students to model the flying discs with stackable cubes before they answer the questions. The example shows a stack of 11 discs made up of 3 colours. It then introduces the addition of the three parts: $^5/_{11} + {}^4/_{11} + {}^2/_{11} = {}^{11}/_{11}$ or 1 whole. The students need to be aware that in a fraction sentence, the numerators (in this case, 5 + 4 + 2) add up to the common denominator (11). This can be easily modelled and provides the basis for a good understanding of what is required in question **2**.

In question **1**, the students need to realise that each stack is a complete set, and therefore the fractions deal with each stack individually and cannot be directly compared with the fractions for the stack before or after.

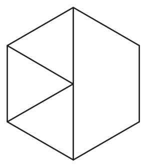
Question **2** asks the students to write addition equations for each of the stacks in question **1**. Have the students refer back to the fraction stories they wrote in words in question **1c**.

Further discussion and investigation
The students may need further practice with this concept of fractions before they fully understand it. Use sets of marbles, counters, or farm animals (from the Beginning School Mathematics materials) and encourage the students to find and write fraction sentences for various sets.

The students could also investigate what fractions of the class are male, have long hair, have fruit in their lunch, and so on to explore this concept.

Use pattern blocks (sets of geometry shapes, including hexagons, triangles, trapezia, and rhombuses) to explore other fraction addition problems. For example, 3 triangles plus 1 trapezium is the same as 1 hexagon.

$$\begin{aligned} {}^3/_6 + {}^1/_2 &= {}^3/_6 + {}^3/_6 \\ &= {}^6/_6 \\ &= 1 \end{aligned}$$



The students could also revisit Stacking Up and find and write fraction subtraction stories.

Page 21: Marble Marvels

Achievement Objectives

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- write and solve story problems which involve halves, quarters, thirds, and fifths (Number, level 2)

Activity

The students will find this activity easier if they have successfully completed the activity on page **20** before they attempt it. For this page, the students need to be able to read and write fractions and understand that fractions are part of a set. They also need to be able to divide and multiply by 2, 3, 4, 5, and 8 and use doubling and halving.

Marble Marvels explores fractions of sets where the number of objects in each set is different. It extends the idea explored on page **20**.

Before the students start work on this activity, prompt them with questions such as:
"Here we have a pile of 12 marbles and a pile of 16 marbles. Would it be better to have $^1/_2$ of the pile of 12 or $^1/_2$ of the pile of 16? Why?"
"Would it be better to have $^1/_3$ of 12 or $^1/_4$ of 16? Why?"
"Would it be better to have $^3/_4$ of 12 or $^1/_2$ of 16? Why?"
"What is $^1/_2$ of 66?"

In question **1**, some students may misread the $^2/_3$ as $^1/_3$, especially since the comparison is with $^1/_4$. To work out $^2/_3$, the students will need to work out $^1/_3$ of the marbles and subtract that from the whole amount to get the $^2/_3$ amount (or multiply the $^1/_3$ by 2). To find the $^1/_4$ amount, they can simply divide by 4.

Question **2** asks the students to consider whether there are other strategies for working out the problem in question **1**. The students' suggestions should be shared and discussed because some methods may not be as efficient as others.

Question **4** requires the students to reverse the process. Starting with 2 numbers, they have to find a fraction of 1 number that produces the same amount as a fraction of the other. There are a number of possible answers, and a variety of strategies could be used to solve this problem:

- Look for patterns in the numbers: 63 and 42 are both divisible by 7. If we find a multiple of 7, we can then decide the fraction of each. If the multiple were 7, the fraction amounts would be $^1/_9$ of 63 and $^1/_6$ of 42.
- Work backwards: If we had all of the 42 marbles in one jar and 42 of the 63 marbles in the other, what fraction of each are these? The first is one whole ($^{42}/_{42}$), and $^{42}/_{63}$ of the other is $^6/_9$ (or $^2/_3$).
- Trial and improvement: Use a table for the guesses:

Number of marbles	Fraction of 63	Fraction of 42	Correct?
60	$^{60}/_{63}$	$^{60}/_{42}$	Not possible
40	$^{40}/_{63}$	$^{40}/_{42}$ ($^{20}/_{21}$)	Yes

Further discussion and investigation
For an interesting discussion, you could ask the students "When can a $^1/_4$ of something be more than $^1/_2$ of something else?"

Pages 22–23: Fraction Frenzy

Achievement Objectives

- classify objects, numbers, and ideas (Mathematical Processes, developing logic and reasoning, level 3)
- find fractions equivalent to one given (Number, level 4)

Activity

Before the students start this activity, you might like to work through two earlier Figure It Out activities with them. Fabulous Folding, *Number*, Figure It Out, Levels 2–3, page 18, uses a simple number line to place fractions in order. (You can extend this later in relation to Fraction Frenzy.) Fun with Fractions, *Number*, Figure It Out, Level 3, page 9, uses paper circles to explore simple equivalent fractions.

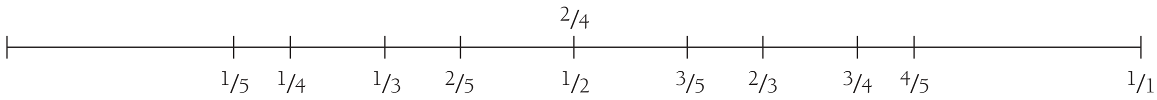
In Fraction Frenzy, the students explore the simple equivalence of fractions. They also discuss the order of fractions. The activity can be extended to investigating whether there is a rule for changing a fraction into an equivalent fraction.

In question **1**, the students sort a set of fraction cards (see the copymaster at the end of these notes) into equivalent sets. In the process, they order them from smallest to largest. If they are not aware of this sequencing, it should become apparent to them in their discussion with a classmate.

There are various ways of helping the students who have difficulty with this question. You could direct them to the fraction wall in question **4** and help them to explore it, or you could use a simpler fraction wall and link a number line to it:

<table>
<tr><td colspan="60">1/1</td></tr>
<tr><td colspan="30">1/2</td><td colspan="30">1/2</td></tr>
<tr><td colspan="15">1/4</td><td colspan="15">1/4</td><td colspan="15">1/4</td><td colspan="15">1/4</td></tr>
<tr><td colspan="20">1/3</td><td colspan="20">1/3</td><td colspan="20">1/3</td></tr>
<tr><td colspan="12">1/5</td><td colspan="12">1/5</td><td colspan="12">1/5</td><td colspan="12">1/5</td><td colspan="12">1/5</td></tr>
</table>

Get the students, perhaps in pairs, to make the fractions on the wall out of strips of paper (they need to be the right lengths), for example, $^1/_5$, $^2/_5$, $^3/_5$, and so on, and then align their strips to create a fraction number line:



A fraction wall such as the one below is a good way to teach the students about different ways of writing fractions:

<table>
<tr><td colspan="360">One whole 1/1</td></tr>
<tr><td colspan="180">One-half</td><td colspan="180">1/2</td></tr>
<tr><td colspan="72">One-fifth</td><td colspan="72">1/5</td><td colspan="72"></td><td colspan="72"></td><td colspan="72"></td></tr>
<tr><td colspan="36">One-tenth</td><td colspan="36">1/10</td><td colspan="36"></td><td colspan="36"></td><td colspan="36"></td><td colspan="36"></td><td colspan="36"></td><td colspan="36"></td><td colspan="36"></td><td colspan="36"></td></tr>
<tr><td colspan="90">One-quarter</td><td colspan="90">One-fourth</td><td colspan="90">1/4</td><td colspan="90"></td></tr>
<tr><td colspan="45">One-eighth</td><td colspan="45">1/8</td><td colspan="45"></td><td colspan="45"></td><td colspan="45"></td><td colspan="45"></td><td colspan="45"></td><td colspan="45"></td></tr>
<tr><td colspan="120">One-third</td><td colspan="120">1/3</td><td colspan="120"></td></tr>
<tr><td colspan="60">One-sixth</td><td colspan="60">1/6</td><td colspan="60"></td><td colspan="60"></td><td colspan="60"></td><td colspan="60"></td></tr>
<tr><td colspan="40">One-ninth</td><td colspan="40">1/9</td><td colspan="40"></td><td colspan="40"></td><td colspan="40"></td><td colspan="40"></td><td colspan="40"></td><td colspan="40"></td><td colspan="40"></td></tr>
</table>

Before they do question **2**, the students need to check their columns from question **1**. They could do this with a classmate. They then go on to place further equivalent fractions into the correct columns. If necessary, lead the students to realise that they can find equivalent fractions by looking for patterns in the numerators and denominators. For example, with $^{25}/_{100}$, 25 goes into 100 four times, so it is equivalent to $^1/_4$. With $^{45}/_{75}$, we can see that both numbers are divisible by 5, so we get $^9/_{15}$. In turn, both these numbers are divisible by 3, so we get $^3/_5$.

Question **4** asks the students to explain how two fractions are equivalent. Have them explain this to another classmate first and then get them to write out their explanation. The writing can be used for teacher assessment or for self- or peer-assessment and will also help the students to internalise the concept.

In question **5**, the students are given only the denominator of the second fraction and have to find its numerator to make it an equivalent fraction. After completing the previous questions, the students should see that a quick way of finding equivalent fractions is to multiply the numerator and denominator by the same number. If they do not understand this, have them make further equivalent fractions with materials and then say and write the names. Rather than giving them the "rule", help the students to find it for themselves.

Question **6** requires a good knowledge of multiplication and division basic facts because the students have to multiply and divide by 3 and 8. They need to search for a link between the two numerators 3 and 375. Encourage them to see that 375 is 3 x 100 + 3 x 25, that is, 3 lots of 125. The students need to identify the relationship between the two numerators:

$$\frac{3\ (\times\ 125)}{8\ (\times\ 125)} = \frac{375}{\square}$$

They then need to apply that relationship to the denominator to maintain the "equivalence" between the fractions.

Further discussion and investigation

Have the students make their own fractions and then place them on number lines. Include fractions greater than 1. Ask them to name fractions equivalent to those shown.

Discuss why $^1/_1$, $^2/_2$, $^5/_5$, and so on are all equal to 1.

Page 24: Surf's Up!

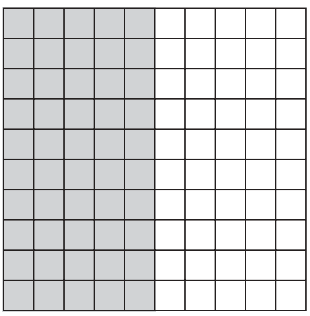
Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- find a given fraction or percentage of a quantity (Number, level 4)
- pose questions for mathematical exploration (Mathematical Processes, problem solving, level 3)

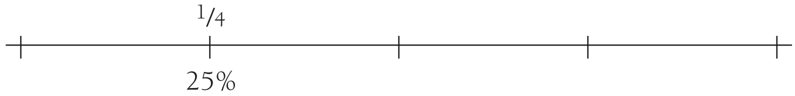
Activity

For this activity, the students need to be familiar with showing a fraction as a percentage, and they need to understand that a percentage is a part of a whole expressed in hundredths. The whole is $^{100}/_{100}$ or 1, which is 100 percent (%). 20 is $^{20}/_{100}$. The students need to be able to multiply and divide by hundreds and tens, especially in terms of money (for example, 10% of \$1.20 is \$0.12). The activity also involves adding and subtracting amounts of money.

Before they do this activity, the students need to have had experience with showing how fractions and percentages are related. They could use a hundreds board or grid for this, covering different amounts with pieces of paper and describing fractions that have 100 as a denominator. Encourage the students to describe these fractions as equivalent fractions, such as $^1/_2$, $^3/_4$, $^9/_{10}$, and so on. (See the activity on the previous page.) For example, the paper covers the fraction $^{50}/_{100}$, which is also $^5/_{10}$ or $^1/_2$ (or 50%).



You could also show the students how fractions and percentages can be drawn on a double number line:



Revise multiplying by 10 and 100 and encourage the students to see that multiplying by 40 is the same as multiplying by 10 and then multiplying by 4 (or vice versa). These skills are necessary when finding 30% of something because a good strategy is to find 10% and then multiply it by 3.

Newspapers or advertising materials are useful sources for the students to explore how businesses advertise reductions of 10%, 25%, and so on.

The questions in this activity focus on a chart showing two different shops with various discounts. The students have to find the sale price of the items in order to complete the table. Encourage the students to notice that most of the percentages are easy to work with. For example, for 30%, they can find 10% and multiply by 3, and 25% is the same as $^1/_4$. The only difficult percentage to be found is 45%. This could be found by first finding 10%, multiplying it by 4 to find 40%, finding $^1/_2$ of 10% to get 5%, and adding this amount to the 40% amount. Make sure the students realise that they have to subtract the percentage amount from the price each time to find the actual cost of those items.

To do question **2**, the students need to be able to compare values and also subtract one from the other. If they have difficulty, get them to calculate the amounts using play money.

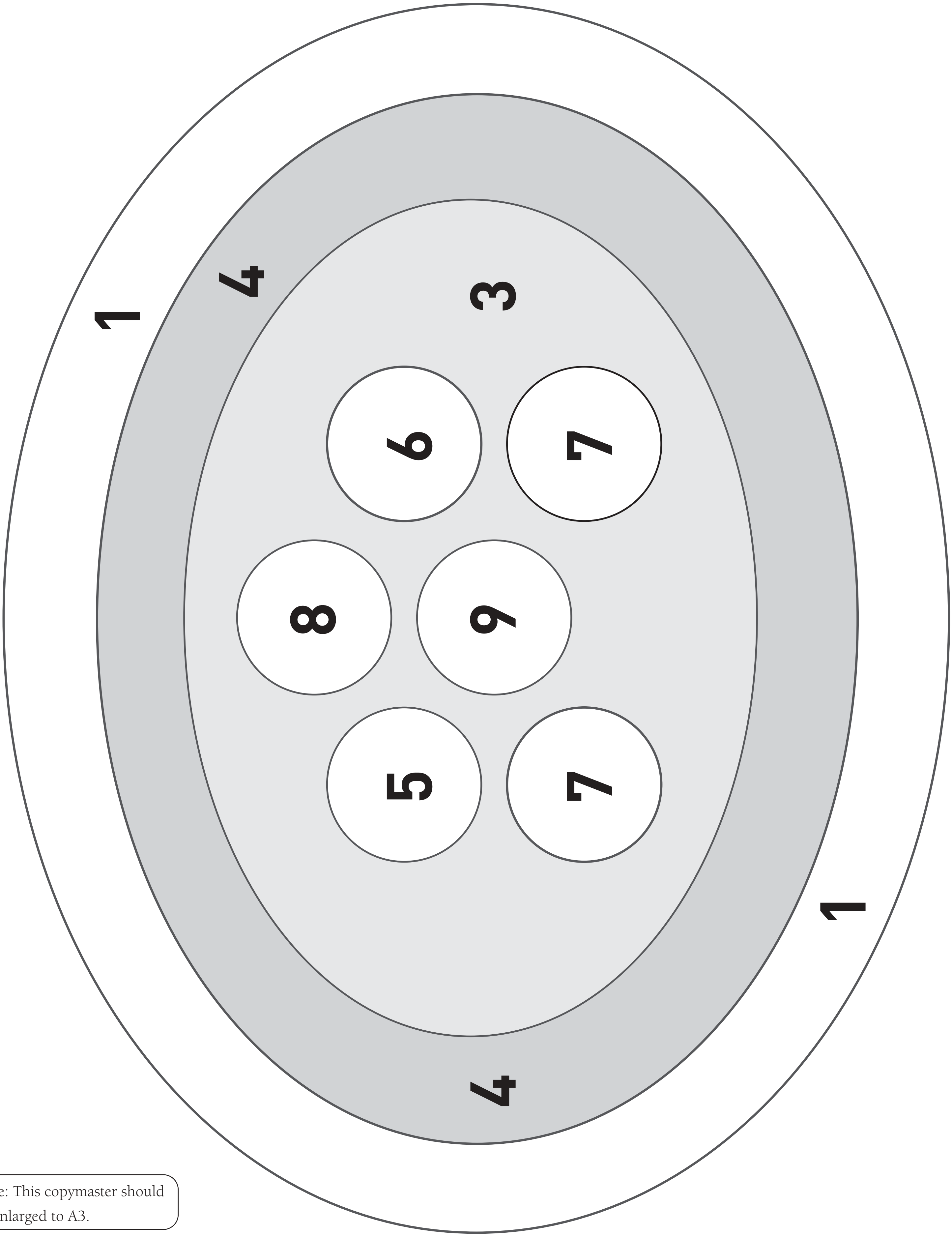
In question **3**, where the students make up their own sale problems, encourage them to make up a chart similar to the one given on the page.

Further discussion and investigation

Collect advertising materials that show percentage reductions and have the students work out the sale prices. Have them find similar items sold by different companies and compare their prices.

Give the students a budget for an activity, such as buying sports equipment for the school or books and CDs for the library. Have them use the advertising material to make a list of realistic retail prices and sale prices. They can then work out the total they would save if they bought all the items at a sale.

Copymaster: Skimming Stones



Note: This copymaster should be enlarged to A3.

Copymaster: Easy Nines

	Using my 10 times table	Down a decade and digits adding up to 9	Using my 3 times table	Answer
6 x 9 =				
9 x 9 =				
3 x 9 =				
5 x 9 =				
2 x 9 =				
8 x 9 =				

$\frac{1}{2}$ $\frac{2}{3}$ $\frac{6}{9}$ $\frac{1}{5}$ $\frac{6}{12}$ $\frac{1}{3}$

$\frac{3}{4}$ $\frac{2}{6}$ $\frac{9}{12}$ $\frac{2}{4}$ $\frac{1}{4}$ $\frac{2}{10}$

$\frac{4}{4}$ $\frac{7}{7}$ $\frac{10}{10}$ $\frac{7}{14}$ $\frac{3}{5}$ $\frac{4}{12}$

$\frac{2}{8}$ $\frac{9}{15}$ $\frac{2}{2}$

$\frac{22}{110}$ $\frac{43}{86}$ $\frac{25}{100}$ $\frac{20}{60}$ $\frac{33}{99}$ $\frac{45}{75}$

$\frac{569}{569}$ $\frac{24}{96}$

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