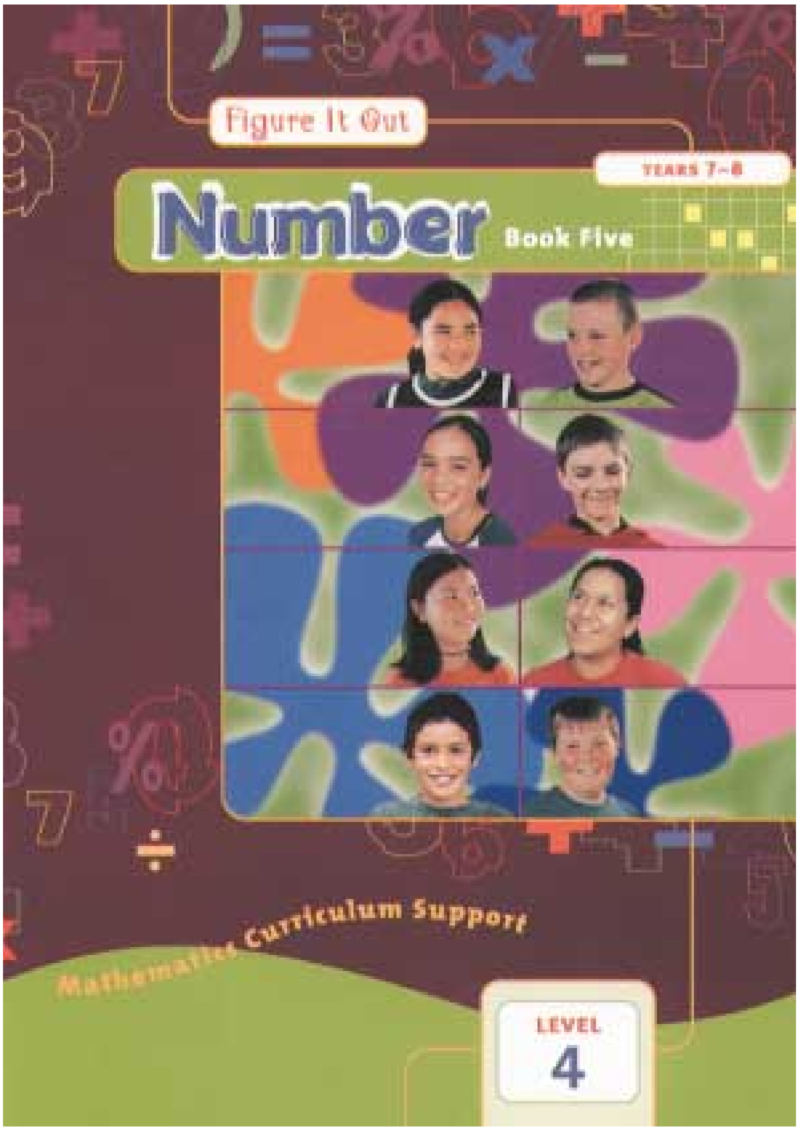
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

CONTENTS

The books for years 7–8 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand year 7–8 classrooms. The books have been developed and trialled by classroom teachers and mathematics educators and follow on from the successful series for levels 2–4 in primary schools.

Student books

The student books in the series are divided into three curriculum levels: levels 2–3 (linking material), level 4, and level 4+ (extension material). All the books are aimed at year 7–8 students in terms of context and presentation.

The following books are included in the series:

Number (two linking, three level 4, one level 4+)
Algebra (one linking, two level 4, one level 4+)
Measurement (one level 4, one level 4+)
Number Sense (one linking, one level 4)
Geometry (one level 4, one level 4+)
Statistics (one level 4, one level 4+)
Themes (level 4): *Disasters, Getting Around*

These 20 books will be distributed to schools with year 7–8 students over a period of two years, starting with the six *Number* books.

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and the life experiences that are meaningful to students aged 11–13 years. The activities can be used as the focus for teacher-led lessons, as independent activities, or as the catalyst for problem solving in groups.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the student books includes full answers and explanatory notes. Students can use them for self-marking, or you can use them for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, comments on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/community

Using Figure It Out in your classroom

Where applicable, each page starts with a list of equipment that the students will need to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Answers Number: Book Five

Page 1 Revisiting Remainders

ACTIVITY

1. a. Kirsty's fraction $\frac{13}{25}$ is correct, but it is not actually a remainder. (263 divides into 25 lots of $10\frac{13}{25}$.)

   Matiu is right, although he could have explained that 13 is the remainder. (There are 10 lots of 25 in 263 with 13 left over.)

   Mei Ling is also right. (263 divides into 25 lots of 10.52.)

   b. Answers may vary, but essentially $\frac{13}{25}$ is equivalent to $\frac{52}{100}$ or 0.52.

2. a. $5\frac{2}{10}$ (or $5\frac{1}{5}$) or 5.2 or 5 r 2

   b. $9\frac{2}{4}$ (or $9\frac{1}{2}$) or 9.5 or 9 r 2

   c. $7\frac{3}{6}$ (or $7\frac{1}{2}$) or 7.5 or 7 r 3

   d. $4\frac{36}{50}$ (or $4\frac{18}{25}$) or 4.72 or 4 r 36

   e. $6\frac{5}{8}$ or 6.625 or 6 r 5

   f. $9\frac{13}{20}$ or 9.65 or 9 r 13

   g. $19\frac{12}{25}$ or 19.48 or 19 r 12

   h. $8\frac{3}{9}$ ($8\frac{1}{3}$) or $8.\dot{3}$ or 8 r 3

   i. $56\frac{8}{9}$ or $56.\dot{8}$ or 56 r 8

3. a.

Divided by	Remainder	Numbers that work
3 and 5	2	17, 32, 47, 62, 77, 92

   b. Answers are likely to vary. All the numbers that work end in 7 or 2. After the first number, 17, all solutions are obtained by adding 15 (3 x 5) to the previous solution. Another way of describing the pattern is adding 2 to each multiple of 15 (15 + 2 = 17, 30 + 2 = 32, and so on).

   c.

Number	÷ 3	÷ 5
17	$5.\dot{6}$	3.4
32	$10.\dot{6}$	6.4
47	$15.\dot{6}$	9.4
62	$20.\dot{6}$	12.4
77	$25.\dot{6}$	15.4
92	$30.\dot{6}$	18.4

   Strategies could include:
   For dividing by 3, consider the nearest multiple of 3 that is lower than the "number that works" and add $0.\dot{6}$ to the other factor that makes that multiple.
   (For example: 3 x 5 = 15. 17 ÷ 3 = $5.\dot{6}$)

   For dividing by 5, go to the nearest multiple of 5 that is lower than the "number that works" and add 0.4 to the other factor that makes that multiple.
   (For example: 5 x 3 = 15. 17 ÷ 5 = 3.4)

Page 2 Remainder Bingo

GAME

A game using division and remainders

Page 3 Give It a Heave!

ACTIVITY

1.

Slingshot	437.13 m
Catapult shot	415.00 m
Flying disc	200.01 m
Cricket ball	121.39 m
Boomerang	120.59 m
Longest gridiron pass	98.72 m
Egg	98.51 m
Javelin (men's 800 g)	98.48 m
Longest gridiron punt	98.36 m
Hammer (men)	86.74 m
Cowpat	80.81 m
Javelin (women's 600 g)	80.00 m
Discus (women's 1 kg)	76.80 m
Discus (men's 2 kg)	74.08 m
Hammer (women)	69.58 m
Gumboot	63.98 m
A playing card	63.52 m
Haggis	55.11 m
Rolling pin	55.08 m
Brick	55.01 m
Shot put (men) indoor	22.67 m
Shot put (women's 4 kg)	22.64 m
Shot put (women) indoor	22.50 m

2.

Karl	50.36
John	50.01
Pare	49.73
Malcolm	48.35
Ian	48.32
Anaru	46.82
Wayne	46.57
Justin	44.93
Vince	44.72
Brendan	38.63
Ross	38.56
Peter	32.19

Page 4 Factorials

ACTIVITY

1. a. 5! = 5 x 4 x 3 x 2 x 1
= 120

b. 6! = 6 x 5 x 4 x 3 x 2 x 1
= 720

c. 10! = 10 x 9 x 8 ...
= 3 628 800

2. You could divide 3 628 800 (10!) by 10 because 10! = 9! x 10. 3 628 800 ÷ 10 = 362 880

3. 7! = 8! ÷ 8
= 40 320 ÷ 8
= 5 040

4. a. 3! ÷ 1! is the same as 3 x 2, 4! ÷ 2! is the same as 4 x 3, and 5! ÷ 3! is the same as 5 x 4.
So the pattern is: multiply the initial factor by the number that is one more than the number shown as the second factorial.

b.–c.
Based on the pattern in **4a**, 6! ÷ 4! is the same as 6 x 5, which is 30, and 10! ÷ 8! is the same as 10 x 9, which is 90.

You can check your predictions based on the following:
1! = 1
2! = 2 x 1
3! = 3 x 2 x 1
4! = 4 x 3 x 2 x 1
5! = 5 x 4 x 3 x 2 x 1
6! = 6 x 5 x 4 x 3 x 2 x 1
7! = 7 x 6 x 5 x 4 x 3 x 2 x 1
8! = 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1
9! = 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1
10! = 10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1

So:
3! ÷ 1! = 6 ÷ 1
= 6, which is 3 x 2

4! ÷ 2! = 24 ÷ 2
= 12, which is 4 x 3

5! ÷ 3! = 120 ÷ 6
= 20, which is 5 x 4

6! ÷ 4! = 720 ÷ 24
= 30, which is 6 x 5

10! ÷ 8! = 362 8800 ÷ 40 320
= 90, which is 10 x 9

You can also set these out using the factors.
For example,

$^{3!}/_{1!} = \frac{3 \times 2 \times 1}{1}$

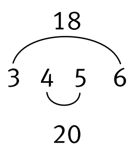
$= 6$

5. a. i. 2 people → 2 ways: ab, ba (2!)
   ii. 3 people → 6 ways: abc, acb, bac, bca, cab, cba (3!)
   iii. 4 people → 24 ways: abcd, abdc, ... (4!)
   b. The number of ways corresponds to the factorial of the number of people being studied.
   c. 3 628 800 ways (10!)

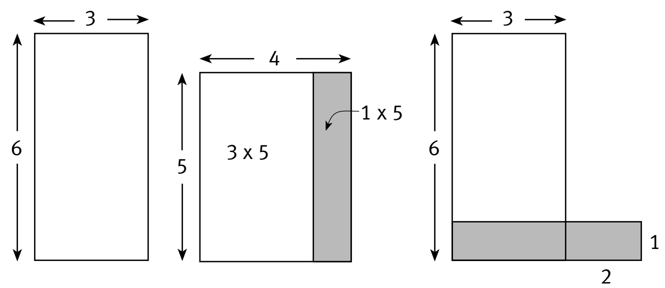
Page 5 True or False?

INVESTIGATION

1. a. True. Each pair of consecutive numbers consists of one odd number and one even number, for example, 2 + 3.
   2 + 3 is 2 + (2 + 1) = 2 x 2 (even) + 1, which is one more than an even number (that is, an odd number).
   b. True. This is because the sum is always three times the middle number, for example,
   6 + 7 + 8 = 21 and 3 x 7 = 21 or
   (7 – 1) + 7 + (7 + 1) = 3 x 7.
   c. False. The product of *any* three consecutive numbers can always be divided by 6. (Each product can also be divided by other numbers, including itself, but 6 is the largest divisor that they all have in common.)
2. a. True
   b. Answers will vary.
   Here is one way to explain this.
   Consider:



20 – 18 = 2. (It works.)
A geometric proof for this is:



Algebraically, you could write this as:
The outside product is n x (n + 3) = n^2 + 3n.
The inside product is (n + 1)(n + 2) = n^2 + 3n + 2 (two greater).

Let the four consecutive numbers be:

n, n + 1, n + 2, n + 3.

Page 6 Order of Operations

ACTIVITY

1. a. i. 16 – 4 x 3 = 4 or (16 – 4) ÷ 3 = 4
   ii. 16 + 4 x 3 = 28
   iii. 16 ÷ 4 x 3 = 12
   iv. 16 + 4 – 3 = 17
   b. 16 ÷ 4 – 3 = 1
   16 – 4 – 3 = 9
   16 x 4 – 3 = 61
   16 ÷ 4 + 3 = 7
   16 x 4 + 3 = 67
   16 – 4 + 3 = 15
2. a. Solutions will vary. Possible solutions include:
   (16 + 8) ÷ 4 – 2 x 3 = 0
   16 + 8 ÷ 4 – 2 x 3 = 12
   16 + 8 ÷ (4 – 2) x 3 = 28
   (16 + 8 ÷ 4 – 2) x 3 = 48
   b. Solutions will vary, but they could include:
   (16 + 8 ÷ [4 – 2]) x 3 = 60
   (Do the inside brackets before the outside brackets.)
   (16 + 8) ÷ (4 – 2) x 3 = 36
   (16 + 8) ÷ (4 – 2 x 3) = ⁻12
   c. Two ways are:
   (16 + 8) ÷ 4 + 2 x 3 = 12 or
   ([16 + 8] ÷ 4 – 2) x 3 = 12
   (Do the inside brackets before the outside brackets.)
3. a. Possible ways include:
   0 = 8 – 4 – 3 – 1
   1 = 8 + 3 – 5 x 2
   2 = 9 – 6 – 2 + 1
   3 = (7 – 4 – 2) x 3
   4 = 4 – 2 + 6 ÷ 3
   5 = (6 – 4 – 1) x 5
   6 = (6 – 3) x (4 – 2)
   7 = (9 – 4 + 2) x 1
   8 = (5 + 7) ÷ 3 + 4
   9 = (6 + 4) ÷ 5 + 7
   10 = (7 + 4 – 6) x 2

b. Possible ways include:
11 = 6 x 4 – 8 – 5
12 = (3 x 2) + (8 – 2)
13 = 2 – 1 + (3 x 4)
14 = 9 – 3 + (4 x 2)
15 = 1 + (3 x 2) + 8
16 = 3 x 1 + 8 + 5
17 = (9 x 2) – 8 + 7
18 = 5 + 9 + (8 ÷ 2)
19 = 5 x (6 – 2) – 1
20 = (5 x 8) ÷ (2 x 1)

Page 7 Operations Checker

GAME

A game using operations and practising basic facts

Page 8 Census Sense

ACTIVITY

1.–2.

a. Note that the under-15 figures differ slightly from those provided in the official 2001 census because the percentages given in the students' book are rounded to the nearest whole number.

Region	Population	People under 15 (to the nearest whole number)
Auckland	1 158 891	266 545
Canterbury	481 431	96 286
Wellington	423 765	93 228
Waikato	357 726	85 854
Bay of Plenty	239 412	57 459
Manawatū/Wanganui	220 089	50 620
Otago	181 542	34 493
Hawke's Bay	142 947	34 307
Northland	140 133	35 033
Taranaki	102 858	24 686
Southland	91 005	20 931
Gisborne	43 974	12 313
Nelson	41 568	8 729
Tasman	41 352	9 511
Marlborough	39 558	8 307
West Coast	30 303	6 970

b. The number of people under 15 will not correspond exactly to the regional order because the percentages vary from 20% to 28%, but the list should be more or less in decreasing order. For example, 20% of Canterbury's total will be a lot more than 25% of Northland's total.

c. 259 575

3. The population in 2001 (3 736 554) is 4.6 times greater than in 1901.

Page 9 Dreaming of Millions

ACTIVITY

1. a. A$1,215
   b. S$1,155
2. China: Y5533.8
   India: Rs31027.5
   Thailand: Bht28323.15
   Vietnam: D9722729.85
   United States: US$668.55
   Greece: EUR691.5
   Philippines: P32,554.2
   United Kingdom: £454.65
3. Vietnam
4. a. $2,243,661.7 (to 1 d.p.)
   b. $3,299,241.2 (to 1 d.p.)
5. Answers will vary.

Page 10 Body Mass

ACTIVITY

a. Toline 18.5
b. Steve 22.3
c. Pānia 20.2
d. Josh 19.8
e. Mei Ling 21.5
f. Aroha 24.8
g. Matiu 19.6
h. George 24.8

Page 11 Saving and Spending

ACTIVITY

1. a. Bank account $12.00
   Tuck shop $6.00
   Clothes $9.00
   Bike repairs $4.00
   Money box $5.00

   b. $162.00.
   ($\frac{1}{3} + \frac{1}{6} + \frac{1}{4} + \frac{1}{9} = \frac{31}{36}$.
   So $22.50 is the remaining $\frac{5}{36}$.
   $22.50 \div 5 = 4.50$ [$\frac{1}{36}$]. $\frac{36}{36} = 4.50 \times 36$
   $= \$162$
   Another way of looking at this is: $22.50 is $4\frac{1}{2}$ times greater than $5, so Pānia would have to earn $4\frac{1}{2}$ times as much. 4.5 x $36 = $162.00)

2. Savings $18.00
   Tuck shop $9.00
   Clothes $13.50
   Skateboard $6.00
   Money box $7.50

3. a. $24

   b. i. Pānia needs to save $\frac{2}{3}$ ($24.00) of her money.

   ii. Budgets will vary.

Pages 12–13 Investigating Digits

ACTIVITY ONE

1. Four subtractions

2. There are seven more, apart from 21 and 12:
   32 and 23
   43 and 34
   54 and 45
   65 and 56
   76 and 67
   87 and 78
   98 and 89

3. 61 and 16
   72 and 27
   83 and 38
   94 and 49
   71 and 17
   82 and 28
   93 and 39

4. 99

ACTIVITY TWO

1. a. Five steps

   b. Three steps

   c. Four steps

2. Possible five-step numbers are 123, 234, 345, 456, 567, 678, 789 or different arrangements of these numbers (for example, 123, 132, 213, 231, 312, or 321). In five-step numbers, the arrangements of the digits to form the smallest possible number is always consecutive, with reverse consecutive numbers for the largest number. The result from the first step is always 198. (So, numbers that cannot be rearranged as consecutive whole numbers will not be five-step numbers.)

INVESTIGATION

a. You should find that all the numbers you try end up as 6 174.

b. When 6 174 is rearranged and subtracted, the result is 6 174.

Page 14 Bargain Bonanza

ACTIVITY

1. $21

2. $20

3. $21

4. $17

5. $30

6. a. $99

   b. $34

Page 15 Bargain Packs

ACTIVITY

1. a. i. $1.20

   ii. $3.60

   iii. $3.60

   iv. $3.00

   v. $20

b. i. 24 cents each

ii. 90 cents each

iii. 60 cents each

iv. $1.00 per kg

v. $4 a pair

2. a. 3 pairs for the price of 2

b. 4 L for the price of 3 L or 2 L for the price of 1.5 L

c. 4 coloured pencils for the price of 3

d. 5 g for the price of 3 g

Page 16 Wooden Projects

ACTIVITY

i. $^3/_4$ m = 750 mm
$^1/_2$ m = 500 mm
$^1/_4$ m = 250 mm

ii. $^3/_8$ m = 375 mm
$^3/_5$ m = 600 mm
$^1/_8$ m = 125 mm

iii. $^2/_5$ m = 400 mm
Each small square is 50 mm x 50 mm.

iv. $^9/_{10}$ m = 900 mm
$^1/_5$ m = 200 mm
$1^1/_4$ m = 1 250 mm

Page 17 Plastic Fantastic

ACTIVITY

1. Room 11, with $3,220
(The other results were: Room 7: $2,380; Room 8: $1,660; Room 9: $2,580; Room 10: $3,040.)

2. Room 10, with 980 items
(The other results were: Room 7: 750; Room 8: 490; Room 9: 770; Room 11: 920.)

3. Answers will vary but must add up to more than $3,220 and less than $3,270.
For example:
$16^1/_2$ boxes of pegs, 9 boxes of sponges, and 10 stacks of buckets, equalling $3,260

4. a. $12,880

b. Various solutions, adding up to at least $2,120. For example: $7^1/_2$ boxes of pegs equalling $900, 8 boxes of sponges equalling $960, and 13 stacks of buckets equalling $260; or 8 boxes of pegs equalling $960, 8 boxes of sponges equalling $960, and 10 stacks of buckets equalling $200.

Page 18 Gentle Giants

ACTIVITY

1. a.

Giant	Actual height (m)
Z.J.	2.459
E.B.	2.362
F.B.	2.343
J.E.	2.337
M.A.C.	2.336
C.W-G.	2.335
S.A.	2.317
J.P.	2.312
M.P.	2.311
R.R.	2.299
B.G.	2.298
B.H.	2.297
E.C.	2.286
J.T.	2.285
P.M.	2.222
D.C.	2.221
J.L.	2.209
P.H.	2.184

b. Daniel Cajanus

c. Sandy Allen. (His claimed height and his actual height are the same.)

d. 2.301 m (to 3 d.p.)

2. a.–b. Answers will vary.

Page 19 Muffin Mania

ACTIVITY

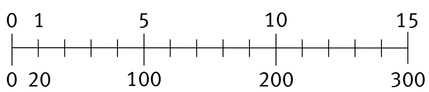
1. $3\frac{1}{2}$ cups plain flour
   1 cup castor sugar
   3 tsp baking powder
   $\frac{2}{3}$ cup butter
   2 eggs
   $1\frac{1}{3}$ cups milk
   grated rind of 2 lemons
   3 cups fresh blueberries
2. $\frac{16}{20}$ (or $\frac{4}{5}$) or 0.8
3. $5\frac{1}{4}$ cups plain flour
   $1\frac{1}{2}$ cups castor sugar
   $4\frac{1}{2}$ tsp baking powder
   1 cup butter
   3 eggs
   2 cups milk
   grated rind of 3 lemons
   $4\frac{1}{2}$ cups of fresh blueberries
4. $10\frac{1}{2}$ cups of flour
   9 tsp baking powder
   2 cups of butter
   4 cups of milk

Page 20 Skateboard Factory

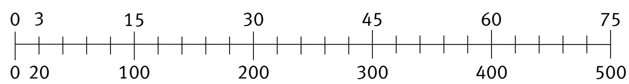
ACTIVITY

(The double number lines shown here are not essential, but you may find them helpful.)

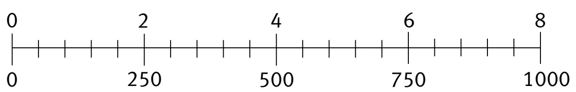
1. 15 (that is, $\frac{1}{20}$ of 300)



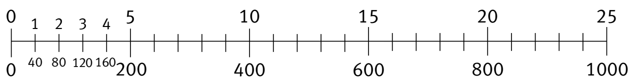
2. 75 (that is, $\frac{3}{20}$ of 500)



3. 2 (that is, $\frac{8}{1000}$ of 250)



4. 120 (that is, $\frac{3}{25}$ of 1 000)



5. a. i. About 18
      ii. About 4
      iii. About 10
   b. 200 skateboards

Page 21 Pick My PIN

ACTIVITY

1. 360. There are six choices of colour for the first peg. After you have chosen a colour for that peg, there are five colours left to choose from, which gives 30 combinations for the first two pegs. There are four choices for the third coloured peg, so there are 120 combinations. That leaves three choices for the last peg. 6 x 5 x 4 x 3 = 360
2. a. 1 296
   b. Because 6 x 6 x 6 x 6 = 6^4, which is 1 296, and 1 296 is a lot more than the 360 you can get using different colours only

GAME

A game of logic and reasoning. Explanations for less than 100 guesses will vary but should mention reasoning strategies such as elimination (NNNN means none of those digits are in), comparison, changing all the digits, changing one digit, and changing the order of digits. Getting feedback from the person with the PIN is also crucial!

Page 22 Percentage Passes

ACTIVITY

1. a. English and maths
   b. Science
   c. Social studies
   d. A table is one way of comparing results:

	Hine	Josh
English	85%	80%
Maths	90%	80%
Science	76%	85%
Social studies	90%	90%

2. Hine's mean percentage score is 85.25% (or 85% rounded to the nearest whole number), and Josh's is 83.75% (or 84% rounded). So Hine scored 1.5% more than Josh (or 2% rounded). (It could have been more accurate and important to use the unrounded scores if the difference had been less than 1%.)

3. a. Yes. $^{19}/_{20}$ is 95%. His mean would then be 86% (rounded).

   b. Hine realised that a science mark of $^{20}/_{25}$ would give her 80% in that subject instead of 76%. This would give her a total of 345, a mean of 86% (rounded).

Page 23 Egyptian Fractions

ACTIVITY

1. a. $^{3}/_{5}$

   b. $^{2}/_{2}$ or 1

   c. $^{3}/_{8}$

   d. $^{5}/_{6}$

   e. $^{8}/_{10}$ or $^{4}/_{5}$

   f. $^{7}/_{8}$

2. Solutions will vary. Possible solutions include:

   a. $^{1}/_{2} + ^{1}/_{5} + ^{1}/_{10}$

   b. $^{1}/_{2} + ^{1}/_{5}$

   c. $^{1}/_{3} + ^{1}/_{9}$

3. Solutions will vary. The possibilities for expressing $^{3}/_{4}$ using fractions no smaller than $^{1}/_{8}$ are:

   $^{1}/_{2} + ^{1}/_{4}$

   $^{1}/_{2} + ^{1}/_{8} + ^{1}/_{8}$

   $^{1}/_{4} + ^{1}/_{4} + ^{1}/_{4}$

   $^{1}/_{4} + ^{1}/_{4} + ^{1}/_{8} + ^{1}/_{8}$

   $^{1}/_{4} + ^{1}/_{8} + ^{1}/_{8} + ^{1}/_{8} + ^{1}/_{8}$

   $^{1}/_{8} + ^{1}/_{8} + ^{1}/_{8} + ^{1}/_{8} + ^{1}/_{8} + ^{1}/_{8}$

   $^{1}/_{6} + ^{1}/_{6} + ^{1}/_{6} + ^{1}/_{4}$

Page 24 Lining Up Numbers

ACTIVITY

1. a. Incorrect total. The correct total is 3 257. Paul's method will find the error, but Toline's will not.

   b. Correct total, although Toline may think there is an error.

   c. Incorrect total. The total is 4 137. Paul's method will not find the error, but Toline's will.

   d. Incorrect total. The correct total is 4 662. Both methods will work, although Toline may miss the error.

2. Toline's method can show a total to be false when it is actually true. This happens when the "round-ups" do not balance the "round-downs". With Paul's method, the units column will usually be correct, but there could be errors in the other columns that Paul's method will not pick up.

Teachers' Notes

Overview **Number: Book Five**

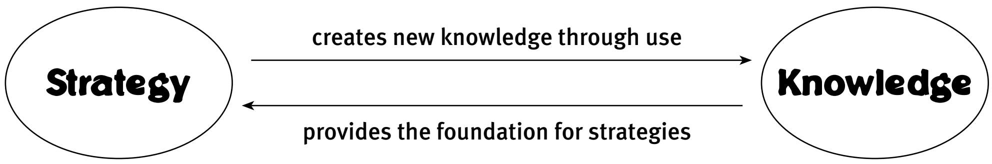
Introduction to Number

There is a remarkable commonality in the way many countries around the world are now teaching arithmetic. Changes in the approaches reflect the evolving demands of everyday life, a greater volume of classroom-based research about how students learn, and a desire to improve general levels of numeracy.

In the past, arithmetic teaching has focused on preparing students to be reliable human calculators. The prevalence of machines in society that calculate everything from supermarket bills to bank balances has meant that students now require a wider range of skills so that they can solve problems flexibly and creatively.

The Figure It Out series aims to reflect these trends in modern mathematics education. A range of books is provided at different levels to develop both number skills and number sense. The *Number* books are aimed at developing students' understanding of the number system and their ability to apply efficient methods of calculation. The *Number Sense* books are aimed at developing students' ability and willingness to apply their number understanding to make mathematical judgments. Teaching number sense requires an emphasis on openness and flexibility in solving problems and the use of communication and interpretation skills.

The development of the Figure It Out series has occurred against the backdrop of a strong drive for improved standards of numeracy among primary-aged students. A key element of this drive has been the creation of the Number Framework, developed as part of the Numeracy Strategy. The framework highlights this significant connection between students' ability to apply mental strategies to solving number problems and the knowledge they acquire.



Learning activities in the series are aimed both at developing efficient and effective mental strategies and at increasing the students' knowledge base. Broadly speaking, the levels given in the six year 7–8 *Number* books can be equated to the strategy stages of the Number Framework in the following way:

Link (Book One):	Advanced counting to early additive part-whole
Link (Book Two):	Advanced additive part-whole
Level 4 (Books Three to Five):	Advanced multiplicative to advanced proportional part-whole
Level 4+ (Book Six):	Advanced proportional part-whole.

Revisiting Remainders

Achievement Objectives

- express a fraction as a decimal, and vice versa (Number, level 4)
- express quantities as fractions or percentages of a whole (Number, level 4)

Other mathematical ideas and processes

- Depending on context, you can express a quotient as a whole number with remainder, as a decimal number, or as a mixed number.
- You can find the decimal equivalent of an ordinary fraction by dividing the numerator by the denominator.
- Number sentences may be open or closed. Closed number sentences may be true or false.

Students will also:

- use the symbol for a recurring decimal number
- detect patterns in numbers.

ACTIVITY

Question **1** shows how the quotient or result in a division operation may be expressed in three different ways:

- as a whole number (with the remainder being left as a remainder: 10 r 13)
- as a mixed number (that is, as a whole number and fraction: $10^{13}/_{25}$)
- as a decimal number (10.52).

The method used depends on the problem or context. For example, if there are 263 people (students, teachers, and adult helpers) going on a trip and each bus has 25 seats, then it makes sense to think in terms of a remainder and order 11 buses. On the other hand, if the 263 were $263 from a restaurant bill to be shared among 25 people, then each person would pay $10.52. In practical terms, it would be unusual today to have a quotient such as $10^{13}/_{25}$ because most of our measurements are metric. But quotients involving mixed numbers could be used when there are common fractions involved, such as halves, quarters, and thirds. For example, if five apples are being shared evenly among four children, then each child will receive $1^1/_4$ apples ($5 \div 4 = 1^1/_4$).

Question **1** deals with decimal amounts that are equivalent to fraction amounts. The students can convert an ordinary fraction into its decimal equivalent by dividing the numerator by the denominator, which can be done easily on a calculator. If the students have not already done so, they could find the decimal equivalents of common fractions such as $^1/_4$, $^1/_2$, $^3/_4$, $^1/_3$, and $^2/_3$ on a calculator. Of course, this problem also requires the students to understand that a fraction such as $^3/_4$ is an alternative way of writing $3 \div 4$ or $4\overline{)3}$.

As equivalent fractions, $^1/_4 = {}^{25}/_{100}$ and $^1/_2 = {}^5/_{10}$
$= 0.25$ $= 0.5$

Matiu's table in question **3** shows how useful patterns are when working with number. The pattern involved in questions **3a** and **3b** is multiples of 15 plus 2, so the next four numbers are (3 x 15) + 2, (4 x 15) + 2, (5 x 15) + 2, and (6 x 15) + 2. When the students understand this, they can find any number in the series. For example, the tenth number would be (10 x 15) + 2, that is, 152.

Similarly, with question **3c**, the students will quickly see that when they divide by 5, the quotient always ends in 0.4, and when they divide by 3, the quotient always ends in 0.666 … or $0.\dot{6}$. Ask them to consider why this happens. Some students will realise that it is because 2 (the remainder) ÷ 5 is 0.4 and 2 ÷ 3 is $0.\dot{6}$.

This is an opportunity to explain to students the symbol used to indicate a recurrent decimal (namely the dot over the 6 in $5.\dot{6}$).

Page 2 Remainder Bingo

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 4)
- recall the basic multiplication facts (Number, level 3)

Other mathematical ideas and processes

Depending on context, you can express a quotient as a whole number with remainder, as a decimal number, or as a mixed number.

GAME

This game calls on the students' knowledge of the basic multiplication facts and their understanding of the link between multiplication and division. It also gives them practice using the terms dividend, divisor, quotient, and remainder. The dividend is the amount being divided, the divisor is what the amount is being divided by, and the quotient is the result, which may include a remainder.

Page 3 Give It a Heave!

Achievement Objectives

- explain the meaning of the digits in decimal numbers with up to 3 decimal places (Number, level 3)
- order decimals with up to 3 decimal places (Number, level 3)

Other mathematical ideas and processes

Students will also:

- investigate the meaning of tenths and hundredths
- investigate the effect of multiplying and dividing numbers by 10, 100, and beyond.

ACTIVITY

In this activity, the students order distances to two decimal places. You may find that you need to check your students' understanding of tenths and hundredths. Some students think that the first column to the right of the decimal point is "oneths", a mirror image of the columns of whole numbers. These students may need practical examples, such as money, to help them realise that this column represents tenths. Using money can be helpful because money amounts are written in decimal form ($1.10). The students who come to see that 10 cents is $^1/_{10}$ of $1, that 50 cents is $^1/_2$ or $^5/_{10}$ of $1, and so on, are less likely to think that 0.35 is bigger than 0.5 because they can now think of the 0.35 as 35 cents and the 0.5 as 50 cents. The use of centimetres on a metre ruler can also be useful. For example, you can help the students understand that 8 centimetres is 8 out of 100 (that is, eight-hundredths or $^8/_{100}$) and is written as 0.08 (no tenths involved), whereas 35 centimetres is $^{35}/_{100}$ (or $^3/_{10}$ and $^5/_{100}$) and is therefore written as 0.35.

Another key understanding that you could develop in the midst of these two activities is that as you move left across the columns, each is 10 times greater than the one before it, and moving right has the opposite effect: each is 10 times less. Thus 5 is 10 times greater than 0.5, and 0.05 is 10 times less than 0.5. This understanding is extremely helpful for mental calculations of the type $2.50 ÷ 10 (which immediately gives $0.25 or 25 cents).

Although it is not required for this activity, you could use this understanding as an opportunity to help the students see that moving across two columns means that a number is 100 times greater or less, moving across three columns means that a number is 1 000 times greater or less, and so on. The students will soon realise that there is a pattern at work here in that the number of places moved relates to the number of zeros and indicates how many times greater or less. They could set this out in a table, for example:

Columns moved	Number of times > or <	Examples Greater (>)	 Less (<)
1	10	6 becomes 60	6 becomes 0.6
2	100	5 becomes 500	5 becomes 0.05
3	1 000	8 becomes 8 000	8 becomes 0.008
4	10 000	4 becomes 40 000	4 becomes 0.0004

Page 4 Factorials

Achievement Objectives

- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, level 4)
- make sensible estimates and check the reasonableness of answers (Number, level 4)

Other mathematical ideas and processes

Students will also:

- investigate the meaning of factorials and the symbol ! in mathematics
- use the identity principle in division
- identify and use patterns in number.

ACTIVITY

Before your students begin this activity, you may need to discuss with them the information given at the top of the page to ensure that they understand what factorials are and how they are written. In factorials, the exclamation mark (!) signifies that the number should be multiplied by consecutively lower whole numbers. Thus 5! (5 factorial) is 5 x 4 x 3 x 2 x 1.

The activity explores factorials as pure numbers and then applies factorials to a real life situation. However, as an extra challenge for the students, you could reverse the order. You could ask them to begin with the bus queue problem and then discuss with them whether there is some clever mathematical short cut to working out problems of this nature.

Although the students could use a calculator to work out solutions to the questions fairly easily, encourage them to look for patterns that will help them solve the problems quickly, without using a calculator. To detect patterns, they will need to understand how the numbers are working. For example, to work out what 9! is when 10! is known, the students need to understand that 10! is 10 times greater than 9! (or that 6! is 6 times greater than 5!, and so on). Similarly, in question **4**, the students should be able to detect a pattern reasonably easily (3! divided by 1! is the same number as 3 x 2, 4! divided by 2! is the same number as 4 x 3, and so on) and to use this pattern to predict the solutions to questions **4b** and **4c**. Ask the students to explain why this pattern works. They might describe it this way:

3! divided by 1! is really $\frac{3 \times 2 \times 1}{1}$

The 1 in the denominator cancels out the 1 in the numerator, so you are left with 3 x 2. So 3! ÷ 1! = 6.

4! divided by 2! is really $\frac{4 \times 3 \times 2 \times 1}{2 \times 1}$

The "2 x 1" parts cancel each other out, so you are left with 4! ÷ 2! = 4 x 3.

5! divided by 3! is really $\frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1}$

The "3 x 2 x 1" parts cancel each other out, so you are left with 5! ÷ 3! = 5 x 4.

Such an understanding will enable the students to work out other examples, such as 8! ÷ 6!, which, in short, is 8 x 7 because it is really $\frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$

This "cancelling out" is the power of the identity principle. The relevant aspect here is the idea that dividing any number by itself results in 1. Take the last example above, $\frac{8 \times 7 \times (6 \times 5 \times 4 \times 3 \times 2 \times 1)}{(6 \times 5 \times 4 \times 3 \times 2 \times 1)}$

The part now shown in brackets is simply 1. It is not even necessary to work out that the bracketed part of the top is 720 and that the bottom is also 720. Logic indicates that they are the same number, whatever that number might be.

Page 5 True or False?

Achievement Objectives

- make conjectures in a mathematical context (Mathematical Processes, developing logic and reasoning, level 4)
- make sensible estimates and check the reasonableness of answers (Number, level 4)

Other mathematical ideas and processes

Students will also:

- investigate patterns involving consecutive whole numbers
- use logic and reasoning to determine the truth or otherwise of mathematical statements and to provide proof.

INVESTIGATION

To determine whether a statement is *true* requires not only checking out several examples that fit the statement but also using logic to justify a position taken. For example, in investigation **1a**, the students could check that the sums of such consecutive numbers as 4 + 5, 15 + 16, 36 + 37 are odd and then use logic to point out that two consecutive numbers are always going to include one even and one odd number and that the sum of an odd and even is always odd, so the statement must be true.

With investigation **1b**, you may need to help the students to realise that adding three consecutive whole numbers results in a sum that is *always* three times greater that the middle number. In the case of the example given (6 + 7 + 8), the students should easily see that this equation is the same as 7 + 7 + 7 (and that the mean or average of the three numbers that add up to 21 is 7). It follows logically that if the sum is three times greater than the middle number, the sum can always be divided evenly by 3.

The key word in investigation **1c** is "any". If this word is ignored, the students may fall into the trap of thinking that the true solution is the product itself. If the statement had been that the largest number that the product of any three consecutive numbers can be divided by is 6, the students would be unable to find any exceptions. To determine whether a statement is *false*, it is only necessary to find one instance that doesn't conform. In this case, 1 x 2 x 3 = 6, so 6 is the largest multiple for the three smallest consecutive whole numbers and is therefore the largest divisor.

Investigation **2** shows once again the power of patterns in number. The students will probably want to try several more examples to see if the pattern holds, but they are then left with the more challenging issue of *why* it works. You will find nice geometric and algebraic explanations or proofs in the Answers. If the students do not discover them for themselves, you could perhaps take them through these explanations. The students could find the algebraic explanation more tricky than the geometric one if they are not familiar with the multiplication involved, which is n x (n + 3) = □ and (n + 1)(n + 2) = □ . The power of this explanation, however, is that it is generalised and represents any instance of four consecutive numbers the students might like to choose.

Page 6 Order of Operations

Achievement Objective

- demonstrate knowledge of the conventions for order of operations (Number, level 4)

Other mathematical ideas and processes

- The order of operations is brackets first, then multiplication and/or division, then addition and/or subtraction. Otherwise, work from left to right.
- A number can have many names.
- The two sides of an equation name the same number.
- In making some equations true, more than one solution may be possible.

ACTIVITY

This activity reinforces the students' knowledge of the conventions for the order of operations, but it is equally important in helping them to expand their understanding of the following algebraic ideas:

- A number can have many names.
- The two sides of an equation name the same number (that is, the equation is an expression, as distinct from a record of a solution involving one or more operations).

If the students are not familiar with using brackets, you may like to work through several examples with them to show the effect of the brackets. For example, in (5 + 2) x 3, you must do the brackets first, so the solution is 21. In 5 + 2 x 3, you do the multiplication first, so the solution is 11.

Question **1** provides numerous examples of how the two sides of an equation name the same number. At the same time, question **1a i** is a nice instance of an equation in which more than one solution is possible.

Question **3** is a good example of how a number can have many names. For example, the number 2 can be 9 – 6 – 2 + 1 or (8 – 5) x 2 ÷ 3 or (6 – 4) x 5 – 8.

Page 7 Operations Checker

Achievement Objective

- demonstrate knowledge of the conventions for order of operations (Number, level 4)

Other mathematical ideas and processes

- The order of operations is brackets first, then multiplication and/or division, then addition and/or subtraction. Otherwise, work from left to right.
- A number can have many names.
- The two sides of an equation name the same number.
- In making some equations true, more than one solution may be possible.

GAME

This game provides the students with a real incentive to operate on a set of three numbers randomly thrown with dice, and in so doing, gain even greater number sense. The numbers on the game board have been kept relatively small (none is greater than 15) so that the students can compute all the operations in their heads. This game, as well as helping to anchor basic facts, reinforces the idea that a number can have many names. For example, one student could place a counter on a 12 square, using the number combination 5, 3, and 4 (that is, 5 + 4 + 3), whereas another could place a counter on one of the other 12 squares, using the number combination 1, 6, and 2 (that is, 6 x 2 x 1).

Page 8 Census Sense

Achievement Objectives

- find a given fraction or percentage of a quantity (Number, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 4)

Other mathematical ideas and processes

Students will also:

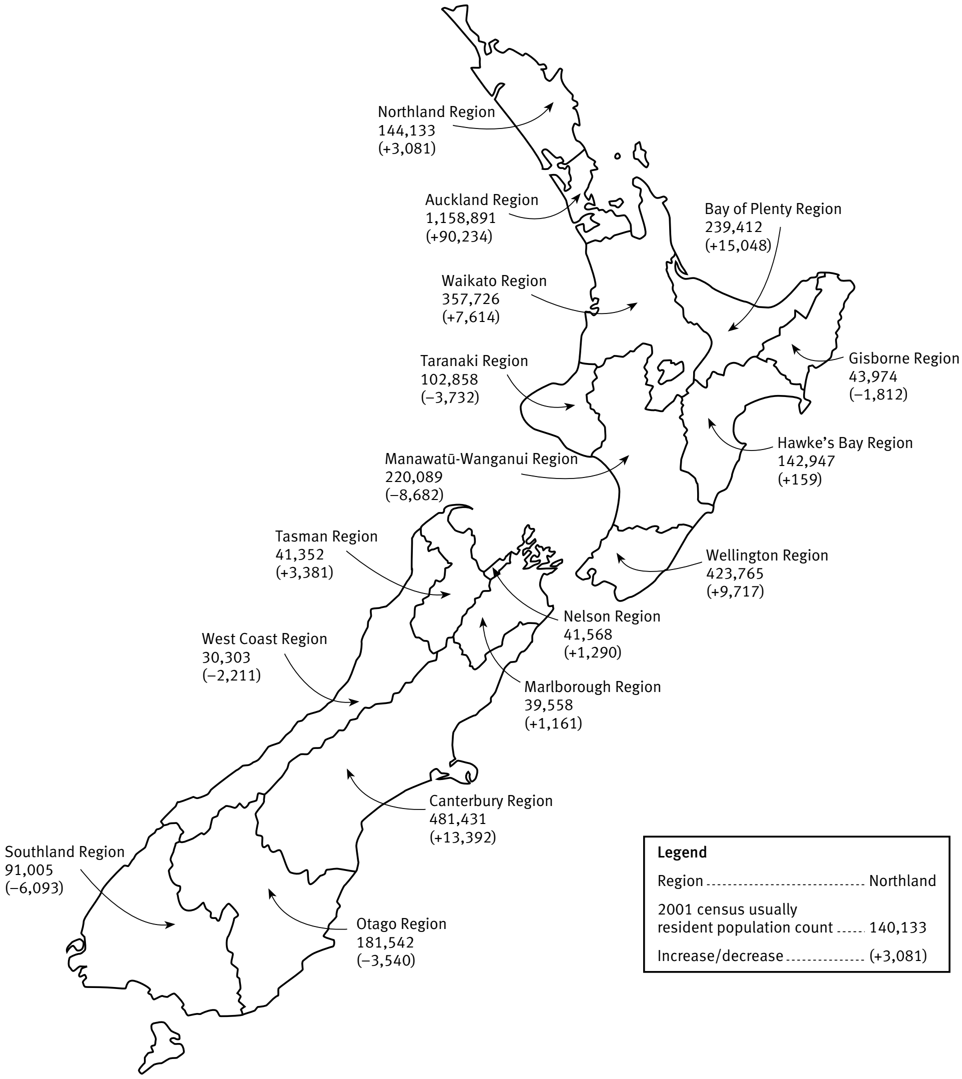
- order large numbers
- calculate percentages
- round to one decimal place.

ACTIVITY

This activity helps the students to develop a better sense of numbers in the tens of thousands and hundreds of thousands. It uses authentic data and therefore reinforces the relationship between mathematics and our lives. After the 2006 census, students will be able to access the latest census data for comparison on the Department of Statistics website www.stats.govt.nz

As an extension, the students could compare the 2001 figures given on the students' page with the previous census figures from 1996. The map on the following page shows the 2001 regional population figures and the increase (+) or decrease (–) since the 1996 census. The students could use this to work out the 1996 population figures, which they could check from the table below the map. You could use the table, which includes the percentage of people under 15 years of age, as the basis for a discussion on changes and possible trends.

2001 Census Usually Resident Population Count of New Zealand's Regional Councils



1996 Regional Population

Region	Population	% under 15
Auckland	1 068 657	23
Bay of Plenty	224 364	25
Canterbury	468 039	20
Gisborne	45 786	28
Hawke's Bay	142 788	25
Manawatū/Wanganui	228 771	25
Marlborough	38 397	22
Nelson	40 278	21
Northland	137 052	26
Otago	185 082	20
Southland	97 098	24
Taranaki	106 590	25
Tasman	37 971	23
Waikato	350 112	25
Wellington	414 048	22
West Coast	32 514	24

In the activity, question **1** involves putting the population figures in order according to size. Most students are likely to put the largest region with the largest population at the top, but it would be just as acceptable for them to put the smallest at the top. This could even be a worthwhile discussion point, with different students giving their views on the matter.

Question **2** requires more in the way of mathematical understanding and calculation. With this problem, you may need to help some students to see that finding a percentage using a calculator requires them to convert the percentage to a decimal and multiply. For example, if their calculator has a percentage button, they can find 23% of 30 303 (people under 15 years old on the West Coast) by entering [30 303] [x] [23] [%]. Alternatively, they could convert 23% to 0.23 and then enter 0.23 x 30 303.

This authentic data and the discussion stemming from question **2b** may lead the students to question the figures. For example, some may ask, "Why is it that people under 15 make up only 20% of the Canterbury population whereas they make up 28% of the population in Gisborne?" This is a social studies issue, but it is likely to involve problem solving using numbers.

Question **3** involves more work than the students may realise at first because they need to add the population figures for each region before they can solve the problem.

Page 9 Dreaming of Millions

Achievement Objectives

- write and solve problems involving decimal multiplication and division (Number, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

Other mathematical ideas and processes

Students will also:

- use proportion
- work with decimals to four places.

ACTIVITY

Working out currency exchange rates can be tricky, so the students may need help to work through the reasoning needed in this problem. One way is to think of the problems as proportion tasks. Ask the students "What has to be done to the NZ$1.00 to get NZ$1,500?" The answer is to multiply it by 1 500. Do the same to the A$0.81 and you get the appropriate exchange figure, namely A$1,215. Questions **1** and **2** can be tackled in the same way.

Question **4** requires a similar strategy. For example, to work out the United States figures, it would be useful to approach the proportion as follows: NZ$1 = US$0.445

NZ$☐ = US$1,000,000

Here the question becomes: What has to be done to the US$0.4457 to make it US$1,000,000? The answer is to multiply it by 2 243 661.6 (1 000 000 divided by 0.4457), which is what you then have to multiply NZ$1 by to get the equivalent exchange figure. Note that for question **4**, the decimal figure obtained on an eight-digit calculator may vary slightly from that obtained on a 10-digit calculator and rounded to one decimal place (2 243 661.7).

Current exchange rates can be found on the Internet at www.currency.co.nz

Page 10 Body Mass

Achievement Objective

- write and solve problems involving decimal multiplication and division (Number, level 4)

Other mathematical ideas and processes

Students will also:

- round to one decimal place
- work with squared numbers
- investigate the notion of "range" in measurement.

ACTIVITY

Ensure that the students understand the formula given at the top of the page. They also need to know how to round to one decimal place. In the case of Toline, her BMI works out to be 18.517821, so the students need to know that to round to one decimal place, they should look at the digit in the hundredths column (1). Given that it is not 5 or more, the BMI rounded to one decimal place is 18.5. In Aroha's case, however, the BMI of 24.779614 is rounded up to 24.8.

This question provides a good opportunity to reinforce how to use the memory function on a calculator. For example, to calculate Toline's BMI, students using basic calculators could square her height and enter this amount in memory. They could then enter her mass, press [÷], recall from memory the squared height [MR], and press [=]. With a scientific calculator, they could enter her mass, press [÷], enter her height, press [x^2] to square the height, and then press [=].

As an extension, you could discuss the notion of "range". The idea of range is especially important when estimating any kind of measure (for example, "I think it was between 4 metres and 5 metres long"; "I should be home between 3.30 p.m. and 4 p.m."), but it is also important in different aspects of health (for example, an acceptable blood pressure ranges from 80–90 on the lower reading and 120–130 on the upper reading). You could also use the statistics for the students' height, mass, and BMI to discuss range. For example, "The height of the students is between 1.30 metres and 1.63 metres. What height might most students of this age be?"

Page 11 Saving and Spending

Achievement Objectives

- express quantities as fractions or percentages of a whole (Number, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)

Other mathematical ideas and processes

Multiplying by a fraction is the same as dividing by its reciprocal.

ACTIVITY

This activity helps the students to develop an understanding of what a fraction means and how to calculate fractional amounts of a whole. Check that your students know that to find one-third of an amount, they need to divide the amount into three equal parts. Concrete materials are useful for helping those who don't know this. You could begin with one whole (for example, one biscuit) and then move on to more than one whole (for example, six biscuits). Although this activity involves only unitary fractions ($^1/_3$, $^1/_4$, and so on) you could, if you wish, extend it to finding other fractions, such as $^2/_3$ or $^3/_4$, of various amounts.

This activity also provides an excellent opportunity for problem solving. Ask the students to discuss, as a class, the strategies they used for questions **1b**, **3a**, and **3b** together with their justifications for their particular strategies and solutions. Justifying solutions is at the heart of working mathematically and is partly why *Mathematics in the New Zealand Curriculum* stresses the importance of students communicating mathematical ideas.

Pages 12–13 Investigating Digits

Achievement Objective

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

Other mathematical ideas and processes

Students will also:

- use patterns in problem solving
- work with multiples of 9
- use subtraction and difference.

ACTIVITIES ONE AND TWO

Both activities continue the theme of patterns in number. Some astute students may ask, "Why, with two-digit numbers, is the final result always 9?" This is a very good question. You could help the students to recognise that the difference between any reversed pairs of two-digit numbers is always a *multiple* of 9 (for example, 35 leads to 53 – 35, which is 18; 69 leads to 96 – 69, which is 27; 81 – 18 is 63, and so on), and that finding the difference between reversed pairs of two-digit multiples of 9 (excluding 99) eventually results in 9 itself.

When the students have tackled questions **2** and **3** of **Activity One**, ask them to share the strategies that they used to determine the number of one-subtraction pairs and to identify the seven two-subtraction pairs. Did they notice, for example, that the one-subtraction pairs all have a difference of 9 and that the difference between the two-subtraction pairs has to be either 45 or 54? A similar approach could be taken with the tasks in **Activity Two** and the **Investigation**.

Page 14 Bargain Bonanza

Achievement Objectives

- express quantities as fractions or percentages of a whole (Number, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

Other mathematical ideas and processes

Students will also:

- calculate percentages involving money
- calculate fractional amounts involving money.

ACTIVITY

This activity provides a range of valuable experiences to do with calculating percentages and fractional amounts that involve money. Retail competition is such that discounts and sales are a regular feature of our lives, so the students will often encounter them.

A good teaching point in this activity is to help the students to realise that the shortest way to calculate the price of the book in question **1** is to use the calculator to find 70% rather than 30% of the $30. This gives the sale price in one step instead of having to calculate and then subtract 30% from the $30. This can also be done mentally by translating the 70% to $^{7}/_{10}$, calculating $^{1}/_{10}$ of 30, and then multiplying it by 7. If this is done on the calculator, the 70% has to be translated to its decimal equivalent, which is 0.7. Then it is simply a matter of multiplying 0.7 x 30 to get 21. Fundamental to this is the notion that finding 0.7 of an amount means *multiplying* the amount by 0.7.

With question **4**, the students may wonder if finding the sale price of two $10 puzzles separately and then adding them would give the same total as finding 85% of double the amount, that is, $20. (It does.) If they do wonder about this, encourage them to investigate this important question too.

Notice that question **5** cleverly reverses the problem. Here the students need to use logic to realise that the $20 spent represents $^{2}/_{3}$ of the original cost. There are different ways of calculating the original price. Perhaps the easiest way is to work out $^{1}/_{3}$ (half of 20 is $10) and multiply this amount by 3, giving $30. Whatever we do to the $^{2}/_{3}$ to get $^{3}/_{3}$ (it has to be multiplied by 1.5), we have to do the same to the $20 (so 20 x 1.5 = 30).

A possible extension of this activity would be for the students to collect local data about sales and special offers (from the media, brochures, circulars, and shops) and pose their own questions for investigation. They could work individually or in pairs to investigate either their own or others' questions.

Page 15 Bargain Packs

Achievement Objectives

- write and solve problems involving decimal multiplication and division (Number, level 4)
- find fractions equivalent to one given (Number, level 4)

Other mathematical ideas and processes

Students will also:

- work with ratios
- use the identity principle.

ACTIVITY

Questions **1a** and **1b** call for multiplication and division respectively. You could encourage the students to do all these equations in their heads and then invite them to say what particular strategies they used to do the mental calculations. If a few students need to jot down some figures to aid their memories, that is fine, but they could explain what they jotted down and why.

Question **2** introduces the level 5 Number objective of working with ratios. At this level, the questions may be more easily understood in terms of simplifying fractions. To simplify fractions, the students need to understand the identity principle, that is, if you divide a number by 1, the number retains its identity. In other words, the number stays the same, although its name may change. For example, question **2a** can be worked out as follows:

$$\frac{6 \div (2) = 3}{4 \div (2) = 2}$$

Dividing by $^2/_2$ is the same as dividing by 1. Note that 1 itself can have many names, including many fractional names. Similarly, question **2c** can be figured out as follows:

$$\frac{36 \div (9) = 4}{27 \div (9) = 3}$$

This translates to four coloured pencils for the price of three.

Take the time to help your students to understand the identity principle because it is a powerful idea that is used in mathematics at higher levels. At this level, it is what enables equivalent fractions to be generated. For example:

$$\frac{3 \times (3) = 9}{4 \times (3) = 12} \quad \text{or} \quad \frac{3 \times (25) = 75}{4 \times (25) = 100}$$

The relevant aspect here is the understanding that a number retains its identity if it is multiplied by 1. So $^3/_4$ is the same fractional number as $^9/_{12}$ or $^{75}/_{100}$, although obviously its name is different.

Page 16 Wooden Projects

Achievement Objective

- express a fraction as a decimal, and vice versa (Number, level 4)

Other mathematical ideas and processes

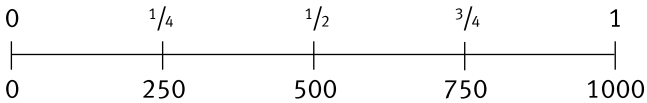
Students will also:

- work with ratio
- use the identity principle
- work with area
- consider the use of "of" when used to mean multiplication.

ACTIVITY

The millimetre is a common unit of measurement for woodworking and building. The students need to understand that there are 1 000 millimetres in 1 metre. Thus, all the fractions in this activity have to be treated as fractions of 1 000.

As with the activity on page **15** of the students' book, there are different ways in which the fractions of a metre can be converted to millimetres. One way is to use a double number line, for example:



Another way is by using ratios, for example, $^3/_4 = {}^{\square}/_{1000}$. Whatever the 4 has to be multiplied by to get 1 000, the 3 at the top has to be multiplied by too, that is, $^3/_4 \times (^{250}/_{250}) = {}^{750}/_{1000}$. Astute students will probably see that this is simply another application of the identity principle.

Alternatively, and still using ratios, the students could reason that whatever the 4 has to be divided by to get 3, the same should be done to the 1 000. Using a calculator, they could soon find that 4 divided by 3 is 1.3333 recurring ($1.\dot{3}$) and dividing 1 000 by the same number gives 750.

Perhaps the easiest way to convert the fractions of metres to millimetres is to use the calculator in a different way. The students could turn each fraction into a decimal (for example, $^3/_4$ is 3 divided by 4, which gives 0.75) and then *multiply* this by 1 000 (0.75 x 1 000 = 750) because it is a decimal fraction *of* 1 000, and "of" in this context means *multiplied* by.

Finally, you may wish to use question **iii** to challenge the students to work out:

- the area of the chessboard (400 mm x 400 mm = 160 000 mm^2)
- the area of each small square (50 mm x 50 mm = 2 500 mm^2).

An additional challenge would be to work out the area in square metres of the chessboard (0.4 m x 0.4 m = 0.16 m^2) and then of each small square (0.05 m x 0.05 m = 0.0025 m^2).
As a check, the students could see if 0.0025 m^2 x 64 (the total number of small squares) gives 0.16 m^2.

Page 17 Plastic Fantastic

Achievement Objectives

- write and solve problems involving decimal multiplication and division (Number, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

Other mathematical ideas and processes

Students will also use addition in a problem-solving context.

ACTIVITY

This activity contains a series of problems that seem relatively simple on the surface but that have some challenges embedded in them. For example, to find solutions to questions **1** and **2**, the students will have to calculate the income and total number of items sold by each of the five classes.

Another challenge is that the problems do not have simple one-step solutions. As in many problem-solving questions, the information needed for a calculation is not always immediately obvious. For example, to calculate the fund-raising effort of Room 7, the students will have to keep in mind that the selling price for the pegs is based on a packet, whereas the quantity sold is listed in boxes. The same is true for the sponges and buckets.

Questions **3** and **4** show that problems can often have more than one solution. The students may find it interesting to hear the solutions that others in the class have come up with and to consider whether those solutions are justified.

One extension to this activity, which illustrates how solutions to problems can vary depending on how aspects of the problem are *defined*, is as follows:
"The owners of a house want to pave the driveway at their new home. The driveway is 20 metres long and 5 metres wide. How many pavers are the owners likely to need, and what cost would they be looking at?"

Solutions will vary according to things such as:

- whether the owners wish to pave the whole width or perhaps just a 3 metre section down the middle (leaving a 1 metre strip either side in grass);
- the size, thickness, and design of the pavers;
- the cost of different pavers, and indeed the cost of the same pavers from different outlets. (They are likely to be cheaper at a manufacturer's yard than at a retailer's premises.)

The extension problem need not be actually solved. You could use it just to explore and discuss the issues involved.

Page 18 Gentle Giants

Achievement Objectives

- explain the meaning of the digits in decimal numbers with up to 3 decimal places (Number, level 3)
- report the distinctive features (outliers, clusters, and shape of data distribution) of data displays (Statistics, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

Other mathematical ideas and processes

Students will also:

- explore the magnitude of decimal numbers to thousandths
- use subtraction, addition, and division
- apply the statistical concept of "mean".

ACTIVITY

For question **1**, the students will need to know how to order decimals and also how to find the difference between two decimal numbers (in this case, the claimed and actual heights).

The students could measure out on the classroom wall the actual heights of the tallest and shortest giants (Zang Jinlian and Paul Hencock respectively). They would then need consider what the ".459 metres" part of Zang Jinlian's height means and what the ".184 metres" part of Paul Hencock's height means.

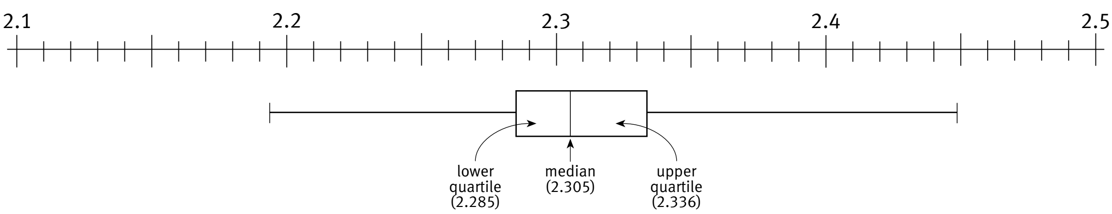
As an extension, you could ask the students to display the data on a stem-and-leaf graph or on a box-and-whisker graph. They could use their graphs to find the median, compare this to the mean, and consider why the two differ.

Stem-and-leaf graph:

2.1 84
2.2 09, 21, 22, (85), 86, 97, 98, 99 (lower quartile)
2.3 11, 12, 17, 35, (36), 37, 43, 62 (upper quartile)
2.4 59

The median here works out at 2.305. It is halfway between 2.299 and 2.311, the two middle scores.

Box-and-whisker graph:



The box-and-whisker graph shows that the giants range in height from 2.184 metres to 2.459 metres and that the height of the 50 percent of giants between the lower quartile and the upper quartile is within the range 2.285 metres to 2.336 metres, a range of just 51 millimetres (or 5.1 centimetres). The other advantage in constructing a box-and-whisker graph is that it requires the students to consider in some depth what heights such as 2.285 metres, 2.305 metres, and 2.336 metres mean in order to locate them on the metre number line.

A possible extension of question **2** is to investigate the mean height of the students in the class, how this differs from the median height, why the median may be a more useful indicator of the average height (it is not affected by extremes [outliers] in the way that a mean score is), and the range of heights for the 50 percent of students who lie between the lower and upper quartiles.

Page 19 Muffin Mania

Achievement Objectives

- express quantities as fractions or percentages of a whole (Number, level 4)
- express a fraction as a decimal, and vice versa (Number, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

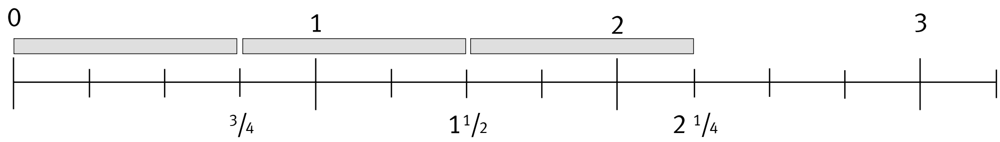
Other mathematical ideas and processes

Students will also:

- look at proportion
- add and multiply fractions
- use logic and reasoning in problem solving.

ACTIVITY

Encourage the students to solve these questions mentally. Materials such as strips of card on a number line could be useful. For example, the following shows that two lots of $^{3}/_{4}$ is $1^{1}/_{2}$ and three lots of $^{3}/_{4}$ is $2^{1}/_{4}$.



Alternatively, two lots of $^{3}/_{4}$ is $^{6}/_{4}$, and $^{6}/_{4}$ is $1^{2}/_{4}$ (or $1^{1}/_{2}$).

If the students don't know what the word "proportion" means, they could think of it as a fraction ("What *fraction* of the total is left?").

The second part of question **2**, expressing the proportion as a decimal fraction, may be represented on a double number line:

0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	$^{1}/_{10}$	$^{2}/_{10}$	$^{3}/_{10}$	$^{4}/_{10}$	$^{5}/_{10}$	$^{6}/_{10}$	$^{7}/_{10}$	$^{8}/_{10}$	$^{9}/_{10}$	1

The number line shows clearly that decimal fractions are numbers that come between consecutive whole numbers (in this instance between 0 and 1).

This may also be represented with Cuisenaire rods, with the orange rod being 1 and each white rod being $^{1}/_{10}$ or 0.1. (Thus, six whites would represent $^{6}/_{10}$ or 0.6.)

In question **4**, the students will have to figure out that 60 muffins is six times more than the number of muffins in the original recipe, so the ingredients will have to be multiplied by 6. Astute students may realise that an easier way is simply to double the results they got for question **3**, assuming that these results are accurate.

Skateboard Factory

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 4)
- find fractions equivalent to one given (Number, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

Other mathematical ideas and processes

Students will also:

- look at proportion
- use the identity principle
- multiply mentally.

ACTIVITY

This is an excellent activity to further develop your students' ideas about proportion. Double number lines are ideal for illustrating equivalent amounts. Some examples are given in the answers for this activity. You can also help the students to see that question **1** involves determining what amount out of 300 is equivalent to 1 out of 20. This can be written as $^1/_{20} = ^{\square}/_{300}$. The students could solve this problem in more than one way. For example, they could use the identity principle (encountered in some earlier activities): $^1/_{20} \times ^{\square}/_{\square} = ^{\square}/_{300}$, which is $^1/_{20} \times ^{15}/_{15} = ^{15}/_{300}$. In other words, $^1/_{20}$ is the same fractional number as $^{15}/_{300}$ because it has been multiplied by 1 in the form of $^{15}/_{15}$.

Alternatively, to retain the same proportion, the students could decide what has to happen to the 20 to get 1 (it needs to be divided by 20) and then apply the same operation to the 300, that is, 300 ÷ 20 = 15.

Using the identity principle, question **3** would look like this: $^8/_{1000} = ^{\square}/_{250}$. However, the students may understand this problem better if they reverse the order: $^{\square}/_{250} = ^8/_{1000}$, which then becomes $^{\square}/_{250} \times (^4/_4) = ^8/_{1000}$.

Encourage the students to do most of these calculations mentally and to use the written forms only to record their thinking.

Question **5** has some really thoughtful challenges in it, and it will be interesting to see which information ($^6/_{150}$ or $^{12}/_{300}$) the students prefer to use to solve **5a i** and **5a ii**. Questions **5a iii** and **5b** require even more thought, that is, $^6/_{150} = ^{\square}/_{250}$ and $^6/_{150} = ^8/_{\square}$. A really good strategy here would be to use the identity principle to simplify the $^6/_{150}$ to $^2/_{50}$ first (by dividing by 1 in the form of $^3/_3$) and then to reframe the problems as $^2/_{50} = ^{\square}/_{250}$ and $^2/_{50} = ^8/_{\square}$ respectively.

Page 21 Pick My PIN

Achievement Objectives

- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

Other mathematical ideas and processes

Students will also:

- work with factorials
- use multiplication
- use logic and reasoning in problem solving.

ACTIVITY

In this activity, the students determine the number of possible combinations of coloured pegs. It is similar in nature to the activity on page **4** of the students' book in that question **1** is a problem involving factorials.

The students may have strategies of their own (for example, a tree diagram) that they wish to try. Having six different-coloured pegs on hand would be helpful because the students will be able to see that there are six possible choices for the first peg in the code; five for the second peg (since one peg has already been allocated to the code); four for the third peg; and three possible choices for the fourth and last peg. The students could tabulate these choices in various ways, but in the end, they would find it helpful if they recognise that factorials are the most powerful and elegant approach, that is, 6 x 5 x 4 x 3 = 360.

Question **2** presents an additional challenge because here the students need to see that not only are there six possible choices for the first peg but six for each of the remaining three pegs as well; in short, the choices are 6 x 6 x 6 x 6, or 6^4, which is 1 296.

GAME

This game uses the ideas involved in the activity above. The students may ask about and wish to investigate the number of possible combinations in a four-digit code that uses all the 10 digits from 0 to 9; it would be 10^4 = 10 000.

Page 22 Percentage Passes

Achievement Objectives

- express quantities as fractions or percentages of a whole (Number, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

Other mathematical ideas and processes

Students will also:

- find the mathematical mean
- round two-place decimals to whole numbers
- calculate mentally.

ACTIVITY

Question **1** asks the students to compare fractions that have different denominators. The students will find these difficult to compare unless they convert the marks to percentages, as is suggested in the hint given on the page in the students' book, or convert the marks to fractions with the same denominator.

Percentages are just decimals expressed as a fraction of 100. Thus, taking Hine's English mark of 17 out of 20 as an example, the students can use a calculator to find what $^{17}/_{20}$ is as a decimal (it is 0.85) and then look at this as a fraction of 100 (it is $^{85}/_{100}$), which in turn is 85 percent. With calculators that have a % button (and most do), the students can key in [17] [÷] [20] [%] and get 85 immediately.

Alternatively, the students could use the identity principle to convert the marks to fractions with the same denominator. For example, $^{17}/_{20} = ^{\square}/_{100}$ becomes $^{17}/_{20} \times ^{5}/_{5} = ^{85}/_{100}$. Encourage the students to use this approach because:

i. it helps them to see that percent really does mean "per hundred";
ii. it enables many of the tasks in this activity to be done mentally (probably all those except the marks out of 30; even out of 40 is not too tough as there are $2^{1}/_{2}$ lots of 40 in 100, so $2^{1}/_{2}$ lots of 32 will give the appropriate percentage);
iii. it helps develop good number sense.

You may wish to discuss with the students whether Hine's and Josh's mean percentage marks should be rounded to whole numbers (85.25 to 85, and 83.75 to 84 respectively). The students may be able to come up with examples of situations where a mean may quite realistically not be a whole number.

In question **3**, the students need to recalculate the science percentages and also the mean percentages for each person. Part of the problem solving that is required involves the students working out that this recalculating is what they need to do.

Page 23 Egyptian Fractions

Achievement Objectives

- find fractions equivalent to one given (Number, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

Other mathematical ideas and processes

Students will also:

- apply the identity principle
- add fractions
- recognise that a variety of solutions is possible for some problems.

ACTIVITY

To tackle the first problem in this activity, the students need to convert most of the fractions into fractions with the same denominator (see the notes for page **22** of the students' book). You may need to discuss with them how to find the lowest common denominator. They will need to convert the $^{1}/_{2}$ and $^{1}/_{4}$ in question **1c** into the equivalent number of eighths, and in question **1d**, they will need to convert the $^{1}/_{2}$ and $^{1}/_{3}$ into the equivalent number of sixths.

Again, the mathematical principle involved is the identity principle. For example $^{1}/_{2} = ^{\square}/_{6}$ can be worked out as $^{1}/_{2} \times (^{3}/_{3}) = ^{3}/_{6}$, and $^{1}/_{3} = ^{\square}/_{6}$ can be multiplied by 1 in the form of $^{2}/_{2}$ like this: $^{1}/_{3} \times (^{2}/_{2}) = ^{2}/_{6}$. Since $^{3}/_{6}$ is equivalent to $^{1}/_{2}$ and $^{2}/_{6}$ is equivalent to $^{1}/_{3}$, $^{1}/_{2} + ^{1}/_{3}$ is the same as $^{3}/_{6} + ^{2}/_{6}$, which is $^{5}/_{6}$.

You may find that some of your students are still confused about the difference between multiplying and adding fractions. When they are multiplying fractions, they must multiply the numerators and the denominators, but when they are adding fractions, they add only the numerators. Spend time with any students who are not certain about what to do to ensure that they understand the processes. They will need to know these for later work in algebra. (The denominator is the fraction's name, for example, thirds. The numerator is the number of parts of that fraction, for example, the number of thirds, quarters, and so on.)

Questions **2** and **3** provide further opportunities for the students to experience the idea that several solutions are possible for some problems, as opposed to there always being just one right answer. In question **2a**, for example, the Egyptians could have written $^4/_5$ as $^1/_5 + ^1/_5 + ^1/_5 + ^1/_5$, $^1/_2 + ^1/_5 + ^1/_{10}$, or $^1/_2 + ^1/_{10} + ^1/_{10} + ^1/_{10}$, and so on. Encourage the students to share and discuss their various solutions.

Page 24 Lining Up Numbers

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

Other mathematical ideas and processes

Students will also:

- use addition
- apply the associative principle
- apply the commutative principle.

ACTIVITY

This activity gives the students valuable practice in adding columns of figures as well as thinking about useful ways of checking their answers.

The students could discuss the following:

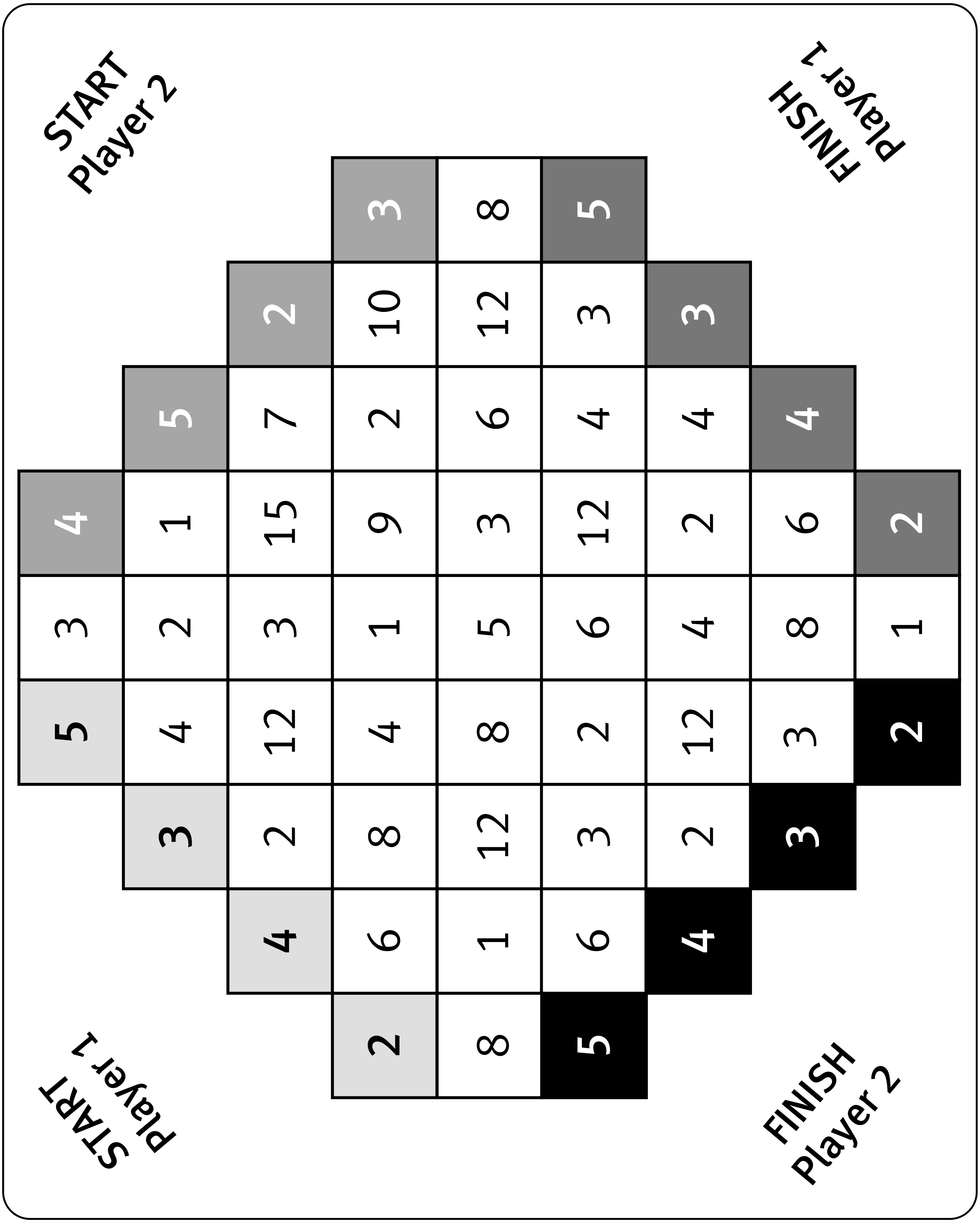
i. When adding up columns of digits, do they just add the numbers from the top to the bottom or do they look for numbers that they can easily add together? For example in the ones column in question **1a**, would they pick out 6 + 4 and 8 + 2?

ii. What strategies do they use to keep track of the digits they are adding when working with a calculator?

If the students do add columns of digits by "making tens", then you may wish to explore with them the mathematics principles that enable them to do so. Take the ones column in question **1a** again: if this were written out horizontally (beginning from the top), it would be 9 + (6 + 4) + 3 + (8 + 2) + 5. The brackets show the making tens strategy. Seen this way, it is really the *associative* (or grouping) principle at work. The *commutative* principle is also being used because the mental addition is actually (6 + 4) + 9 ... In other words, after associating or grouping 6 and 4, the order of the numbers has been changed.

These associative and commutative principles (or properties or laws) work for addition and multiplication but not for subtraction and division. The students may be able to suggest other more difficult instances where they could make use of these principles.

Operations Checker Game Board



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