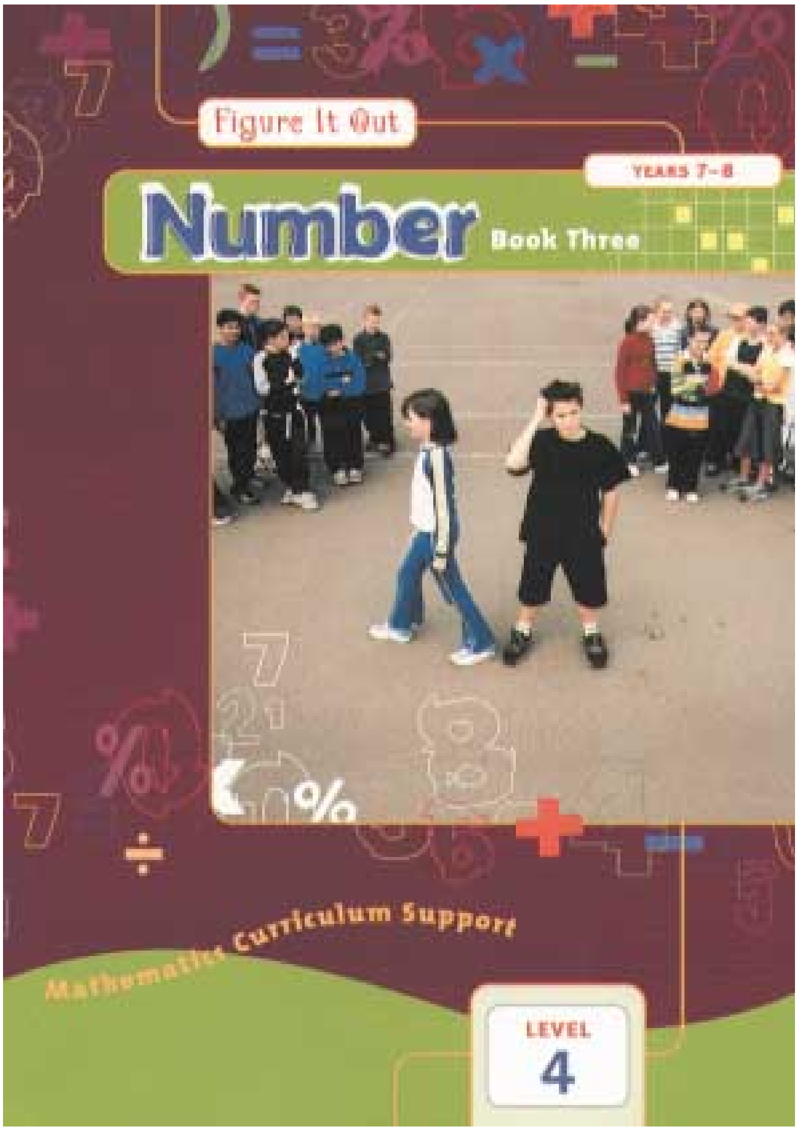
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

CONTENTS

The books for years 7–8 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand year 7–8 classrooms. The books have been developed and trialled by classroom teachers and mathematics educators and follow on from the successful series for levels 2–4 in primary schools.

Student books

The student books in the series are divided into three curriculum levels: levels 2–3 (linking material), level 4, and level 4+ (extension material). All the books are aimed at year 7–8 students in terms of context and presentation.

The following books are included in the series:

Number (two linking, three level 4, one level 4+)
Algebra (one linking, two level 4, one level 4+)
Measurement (one level 4, one level 4+)
Number Sense (one linking, one level 4)
Geometry (one level 4, one level 4+)
Statistics (one level 4, one level 4+)
Themes (level 4): *Disasters, Getting Around*

These 20 books will be distributed to schools with year 7–8 students over a period of two years, starting with the six *Number* books.

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and the life experiences that are meaningful to students aged 11–13 years. The activities can be used as the focus for teacher-led lessons, as independent activities, or as the catalyst for problem solving in groups.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the student books includes full answers and explanatory notes. Students can use them for self-marking, or you can use them for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, comments on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/community

Using Figure It Out in your classroom

Where applicable, each page starts with a list of equipment that the students will need to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Answers Number: Book Three

Page 1 Linking Lollies

ACTIVITY

a. Each set of four has one lime and three orange.
$^{1}/_{4}$ of 36 is 9, so she uses nine lime.
$^{3}/_{4}$ of 36 is 27, so she uses 27 orange.

b. Each set of five has two raspberry and three blueberry.
$^{2}/_{5}$ of 60 is 24, so she uses 24 raspberry.
$^{3}/_{5}$ of 60 is 36, so she uses 36 blueberry.

c. Each set of three has two blueberry and one orange.
$^{2}/_{3}$ of 24 is 16, so she uses 16 blueberry.
$^{1}/_{3}$ of 24 is 8, so she uses eight orange.

d. Each set of six has three lime, one raspberry, and two blueberry.
$^{3}/_{6}$ of 72 = $^{1}/_{2}$ of 72, which is 36, so she uses 36 lime.
$^{1}/_{6}$ of 72 is 12, so she uses 12 raspberry.
$^{2}/_{6}$ of 72 = $^{1}/_{3}$ of 72, which is 24, so she uses 24 blueberry.

Page 2 Conversion Cousins – Part 1

ACTIVITY

1. a. $28

b. Yes. The other presents cost $10 (Fergus's brother), $20 (Percy's dad), $15 (Dewey's dad), and $25 (Dewey's mum). This adds up to $70, leaving $30 for Grandad's present.

2.

	Fergus's brother	Percy's dad	Dewey's dad	Dewey's mum	Grandad	Total cost
Fergus	$^{1}/_{10}$	$^{20}/_{100}$ (or $^{1}/_{5}$)	$^{15}/_{100}$ (or $^{3}/_{20}$)	$^{25}/_{100}$ (or $^{1}/_{4}$)	$^{28}/_{100}$ (or $^{7}/_{25}$)	$^{98}/_{100}$ (or $^{49}/_{50}$)
Percy	10%	20%	15%	25%	28%	98%
Dewey	0.1	0.2	0.15	0.25	0.28	0.98
Cost	$10	$20	$15	$25	$28	$98

Page 3 Conversion Cousins – Part 2

ACTIVITY

1. a.

	Cheese	Banana	Chocolate	Raspberry	Blueberry	Apricot	Total
Fraction	$^{1}/_{4}$	$^{1}/_{5}$	$^{1}/_{10}$	$^{1}/_{3}$	$^{15}/_{100}$ (or $^{3}/_{20}$)	$^{5}/_{100}$ (or $^{1}/_{20}$)	$^{325}/_{300}$ (or $^{65}/_{60}$)
Percentage	25%	20%	10%	$33^{1}/_{3}$%	15%	5%	$108^{1}/_{3}$%
Decimal	0.25	0.2	0.1	$0.\dot{3}$	0.15	0.05	$1.08\dot{3}$
Number of muffins	15	12	6	20	9	3	65

Note: $^{108}/_{100}$ is equivalent to $^{65}/_{60}$

b. Five extra muffins

c. 15

2. 45 muffins

3. Answers will be similar to:
Fergus Fraction: "We got $^{1}/_{4}$ off."
Percy Percentage: "We got a 25% discount."
Dewey Decimal: "We got 0.25 off."

Page 4 Football Fractions

ACTIVITY

1. a. 2
   b. 2
2. a. 6
   b. 1
   c. 1
3. Bay Bruisers 38, Hokianga Hecklers 36

Page 5 Zax Tax

ACTIVITY ONE

1. 250 zax
2. a. 64 000 zax
   b. 16 000 zax
3. a. 50 000 zax
   b. Answers may vary. Someone who earned 49 000 zax would pay 24 010 zax in tax, leaving them 24 990 zax. Someone on a salary of 51 000 zax would pay 26 010 zax in tax, also leaving them 24 990 zax. (Take-home pay from 51 000 zax on becomes less and less.)

ACTIVITY TWO

1. 125 zax
2. 42 000 zax
3. 100 000 zax. On a salary of 100 000 zax, you would pay 50 000 in tax (50%). On a salary of 101 000 zax, you would pay 50.5% in tax, which is more than half.
4.

New salary	Tax rate	Tax	Your share
110 000	55%	60 500	49 500
120 000	60%	72 000	48 000
130 000	65%	84 500	45 500

Page 6 Involving Interest

ACTIVITY

1. Kaylene would get $1,771.56 from the Invercargill Investment Bank, $1,750.00 from the Titahi Bay Local Bank, and $1,760.91 from Martinborough Municipal Bank. (If you rounded these to the nearest whole dollar, the answers would be $1,772, $1,750, and $1,761 respectively.)
2. To get the most money back, Kaylene should bank her money with the Invercargill Investment Bank.

INVESTIGATION

Answers will vary.

Page 7 Frantic Fund-raising

ACTIVITY

1. a. $276.50
   b. $27.65
2. $1.50
3. a. 375
   b. $187.50
   c. $18.75
4. $128.60
5. a. 2 000
   b. $300
6. a. $691.50
   b. $69.15

Page 8 Crossing Out Singles

GAME

A game using addition and multiplication facts

Page 9 Cover Up

GAME

A game using order of operations

Page 10 Team Leaders

ACTIVITY

1. **a.** You could set out your answer like this:

Children	Groups	Remainder
50	8 x 6 = 48	2
48	6 x 7 = 42	6
42	8 x 5 = 40	2
40	6 x 6 = 36	4
36	4 x 8 = 32	4
32	3 x 9 = 27	5
27	3 x 7 = 21	6
21	3 x 6 = 18	3
18	3 x 5 = 15	3
15	1 x 8 = 8	7
8	1 x 5 = 5	3
5	The leaders!	

b. Answers will vary. One solution, with two fewer moves, is:

Children	Groups	Remainder
50	5 x 9 = 45	5
45	5 x 8 = 40	5
40	5 x 7 = 35	5
35	3 x 9 = 27	8
27	3 x 7 = 21	6
21	2 x 8 = 16	5
16	2 x 6 = 12	4
12	1 x 8 = 8	4
8	1 x 5 = 5	3
5	The leaders!	

2. Answers will vary. One solution is:

Children	Groups	Remainder
50	6 x 8 = 48	2
48	5 x 9 = 45	3
45	5 x 8 = 40	5
40	6 x 6 = 36	4
36	4 x 8 = 32	4
32	3 x 9 = 27	5
27	6 x 4 = 24	3
24	4 x 5 = 20	4
20	2 x 7 = 14	6
14	2 x 5 = 10	4
10	3 x 3 = 9	1
9	1 x 6 = 6	3
6	1 x 4 = 4	2
4	The leaders!	

3. Answers will vary. One solution is:

Children	Groups	Remainder
40	5 x 7 = 35	5
35	5 x 6 = 30	5
30	3 x 8 = 24	6
24	4 x 5 = 20	4
20	2 x 7 = 14	6
14	2 x 5 = 10	4
10	1 x 7 = 7	3
7	1 x 5 = 5	2
5	The leaders!	

Page 11 Purchasing Payments

ACTIVITY

1. **a.** The Loud Noise Company offers the best time-payment deal (option 1). (25% deposit and $27.42 per month for 12 months = $435.29.)

   b. It is cheaper than the others by:
   $56.11 (Base Control Ltd, $491.40, option 1)
   $34.12 (Base Control Ltd, $469.41, option 2)
   $59.71 (Booms, $495, option 1)
   $51.01 (Booms, $486.30, option 2)
   $84.31 (Loud Noise Company, $519.60, option 2).

2. Various answers, as long as they are close to $435.29 in total (to be the best deal). For example, no deposit with $12.10 per month for 36 months is $435.60, or 10% deposit ($43.50) and $16.31 per month for 23 months, with a final payment of $16.37, is $435.00.

Page 12 Going for Gold!

ACTIVITY

1. Lane 1 James
   Lane 2 Pete
   Lane 3 Bill
   Lane 4 Anaru
   Lane 5 Matiu
   Lane 6 Tama
   Lane 7 Lain
   Lane 8 Karl

2. **a.** Levin (116.547)
   Hastings (117.268)
   Gisborne (118.030)
   Hamilton (118.754)
   Manakau (119.183)
   Taupō (119.639)
   Pakuranga (120.219)
   Bulls (122.420)

   b. They had to make up more than 1.483 seconds to win. (For example, they would win if they swam 1.484 seconds faster than Levin over the last two legs.)

Page 13 Awesome Athletes

ACTIVITY

1. **a.**

Place	100 m (s)	400 m (s)	Long jump (m)	High jump (m)
1st	Jane 14.84	Jane 80.07	Nikki 4.67	Sarah 1.32
2nd	Nikki 15.02	Sarah 80.15	Mere 4.60	Nikki 1.30
3rd	Olive 15.06	Leilani 80.60	Jane 4.54	Mere 1.29
4th	Sarah 15.12	Nikki 81.30	Olive 4.15	Leilani 1.27
5th	Leilani 15.35	Olive 81.35	Sarah 4.06	Jane 1.26
6th	Mere 16.10	Mere 82.41	Leilani 3.98	Olive 1.19

   b. Jane and Nikki are first equal (7 points each).

2. Nikki would be the champion. The results would look like this:

Nikki	5 + 3 + 6 + 5	=	19
Jane	6 + 6 + 4 + 2	=	18
Sarah	3 + 5 + 2 + 6	=	16
Mere	1 + 1 + 5 + 4	=	11
Olive	4 + 2 + 3 + 1	=	10
Leilani	2 + 4 + 1 + 3	=	10

3. Answers will vary. Two suggestions are:
   - 1st = 10, 2nd = 6, 3rd = 4, 4th = 3, 5th = 2, 6th = 1 (This would mean Jane had 26 points and Nikki 25.)
   - Points are awarded only to the first four places: 1st = 10, 2nd = 6, 3rd = 4, 4th = 3. (In this case, Nikki would have 25 and Jane 24.)

Page 14 New Car Capers

ACTIVITY

1. **a.** End of 1st year: $24,312
   End of 2nd year: $19,693
   End of 3rd year: $17,527
   End of 4th year: $16,475
   End of 5th year: $15,157

   b. 1st year: $640 (rounded) per month
   2nd year: $385 per month
   3rd year: $181 per month
   4th year: $88 per month
   5th year: $110 per month

2. Answers will vary. Teacher to check

Page 15 PE Problems

ACTIVITY

1. **a.** Team A: 2
   Team B: 8
   Team C: 4
   Team D: 6

   b. 20

2. A + B and C + D

3. **a.** Team A: 4
   Team B: 10
   Team C: 5
   Team D: 8

   b. No. (27 is an uneven number.)

4. 34 if you are allowed two teams of 6 (Team A: 6; Team B: 12; Team C: 6; Team D: 10)

   41 if each team needs to have a different number of students (Team A: 8; Team B: 14; Team C: 7; Team D: 12)

Page 16 Egyptian Multiplication

ACTIVITY

1. a.

Doubling	Multiple of 48
48	1
96	**2**
192	4
384	8
768	**16**
1 536	32

96 + 768 = 864 (18 x 48 = 864)

b.

Doubling	Multiple of 745
745	**1**
1 490	**2**
2 980	**4**
5 960	8
11 920	**16**
23 840	32

745 + 1 490 + 2 980 + 11 920 = 17 135
(23 x 745 = 17 135)

2. Answers will vary.

3. The Egyptian method works by spreading out (distributing) the multiplication and then adding the products. For example, to work out 24 x 12, they would spread out the 12 into 4 + 8:

   12 x 24 = 24 x 12
   = 24 x (4 + 8)
   = (24 x 4) + (24 x 8)
   = 96 + 192
   = 288

   To work out 23 x 745, they would spread out the 23 into 1 + 2 + 4 + 16:

   23 x 745 = 745 x 23
   = 745 x (1 + 2 + 4 + 16)
   = 745 x 1 + 745 x 2 + 745 x 4 + 745 x 16
   = 745 + 1 490 + 2 980 + 11 920
   = 17 135

Page 17 Kapa Haka Hāngi

ACTIVITY

1. a.–b. (Figures are rounded to 1 d.p.)

	50 people	70 people
Potatoes	20 kg	28 kg
Pumpkin	5 large	7 large
Cabbage	10 large	14 large
Kūmara	13.3 kg	18.7 kg
Lamb	15 kg	21 kg
Pork	8.3 kg	11.7 kg
Chicken	11.7 kg	16.4 kg (need to round up to have enough)

2. a. Answers will vary.

   b. Answers will vary.

Page 18 Digit Challenge

ACTIVITY ONE

61

ACTIVITY TWO

There appears to be only one possible solution:
3 8 1 6 5 4 7 2 9

Page 19 Tasty Treats

ACTIVITY

1. 250 g butter
   $^2/_3$ cup honey
   1 cup brown sugar
   $1^1/_2$ cups wholemeal self-raising flour
   $1^1/_2$ cups plain flour
   $^1/_2$ tsp coriander
   1 tsp cinnamon
   $^1/_2$ tsp nutmeg
   2 pinches ground cloves
   $^1/_2$ cup raisins
   $^1/_2$ cup almonds

2. 1.25 kg butter
   $3\frac{1}{3}$ cups honey
   5 cups brown sugar
   $7\frac{1}{2}$ cups wholemeal self-raising flour
   $7\frac{1}{2}$ cups plain flour
   $2\frac{1}{2}$ tsp coriander
   5 tsp cinnamon
   $2\frac{1}{2}$ tsp nutmeg
   10 pinches ground cloves
   $2\frac{1}{2}$ cups raisins
   $2\frac{1}{2}$ cups almonds
3. **a.** $19.50

   b. $97.50 if no savings were made from buying larger quantities of ingredients. If savings were made, as is likely, the profit would increase.

Pages 20–21 Cycling On ...

ACTIVITY ONE

1. **a.–b.**

Stage	Time taken (min.)	Distance (km)
1	8	3.6
2	8	3.6
3	13	4.7
4	10	4.3
5	4	1.2
6	16	7.4
7	21	7.2

2. 3 hrs
3. **a.** 80 min. (1 hr, 20 min.)

   b. $\frac{4}{9}$
4. **a.** 24 km/h

   b. Stage 6

   c. Answers will vary. Possible answers could include the impact of tiredness, hills, gravel roads, corners, and compulsory stops.

ACTIVITY TWO

1. 24.6 km/h, which rounds to 25 km/h
2. **a.** 6.5 km

   b. 22.9 km/h, which rounds to 23 km/h
3. 23.8 km/h, which rounds to 24 km/h

ACTIVITY THREE

1. 11.47 a.m.
2. **a.** 23 min.

   b. 21.39 km/h, which rounds to 21.4 km/h (1 d.p.)

ACTIVITY FOUR

1. **a.** 10.31 a.m.

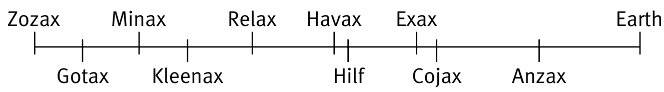
   b. 18.57 km/h, which rounds to 19 km/h
2. 17.64 km/h, which rounds to 18 km/h
3. **a.** Group A (6 km/h faster)

   b. $33\frac{1}{3}$% faster, which rounds to 33% (to the nearest whole number)
4. Answers will vary. Group A ride further than the other two groups (this could be considered an advantage or a disadvantage). Group C were 13 minutes slower than group A over the same route, so biking at the end of the camp may have been more tiring than biking at the beginning.

Page 22 Exploration to Earth

ACTIVITY

1. **a.** A space line might look like this:



   b. Taking each distance from Zozax away from 1 000 000 gives the distance of each space stop from Earth:

Anzax	188 000 kz
Cojax	359 250 kz
Exax	391 014 kz
Gotax	916 743 kz
Kleenax	750 962 kz
Havax	506 975 kz
Minax	826 531 kz
Relax	664 856 kz
Hilf	500 000 kz

2. **a.** The distances between space stops are:
   Zozax–Gotax: 83 257 kz
   Gotax–Minax: 90 212 kz
   Minax–Kleenax: 75 569 kz
   Kleenax–Relax: 86 106 kz
   Relax–Havax: 157 881 kz
   Havax–Hilf: 6 975 kz
   Hilf–Exax: 108 986 kz
   Exax–Cojax: 31 764 kz
   Cojax–Anzax: 171 250 kz
   Anzax–Earth: 188 000 kz

b. The number of sleeps can be worked out by finding how many 25 000 kz fit into the distance from one space stop to the next. (Any part distance would mean a sleep or a break at the next space stop.)
Zozax–Gotax: 3 sleeps (83 257 kz)
Gotax–Minax: 3 sleeps (90 212 kz)
Minax–Kleenax: 3 sleeps (75 569 kz)
Kleenax–Relax: 3 sleeps (86 106 kz)
Relax–Havax 6 sleeps (157 881 kz)
Havaz–Hilf: 0 sleeps (6 975 kz)
Hilf–Exax: 4 sleeps (108 986 kz)
Exax–Cojax: 1 sleep (31 764 kz)
Cojax–Anzax: 6 sleeps (171 250 kz)
Anzax–Earth: 7 sleeps (188 000 kz)

3. Minax, Relax, Hilf, Cojax, Anzax

Page 23 Orchard Antics

ACTIVITY

1. The apple trees are planted 1 metre in from the fence in a horizontal and vertical direction. They are planted 2 metres apart, horizontally and vertically.

   The pear trees are planted 2 metres in from the fence in a horizontal and vertical direction. They are planted 4 metres apart, horizontally and vertically.

2. a. i. Using the planting rules from question **1**, they could plant 468 apple trees (18 rows of 26 trees per row).

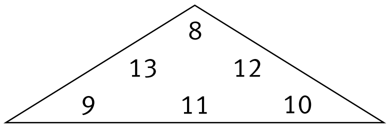
   ii. Using the planting rules from question **1**, they could plant 117 pear trees (9 rows of 13 trees per row).

   b. Using the planting rules from question **1**, there are two possible answers, depending on whether the land is divided in half vertically (that is, into two pieces 26 m x 36 m) or horizontally (that is, into two pieces 52 m x 18 m). For the vertical split, they could plant 288 trees: 234 apple trees (13 rows of 18 trees per row) and 54 pear trees (6 rows of 9 trees per row). For the horizontal split, they could plant 286 trees: 234 apples (9 rows of 26 trees per row) and 52 pears (4 rows of 13 trees per row).

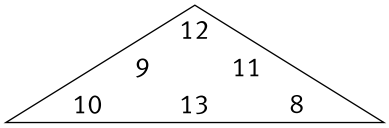
Page 24 Dive Tank Dilemma

ACTIVITY ONE

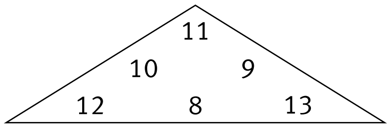
1. A possible arrangement is:



2. A possible arrangement is:

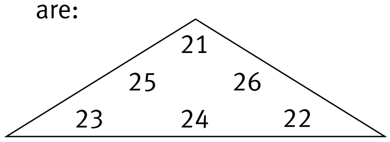


3. A possible arrangement is:

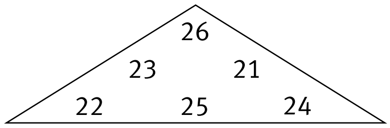


4. a. 21, 22, 23, 24, 25, 26

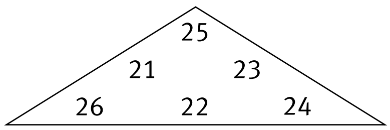
   b. Answers will vary. Some possible arrangements are:



(side totals of 69)



(side totals of 71)

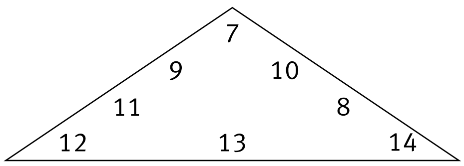


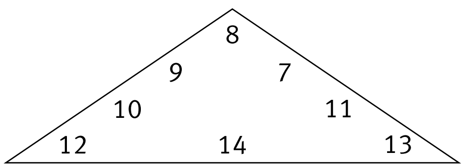
(side totals of 72)

ACTIVITY TWO

a. 7, 8, 9, 10, 11, 12, 13, 14

b. Two possible arrangements are:





Teachers' Notes

Overview Number: Book Three

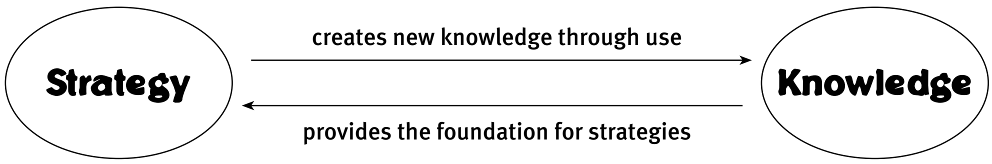
Introduction to Number

There is a remarkable commonality in the way many countries around the world are now teaching arithmetic. Changes in the approaches reflect the evolving demands of everyday life, a greater volume of classroom-based research about how students learn, and a desire to improve general levels of numeracy.

In the past, arithmetic teaching has focused on preparing students to be reliable human calculators. The prevalence of machines in society that calculate everything from supermarket bills to bank balances has meant that students now require a wider range of skills so that they can solve problems flexibly and creatively.

The Figure It Out series aims to reflect these trends in modern mathematics education. A range of books is provided at different levels to develop both number skills and number sense. The *Number* books are aimed at developing students' understanding of the number system and their ability to apply efficient methods of calculation. The *Number Sense* books are aimed at developing students' ability and willingness to apply their number understanding to make mathematical judgments. Teaching number sense requires an emphasis on openness and flexibility in solving problems and the use of communication and interpretation skills.

The development of the Figure It Out series has occurred against the backdrop of a strong drive for improved standards of numeracy among primary-aged students. A key element of this drive has been the creation of the Number Framework, developed as part of the Numeracy Strategy. The framework highlights this significant connection between students' ability to apply mental strategies to solving number problems and the knowledge they acquire.



Learning activities in the series are aimed both at developing efficient and effective mental strategies and at increasing the students' knowledge base. Broadly speaking, the levels given in the six year 7–8 *Number* books can be equated to the strategy stages of the Number Framework in the following way:

Link (Book One):	Advanced counting to early additive part-whole
Link (Book Two):	Advanced additive part-whole
Level 4 (Books Three to Five):	Advanced multiplicative to advanced proportional part-whole
Level 4+ (Book Six):	Advanced proportional part-whole.

Page 1 **Linking Lollies**

Achievement Objectives

- find fractions equivalent to one given (Number, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)
- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, level 4)

Other mathematical ideas and processes

Students will also use algebraic thinking to identify patterns.

ACTIVITY

In this activity, the students need to recognise a pattern and then use fractions to describe the pattern.

Some students may need to use multilink cubes, counters, or diagrams to help them identify the pattern, for example, LOOOLOOOLO ..., with L representing a lime lolly and O representing an orange lolly. A table such as the one below may also help.

Number of patterns	Number of lime	Number of orange	Total lollies	Diagram
1	1	3	4	LOOO
2	2	6	8	LOOOLOOO
3	3	9	12	LOOOLOOOLOOO
4				
⋮				

By completing the first part of the table, the students should realise that the number of lime lollies is the same as the number of patterns, the number of orange lollies matches the answers to the three times table, the total number of lollies matches the four times table, and so on. Once they have made these connections, the students can consider short cuts or quicker, more efficient ways of working. You could encourage them with comments such as " Do you need to draw a diagram for the next one, or can you figure it out?" From the table, they can see that one out of every four (that is, $^1/_4$) of the lollies are lime and three out of every four (that is, $^3/_4$) of the lollies are orange. Students who have trouble finding one-quarter of 36 need help seeing the connections between 36 divided by 4, $^1/_4$ of 36, sharing 36 into four equal groups, and 4 x $\boxed{}$ = 36.

At the end of the lesson, have the students share and discuss their methods of working. Students generally benefit from explaining their own methods and listening to other students explain theirs. You could set follow-up activities so that the students could try out the other methods. They could then discuss which is the most efficient method for them.

Other ways of extending this activity could include:

- developing more ula for the students to work out
- having the students develop an ula for their classmates to work out
- asking the students "What if ..." questions, such as:
  "What if the same pattern were used for an ula twice as long: how many lollies of each flavour would you need?"
  "What if the ula were 100 lollies long?"
  "What if there were 60 lime lollies: how many orange lollies would you need?"
  "What if the same number of each type of lolly were used: what other patterns could you make?"

Page 2 Conversion Cousins – Part 1

Achievement Objectives

- express a fraction as a decimal, and vice versa (Number, level 4)
- express a decimal as a percentage, and vice versa (Number, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)

Other mathematical ideas and processes

Students will also combine quantities expressed in different forms (for example, fractions, decimals, and percentages).

ACTIVITY

To do this activity, students need to be familiar with decimals, fractions, and percentages and the relationship between them.

Students who have trouble with this might find it helpful to complete a table like the one following. Encourage them to use what they already know as the starting point, such as 50% and $^{1}/_{10}$. They can use the completed table to find the answers, although the challenge always needs to be "Can you work it out without looking at the table?"

Percentage	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Decimal	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Fraction	$^{0}/_{10}$	$^{1}/_{10}$	$^{2}/_{10}$	$^{3}/_{10}$	$^{4}/_{10}$	$^{5}/_{10}$	$^{6}/_{10}$	$^{7}/_{10}$	$^{8}/_{10}$	$^{9}/_{10}$	$^{10}/_{10}$

The students also need to understand how to add amounts in different forms. For example, to answer question **1b**, they need to work out $^{1}/_{10}$ + 20% + 0.15 + 0.25. Discuss with the students how they might do this. Some may prefer to convert all the amounts to fractions with a common denominator ($^{10}/_{100} + {}^{20}/_{100} + {}^{15}/_{100} + {}^{25}/_{100}$), some may convert them all to percentages (10% + 20% + 15% + 25%), and others may convert them all to decimals (0.1 + 0.2 + 0.15 + 0.25).

Page 3 Conversion Cousins – Part 2

Achievement Objectives

- express a fraction as a decimal, and vice versa (Number, level 4)
- express a decimal as a percentage, and vice versa (Number, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)

Other mathematical ideas and processes

Students will also combine quantities expressed in different forms (for example, fractions, decimals, and percentages).

ACTIVITY

For question **1**, the students need to be able to find a fraction, a decimal, and a percentage of a whole number to complete the table and find the number of muffins each family ordered.

There are several ways to find the number of muffins. The students can choose whether to use fractions, ($^{1}/_{4}$ of 60), percentages (25% of 60), or decimals (0.25 of 60).

The following examples show how the students could approach solving these problems.

Example 1: What is 25% of 60?
The student might know that 50% is $^1/_2$ or 0.5. Therefore 50% of 60 is 30, the same as half of 60. Also, half of 50% is 25%, so 25% of 60 is half of 30, that is, 15. In this example, the students use their knowledge of doubles and halves.

Example 2: What is $^1/_{10}$ of 60?
The students might know that to find $^1/_{10}$ of any number, you divide the number by 10. So $^1/_{10}$ of 60 is the same as sharing 60 evenly between 10 groups, 60 ÷ 10, or 10 x $\square$ = 60.

This might be a good time to ask the students to look for a short cut for finding 10% of any number. If they can figure it out for themselves, they are much more likely to understand it, remember it, and use it. Reinforce this by getting them to look for a short cut to help them find the answers to calculations such as: 10% of 50, 10% of 90, 10% of 120, 10% of 470, 10% of 2 360, and 10% of 10. Don't forget to include numbers that involve decimals, such as 10% of 8, 10% of 12, 10% of 56.4, 10% of 4.70, 10% of 0.2, and 10% of 12.34.

Example 3: What is $^1/_3$ of 60?
The students might know that $^1/_3 + {}^1/_3 + {}^1/_3 = 1$ (or a whole or all of it). Following on from this, the whole, or 60, is divided up into three equal parts. Therefore we are looking for $\square + \square + \square$ = 60. 20 + 20 + 20 = 60, so $^1/_3$ of 60 is 20. So if $^1/_3$ of 60 is 20, $^2/_3$ of 60 is 40 (two lots of 20), $^3/_3$ of 60 is 60 (three lots of 20), $^4/_3$ of 60 is 80 (four lots of 20), and so on.

Question **2** will help the students to see and understand the connection between fractions and ratios. The problem can be represented and worked out as a ratio or as a fraction. In the ratio approach, 3:1 means that for every three muffins, one is free. This gives four in total, three paid for and one free. If six muffins are paid for, then a further two are free, a ratio of 6:2, which is equivalent to 3:1. If 12 muffins are paid for, then four more are free, a ratio of 12:4, which is equivalent to 3:1 and 6:2. This method of doubling is a good way to help the students to see the equivalent nature of some ratios. However, in this particular case, doubling does not lead to the required answer of 60 muffins, so the students need to do some further thinking. A table such as the one below clearly shows the relationships and how they lead to too many muffins.

Ratio	Paid for	Free	Total number
3:1	3	1	4
6:2	6	2	8
12:4	12	4	16
24:8	24	8	32
48:16	48	16	64

If you subtract one group of four muffins (three paid for and one free), you get to the desired total number, 60. 48 – 3 = 45 and 16 – 1 = 15, so 45:15 is the equivalent ratio with a total of 60. So if you pay for 45 muffins, you get 15 free.

Another way of thinking is to see how many groups of four are in the 60 muffins: 60 ÷ 4 = $\square$, $\square$ x 4 = 60, or "If 60 is divided into groups of four, how many groups of four would there be?" There are 15 groups of four in 60, therefore 15 are free (one out of each group of four) and there are 45 to be paid for (three out of every group of four, and there are 15 groups, so 15 x 4 = 45 + 15).

In terms of the special offer, you could discuss with the students how many muffins the cousins would have paid for if they had filled each family's initial requests (47).

Page 4 Football Fractions

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)

Other mathematical ideas and processes

Students will also use basic multiplication facts and algebraic thinking to find unknown variables.

ACTIVITY

This activity challenges the students to use a variety of skills and strategies. First, they need to find fractions of amounts, and then they have to use basic multiplication facts, algebraic thinking, and problem-solving strategies to find out how many points were from converted and unconverted tries, penalties, and drop goals.

Students at this level should have several strategies for finding fractions of amounts. For example, to find two-thirds of the Hecklers' score, they could divide 36 by 3 and then multiply the result by 2. Or to find three-fifths of the Bruisers' score, they might convert three-fifths to 0.6 and then multiply 30 by 0.6.

For question **1**, they'll find that two-thirds of 36 is 24. This is made up of converted tries (seven points each) and unconverted tries (five points each). So the students need to work out what combination of sevens and fives will make 24. They will probably do this using trial and improvement. If there is one converted try (worth seven points), the remaining 17 points must come from unconverted tries (worth five points each). Five does not divide evenly into 17, so this can't be the answer. If there are two converted tries (2 x 7 = 14), the remaining 10 points can come from two unconverted tries (2 x 5 = 10). Two converted and two unconverted tries gives the correct answer. The students will find that there are no other correct answers.

A table is a useful way of recording trial and improvement.

Tries				Total
Unconverted		Converted		
No.	Points	No.	Points	
–	–	3	21	21
1	5	2	14	19
2	10	2	14	24 ✓
3	15	1	7	22
4	20	–	–	20

They can use similar strategies to answer questions **2** and **3**.

Page 5 Zax Tax

Achievement Objectives

- express quantities as fractions or percentages of a whole (Number, level 4)
- effectively plan mathematical exploration (Mathematical Processes, problem solving, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)

Other mathematical ideas and processes

Students will also use problem-solving strategies.

For the activities on this page, the students have to make multiple calculations involving percentages and large numbers. They need to clearly organise the information about the zax taxes so that they can see patterns that may help them to answer the questions.

ACTIVITY ONE

You will probably need to have a discussion with the students before they start this activity. They need to understand:

- the concept of taxes, including the difference between salary before and after tax
- how to work out percentages of large numbers
- the need to have an organised way of recording the information they work out.

A table such as the one below would be a good way of setting out their work.

Salary	Percentage in tax	Tax payable	After-tax income	Check
$40,000	40%	$16,000	$24,000	$16,000 + $24,000 = $40,000
$80,000	80%			

Students who have difficulty seeing any pattern with the taxes and salaries or who struggle to understand what is happening as the salaries get higher may need to organise the salaries into numerical order, for example, $0, $10,000, $20,000, $30,000, $40,000, and so on to $100,000, and work each one out separately.

Salary	Percentage in tax	Tax payable	After-tax income	Check
$0	0%	$0	$0	$0 + $0 = $0
$10,000	10%	$1,000	$9,000	$1,000 + $9,000 = $10,000
$20,000	20%	$4,000	$16,000	$4,000 + $16,000 = $20,000
$30,000	30%			
$40,000	40%			
$50,000				
⋮				

A table like this helps the students to find the answer to question **3a**. They need to realise that the amount of after-tax income doesn't continue to increase. They also need to realise that, for all before-tax incomes over $50,000, the after-tax income actually decreases. You could ask the more able students to try to estimate the answer to question **3a** using logic and reasoning instead of a table. They should see that for before-tax incomes of over $50,000, people are paying more than 50% (that is, more than half their income) in tax and so are paying more in tax than they are getting in after-tax income. So, people with before-tax incomes of over $50,000 (who are paying more than half their income in tax) are unlikely to be left with more money than other workers after they have paid tax.

If the students are comfortable working out the after-tax income by calculating the amount of tax paid and then subtracting this from the before-tax salary, challenge them to work out the after-tax income in just one calculation. For example, if an official earns $80,000 before tax and pays 80% tax, their after-tax income can be calculated by finding 20% of $80,000. In order to understand this, the students need to understand that 80% plus 20% is 100%, which is one whole. So if they take 80% away from the whole, they are left with 20%.

You could use this activity to teach or reinforce how to use a calculator to find a percentage. For 80% of $80,000, on most basic calculators you key in $\boxed{80000}$ $\boxed{x}$ $\boxed{80}$ $\boxed{\%}$. You could also convert the 80% to 0.8 and key in $\boxed{80000}$ $\boxed{x}$ $\boxed{0.8}$ $\boxed{=}$

ACTIVITY TWO

This activity is similar to the first activity, and the same knowledge and organisation of information is needed.

The following questions could be asked at the beginning of the activity to get the students thinking and looking for more efficient ways of working:
"What was the best salary to earn in the previous activity? Why?"
"Is it the figure you thought it would be?"

"What effect did the tax have on this figure?"
"Will the best salary in **Activity Two** be a high or low amount?"
"Do you need to calculate all the possible salaries?"
"What salaries will you start with and why?"
"How many salaries and taxes do you need to work out to show that your answer is correct?"
"What are some quick ways of working out the percentage?"

Page 6 Involving Interest

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)

Other mathematical ideas and processes
Students will also work with compounding interest.

ACTIVITY

Kaylene reinvests the interest she earns back into her savings account, and so this activity introduces students to the concept of compounding interest, that is, interest paid on the interest. This has a surprisingly large effect on the investment, and Kaylene ends up with approximately 175% of her initial amount. Ensure that the students understand how compounding interest works before they start this activity.

The students will need to have a system for keeping track of what is going on year by year. Using a table such as the one below is a good way of organising the information. You may like to discuss with the students how to round sensibly.

	Invercargill Investment Bank	Titahi Bay Local Bank	Martinborough Municipal Bank
Money banked	$1,000	$1,000	$1,000
Amount after 1 year	$1,100	$1,000	$1,105
Amount after 2 years	$1,210	$1,000	$1,102.50
Amount after 3 years	$1,331	$1,000	$1,212.75
Amount after 4 years	$1,464.10	$1,000	$1,334.03
Amount after 5 years	$1,610.51	$1,000	$1,467.43
Amount after 6 years	$1,771.56	$1,750	$1,760.91

Using a spreadsheet on a computer would also be a good way of organising the information. You could extend this activity by having the students develop a spreadsheet that will work out each end-of-year total automatically.

INVESTIGATION

As well as using the concepts developed in the activity, the students will see in the investigation how interest rates vary depending on the length of the investment. You could discuss with them why this happens.

Other useful discussion points for this page are:

- What is the effect of the frequency of interest payments? For example, will Kaylene earn more money in her investment if interest is paid monthly or yearly?
- What is the effect of compound interest on retirement savings, and why are people encouraged to start saving for their retirement early?
- What is the effect of compound interest on money borrowed? (Over a long period, such as a 25 year term, the interest paid will generally add up to more than the amount borrowed.)

Page 7 Frantic Fund-raising

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- write and solve problems involving decimal multiplication and division (Number, level 4)

Other mathematical ideas and processes

Students will also:

- use number to explore real-life problems
- investigate the results of multiplying and dividing by powers of 10.

ACTIVITY

In this activity, the students need to keep track of the funds raised for the netball trip.

Question **1** is a fairly simple division problem: 300 ÷ 200 or 3 ÷ 2.

Question **2** requires the students to use their knowledge of multiplication. They need to consider the issue of multiplying by cents or dollars.

Drawing a table like the one below is a good way of helping the students to see why 5 cents is $0.05, not $0.5. They should already know that 100 cents equals $1, so this is a good starting point.

Cents	Dollars
100	1
50	0.5
40	0.4
20	0.2
10	0.1
5	0.05

The last part of this question and question **4** require an understanding of what an average is. Helping the students to see the links between sharing equally, division, and average is important.

You could extend question **4** by asking "How much could each girl have earned for each of the 4 weeks to reach the total of $1,286? Give five different possible correct answers."
"What if …
they earned twice as much in the second week as the first week?
they didn't earn any money in the second week?
each week, they each made more money than they had the week before?"

Question **5** helps the students to understand the concept of grouping and its connections to multiplication. Avoid telling them short cuts or rules such as "just add a zero to the end when multiplying by 10". Students who understand the concept and are encouraged to look for short cuts will find the zero rule themselves. Students who learn short cuts but have no understanding soon become stuck and unable to continue. One of your key roles is to develop understanding.

Discussion questions for question **6** could be:
"Is it better to work out the totals for the whole group and then divide by the 10 girls or to divide each week's profit into each girl's share and then work out the totals?"

You could extend this activity by asking:
"What was the most profitable fund-raising activity?"
"Which activity do you think was the most practical (or the most enjoyable) fund-raising activity?"

Page 8 Crossing Out Singles

Achievement Objectives

- make conjectures in a mathematical context (Mathematical Processes, developing logic and reasoning, level 4)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

Other mathematical ideas and processes

Students will also use mental strategies for efficient addition and multiplication.

GAME

This game is a good context for the students to practise their basic addition and multiplication facts and to practise using efficient mental strategies for addition and multiplication. For example, when adding the first four numbers on Geoff's board, the students could look for numbers that add up to 10 (3, 5, and 2 in the second row or 6, 3, and 1 in the third row), or they could look for doubles (in the bottom row, double 3 is 6, double 6 is 12). When they are multiplying by the fifth number in each row, for example, 17 x 5 in the bottom row, they may calculate it as (20 x 5) – (3 x 5) or (10 x 5) + (7 x 5). Alternatively, they may calculate it as half of 17 x 10.

After the students have played the game several times, encourage them to start thinking about strategies for winning the game. This strategic thinking is excellent mathematical work.

Many variations of this game are possible with only minor changes:

- Change the dice: use a 10-sided dice, a 10, 20, 30, 40, 50, and 60 dice, or, to simplify the game, a dice with two ones, two twos, and two threes.
- Change the rules so that the lowest score wins.
- Change the size of the board to make it more complex or simplify it to suit different groups of students.

Page 9 Cover Up

Achievement Objective

- demonstrate knowledge of the conventions for order of operations (Number, level 4)

Other mathematical ideas and processes

Students will also:

- maintain basic facts
- devise and use strategies for estimating results.

GAME

Clear, coloured counters are ideal for this game, in which the students use their basic facts along with their understanding of the order of operations. They need a good understanding of the order of operations conventions.

The acronym BEDMAS is often used to help students remember the order of operations. Each letter stands for an operation, and the order of letters in the acronym is the order in which to perform the operations.

In BEDMAS, B refers to anything inside brackets in the equation. E refers to exponents, that is, powers, square roots, and so on. D and M refer to division and multiplication, including finding the percentage of numbers. Division and multiplication are treated together and completed as they are encountered in the equation going from left to right. A and S refer to addition or subtraction, which are treated together and completed as they are encountered in the equation going from left to right.

An example is:
$(9 - 6) + 2^3 \times 3$

Step 1 (brackets)	$3 + 2^3 \times 3 =$
Step 2 (exponents: 2^3)	$3 + 8 \times 3 =$
Step 3 (multiplication: 8×3)	$3 + 24 =$
Step 4 (addition)	27

Some calculators follow these conventions for ordering operations, and some calculators do not. Have the students check out several calculators and decide whether they follow these conventions. You could use something like $8 - 2 \times 3 =$ as a quick test. If $8 - 2 \times 3 = 18$, the calculator does not follow the normal conventions. If $8 - 2 \times 3 = 2$, it does. A more advanced test is the equation above.

Page 10 Team Leaders

Achievement Objectives

- devise and follow a set of instructions to carry out a mathematical activity (Mathematical Processes, communicating mathematical ideas, level 4)
- recall the basic multiplication facts (Number, level 3)

Other mathematical ideas and processes
Students will also work with remainders.

ACTIVITY
In this activity, the students use their knowledge of multiplication and division basic facts to work out remainders. Make sure that the students understand the concept of division and remainders. Acting out the situation with the group would be a good way to ensure that the students understand the context of this activity. It is also a fun activity that they are likely to enjoy enacting. Alternatively, the students could represent the activity with counters to help them see what's happening.

To answer question **1b**, the students need to choose the leaders in fewer turns. You could discuss with them how they might do this. They should be able to work out that the higher the remainder, the sooner the number of children left reduces. For example, when Mr Randall begins by calling out "Six!", the children form eight groups of six, which leaves two over. But if Mr Randall had called out "Nine!", the children would have formed five groups of nine, and five children would have been left over. The students can use this approach of aiming for the highest possible remainder when they work out the answers to questions **2** and **3**.

Page 11 Purchasing Payments

Achievement Objectives

- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)
- write and solve problems involving decimal multiplication and division (Number, level 4)

Other mathematical ideas and processes

Students will also:

- develop number sense in relation to real-life situations
- use number to explore real-life situations.

ACTIVITY

In this activity, the students use their understanding of percentages in a hire purchase context. This activity is a good opportunity for the students to use calculators, but they also need to use their number sense and estimation skills. Hopefully, the following comments will be overheard as the students work:
"The answer will be about ..."
"That can't be right, it's too much."
"That's easy. I don't need the calculator for that."
Encourage the students to think about when, how, and why they use a calculator.

The students should notice that in all the repayment options, the amount repaid is more than the original retail price (between $10 and $96 more). This is because interest or service fees are charged on hire purchases. Some students may decide that they would rather save up all the money needed and buy the stereo for cash to avoid paying interest. Other students may decide that the extra $10 they pay (if they take Loud Noise Company's best deal) is worth it because they get the stereo straight away.

As an extension, the students could investigate hire purchase, rental, and mortgage and loan arrangements, using local information and situations. They could look at a range of items, such as cars, home appliances, houses, rental property, credit, and so on.

Page 12 Going for Gold!

Achievement Objectives

- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)
- order decimals with up to 3 decimal places (Number, level 3)

ACTIVITY

In this activity, the students order and add decimals to thousandths. Bridging material on decimals to three decimal places can be found in *Number*, Book Two of this series, on pages **17**, **18–19**, and **20**.

Care is needed in distinguishing time and decimals because time is not always a decimal. For example, 5 hours 57 minutes is not 5.57 because there are 60 minutes in one hour. 5 hours and 57 minutes is 5 hours and 57 out of 60 minutes, that is, $5^{57}/_{60}$ hours. This is almost 6 hours. $^{57}/_{60}$ as a decimal is 0.95, so 5 hours 57 minutes as a decimal is 5.95 hours. In this activity, tenths, hundredths, and thousandths of seconds are decimals and can be treated as any other decimal.

In a swimming medley relay, four swimmers race as a team. For example, the first swimmer swims 100 metres backstroke, the second swimmer 100 metres butterfly, the next swimmer 100 metres breaststroke, and the final swimmer 100 metres freestyle. To answer question **2**, the students need to add the times of the two legs to find the order after the butterfly leg and then find the difference between the times of the fastest team and Gisborne at this stage to find out how much time Gisborne has to make up.

You could extend this activity by asking "Suppose the eight swimmers in question **1** who did not make the final want to race and see who would get places ninth, tenth, eleventh, twelfth, and so on. Using the same method, what lane order would they be in?"

The students could design a computer spreadsheet for recording a medley competition and other swimming events. Challenge them to design the spreadsheet so that it will work out the lane positions automatically.

Page 13 Awesome Athletes

Achievement Objectives

- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)
- order decimals with up to 3 decimal places (Number, level 3)

Other mathematical ideas and processes

Students will also devise and use strategies for estimating results of computations.

ACTIVITY

The context for this activity is likely to be familiar for students, although a brief discussion would be useful to make sure the students realise that the fastest time (that is, the lowest number) is the winner for the running events and the highest or longest jump is the winner for the jumping events. The following questions could be used for this:
"Who wins in running events: fastest time or slowest time?"
"Who wins in a jumping event: shortest or longest distance?"
"Which is faster: 15.12 second or 15.21 seconds? Why?"

Alternatively, you could begin by putting forward the following scenario:
"Luke suggests that the best way to work out who is the champion is to add up the times and distances of each event for each competitor, and the one with the most points wins. Luke would get 104.4 points (16.1 + 82.41 + 4.6 + 1.29 = 104.4). Is this a good way to work out the winner? Why or why not?"

The students could experiment with several different scoring systems to answer question **3**. As an extension, they could investigate scoring systems in athletics and sports such as rugby and cricket and discuss whether they think the systems are fair.

Page 14 New Car Capers

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)

Other mathematical ideas and processes

Students will also talk about the use of percentages in an everyday context.

ACTIVITY

In this activity, the students need to use their knowledge of percentages to work out the depreciation of a car over a period of time. They need similar skills to those used for the activity on page **5** of the student book.

Using a calculator for this activity would allow the students to focus on the meaning and reasonableness of their answers and not get bogged down with computations. However, the students do need to be able to check that the answer on the calculator is likely to be correct. Quickly estimating the answer before using the calculator is a good way to do this. For example, 50% is the same as a half, which is the same as dividing by 2. So 50% of $12,000 is $6,000. 25% is half of 50%, so half and then half again gives you 25%. So for 25% of $12,000, half and then half again equals $3,000.

If you know that 10% of 12 000 is 1 200, then you can work out:

- 5% by halving the 1 200, so 5% of 12 000 is 600;
- 20% by doubling it, so 20% is 1 200 + 1 200 = 2 400;
- 30% by adding the amount three times, so 30% is 1 200 + 1 200 + 1 200 = 3 600;
- 90% by taking 10% from the whole amount, so 90% is 12 000 – 1 200 = 10 800; and so on.

The usual approach taken to working out depreciation is to first work out the percentage depreciated and then to subtract that from the total. This approach is recommended as the students come to understand the concept. This is a good activity to use to introduce the memory button on a calculator if the students are not familiar with its use. Once the students have a good understanding, challenge them to think of more efficient methods. Asking a question such as "Is finding 75% of a quantity the same as finding 25% and subtracting it from the quantity?" could be used to get them thinking.

Questions to promote more thinking could include:
"Why do you think the depreciation rate starts to go up after 5 years?"
"Is it likely to keep going up?"
"Could the depreciation rate ever be 110%?"
"After how many years will the car be worth nothing?"

Challenge the students to design a spreadsheet that will automatically work out the depreciation once they have entered the car's original cost.

The value of the car in question **1a** could be worked out like this:
Value after 1 year = $31,990 x 0.76
= $24,312
Value after 2 years = $24,312 x 0.81
= $19,693
and so on.

For question **1b**, the yearly decrease needs to be divided by 12 to get the monthly average. For example, $31,990 x 0.24 = $7,678 (the decrease for the year). $7,678 ÷ 12 = $640, the average monthly decrease over that period of time. For the fifth year, the decrease is ($16,475 – $15,157) ÷ 12, which is $110.

Page 15 PE Problems

Achievement Objective

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

Other mathematical ideas and processes

Students will also use, create, and describe formulae derived from practical concepts, using words and symbols.

ACTIVITY

The students should spend some time thinking about how they will approach this problem. Although basic facts will be used to find the answer, how to even start could be a problem for some students. The four-step method outlined in the problem-solving section of www.nzmaths.co.nz provides a good framework to work with:
Step 1: Understand and explore the problem.
Step 2: Find a strategy.
Step 3: Use the strategy to solve the problem.
Step 4: Look back and reflect on the solution.

The following problem-solving strategies could be considered as students work through the first few steps:

- Guess, then check and improve.
- Use equipment.
- Act it out.
- Draw a diagram or picture.
- Make a list or table.
- Use logic and reasoning.
- Use algebraic thinking.

For this particular activity, you could suggest to a struggling student that they first make an assumption, for example, team D is the smallest, and start working from there. If this doesn't give the correct answer, then have them try assuming that another team is the smallest. Encourage your students to use logic and reasoning to find which teams could be the smallest.

The students can use the same problem-solving approach for question **3**, but those who are comfortable using algebraic notation might like to solve it algebraically. For example, all the team sizes can be expressed in relation to team B:

$a = b - 6$
$c = b \div 2$
$d = b - 2$

We know that $a + b + c + d = 27$, so using the information above, this can be rewritten as:

$$
\begin{aligned}
b - 6 + b + (b \div 2) + b - 2 &= 27 \\
b + b + (b \div 2) + b &= 35 && \text{adding 8 to each side of the equation} \\
3b + (b \div 2) &= 35 && b + b + b = 3b \\
3b + 0.5b &= 35 && \text{rewriting } b \div 2 \text{ as } 0.5b \\
3.5b &= 35 \\
b &= 10 && \text{dividing both sides by 3.5}
\end{aligned}
$$

A table as a problem-solving approach could look like this:

A b – 6	B b	C b ÷ 2	D b – 2	Total
2	8	4	6	20
4	10	5	8	27 ✓
6	12	6	10	34

Page 16 Egyptian Multiplication

Achievement Objectives

- devise and follow a set of instructions to carry out a mathematical activity (Mathematical Processes, communicating mathematical ideas, level 4)
- explain satisfactory algorithms for addition, subtraction, and multiplication (Number, level 4)

Other mathematical ideas and processes

Students will also extend their understanding of the number system.

ACTIVITY

In this activity, students learn another way of multiplying. They must follow the directions first and then consider the challenge "How does it work?"

To understand the Egyptian method, the students need to understand and be able to use the distributive law and part-whole thinking. The distributive law acknowledges that numbers that are multiplied together can be broken into parts and the parts multiplied and then added together to give the same answer.
For example, 7 x 14 = 98 gives the same answer as 7 x (10 + 4) = (7 x 10) + (7 x 4)
= 70 + 28
= 98

This thinking or law is the basis of the way many students are taught to multiply. The standard algorithm works because of this thinking, for example:

$$\begin{array}{rl} 14 & \\ \times\ 7 & \\ \hline 28 & \text{from } 7 \times 4 \\ +\ 70 & \text{from } 7 \times 10 \\ \hline 98 & \end{array}$$

In this type of algorithm, the whole is broken into parts such as ones, tens, hundreds, and thousands or tenths and hundredths, but in the Egyptian method, the whole is broken into multiples of the larger number in the multiplication statement. These multiples are worked out by doubling.

This would be a good activity for introducing or practising the use of the constant function on a calculator. Once the students are familiar with working out the doubles, using the constant function would speed up the work. The constant function on a calculator allows you to instruct the calculator to keep repeating the function you set each time you push the equals button. The function required for the Egyptian method is doubling, which is multiplying by 2.

On most basic calculators, to multiply 24 by 2 and keep doubling the answer, push $\boxed{24}$ $\boxed{x}$ $\boxed{2}$ $\boxed{=}$ $\boxed{=}$ …
(On some calculators, you need to push $\boxed{2}$ $\boxed{x}$ $\boxed{x}$ $\boxed{24}$ $\boxed{=}$ $\boxed{=}$…) The small k on the screen is instructing the calculator to multiply by 2 each time the equals button is pushed (48, 96, 192, 384, 768, 1 536, 3 072, 6 144, 12 288, and so on).

Page 17 Kapa Haka Hāngi

Achievement Objectives

- effectively plan mathematical exploration (Mathematical Processes, problem solving, level 4)
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- write and solve problems involving decimal multiplication and division (Number, level 4)

Other mathematical ideas and processes

Students will also:

- use number to explore events in their own lives
- explore equivalent fractions.

ACTIVITY

In this activity, the students will be multiplying and dividing with decimals to find unknown quantities.

The students could use the problem-solving approach suggested in the notes for page **15** when working on this activity.

Encourage the students to estimate answers from the information they know. For example, feeding 30 adults requires 12 kilograms of potatoes, and so feeding 60 adults requires 24 kilograms (doubling). This means that feeding 50 adults requires a little less than 24 kilograms and for 70 adults you must add a little more. This information will help the students as they work towards the correct answer.

To work out the specific amounts, the students need to recognise the relationship between the amount of food and the number of adults. They are given the amount of food for 30 adults. There are several ways they could find the amount of food needed for 50 or 70 adults. For example:

- They could divide the amount of food needed for 30 adults by 3 to find the amount of food they need for 10 adults. They can then multiply this new amount by 5 to find the food needed for 50 adults and by 7 to find the food needed for 70 adults. This will be straightforward for food amounts that are easily divided by 3 (potatoes, kamokamo, cabbage, and lamb). Other food amounts will be a little more complex. Eight kilograms (or 8 000 grams) of kūmara will become $2.\dot{6}$ kilograms for 10 people, $13.\dot{3}$ for 50 people, and $18.\dot{6}$ kilograms for 70 people.

- They could work out what they need to multiply the 30-adult amounts by to get 50-adult amounts:

  $$30 \times \square = 50$$
  $$\square = 50 \div 30$$
  $$= 1.\dot{6}$$

  So they multiply all the 30-adult amounts by $1.\dot{6}$ to get the 50-adult amounts. They use the same process to find the 70-adult amounts.

To work out the amount of chicken, the students need to apply some common sense when rounding. The chicken for 50 people rounds nicely to 11.7 kilograms. For 70 people, however, the actual figure is $163.\dot{3}$, which should be rounded up to 163.4 so that there is slightly more chicken available rather than slightly less.

Page 18 Digit Challenge

Achievement Objective

- effectively plan mathematical exploration (Mathematical Processes, problem solving, level 4)

Other mathematical ideas and processes

Students will also:

- maintain basic facts
- generalise mathematical ideas and conjectures
- critically follow a chain of reasoning.

ACTIVITY ONE

You could suggest to the students who are struggling to get started that the problem-solving strategy of eliminating is a good way to begin. They need to make their own discoveries, but the most important clue is the remainder of 1. The students may realise that a good place to start is with numbers ending in 1 and 6 because numbers divisible by 5 always end in 0 or 5, and the 1 or the 6 would give the remainder of 1. (Matiu's grandmother is likely to be between 40 and 100.) They can then eliminate all the numbers ending in 6 because they are even. Finally, the students could check which of the remaining numbers divides by 3, 4, and 6 with a remainder of 1.

ACTIVITY TWO

The students will use a problem-solving approach and their knowledge of patterns in multiples to solve the digit challenge.

Ask the students to think of all the patterns in multiples of numbers that they know. If they are having trouble thinking of patterns, they could use the calculator's constant function (see the notes for page **16**) to try to identify patterns. Patterns that they might come up with are:

- Multiples of even numbers are always even.
- Multiples of 5 always end in 5 or zero.
- The last two digits in a number divisible by 4 are themselves divisible by 4.
- The digits in multiples of 3 always add up to a number divisible by 3.
- The digits in multiples of 9 always add up to a number divisible by 9.
- The last three digits in a number divisible by 8 are themselves divisible by 8.
- The digits in multiples of 6 always add up to a number divisible by 3, and the number is always even.

Most of the patterns help to solve the challenge, but some do not. For example, the pattern in the multiples of 8 does not help.

The students could use trial and improvement to solve the challenge, but it's a fairly tedious process. They could use a table to keep track. If a computer is available, they could use a spreadsheet and list all the possible two-digit numbers, then the matching three-digit numbers divisible by 3, and so on. They should be able to work out that any nine-digit number with every digit different will add up to a number divisible by 9 (that is, 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45, which is divisible by 9). For divisibility by 8, they then need only to place the sole remaining even digit and check by division. 7 is the problem, and every number has to be checked by division. The students may be able to work out these steps for themselves.

Tasty Treats

Achievement Objective

- effectively plan mathematical exploration (Mathematical Processes, problem solving, level 4)

Other mathematical ideas and processes

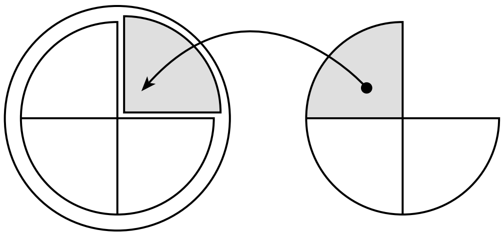
Students will also solve problems involving fractions.

ACTIVITY

In this activity, the students use their knowledge of doubling and of fractions to increase the quantities of a recipe. They also need to work out the cost and profit of selling the honey crunch bars.

The doubling of the quantities is likely to be straightforward for the students, although some may need to rely on diagrams or materials to represent the quantities to help them.

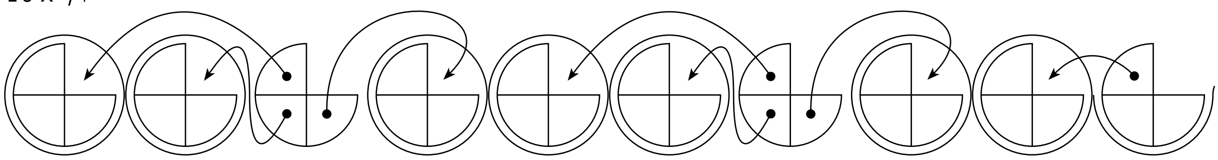
For example, doubling $^3/_4$ can be represented by the following diagram:
$2 \times {}^3/_4$



Moving one quarter to fill up one circle leaves two quarters. This means that $2 \times {}^3/_4 = 1^2/_4$ or $1^1/_2$.

There are six quarters or $^6/_4$, which is the equivalent to $1^2/_4$ or $1^1/_2$.

Multiplying the quantities could be treated in a similar way:
$10 \times {}^3/_4$



Moving quarters to fill up circles gives seven full circles and two quarters, which is equivalent to $7^2/_4$ or $7^1/_2$.

These diagrams should help the students to see the connection between this modelling and the more efficient way of working the quantities out: $10 \times {}^1/_4 = {}^{10}/_4$, which is the same as $2^2/_4$ or $2^1/_2$.

The students should know how to calculate profit (that is, money made minus the costs), but you may want to check this before they start question **3**.

Pages 20–21 Cycling On ...

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- write and solve problems involving decimal multiplication and division (Number, level 4)

Other mathematical ideas and processes

Students will also use number to explore events in their own lives and cultures.

ACTIVITIES ONE TO THREE

In these activities, the students have to work with a lot of data as they find differences between distances, find differences between times, find fractions, calculate average speeds in kilometres per hour, and calculate percentages.

The students need to be aware that although both the data about time and the data about distances are presented in the same decimal format, they actually represent different things. The distance data is in standard decimal format, that is, the figure to the right of the decimal point represents tenths. But the time data is not in this same format. Here, the figures to the right of the decimal place represent the number of minutes, that is, sixtieths. So when the students want to find the distance travelled in stage 4, they can calculate 16.2 – 11.9. But in finding out how long this stage took to cycle, they will get the wrong answer if they calculate 11.09 – 10.59. Instead, they will have to work out that the difference in time between 10.59 and 11.00 is 1 minute, and then there are another 9 minutes to get to 11.09, a total of 10 minutes altogether. You might like to ask the students to compare this way of recording time with the method used on page **12** of the students' book.

Several questions in these activities ask the students to find an average speed in kilometres per hour. This rate shows how far someone would travel in 1 hour if their speed was evened out over the whole hour.

For **Activity Four**, question **3a**, the students need to use the information from earlier questions in this activity and from **Activity Two**. From their earlier answers, they will quickly work out that the group A riders were 13 minutes faster over the same distance than the group C riders.

One way of working out the percentage for question **3b** is to think of it in terms of how much faster group A were than group C. Using the average speeds, 24 km/h ÷ 18 km/h = $1.\dot{3}$, so group A were $^1/_3$ or $33^1/_3$% faster than group C.

Exploration to Earth

Achievement Objectives

- effectively plan mathematical exploration (Mathematical Processes, problem solving, level 4)
- make sensible estimates and check the reasonableness of answers (Number, level 4)
- explain the meaning of digits in any whole number (Number, level 3)

ACTIVITY

In this activity, the students use and think about large numbers.

Their use of the space line for question **1a** will reveal how well the students understand large numbers. Having them place the numbers they know, such as 100 000, 200 000, and 300 000, onto the space line first may help them to figure out the other numbers. The question asks only for the approximate position on the space line for each distance. Having the students explain why they placed a space stop where they did would reveal the depth of their understanding.

Question **1b** requires the students to subtract the distance from each space station from 1 000 000. To help the students to fully develop an understanding for large numbers and addition and subtraction, you could have them use at least three different methods of working these distances out. Here are several methods that they could use:

Earth to Hilf is 500 000 kz.
Method: Using already known knowledge, that is, half of a million is 500 000

Earth to Anzax is 188 000 kz.
Method: Mental calculations, using adding on
812 thousand + 100 thousand is 912 thousand.
912 thousand + 80 thousand is 992 thousand.
992 thousand + 8 thousand is 1 000 thousand.
Therefore, the answer is 100 + 80 + 8 thousand or 188 000.

Earth to Kleenax is 750 962 kz.
Method: Mental calculations, working to simpler numbers

249 038 + [something] = 250 000	38 + [962] = 1 000
250 000 + [something] = 1 000 000	250 000 + [750 000] = 1 000 000

Therefore, the answer is 962 + 750 000 or 750 962.

Earth to Relax is 664 856 kz.
Method: Calculator
1 000 000 – 335 144 = 664 856

Page 23 Orchard Antics

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)
- recall the basic multiplication facts (Number, level 3)

ACTIVITY

In this activity, the students use their knowledge of multiplication arrays and area to work out the number of trees that can be planted in an orchard.

The students could fairly easily count each tree on the 12 metre by 12 metre piece of land or at least count the number of rows and the number of trees in each row and multiply them together. But they need to find more efficient strategies to find the number of trees that could be planted in question **2**.

The simplest way to approach a 52 metre by 36 metre plot is to divide each measurement by 2 for the apple trees and by 4 for the pear trees (so 26 by 18 apple trees and 13 by 9 pear trees). For sides not divisible evenly by 4, the students will need to realise that some land will be spare if the trees are to have the required amount of growing space.

Some possible extension questions are:
"How many apple trees could be planted on a rugby field?"
"How many pear trees could be planted on your school field?"

Page 24 Dive Tank Dilemma

Achievement Objective

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

ACTIVITY

There are several strategies that could be used for this activity. Some are more efficient and involve more mathematical thinking than others. Your role is to help the students move to more efficient methods as their confidence and ability increase. Discussions about the strategies used is a good way to help the students think about how they solve problems, and it also exposes them to strategies they may not have considered using.

Possible strategies:

- Guess and check.

Guess: 8, 9, 11	8 + 9 + 11 = 28	not enough, try other numbers

- Guess and improve.

Guess: 8, 9, 11	8 + 9 + 11 = 28	not enough, try a number 2 bigger, that is, swap 11 with 13
New guess: 8, 9, 13	8 + 9 + 13 = 30	correct answer

- Make a table and systematically try all the possibilities.

  Start with one of the numbers being 8:

8	9	10	no
8	9	11	no
8	9	12	no
8	9	13	yes
8	10	12	yes
8	11		not possible

  Then try with 9:

9	8	13	yes, already have above
9	10	11	yes
9	10	12	no
9	10	13	no

  (Hopefully, the students will see that there is no point trying this combination.)

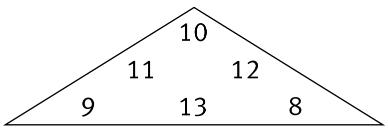
9	11	10	yes, already have above
9	11	12	no
9	11	13	no

  and so on.

- Use logic and reasoning.

  If 13 is one of the numbers, the other two numbers must add up to 17. 8 + 9 = 17, so 13, 9, 8 is one group of three. No other pair of numbers adds up to 17 because 8 is the smallest number allowed. Therefore, 13 cannot be at a corner; it must be in the middle. And so on.

  Another method using logic and reasoning is this: the average mass of the one tank in a row must be 10. So one row could include one fewer and one more, that is, 9, 10, 11. Another row could include two fewer and two more, that is, 8, 10, 12. 10 is in both sets, so it must be a corner. The only tank left is the 13, which needs 8 and 9 to make 30. So 8 and 9 must be corners too. This gives:



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