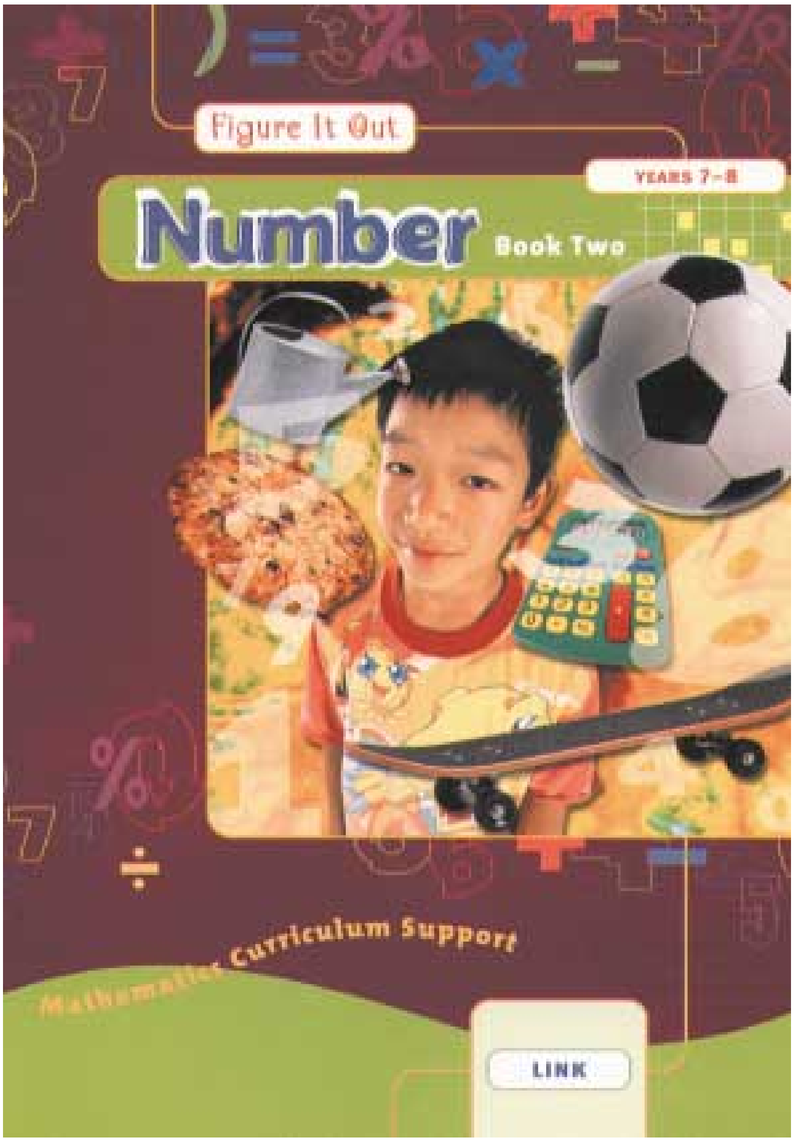
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

CONTENTS

The books for years 7–8 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand year 7–8 classrooms. The books have been developed and trialled by classroom teachers and mathematics educators and follow on from the successful series for levels 2–4 in primary schools.

Student books

The student books in the series are divided into three curriculum levels: levels 2–3 (linking material), level 4, and level 4+ (extension material). All the books are aimed at year 7–8 students in terms of context and presentation.

The following books are included in the series:

Number (two linking, three level 4, one level 4+)
Algebra (one linking, two level 4, one level 4+)
Measurement (one level 4, one level 4+)
Number Sense (one linking, one level 4)
Geometry (one level 4, one level 4+)
Statistics (one level 4, one level 4+)
Themes (level 4): *Disasters, Getting Around*

These 20 books will be distributed to schools with year 7–8 students over a period of two years, starting with the six *Number* books.

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and the life experiences that are meaningful to students aged 11–13 years. The activities can be used as the focus for teacher-led lessons, as independent activities, or as the catalyst for problem solving in groups.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the student books includes full answers and explanatory notes. Students can use them for self-marking, or you can use them for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, comments on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/community

Using Figure It Out in your classroom

Where applicable, each page starts with a list of equipment that the students will need to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Answers Number: Book Two

Page 1 Pizza Party

ACTIVITY

1. a. 8 out of 24 ($^8/_{24}$, which is $^1/_3$)
   b. 12 out of 24 ($^{12}/_{24}$, which is $^1/_2$)
   c. 16 out of 24 ($^{16}/_{24}$, which is $^2/_3$)
   d. 4 out of 24 ($^4/_{24}$, which is $^1/_6$)
   e. 8 out of 24 ($^8/_{24}$, which is $^1/_3$)
2. a. 12
   b. i. $^1/_3$ (24 out of 72 pieces)
      ii. 4 ($^1/_3$ of 12)

Pages 2–3 Table Tricks

ACTIVITY

1. a. 2 x 7 = 14, so 4 x 7 = **28** (or double 14 to get 28)
   b. 2 x 9 = 18, so 4 x 9 = **36**. (18 + 18 = 36)
   c. 2 x 6 = **12**, so 4 x 6 = **24**. (12 + 12 = 24)
   d. 2 x 8 = **16**, so 4 x 8 = **32**. (16 + 16 = 32)
   e. 2 x 5 = **10**, so 4 x 5 = **20**. (10 + 10 = 20)
   f. 2 x 4 = **8**, so 4 x 4 = **16**. (8 + 8 = 16)
2. a. 2 x 6 = **12**, so 3 x 6 = **18**. (12 + 6 = 18)
   b. 2 x 8 = **16**, so 3 x 8 = **24**. (16 + 8 = 24)
   c. 2 x 9 = **18**, so 3 x 9 = **27**. (18 + 9 = 27)
   d. 2 x 5 = **10**, so 3 x 5 = **15**. (10 + 5 = 15)
   e. 2 x 7 = **14**, so 3 x 7 = **21**. (14 + 7 = 21)
   f. 2 x 4 = **8**, so 3 x 4 = **12**. (8 + 4 = 12)
3. a. 3 x 5 = 15, so 6 x 5 = **30**. (15 + 15 = 30)
   b. 3 x 8 = **24**, so 6 x 8 = **48**. (24 + 24 = 48)
   c. 3 x 9 = **27**, so 6 x 9 = **54**. (27 + 27 = 54)
   d. 3 x 4 = **12**, so 6 x 4 = **24**. (12 + 12 = 24)
   e. 3 x 7 = **21**, so 6 x 7 = **42**. (21 + 21 = 42)
   f. 3 x 6 = **18**, so 6 x 6 = **36**. (18 + 18 = 36)
4. a. 10 x 7 = 70, so 9 x 7 = **63**. (70 – 7 = 63)
   b. 10 x 4 = 40, so 9 x 4 = **36**. (40 – 4 = 36)
   c. 10 x 8 = **80**, so 9 x 8 = **72**. (80 – 8 = 72)
   d. 10 x 9 = **90**, so 9 x 9 = **81**. (90 – 9 = 81)
   e. 10 x 3 = **30**, so 9 x 3 = **27**. (30 – 3 = 27)
   f. 10 x 6 = **60**, so 9 x 6 = **54**. (60 – 6 = 54)
5. Answers may vary. The 7 times table is the 5 times table plus the 2 times table.
   For example: 7 x 8 = (5 x 8) + (2 x 8)
   = 40 + 16
   = 56
   The 8 times table is twice the 4 times table.
   For example: 4 x 6 = 24, so 8 x 6 = 48

Page 4 A Million Grains of Rice

INVESTIGATION

1. Results will vary.
2. a. Answers will vary. You will need to know the approximate area of your house. For a house that is 90 m^2, you would need 900 000 cubes. A larger house would need more than a million cubes.
   b. Answers will vary. You will need to measure your classroom. For a classroom with an area of 50 m^2, you would need 500 000 cubes.

Page 5 Factor Fun

GAME

A game using factors

Pages 6–7 Planting with the Whānau

Activity

1. You should find that each method gives an answer of 95 kūmara plants.
   Māni's method:
   10 x 5 = 50
   9 x 5 = 45
   50 + 45 = 95
   Ruawai's method:
   5 x 20 = 100
   100 – 5 = 95
   Huia's method:
   19 x 10 = 190
   190 ÷ 2 = 95

2. a. There are 234 potato plants.
   Māni's method:
   9 x 20 = 180
   9 x 6 = 54
   180 + 54 = 234
   Ruawai's method:
   8 x 25 = 200
   200 + 25 = 225
   225 + 9 = 234
   Huia's method:
   10 x 26 = 260
   260 – 26 = 234

   b. Answers will vary. With these numbers, each method is equally efficient.

3. Answers will vary. Some possible ways to find 4 x 18 are:

   i. (4 x 10) + (4 x 8)
   = 40 + 32
   = 72

   ii. 4 x 20 = 80, 4 x 2 = 8, 80 – 8 = 72

   iii. 4 x 18 = 8 x 9 (double one factor, halve the other)
   = 72

   iv. 5 x 18 = 90, 90 – 18 = 72

   v. 3 x 4 = 12, so 6 x 4 = 24 and 12 x 4 = 48.
   48 + 24 = 72

   vi. 9 x 4 = 36, so 18 x 4 = 72.

Page 8 Coupon Combinations

Activity

1. 285 + 400 = 685
   396 is 4 less than 400
   685 – 4 = 681 tokens

2. a. 523 + 400 = 923
   923 + 40 = 963
   963 + 8 = 971 tokens

   b. Two ways are:
   i. 448 is 2 less than 450
   523 + 450 = 973
   973 – 2 = 971 (rounding and compensating)
   ii. 500 + 400 = 900
   23 + 48 is close to 25 + 50 = 75
   75 – 2 – 2 = 71
   900 + 71 = 971 (place value and rounding)

3. Lawrence and Charlotte can combine their tokens: 285 + 523 = 808. Emeli and Simon can combine their tokens: 448 + 396 = 844.

Page 9 Seeing Double

Activity

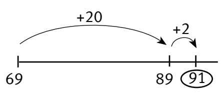
1. $^1/_4$
2. $^3/_5$
3. $^1/_8$
4. $^5/_8$
5. $^5/_2$ or $2^1/_2$
6. $^7/_4$ or $1^3/_4$

Pages 10–11 Empty Lines

Activity

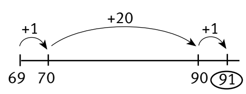
The arrows and the +/– signs are not essential. Use them if you find them helpful.

1. Some possible strategies are:

   a. 

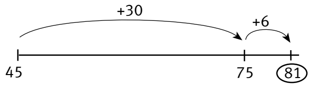
   69 + 20 + 2
   = 89 + 2
   = 91

   or



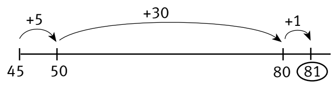
   69 + 1 + 20 + 1
   = 70 +21
   = 91

b.



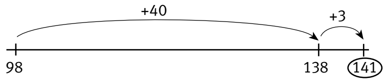
$45 + 30 + 6$
$= 75 + 6$
$= 81$

or



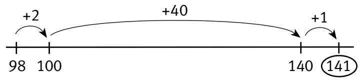
$45 + 5 + 30 + 1$
$= 50 + 31$
$= 81$

c.



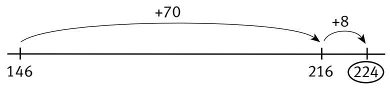
$98 + 40 + 3$
$= 138 + 3$
$= 141$

or



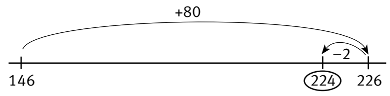
$98 + 2 + 40 + 1$
$= 100 + 41$
$= 141$

d.



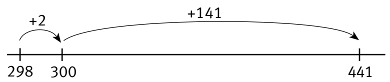
$146 + 70 + 8$
$= 216 + 8$
$= 224$

or



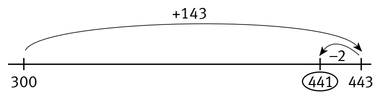
$146 + 80 - 2$
$= 226 - 2$
$= 224$

e.



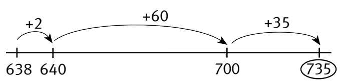
$298 + 2 + 141$
$= 300 + 141$
$= 441$

or



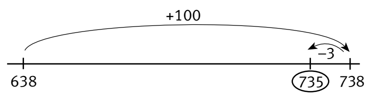
$300 + 143 - 2$
$= 443 - 2$
$= 441$

f.



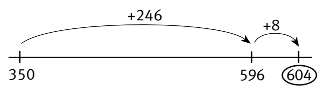
$638 + 2 + 60 + 35$
$= 640 + 95$
$= 735$

or



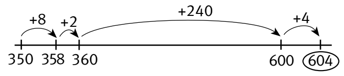
$638 + 100 - 3$
$= 738 - 3$
$= 735$

g.



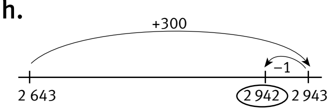
$350 + 246 + 8$
$= 350 + 254$
$= 604$

or



$350 + 8 + 2 + 240 + 4$
$= 360 + 244$
$= 604$

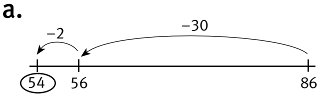
h.



$2\,643 + 300 - 1$
$= 2\,943 - 1$
$= 2\,942$

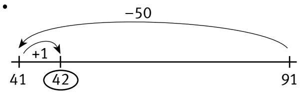
2. Some possible strategies are:

a.



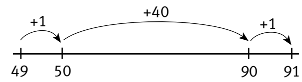
$86 - 30 - 2$
$= 56 - 2$
$= 54$

b.



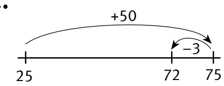
$91 - 50 + 1$
$= 41 + 1$
$= 42$

or



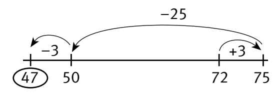
$49 + \square = 91$
$\square = 42$
$(1 + 40 + 1 = 42)$

c.



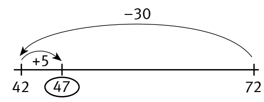
$25 + \square = 72$
$\square = 47$
$(50 - 3 = 47)$

or



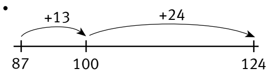
$72 + 3 - 25 - 3$
$= 75 - 25 - 3$
$= 47$

or



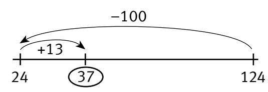
$72 - 30 + 5$
$= 42 + 5$
$= 47$

d.



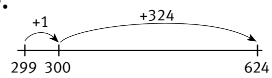
$87 + \square = 124$
$\square = 37$
$(13 + 24 = 37)$

or



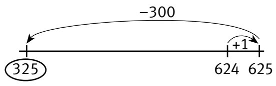
$124 - 100 + 13$
$= 24 + 13$
$= 37$

e.



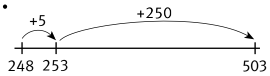
299 + □ = 624
□ = 325
(1 + 324 = 325)

or



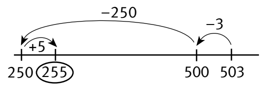
624 + 1 − 300
= 625 − 300
= 325

f.



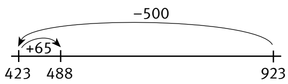
248 + □ = 503
□ = 255
(5 + 250 = 255)

or



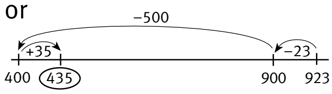
503 − 3 − 250 + 5
= 500 − 250 + 5
= 250 + 5
= 255

g.



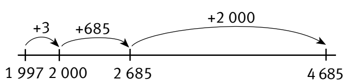
923 − □ = 488
923 − 500 + 65 = 488
□ = 435
(488 − 423 = 65, so you add 65 at the end.)

or



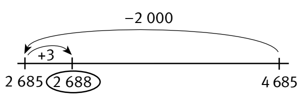
923 − 23 − 500 + 35
= 900 − 500 + 35
= 400 + 35
= 435
(523 − 488 = 35, so you add 35 at the end.)

h.



1 997 + □ = 4 685
□ = 2 688
(3 + 685 + 2 000 = 2 688)

or



4 685 − 2 000 + 3
= 2 685 + 3
= 2 688

Page 12 Problems Galore

ACTIVITY

Answers and possible methods for each problem are:

1. 216 people
   514 − 298 = 216 (514 − 300 + 2 = 216 or
   298 + 2 + 214 = 514
   2 + 214 = 216)
2. 88 eels
   237 − 149 = 88 (250 − 150 − 13 + 1 = 88 or
   149 + 1 + 50 + 37 = 237
   1 + 50 + 37 = 88)
3. 35 balloons
   100 − 29 − 36 = 35 or 100 − 30 − 35 = 35
4. $213
   □ − 87 = 126
   87 + 126 = 213 (90 + 123 = 213)
5. 226 people
   824 − □ = 598
   824 − 598 = 226 (824 − 600 + 2)
6. 104 cards
   163 − 45 = 118 (165 − 45 − 2 = 118)
   118 ÷ 2 = 59 (Sam)
   59 + 45 = 104 (Alisha)

Page 13 Fun Times

ACTIVITY

Answers and a possible method for each problem are:

1. 95 biscuits
   5 x 20 = 100
   so 5 x 19 = 100 − 5
   = 95
2. 17 weeks
   100 ÷ 25 = 4
   so 400 ÷ 25 = 16
   420 − 400 = 20 (1 more week's pay)
   16 + 1 = 17
3. 1 344 muffins
   4 x 48 x 7:
   4 x 50 = 200
   so 4 x 48 = 200 − 8
   = 192
   200 x 7 = 1 400
   so 192 x 7 = 1 400 − (8 x 7)
   = 1 400 − 56
   = 1 344

4. 261 skateboards
□ x 4 = 1 044
25 x 4 = 100
so 250 x 4 = 1 000
11 x 4 = 44
so □ = 250 + 11
= 261

5. 336 outfits
20 x 16 = (20 x 10) + (20 x 6)
= 200 + 120
= 320
so 21 x 16 = 320 + 16
= 336

6. 22 seats
500 ÷ 25 = 20
50 ÷ 25 = 2
so 550 ÷ 25 = 20 + 2
= 22
or
25 x □ = 550
25 x 4 = 100, so 25 x 20 = 500
25 x 2 = 50
20 + 2 = 22

Page 14 Divisive Tactics

ACTIVITY

Answers and possible methods are:

1. 42 teams
252 ÷ 6 = □
6 x 4 = 24, so
240 ÷ 6 = 40
12 ÷ 6 = 2
40 + 2 = 42

2. 11 pork chops in each pack
132 ÷ □ = 12. Change to:
12 x □ = 132
12 x 10 = 120, so 12 x 11 = 132

3. 102 doughnuts (with two left over from the four-packs)
□ ÷ 6 = 17 □ ÷ 4 = 25
6 x 17 = 102 4 x 25 = 100

INVESTIGATION

1. There are 22 west-to-east streets.
176 ÷ 8 = □
8 x 2 = 16, so
160 ÷ 8 = 20
16 ÷ 8 = 2
20 + 2 = 22

2. 30 tracks
One of each track length gives 2 + 3 + 4 = 9 minutes.
90 ÷ 9 = 10. So they can get 10 x 3 = 30 tracks on the tape.

Page 15 More Problems

ACTIVITY

Answers and possible methods are:

1. 733 cans
247 + 486
Try: 250 + 500 = 750
750 – 3 = 747
747 – 14 = 733
or
247 + 400 = 647
647 + 80 = 727
727 + 6 = 733

2. 300 letters
74 + 26 = 100, 38 + 62 = 100, 47 + 53 = 100
100 + 100 + 100 = 300
or (using rounding)
70 + 40 + 50 + 60 + 30 + 50 = 300
+ 4 – 2 – 3 + 2 – 4 + 3 = 0
300 – 0 = 300

3. 361 stamps
345 + □ = 706
345 + 55 = 400, 400 + 300 = 700, 700 + 6 = 706
55 + 300 + 6 = 361

4. $266
□ + 247 = 513
247 + 3 = 250, 250 + 250 = 500, 500 + 13 = 513
3 + 250 + 13 = 266

5. Goran got 13 points. (Nina got 19 points.)
97 – (14 + 28 + 23) = 97 – 65
= 32
32 = 6 + (2 x Goran's score)
32 – 6 = 2 x Goran's score
26 = 2 x Goran's score
13 = Goran's score

Page 16 Expanding Your Horizons

ACTIVITY

1. a. 78
   b. 278
   c. 170
   d. 3 978
   e. 5 978
   f. 5 000
2. a. 56 + 34 or 54 + 36
   b. 178 + 254, 174 + 258, 158 + 274, or 154 + 278
   c. 3 170 + 5 930, 3 130 + 5 970, 3 970 + 5 130, or 3 930 + 5 170
   d. 7 206 + 5 904, 7 204 + 5 906, 7 906 + 5 204, 7 904 + 5 206, 7 136 + 5 974, 7 134 + 5 976, 7 176 + 5 934, 7 174 + 5 936, 7 936 + 5 174, 7 934 + 5 176, 7 976 + 5 134, or 7 974 + 5 136
3. a. [+] 820
   b. [+] 170
   c. [+] 744
   d. [−] 2 744
   e. [−] 1 918

Page 17 Expanding with Decimals

ACTIVITY

1. Answers will vary. You should have five out of each of the following, in the order listed.
   a. 0.5, 0.51, 0.512, 0.514, 0.517, 0.56, 0.562, 0.564, 0.567, 0.58, 0.582, 0.584, 0.587
   b. 7.9, 7.91, 7.912, 7.914, 7.917, 7.96, 7.962, 7.964, 7.967, 7.98, 7.982, 7.984, 7.987, 8.01, 8.012, 8.014, 8.017, 8.06, 8.062, 8.064, 8.067, 8.08, 8.082, 8.084, 8.087
   c. 3, 3.002, 3.004, 3.007, 3.01, 3.012, 3.014, 3.017, 3.06, 3.062, 3.064, 3.067, 3.08, 3.082, 3.084, 3.087, 3.3, 3.302, 3.304, 3.307, 3.502, 3.504, 3.507, 3.31, 3.312, 3.314, 3.317, 3.36, 3.362, 3.364, 3.367, 3.38, 3.382, 3.384, 3.387, 3.5, 3.51, 3.512, 3.514, 3.517, 3.56, 3.562, 3.564, 3.567, 3.58, 3.582, 3.584, 3.587
2. a. 7.5 + 3.3 or 7.3 + 3.5
   b. 0.567 + 0.312, 0.562 + 0.317, 0.517 + 0.362, or 0.512 + 0.367
   c. 8.56 + 7.31, 8.51 + 7.36, 8.36 + 7.51, or 8.31 + 7.56
   d. 3.56 + 8.98, 3.58 + 8.96, 3.96 + 8.58, or 3.98 + 8.56
3. a. [+] 4.8
   b. [−] 5.45
   c. [+] 0.17
   d. [−] 0.623
   e. [+] 0.62

Pages 18–19 Placing Points

ACTIVITY ONE

1. a. 7.436
   b. 5.28
   c. 0.0059 (Some calculators may show this as 5.9^{-03})
   d. 0.0384
2. The digits move one place value to the right. (You can see this very clearly on a calculator.)

ACTIVITY TWO

a. 82.35
b. 0.0784
c. 0.00946 (Some calculators may show this as 9.46^{-03})

ACTIVITY THREE

The simplest calculations would be:
a. 54.83 [÷] 10 [=] 5.483
b. 77.7 [x] 100 [=] 7 770
c. 9.468 [÷] 100 [=] 0.09468
d. 116.42 [÷] 100 [=] 1.1642

ACTIVITY FOUR

a. 48.6 [x][5][=] (because 48.6 x 5 = 243)
b. 3 210 [÷][5][=] (because 3 210 ÷ 5 = 642)
c. 0.2 x [5][x][5][=] (because 0.2 x 5 x 5 = 5)
d. 1[÷] 5 [÷][5][=] (because 1 ÷ 5 ÷ 5 = 0.04) or 1[÷][5][=][=] (using the constant function)

Page 20 Getting the Point

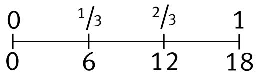
ACTIVITY

1. a. $\frac{70}{100}$ or $\frac{7}{10} = 0.7$ $\frac{30}{100}$ or $\frac{3}{10} = 0.3$
   b. $\frac{40}{100}$ or $\frac{4}{10} = 0.4$ $\frac{60}{100}$ or $\frac{6}{10} = 0.6$
   c. $\frac{25}{100}$ or $\frac{1}{4} = 0.25$ $\frac{75}{100}$ or $\frac{3}{4} = 0.75$
   d. $\frac{63}{100} = 0.63$ $\frac{37}{100} = 0.37$
2. No, because 100 will not divide evenly by 3.

Page 21 Mystery Fractions

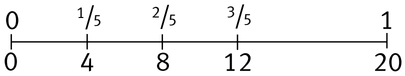
ACTIVITY

1. Felicity's jar: 12 are red.



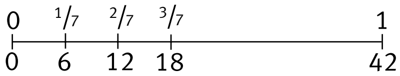
Fraction	1	$\frac{1}{3}$	$\frac{2}{3}$
Number	18	6	12

Ro's jar: 12 are green.



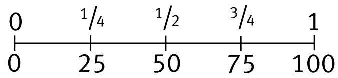
Fraction	1	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$
Number	20	4	8	12

Tama's jar: 18 are white.



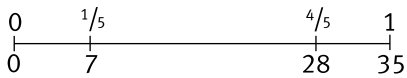
Fraction	1	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{3}{7}$
Number	42	6	12	18

Min-Lee's jar: 75 are yellow.



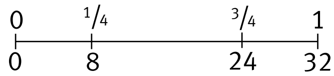
Fraction	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$
Number	100	50	25	75

2. Answers will vary.
3. a. 28



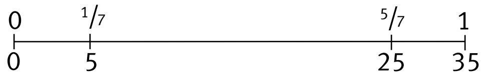
Fraction	1	$\frac{1}{5}$	$\frac{4}{5}$
Number	35	7	28

b. 24



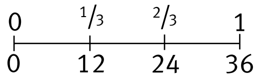
Fraction	1	$\frac{1}{4}$	$\frac{3}{4}$
Number	32	8	24

c. 25



Fraction	1	$\frac{1}{7}$	$\frac{5}{7}$
Number	35	5	25

d. 24



Fraction	1	$\frac{1}{3}$	$\frac{2}{3}$
Number	36	12	24

Pages 22–23 Classy Courtyards

ACTIVITY ONE

1. a. $\frac{2}{4}$ or $\frac{1}{2}$
   b. $\frac{1}{4}$
   c. $\frac{1}{4}$
2. a. $\frac{8}{16}$ or $\frac{1}{2}$
   b. $\frac{4}{16}$ or $\frac{1}{4}$
   c. It is $\frac{1}{4}$ of each design and therefore equal.
3. a. $\frac{8}{16}$ or $\frac{1}{2}$
   b. Both are equal: $\frac{4}{16}$ or $\frac{1}{4}$
   c. The tiled area of each courtyard is half of the total area: $\frac{2}{4}$ and $\frac{8}{16}$. These are equivalent fractions.
4. a. $\frac{32}{64}$ or $\frac{1}{2}$
   b. $\frac{16}{64}$ or $\frac{1}{4}$
   c. $\frac{16}{64}$ or $\frac{1}{4}$
5. The herb gardens are $\frac{1}{4}$, $\frac{4}{16}$, and $\frac{16}{64}$. They are all equivalent fractions.

ACTIVITY TWO

1. $^{3}/_{10}$
2. $^{5}/_{10}$ or $^{1}/_{2}$
3. $^{10}/_{40}$ or $^{1}/_{4}$
4. Answers will vary.

Page 24 Boxed Biscuits

ACTIVITY

1. a.–b.

Packs	Mixture	Fraction	
		Apricot	Berry
2	6 apricot, 6 berry	$^{6}/_{12}$ or $^{1}/_{2}$	$^{6}/_{12}$ or $^{1}/_{2}$
3	4 apricot, 4 berry	$^{4}/_{8}$ or $^{1}/_{2}$	$^{4}/_{8}$ or $^{1}/_{2}$
6	2 apricot, 2 berry	$^{2}/_{4}$ or $^{1}/_{2}$	$^{2}/_{4}$ or $^{1}/_{2}$
12	1 apricot, 1 berry	$^{1}/_{2}$	$^{1}/_{2}$

2. 27 box

Packs	Fractions	
	Chocolate	Berry
3	$^{6}/_{9}$ or $^{2}/_{3}$	$^{3}/_{9}$ or $^{1}/_{3}$
9	$^{2}/_{3}$	$^{1}/_{3}$

36 box

Packs	Fraction	
	Date	Apricot
3	$^{9}/_{12}$ or $^{3}/_{4}$	$^{3}/_{12}$ or $^{1}/_{4}$
9	$^{3}/_{4}$	$^{1}/_{4}$

100 box

Packs	Fractions			
	Chocolate	Date	Berry	Apricot
2	$^{20}/_{50}$ or $^{2}/_{5}$	$^{15}/_{50}$ or $^{3}/_{10}$	$^{10}/_{50}$ or $^{1}/_{5}$	$^{5}/_{50}$ or $^{1}/_{10}$
5	$^{8}/_{20}$ or $^{2}/_{5}$	$^{6}/_{20}$ or $^{3}/_{10}$	$^{4}/_{20}$ or $^{1}/_{5}$	$^{2}/_{20}$ or $^{1}/_{10}$
10	$^{4}/_{10}$ or $^{2}/_{5}$	$^{3}/_{10}$	$^{2}/_{10}$ or $^{1}/_{5}$	$^{1}/_{10}$

Teachers' Notes

Overview Number: Book Two

Title	Content	Page in students' book	Page in teachers' book
Pizza Party	Expressing proportions as fractions	1	13
Table Tricks	Learning multiplication basic facts	2–3	14
A Million Grains of Rice	Understanding large numbers	4	14
Factor Fun	Practising multiplication facts	5	15
Planting with the Whānau	Solving multiplication and division problems	6–7	16
Coupon Combinations	Adding whole numbers	8	16
Seeing Double	Solving decimal problems using double number lines	9	17
Empty Lines	Using empty number lines to show thinking strategies	10–11	18
Problems Galore	Solving whole-number addition and subtraction problems	12	19
Fun Times	Solving whole-number multiplication and division problems	13	20
Divisive Tactics	Solving whole-number multiplication and division problems	14	20
More Problems	Solving whole-number addition and subtraction problems	15	21
Expanding Your Horizons	Understanding the meaning of digits in whole numbers	16	22
Expanding with Decimals	Understanding the value of digits in decimals	17	23
Placing Points	Multiplying and dividing decimal numbers by tens and fives	18–19	23
Getting the Point	Writing fractions as decimals	20	24
Mystery Fractions	Finding fractions of whole-number amounts	21	25
Classy Courtyards	Using equivalent fractions	22–23	25
Boxed Biscuits	Finding equivalent fractions	24	26

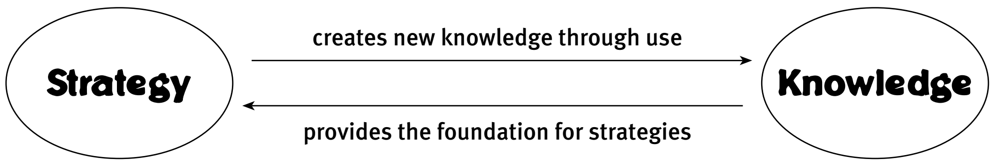
Introduction to Number

There is a remarkable commonality in the way many countries around the world are now teaching arithmetic. Changes in the approaches reflect the evolving demands of everyday life, a greater volume of classroom-based research about how students learn, and a desire to improve general levels of numeracy.

In the past, arithmetic teaching has focused on preparing students to be reliable human calculators. The prevalence of machines in society that calculate everything from supermarket bills to bank balances has meant that students now need a wider range of skills so that they can solve problems flexibly and creatively.

The Figure It Out series aims to reflect these trends in modern mathematics education. A range of books is provided at different levels to develop both number skills and number sense. The *Number* books are aimed at developing students' understanding of the number system and the ability to apply efficient methods of calculation. The *Number Sense* books are aimed at developing students' ability and willingness to apply their number understanding to make mathematical judgments. Teaching number sense requires an emphasis on openness and flexibility in solving problems and the use of communication and interpretation skills.

The development of the Figure It Out series has occurred against the backdrop of a strong drive for improved standards of numeracy among primary-aged students. A key element of this drive has been the creation of the Number Framework, developed as part of the Numerary Strategy. The framework highlights this significant connection between students' ability to apply mental strategies to solving number problems and the knowledge they acquire.



The learning activities in the series are aimed at both the development of efficient and effective mental strategies and increasing the students' knowledge base. Broadly speaking, the levels given in the six year 7–8 *Number* books can be equated to the strategy stages of the Number Framework in the following way:

Link (Book One):	Advanced counting to early additive part-whole
Link (Book Two):	Advanced additive part-whole
Level 4 (Books Three to Five):	Advanced multiplicative to advanced proportional part-whole
Level 4+ (Book Six):	Advanced proportional part-whole.

Page 1 Pizza Party

Achievement Objectives

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- find fractions equivalent to one given (Number, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, levels 1–3)

ACTIVITY

Before the students begin this activity, discuss the relationships that are shown in the Venn diagram. This will help them to develop the logic and reasoning processes that they will need to use in this activity.

The students need to identify the number of students in "and", "or", and "not" relationships. For example, ask the students to point out the people who want:

- mushroom and pineapple toppings
- onion and pineapple but not mushroom
- pineapple or mushroom but not both
- pineapple but nothing else.

Ensure that the students notice that Ms Martell is included in the "whole" of 24 people. Encourage them to express the fractions clearly in words (eight out of twenty-four), in numerals ($^{8}/_{24}$), and as simplified equivalent fractions ($^{1}/_{3}$).

Students who are having difficulty with question **1** could represent their equivalent fractions with materials, drawings, or number forms. For materials, they could use class members to act out the scenarios or use labels to represent each choice and manipulate these. For example, m = mushroom, p = pineapple, and o = onion. The 24 people could be labelled along a number line or along a strip graph.

m	mp	mp	mp	mo	mo	mo	mpo	p	p	p	p	p	p	p	p	po	po	po	po	o	o	o	o
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24

Eight people have mushroom, and 8 + 8 + 8 = 24, so one section of 8 is one part out of the three parts that make 24. So $^{8}/_{24} = ^{1}/_{3}$.

Question **2** renames the whole in two further ways: as 72 because this is the total number of pieces needed for the 24 people and as 12 because this is the total number of pizzas. A ratio table may be a useful tool to show the students the connections between these:

People	24	16	12	8	4	2	0
Pieces	72	48	36	24	12	6	0
Pizzas	12	8	6	4	2	1	0
Unit	1	$^{2}/_{3}$	$^{1}/_{2}$	$^{1}/_{3}$	$^{1}/_{6}$	$^{1}/_{12}$	0

Pages 2–3 Table Tricks

Achievement Objectives

- demonstrate the ability to use the multiplication facts (Number, level 2)
- recall the basic multiplication facts (Number, level 3)
- make sensible estimates and check the reasonableness of answers (Number, level 3)

ACTIVITY

Each question in this activity encourages students to look *within* the array and partition it in such a way that they can easily find the answer. For example, 7 x 2 can be seen as the same as 5 x 2 plus 2 x 2, so a student who doesn't already know the answer can use this idea to work it out. You might also need to explain to those students who already know that 7 x 2 = 14 that partitioning is an easy way to confirm an answer.

Each question contains a pair of related equations. This encourages the students to see a connection between the two parts of the question and to use this to find the answer. Some students may find this hard to do at first, but it's a great way to improve their thinking skills. Many students are so used to treating each calculation as a separate question that they may feel strange using one question to solve another. Keep encouraging them by asking them to look for the things that are the same and the things that are different in the pairs of equations. Ask them how they could use the sameness and differences to work out the answer. For example, in question **1**, they will see that the difference is that the "2" rows have become "4". The 2 has been doubled, so the answer must be doubled because everything else is the same.

The end result of using these patterns and connections is that students will develop an intuitive feel for the distributive property of multiplication and division as well as learning their facts.

Students who find the dot arrays too simple could use the same thinking to multiply larger numbers, for example, 15 x 8 = 120, so 15 x 16 = □. The students could extend this thinking further by using it in related division problems. For example, 15 ÷ 5 = 3, so 30 ÷ 5 = □ or 120 ÷ 15 = 8, so 240 ÷ 15 = □.

Page 4 A Million Grains of Rice

Achievement Objectives

- explain the meaning of the digits in any whole number (Number, level 3)
- make sensible estimates and check the reasonableness of answers (Number, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–4)

ACTIVITY

Explore with your students the relationships that exist between the place values of the digits in seven-digit numbers. Encourage them to make statements such as:
"The place value increases 10 times as we go one place to the left."
"The place value increases 100 times as we go two places to the left."

It would also be worthwhile to establish some key facts about what makes up a million. Questions such as "How many tens, hundreds, and/or thousands are there in one million?" would help to establish the necessary knowledge for the problems.

Ask the students to evaluate the problem-solving strategies suggested on the page. Two of the suggestions will result in the students seeing the million grains from a *volume* perspective, whereas the last suggestion will result in an *area* point of view. They could also explore a strategy for developing a *linear* view of the million grains. These strategies make this activity a wonderful opportunity for integrating number learning outcomes with measurement ones.

In question **1a**, the students look at the mass and area of a million grains of rice. They use the 500 gram packet or the A4 sheet as one unit, and then they have to find out how many grains of rice are in this one unit. They then need to find out how many of the units are needed for one million grains of rice.

Encourage the students to use sensible factors when working out how many grains of rice are in the unit. For example, 10 x 50 = 500, so the students could weigh out 10 grams of rice, count the grains, and then multiply by 50 to find the number of grains in 500 grams. For the A4 sheet, the students could use fractions by folding the sheet into eighths, covering $^1/_8$ of the sheet with grains, and then counting the grains and multiplying the answer by 8.

Question **2** changes the item being worked with from rice to centimetre cubes. These cubes are used in an *area* context, so point out to the students that it is a square centimetre that is the issue here, not cubic centimetres. By using cm^2, this question provides the opportunity to explore the connections between metric units of area, particularly that 10 000 cm^2 = 1 m^2. This fact can be expanded to show that a million cubes cover an area of 100 m^2.

As an extension, the students could make a scale drawing of their house, cut the drawing up into separate rooms, and then see if they can rearrange all or some of the pieces into an area of 100 m^2.

Page 5

Factor Fun

Achievement Objective

- recall the basic multiplication facts (Number, level 3)

ACTIVITY

This is an ideal basic facts game because it's fun and easy to set up and explain, and it can be easily varied by changing the rules. For example, instead of covering four in a square, the students could cover four in a line or find four that have a common factor.

A basic facts game on its own does not ensure that students will recall basic facts. Encourage your students to use the facts they know to work out the ones they are unsure of. For example, if they don't know 7 x 8, they might use 7 x 7 and add 7 or 8 x 8 and subtract 8. Your students can use this game to help them to identify the facts that they don't know. The game will also encourage them to learn their facts so that they become more proficient at playing it.

Any memorisation programme you use with the students should encourage multi-sensory repetition of the fact as well as finding or inventing triggers, such as patterns, to establish long-term recall.

Pages 6–7 Planting with the Whānau

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 3)
- recall the basic multiplication facts (Number, level 3)
- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- write and solve story problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

ACTIVITY

The gardens used in the activity are set out as multiplication arrays. The students will use a range of strategies and a range of operations as they answer the questions.

Discuss with your students the strategies that Māni, Ruawai, and Huia have used. Ask them what facts the characters would have to know to choose the strategies they did. (Māni saw 19 as 10 and 9, Ruawai saw 19 as 20 – 1, and Huia saw 5 as 10 ÷ 2.) Point out that all the characters were looking for an easy way to find a solution, and this is what your students should do as well. Asking them to mentally solve questions such as "If 8 x 5 = 40, then 16 x 5 = □" will help them to see easier ways to find the solution. Initially, they may find this difficult because they may not be used to this type of question, but encouragement and regular practice will improve their thinking dramatically.

Page 8 Coupon Combinations

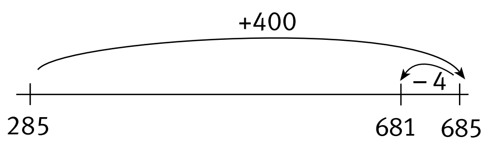
Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 3)
- mentally perform calculations involving addition and subtraction (Number, level 2)
- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)

ACTIVITY

This activity helps students to develop strategies they can use when solving addition problems. Many older students need to revisit the strategies used for addition and subtraction rather than simply applying a memorised routine that gets them the correct answer. Mathematicians are able to see relationships and connections between the problem they are solving and ideas they have met before. So, you need to encourage students learning mathematics to think mathematically.

Encourage the students to work mentally with the problems rather than solve them using a vertical algorithm because they will not use a thinking strategy if they use the vertical algorithm. If students are not able to keep all the numbers in their heads, model a recording method that reflects the thinking strategy used. For example, Simon's thinking could be recorded as:



Page 9 Seeing Double

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 3)
- express a fraction as a decimal, and vice versa (Number, level 4)

ACTIVITY

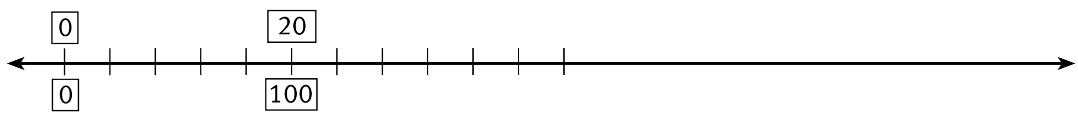
The double number line is a powerful tool for understanding the equivalence of two related numbers. It's called "double" because each mark on the line has two sets of numbers matched to it. The top row of numbers describes the whole represented by the line in one way and the bottom row describes the whole in another way. Because the whole line is the same, it's possible to see the equivalences between the rows of numbers at any point on the line. For example, in question **1**, it's easy to see that 25 out of 100 along the top row is equivalent to 0.25 out of 1 on the bottom row of numbers.

To use a double number line, the students need to know that the starting number (usually 0) for both rows of numbers needs to be at the same point on the line and there needs to be another common point that has two values attached to it. The relationship between these values holds the key to finding other equivalent values.

Question **1** has the second common point at the end of the line where 100 on the top matches 1 on the bottom. Ask the students to use this pair of numbers to explain the relationship along the double number line. Students who are thinking multiplicatively may explain this relationship as "the top number is 100 times greater than the number on the bottom line". If students say "the top line is 99 more than the bottom line", they are thinking additively about the relationship, which is incorrect.

Note the extension in question **5**, where the row of numbers does not show the position for 1. Encourage the students who need help with this to find the position for 1 before they work out the required answer.

A copymaster of empty number lines is provided at the back of this booklet so that students can practise other double number line relationships. The students could work in pairs, with one student placing a zero at some point on the top and bottom of the line and then two other numbers somewhere else. The other person must complete both rows of numbers at each point on the line. For example:



Pages 10–11 Empty Lines

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 3)
- mentally perform calculations involving addition and subtraction (Number, level 2)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, levels 2–4)

ACTIVITY

The important part of this activity is for students to develop thinking strategies and recording methods that are as flexible as the thinking used. Whether or not the addition and subtraction problems are difficult is not important. The key phrase in this activity is "draws his/her thinking".

The empty line differs from the conventional number line in that the spacing between numbers does not have to use steps of equal size. The empty line is not used to measure or count but to show the relationship between the numbers when an operation is performed. The operation may have a number of steps. Each step is shown as a number with its operation sign, with an arrowed line showing the direction as on any number line.

The students will need to see how each of the six characters have called on their own knowledge in deciding which strategy to use. For example, Michael is using his knowledge of the place value of 38 as 3 tens and 8 ones to choose to add 30 first and then 8. This "front-on" method is perfectly logical and appropriate. Hine sees that she can round 47 up to 50 by adding 3, leaving 30 and 5 to be added. Her knowledge of addition facts that make 10 is the basis for choosing this method.

Have the students look for number relationships in the question that might help them to choose a good strategy. In question **1b** (45 + 36), they may see that 45 can be rounded up to 50 by adding 5, and therefore they can use a compensation strategy of adding 5 to one addend and subtracting 5 from the other. On the other hand, they may recognise that both addends are part of the nine times table, so 5 nines are added to 4 nines, and this must be 9 nines, and so the answer is 81.

The students who do not have a good knowledge of subtraction basic facts may not be able to find relationships in question **2** that can be turned into operational strategies. You may need to revise these subtraction basic facts before starting the problems. Include some extended facts as well. Facts such as 100 – 50 = 50 and 600 – 300 = 300 can be related to 10 – 5 = 5 and 6 – 3 = 3. Some students may not be aware of connections like these.

Encourage your students to solve the problems in the way that is easiest for them. Show the students that you value each strategy equally and that the aim is for them to seek the most efficient strategy that they can use, not merely to memorise a strategy that they have been shown.

Page 12 Problems Galore

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, levels 2–4)

ACTIVITY

The students will need to read the problems in this activity carefully to correctly interpret the relationships between the given quantities. The ideas in the bulleted list below will help them to focus on finding these relationships before they think about doing a calculation. Give the students sufficient time to try things out and self-correct before telling them whether they are correct. Encourage them to follow these steps:

- Explore each problem by retelling it in their own words.
- Try to write an equation that shows the quantities in the correct relationship for the problem.
- Check the equation against the question to see if it matches the intention of the problem.
- Rearrange the correct equation in a way that makes it easy and efficient to get a result.
- Estimate before calculating exactly and then ask themselves "Is this a sensible answer?"

The students can then devise and share their own strategy for solving each equation. This is an ideal activity for them to work on in problem-solving groups. Four students per group is a good size.

Problem **1** is presented in a way that directly matches the traditional subtraction equation $514 - 298 = \square$. The students can choose to solve the equation by adding on or by subtracting 300 and adding 2.

Problem **2** may be initially interpreted as $237 - \square = 149$. This can then be adjusted to $237 - 149 = \square$, using knowledge about relationships in equations.

Problem **6** will challenge the students because they may need to come up with two or three equations to explain the relationships in the problem. They may find it helpful to use the characters' names in their first attempts to record an equation. For example, Sam = Alisha – 45 and Sam + Alisha = 163. At this stage, you may need to model the problem-solving strategy of using a simpler example to help your students to see the relationships. For example, "If Sam = Alisha and Sam + Alisha = 10, what would you do?" or "If Sam = Alisha + 2 and Sam and Alisha = 10, what would you do?" If the students can solve this, they should then be able to relate their strategy to the larger quantities in the problem.

The students could start abbreviating names to first letters to make recording the equation more efficient. This would help many students to understand the role of pronumerals in equations.

Page 13 Fun Times

Achievement Objectives

- explain the meaning of the digits in any whole number (Number, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

ACTIVITY

This activity is similar to that on page **12**, but it uses multiplication and division contexts. As in the activity on page **12**, the thinking strategies used to get to the answers are the most important part of the questions.

The students could again work in co-operative, problem-solving groups of four.

Question **1** is a conventional "sets of" context, so the likely equation will be 5 x 19 = □. Emphasise to your students that their job as mathematicians is to find an easy and efficient way to handle the problem but that there is more than one way to do this. Thinking strategies that involve counting or addition, such as adding 5 nineteen times, may be easy, but they are not efficient. Strategies such as knowing that 5 x 20 is 100 and recognising that 5 x 19 is one 5 fewer, so the answer is 95, or that 10 times 19 is 190, so 5 times 19 is half of this, show that a student has developed more advanced thinking skills and will have more choices of methods to solve problems.

The equation for question **3** will have three factors, 4 x 7 x 48. Encourage the students to reorganise the factors in a way that they find easy. They may find that it is easier to multiply the 48 x 4 and then multiply by 7. If the equation were turned into prime factors, the students would get 7 x 3 x 2 x 2 x 2 x 2 x 2 x 2. This means that 21 (the 7 x 3) is doubled six times (42, 84, 168, 336, 672, and 1 344). The answer is 1 344. You may wish to share this idea with your students.

One of your most important duties as a teacher of mathematics is helping students to see the connections between what they already know and what they have to find out.

Page 14 Divisive Tactics

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, levels 2–3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)

ACTIVITY

Like those on the previous two pages, this activity helps students to develop thinking strategies. Allow the students to ask for the answer to a number fact that they need to help them with their thinking. This fact can be recorded for memorisation at a later date. Keep the students focused on looking for connections between what they know and what the problem is asking them to do.

For question **1**, which involves the equation 252 ÷ 6 = □, the students may see that it's useful to break 252 into 240 + 12 because of the fact that 24 ÷ 6 = 4, so 240 ÷ 6 = 40. They will be more likely to see this connection for themselves if you revise multiplying and dividing a number by 10 in the maintenance part of the lesson before the students attempt these problems.

For question **3**, the students will need to find two equations and compare them: □ ÷ 6 = 17 and □ ÷ 4 = 25. By changing these to multiplication, they will find two answers, 102 and 100. You may need to direct them back to the question, which asks for the lowest number needed to fit both situations.

INVESTIGATION

For question **1**, the students may need to model a similar situation with easier numbers to see the multiplication relationship between the number of streets in each direction and the number of intersections.

Question **2** has a number of interesting possibilities, but some students may feel overwhelmed by the three sets of relationships involved, namely the 2 minute, 3 minute, and 4 minute tracks. If they get frustrated, suggest that they explore the number of tracks if a 2 minute track was followed by a 3 minute track and this was followed by a 4 minute track. This should help them see that it takes 9 minutes to run one track of each length, and so, after 90 minutes, they will have done 10 tracks of each type. At this stage, they may need to look at the question again to see that it asks for the total number of tracks, which is 30.

Page 15 More Problems

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, levels 2–3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)

ACTIVITY

Once again, the instruction not to use a calculator encourages students to focus on developing and recording strategies rather than on answers. The equations and diagrams that the students produce should reflect their actual thinking and may not be the conventional working forms. You should encourage this. As with the earlier pages, you could ask the students to work in problem-solving groups of four to encourage peer teaching and maximum involvement in the class.

The strategies shown on pages **10–11** of the student book would be good models for this activity.

The hint in question **2** is intended to encourage the students to find pairs that make 100 (that is, 74 and 26, 38 and 62, and 47 and 53), but some students may choose to pair up 74 with 47 to get 121 and 62 and 26 to get 88.

Question **5** presents a challenging set of relationships. There are two unknowns, Nina's and Goran's scores, so the relationship between them will need to be examined after the other scores are subtracted. Those students who are having difficulty may need hints such as "How much did Nina and Goran score together?" and "If they had scored the same amount, how much would they have each scored?" At this stage, you could present the students with a table that shows their findings and the relationship between them.

Nina		Goran		Total	Difference between them
16	+	16	=	32	0
17	+	15	=	32	2
☐	+	☐	=	32	6

As an extension, you could ask the students more questions that follow the model of questions **3** and **4**. These present a different cognitive challenge to students than the more common result-unknown problems.

Page 16

Expanding Your Horizons

Achievement Objectives

- explain the meaning of the digits in any whole number (Number, level 3)
- make sensible estimates and check the reasonableness of answers (Number, levels 2 and 3)
- recall the basic addition and subtraction facts (Number, level 2)

ACTIVITY

A copymaster of compact numeral cards is provided at the end of these notes.

Make sure that the students know how to use the cards. The arrow on the cards helps the students to align the numeral cards correctly so that the compact number is visible. You may need to show some students how to organise the cards to make a number by placing the card with the largest number on the bottom and then the other cards on top in ascending order. Have the students organise their set of cards in place value order and in size order.

Before the students attempt the questions, have them experiment with the cards to make numbers using different combinations. Ensure that they vary the number of cards so that they see the "0" used as a placeholder. Watch out for students who try to use two cards from the same column because this shows that they do not understand the cards.

Question **1** is a good activity to develop a sense of number size and place value.

You could extend question **2** by challenging the students to find all the possible ways to make the numbers. This would encourage them to use a systematic approach to finding all the combinations.

In question **3**, the students need to work out the quantity as well as the operation required to change the display. Have them estimate a sensible answer before calculating the result.

Page 17 Expanding with Decimals

Achievement Objectives

- explain the meaning of the digits in decimal numbers with up to 3 decimal places (Number, level 3)
- order decimals with up to 3 decimal places (Number, level 3)
- make sensible estimates and check the reasonableness of answers (Number, level 3)

ACTIVITY

A copymaster of decimal compact numeral cards is provided at the end of these notes.

This activity builds on the last activity by introducing decimals. Note that the arrows are now on the left side so that the digits can be aligned from the ones place. Note also that the card with the smallest number (which has the most digits) goes on the bottom when a compact numeral is made.

Question **1** challenges students' perceptions of the relationship between the number of digits and the size of decimal numbers. Many students judge the size of numbers by the number of digits they contain. This will lead to misunderstandings because a one-digit decimal can be larger than a three-digit decimal. Ask the students "What do you look for to judge the size of a decimal number?" A good response would indicate that the first significant digit *to the right* of the decimal point is a useful place to start (for example, the 2 in 57.23).

Point out to your students that the job of the decimal point is to indicate the position of the ones place. They can then ascertain the value of all other digits. Make sure that the students do not use a column space for the decimal point. If they do, they often lose the relationship between the ones place and the tenths place.

In question **3**, the students could approach the comparisons as difference questions initially to find the quantity involved. They could then look at the relationship suggested by the arrow to see if the quantity should be added or subtracted.

Pages 18–19 Placing Points

Achievement Objectives

- explain the meaning of the digits in decimal numbers with up to 3 decimal places (Number, level 3)
- explain the meaning of the digits in any whole number (Number, level 3)
- make sensible estimates and check the reasonableness of answers (Number, level 3)

ACTIVITIES ONE AND TWO

An important learning outcome of these activities is that students understand the effect on place value of multiplying and dividing by 10 and multiples of 10.

The students need to focus on the way the digits change place. The common rule often given to students to "move the decimal point" should not be introduced too soon as students who learn this as a rote rule don't develop the essential understanding of the effect on the digits. The explanations that the students give in question **b** of **Activity One** should indicate that they understand that:

- multiplication by 10 moves digits one place to the left
- division by 10 moves digits one place to the right
- multiplication by 100 moves digits two places to the left
- division by 100 moves digits two places to the right.

These rules can then be extended to include 1 000 as a factor.

ACTIVITY THREE

The relationship between the pairs of numbers throughout this activity and **Activity Four** is multiplicative. If the students look for an additive relationship, show them the difference between additive views and multiplicative views by looking at two numbers like 3 and 30. Additive thinkers will see the relationship as "27 more" while multiplicative thinkers will see it as "10 times greater". The students who cannot understand the multiplicative view will find these activities difficult.

An interesting extension would be to allow the students to use the decimal point button on their calculator. By doing this, they would have a choice of either dividing by 0.1 or multiplying by 10. You can extend this to show the equivalence between multiplying by 0.01 and dividing by 100.

ACTIVITY FOUR

The students should realise from the keys given that all the relationships must involve multiples of 5.

The change in question **c** requires the students to multiply by 5 and then multiply by 5 again. You can help them to see the connections in this change if you rewrite the 5 as 5.0 and then compare it to 0.2. In a vertical mode, this would look like this:
5.0
0.2
They should see from this that multiplication will be needed to turn the 0.2 into the whole number, 5.

The change in question **d** requires repeated division by 5. From the vertical form
1.00
0.04
the students should realise that the 1 will need to be divided to get the smaller changed number.

You could use the change in question **c** to discuss how the constant function on a calculator works. When multiplication is involved in a constant function arrangement on a calculator, the factor that is to be repeated must be entered first. To get the correct answer of 5, they would need to press [5] [x] [0.2] [=] [=]. (If 0.2 is entered first, [0.2] [x] [5] [=] [=], they would get an answer of 0.2 because 0.2 would be the factor being repeated.)

Page 20 Getting the Point

Achievement Objectives

- explain the meaning of the digits in decimal numbers with up to 3 decimal places (Number, level 3)
- express a fraction as a decimal, and vice versa (Number, level 4)

ACTIVITY

The important feature of this activity is that the whole (in this case, a beads frame) is separated into two parts in the corners. These parts can therefore be represented as a fraction of 100 if the whole is considered to be 100 out of 100.

Make sure that the students notice that the fractions shown in question **1** are complementary because they always add to 1 or $^{100}/_{100}$. A useful warm-up activity may be to have the students make a chart showing some addition complements to 100, for example:

10 + 90 = 100, 20 + 80 = 100, 30 + 70 = 100, …
5 + 95 = 100, 15 + 85 = 100, 25 + 75 = 100, …

You could also explore question **2** by using a calculator as well as the abacus mentioned in the question. If the students press [1][÷][3] and [=] on a calculator, the display will show 0.3333333. If they then multiply this by 3, the calculator display will show 0.9999999 rather than 1. This rounding error is caused by the problem that 1 cannot be divided by 3 exactly in a decimal system, and so a recurring decimal is formed.

Page 21 Mystery Fractions

Achievement Objective

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)

ACTIVITY

This activity examines the use of a ratio table or double number line as a tool to find the fractional amounts of a set. The students need to recognise the "whole" to which the fractions refer. The whole in this case is the total number of cubes in the jar, not the jar itself.

Some students may find fractions with a numerator greater than 1 difficult to handle. This is usually because they do not fully understand the meaning of the fractional notation. Check that the students do understand $^3/_4$ as three lots of one-quarter and $^3/_5$ as three lots of one-fifth.

Observe how the students attempt the problem. Look to see if they know to share equally by the number indicated in the denominator. For example, $^2/_3$ will involve equal sharing by 3. If they can make equal shares, do they then use the numerator to decide how many equal shares to consider? Some students may use addition to find the complete fraction while others use multiplication. Have the students share their strategies.

Notice the ratio table device used by Simon. This is an excellent tool for developing proportional thinking in students. This ratio table does not use distance to show the relationships, as the double number line does; it is simply a number table.

Pages 22–23 Classy Courtyards

Achievement Objectives

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- find fractions equivalent to one given (Number, level 4)

These activities encourage students to visualise equivalent fractions of the whole as well as of the parts within the whole. You may need to introduce the students to the convention of changing a fraction to its simplest equivalent form.

ACTIVITY ONE

Before your students attempt the questions, draw their attention to the courtyard in the first four designs. They should identify this as the "whole" to which the fractional parts refer. They should also note the equivalent fractions shown for the whole in each design. The first design shows the whole as $^4/_4$, the second and third designs show the whole as $^{16}/_{16}$, and the fourth design shows the whole as $^{64}/_{64}$. These are all equivalent forms of 1.

As an extension, the students could explore other equivalent fractions for one whole. When you are working with the students on these fractions, record them in a random number order, such as $1 = {}^3/_3 = {}^5/_5 = {}^2/_2 = {}^{10}/_{10} = {}^4/_4$, rather than in a sequential pattern that can result in the misunderstanding that the size is increasing, as in $1 = {}^2/_2 = {}^3/_3 = {}^4/_4 = {}^5/_5 = {}^{10}/_{10}$.

As the students complete all the questions for each design, it would be a good idea to have them combine the fractions for each part so that they can see that it makes the whole.

Design 1 will have $^2/_4 + ^1/_4 + ^1/_4 = 1$, which simplifies to $^1/_2 + ^1/_4 + ^1/_4 = 1$.
Design 2 will have $^8/_{16} + ^4/_{16} + ^4/_{16} = 1$, which simplifies to $^1/_2 + ^1/_4 + ^1/_4 = 1$.
Design 3 will have $^8/_{16} + ^4/_{16} + ^4/_{16} = 1$, which simplifies to $^1/_2 + ^1/_4 + ^1/_4 = 1$.
Design 4 will have $^{32}/_{64} + ^{16}/_{64} + ^{16}/_{64} = 1$, which simplifies to $^1/_2 + ^1/_4 + ^1/_4 = 1$.

This may help them to see the equivalence connections and how the simplified form makes the equivalences obvious.

Question **5** asks the students to compare the fractions planted in herbs. This could be recorded in chart form:

Design	Fraction shown	Simple form
1	$^1/_4$	$^1/_4$
2	$^4/_{16}$	$^1/_4$
3	$^4/_{16}$	$^1/_4$
4	$^{16}/_{64}$	$^1/_4$

All of them show that $^1/_4$ of the courtyard is planted in herbs.

ACTIVITY TWO

This activity uses a non-square visualisation of a whole and enables denominators that are not square numbers to be imaged.

For question **4**, you could ask the students to record the fractions of the parts as $^6/_{18}$ and as $^1/_3$. As with David's other designs, the students' designs will not be square. They could also record an equation such as $^1/_3 + ^1/_3 + ^1/_3 = 1$ to show the whole courtyard.

Page 24 Boxed Biscuits

Achievement Objectives

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- find fractions equivalent to one given (Number, level 4)

ACTIVITY

In the warm-up phase of the lesson, before the students begin this activity, have them review factors. They could list the factors of 24, 27, 36, and 100 on a chart. For example, the factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

Introduce the activity and ask the students to discuss how the factors of the numbers on the boxes can help them to decide the size and number of the smaller packs of biscuits.

They could start by listing the combinations. For example:

Total number	Number of packs	Biscuits in each pack
24	4	6
24	6	4
24	8	3
24	3	8
24	12	2
24	2	12

The students may need help to develop a chart to solve the problems. When they have worked out the number of biscuits in each pack, they need to focus on the fractions of each kind of biscuit. For example, in the list above, there are only two kinds of biscuit, so the three biscuits in eight packs won't work.

Ensure that they have headings like

Packs	Mixture	Fraction

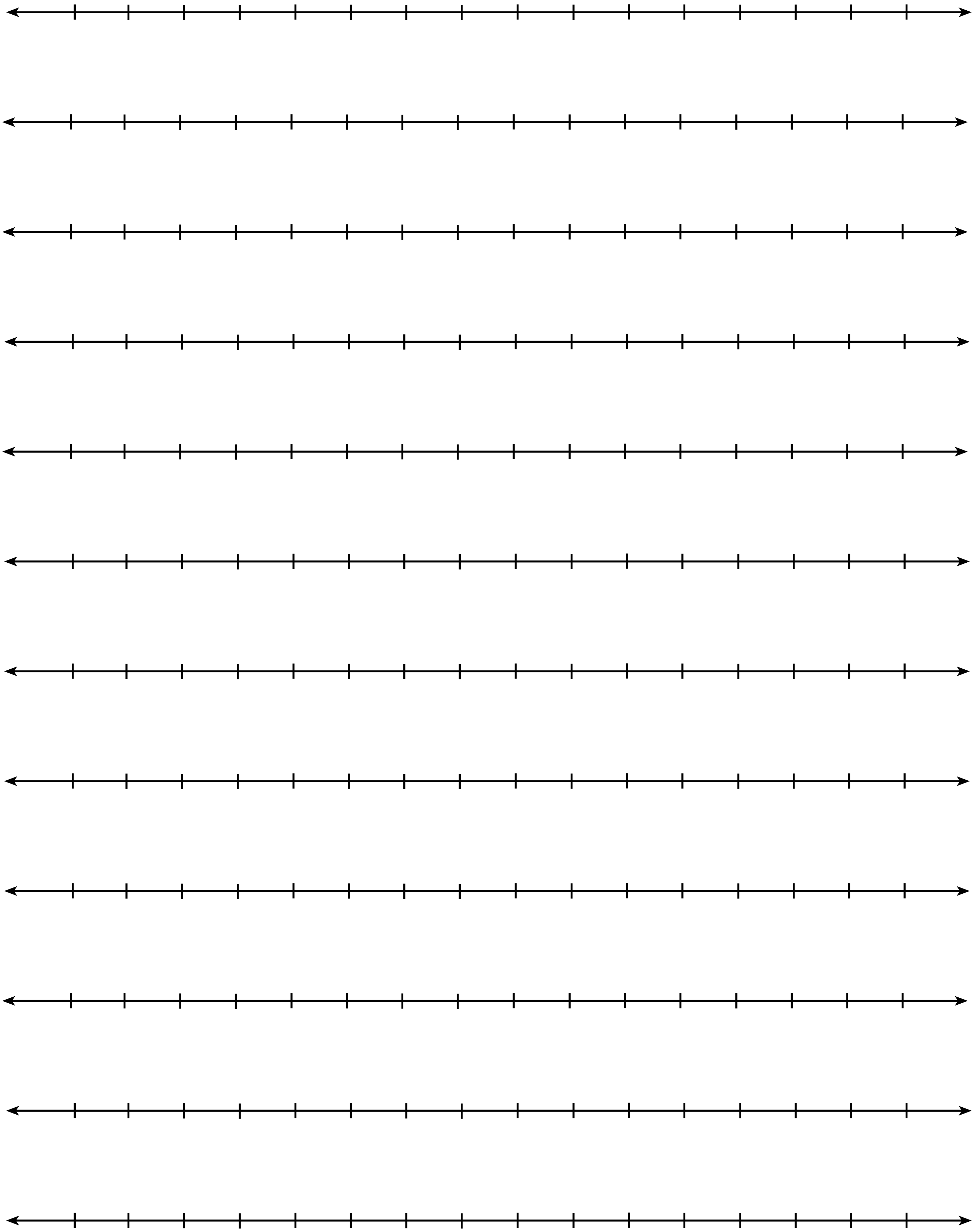
The students may like to use a spreadsheet to make their chart.

This activity features a "set" model for the whole. The size of the whole changes with each box. This is an opportunity to show how a fraction like $^1/_2$ can be different in size as the whole set changes. A fraction is always relative to the whole. So, despite the change in size of the packs in the 24 box, the proportion that is apricot is still $^1/_2$.

Students who can explain equivalent fractions using materials and images should be challenged to see how to find equivalences using numbers. This will help them to use and understand the number property used to calculate equivalences. The key idea here is that the relationship between the fractions needs to be kept equivalent, so the operation used to change the numbers is multiplication or division by 1. The factor of 1 needs to be written in a suitable equivalent fraction form. For example, to change $^1/_4$ to an equivalent such as $\square/_{20}$, use $^1/_4$ x $^5/_5$ = $^5/_{20}$. Point out that $^5/_5$ is a form of 1, so $^1/_4$ is being multiplied by 1 to keep the equivalence.

Copymaster

Double Number Lines



Copymaster **Compact Numeral Cards**

9	8	7	6	5	4	3	2	1
90	80	70	60	50	40	30	20	10
900	800	700	600	500	400	300	200	100
9000	8000	7000	6000	5000	4000	3000	2000	1000

Decimal Compact Numeral Cards

0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1.	2.	3.	4.	5.	6.	7.	8.	9.

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