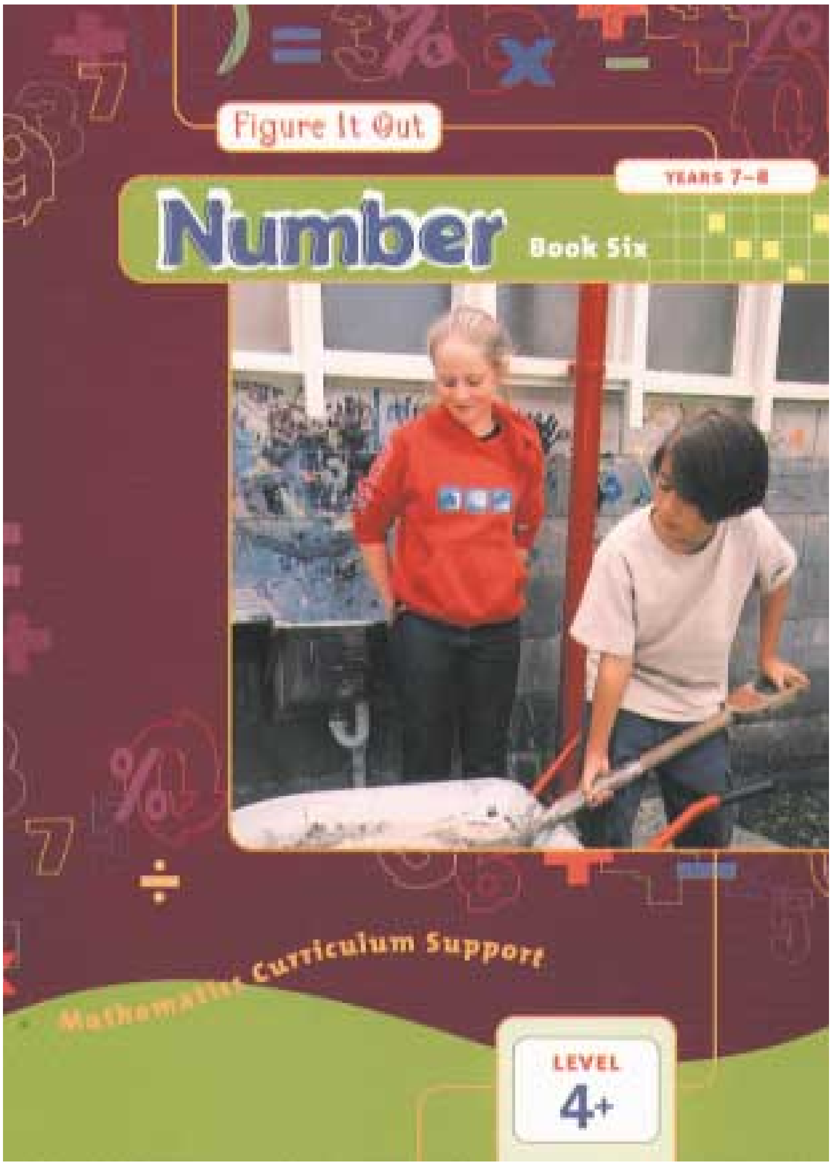
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

CONTENTS

The books for years 7–8 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand year 7–8 classrooms. The books have been developed and trialled by classroom teachers and mathematics educators and follow on from the successful series for levels 2–4 in primary schools.

Student books

The student books in the series are divided into three curriculum levels: levels 2–3 (linking material), level 4, and level 4+ (extension material). All the books are aimed at year 7–8 students in terms of context and presentation.

The following books are included in the series:

Number (two linking, three level 4, one level 4+)
Algebra (one linking, two level 4, one level 4+)
Measurement (one level 4, one level 4+)
Number Sense (one linking, one level 4)
Geometry (one level 4, one level 4+)
Statistics (one level 4, one level 4+)
Themes (level 4): *Disasters, Getting Around*

These 20 books will be distributed to schools with year 7–8 students over a period of two years, starting with the six *Number* books.

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and the life experiences that are meaningful to students aged 11–13 years. The activities can be used as the focus for teacher-led lessons, as independent activities, or as the catalyst for problem solving in groups.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the student books includes full answers and explanatory notes. Students can use them for self-marking, or you can use them for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, comments on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/community

Using Figure It Out in your classroom

Where applicable, each page starts with a list of equipment that the students will need to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Answers Number: Book Six

Page 1 Square Sums

INVESTIGATION

Apart from 0 and 1, Fermat's conjecture works for all the whole numbers to 100, including all the square numbers. Several solutions are possible for many numbers. For example:
50 = 49 + 1 or 36 + 9 + 4 + 1
71 = 36 + 25 + 9 + 1 or 49 + 9 + 9 + 4

Pages 2–3 Digital Delights

INVESTIGATION ONE

1. 3 x 3 x 3 x 2 = 54
2. a. Answers will vary.
   b. A general approach is to use combinations of the lowest prime numbers, 2 and 3. For example, for 13:
   2 x 2 x 2 x 2 x 2 x 3 = 96
   2 x 2 x 3 x 3 x 3 = 108

INVESTIGATION TWO

1. The end result is always 11. For example, with 2 and 8:
   82 + 28 = 110
   110 ÷ 10 = 11
2. Answers will vary. For example, if you choose the digits 2 and 7, you have 27 + 72. Each digit features once in the tens place and once in the ones place, so there are 11 lots of each digit. This means you have 11 lots of 2 and 7. 11 x (2 + 7) = 99. The same thing will occur with any digit you choose.

INVESTIGATION THREE

1. The result is always 222. For example, with 1, 4, and 7 as the digits:
   147
   174
   417
   471
   714
   741
   ______
   2 664
   1 + 4 + 7 = 12
   2 664 ÷ 12 = 222
2. As with **Investigation Two**, except that this time, there are 222 lots of each digit you choose. Suppose you choose 1, 4, and 7:
   147 = (100 x 1) + (10 x 4) + 7
   174 = (100 x 1) + (10 x 7) + 4
   417 = (100 x 4) + (10 x 1) + 7
   471 = (100 x 4) + (10 x 7) + 1
   714 = (100 x 7) + (10 x 1) + 4
   741 = (100 x 7) + (10 x 4) + 1
   Altogether, there are (200 x 7) + (20 x 7) + (2 x 7) = 222 x 7. This is also true for the other digits.

INVESTIGATION FOUR

1. a. 33, 55, 77, and 99
   b. They are all multiples of 11.
   c. Answers may vary. You may have realised that you can eliminate prime numbers and all other odd numbers except those that are multiples of 11.
2. There are five: 22, 24, 36, 44, and 48.

Page 4 Powerful Thought

ACTIVITY

1. a. 2^6 can be regrouped as
   (2 x 2 x 2) x (2 x 2 x 2) = 8 x 8
   = 64
   b. 3^4 = (3 x 3) x (3 x 3)
   = 9 x 9
   = 81
2. a. Yes, all the equations are true.
   i. 2^1 x 2^5 = 2 x 32
   = 64, which is 2^6
   ii. 2^3 x 2^4 = 8 x 16
   = 128, which is 2^7
   iii. 2^5 x 2^5 = 32 x 32
   = 1 024, which is 2^{10}

b. Adding exponents gives the exponent of the product:
$2^a \times 2^b = 2^{(a + b)}$
For example: $2^2 \times 2^3 = 2^{(2 + 3)}$
$= 2^5$

3. a.

3^1	3^2	3^3	3^4	3^5	3^6	3^7	3^8	3^9	3^{10}
3	9	27	81	243	729	2 187	6 561	19 683	59 049

b. i. Yes. $3^2 \times 3^4 = 9 \times 81$
$= 729$, which is 3^6

ii. Yes. $3^5 \times 3^3 = 243 \times 27$
$= 6\ 561$, which is 3^8

In both cases, the exponents of the factors add up to the exponent in the product. Working out the equation shows this to be true.

Page 5 Sunburst

ACTIVITY

1.

	Standard	Normal form
Venus	1.396×10^8	139 600 000
Earth	1.496×10^8	149 600 000
Meteorite	2.374×10^8	237 400 000
Asteroid	5.123×10^8	512 300 000
Jupiter	7.783×10^8	778 300 000
Saturn	1.496×10^9	1 496 000 000
Halley's comet	5.736×10^9	5 736 000 000
Pluto	5.9×10^9	5 900 000 000
Alpha Centauri	4.064×10^{13}	40 640 000 000 000
Betelgeuse	2.93×10^{15}	2 930 000 000 000 000

2. The radiation would travel 5.826×10^9 km before it was no longer life-threatening. So Pluto, Alpha Centauri, Betelgeuse, and any other star that is further than 5.826×10^9 kilometres away from the Sun could be considered.

Page 6 Number Returns

ACTIVITY

1. a. You get your original three-digit number.

b. You get your original three-digit number because you are dividing by 1 001 (7 x 11 x 13) altogether. When you multiply a three-digit number by 1 001, you get a repeat pattern, for example:

$$\begin{array}{r} 527 \\ \times\ 1\ 001 \\ \hline 527 \\ 527\ 000 \\ \hline 527\ 527 \end{array}$$

2. a. 2 345

b. 4 568

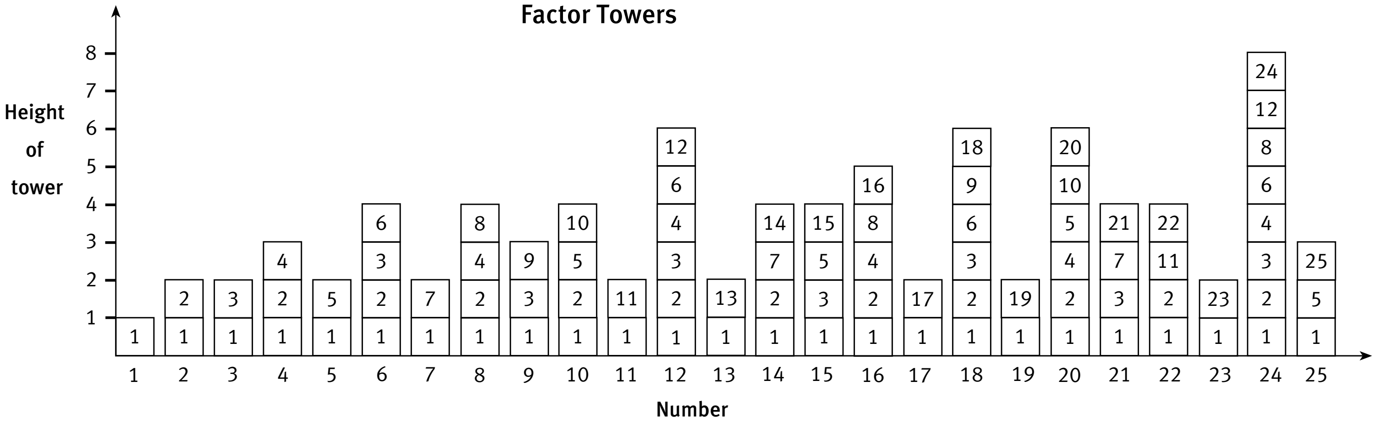
c. Descriptions of patterns will vary, but the result is always seven digits. You might notice that the first two digits and the last two digits of the answer are the same as the first two digits and the last two digits of the original number. The middle digits vary according to whether the first and last digits of the original number add up to 10 or more.

3. 10 001

Page 7 Factor Towers

ACTIVITY

1. Practical activity. Your factor tower city should be similar to this one:



2. a. 2, 3, 5, 7, 11, 13, 17, 19, 23

   b. Three factors: 4, 9, 25
   Four factors: 6, 8, 10, 14, 15, 21, 22
   Five factors: 16
   Six factors: 12, 18, 20
   Eight factors: 24

3. a. Responses will vary. They could include:
   The numbers with two factors are prime numbers.
   The numbers with three factors are square numbers.
   For the numbers with four factors, the first three factors (in order) multiply together to give the fourth factor. For example, factors of 10 are 1, 2, 5, and 10.
   1 x 2 x 5 = 10

   b. Responses will vary. They could include:
   All numbers have at least 1 and themselves as factors.
   Numbers with an odd number of factors are square numbers.
   24 has the most factors (8), and 1 has the least (1).

INVESTIGATION

Yes, it does. For example, $10^2 = 100$, which is the same as $1^3 + 2^3 + 3^3 + 4^3 = 100$.

Page 8 Tiling Teasers

ACTIVITY

1.

Side length (cm)	Area (cm^2)
10	100
11	121
12	144
15	225
20	400
25	625
60	3 600

2. a. 30 cm

   b. 50 cm

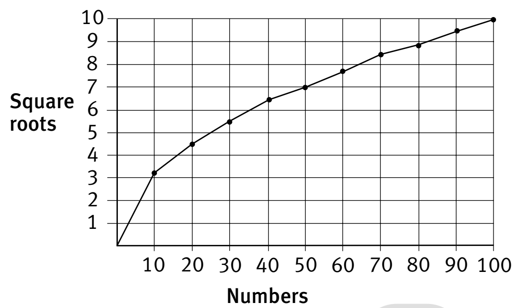
3. a. 40 cm

   b. i. area: 800 cm^2
   side lengths: 28.3 cm (1 d.p.)

   ii. The second smallest square has an area of 400 cm^2 and side lengths of 20 cm. The smallest square has an area of 200 cm^2 and side lengths of 14.1 cm (1 d.p.).

4. 5, 6, 7, 8, 9, 10, 0, 1, 2, and 3

5. a. Points will vary, depending on the numbers chosen.
   An example is:



b. i. Estimates may vary.
Actual square root: 5.92 (2 d.p.)

ii. Estimates may vary.
Actual square root: 7.87 (2 d.p.)

iii. Estimates may vary.
Actual square root: 4.24 (2 d.p.)

iv. Estimates may vary.
Actual square root: 8.72 (2 d.p.)

Note: Other sensible rounding or truncating solutions are also acceptable. Check with your teacher.

Page 9 Pascal's Patterns

ACTIVITY

1.

```
          1   5   10   10   5   1
        1   6   15   20   15   6   1
      1   7   21   35   35   21   7   1
    1   8   28   56   70   56   28   8   1
```

2. a. 10, 15, 21, 28, 36, 45

b. Explanations will vary. For example:
- The difference between the numbers is increasing by 1 each time.
- The numbers are triangular numbers:

```
   .       1
  . .      3
  . . .    6
 . . . .   10
 . . . . . 15
. . . . . . 21
```

3. Comments on the patterns will vary. You should notice that a different pattern emerges with each different factor.

INVESTIGATION

Comments will vary.

Page 10 Spending on Sport

ACTIVITY

1. a. $103.96

b. $79.88

c. $125.04

d. $48.38

e. $58.27

f. $83.20

g. $53.96

h. $62.98

i. $83.97

j. $8.96

k. $42.46

l. $79.67

2. a. Solutions will vary. One possibility is:
flippers $58.27
basketball $48.38
soccer ball $42.46
racquet $103.96
baseball and mitt $62.98
Total $316.05

b. Solutions will vary. For the example given above, the savings would be:
flippers $47.68
basketball $16.12
soccer ball $7.49
racquet $25.99
baseball and mitt $62.97
Total discount $160.25

c. Percentages will vary. For the example given above, the percentage can be worked out by:
162.25 ÷ (316.05 + 160.25) x 100 = 33.64%,
which rounds to 33.6%.

3. Explanations will vary. For example, to find 40% of $139.95: The discount is 40%, so Philip would pay 60% of the full price. You can use your calculator to work out 60% of $139.95 like this if you have a % button on your calculator: 139.95 x 60%. If you do not have a % button, you can go 60 ÷ 100 x 139.95 or 0.6 x 139.95. The same principle applies when you are finding discounted prices involving fractions.

Page 11 Accident-prone

ACTIVITY

1. a. Estimates may vary, but they should be close to the calculated answers. The actual answers are:

i. 75

ii. 20

iii. 100

iv. 78

v. 304

vi. 410

vii. 120

viii. 163

ix. 37

x. 56

xi. 16

b. Other sports: 0.3105 of total accident claims and 621 people

2. a. 296

b. 148

3. $392.79 if based on full amount and $392.59 if based on whole $100 units

Pages 12–13 Alien Counting

ACTIVITY ONE

1. Explanations need to include the fact that the Cartoks see the moons as one lot of 6 and 3 left over, as below.

❂ ❂ ❂ ❂ ❂ ❂ ❂ ❂ ❂
1 3

2. Explanations need to include the fact that the Cartoks see the planets as two lots of 6 and 1 left over. For example, two of their sixes is like two of our tens:

❂ ❂ ❂ ❂ ❂ ❂ ❂
❂ ❂ ❂ ❂ ❂ ❂
2 1

The Cartoks' counting system is a base 6 number system.

3. a. 12_6

b. 25_6

c. 42_6

d. 50_6

4.

1	6	36	216	1 296	7 776	46 656	…
6^0	6^1	6^2	6^3	6^4	6^5	6^6	…

5. a. 27

b. 32

c. 52. (1 x 36 + 2 x 6 + 4 x 1 = 52 or 1 x 6^2 + 2 x 6 + 4 x 1 = 52)

d. 321. (1 x 216 + 2 x 36 + 5 x 6 + 3 x 1 = 321 or 1 x 6^3 + 2 x 6^2 + 5 x 6 + 3 x 1 = 321)

ACTIVITY TWO

1. 11_8 moons in question **1** and 15_8 planets in question **2**
The Vanan counting system is a base 8 number system. So 21 in Vanan counting would look like this:

❂ ❂ ❂ ❂ ❂ ❂ ❂ ❂ ❂
❂ ❂ ❂ ❂ ❂ ❂ ❂ ❂
2 1

2. a. 10_8

b. 21_8

c. 32_8

d. 36_8

3. Responses will vary.

Pages 14–15 Judo Competition

ACTIVITY ONE

1. a. Jason 7, Emily $^-3$, Ruth 8, Ben $^-6$, Kate 7, Tom 11

b. Strategies will vary. One strategy could be: "I added all the positives together, then added all the negatives together, and then found the difference."

2. a. Jason's, Emily's, and Kate's scores

b. 17 points. (Tom has 11, and Ben is on $^-6$.)

3. $^-9$

4. 24

ACTIVITY TWO

1. a. Jason 12, Emily $^-34$, Ruth 24, Ben 6, Kate 14, Tom 40

b. 74

2. a. 62

b. 86

ACTIVITY THREE

1.–3. Responses will vary. Teacher to check.
A possible score sheet could be:
Jason: ippon, chui (24); Ruth: yuko, koka (9);
Ben: waza-ari, shido (12); Kate: chui, koka (⁻3);
Tom: ippon, koka (33).
Score for the round: 75
Total score: 86 + 75 = 161

Page 16 Video Viewing

ACTIVITY

1. a. 33 min. and 46 s

b. ⁻0:33:28

c. ⁻0:29:03

2. a. 26 min. and 26 s

b. 0:01:42

3. a. 2 hrs, 1 min., and 15 s

b.

Start	Stop
⁻1:44:33	⁻1:42:33
⁻1:27:33	⁻1:25:33
⁻1:10:33	⁻1:08:33
⁻0:53:33	⁻0:51:33
⁻0:36:33	⁻0:34:33

(0:29:00 will occur during the next 15 minute viewing period.)

Page 17 Hypertufa Tiles

ACTIVITY ONE

1. a. $^1/_4$. (This can be written as 25%.)

b. 1:3

2. 5 spade loads of cement
$2^1/_2$ spade loads of aggregate

3. a. 14 spade loads of cement
7 spade loads of sand
7 spade loads of aggregate

b. 7 spade loads of cement and $3^1/_2$ each of sand and aggregate

ACTIVITY TWO

1. a. 8 parts cement
4 parts sand
1 part shell
1 part pumice
2 parts peat
water to mix

b. 6 spade loads cement
3 spade loads sand
$^3/_4$ spade load shell
$^3/_4$ spade load pumice

2. a. 15 spade loads cement
$7^1/_2$ spade loads sand
$1^7/_8$ spade loads shell
$1^7/_8$ spade loads pumice
$3^3/_4$ spade loads peat

b. These quantities would not be easily measured, so they would probably decide to make more than they actually needed, to keep the proportions as accurate as possible. The nearest easily measured mixture would be these spade loads: 16 of cement, 8 of sand, 2 of shell, 2 of pumice, and 4 of peat.

Pages 18–19 Squaring Off

ACTIVITY

1. **a.** The horizontal sides are decreasing by 1 each time, and the vertical sides are increasing by 1 each time. The area also decreases each time.

   b. i.

Size	Area	Difference from previous area
6 x 6	36	
5 x 7	35	1
4 x 8	32	3
3 x 9	27	5
2 x 10	20	7
1 x 11	11	9

   ii. Answers may vary. The difference is going up in 2s, and the differences are consecutive odd numbers.

   c. The area of the initial square will vary, but the pattern of differences will be the same.

   For example:

Size	Area	Difference from previous area
8 x 8	64	
7 x 9	63	1
6 x 10	60	3
5 x 11	55	5
4 x 12	48	7

Size	Area	Difference from previous area
9 x 9	81	
8 x 10	80	1
7 x 11	77	3
6 x 12	72	5
5 x 13	65	7

2. **a.** A table of rectangles up to 88 x 112 should look like this:

Size	Area	Difference from previous area
100 x 100	10 000	
99 x 101	9 999	1
98 x 102	9 996	3
97 x 103	9 991	5
96 x 104	9 984	7
95 x 105	9 975	9
94 x 106	9 964	11
93 x 107	9 951	13
92 x 108	9 936	15
91 x 109	9 919	17
90 x 110	9 900	19
89 x 111	9 879	21
88 x 112	9 856	23

   b. i. 9 996

   ii. 9 964

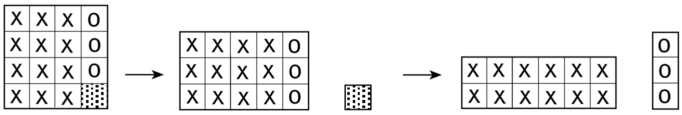
   iii. 9 856

3. a. Yes, it will work with your other patterns.
For example:

Size	Area	Difference from previous area	Difference in area from original
7 x 7	49		
6 x 8	48	1	1
5 x 9	45	3	4
4 x 10	40	5	9
3 x 11	33	7	16

b. As well as a table, there are various other ways, including:

- pictorial charts, for example:

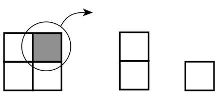


4 x 4 3 x 5 left over 2 x 6 left over

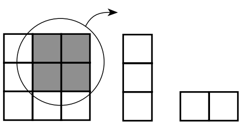
- various algebraic formulae, for example: $n \times n = n^2$
  $(n - 1)(n + 1) = n^2 - 1$
  $(n - 2)(n + 2) = n^2 - 4$
  $(n - 3)(n + 3) = n^2 - 9$
  $(n - a)(n + a) = n^2 - a^2$

4. a. Answers will vary. For example:

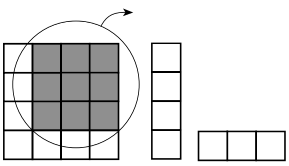
b. $11^2 - 10^2 = 11 + 10$



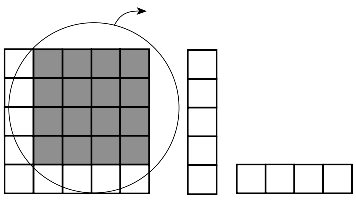
$2^2 - 1^2 = 2 + 1$



$3^2 - 2^2 = 3 + 2$



$4^2 - 3^2 = 4 + 3$



$5^2 - 4^2 = 5 + 4$

c. i. 173

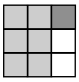
ii. 207

iii. 1 123

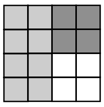
iv. 4 175

d. Explanations will vary. As shown in the diagrams for question **4a**, if the total of the second squared number is removed, you are always left with the sum of the first number plus the second number.

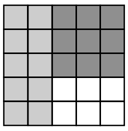
5. **a.** Diagrams will vary. For example:



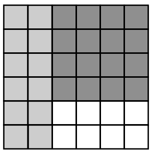
$3^2 - 1^2 = 2 \times 3 + 2 \times 1$ (where the 3^2 refers to the whole square, the 1^2 to the dark grey square, and the 2 x 3 and 2 x 1 to the light grey and white squares respectively).



$4^2 - 2^2 = 2 \times 4 + 2 \times 2$



$5^2 - 3^2 = 2 \times 5 + 2 \times 3$



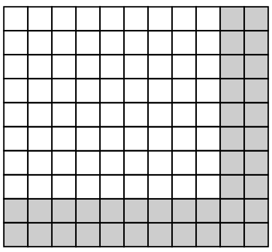
$6^2 - 4^2 = 2 \times 6 + 2 \times 4$

Explanations will vary. Add together one side length from each square and then double your total. For example, for $6^2 - 4^2$: $2 \times (6 + 4) = 20$.
Another explanation is: Find the number in between the side length numbers and multiply by 4. For example, for $6^2 - 4^2$: $4 \times 5 = 20$.
Algebraically, this can be shown as:

$$\begin{aligned}(n + 2)^2 - n^2 &= n^2 + 4n + 4 - n^2 \\ &= 4n + 4 \\ &= 4(n + 1)\end{aligned}$$

where n is the side length of the smaller square.

b. 36.



$10^2 - 8^2 = 2 \times 10 + 2 \times 8$
Explanations will vary. For example, using the pattern described above: add 10 and 8 and double the total ($2 \times [10 + 8] = 36$) or find the number between 8 and 10 and multiply it by 4 ($4 \times 9 = 36$).

Page 20 Alien Bacteria

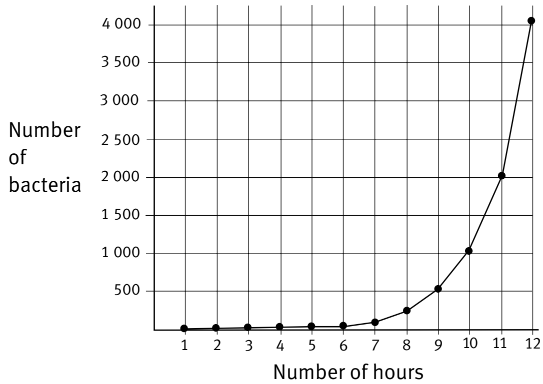
ACTIVITY

1. **a.**

Hours	Number of bacteria
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1 024
11	2 048
12	4 096

b. 10 hours (to the nearest hour)
c. 17 hours (to the nearest hour)
d.

First 12 Hours of Growth



2. **a.** The formula does work for other values.
   b. The number of bacteria in 10 hours is 2^{10}, that is, 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2, which is 1 024 bacteria.
3. 16 777 216 bacteria
4. 29 hours

Page 21 Tasty Division

ACTIVITY

1. Day 1: 6
   Day 2: 6
   Day 3: 6
   Sweets left: 6
2. Day 1: 6
   Day 2: 8
   Day 3: 4
   Day 4: 4

Day 5: 4
Day 6: 4

3. She had 36 sweets to start with. (Working backwards, Thursday's 2 was $\frac{1}{3}$ of Wednesday's 6, which was $\frac{1}{3}$ of Tuesday's 18, which was $\frac{1}{2}$ of Monday's 36.)

4. She had 40 sweets to start with. (Working backwards, Saturday's 6 was $\frac{2}{3}$ of Friday's 9, which was $\frac{1}{2}$ of Thursday's 18, which was $\frac{3}{4}$ of Wednesday's 24, which was $\frac{4}{5}$ of Tuesday's 30, which was $\frac{3}{4}$ of Monday's 40.)

5. Problems and solutions will vary.

Page 22 Sign of the Times

ACTIVITY ONE

1. The patterns continued are:

   a. 3 x 0 = 0
   3 x $^-1$ = $^-3$
   3 x $^-2$ = $^-6$
   3 x $^-3$ = $^-9$
   3 x $^-4$ = $^-12$
   Each result is 3 less than the previous one.
   The second half of the pattern is the 3 times table extended backwards.

   b. $^-1$ x 7 = $^-7$
   $^-2$ x 7 = $^-14$
   $^-3$ x 7 = $^-21$
   $^-4$ x 7 = $^-28$
   $^-5$ x 7 = $^-35$
   Each result is 7 less than the previous one.
   The second half of the pattern is the 7 times table extended backwards.

   c. $^-2$ x $^-2$ = 4
   $^-3$ x $^-2$ = 6
   $^-4$ x $^-2$ = 8
   $^-5$ x $^-2$ = 10
   $^-6$ x $^-2$ = 12
   Each result is 2 more than the previous one.
   The answers in the second half of the pattern look exactly like the 2 times table multiplied by positive factors.

2. a. $^-1$ x 7 = $^-7$ $^-2$ x 7 = $^-14$ $^-3$ x 7 = $^-21$

   ⊖⊖⊖⊖⊖⊖⊖

   ⊖⊖⊖⊖⊖⊖⊖
   ⊖⊖⊖⊖⊖⊖⊖
   7 lots of $^-2$

   ⊖⊖⊖⊖⊖⊖⊖
   ⊖⊖⊖⊖⊖⊖⊖ …
   ⊖⊖⊖⊖⊖⊖⊖
   7 lots of $^-3$

   b. $^-1$ x $^-2$ = 2 can only be imagined because the second negative quantity cannot be shown.

ACTIVITY TWO

1.

x	$^-3$	$^-2$	$^-1$	0	1	2	3
3	$^-9$	$^-6$	$^-3$	0	3	6	9
2	$^-6$	$^-4$	$^-2$	0	2	4	6
1	$^-3$	$^-2$	$^-1$	0	1	2	3
0	0	0	0	0	0	0	0
$^-1$	3	2	1	0	$^-1$	$^-2$	$^-3$
$^-2$	6	4	2	0	$^-2$	$^-4$	$^-6$
$^-3$	9	6	3	0	$^-3$	$^-6$	$^-9$

2. Answers may vary. Multiplying a positive integer by a negative integer gives a negative product. Multiplying a negative integer by a negative integer gives a positive product.

Page 23 Pulley Power

ACTIVITY

1. The pulley wheel will turn twice in the opposite (anticlockwise) direction.

2. a.–b. i. Three turns anticlockwise

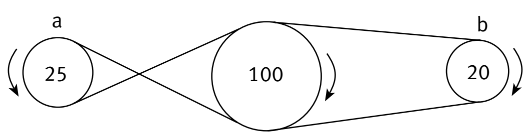
   ii. Four turns anticlockwise

   iii. Eight turns clockwise

   iv. The 16 pulley wheel will make three turns clockwise. The 12 pulley wheel will make four turns anticlockwise.

3. a. Wheel a should have a circumference of 25. Wheel b should have a circumference of 20.

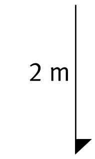
   b. Drawings will vary. They should be similar to this one:



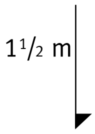
Page 24 What You See …

ACTIVITY

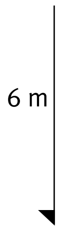
1. The image would be 2 m tall because it's twice the distance from the pinhole than the flag is.
2. i.



ii.



iii.



Teachers' Notes

Overview
Number: Book Six

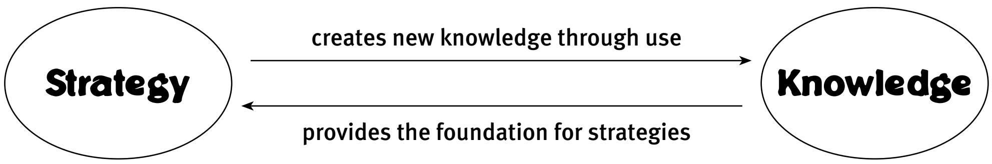
Introduction to Number

There is a remarkable commonality in the way many countries around the world are now teaching arithmetic. Changes in the approaches reflect the evolving demands of everyday life, a greater volume of classroom-based research about how students learn, and a desire to improve general levels of numeracy.

In the past, arithmetic teaching has focused on preparing students to be reliable human calculators. The prevalence of machines in society that calculate everything from supermarket bills to bank balances has meant that students now need a wider range of skills so that they can solve problems flexibly and creatively.

The Figure It Out series aims to reflect these trends in modern mathematics education. A range of books is provided at different levels to develop both number skills and number sense. The *Number* books are aimed at developing students' understanding of the number system and their ability to apply efficient methods of calculation. The *Number Sense* books are aimed at developing students' ability and willingness to apply their number understanding to make mathematical judgments. Teaching number sense requires an emphasis on openness and flexibility in solving problems and the use of communication and interpretation skills.

The development of the Figure It Out series has occurred against the backdrop of a strong drive for improved standards of numeracy among primary-aged students. A key element of this drive has been the creation of the Number Framework, developed as part of Numeracy Strategy. The framework highlights this significant connection between students' ability to apply mental strategies to solving number problems and the knowledge they acquire.



The learning activities in the series are aimed both at developing efficient and effective mental strategies and at increasing the students' knowledge base. Broadly speaking, the levels given in the six year 7–8 *Number* books can be equated to the strategy stages of the Number Framework in the following way:

Link (Book One):	Advanced counting to early additive part-whole
Link (Book Two):	Advanced additive part-whole
Level 4 (Books Three to Five):	Advanced multiplicative to advanced proportional part-whole
Level 4+ (Book Six):	Advanced proportional part-whole.

Page 1 Square Sums

Achievement Objectives

- explain the meaning and evaluate powers of whole numbers (Number, level 4)
- generalise mathematical ideas and conjectures (Mathematical Processes, developing logic and reasoning, level 5)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, levels 4–5)

Other mathematical ideas and processes

Students will also:

- investigate Fermat's conjecture that every whole number is the sum of two, three, or four square numbers
- investigate the properties of square numbers
- carry out a systematic investigation.

INVESTIGATION

A dictionary definition of a conjecture is that it is an unproven assertion. Basically, a conjecture is a mathematical idea that holds for many instances but may not hold for some number that nobody has yet investigated. Mathematicians have not yet found any way to demonstrate beyond all doubt that the idea in question logically holds for all possible instances. In other words, no proof is yet available to allow it to be accepted as a full theorem.

The students will find that there are several solutions for some numbers. They could compare their solutions with a classmate's to find other possible solutions. This reinforces the idea that in many mathematical problems there is no one "right answer".

A possible extension would be to ask the students to find other conjectures of Fermat's and investigate one or more of them. For example, Fermat's last conjecture, called "Fermat's last theorem", is that the equation $x^n + y^n = z^n$ can have no solution when n is an integer greater than 2. Fermat wrote in the margin of a book on equations that there was not enough space for him to record what he called his "truly wonderful proof", and unfortunately he died before he recorded it anywhere else. Students who can work competently with exponents could try a few examples to see whether Fermat's idea that no solution is possible seems to hold true, for example, $2^3 + 3^3 = \square^3$ or $4^4 + 5^4 = \square^4$.

Pages 2–3 Digital Delights

Achievement Objectives

- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, levels 4–5)
- report the results of mathematical explorations concisely and coherently (Mathematical Processes, communicating mathematical ideas, levels 4–5)

Other mathematical ideas and processes

Students will also:

- investigate prime numbers
- investigate visible factor numbers
- investigate patterns in number
- use the distributive principle.

INVESTIGATION ONE

This investigation is structured so that the students explore the mathematical patterns in several examples and are then invited to see if they can find some generalisation that would apply to all examples. You may like to discuss the hint given in the speech bubble. This will help them with their answer to question **2b**. Using combinations of the lowest prime numbers that are addends of any number is the best method for trying to achieve the greatest product. Take the number 14, for example. The addends 3 + 3 + 3 + 3 + 2, when multiplied, give 162, which is greater than any other combination.

INVESTIGATION TWO

This investigation is similar to **Investigation One** in that exploration of the patterns in individual examples can lead to a more generalised explanation. One way of explaining the pattern is provided in the Answers. You could also explain it like this:

27 + 72

2 tens 2 ones 2(10 + 1) or 2 x 11

7 ones 7 tens 7(1 x 10) or 7 x 11

So 27 + 72 = 2 x 11 + 7 x 11 or 11(2 + 7)
= 99

No matter which two digits are chosen, there will always be 11 lots of the sum of the two digits. It therefore follows that dividing the total of the expression by the sum of the two digits will always give a result of 11.

INVESTIGATION THREE

The students should be able to build on the pattern in **Investigation Two** to find the pattern in this investigation.

With any three digits, for example, 7, 4, and 1, the students can make six different three-digit numbers. Each digit will appear twice in the hundreds column, twice in the tens column, and twice in the ones column. One way of showing this is provided in the Answers. Another way of adding these together is:
(7 x 100) + (7 x 100) + (7 x 10) + (7 x 10) + (7 x 1) + (7 x 1), and
(4 x 100) + (4 x 100) + (4 x 10) + (4 x 10) + (4 x 1) + (4 x 1), and
(1 x 100) + (1 x 100) + (1 x 10) + (1 x 10) + (1 x 1) + (1 x 1).
This can be shortened, again using the distributive property, to (7 x 200) + (7 x 20) + (7 x 2), and (4 x 200) + (4 x 20) + (4 x 2), and (1 x 200) + (1 x 20) + (1 x 2). Collecting all this together and using the commutative property to change the order of the terms gives
200 x (7 + 4 + 1) + 20 x (7 + 4 + 1) + 2 x (7 + 4 + 1) = 222 x (7 + 4 + 1).

This can also be shown as:

$$741 + 714 + 471 + 417 + 174 + 147$$
$$= 7(100 + 100 + 10 + 1 + 10 + 1)$$
$$+ 4(10 + 1 + 100 + 100 + 1 + 10)$$
$$+ 1(1 + 10 + 1 + 10 + 100 + 100)$$
$$= 7(222) + 4(222) + 1(222)$$
$$= (7 + 4 + 1)(222)$$

Thus, it can be seen that the result will be equal to 222 times the sum of the three digits and that dividing the result by the sum of the three digits will always yield 222.

INVESTIGATION FOUR

This investigation introduces the idea of "visible factor" numbers, that is, numbers that can be divided evenly by each of the digits in the number. For question **1a**, the students may realise, after they have found 33 and 55, that the four possible visible factor odd numbers that are greater than 20 and less than 100 are multiples of 11. They may need to confirm this by trialling and eliminating the other odd numbers. The students may also realise that these odd numbers cannot be in the even decades (20s, 40s, 60s, and 80s) because odd numbers cannot be divisible by the evens tens digit. That leaves only the 30s, 50s, and 70s to search.

Page 4 Powerful Thought

Achievement Objectives

- explain the meaning and evaluate powers of whole numbers (Number, level 4)
- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, levels 4–5)

Other mathematical ideas and processes

Students will also:

- apply the associative principle
- investigate exponents
- learn that the product of two equal base numbers with exponents can be found by adding the exponents.

ACTIVITY

Some students may need to revise their knowledge of exponents. You could help them by asking whether they know of a short way to represent, say, the volume of a cube that measures 4 centimetres by 4 centimetres by 4 centimetres, that is, 4 x 4 x 4 (having one on hand with the squares drawn on the outside faces would be very useful). Some may want to say that this is three 4s instead of 4^3, in which case you may need to go back to using squares. With a 4 by 4 square ruled on card, the students will be able to see that 4 by 4 is not the same as two 4s because two 4s is 8, not 16. You may need to show them the conventional way of writing the squared figure of 2 (that is, 4^2). Then they could learn that there are two ways of saying this, namely 4 squared (after all, it is a square) or 4 to the power of 2. This may give them a better basis to deal with the 4 by 4 by 4 cube and to learn that the numeral 4^3 can be called 4 cubed (it is a cube, so this should make sense) or 4 to the power of 3. "To the power of" thus means the number of times that a number is multiplied by itself. It is worth commenting to the students that with powers higher than 3, the only way to express the power is "to the power of …", for example, to the power of 7.

In activity **1a**, the product of 8 x 8 stems from the use of the associative or grouping principle, which works for both multiplication and addition. In this case, 2 x 2 x 2 x 2 x 2 x 2 has actually been grouped as (2 x 2 x 2) x (2 x 2 x 2). If necessary, help the students to recognise that this is the associative principle in action and that it can provide a powerful means of mental calculation. It can also be used in other multiplication settings, such as 15 x 7 x 2 x 10. Combining the commutative and associative principles allows this to be done mentally, that is: (2 x 15) x 7 x 10.

Question **2** provides an opportunity for the students to identify (if they have not already done so) the pattern in which exponents are added to find the product of two equal numbers. For example, by working out that $2^2 \times 2^4$ is 64 or the same as 2^6, they may begin to see that adding the exponents, in this case $^{2+4}$, gives the exponent of the product.

With question **3**, suggest to the students that they can generate the powers of 3 (or any other number) on the calculator by using the constant function. By keying in $\boxed{3}\boxed{x}\boxed{=}\boxed{=}\boxed{=}$ or, on some calculators, $\boxed{3}\boxed{x}\boxed{x}\boxed{=}\boxed{=}\boxed{=}$, they will get the next power of 3 each time that they press the $\boxed{=}$ button. To ensure that the students understand what is happening here, you may need to ask, “How many times have you multiplied 3 by itself now?” If the students have scientific calculators, you could show them how to use the x^y button, where x is the base number and y is the exponent. For example, for 3^4, they would key in $\boxed{3}\boxed{x^y}\boxed{4}\boxed{=}$.

Page 5

Sunburst

Achievement Objectives

- convert numbers expressed in standard form to ordinary form, and vice versa (Number, level 5)
- write and solve problems involving decimal multiplication and division (Number, level 4)

Other mathematical ideas and processes

Students will also:

- apply the normal form of writing numbers
- apply the standard (or scientific) form of writing numbers
- work with decimal numbers
- investigate the magnitude of very large numbers
- multiply by adding exponents.

ACTIVITY

In this activity, students convert numbers from normal form into standard form (and vice versa) and order very large numbers.

When numbers are converted to standard form (also known as scientific notation), they are always written as the product of a number between 1 and 10 and a power of 10. For example, 2 342.6 written in standard form is 2.3426×10^3. The 2 is a whole number between 1 and 10 and 10^3 denotes 1 000. As another example, 0.0042 expressed in standard form would be 4.2×10^{-3}. The standard form of numbers becomes efficient when dealing with very large numbers (as with the distances in this activity) or with very small numbers (such as the wavelength of sodium light in a street lamp). The students will probably soon realise the value of expressing very large numbers in standard notation as they work on question **1**, especially when they come to write the distance for Betelgeuse, which turns out to be 2 930 000 000 000 000 kilometres from the Sun. Calculators often work in standard form, although the way this is expressed varies with different calculators.

In standard form, the students will need to consider both the first factor (that is, the number between 1 and 10) and the second factor (the particular power of 10) when determining just how big the number is. In the data for this activity, they will find that the distances for Earth and Venus both have a second factor of 10^8 and a first factor of 1 point something. They will therefore have to look closely at the decimals that follow the ones digit of the first factor (0.496 and 0.396 respectively) if they are to place these two planets accurately in order of distance. At a distance of 1.396×10^8 or 139 600 000 kilometres, Venus is the closest celestial body to the Sun of those listed in the activity. The requirement in question **1** that the distances be listed in both standard and normal form should help the students begin to understand the relationship between normal and standard form.

When working on question **2**, some students may need help (perhaps from other students) to see that converting the velocity of the radiation to distance can be done most effectively by writing the 200 hours as 2×10^2. The expression then becomes $2.913 \times 10^7 \times 2 \times 10^2$, which can be written as $2.913 \times 2 \times 10^9$ (the 10^9 being obtained by adding the exponents ^{7} and ^{2}, as in the activity on the previous page of the student book). In the form 5.826×10^9, the distance can then be compared with the distance (in standard form) of the various celestial bodies named in the activity.

Page 6

Number Returns

Achievement Objectives

- generalise mathematical ideas and conjectures (Mathematical Processes, developing logic and reasoning, level 5)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 4–5)
- make sensible estimates and check the reasonableness of answers (Number, level 4)

Other mathematical ideas and processes

Students will also:

- investigate repeating sets of digits
- investigate patterns in numbers
- use prime numbers
- use logic and reasoning to solve problems
- apply their knowledge of place value and multiplication.

ACTIVITY

In question **1**, which involves three-digit numbers, the students will find that they always get their original number, no matter which three numbers they try, after dividing consecutively by 7, 11, and 13. The question, of course, is why this happens. The boy wondering about 7 x 11 x 13 provides a clue. In effect, when dividing consecutively by 7, 11, and 13, they have been dividing by 1 001. If they try multiplying a three-digit number by 1 001, they may get a better idea of what is happening. For example, 368 x 1 001 effectively gives one lot of 368 and one thousand lots of 368 (368 000). Combined, this gives 368 368, the repeating type of pattern they have been investigating.

Another way of looking at this is:

368 368
= (300 x 1 000) + (300 x 1) + (60 x 1 000) + (60 x 1) + (8 x 1 000) + 8 x 1)
= (300 + 60 + 8) x 1 000 + (300 + 60 + 8) x 1
= 368 x 1 000 + 368 x 1
= 368 x 1 001

You could challenge your students to consider what they would need to multiply a two-digit number by to achieve a repeating pattern (it turns out to be 101: for example, with 75, it's one lot of 75 and 100 lots of 75, which altogether is 7 575) and even what they have to multiply a one-digit number by for it to repeat (multiply by 11).

When the students move on to question **2**, they will see that multiplying a four-digit number by 1 001 does not give a product with repeated digits. Encourage them to use logic and reasoning and their knowledge of multiplication and place value, rather than a calculator, to find the numbers that Mike started with. (They could use a calculator to check their reasoning.)

After multiplying by 1 001, Mike has a product of 2 347 345, which is one thousand lots of the four-digit number plus one lot of the four-digit number, that is:

$$\begin{array}{r} \square\square\square\square \\ \times 1\,0\,0\,0 \\ \hline \square\square\square\square \\ \square\square\square\square 0\,0\,0 \\ \hline 2\,3\,4\,7\,3\,4\,5 \end{array}$$

From here, it is relatively straightforward to work out most of the missing digits:

$$\begin{array}{r} \square\square\square\square \\ \times 1\,0\,0\,0 \\ \hline \square\boxed{3}\boxed{4}\boxed{5} \\ \boxed{2}\boxed{3}\boxed{4}\square 0\,0\,0 \\ \hline 2\,3\,4\,7\,3\,4\,5 \end{array}$$

The students can use logic and reasoning to see that the two missing digits that add up to 7 will be 2 and 5, and so the original four-digit number was 2 345.

$$\begin{array}{r} \boxed{2}\boxed{3}\boxed{4}\boxed{5} \\ \times 1\,0\,0\,1 \\ \hline 2\,3\,4\,5 \\ 2\,3\,4\,5\,0\,0\,0 \\ \hline 2\,3\,4\,7\,3\,4\,5 \end{array}$$

They can approach question **2b** in the same way:

$$\begin{array}{r} \square\square\square\square \\ \times 1\,0\,0\,1 \\ \hline \square\square\square\square \\ \square\square\square\square\,0\,0\,0 \\ \hline 4\,5\,7\,2\,5\,6\,8 \end{array}$$

which leads to:

$$\begin{array}{r} \square\square\square\square \\ \times 1\,0\,0\,1 \\ \hline \square 5\,6\,8 \\ 4\,5\,\square\square\,0\,0\,0 \\ \hline 4\,5\,7\,2\,5\,6\,8 \end{array}$$

They know from question **2a** that the first and last digit add up to the middle digit:

$$\begin{array}{r} 4\,5\,6\,8 \\ 4\,5\,\square 8\,0\,0\,0 \\ \hline 4\,5\,7\,2\,5\,6\,8 \end{array}$$

From here, they can use their knowledge of addition to fill in the last gap and find the original four-digit number (4 568).

The students will use the understanding gained in questions **1** and **2** to answer question **3**. For the digits to repeat, without any of them changing by being added, they must have:

□□□□	multiply by 1
□□□□ 0 0 0 0	multiply by 10 000

So the four-digit number must be multiplied by 10 001.

Page 7 Factor Towers

Achievement Objectives

- explain the meaning and evaluate powers of whole numbers (Number, level 4)
- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, levels 4–5)
- generalise mathematical ideas and conjectures (Mathematical Processes, developing logic and reasoning, level 5)

Other mathematical ideas and processes

Students will also:

- investigate factors
- use exponents and triangular numbers
- investigate the relationships between triangular, squared, and cubic numbers.

ACTIVITY

This is a good activity for the students to work in pairs to find and discuss patterns. The real fascination with the activity will probably emerge in question **3**, where the students find patterns in the lists of numbers and factors. They may notice, for example, that the numbers with only two factors are all the prime numbers (which makes sense when they think about it). They may also notice that the numbers with three factors are all square numbers and that the five-factor number (16) is also a square number. The students could investigate how many factors the next square number (36) has. (It has 9.) They may notice that in the case of numbers with four factors, the first three always multiply together to give the fourth factor. For example, with 15, the factors are 1, 3, 5, and 15 and 1 x 3 x 5 = 15.

INVESTIGATION

The students will hopefully be intrigued by the relationship between squares of triangular numbers and cubed numbers. This is an investigation into pure number because there is no ready way that the relationship can be modelled with materials. The students will need to list some of the other triangular numbers to tackle this. If they sketch the numbers like this:

```
    *
   * *
  * * *
 * * * *
* * * * *
```

they should see that the fifth triangular number is 15. They could write the relationship between the square of this number and cubed numbers like this:

$$\begin{aligned}(1 + 2 + 3 + 4 + 5)^2 &= 15^2\\ &= 225\\ &= 1^3 + 2^3 + 3^3 + 4^3 + 5^3.\end{aligned}$$

Again, to find the cube of the higher numbers (such as the 4 and 5), the students could simply use the constant function on their calculator (that is, $\boxed{4}\boxed{x}\boxed{=}\boxed{=}$ or $\boxed{4}\ \boxed{x}\ \boxed{x}\boxed{=}\boxed{=}$) or the x^y button on a scientific calculator, as was suggested in an earlier activity.

Page 8

Tiling Teasers

Achievement Objectives

- express the values of square roots in approximate and exact forms (Number, level 5)
- make sensible estimates and check the reasonableness of answers (Number, level 4)

Other mathematical ideas and processes

Students will also:

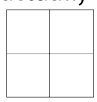
- calculate areas of rectangles from measurements of length
- find squares of numbers
- estimate within a range (from a line graph)
- use logic to solve problems.

ACTIVITY

The students need to be comfortable finding the area of a square and working with square roots in order to do this activity.

Question **1** provides a basis for question **2**, which some students will be able to calculate mentally. Those who can't, could use the $\sqrt{}$ key on their calculator. Question **3** is where the real challenge begins, although **3a** can be figured out in the same manner as question **2**.

For question **3b i**, some students may work out the length of each side of the first inner square on the tile by using Pythagoras' theorem that the sum of the squares on two sides of a right-angled triangle is equal to the square of the hypotenuse. Other students will probably work out that the first inner square is actually half the area of the whole tile. You could help them to understand this by dividing the whole tile into quarters and imposing the first inner square on this or by folding a square of paper in at the corners. Similarly, they may be able to see that the second inner square is one-half of the area of the first inner square or one-quarter of the area of the whole tile, and that the area of the smallest inner square is half that of the previous one. In other words, the area keeps halving as the squares get progressively smaller.



The students can then find the length of the sides of the squares by finding the square root of the area of each square. For example, the side lengths of the square with an area of 800 cm^2 are each $\sqrt{800}$, which is 28.3 (to one decimal place).

Alternatively, some students may calculate the length of the sides of the first inner square by using Pythagoras' theorem, based on the calculation that the sides of the whole tile are each 40 centimetres in length. The students may notice that one side of this first inner square is actually the hypotenuse of the triangle in the outer corner of the tile. Thus, the length of the hypotenuse is the square root of $20^2 + 20^2$, which is $\sqrt{400 + 400} = \sqrt{800}$, which is 28.3 centimetres to one decimal place. The students can then use logical reasoning to figure out that the length of the sides of the smallest inner square is just half that of the first inner square (that is, 14.14 centimetres or 14.1 to one decimal place) and that the length of the sides of the second inner square is half that of the tile itself (that is, 20 centimetres).

In question **4**, the students will find it easier if they draw a fairly large graph of the data shown so that the estimates required in **4b** are more manageable. As well as using their graph to estimate the square roots, they can estimate by finding the nearest square number. For example, 35 is close to the square number 36, and the square root of 36 is 6, so the square root of 35 will be a little less than 6. They can then use a calculator to find the actual square root.

Page 9 Pascal's Patterns

Achievement Objective

- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, levels 4–5)

Other mathematical ideas and processes

Students will also:

- identify multiples and multiples of prime numbers
- investigate patterns.

ACTIVITY AND INVESTIGATION

A copymaster for Pascal's triangle is provided at the end of these notes. Don't give the students the photocopies of Pascal's triangle until they have done question **1** as it will give them the answers. After doing page **7** of the students' book, the students should recognise the pattern in question **2** (1, 3, 6, 10, 15, and 21) as being triangular numbers.

The students will have various ideas about the "patterns" made by the odd numbers, multiples of 3, multiples of 5, and multiples of a prime number. The **Investigation**, which involves extending Pascal's triangle, might provide them with some further clues to possible patterns.

The students may be interested to know that Pascal's triangle originated from a question posed to Pascal by Chevalier de Mère, an acquaintance who was a gambler. He wanted to know what the chances were of a double six coming up if two dice were tossed 24 times. Pascal's triangle is a chart of probabilities.

Page 10 Spending on Sport

Achievement Objectives

- express a decimal as a percentage, and vice versa (Number, level 4)
- express quantities as fractions or percentages of a whole (Number, level 4)

Other mathematical ideas and processes

Students will also:

- use percentages
- use equivalent fractions and decimals
- use a calculator efficiently for finding complementary percentages and decimals
- round to two decimal places
- use logic and reasoning to solve problems.

ACTIVITY

For students who do not have a % button on their calculator or do not know how to use it, the key to solving question **1** on their calculator is being able to convert percentages and fractions to decimals. For example, with the archery set advertised as having 40% off, the students need to know that 40% is the same as 0.4. Similarly, with the stopwatch advertised as having one-quarter off, they need to know that $^1/_4$ is the same as 0.25. (Some students may prefer to key in percentages such as 40% as $\boxed{x}$ $\boxed{40}$$\boxed{\div}$$\boxed{100}$, even though it is not as efficient as converting to decimals.)

Another strategy for doing the calculations readily on their calculators is to use the complementary percentage or decimal. For example, with the archery set mentioned above, it is much easier to think of the sale price as being 60% or 0.6 of the original price. This means that all the students need to do is key in $\boxed{0.6}\,\boxed{\times}\,\boxed{139.95}$ and they immediately get the sale price, namely $83.97, in a single step.

The trickiest one is the skateboard, which has $^1/_3$ off the advertised price. Using the thinking suggested, the students will see that the sale price is $^2/_3$ of the original price, but to get an accurate sale price, they will need to key into their calculators $\boxed{2}\,\boxed{\div}\,\boxed{3}\,\boxed{\times}\,\boxed{119.50}\,\boxed{=}$, which will give them 79.666658 or $79.67 (rounded to two decimal places). If they key in just 0.67 for the $^2/_3$, then they will get a sale price of $80.07, which is close but not accurate.

In question **2**, the students' solutions will vary. This will demonstrate for them yet again that in realistic problems, there is seldom one right answer. As with some previous problems, you can encourage your students to share their results for this activity with the class and to justify to others that their solutions are valid.

Question **3** challenges the students to use their calculators efficiently. The methods discussed above are all examples of efficient calculator use. Encourage the students to use these methods rather than the less efficient two-step methods such as finding the discounted price of the skateboard by dividing 119.50 by 3, recording the answer (on paper or in the calculator memory), clearing the calculator window, and keying in 119.50 – 39.83.

The students could set out their working as a table:

	Actual price	Sale price	Discount
a.			
b.			
c.			
d.			

Page 11 Accident-prone

Achievement Objectives

- express quantities as fractions or percentages of a whole (Number, level 4)
- make sensible estimates and check the reasonableness of answers (Number, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, levels 4–5)

Other mathematical ideas and processes

Students will also:

- work with decimal fractions of quantities
- work with decimal fractions to three or more places
- estimate within a range
- use logic and reasoning to solve problems
- use the memory function on a calculator.

ACTIVITY

To undertake the estimation requirement in question **1a**, the students need to understand what the decimal fractions mean in terms of number per hundred. An estimate and the actual figure for athletics injuries are provided, so you could use this as a teaching example. For the decimal fraction of claims relating to athletics, the students will need to realise that the 0.03 something means at least three per hundred.

Since there is a total of 2 000 claims, this means that there are 20 hundreds altogether so, if the figure were just 0.03, the number of people would be 60 (3 x 20).

The second kind of understanding needed is that of the magnitude of fractions. In the athletics example, the students need to realise that while the 0.03 means three per hundred, it is actually more than three but less than four (although closer to four than three). This leads to the notion of range. There are several reasons why it is more useful to be able to estimate within a range than to estimate a single number. In life, we often estimate within a range, for example, when we say we expect to arrive in a particular place between 4 p.m. and 4.30 p.m. Estimating within a range also takes away from the students the sense of failure they tend to experience when their single estimate turns out to be "wrong". In the athletics example, the students should be able to follow the reasoning that the number will be somewhere between 70 and 80 people.

A technical strategy that students may find helpful in using the calculator to work out the number of people involved is to put the figure of 2 000 (accident claims) into memory. Then, instead of having to key it in for each calculation, they can simply key in the decimal, then $\boxed{\text{x}}$, then $\boxed{\text{RM}}$ to recall the 2 000, and press $\boxed{=}$. Alternatively, they could put 2 000 in as a constant.

In question **3**, some simple explanation may be needed about what a "non-work premium" is (basically, an amount of money that is over and above what self-employed people normally pay for work-related injuries). The students also need to see that $36,518 is 365 lots of $100, or to be really accurate, it is 365.18 lots of $100. When the students understand this, they may wonder whether they should multiply $1.0756 by 365 or 365.18. The answer is that it depends on how the issue is interpreted. This is another example of a problem that can have more than one valid solution, depending on definitions.

Pages 12–13 Alien Counting

Achievement Objectives

- explain the meaning and evaluate powers of whole numbers (Number, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, levels 4–5)

Other mathematical ideas and processes

Students will also:

- explore numbers in other bases
- convert between bases
- revisit the meaning of place value
- express numbers in powers/exponents.

ACTIVITIES ONE AND TWO

With the advent of New Maths in the 1960s, numbers in other bases came to prominence because it was thought that study of these would give students insights into the way our base 10 (written as base_{10}) place value system works. On the whole, it didn't. Instead, it tended to confuse learners more than it enlightened them, but that was probably because it was introduced at too early an age.

However, there are good reasons for studying numbers in other bases. Approached sensibly, they can lead to a better understanding of how our base_{10} number system works. Another reason is that base_2 is part of our lives because it is fundamental to the way our electronic communications work. Electric current can be either *on* or *off*, so this property is used to denote the digits 0 and 1, which in turn allows quite complex communication. Our calculators and computers work on the base_2 system, so the whole of communication technology depends on this base. A further reason that numbers in other bases are worth studying is that, once again, the notion of there always being one right answer in mathematics is undermined.

In **Activity One**, question **2**, it should soon become clear, for example, that 7 + 6 can be 21 as well as 13 (although the 21, admittedly, should be written 21_6). What the correct sum is depends on the number system being used.

Questions **1** and **2** in this activity lead students to the understanding that the foundation of the place value system is that digits get progressively greater by powers of the base number as they move left and, conversely, get progressively smaller as they move right.

To illustrate: Take a number in base_{10} (the base at the heart of our decimal system), such as 5. As the 5 moves left one column into the tens column, it becomes 10 times greater, and as it moves left one further column into the hundreds column, it becomes 10 times greater again (and hence 100 times greater than the original 5). Conversely, if the original 5 moves right one place to the tenths column, it becomes 10 times smaller.

Teachers in earlier years have usually helped students to understand that in our usual base_{10} place value system, the first column is ones, the next to the left is tens, the next is hundreds, and so on. What your students now need to understand is that the first column is "lots of 1", the second is "lots of 10", the third is "lots of 100", and so on. They also need to understand that these can be expressed in powers of 10. For example, 100 is 10^2, 1 000 is 10^3, and so on. The students could help construct a table of these understandings, using a number such as 6 452, perhaps as follows:

Lots of 1 000 000	Lots of 100 000	Lots of 10 000	Lots of 1 000	Lots of 100	Lots of 10	Lots of 1
10^6	10^5	10^4	10^3	10^2	10^1	10^0
			...	10 x 10	10 x 1	
			6	4	5	2

The useful thing about setting the table out in this form is that it mirrors the way we write numbers. (Note that this is the reverse of the way the table is set out in **Activity One**, question **4**.) There, the table shows the potential for going on to higher place values. The setting out here focuses on the place values for a specific number. It shows clearly what the digits in 6 452 (in the bottom row of the table) stand for. The 6 signifies six lots of 1 000 or six lots of 10^3; the 4 represents four lots of 100 or four lots of 10^2; the 5 is five lots of 10 or five lots of 10^1; and the 2 is two lots of 1 or two lots of 10^0.

You can approach the understanding of numbers in other bases in a similar way. For example, base_5 numbers would look like this:

Lots of 5^6	Lots of 5^5 ...	Lots of 5^4 or 625	Lots of 5^3 or 125	Lots of 5^2 or 25	Lots of 5^1 or 5	Lots of 5^0 or 1

The other useful thing about setting out the tables as shown above is that it can lead relatively easily to considering decimal fractions in terms of powers of 10. Thus tenths are denoted as 10^{-1}, hundredths as 10^{-2}, and so on. It then becomes possible to multiply (and divide) numbers involving decimal fractions by operating on the powers of the numbers, as in the activity on page **4** of the student book.

You could have an interesting discussion with your students about early counting strategies. They should have realised that the number of fingers each type of alien has is related to the number base that they use. Most humans use a base_{10} system.

Pages 14–15 Judo Competition

Achievement Objectives

- explain the meaning of negative numbers (Number, level 4)
- solve problems involving positive and negative numbers, using practical activities or models if needed (Number, level 5)

Other mathematical ideas and processes

Students will also:

- use integers (positive and negative numbers)
- use logic and reasoning to solve problems.

ACTIVITIES ONE AND TWO

For students who are not familiar with how negative numbers work, at least for addition and subtraction, a number line could be a good means of helping to develop their understanding.

$^{-}8$ $^{-}7$ $^{-}6$ $^{-}5$ $^{-}4$ $^{-}3$ $^{-}2$ $^{-}1$ 0 1 2 3 4 5 6 7 8

Integers are also known as "signed" numbers, but notice the convention regarding the use of signs: a negative sign denotes a negative number, but the accepted assumption is that a number without a sign is a positive number.

The number line can help the students to see that:

i. addition usually means go to the *right* along the number line, for example, $2 + 5 = 7$ (begin at 2, add 5, and you end up at 7). The same happens if you start on a negative number, for example, $^{-}6 + 9 = 3$ (begin at $^{-}6$, add 9, and you end up at 3).

ii. subtraction usually means to go *left* along the number line, for example, $7 – 3 = 4$ (begin at 7, subtract 3, and you end up at 4). Another example would be $2 – 5 = {}^{-}3$ (begin at 2, subtract 5, and you end up at $^{-}3$). A third example is $^{-}2 – 4 = {}^{-}6$ (begin at $^{-}2$, subtract 4, and end up at $^{-}6$).

Dealing with a negative means moving in the opposite direction. For example:

3 + 5 = 8
Start at 3 move 5 right end at 8.

3 + $^{-}5$ = $^{-}2$
Start at 3 move 5 *left* end at $^{-}2$.

$^{-}2$ – 4 = $^{-}6$
Start at $^{-}2$ move 4 *left* end at $^{-}6$.

$^{-}2$ – $^{-}4$ = 2
Start at $^{-}2$ move $^{-}4$ right end at 2.

You need to emphasise the "0" point on a number line and the fact that a number and its opposite will always add up to 0. For example, $3 + {}^{-}3 = 0$, so $3 + {}^{-}5$ is $3 + {}^{-}3 + {}^{-}2$, which is $0 + {}^{-}2$ or $^{-}2$.

The students may be able to summarise all this along the following lines as a basis for tackling the judo competition tasks:

Operation	Direction along number line	Example
Add a positive	Go right	$^{-}2 + 4 = 2$
Subtract a positive	Go left	$2 – 4 = {}^{-}2$
Add a negative	Go left	$^{-}2 + {}^{-}4 = {}^{-}6$
Subtract a negative	Go right	$2 – {}^{-}4 = 6$

The other understanding required of students for these activities is what happens when integers are multiplied by other integers. This can be summarised as:

Integer x integer	Example
$^+ \times {}^+ = {}^+$	$3 \times 4 = 12$
$^- \times {}^- = {}^+$	$^-3 \times {}^-4 = 12$
$^+ \times {}^- = {}^-$	$3 \times {}^-4 = {}^-12$
$^- \times {}^+ = {}^-$	$^-3 \times 4 = {}^-12$

Note that only $^+ \times {}^+$ and $^+ \times {}^-$ are involved in this activity.

Page 16 Video Viewing

Achievement Objectives

- explain the meaning of negative numbers (Number, level 4)
- solve problems involving positive and negative numbers, using practical activities or models if needed (Number, level 5)

Other mathematical ideas and processes

Students will also:

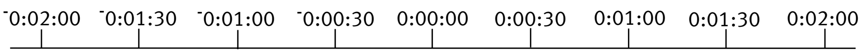
- add and subtract integers
- use logic and reasoning to solve problems
- calculate time to seconds
- realise the importance of the number 60 when calculating time periods.

ACTIVITY

This activity requires the addition and subtraction of positive and negative numbers, as on the previous page, but it also requires the students to keep in mind that they are working with clock time. This means that 60 becomes a crucial number for both seconds and minutes. For example, as soon as a figure goes beyond 60 seconds, it becomes an extra minute and the seconds start again.

The activity is reasonably challenging, and the students should approach it in a considered and methodical way. Again, a number line may be a very useful way to tackle these problems, but, as indicated above, it will need to show time rather than the usual integers. The students, as a class, could discuss and draw such a time number line. It would be something like the one below, although considerably extended.

The students could draw two time lines, the second showing much larger intervals of time. This would be useful for question **4**, where 2 minute and 15 minute periods need to be calculated. For example:

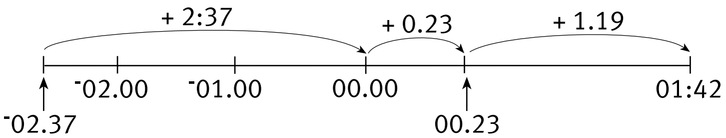


The notes for the previous page explain the principles of adding and subtracting integers. The students should be able to apply that learning to the questions on this page.

For example, for question **2b**, the students need to add 4 minutes and 19 seconds to the ˉ0:02:37 given in the previous question, that is, ˉ0:02:37 + ˉ0:04:19. One way to show this is:

ˉ0:02:37
0:00:00 2:37
0:00:23 0:23 (a total of 3 min.)
0:01:42 1:19 (4 min. 19 s)

They could also show this as a number line:



This activity will help the students to recognise that there are realistic circumstances in our lives where we need to be able to work with negative numbers. It could also be helpful to encourage them to discuss other circumstances where this is so, for example, students' older family members may have overdrafts or bank loans, mortgages on houses or property, or loans for vehicles and household appliances. The students may find it interesting that a person who has very little money may be much better off than someone who has a loan to repay.

Page 17 Hypertufa Tiles

Achievement Objectives

- express quantities as fractions or percentages of a whole (Number, level 4)
- share quantities in given ratios (Number, level 5)

Other mathematical ideas and processes

Students will also:

- work with proportion and ratio
- use percentages and fractions
- use logic and reasoning to solve problems.

ACTIVITY

Ratios and proportions can be a bit confusing, and this activity helps address confusions that arise about these maths ideas.

In **Activity One**, the students need to recognise that the mixture contains four parts in total and that just one of these is sand. The sand therefore accounts for one-quarter or 25% of the parts. However, in terms of ratios, the mixture comprises one part sand to three parts of other ingredients (cement and aggregate) so that sand as a ratio is written 1:3. *Proportion* then has to do with comparison to the *total*, whereas *ratio* has to do with comparison to the other constituent parts. To illustrate this further, concrete can be made from one part cement and four parts builders' mix. This means that the proportion of cement is 1 out of 5 or 20%, whereas the ratio is 1:4.

For the remainder of the problems in **Activity One** and for those in **Activity Two**, the students will be working with the idea of proportion, including its application to fractions. In **Activity One**, question **2**, for example, they need to reason that if $2\frac{1}{2}$ spade loads of sand represents one part of the mixture, then you need $2\frac{1}{2}$

spade loads of aggregate and twice that of cement (that is, five spade loads). In question **3a** of this activity, they need to reason backwards: if a quantity of 28 spade loads is mixed, then one-quarter (seven spade loads) will be sand, the same will be aggregate, and half (14 spade loads) will be cement. The students need to note that in question **3b** they are required to work out how many *extra* spade loads of each ingredient are required. They may be surprised to find that they are getting into fractions with this question. That is, with 14 extra spade loads needed, half (seven spade loads) will obviously be cement, but the one-quarter each of sand and aggregate both work out at $3\frac{1}{2}$ spade loads.

Activity Two is a nice extension on question **3a** and has the students using a fraction (that is, one-quarter shell, and so on) as the base unit of 1. Students who have difficulty with this problem may find it useful to rewrite the mixture, perhaps as follows:

Mixture	Equivalent to
2 parts cement	8 parts cement
1 part sand	4 parts sand
$^1/_4$ part shell	1 part shell
$^1/_4$ part pumice	1 part pumice
$^1/_2$ part peat	2 parts peat

Pages 18–19 Squaring Off

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 4–5)
- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, levels 4–5)
- make sensible estimates and check the reasonableness of answers (Number, level 4)

Other mathematical ideas and processes

Students will also:

- investigate square numbers and the difference between square numbers
- use patterns to solve problems.

ACTIVITY

This extensive but very interesting activity can help the students to develop strategies that will assist them with mental calculations. It may also give them a further sense of the fascination of numbers.

The particular number patterns that can be derived from this activity and that also provide powerful mental strategies are as follows:

1. The product of numbers that are equidistant from a middle number whose square is known can be calculated from this square number. To take some simple examples: 9 x 11 is the same as $10^2 - 1^2$; 8 x 12 is the same as $10^2 - 2^2$; 7 x 13 is the same as $10^2 - 3^2$; and so on. This idea can be applied to much larger numbers, such as 26 x 34. This is the same as $30^2 - 4^2$, which the students could work out in their heads as 900 – 16, which is 884. They could even tackle 47 x 53 mentally (that is, $50^2 - 3^2$ = 2 500 – 9, which is 2 491) and 95 x 105 ($100^2 - 5^2$ = 10 000 – 25, which is 9 975). The students are usually amazed at what they can accomplish mentally through using the power of patterns.

2. Finding the difference between adjacent square numbers can be done by simply adding the numbers. For example, $6^2 - 5^2$ is the same as 6 + 5. Again, this pattern can be applied to relatively large numbers, such as $100^2 - 99^2$ (that is, 100 + 99 = 199) and $2\,025^2 - 2\,024^2$ (that is 2 025 + 2 024 = 4 049).
3. If the difference between a pair of numbers is 2, then the difference between the squares of these two numbers is double the sum of the numbers. For example, $8^2 - 6^2$ is the same as (2 x 8) + (2 x 6) or 2 x 14, which is 28. Likewise, $51^2 - 49^2$ is the same as 2 x (51 + 49), which is 200.
4. The difference between pairs of squared numbers that are distant by 2 from each other can *also* be found by multiplying the intervening number by 4. In the examples above, $8^2 - 6^2$ is the same as 4 x 7 (because 7 is between 8 and 6) and $51^2 - 49^2$ is the same as 4 x 50 (because 50 is between 51 and 49).

The beauty about patterns such as those above is that some of the mental calculations they permit take less time than trying to do the original calculation on a calculator. In the last example, the calculator operation would involve squaring 49 and putting the product into memory, then squaring 51, and finally subtracting from it the recalled square of 49. It seems much easier to double the sum of 49 and 51 (which in this case is a very convenient doubling of 100) or to work out 4 x 50.

The activities and suggested mathematical models that allow the students to gain understanding of the patterns are set out in this activity in a nice developmental way. The answers for this activity provide additional possibilities for helping to facilitate such understanding.

Page 20 Alien Bacteria

Achievement Objectives

- explain the meaning and evaluate powers of whole numbers (Number, level 4)
- round numbers sensibly (Number, level 5)
- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, levels 4–5)

Other mathematical ideas and processes

Students will also:

- use powers of 2
- work with exponents
- use a calculator to generate powers
- round to the nearest million
- use logic and reasoning to solve problems.

ACTIVITY

This activity looks at exponential growth and shows how quickly a population increases if it doubles every hour. The table in question **1** can be worked out quite easily using a calculator by keying in $\boxed{2}\boxed{x}\boxed{=}\boxed{=}\boxed{=}$ and so on or by using the x^y button on a scientific calculator. This actually replicates the table on page **4** of the student book, and so the students are likely to realise that the number of bacteria in 10 hours is 2^{10}, that is, 1 024 bacteria. If the students have only an eight-digit calculator, they could encounter difficulty with question **4**, which asks them to determine how many hours scientists have to find and destroy the bacterium before it reaches a count of 1 000 000 000. The students will be able to get as far as 26 hours on their eight-digit calculator (67 108 864). To go beyond 26 hours, they could simply round the 67 108 864 to 67 million, then double it to 134 million for 27 hours, double it again to 268 million for 28 hours, double it once more to 536 million for 29 hours, and finally double that 1 072 million for 30 hours (which is too late, as it is over the 1 000 000 000 threshold given in question **4**). The students could try to do this on their calculators to see what happens. Some calculators will give the answer to 2^{27} as 1.34217 08, that is, 1.34217×10^8, while others may display it as 1.34217E + 8. (This isn't an exact answer. Some rounding has taken place because the calculator cannot fit all the digits in the window.) This is standard form, which the students investigated on page **5** of the student book.

The students will be able to see that scientists have just 29 hours to find and destroy the bacterium before it proves to be impossible to contain.

In question **1c**, the students are asked to construct a graph of the first 12 hours of growth of the deadly alien bacterium. They may need to discuss and perhaps be helped to decide what scale they should have on the vertical axis, which denotes the number of bacterium. A discussion about the effect of the vertical scale, especially in relation to the first 6 hours, would also be useful. A scale that may be useful is one that goes up in increments of 500 and ranges up to 4 500. Such a scale would allow for the whole 12 hours of growth to be recorded as a total of 4 096 bacteria are generated in the twelfth hour.

Page 21

Tasty Division

Achievement Objectives

- find a given fraction or percentage of a quantity (Number, level 4)
- share quantities in given ratios (Number, level 5)

Other mathematical ideas and processes

Students will also:

- use proportions
- find fractions of whole numbers
- use logic to solve problems
- find fraction quantities via unit fractions.

ACTIVITY

The students will need to work systematically to find the number of sweets in each problem because there aren't any shortcuts they could take. For example, in the first problem, the fractions have to be taken one at a time. On the first day, one-quarter of the 24 sweets were eaten, so this means that six were eaten and 18 were left. On the second day, one-third of the 18 were eaten (that is, another six), leaving 12. On the third day, half of the sweets were eaten, which means that six were left. Over the period of 3 days, therefore, Hazel ate 18 sweets altogether, and there were six left.

In questions **3** and **4**, the students will have to work backwards because, rather than being given the initial number of sweets, they are now given just the number left. One possible "working backwards" method is given in the Answers.

You could encourage the students to do most of the problem solving involving fractions in their heads. A strategy that they are likely to find useful in carrying out these mental calculations is to consider what the unit value is. For example, in question **4**, the six sweets remaining for Saturday represent two-thirds of what was there first thing on Friday. If six represents two-thirds, then one-third must have been three, and therefore three-thirds must have been nine. Likewise, the 30 that were left over from Monday represent three-quarters of Monday's initial stock of sweets. It follows that one-quarter would be 10, and therefore four-quarters would be 40.

Page 22 Sign of the Times

Achievement Objectives

- explain the meaning of negative numbers (Number, level 4)
- solve problems involving positive and negative numbers, using practical activities or models if needed (Number, level 5)

Other mathematical ideas and processes

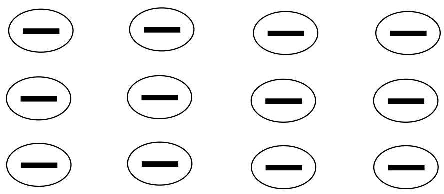
Students will also:

- work with negative numbers
- multiply integers
- use mathematical modelling
- use a calculator to determine a pattern.

ACTIVITIES ONE AND TWO

These activities extend the activities on pages **14–15** and **16** of the students' book from the addition and subtraction of integers to the multiplication of integers (or signed numbers).

Using the model of "negative buttons" may help the students to see that multiplying a negative by a whole number can be thought of in much the same way as ordinary multiplication of rational numbers (whole numbers and fractions). It involves taking a repeated addition approach. For example, $4 \times {}^{-}3$ can be shown as four lots of negative 3 buttons:



The diagram shows that the result is ${}^{-}12$.

However, when it comes to equations such as ${}^{-}4 \times {}^{-}3 = 12$, it is impossible to model this because there is no way that the second negative number can be shown. The students could draw the patterns they found in **Activity One**, question **1**, to find the product of a negative number multiplied by a negative number. For example, the products in **1c** increase by 2, that is:

$3 \times {}^{-}2 = {}^{-}6$
$2 \times {}^{-}2 = {}^{-}4$
$1 \times {}^{-}2 = {}^{-}2$
$0 \times {}^{-}2 = 0$
${}^{-}1 \times {}^{-}2 = 2$
${}^{-}2 \times {}^{-}2 = 4$
${}^{-}3 \times {}^{-}2 = 6$
${}^{-}4 \times {}^{-}2 = 8$
${}^{-}5 \times {}^{-}2 = 10$
${}^{-}6 \times {}^{-}2 = 12$

This shows that a negative number multiplied by a negative number gives a positive product. The students can use this rule or the strategy of continuing patterns to complete the table in **Activity Two**. They could also draw a table that summarises the four possibilities:

Multiplying Integers
(examples in brackets)

x	**Positive** (4)	**Negative** ($^-4$)
Positive (3)	+ (12)	– ($^-12$)
Negative ($^-3$)	– ($^-12$)	+ (12)

This would provide a full answer to the question posed in **Activity Two**, question **2** and would perhaps enable the students to work on **Activity Two**, question **1** with greater understanding.

The students could also use their calculators to investigate what they get when they multiply a negative by a negative.

Page 23 Pulley Power

Achievement Objective

- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, levels 4–5)

Other mathematical ideas and processes

Students will also:

- use spatial visualisation
- apply inverse proportion.

ACTIVITY

The pedals, chain, and sprockets on a bike are a form of pulley system. The big sprocket with pedals is equivalent to the motor, the chain is the same as the belt, and the small sprockets on the back wheel are equivalent to the pulley. (See Gearing Up, pages 22–23, in *Sport*, Figure It Out, levels 3–4.) It may therefore be useful to first link this activity to the action of pedalling a bike. The students probably haven't previously considered the physics of pedalling, so this activity could provide them with some interesting insights.

This activity requires not only some maths calculations but also some spatial visualisation. To determine which way the pulley will turn in each problem, the students will have to mentally trace the path of the belt from the motor wheel to the pulley wheel. If this proves to be a challenge for some students, they could physically trace the path with their finger. After they have tried one or two examples, the students may come to see that:

i. when there is a direct pathway (as with a bike chain, and as in question **2b ii**), the pulley turns in the same direction as the motor wheel;
ii. when there is one loop in the belt (as in question **2b i**), the pulley turns in the opposite direction to the motor wheel.

The next task is to determine the speed of the pulley wheel in each case. Here, you can help the students to see that the speed is in inverse proportion to the size of the motor and pulley wheels. Thus, when the motor wheel is, say, 50 centimetres in diameter and the pulley wheel is 25 centimetres wide (that is, the motor wheel is twice the diameter of the pulley), the speed of the pulley will be twice that of the motor wheel. To illustrate further, if the motor wheel was twice as large again (100 centimetres in diameter) and the pulley wheel remained at 25 centimetres, the motor wheel would now be four times the diameter of the pulley wheel and so the speed of the pulley wheel would be four times as great as the speed of the motor wheel. Perhaps the students could summarise this relationship as follows:

Circumference of motor wheel compared with pulley wheel	Speed of pulley wheel compared with motor wheel
Same	Same
Twice the size	2 times
3 times the size	3 times
4 times the size	4 times

Page 24 What You See …

Achievement Objectives

- find a given fraction or percentage of a quantity (Number, level 4)
- share quantities in given ratios (Number, level 5)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, levels 4–5)

Other mathematical ideas and processes

Students will also:

- apply proportion
- explore negative enlargement
- use spatial visualisation
- apply the identity principle.

ACTIVITY

This activity has similarities with the activity on the previous page because they both deal with spatial visualisation and proportion.

Spatial visualisation is needed to mentally reorientate the flags into an upside-down and reversed position. Those students who find the mental manipulation of the flags a bit difficult could perhaps begin by using small model flags and physically changing their positions.

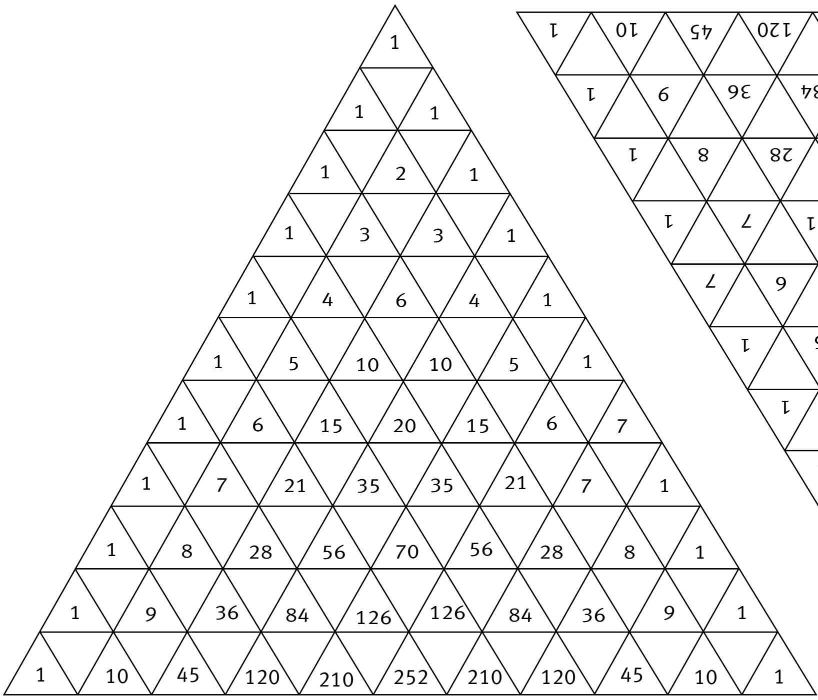
The concept of proportion that the students need to understand is that the size of the image relates to its distance from the pinhole compared with the distance of the flag itself from the pinhole. For example, in question **1**, the image is 10 metres from the pinhole, whereas the flag is just 5 metres from it. This means that the image is twice the distance from the pinhole that the flag is, so the height of the image will be twice that of the flag. This can be shown mathematically as: $\frac{1}{5} = \frac{?}{10}$

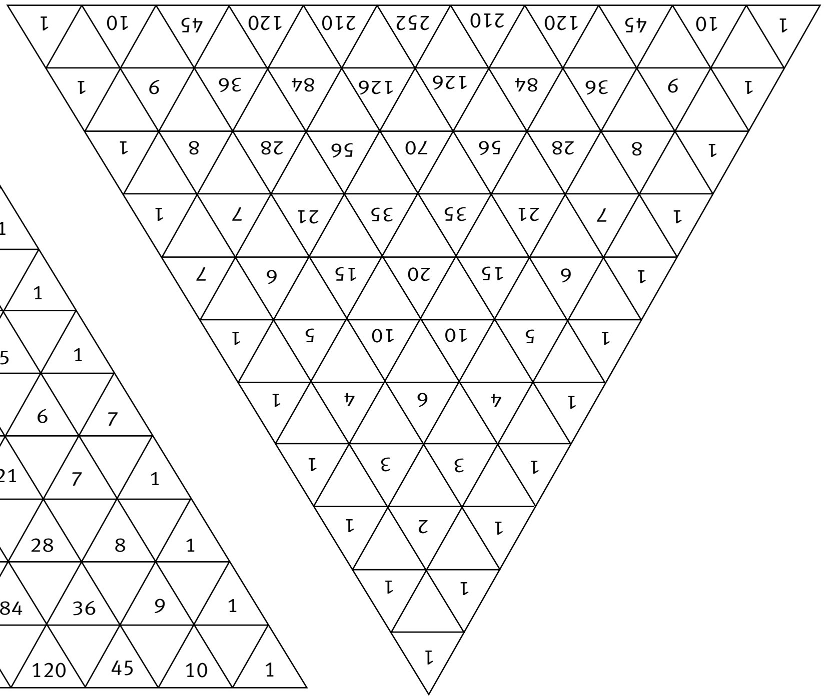
Although it may be reasonably obvious to most students that the image will be 2 metres, it may be useful to review the maths involved in determining this. Once again, it is the identity principle involved (in this case, multiplying by 1) and can be set out as $^1/_5 \times {}^2/_2 = {}^2/_{10}$.

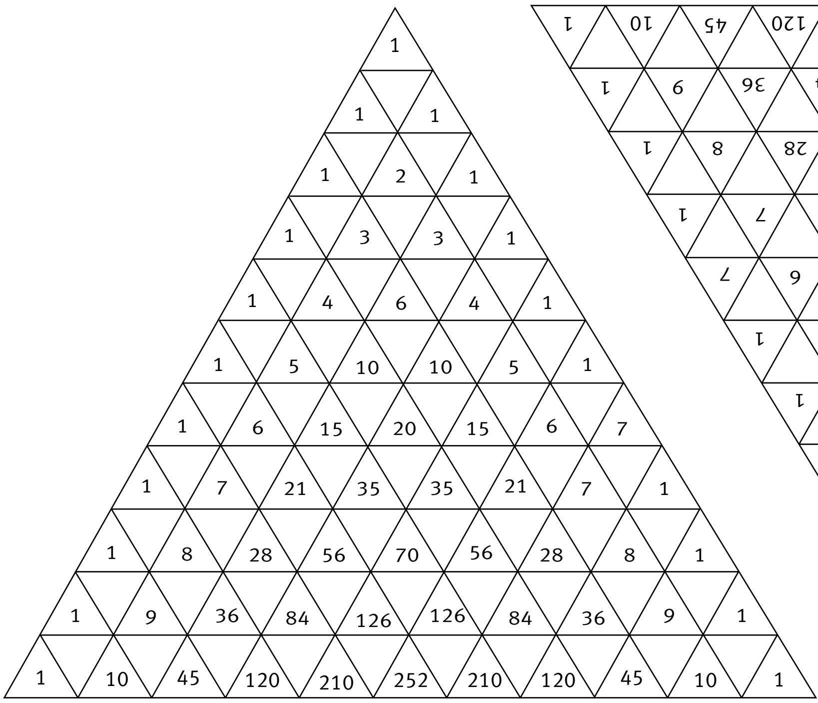
Notice that in question **2i**, the distance of the flag and image from the pinhole are identical, which means that the height of the image is the same as the height of the flag.

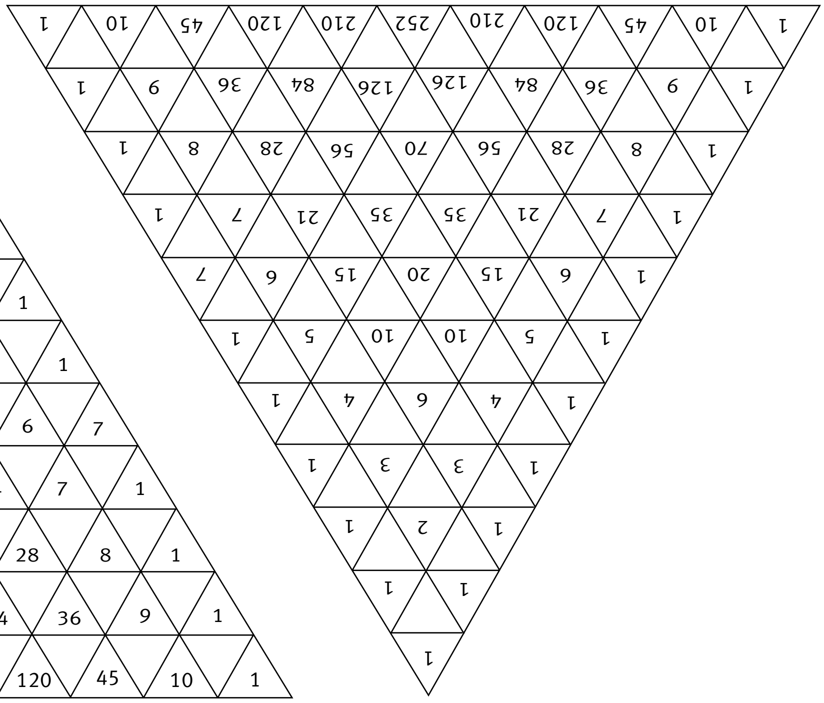
The students may wonder about the height of an image being greater than that of the original flag (question **2iii**) as this would not normally be the case in a pinhole camera. However, if they can imagine the back of the camera being extended backwards a considerable distance (and much enlarged), they might see that an image can indeed be much bigger than the original object.

Copymaster **Pascal's Triangle**









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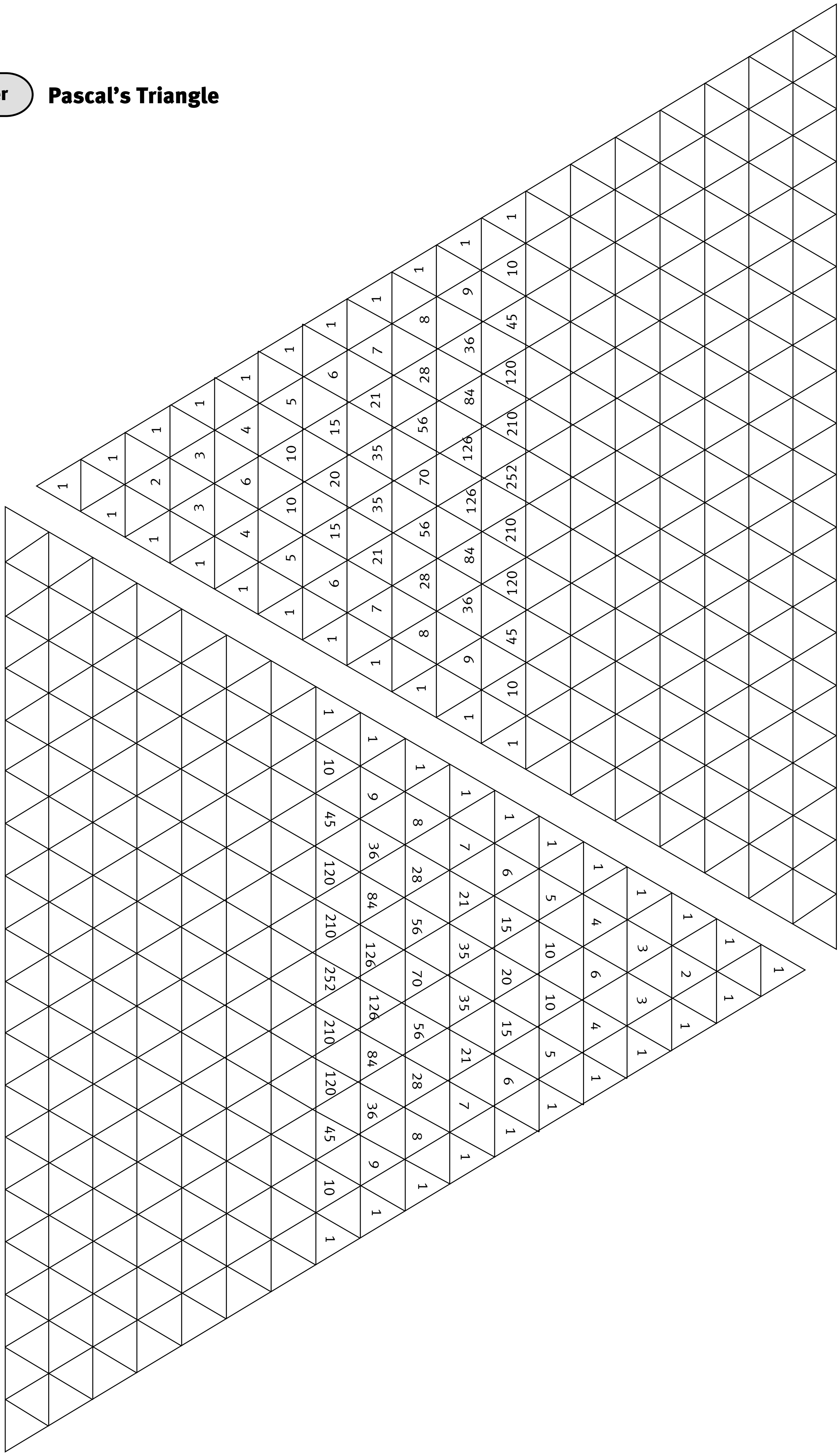
Pascal's Triangle



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