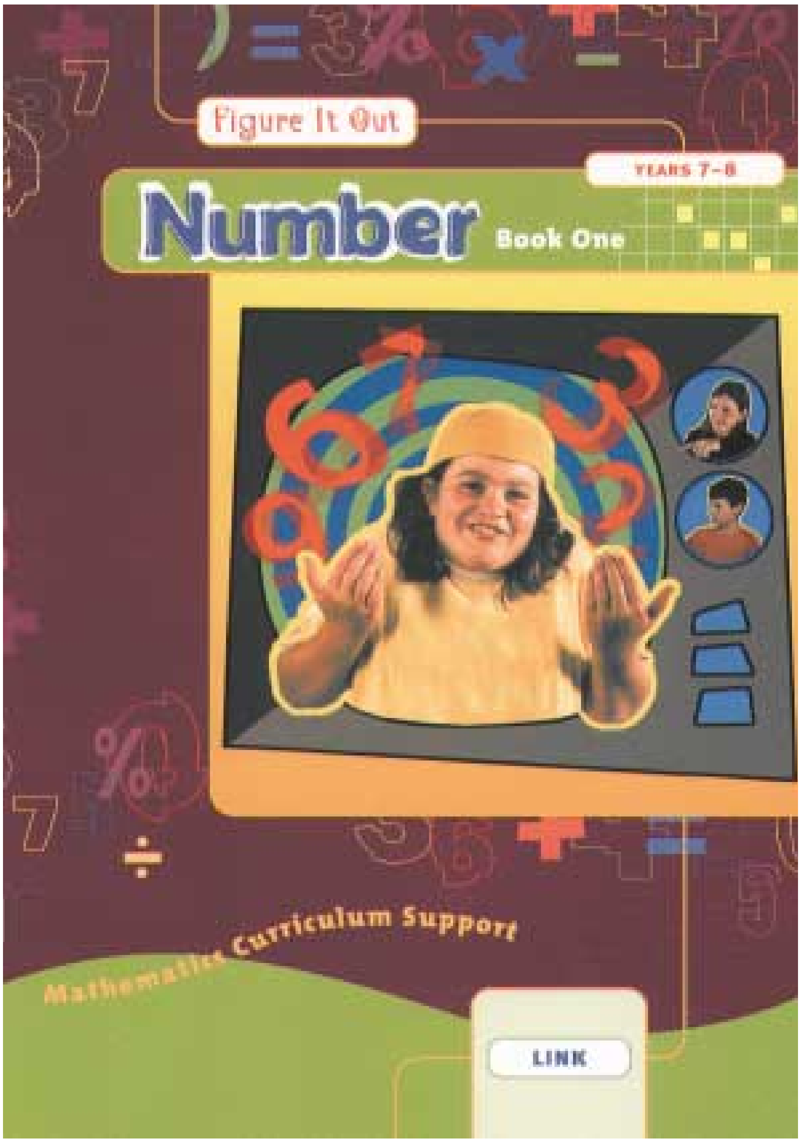
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

CONTENTS

The books for years 7–8 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand year 7–8 classrooms. The books have been developed and trialled by classroom teachers and mathematics educators and follow on from the successful series for levels 2–4 in primary schools.

Student books

The student books in the series are divided into three curriculum levels: levels 2–3 (linking material), level 4, and level 4+ (extension material). All the books are aimed at year 7–8 students in terms of context and presentation.

The following books are included in the series:
Number (two linking, three level 4, one level 4+)
Algebra (one linking, two level 4, one level 4+)
Measurement (one level 4, one level 4+)
Number Sense (one linking, one level 4)
Geometry (one level 4, one level 4+)
Statistics (one level 4, one level 4+)
Themes (level 4): *Disasters, Getting Around*
These 20 books will be distributed to schools with year 7–8 students over a period of two years, starting with the six *Number* books.

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and the life experiences that are meaningful to students aged 11–13 years. The activities can be used as the focus for teacher-led lessons, as independent activities, or as the catalyst for problem solving in groups.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the student books includes full answers and explanatory notes. Students can use them for self-marking, or you can use them for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, comments on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/community

Using Figure It Out in your classroom

Where applicable, each page starts with a list of equipment that the students will need to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

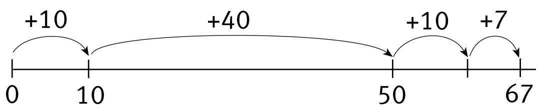
Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Answers Number: Book One

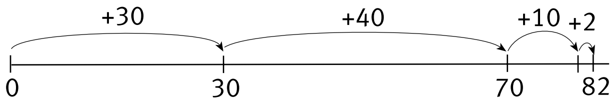
Page 1 Archery Addition

ACTIVITY

1. Toline Round 2:



Round 3:



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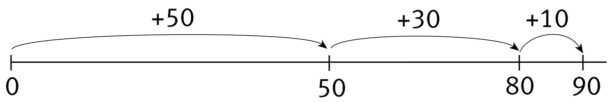

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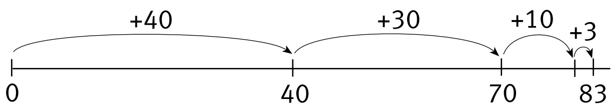
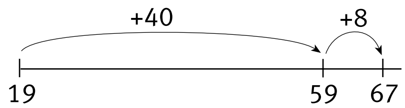


Michael Round 2:



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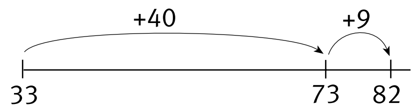

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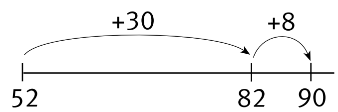
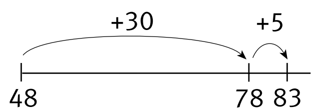


Round 5:



2. 37

Pages 2–3 That's Odd!

ACTIVITY ONE

1. They cannot make a complete set of pairs. There is always one left over.

2. Even. The odd (remainder) units combine to form a unit of 2 (a pair).

3. a. i. 18

ii. 22

iii. 104

iv. 14

v. 24

b. Two odd numbers add up to an even number.

4. a. The answer is even.

b. The answer is odd.

5. a. 4. Odd – odd gives even.

b. 6. Even – even gives even.

c. 7. Odd – even gives odd.

d. 7. Even – odd gives odd.

ACTIVITY TWO

1. Even. Each set of eight can be divided evenly by 2.

2. a. i. 14 (even)

ii. 30 (even)

iii. 32 (even)

iv. 48 (even)

v. 27 (odd)

b. Even x even = even

Odd x odd = odd

Even x odd = even

Odd x even = even

ACTIVITY THREE

1. **a.** Because they are all even numbers

   b. The remainder is 1. 7, 19, 35, and 4 963 are all odd numbers. When odd numbers are divided by 2, the remainder is always 1.

2. **a.** 28 **c.** 96 **e.** 44

Pages 4–5 Fives and Tens

ACTIVITY

1. **a.** **i.** 20

   ii. 20

   b. **i.** 30

   ii. 30

   c. Each pair has the same answer. It works because there is the same number of shapes in each array, but they are arranged differently.

   d. 4 x 10

2. **a.** 4 x 5 = 20, 4 x 6 = 24

   b. 6 x 5 = 30, 6 x 6 = 36

   c. 8 x 5 = 40, 8 x 6 = 48

   d. 7 x 5 = 35, 7 x 6 = 42

   e. 5 x 5 = 25, 5 x 6 = 30

3. **a.** 3 x 10 = 30, 3 x 9 = 27

   b. 4 x 10 = 40, 4 x 9 = 36

   c. 7 x 10 = 70, 7 x 9 = 63

   d. 9 x 10 = 90, 9 x 9 = 81

   e. 5 x 10 = 50, 5 x 9 = 45

   f. 8 x 10 = 80, 8 x 9 = 72

4. **a.** 4 x 5 = 20, 4 x 4 = 16

   b. 6 x 5 = 30, 6 x 4 = 24

   c. 9 x 5 = 45, 9 x 4 = 36

   d. 7 x 5 = 35, 7 x 4 = 28

   e. 3 x 5 = 15, 3 x 4 = 12

Page 6 Fund-raising

ACTIVITY

1. 1 000

2. **a.** 80 to catch up with James, 140 to catch up with Atareita, and 160 to catch up with Frances

   b. 160 more than Mira, 140 more than Paora, 80 more than James, and 20 more than Atareita

3. 170

4. **a.** 17 cartons and three extra boxes

   b. $1,730

Page 7 Jungle Land

ACTIVITY

1. 600

2. 400

3. **a.** 300

   b. 700

4. **a.** Stephanie: 300; Jamie: 100; Tristan: 400

   b. Stephanie: waterfall, swimming hole, or Up the Creek; Jamie: minigolf; Tristan: 4WD Safari

5. Answers will vary.

Page 8 Space Zapper

GAME

A game of counting in 100s and 1 000s

Page 9 Weigh to Go

ACTIVITY

1. red

2. blue

3. **a.** yellow

   b. white

   c. white

   d. white

   e. Di Etting: $9.50; Mr E. L. Ness: $15; Sarah Fit: $17; G. Nome: $24

4. Answers will be one of:
   - two fruit bars
   - one herb box, one muesli bar
   - one herb box, one fruit bar
   - two herb boxes.
5. a. Answers will vary. For example:
      - honey, yoghurt mix, muesli bar (375 g)
      - yoghurt mix, muesli bar, fruit bar (225 g)
      - muesli bar, spice box, fruit bar (250 g).

   b. Answers will vary. For example:
      - two honey, one herb box (425 g)
      - two yoghurt mixes, one carob brick (350 g)
      - two carob bricks, one muesli bar (375 g).

   c. Answers will be one of:
      - three carob bricks (450 g)
      - three spice boxes (375 g)
      - three yoghurt mixes (300 g)
      - three muesli bars (225 g)
      - three fruit bars (150 g).

Page 10 Resolving Differences

ACTIVITY

1. a. i. 48 and 38; 22 and 12

      ii. 30 and 29; 23 and 22

      iii. 43 and 41; 14 and 12

      iv. 48 and 43; 43 and 38; 34 and 29; 22 and 17; 17 and 12

   b. 36 (the difference between 48 and 12)

   c. 48 and 12; 38 and 22; 43 and 17

2. a.

82	86	96
60	68	76
44	46	58
24	28	34

   b. i. 96 and 86; 86 and 76; 68 and 58; 44 and 34; 34 and 24

      ii. 82 and 76; 34 and 28

      iii. 60 and 58; 46 and 44

   c. Yes. They are all even numbers, so any differences are also even (even – even = even). (All even and odd numbers, when multiplied by 2, are even.)

3. a.

410	430	480
300	340	380
220	230	290
120	140	170

   b. i. 480 and 380; 220 and 120

      ii. 480 and 430; 430 and 380; 340 and 290; 220 and 170; 170 and 120

      iii. 430 and 410; 140 and 120

      iv. 300 and 290; 230 and 220

4. Answers will vary. The answers in **1a** are similar to those in **3b** except for the zeros at the end. This is because the new numbers in **3b** are 10 times larger than those given in **1a**.

Page 11 Table Talk

ACTIVITY

1. Answers may vary. A chart for the least tables required would be:

	Number of tables	Chairs
Lee	3	7
Hugh	5	11
Pickle	1	3
Jellyman	4	9
Stew	2	5

2. Yes. Emeli will need 14 tables for the bookings that she already has and three tables for the new group of eight, a total of 17.
3. Yes. One large table will seat eight, but at present, Emeli needs three small tables to seat eight. So, she could seat 72 people in groups of eight (9 x 8) on the large tables but only 48 (6 x 8) using three small tables for each group.

Page 12 Sums and Products

ACTIVITY

1. a. 1 and 4
   b. 2 and 5
   c. 4 and 3
   d. 2 and 6
   e. 2 and 8
   f. 7 and 5
2. a. Answers will need to be two of the following:
   6 + 4. 6 + 4 = 10, 6 x 4 = 24;
   3 + 8. 3 + 8 = 11, 3 x 8 = 24;
   2 + 12. 2 + 12 = 14, 2 x 12 = 24;
   1 + 24. 1 + 24 = 25, 1 x 24 = 24.
   b. Answers will need to be two of the following:
   5 + 6. 5 + 6 = 11, 5 x 6 = 30;
   3 + 10. 3 + 10 = 13, 3 x 10 = 30;
   2 + 15. 2 + 15 = 17, 2 x 15 = 30;
   1 + 30. 1 + 30 = 31, 1 x 30 = 30.
   c. Answers will vary.
3. Answers will vary. For example, any number multiplied by 1 will fit:
   3 and 1 because 3 + 1 = 4 and 3 x 1 = 3;
   78 and 1 because 78 + 1 = 79 and 78 x 1 = 78

Page 13 Container Contents

ACTIVITY

1. 8
2. a. 8
   b. 9
   c. 6
   d. 8
3. Answers will vary. You could use a table to show the various options:

Number of rows of vans	Number of rows of cars	Number of vans in container	Number of cars in container	Total vehicles in container
1	5	3	20	23
2	4	6	16	22
3	3	9	12	21
4	2	12	8	20
5	1	15	4	19

Page 14 Caught on Camera

ACTIVITY

1. a. 35 km/h
   b. 28 km/h
   c. 14 km/h
   d. 41 km/h
2. Answers will vary. For example:
   - 36 km/h because that is the fine for the more serious offence
   - The average of both tickets (61 km/h), so 39 km/h as a penalty
   - 28 km/h because the total speed over 50 km/h was 22 km/h (8 + 14)
   - Do the new speed for one ticket for the first month and for the other ticket the next month.
3. The person wouldn't be able to drive at all. He or she was 54 km/h over the speed limit.
   50 – 54 = ⁻4, and it wouldn't be practical to drive backwards at 4 km/h.
4. a. 103 km/h
   b. 96 km/h
   c. 89 km/h
   d. 112 km/h

Page 15 King of the Castle

GAME

A game using addition strategies

ACTIVITY

1. a. Answers may vary. The best strategy is to put the largest numbers in the middle squares so that they can be added to both the left and the right.
   b. This time, the best strategy is to put the largest numbers on the outside.
2. Answers will vary. To get the highest possible number in the top layer, you need to have the 8 in the centre of the bottom row and the lowest numbers (4 and 5) on the outside. Possible orders are: 4, 7, 8, 6, 5; 5, 7, 8, 6, 4; 4, 6, 8, 7, 5; 5, 6, 8, 7, 4. With all these options, you get 109 in the top layer.

Pages 16–17 Firewood Fever

ACTIVITY

1. a.

Sticks found	Bundles of 10	Loose sticks	Total
8		8	8
6	1	4	14
7	2	1	21
8	2	9	29
16	4	5	45
9	5	4	54
17	7	1	71
6	7	7	77

   b. 23

2. 92

3. a. 85

   b. 72

   c. 63

   d. 38

   e. 22

   f. 17

4. One bundle and seven single sticks

Page 18 Down with Darts

ACTIVITY

1. Charlotte. (She has 55, Lawrence has 56, and the lower score is winning.)

2. a.

	Charlotte	Lawrence
Turn 5	28	42
Turn 6	8	24

   b. Charlotte: double 4; Lawrence: double 12 or triple 8

Page 19 Absolutely Abseiling

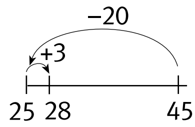
ACTIVITY

1. 16 m

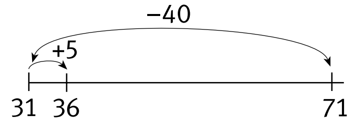
2. 28 m

3. Answers and possible number lines are:

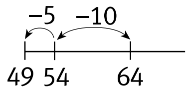
   a. 17 m



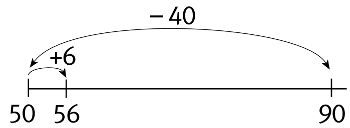
   b. 35 m



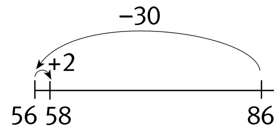
   c. 15 m



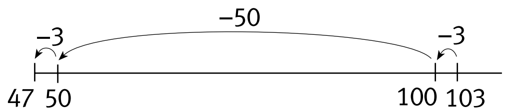
   d. 34 m



   e. 28 m



   f. 56 m



Page 20 Piece of Cake

ACTIVITY

1. 3

2. a–b. Pictures will vary. Possible fraction sentences are:

   i. $^1/_6$ of 18 = 3 (so the picture should show three candles on each sixth)

   ii. $^1/_7$ of 21 = 3 (so the picture should show three candles on each seventh)

   iii. $^1/_{12}$ of 60 = 5 (so the picture should show five candles on each 12th)

   iv. $^1/_5$ of 25 = 5 (so the picture should show five candles on each fifth)

   v. $^1/_{10}$ of 100 = 10 (so the picture should show 10 candles on each 10th)

3. a. i. $^1/_2$

   ii. $^2/_3$

   iii. $^3/_5$

   iv. $^2/_6$ or $^1/_3$

b. i. 6

ii. 8

iii. 9

iv. 4

Page 21 Helping with the Hāngi

ACTIVITY

1. a. Five packs

   b. One potato, half a kūmara (two pieces), half a kamokamo (five pieces)

2. Four potatoes, two kūmara, one slab of meat, no extra kamokamo

3. a. 1

   b. Two quarter pieces of kūmara: $^2/_4 = {}^1/_2$; one piece of kamokamo: $^1/_{10}$

4. a. 10

   b. $^1/_4$

Page 22 Chocolate Chip Feast

ACTIVITY

1. a.–b. Two possible ways are:
   $^1/_2 + {}^1/_4$ or $^1/_4 + {}^1/_4 + {}^1/_4$ (or $3 \times {}^1/_4$)

2. a. The Superchoc sign ($^3/_4$)

   b. The Nibbler sign ($^1/_4$)

   c. Superchoc: tray iv

   Freckles: tray i

   Spotties: tray ii

   Nibbler: tray v

   Chocaholics: tray iii

Page 23 All Bottled Up

ACTIVITY

1. a. Two cups

   b. Four cups

   c. Six cups

   d. Three and a half ($3^1/_2$) cups

   e. 15 cups

   f. 40 cups

2. Four cups

Page 24 Bits and Pieces

ACTIVITY

Answers and some possible strategies include:

1. 15. ($30 \div 2 = 15$. Half of 30 is 15.)

2. 21. ($^1/_4$ of 28 is 7, so $^3/_4$ is $3 \times 7 = 21$.
   Or: $28 \div 4 = 7$. $3 \times 7 = 21$)

3. $^{10}/_{15}$, which is $^2/_3$.

4. 30. ($50 = 5 \times 10$, so 5×6 are liquorice.
   $5 \times 6 = 30$)

5. a. 7. ($35 \div 5 = 7$)

   b. $^2/_5$ (the same proportion as in one packet)

6. $^2/_3$. ($12 + 6 = 18$. $^{12}/_{18} = {}^2/_3$)

Teachers' Notes

Overview **Number: Book One**

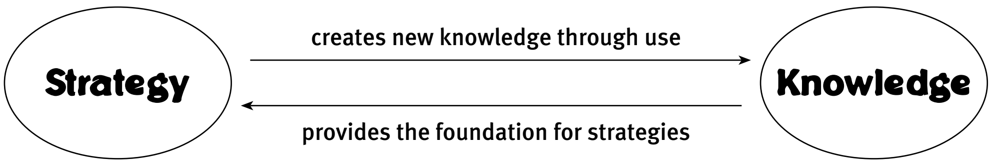
Introduction to Number

There is a remarkable commonality in the way many countries around the world are now teaching arithmetic. Changes in the approaches reflect the evolving demands of everyday life, a greater volume of classroom-based research about how students learn, and a desire to improve general levels of numeracy.

In the past, arithmetic teaching has focused on preparing students to be reliable human calculators. The prevalence of machines in society that calculate everything from supermarket bills to bank balances has meant that students now require a wider range of skills so that they can solve problems flexibly and creatively.

The Figure It Out series aims to reflect these trends in modern mathematics education. A range of books is provided at different levels to develop both number skills and number sense. The *Number* books are aimed at developing students' understanding of the number system and their ability to apply efficient methods of calculation. The *Number Sense* books are aimed at developing students' ability and willingness to apply their number understanding to make mathematical judgments. Teaching number sense requires an emphasis on openness and flexibility in solving problems and the use of communication and interpretation skills.

The development of the Figure It Out series has occurred against the backdrop of a strong drive for improved standards of numeracy among primary-aged students. A key element of this drive has been the creation of the Number Framework, developed as part of the Numeracy Strategy. The framework highlights this significant connection between students' ability to apply mental strategies to solving number problems and the knowledge they acquire.



The learning activities in the series are aimed both at developing efficient and effective mental strategies and at increasing the students' knowledge base. Broadly speaking, the levels given in the six year 7–8 *Number* books can be equated to the strategy stages of the Number Framework in the following way:

Link (Book One):	Advanced counting to early additive part-whole
Link (Book Two):	Advanced additive part-whole
Level 4 (Books Three to Five):	Advanced multiplicative to advanced proportional part-whole
Level 4+ (Book Six):	Advanced proportional part-whole.

Note: Fraction circle diagrams and double number lines are useful for teaching fractions. Copymasters for these are provided at the end of these notes.

Page 1

Archery Addition

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, levels 2–4)

ACTIVITY

Some students in year 7 and 8 classes solve problems like these using only counting strategies. Many others use a ritualistic memorisation of the vertical working form (algorithm). Although they can find the correct answer, they may have missed the opportunity to develop the true additive thinking strategies that are vital to their mathematical development.

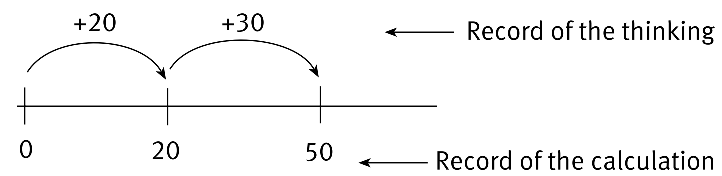
These thinking strategies are modelled in the diagrams and speech bubbles in this activity. They are examples of the ways in which students can use their knowledge of number relationships to find easy and efficient ways of calculating.

Avoid instructing your students to simply copy these ways of thinking because this would result in them trying to memorise another "way" of doing the sum. Encourage them to think about different ways of solving the problems, as illustrated by Toline and Michael, and ensure that they can communicate their understanding of these through their discussion and use of diagrams. The strategies that the students then use for themselves become a matter of personal choice, a choice that is based on knowledge and understanding.

Toline is modelling what is called "front-end addition" because she is starting at the front by adding the tens. This is a very common mental strategy among adults, despite their being taught to start from the ones place at school. Try extending the use of Toline's method to add three- and four-digit numbers.

Michael's method extends the counting-on idea by starting from a quantity, breaking up the other addend into useable parts, and then adding on those parts. You can extend this idea by trying to use it to join three or more addends.

The open number lines record both the thinking and the calculation in a way that reflects what the student is doing. Unlike a traditional number line, the spaces do not have to be separated in the correct proportion to match the quantities; it is more a schematic drawing of the thinking steps.



Pages 2–3

That's Odd!

Achievement Objectives

- classify objects, numbers, and ideas (Mathematical Processes, developing logic and reasoning, levels 2–4)
- use words and symbols to describe and continue patterns (Mathematical Processes, developing logic and reasoning, levels 2–4)

Activity One

In this activity, students explore the properties of odd and even numbers. This helps them to develop their understanding of number.

Using tens frames like those in the illustrations, the students arrange the numbers in pairs. This makes it easier to see the difference between odd and even numbers. Ask the students to visualise and then draw or show other odd and even numbers.

Have the students discuss and record on a wall chart or large piece of paper their description of what makes an odd number into an even number. Statements such as "an odd number is an even number plus or minus one" or, conversely, "an even number is an odd number plus or minus one" would demonstrate a good understanding of the relationship.

The pairs arrangement of even numbers shows that even numbers all have a common factor of 2. Encourage your students to see that all even numbers belong to the 2 times table.

A good way to record the different ways of joining odd and even numbers would be on a 2 by 2 matrix.

+/−	Even	Odd
Even	Even	Odd
Odd	Odd	Even

x	Even	Odd
Even	Even	Even
Odd	Even	Odd

Activity Two

Encourage your students to notice that in the multiplication grid (above) for odd and even numbers, three of the options lead to an even result. Whenever one factor is the number 2, the result must be even because the 2 ensures a doubles or paired relationship.

A useful approach to question **1** is to have the students turn 6 x 4 and 8 x 3 into prime factors. They can then compare 3 x 2 x 2 x 2 with 2 x 2 x 2 x 3 to see that, in essence, they are the same.

In question **2**, they should realise that any number that has a factor of 2 must be even, and indeed, any number that does not have 2 as a prime factor must be odd.

Activity Three

A good extension for studying even and odd numbers is to list a sequence of the numbers and find the general rule.

Sequence term	1	2	3	4	5	...	10	...	20	n
Even number	2	4	6	8	10	...	20	...	40	2n

Sequence term	1	2	3	4	5	...	10	...	20	n
Odd number	1	3	5	7	9	...	19	...	39	2n – 1

If they can see that the general rule for the nth term of even numbers is 2n (two times the number in the term sequence), the factor of 2 is clearly paramount, whereas with the odd numbers, the rule is 2n – 1 or the even number minus 1.

Pages 4–5 Fives and Tens

Achievement Objectives

- demonstrate the ability to use the multiplication facts (Number, level 2)
- recall the basic multiplication facts (Number, level 3)

ACTIVITY

The array pattern used here to model multiplication is an important pattern for students. It illustrates the "sets of" scenarios in which students should use multiplication to find the total. It's an effective way of showing the relationships between the factors and the products (the family of facts).

The array pattern above shows a 4 by 3 arrangement that can also be described as a 3 by 4 arrangement. If we look at the rows, we can see how many fours there are in 12, while the columns show how many threes there are in 12. Encourage your students to see that a 4 by 3 view and a 3 by 4 view are different ways of looking at the same product of 12. This demonstrates a deeper understanding than simply saying that 3 x 4 is the same as 4 x 3.

When two factors are combined, as in the array above, the shape of the arrangement is often a rectangle. Conversely, all rectangles can be described by two factors, which represent the lengths of the sides. The product will be the area of the rectangle.

Prime factors are very useful in exploring array patterns. You may need to show the students how to turn the array factors into prime factors. For example, in question **1**, 4 x 5 will be 2 x 2 x 5. By recombining these factors in different ways, the students will see all the possible arrays that they can make. In this case, (2 x 2) x 5 = 4 x 5, whereas 2 x (2 x 5) = 2 x 10. This would be useful in question **1d**, where 8 x 5 expressed as prime factors is (2 x 2 x 2) x 5, which can be regrouped as (2 x 2) x (2 x 5) to become 4 x 10. This work will give the students a powerful appreciation of the associative property for multiplication.

Another useful strategy is to use doubling and halving. For example, in 8 x 5 = 40, you can halve the 8 and double the 5 to get 4 x 10 = 40.

In questions **2** and **3**, the students derive unknown facts from known ones. After they see the connections modelled here, the students should see that this deriving strategy can be used for any multiplication situation. For example, 4 x 19 can be derived from 4 x 20 by subtracting a 4.

Students who have only a "sets of" interpretation for multiplication equations may have difficulty understanding this strategy.

Ensure that your students understand how to change their view of the first equation to match up to the second equation. For example, to use 6 x 5 = □ to work out 6 x 6 = □, they may have to change their interpretation of 6 x 5 as six lots of 5 to the equivalent form of 5 x 6 as five lots of 6 to enable them to see the connection with 6 x 6 as six lots of 6.

Page 6 Fund-raising

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- explain the meaning of the digits in any whole number (Number, level 3)

ACTIVITY

This activity does more than ask students to solve problems using tens and hundreds. It also provides an opportunity for them to understand how the base 10 nature of our number system can be used to solve problems that involve numbers that are factors of 10.

The students may solve question **1** by using addition or by using a combination of addition and multiplication. They may see that 10 x 10 is 100 and then add 100 a further nine times to get 1 000. Make sure that they can see the connection between this and working it out as 10 x 10 x 10.

Mira has used a counting-on-in-tens strategy in question **2**. Discuss and share other strategies, such as using basic facts and place value ideas to add or subtract. Another strategy may be to use rounding. Mira could have rounded her total up by 30 to get to 100 and then subtracted 10 from that rounding to find that Paora's 90 was 20 more than her 70.

It's important to share these strategies and to encourage your students to understand them, but they should not be trying to memorise them. The strategies will be of little value to the students if they become just another learned ritual. Students who are encouraged to look for efficient ways of solving problems are more likely to see the value of adopting some of the shared strategies.

Question **4** has some important connections for students to appreciate. In **4a**, the students may see that multiplying or dividing by 100 is a matter of adding or subtracting zeros. Examine this rule with your students and make sure that the idea of multiplying by 100 is seen as more than simply adding two zeros. (Adding zeros to the front of a number makes no difference to a number.) Multiplying by 100 is placing two zeros on the right end because the digits have moved two places to the left. The reverse occurs for division by 100.

In question **4b**, you can highlight the connection between multiplying by $^1/_{10}$ and dividing by 10. The 10 cents can be used as $^1/_{10}$ of a dollar or as a dollar divided by 10.

Page 7 Jungle Land

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- explain the meaning of the digits in any whole number (Number, level 3)

ACTIVITY

This activity continues the focus on the base 10 nature of our number system by encouraging students to extend their addition combinations to those that make 1 000. This is a useful extension to their earlier facts that add up to 10 and 100. These facts will enable them to use regrouping strategies when adding or subtracting three-digit numbers.

After the students have explored the impact on ticket points of various activities, question **4b** asks them to select just *one* activity for each child to use up all the points they have left. The students' answers, of course, will depend on whether they have correctly calculated the answers to the previous questions.

Question **5** could easily be discussed in pairs or groups. As an extension, you could challenge your students to plan a programme that uses two tickets. They could also explore the effect of a 20 percent increase in points on all activities to find out how many activities could be done with one ticket and whether the children could still use up the exact number of points on their tickets.

Page 8

Space Zapper

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)

GAME

A copymaster of the game board is provided at the end of these notes.

This game encourages students to develop addition or subtraction strategies for combining hundreds and thousands. As they play the game, take the opportunity to discuss their thinking strategies. Questions that will encourage thinking may include "How many points do you have so far?" followed by "How did you work that out?"

Quick games like this one are ideal as independent activities, either as a practice activity or as a reward for effort in other mathematics work.

Few rules are listed for this game. This allows you to adapt the game to cater for the interests and challenge levels of your students.

Try incorporating rules such as these, singly or in combinations:

- Each player must visit at least one of each type of ship and power source.
- Throwing a double means you have been struck by a meteorite. You must miss a turn.
- You must reach a power source before you have moved 12 spaces, or you will be stranded in space without power.
- You cannot land on the space station until you have collected 1 000 points.
- You must throw an exact amount to land on an alien ship and/or on a power source.

These rules will require the students to use strategies other than dashing down the shortest route possible. The extended rules will also ensure that the game retains its interest if you want it to be a long-running feature of class or group activities.

Page 9 Weigh to Go

Achievement Objective

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)

ACTIVITY

This activity uses the measurement of mass as a context for number problems. You can integrate number more fully by having your students act out the scenario. They could estimate the mass of a variety of objects and match each object to a red, yellow, blue, white, or green tag. The students could then weigh each object to check their estimations and tag selections.

You would need to organise the objects to be weighed beforehand and label them with numbers or letters so that they are easily distinguished during discussion. Objects to be weighed could include plastic bags of beans, old socks with stones in them, or plastic bottles of water. You could choose objects that do not need to be in containers, but you would need to use sensible things that could be weighed easily on kitchen scales or in a bucket balance.

When introducing question **4**, ask a question such as "Can the two items be less than 100 grams in total?" to explore the logical implications here. It may be helpful for both questions **4** and **5** to have the students reorganise the list of health food orders in descending order of mass. Putting the items on a spreadsheet would present them with the opportunity to appreciate the "sort a list" function.

Page 10 Resolving Differences

Achievement Objective

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)

ACTIVITY

Many students have only seen subtraction in "take-away" contexts where a subset is separated from a set. This activity focuses on differences. Difference contexts may be recorded as subtraction equations, but their actual purpose is to compare two sets. No separation has to occur.

In difference situations, it doesn't matter if you ask for the difference between 4 and 7 or between 7 and 4, the result is still 3. Note that the only subtraction recording that suits either of these scenarios is 7 – 4 =3. This means that while the order of choosing the numbers in question **1** does not matter, if students record these quantities in a subtraction equation, they will have to write the larger quantity first.

Question **2c** revisits the property of even numbers that is developed on pages **2** and **3** of the students' book. All even numbers have a prime factor of 2. All the numbers in the grid have been multiplied by 2, so they must be even.

Question **4** is asking students to see the connection between the grids. If each of the numbers in the grid has been multiplied by 10, the differences, though 10 times greater than those in question **1**, must still be in the same position on the grid.

Table Talk

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–4)
- continue a sequential pattern and describe a rule for this (Algebra, level 2)

ACTIVITY

You can use the context provided in this activity as an opportunity to develop problem-solving skills as well as an opportunity to use addition and subtraction strategies. This activity also integrates some algebra with the number learning outcomes.

Question **1** encourages the students to explore possible ways of arranging the tables and how this relates to the number of people who can be seated in a group.

Help the students by asking questions such as “What happens when the tables are grouped in a line?”

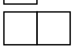
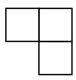
Work with them to arrange a pattern like this:

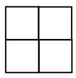
Number of tables	1	2	3	4	5
Number of chairs	4	6	8	10	12

Similarly, with a question such as “What happens when all the tables are separated?”, you can arrange a pattern like this:

Number of tables	1	2	3	4	5
Number of chairs	4	8	12	16	20

The students’ discussion should show that when the tables are put together in a line, there are two chairs less for each additional table than when the tables are separated out. Encourage the students to explore further with questions such as “Are there other ways of grouping tables besides putting them in a line?”

One table can always seat four. 
Two tables can always seat six. 
Three tables can be arranged in a line to seat eight. Alternatively, they can be arranged in an L shape, but the students should consider whether two people can fit easily into the inside corner.
An L-shaped arrangement is more likely to seat seven comfortably: 

Four tables can be arranged in a square, but this will still only seat eight: 

Question **3** extends the investigation into the arrangement of patterns further still. Ensure that the students appreciate that 18 small tables arranged separately can seat 72. This is the same as nine large tables arranged separately, so it is the table groupings that cause the differences.

Arrangement of separate large tables:

Number of large tables	1	2	3	4	5	6	7	8	9
Number of chairs	8	16	24	32	40	48	56	64	72

Large tables in a line:

Number of large tables	1	2	3	4	5	6	7	8	9
Number of chairs	8	12	16	20	24	28	32	36	40

Note that by joining the large tables, you get four positions less with each join, which is less efficient than separate groups of eight. An interesting discussion point would be to consider the L-shaped arrangement of tables again in relation to the large tables. In this case, the configuration means that Emeli would lose as many seats as she gains.

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Sums and Products

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- demonstrate the ability to use the multiplication facts (Number, level 2)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–4)

ACTIVITY

Understanding that the term "sum" describes an addition relationship and the term "product" describes a multiplication relationship is vital with this problem. Students need to see that although the numbers used are the same, the relationship between them makes for a different result.

Have the students clarify the problem by restating it in their own words. Let them attempt two or three examples on their own and then bring them together to discuss the strategies they used. Most will be using a guess-and-check strategy. Ask them if they can find a more efficient way. If necessary, suggest that they explore the number of possible addends that can make the sum and the number of whole-number factors that can make the product.

A systematic approach would be to draw up a table comparing addends and factors. For question **1b**, it would look like this:

Addends that sum to 7	Factors that make a product of 10
0 + 7	1 x 10
1 + 6	2 x 5
2 + 5	
3 + 4	

The students may be able to see that while there are four possible additions for this problem, there can only be two multiplications. It would therefore be more efficient to find the factors that make the product and then use this list to find the addends that sum to the correct total.

So in question **1f**, if the students listed the factors for 35 as 1 x 35 = 35 and 5 x 7 = 35 and then looked to see which numbers were used again to make a sum of 12, they would quickly discover that 5 + 7 is the answer.

You can use question **2c** as a challenge to see who can make up 10 sum-and-product questions for their group to solve. The students must know the answers before they ask their classmate to solve the problem, and they must be able to explain how the problems work.

Question **3** is there to challenge the incorrect view held by most students that multiplication always results in products that are greater than the factors used. To find a sum that is larger than the product, the students need to use a factor that is equal to or smaller than 1.

Make sure that your students understand the identity element for multiplication. They should be able to explain clearly that when a number is multiplied by 1, the result stays the same.

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Container Contents

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- demonstrate the ability to use the multiplication facts (Number, level 2)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–4)

ACTIVITY

This activity asks the students to visualise arrays of cars stored in the big containers. They also need to realise that an array is the product of two factors. For questions **1** and **2**, the students will know the total size of each array because it is given each time as the number of vehicles in the container. They also know one of the factors in the array because they can see one of its sides. Their task is to find the other factor, which is the other side of the array.

One way of describing the problems in questions **1** and **2** is to write them as start-unknown equations, for example, $\square \times 4 = 32$. The students can solve this through multiplication as "What times 4 equals 32?" or through division as "32 divided by 4 equals what?"

Students who have difficulty visualising the arrays could model them using cubes or square tiles as vehicles. They should see that each array starts with a particular number of cubes (vehicles) and builds up in rows of the same number of cubes until it reaches the total number.

Question **3** is a great open-ended question that will encourage visual thinking. Have your students discuss the information provided and draw conclusions about the implications of the facts. If they are struggling, suggest that they draw a diagram to show what they know.

There are three certain known facts: there are six rows of vehicles, there is at least one row of cars, and there is at least one row of vans. However, somewhere within the container, the number of vehicles in the row changes from three to four. This means that there could be five possible arrangements. A table or diagrams to show all the possible ways of arranging the cars and the vans would be a good strategy and would enable the students to visualise what is happening.

Another approach is to ask the students to find how many rows of vans would give them the most vehicles (one) and how many rows of vans would give the least vehicles (five).

Caught on Camera

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical processes, problem solving, levels 2–4)

ACTIVITY

You can have some fun with this interesting scenario. As a side topic, you might like to discuss ways of ensuring that drivers do keep to their correct speeds. The students may have heard about governors that can be attached to motors to keep them below a certain speed. Possibly the cars could have a large sign displayed that indicates their maximum speed so that other drivers know the speed they should be travelling at.

After the students have warmed up with question **1,** have them work in problem-solving groups of four to find as many possible options for question **2** as they can.

In question **3,** the students need to apply some common sense rather than simply calculating and coming up with a negative number that would have the driver moving backwards at 4 km/h. You could use this as an opportunity to discuss negative numbers. The students may not be aware that $^-4$ km/h means moving backwards.

Question **4** changes the focus of the activity. These are called part-unknown questions because the equations that directly match the question have one part missing, as in $57 + \square = 80$.

Such questions encourage the students to think carefully about what is happening in order to come up with appropriate equations. The students may solve these questions by counting on or adding on. Alternatively, they could reorganise the elements into the common subtraction form of $80 - 57 = \square$.

These questions also have two parts to them. First, the students will find out how much faster than the speed limit the driver was travelling. When they have calculated this, they will use the information to work out the actual speed that the driver was travelling at. For example, for question **4a,** it would be $57 + \square = 80$ and then $80 + 23 = \square$.

An interesting extension would be to have the students add the speed the drivers were doing to their penalty speed and then divide by 2. This will give them the speed limit. If the students wonder why they need to divide by 2, you could work through one example with them:
To get the driver's speed, note that 80 – 57 = 23, so the driver was going 23 km/h over the speed limit and so was travelling at 80 + 23 = 103 km/h.
$(80 - 57) + 80 = 103$
So $80 + 80 = 103 + 57$ (that is, speed limit x 2 = speed + penalty)
So $80 = \frac{1}{2}(103 + 57)$ or $(103 + 57) \div 2$

In each part of question **4**, this is:

4a $(103 + 57) \div 2 = 80$

4b $(96 + 64) \div 2 = 80$

4c $(89 + 71) \div 2 = 80$

4d $(112 + 48) \div 2 = 80$

They could use this method to check their answers.

Page 15 King of the Castle

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–4)

ACTIVITY

The game that precedes this activity provides good practice in cumulative addition, but its real purpose is to challenge students to find and use effective strategies. The game is easy to set up. It is best to have the students play the game a few times before they attempt the questions in the activity. This will give them a chance to become familiar with the structure of the cumulative addition as they go up the pyramid.

Avoid telling the students the best strategy, but if they need hints, try questions like these:
"Have you tried putting the numbers in different places at the start?"
"Is there anything the same about the position of the small numbers in those patterns that give the largest number?"

When they realise that the highest number at the top is achieved by having the largest numbers in the middle and the smallest numbers on the outside, make sure that they discuss why this makes a larger result than other ways.

They should realise that the numbers in the centre are added to both left and right sides above them, whereas the numbers on the end are added only one way, so the centre numbers are used as addends more times than the end ones.

The game is extended to more layers in question **2** of the activity. Further extensions can occur by using larger numbers as well as by adding more layers.

Pages 16-17 Firewood Fever

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- explain the meaning of the digits in 2- or 3-digit numbers (Number, level 2)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–4)

ACTIVITY

Year 7–8 students are likely to relate better to the firewood context of this activity than to being asked to make bundles of 10 with ice cream sticks and rubber bands as they probably were in earlier years, though the underlying mathematical idea is exactly the same. The students need to see the *oneness* of a bundle of 10 and use this to help solve addition and subtraction problems.

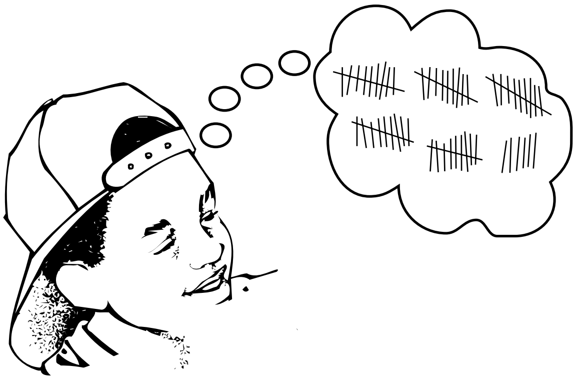
Draw attention to Cathy's style of recording (number line jumps) as shown in her first two thought bubbles. These demonstrate how she used her knowledge of number relationships, rather than counting, to solve the problem. Encourage your students to find their own ways of using number relationships to do the cumulative additions. They could also record their thinking using Cathy's model.

Allow the students to add the tens before the ones if they want to, as this is a valid strategy. It also shows that they have a thorough understanding of the relationship between tens and ones.

Note that Cathy's truck holds 10 bundles of 10, which is 100 sticks. You can extend this to three-digit quantities using a visual model of trucks, bundles, and sticks. For example, 347 sticks will produce three truckloads, four bundles, and seven loose sticks.

For students who need more help to see 10 or 100 as a unit, relate the firewood groupings to the groupings of the cubes, rods, and squares in place value blocks.

Encourage the students to develop mental images of two- or three-digit quantities, using firewood or other models, and then have them show these images in drawings as well as numbers.



Challenge the students to find things around their homes that are packed in tens. For example, they might find a packet of chocolate marshmallow biscuits that holds 10 biscuits. The students can then make up and solve problems based on contexts involving those materials, for example: "Room One took eight packets of chocolate marshmallow biscuits to camp. By day 2, there were only three full packets and one opened packet with six left in it. How many biscuits had been eaten?"

Page 18 Down with Darts

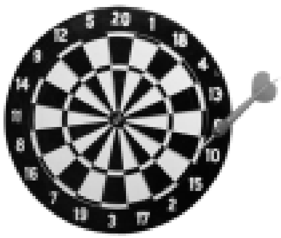
Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–4)

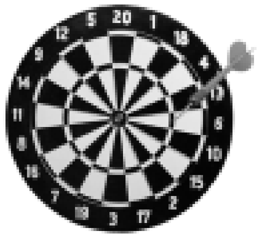
ACTIVITY

This activity is based on the popular darts game called 301 in which contestants have to reduce their total from 301 until they get to exactly zero. They must start and finish by throwing a double or a triple. This game is often credited with helping to develop quick mental strategies in people who failed to learn such strategies at school.

Make sure that the students understand how a double or triple is scored on a dartboard so that they can understand how Charlotte scored a double 8. It may also help them to appreciate the challenge involved in question **2b**, when players are asked to score a double or a triple to finish. (To get double the score, the dart must lodge in the outer ring. To get triple the score, the dart must lodge in the inner ring. Otherwise, your score is the number on the outside of the rim unless you score either 25 or 50 from the bullseye.)



Double 10



Triple 6

Discuss the thinking strategies used by Charlotte and Lawrence. Ask a question such as "Why did Charlotte decide to take away 20?" followed by "How did she know she would have the correct score?" You could then ask: "Charlotte made the subtraction easy by changing the amount she took away, but Lawrence chose a different strategy. What did he change, and how did it work for him?"

Encourage the students to discuss strategies for doing the subtractions quickly and doing them mentally rather than by working them out on paper.

You could challenge your students to find the least number of dart throws needed to win when starting from 100, apart from two red bullseyes. The answer is two throws because the first could score a triple 20, which leaves 40 to go, and this can be scored with a double 20 to finish on the next throw.

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Absolutely Abseiling

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–4)

ACTIVITY

The cliff-face context of this activity is easily visualised as a vertical number line. The activity builds on the mental strategies for subtraction developed in the activity on the previous page.

Discuss the strategies presented in the speech bubbles. Have the students explain how the strategies chosen relate to a number line and why each one could be an easy way of working out the distances mentally. If the students want to work out the problems on paper, remind them that it would not be possible to do this while abseiling, so they should put themselves in Edmund's place and work the distances out mentally.

Challenge the students to come up with two or three different ways of working out the jumps. Ideas might include:
For question **3a**: "45 metres take away 20 metres is 25 metres, so going back 3 metres to get to 28 metres means a total of 17 metres."

Another way could be: "45 metres take away 10 metres is 35 metres, another 5 metres makes 30 metres and another 2 makes 28. So, I jumped 10 and 5 and 2, which makes 17 metres."

For question **3b**: "70 take away 35 is 35, so 71 metres take away 35 metres would be 36 metres. So the answer is 35 metres."

Another way could be: "71 minus 1 makes 70, minus another 30 makes 40, and another 4 takes it to 36 metres. So 1 + 30 + 4 = 35 metres."

A third way may be: "71 take away 40 is 31, go back 5 makes 36. So 40 – 5 = 35 metres."

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Piece of Cake

Achievement Objectives

- write and solve story problems which involve halves, quarters, thirds, and fifths (Number, level 2)
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–4)

ACTIVITY

This activity develops the idea of finding a fraction of a set of objects by connecting the fraction of a region concept, modelled as a cake, with the idea of sets, as modelled by the candles.

Emphasise that the whole to which the fractions refer is the cake and that, in this scenario, the candles will be placed evenly over the cake so that each equal-sized piece will have the same number of candles.

The students need to understand how the numbers combine in an integrated manner when they are recording the notation for fractions. In the notation recorded as $^2/_5$, the students need to know that the numerator, 2, refers to the number of parts being considered. The denominator, 5, refers to how many parts the whole (unit) is being divided into.

The drawings in question **2** do not need to be accurate, but they should be reasonable approximations for the correct number of equal parts. Have the students write their equations using the word "of" as well as the "x" symbol so that they can make a meaningful connection. For example, $^1/_2$ of 16 = 8 and $^1/_2 \times 16 = 8$.

Question **3a** can also be used to show equivalent fractions. In problem **3a i**, one-half of the cake has six out of 12 candles. This is recorded as $^1/_2 = {^6/_{12}}$. In problem **3a ii**, two-thirds of the cake has eight out of 12 candles: $^2/_3 = {^8/_{12}}$.

Make sure that the students understand that one whole cake can also be described as a fraction by asking them to combine the parts eaten with the parts left as a way of checking their calculations. These can be recorded as addition equations, for example, for **3b i**, $^6/_{12} + {^6/_{12}} = {^{12}/_{12}} = 1$ and for **3b iv**, $^8/_{12} + {^4/_{12}} = {^{12}/_{12}} = 1$.

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Helping with the Hāngi

Achievement Objectives

- write and solve story problems which involve halves, quarters, thirds, and fifths (Number, level 2)
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–4)

ACTIVITY

This activity challenges students to see fractions of different wholes (the food items) and to recombine them into a different whole (the hāngi pack).

If the students are having difficulty understanding what is happening in this context, have them make a chart that shows the relationships between the whole food item and the number of pieces:

The whole	Number of pieces	One piece as a fraction
Potato	2 pieces	$^1/_2$
Kūmara	4 pieces	$^1/_4$
Kamokamo	10 pieces	$^1/_{10}$
Meat slab	5 pieces	$^1/_5$

In question **1**, the students could explore the baskets of ingredients in relation to hāngi packs on another chart:

Items in the basket	Number of hāngi packs each item could go into
6 potatoes	6 packs
3 kūmara	6 packs
1 kamokamo	10 packs
1 slab of meat	5 packs

From this, they can see that this basket of ingredients can only make five complete packs, but there will be some pieces of vegetables left over to use in other packs. The students will need to use this information to answer question **1b**.

Question **4** refers to the hāngi packs that Māni prepares in question **2**. The question presents an opportunity to introduce vulgar fractions (fractions expressed by numerators and denominators rather than decimally) and mixed fractions because there will be 10 pieces of bread, and when we compare this to the number of loaves used, we can say that Māni needs $^{10}/_8$ loaves, which is one whole and $^2/_8$ of a second loaf.

Page 22

Chocolate Chip Feast

Achievement Objectives

- write and solve story problems which involve halves, quarters, thirds, and fifths (Number, level 2)
- solve practical problems which require finding fractions of whole number and decimal amounts Number, level 3)

ACTIVITY

The way in which each student responds to an activity that requires them to order fractions gives a good indication of their understanding of fractional notation.

Unit fractions (fractions that have a numerator of 1) should be ordered first. The measuring cups in question **1** provide an opportunity to check this ability. Ask the students to order the cups from smallest to largest. The correct order would be $^1/_5$, $^1/_4$, $^1/_3$, and $^1/_2$. If the students reverse the correct order, it indicates that they do not understand the role of the numeral in the denominator. Explanations such as “the bigger the number in the denominator, the smaller the pieces that make up the unit” would show sound understanding.

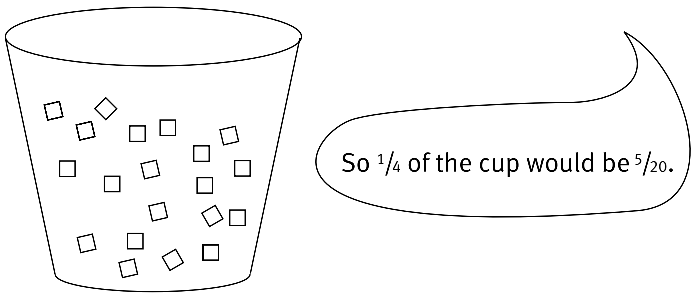
Question **2** extends the ordering of fractions to non-unit fractions. Many students find this difficult because they do not understand the role of the numerator. These students need to see the connection between the unit fraction and the non-unit fraction. For example, three-quarters is three lots of one-quarter.

Have the students estimate the order of the fractions in question **2i–v** by listing them from smallest to largest. Highlight the fact that the whole (a cup of chocolate chips) is the same for each tray. The students need to understand this in order to be able to compare the fractions for size.

Question the students to check for understanding, for example: “How do you know that $^2/_5$ is smaller than $^1/_2$?” Accept answers such as “A half of five is two and a half, so two-fifths (two out of five) must be less.”

You may wish to explain equivalent fractions based on a common denominator. For example, comparing halves, quarters, and fifths requires a common denominator of 20.

The students could explore the size of each fraction by using small place value blocks to represent chocolate chips and placing 20 of these in a paper cup.



As they order the fractions, they should find that $^1/_4 = {}^5/_{20}$; $^2/_5 = {}^8/_{20}$; $^1/_2 = {}^{10}/_{20}$; and $^3/_4 = {}^{15}/_{20}$. If you wanted to include $^1/_3$ in this discussion, the students would need to use 60 blocks.

Page 23 All Bottled Up

Achievement Objectives

- write and solve story problems which involve halves, quarters, thirds, and fifths (Number, level 2)
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)

ACTIVITY

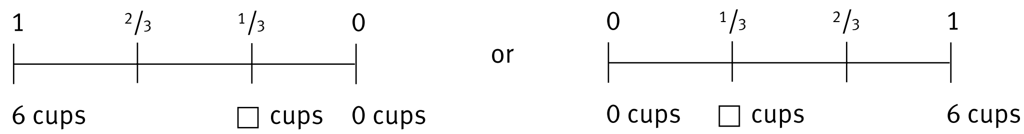
The capacity context of this activity allows students to explore the concept of the fraction of a set more fully. Use the first diagram in each pair shown in question **1** to help the students to appreciate the capacity of the whole container as an equivalent fraction for 1. Have the students record these fractions for 1 for each part of the question. They could record these fractions in a table.

Question	**1a**	**1b**	**1c**	**1d**	**1e**	**1f**
Whole container as cups	6/6	10/10	8/8	7/7	24/24	100/100

Question **1e** will trap the unwary if they do not read the caption carefully. Question **1e** also focuses the students on the complementary relationship between *full* and *empty*. After the students have answered question **1e**, you could make this relationship quite explicit for each part of question **1** to reinforce the equivalent fractions for 1. Another table could be useful:

Question	**1a**	**1b**	**1c**	**1d**	**1e**	**1f**
Fraction shown	1/3	2/5	3/4	1/2	3/8	4/10
Fraction needed to make the whole	2/3	3/5	1/4	1/2	5/8	6/10

This is a good activity to introduce the double number line as a tool for visualising the relationship between the fraction and the set. For question **1a**, the number line could look like this:



Question **2** shows the effect of halving again and again. A ratio table would show this very powerfully:

Number of cups left	16	8	4	2	1
Fraction left	1	1/2	1/4	1/8	1/16

(The shaded part shows what happens if you continue halving.)

Page 24 **Bits and Pieces**

Achievement Objectives

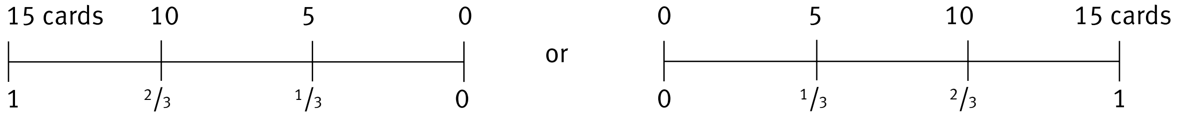
- write and solve story problems which involve halves, quarters, thirds, and fifths (Number, level 2)
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)

ACTIVITY

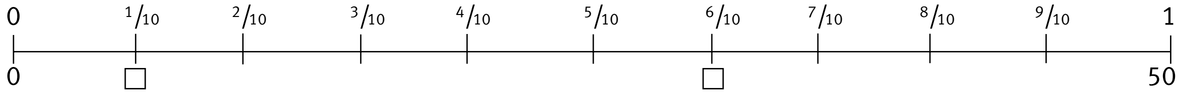
This activity encourages students to develop mental strategies for solving fraction problems.

For example, question **2** may be solved by working out that $\frac{1}{4}$ of 28 is 7 and then multiplying 7 by 3 to find $\frac{3}{4}$ of 28. Alternatively, the 7 (which is $\frac{1}{4}$) could be taken away from 28, leaving $\frac{3}{4}$. Ensure that the students connect the way that the fractions are used with the quantities of the set involved. In this case, 28 is $\frac{4}{4}$ of the cars, so $\frac{1}{4}$ of the cars is 7. As $\frac{4}{4} - \frac{1}{4}$ is $\frac{3}{4}$, then 28 – 7 = 21 white vehicles.

Students who cannot develop a mental strategy for solving the problems might find a double number line useful. For question **3**, the number line could look like this:

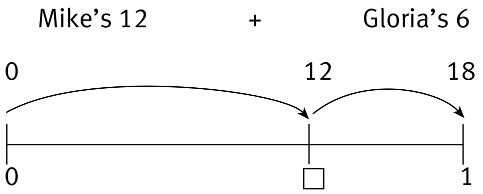


In question **4**, the students need to explore the equivalence between $\frac{6}{10} = \frac{\square}{50}$. Again, a double number line may help those who find it hard to form a mental strategy:

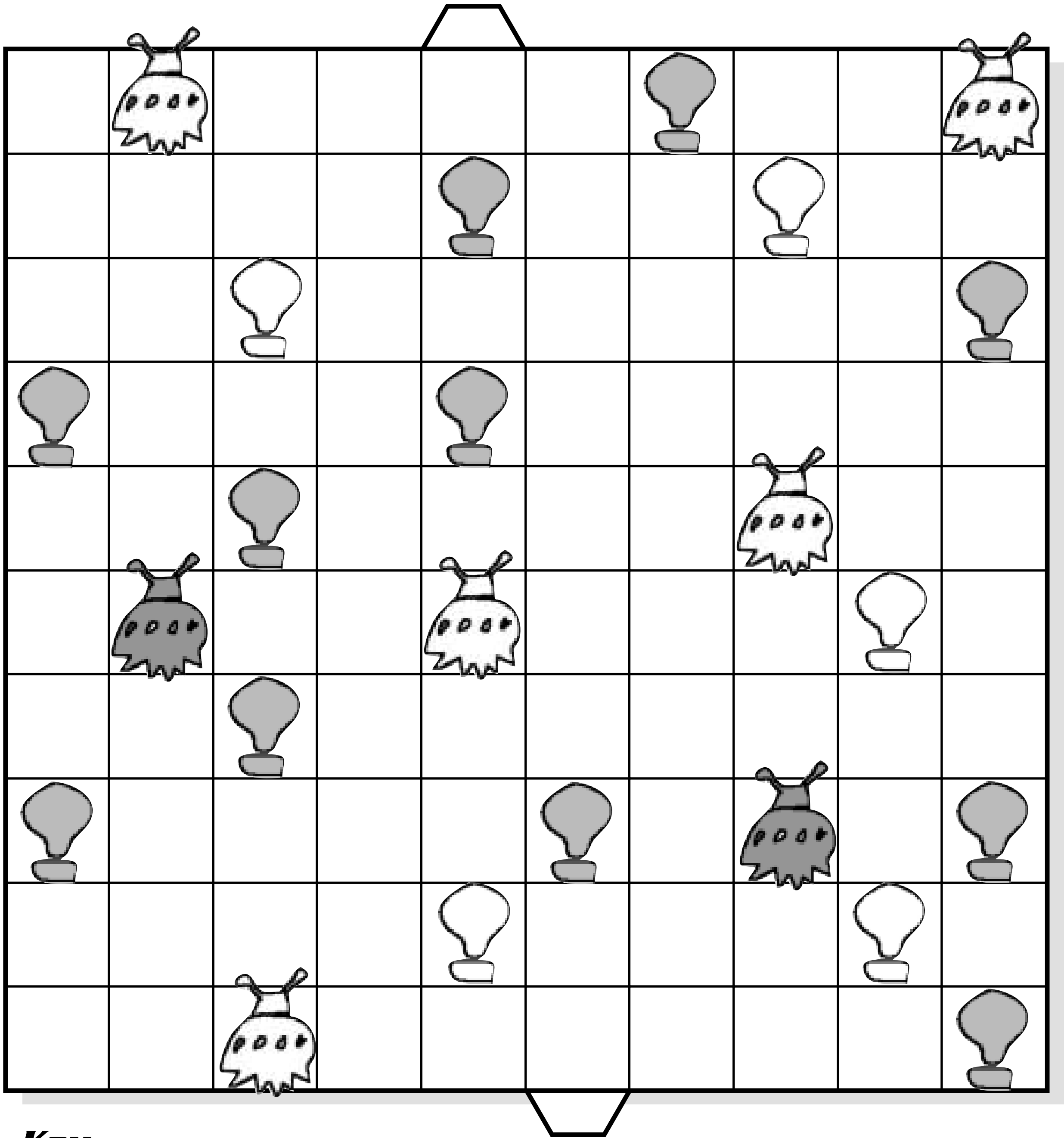


Some students will choose to work out $\frac{1}{10}$ of the lollipops first and then multiply this by 6 to find $\frac{6}{10}$, while others may find $\frac{5}{10}$ of the lollipops and add this amount to $\frac{1}{10}$ to make the $\frac{6}{10}$.

Question **6** uses the ratio idea to find the proportion of the marbles. The students may find this question difficult unless they understand how to visualise all the marbles. They will need to see that Mike has one part and Gloria has the rest and together this makes the whole set.



Space Station



Base Station

Key

 blue alien ship

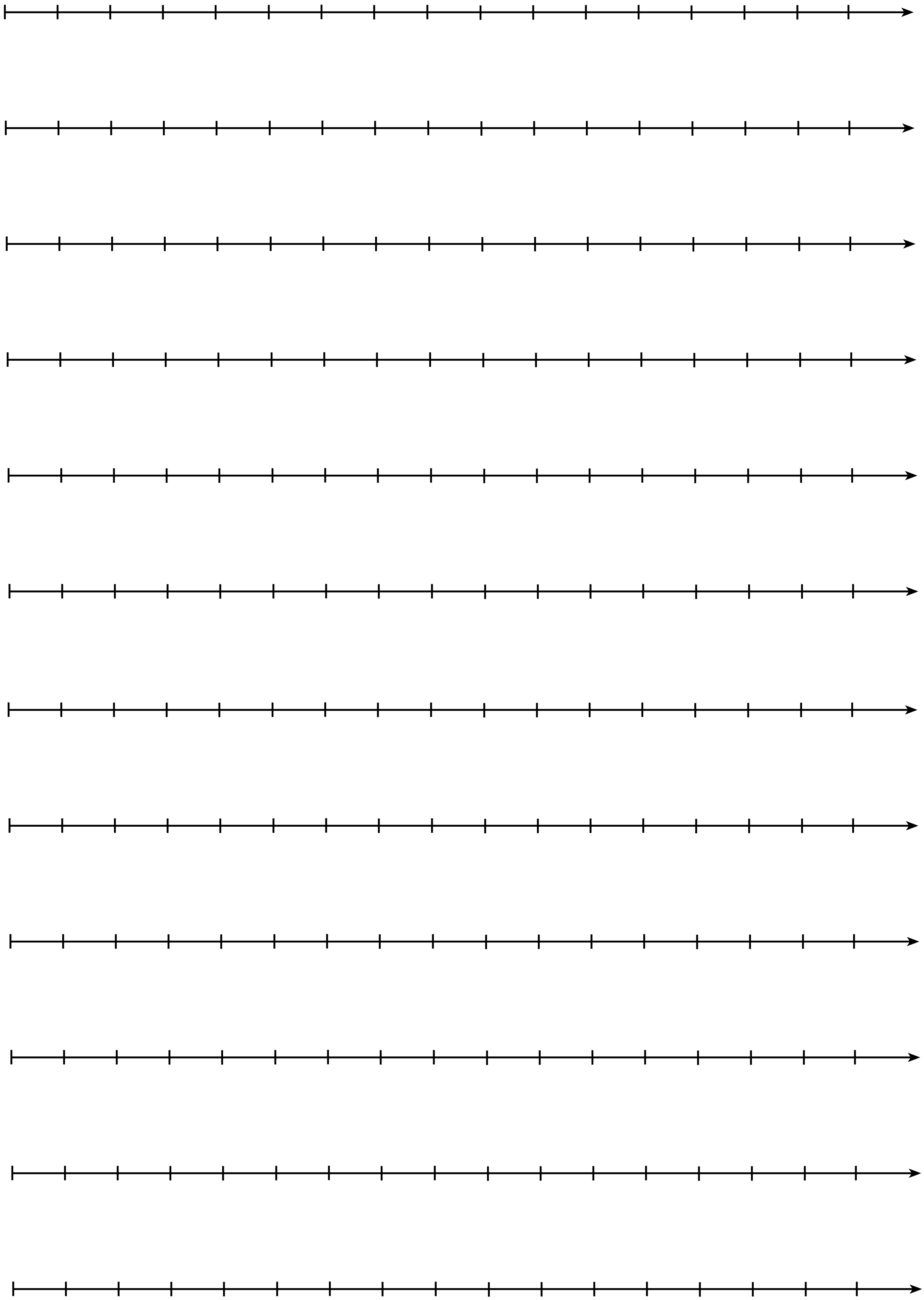
 yellow alien ship

 red alien power source

 green alien power source

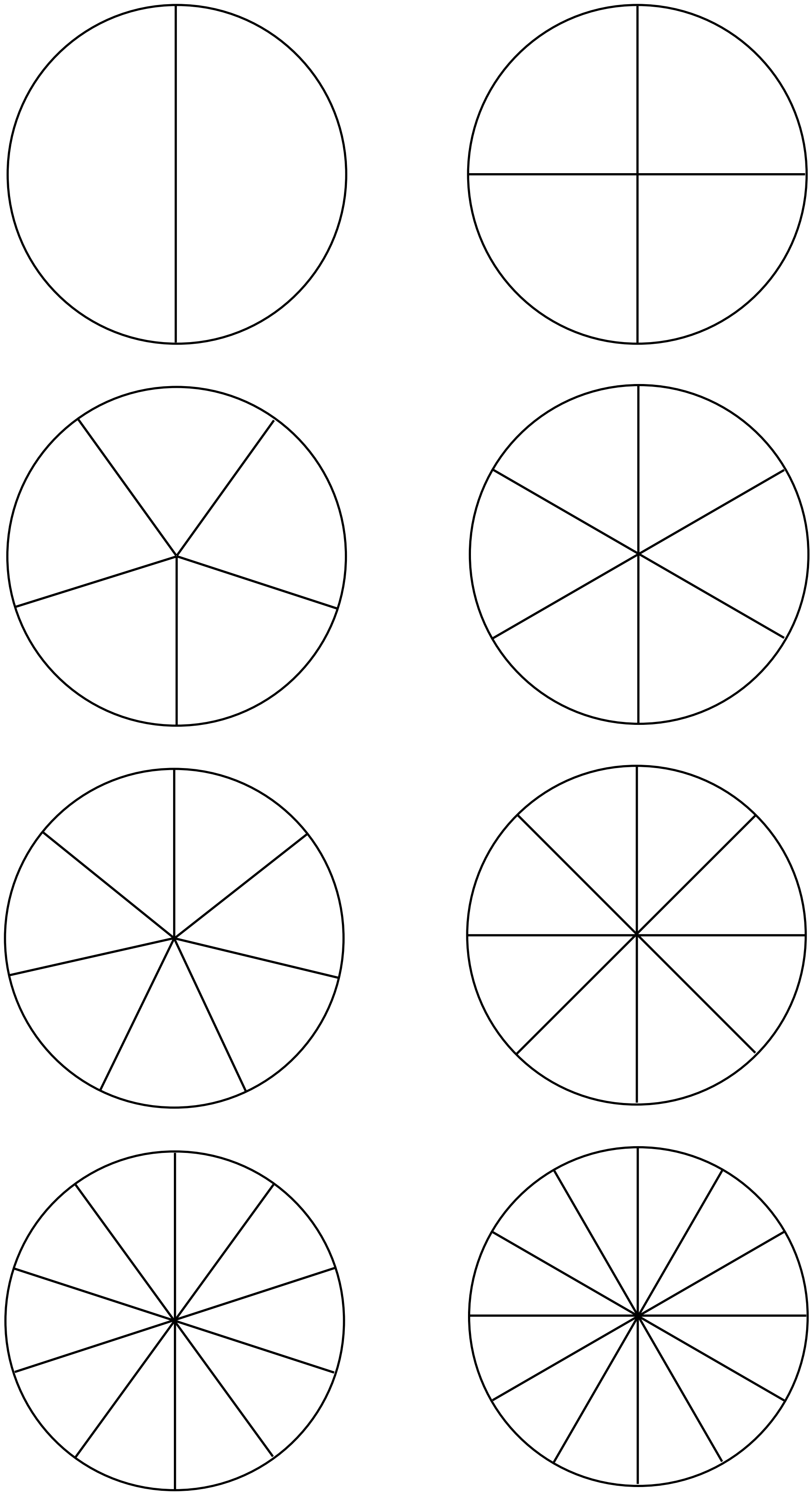
Note: Colour in the ships and power sources the same colours as on page 8 before you start playing.

Double Number Lines



Copymaster

Fraction Circles



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