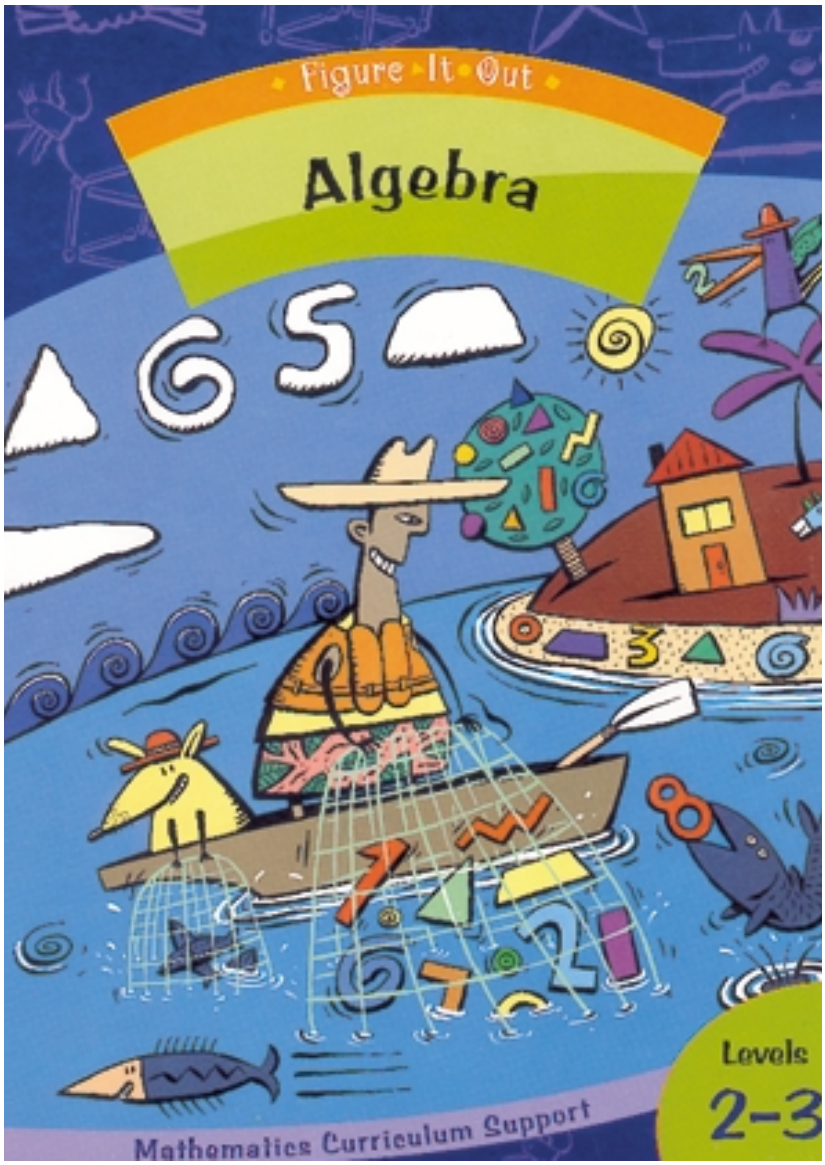
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Contents

Introduction

The Figure It Out series is designed to support *Mathematics in the New Zealand Curriculum*. The booklets have been developed and trialled by classroom teachers and mathematics educators. The series builds on the strengths of a previous series of mathematics booklets published by the Ministry of Education, the School Mathematics supplementary booklets.

Figure It Out is intended to supplement existing school mathematics programmes and can be used in various ways. It provides activities and investigations that students can work on independently or co-operatively in pairs or groups. Teachers can select particular activities that provide extension to work done in the regular classroom programme. Alternatively, teachers may wish to use all or most of the activities in combination with other activities to create a classroom programme. The booklets can be used for homework activities, and the relevant section in the teachers' notes could be copied for parents. These notes may also provide useful information that could be given as hints to students.

There are eight booklets for levels 2–3: one booklet for each content strand, one on problem solving, one on basic facts, and a theme booklet. Each booklet has its own *Answers and Teachers' Notes*. The notes include relevant achievement objectives, suggested teaching approaches, and suggested ways to extend the activities. The booklets in this set (levels 2–3) are suitable for most students in year 4. However, teachers can decide whether to use the booklets with older or younger students who are also working at levels 2–3.

The booklets have been written in such a way that students should be able to work on the material independently, either alone or in groups. Where applicable, each page starts with a list of equipment that the students will need in order to do the activities. Students should be encouraged to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Teachers could encourage students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Where possible, suggestions have been made to encourage discussion and oral presentation of answers, and teachers may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum*, page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. Successful ways of solving problems should be acknowledged, and where more effective or efficient processes can be used, students can be encouraged to consider other ways of solving the problem.

Figure It Out

Algebra

Answers

Page 1: Pick the Pattern

Activity One

1. **a.** purple circle

   b. red trapezium

   c. P

   d. R

   e. 14

   f. 

   g. Pink / Black

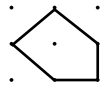
2. **a.** purple circle

   b. orange hexagon

   c. orange sideways

   d. 

   e. 24

   f. 

   g. Pink

3. Answers will vary.

Activity Two

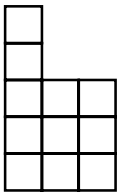
1. Answers will vary.
2. Answers will vary.

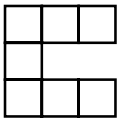
Page 2: The Mystery of the Vanishing Pattern

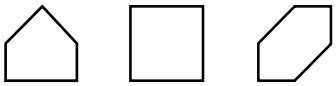
Activity One

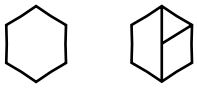
a. 
large pink square


small green triangle

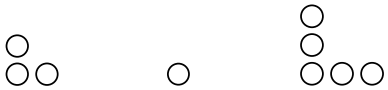
b. 
blue


blue

c. 

d. 

e. 0 6 15

f. 
yellow yellow yellow

Note: the pattern is green, yellow, (multiples of two); the number pattern is 1, 3, 5.

Activity Two

Answers will vary.

Page 3: Punching Numbers

Activity One

1. Answers could include: The counters are in a vertical pattern.

2. Answers could include: The counters are vertical; they are in a straight line going up.

3. Answers will vary.

Activity Two

1. 8 x 2 or 

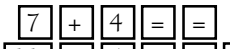
or 

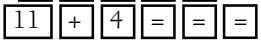
Each number is twice as large as the previous number.

2. 27 x 3 or 

or 

Each number is three times as large as the previous number.

3. Either 

…
or 

…

4. Answers will vary.

Page 4: Follow That Arrow

Activity One

	Missing number	Rule
1.	8	+ 4
2.	1	– 5
3.	14	x 2
4.	2	÷ 5
5.	5	x 1 or + 0
6.	12	÷ 2

Activity Two

	Relationship		
1.	+ 4	6	→ 10
		11	→ 15
		18	→ 22
2.	x 4	3	→ 12
		5	→ 20
		6	→ 24
3.	– 2	5	→ 3
		6	→ 4
		7	→ 5
4.	x 4	2	→ 8
		4	→ 16
		6	→ 24

Page 5: Sticky Moments

Activity One

a. 31 matchsticks

b. 42 matchsticks

c. 41 matchsticks

d. 54 matchsticks

Activity Two

a. 61 matchsticks

b. 82 matchsticks

c. 81 matchsticks

d. 104 matchsticks

Activity Three

Answers will vary.

Page 6: Fair and Square

Activity

1. 31 yellow squares

2. 22 orange squares

3. 20 blue squares

4. **a.** 2 green, 9 white

b. 3 green, 12 white

c. 8 green

5. **a.** 6 orange

b. 3 blue, 9 orange

c. 30 orange

Page 7: Table Time

Activity

1.

People	1	2	3	4	5	6
Gumboots	2	4	6	8	10	12

2.

Cars	1	2	3	4	5	6
Tyres	4	8	12	16	20	24

3.

Cars	1	2	3	4	5	6
Tyres	5	10	15	20	25	30

4.

Customers	1	2	3	4	5	6	7
Books left	27	24	21	18	15	12	9

5.

Groups	1	2	3	4	5	6	7	8
Seats left	35	30	25	20	15	10	5	0

Page 8: Puzzling Picnics

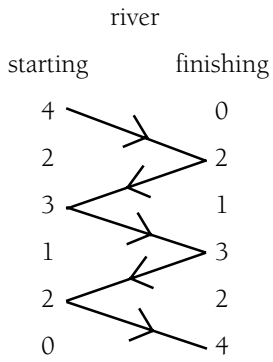
1.

a. 3 trips

b. 5 trips

c. 7 trips

d. 9 trips



Each arrow shows one trip, so four people need five trips to get everyone across the river.

2. Two more trips are needed for each extra person after the first two people:

number of people	1	2	3	4	5	6
number of trips	1	1	3	5	7	9

3. 17 trips

4. The pattern would be a repeating pattern of odd numbers, with two more trips for each extra two people after the first trip:

- 1 trip for 1, 2, or 3 people
- 3 trips for 4 or 5 people
- 5 trips for 6 or 7 people.

Page 9: A Spot of Sickness

Activity One

1. **a.** 9

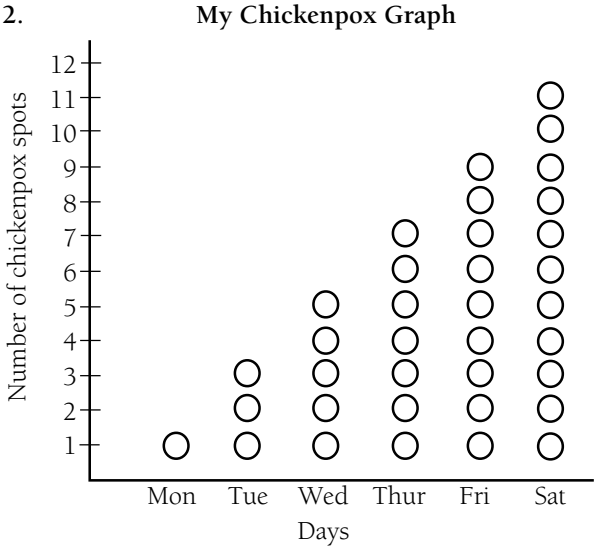
b. Each time, Jerry adds two more spots.

2. It will have 11 spots on it.

Activity Two

1. Answers will vary.

2.



3. Answers will vary. One idea is to mark dots on the paper where the counters were.

4. Two extra spots each day make a diagonal pattern.

Page 10: Building Graphs

Activity One

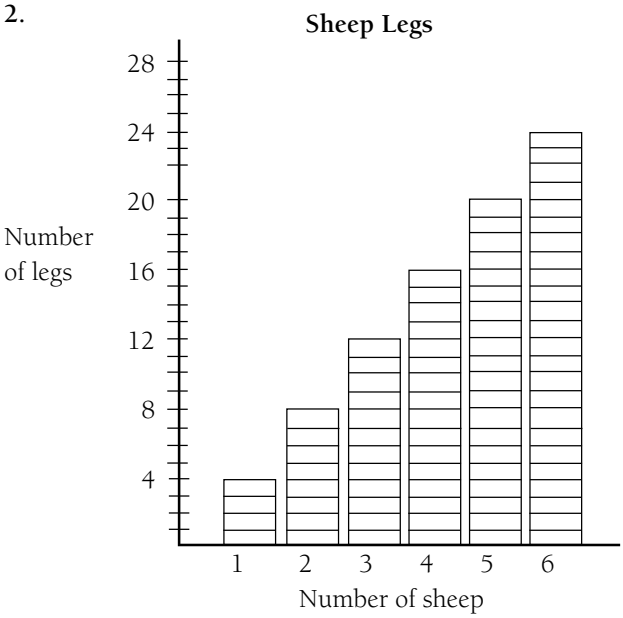
1. 3 sheep, 12 legs

4 sheep, 16 legs

5 sheep, 20 legs

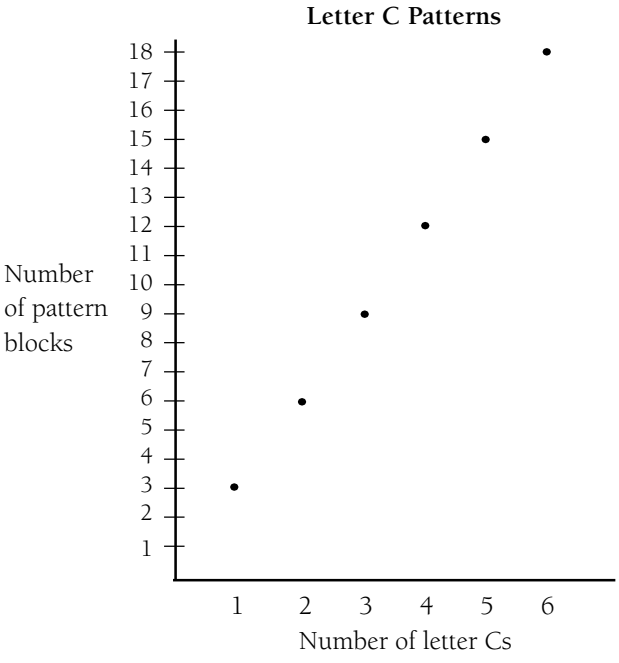
6 sheep, 24 legs

2.



3. Answers could include: The pattern is diagonal.

Activity Two



Activity Three

Answers will vary.

Page 11: A Smile a Day

Activity One

Answers will vary. Some things make Jack very happy (for example, finding gold), and other things make him sad.

Activity Two

Answers will vary.

Page 12: The Big Shake

1. Answers will vary.
2. Answers will vary. The height of the waves needs to fit the description (teacher to check).

Page 13: Story Plots

Activity

1. and **2.** Answers will vary.

Pages 14 and 15: Shoot the Hoop

Game

A game using digit cards

Page 16: Number Nibbles

Activity

1. 2 blue, 4 green, 7 red
2. Answers will vary. For example, 12 green, 4 blue, 4 red
3. 14 blue, 7 red, 2 yellow, 2 green
4. 6 blue, 5 red, 3 green
5. Answers will vary. For example, 6 yellow, 2 blue, 4 red
6. Answers will vary.

Page 17: Crunch Machine

Activity One

+ 5 2 + 5 = 7
4 + 5 = 9
6 + 5 = 11
8 + 5 = 13
10 + 5 = 15

– 8 8 – 8 = 0
12 – 8 = 4
16 – 8 = 8
20 – 8 = 12

x 3 7 x 3 = 21
8 x 3 = 24
9 x 3 = 27
10 x 3 = 30

÷ 5 5 ÷ 5 = 1
10 ÷ 5 = 2
15 ÷ 5 = 3
20 ÷ 5 = 4

Activity Two

1. 6 ÷ 3 = 2 or 6 = 3 x 2
2. 50 = 10 + 10 + 30, 50 – 10 = 10 + 30, or 50 – 10 – 10 = 30
3. 15 + 21 + 4 = 40
4. 24 = 6 x 4 or 24 ÷ 6 = 4
5. 16 + 17 = 3 x 11
6. 10 = 2 x 3 + 4 or 10 + 2 = 3 x 4
7. 12 = 3 x 2 x 2 or 12 ÷ 3 = 2 x 2
8. 40 + 3 + 7 = 50

Activity Three

Answers will vary.

Page 18: Perfect Patterns

Activity One

a. Yes.

b. The pattern is + 1.

Activity Two

a. 5 + 0 = 5

b. 5 – 0 = 5

c. 5 x 1 = 5

d. 5 + 5 = 10

e. 5 + 6 = 11

f. 5 = 7 – 2

g. 5 ÷ 5 = 1

Answers will vary for the equations further down for each pattern.

Activity Three

Answers will vary.

Page 19: Flower Power

24 flower Use the ÷ bag. 24 ÷ 4 = 6, 24 ÷ 12 = 2, 24 ÷ 2 = 12, 24 ÷ 8 = 3, 24 ÷ 3 = 8, 24 ÷ 6 = 4

3 flower Use the x bag. 3 x 6 = 18, 3 x 7 = 21, 3 x 1 = 3, 3 x 2 = 6, 3 x 3 = 9, 3 x 5 = 15, 3 x 4 = 12

37 flower Use the – bag. 37 – 1 = 36, 37 – 23 = 14, 37 – 7 = 30, 37 – 16 = 21, 37 – 20 = 17, 37 – 28 = 9, 37 – 37 = 0

60 flower Use the ÷ bag. 60 ÷ 2 = 30, 60 ÷ 3 = 20, 60 ÷ 6 = 10, 60 ÷ 5 = 12, 60 ÷ 10 = 6, 60 ÷ 12 = 5, 60 ÷ 4 = 15

5 flower Use the + bag. 5 + 12 = 17, 5 + 3 = 8, 5 + 10 = 15, 5 + 15 = 20, 5 + 22 = 27, 5 + 6 = 11, 5 + 26 = 31

Page 20: What Goes Where?

Activity One

1. **a.** =

b. <

c. 7 or higher

d. >

e. <

f. 1 or 0

g. >

h. 4 or more

2. Answers will vary. They could include: "I worked out the numbers on each side of the ☐ symbol and then decided which symbol (<, >, or =) to use."

3. Answers will vary.

Activity Two

1. Tama belongs to Sharks.

2. Va'e belongs to Dolphins.

3. Jesse belongs to Sea horses.

4. Rama belongs to Sharks.

Page 21: Coin Control

Activity One

1. 20

2. 5 x 4 is quicker.

Activity Two

Lara Loot has 40 gold coins.

Gerri Gem has 15 gold coins.

Carlos Credit has 16 gold coins.

Donny Dosh has 15 + 25 = 40 gold coins.

Activity Three

Answers will vary.

Page 22: Something Fishy

Activity One

1. 3 fish, 4 hot dogs, 3 scoops of chips

2. "d" could stand for anything, for example, double burger, but there is a doughnut in one basket.

3. Jones: 3 fish, 5 hot dogs, 4 scoops of chips
Paki: 2 hot dogs, 2 fish, 2 d, 3 scoops of chips
Patel: 4 d, 2 fish, 4 hot dogs, 1 scoop of chips
Sang: 3 fish, 4 hot dogs, 4 d, 6 scoops of chips

Activity Two

Answers will vary.

Page 23: Train Talk

Activity One

12 centimetre trains: 6 x 2 centimetre rods
2 x 6 centimetre rods
12 x 1 centimetre rods

Activity Two

1. 16 centimetre trains: 16 x 1 centimetre rods
8 x 2 centimetre rods
4 x 4 centimetre rods
2 x 8 centimetre rods

24 centimetre trains: 24 x 1 centimetre rods
12 x 2 centimetre rods
8 x 3 centimetre rods
6 x 4 centimetre rods
4 x 6 centimetre rods
3 x 8 centimetre rods

18 centimetre trains: 18 x 1 centimetre rods
9 x 2 centimetre rods
6 x 3 centimetre rods
3 x 6 centimetre rods
2 x 9 centimetre rods

2. Answers need to include the fact that it is only possible to make one train for each length of 17 centimetres, 13 centimetres, and 23 centimetres, and in each case, that train is from 1 centimetre rods.

3. This is because these numbers have only two factors, themselves and one. They are prime numbers.

Page 24: Balancing Act

Activity One

1. 12 green balance 2 blue.

2. 12 red balance 4 black.

Activity Two

Answers will vary.

Figure It Out

Algebra

Teachers' Notes

Overview: Algebra

Title	Content	Page in students' book	Page in teachers' notes
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The Mystery of the Vanishing Pattern	Finding rules for sequential patterns	2	11
Punching Numbers	Number patterns	3	12
Follow That Arrow	Arrow diagrams	4	14
Sticky Moments	Finding rules for sequential patterns	5	15
Fair and Square	Finding relationships	6	17
Table Time	Using tables to show patterns	7	18
Puzzling Picnics	Describing relationships	8	19
A Spot of Sickness	Patterns and graphs	9	20
Building Graphs	Graphing relations on number planes	10	21
A Smile a Day	Drawing relationships	11	21
The Big Shake	Showing relationships	12	22
Story Plots	Describing graphs	13	22
Shoot the Hoop	Using >, <, =, or "between"	14–15	23
Number Nibbles	Number relationships	16	23
Crunch Machine	Patterns of equations	17	24
Perfect Patterns	Number relationships	18	25
Flower Power	Number relationships	19	25
What Goes Where?	Number relationships	20	26
Coin Control	Linking x with repeated +	21	27
Something Fishy	Writing algebraic expressions	22	27
Train Talk	Linking repeated + and x	23	28
Balancing Act	Extending relationships	24	28

Page 1: Pick the Pattern

Achievement Objective

- continue a sequential pattern and describe a rule for this (Algebra, level 2)

Activities One and Two

Using a table to organise the attributes of members in a sequential pattern is an important strategy in solving problems of the type found in these activities. This is particularly true where several attributes are involved in a single pattern. Another useful strategy is to identify the repeating element in the pattern.

For example, pattern **a** has an element of R – P, which means that multiples can be used to predict the tenth and twentieth object. Since P occurs at every multiple of two, both the tenth and twentieth objects must be purple circles. Similarly, the repeating element in pattern **b** is R O P, so the multiples of three can be applied. The ninth member of the pattern is P, so the tenth member must be R. The eighteenth member of the pattern is P, so the twentieth member must be O.

In pattern **c**, two attributes are involved, so it is useful to make a table.

Number	1	2	3	4	5	6	7	…
Colour	Purple	Orange	Green	Purple	Orange	Green	Purple	…
Position	Up	Side	Up	Side	Up	Side	Up	…

Once students can see a pattern of colour and position in the table, they can use the multiples. The colour pattern repeats in an element of three, the position repeats in an element of two. Therefore, the tenth member will be on its side and purple, and the twentieth member will be upright and orange.

The tenth and twentieth objects in the other patterns can be established by similar means.

Page 2: The Mystery of the Vanishing Pattern

Achievement Objective

- continue a sequential pattern and describe a rule for this (Algebra, level 2)

Activities One and Two

Students can expand on the strategies used on page 1 to complete the problems given on this page. For example, pattern **a** can be analysed using a table:

Position	1	2	3	4	5	6	7	8
Size	Small	Large	Small	Large	Small	Large	Small	Large
Shape	Square	Triangle	Circle	Square	Triangle	Circle	Square	Triangle
Colour	Pink	Green	Yellow	Pink	Green	Yellow	Pink	Green
				⌂	⌂			
				Unknown	Unknown			

Similarly, students can develop the following tables to help them identify the other patterns:

b.

Position	1	2	3	4	5	6	7
Shape							
Colour	Yellow	Blue	Yellow	Blue	Yellow	Blue	Yellow

c.

Position	1	2	3	4	5	6	7	8
Shape	Triangle	Square	Pentagon	Hexagon	Triangle	Square	Pentagon	Hexagon

d.

Position	1	2	3	4	5	6	7	8
Shape								

e.

Position	1	2	3	4	5	6	7	
Sequence	0	3	6	9	12	15	18	(Multiples of 3)

f.

Position	1	2	3	4	5	6	7	8
Shape								
Colour	Green	Yellow	Green	Yellow	Green	Yellow	Green	Yellow

Page 3: Punching Numbers

Achievement Objective

- continue a sequential pattern and describe the rule for this (Algebra, level 2)

The aim of this page is to encourage students to connect rules involving repeated addition or multiplication to their geometrical representation on a hundreds board. Students will need copies of the hundreds board to record their patterns.

Activity One

1. The sequence of numbers will be 1, 3, 5, 7, 9, 11, 13, 15, …

This can be generated on most calculators by keying in 1 + 2 = = = = = = … (Teachers will need to check that this works on the calculators used by students. Some scientific calculators are not suitable for this.)

The geometric pattern is that every second column is shaded:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
. .	. .	. .	. .	. .	. .	. .	. .	. .	. .

2. The sequence of numbers will be 5, 10, 15, 20, 25, 30, …

This can be generated on most calculators by keying in: [+] [5] [=] [=] [=] [=] …

The geometric pattern is:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
. .	. .	. .	. .	. .	. .	. .	. .	. .	. .

You will need to show students how to use the constant capability of the calculator. For instance, [2] [+] [3] [=] [=] [=] [=] … will generate the sequence 5, 8, 11, 14, 17, … (or, on some calculators, 5, 7, 9, 11, …)

Activity Two

This activity requires students to "work backwards" from the number sequence to establish the rule. Students could check the rule by repeatedly keying in the "x 2", for example, or they could generate the number sequence as follows:

1. The next number will be 16 as the function keyed in was [2] [x] [=] [=] [=], making the next number double the previous number. (You could also use [2] [x] [1] [=] [=] [=] [=] .)

2. Similarly, the next number in Finlay's pattern will be 81 as the next number is three times the previous number. This sequence can be created by keying in [3] [x] [=] [=] [=] or [3] [x] [1] [=] [=] [=] [=] .

3. Miffy's pattern is growing by four, which indicates a [+] [4] constant. The pattern starts at eleven, so either [11] [+] [4] [=] [=] … or [7] [+] [4] [=] [=] [=] are acceptable.

Note: As with other calculator function activities, you will need to check that these work as suggested on the calculators used by the students.

Page 4: Follow That Arrow

Achievement Objective

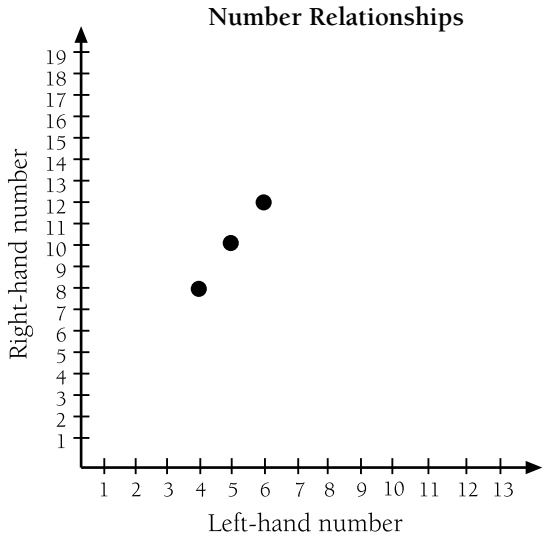
- use graphs to illustrate relationships (Algebra, level 2)

Activities One and Two

Although students are not asked to draw graphs in these activities, you may wish to ask your students to do so. The important concept being developed is that of a relation: how the value of one variable (the number on the left) maps onto the values of another variable (the numbers on the right).

For example, in **Activity One**, question **3**, 4 maps onto 8, 5 maps onto 10, and 6 maps onto 12, which suggests that students need to double the numbers on the left to map them onto the numbers on the right. This could be graphed as:

You can ask students to look for a pattern in the points on a graph and use this pattern to predict other values.



Students will probably use informal rules about the operations, such as "the numbers on the right are larger than the numbers on the left, so either addition or multiplication is used". Although these informal rules are useful in some problems, they can prove troublesome in later study. For example, adding a negative integer gives an answer less than the starting addend, as does multiplication by a fraction less than one.

Page 5: Sticky Moments

Achievement Objective

- continue a sequential pattern and describe a rule for this (Algebra, level 2)

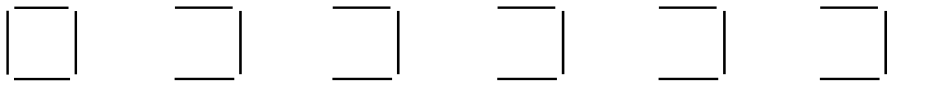
Activities One and Two

Many students will need to use matchsticks or similar equipment to build the patterns. Copying is an important initial behaviour if visual patterns are to help develop algebraic thinking.

Encourage students to look for systematic counting strategies to find the total number of matchsticks in each pattern. Below are some useful methods commonly used by students:

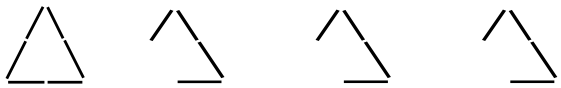
1. Add-on counting

a.



4 + 3 + 3 + 3 + 3 + 3 + …

b.



6 + 4 + 4 + 4 + …

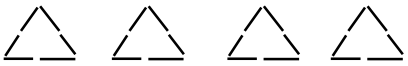
2. Using the whole figure but taking off the joins

a.

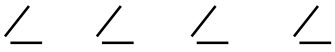
4 + 4 + 4 + 4 + …

take away 1 + 1 + 1 + 1 to allow for joins

b.



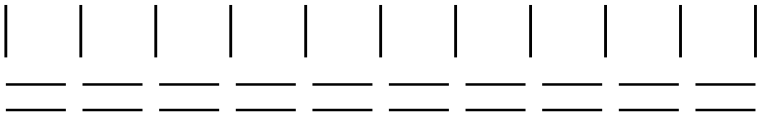
6 + 6 + 6 + 6 + …



take away 2 + 2 + 2 + 2 …

3. Parting the whole figure

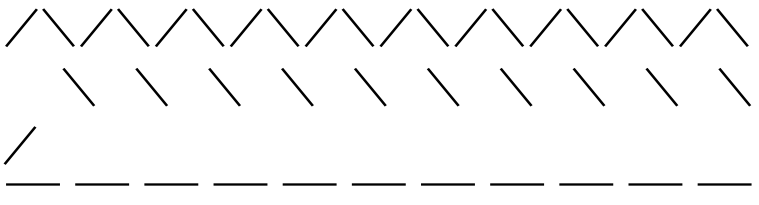
a.



11 vertical

10 + 10 horizontal

b.



20 caps

10 right side

1 left side

11 bottoms

Where students use methods you are unsure of, ask them to explain their reasoning: "Where did you get these numbers from? Can you show me how you did it?"

You can use a variety of strategies to find the number of matchsticks needed to build a 20-object pattern. Such strategies include:

1. Use matchsticks to build the actual pattern

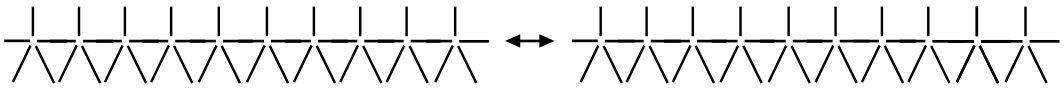
2. Use a table and look for the pattern:

Squares	1	2	3	4	5	…	20
Matchsticks	4	7	10	13	16	…	61

3. Write an equation:

$$\underbrace{4 + 3 + 3 + 3 + 3 + \ldots + 3 + 3}_{\text{19 threes}} = 61$$

Be aware that some students will make a "ratio error", believing that the number of matchsticks required to make a 20-object pattern will be twice that required for a 10-object pattern. The error ignores the effect of joining, but you can correct it by modelling with equipment. For example:



41 matchsticks 41 matchsticks

Joining these two sets gives a total of 81 matchsticks (rather than 41 + 41 = 82) because one middle arm is lost in the join.

Page 6: Fair and Square

Achievement Objectives

- continue a sequential pattern and describe the rule for this (Algebra, level 2)
- use graphs to illustrate relationships (Algebra, level 2)

Activity

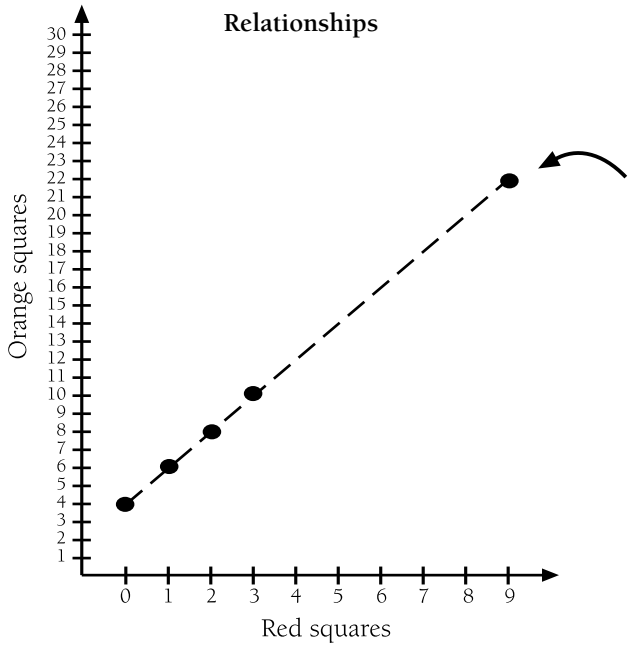
Students will need square tiles to build the patterns, or you can get them to draw the tile patterns on square grid paper. Although "building on" methods using equipment can solve the problems quite effectively, encourage students to look for rules that may help.

For example, with pattern **1**, students may notice that three yellow squares are added to the pattern each time a black square is added. This may be shown in a table:

Black Squares	1	2	3	4	5	6	7	8	9	10
Yellow Squares	4	7	10	13	16	19	22	25	28	31

goes up in threes

Students can check the predictions they made using tables or equations by continuing the pattern to the required length. In theory, future predictions could be found by graphing the relations involved. For example, the graph for pattern **2** could be extended to find the tenth term:



Since the points lie in a line (in other words, it is a linear relation), the imaginary path of points can be extended to (10, 22)

In practice, however, finding future values by extending a linear graph can be troublesome because one small slip in the line will lead to incorrect values. However, this unreliability is a useful teaching point.

Some students may find direct rules linking the values of the variables in the relation. For example, for pattern **3**, this might be described as "The number of blue squares, add one, gives the number of pink squares." This is a powerful method since it allows students to predict elements in a pattern without building the pattern or continuing a table. For example, in pattern **2**, the number of oranges squares is twice the number of red squares plus two. Therefore, for ten red squares, the number of orange squares is given by $(10 \times 2) + 2 = 22$.

Page 7: Table Time

Achievement Objective

- continue a sequential pattern and describe the rule for this (Algebra, level 2)

Activity

This activity is designed to highlight the usefulness of tables as a means of organising number sequences and looking for patterns. All these patterns involve multiples, so the addition constant on a calculator could be used. Some students will need to use counters as a physical representation of the problems.

Extending the table of values is an excellent way of predicting future terms, which some questions ask for. For example, in pattern **1**, the table can be extended as shown to find how many people buy 24 gumboots:

People	1	2	3	4	5	6	7	8	9	10	11	12
Gumboots	2	4	6	8	10	12	14	16	18	20	22	24

Encourage students to look for patterns "down the table", that is, the rule directly connecting each variable in the top row of the table with the corresponding variable in the bottom row, as well as patterns across. For example, in pattern **3**, the table would be:

pattern down is "times five"

Cars	1	2	3	4	5	6
Tyres	5	10	15	20	25	30

pattern across is "plus five"

In pattern **4**, the pattern down the table is difficult and involves subtracting three times the number of customers from 30 to get the number of books left. For example:

Customers	1	2	3
Books left	27	24	21
	⇧	⇧	⇧
	30 – (1 x 3)	30 – (2 x 3)	30 – (3 x 3)

Note: This question assumes that customers each buy only one book.

Page 8: Puzzling Picnics

Achievement Objectives

- continue a sequential pattern and describe a rule for this (Algebra, level 2)
- use graphs to illustrate relationships (Algebra, level 2)

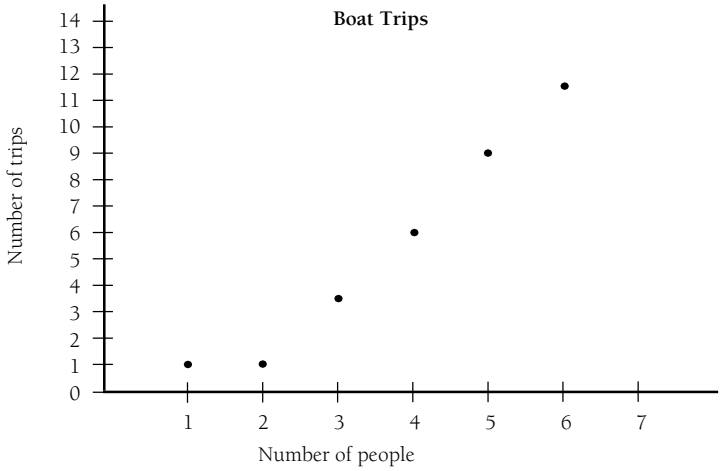
Activity

The boat-crossing problem is an excellent way of getting students to describe relationships. The students will need to carefully find the correct number of trips, using counters, physically moving themselves and other classmates across the room, or drawing diagrams (see diagram in the Answers section).

Organising the information into a table is a useful way to look for patterns.

Number of people	1	2	3	4	5	6
Number of trips	1	1	3	5	7	9

The pattern for the first two people is misleading, but after that, the pattern settles down to two more trips being required for each extra person. On a relationship graph, this appears as a linear (straight-line) pattern, which can be used to predict unknown values:



Students may use the "going up by two" rule to extend the pattern and find the number of trips required for 10 people.

Number of people	7	8	9	10
Number of trips	11	13	15	17

Given the larger boat with a capacity of three people, the number of trips required becomes fewer and follows the pattern:

Number of people	1	2	3	4	5	6	7
Number of trips	1	1	1	3	3	5	5

The repeating pattern of odd numbers is due to the net capacity of the boat being two people. That is, although the boat carries three people, one person must row the boat back to the other side, so only two people have completed their trip across the river.

Page 9: A Spot of Sickness

Achievement Objectives

- continue a sequential pattern and describe a rule for this (Algebra, level 2)
- use graphs to illustrate relationships (Algebra, level 2)

Activity One

Counters and faces drawn on pieces of card are essential for this activity. They enable students to construct the graph in **Activity Two**.

Students will need to count the number of chickenpox spots carefully as any errors will obscure the pattern. Organising the information in a table is helpful.

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Number of chickenpox spots	1	3	5	7	9

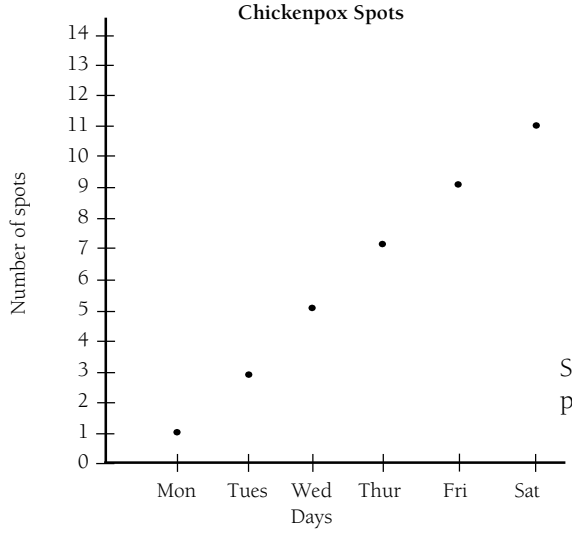
The number pattern makes it clear that the number of chickenpox spots is growing by two each day. Other answers are possible as long as they can be justified. For example:

"I think Jerry has six chickenpox spots on Friday because he is getting better, and he loses the first chickenpox spot he got."

Activity Two

This activity requires students to draw a graph of the relationship between the days of the week and the number of chickenpox spots on Jerry's face. It is conventional to show such relations as points on a number plane, as in the example below. This means that a point at the top of each column of counters is sufficient. It is also conventional to have the independent variable (the one we have knowledge of) on the horizontal axis and the dependent variable (the one we are trying to find out about) on the vertical axis.

In this activity, students' graphs will vary, but encourage them to explain their graphs to others to facilitate peer assessment. It is vital that students explain the position of points to establish whether or not they understand ordered pairs.



Students should note that the points climb on a linear path.

Page 10: Building Graphs

Achievement Objective

- use graphs to illustrate relationships (Algebra, level 2)

Activities One, Two, and Three

As with the graphs formed for page 9, these relations can be shown as sets of points. These points can be written as ordered pairs, such as {(1,4), (2,8), (3,12), (4,16), …}, where the first number (the independent variable) gives the number of sheep and the second number (the dependent variable) gives the number of legs in **Activity One**.

The two graphs required are shown in the Answers section.

With both graphs, the sets of points lie along a line because the patterns increase by a regular number between points. For example, in the graph for **Activity One**, there are four more legs for each extra sheep.

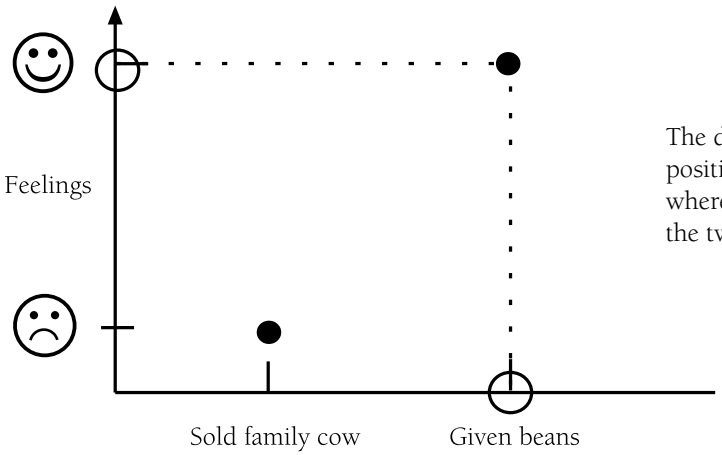
Page 11: A Smile a Day

Achievement Objective

- use graphs to illustrate relationships (Algebra, level 2)

Activities One and Two

These activities involve informal relations where one or more of the variables is difficult to quantify. Discuss Jack's graph to help students interpret the information in it. A rectangular piece of paper can make reading the graph easier.



The dotted line shows how to position the paper to show where the point intersects with the two axes.

In **Activity Two**, students' graphs will vary, but encourage them to explain their graphs to others to facilitate peer assessment. It is vital that students explain the position of points to establish whether they understand ordered pairs.

Page 12: The Big Shake

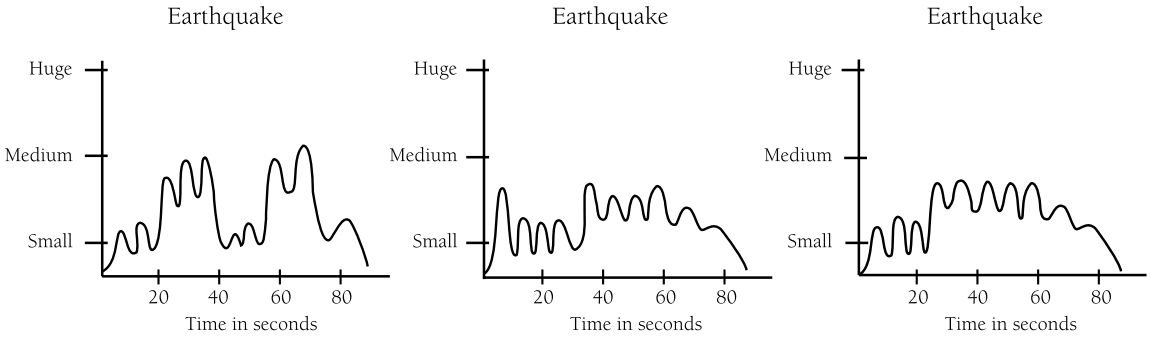
Achievement Objective

- use graphs to illustrate relationships (Algebra, level 2)

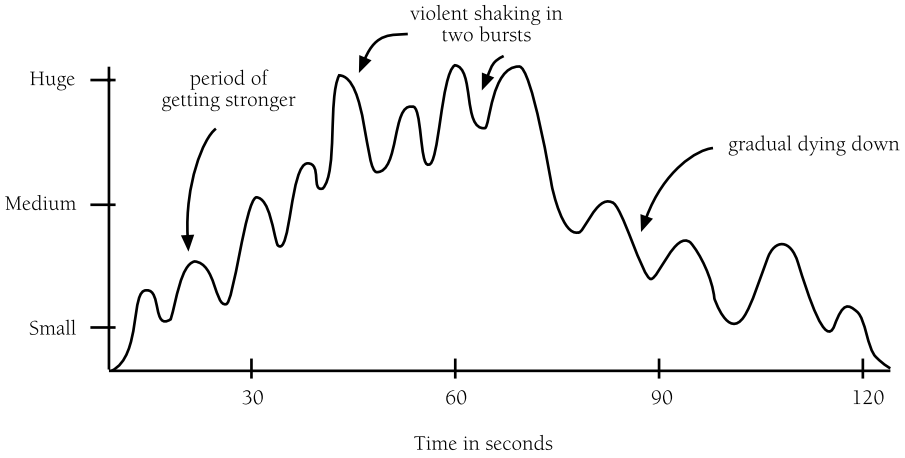
Activity

Students will need to recognise that the greater the height of the waves in the graph, the greater the intensity of the earthquake. Although the graphs students draw for each scenario may appear different, look for the height of the waves.

For instance, all three graphs below would match the first scenario.



Encourage students to tell the story of their graph to explain the patterns in it. For example, this graph matches scenario **c**:



Page 13: Story Plots

Achievement Objective

- use graphs to illustrate relationships (Algebra, level 2)

Activity

The activity focuses on relations where both variables could be quantified but where the dependent variable is simplified. For example, the temperature could be given in degrees Celsius, but a three-point scale has been used instead.

If students have difficulty, focus on particular points on the graph and discuss what they mean. Piece together the story of A Spring Day and then ask the students to write their version of the complete story.

A suitable story might be:

"First thing in the morning, the sun came out, and the temperature gradually increased until it was hot. After 1 p.m., a cold wind blew in while clouds covered the sun, so the temperature dropped. At about 3 p.m., the skies cleared, and it got hotter for an hour. The cloud returned at 5 p.m., and the temperature got colder and colder until 10 p.m."

A model of a car travelling to Grandma's house may help students interpret the distance/time graph of **1**.**b**. The slope of the line at given intervals tells the speed of the car. The two horizontal portions indicate that distance is not increasing over two periods of time, so we can presume the vehicle is stationary at those times. A suitable story might be:

"It took us from 8 a.m. to 9 a.m. to get out of Auckland, and then we travelled fast on the motorway until 10 a.m.. We stopped at Pokeno for an ice cream for half an hour, and then Sue drove. She is just learning, so she went really slowly. Finally we got to Grandma's place at 12 p.m."

Pages 14 & 15: Shoot the Hoop

Achievement Objective

- use the mathematical symbols =, <, > for the relationships "is equal to", "is less than", and "is greater than" (Algebra, level 2)

Game

This game uses digit cards and encourages the students to think about the symbols and relationships outlined in the achievement objective.

Page 16: Number Nibbles

Activity

Students can use counters to experiment to find the correct solutions. Getting students to work in co-operative groups will help those students who have reading difficulties.

The activity focuses on finding unknowns. The clues could be described using algebraic symbols, but this is not the purpose at this level. (For example, "There are three more red stones than green" could be written as g + 3 = r.)

Encourage students to prioritise the clues in order of significance. For instance, in recipe **4**, the clue "There are six blue stones" defines a key part of the solution from which the other clues develop.

The progression below illustrates a possible path for solving recipe **3**.

"There are 25 stones in total." [Student draws 25 circles on paper.]

"At least half the stones are blue." [Blue could occupy 13 or more circles.]

"There are twice as many blue stones as red stones." [There must be an even number of blue stones, so the possibilities are 14 blue and 7 red (21 in total) or 16 blue and 8 red (24 in total).]

"There are the same number of yellow stones as green stones." [Only the 14 blue and 7 red possibility will work, so there are 2 yellow and 2 green stones as well.]

With recipe **6**, restrict students to a maximum of 24 stones. This may curb their inclination to use large numbers, making the equipment difficult to use. Students will need to realise that it is easier to create these problems than solve them!

Page 17: Crunch Machine

Achievement Objectives

- continue a sequential pattern and describe the rule for this (Algebra, level 2)
- use the mathematical symbols =, <, > for the relationships "is equal to", "is less than", and "is greater than" (Algebra, level 2)

Activity One

Organising the results in an in/out table may help students to recognise patterns:

(+ 5) In	Out
2	7
4	9
6	11
8	13
10	15

There is a difference of two between consecutive "in" numbers and a difference of two between consecutive "out" numbers. The "out" number is five more than the "in" number.

(– 8) In	Out
8	0
12	4
16	8
20	12

There is a difference of four between consecutive "in" numbers and a difference of four between consecutive "out" numbers. The "out" number is eight less than the "in" number.

(x 3) In	Out
7	21
8	24
9	27
10	30

The "in" numbers go up in ones.
The "out" numbers go up in threes.

(÷ 5) In	Out
5	1
10	2
15	3
20	4

The "in" numbers go up in fives.
The "out" numbers go up in ones.

Activity Two

Some students may need a calculator to help them, although processing the problems mentally is more efficient.

Realising that the = sign must go in one of the vacant squares is a clue to how the problems can be solved in an organised way.

For example, question **3** offers three possibilities:

15 = 21 ❑ 4 ❑ 40 [This looks unlikely.]

15 ❑ 21 = 4 ❑ 40 [More likely, although + and – are the only likely operators.]

15 ❑ 21 ❑ 4 = 40 [Using addition signs gives the correct solution.]

Students may enjoy making up their own problems, although this is another case where creating problems is much easier than solving them.

Page 18: Perfect Patterns

Achievement Objectives

- continue a sequential pattern and describe the rule for this (Algebra, level 2)
- use the mathematical symbols =, <, > for the relationships "is equal to", "is less than", and "is greater than" (Algebra, level 2)

Activities One, Two, and Three

The aim of these activities is for students to focus on the relationships between numbers in different positions within a family of equations. Most students are likely to see relationships down the equations rather than within the equations themselves. For example, consider the set:

$1 = 3 - 2$

$2 = 4 - 2$

$3 = 5 - 2$

$4 = 6 - 2$

Students will see that the left-hand numbers are "increasing by one" as are those immediately to the right of the equals sign. The "subtract two" part of each equation remains constant. This makes writing the next equation a patterning exercise.

The problem of writing an equation much further down encourages students to look for relationships within the equations and is much more difficult. Many students will wish to keep writing equations down in the pattern; encourage them to think around this. For example, with the pattern above, they might note that the number immediately to the right of the equals sign is always two greater than the left-hand number. This enables any number of equations in this pattern to be written:

$20 = 22 - 2$

⇧

two greater than 20

Page 19: Flower Power

Achievement Objective

- use the mathematical symbols =, <, > for the relationships "is equal to", "is less than", and "is greater than" (Algebra, level 2)

Activity

It is best to work through the first example with the class so that students understand that they must find a single operation that relates the numbers on the petal to the number at the centre of the flower. Note how all the petal numbers in this example can be expressed as two multiplied by another factor.

Students will need to apply their knowledge of the structure of the operations addition, subtraction, multiplication, and division. For example, with flower 37, the petal numbers are less than the centre number. This suggests that the operation is either division or subtraction since only whole numbers are involved. Students may realise that 37 is a difficult number to divide evenly because it is a prime. Subtraction is the only plausible operation left.

As an extension, encourage students to create their own flowers. These may include fractions as well as whole numbers.

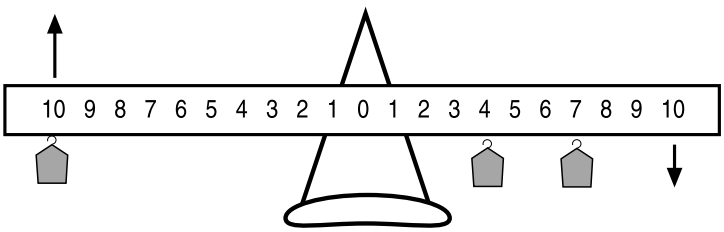
Page 20: What Goes Where?

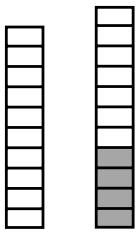
Achievement Objectives

- use the mathematical symbols =, <, > for the relationships "is equal to", "is less than", and "is greater than" (Algebra, level 2)

Activity One

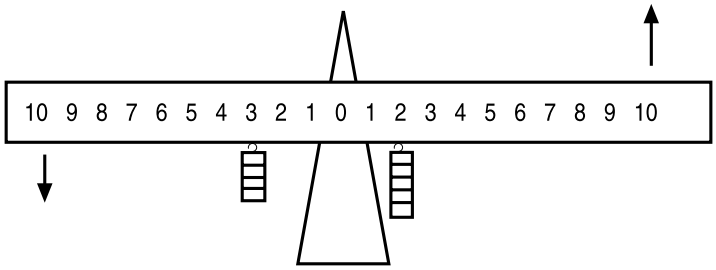
Some students may find the examples of inequalities in this activity difficult, and they will need prior experience with simpler inequalities, for example, 10 ❑ 4 + 7. Physical models, such as number balances and towers of connecting cubes, can help develop the students' understanding.

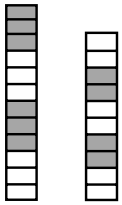




10 < 4 + 7

These models still apply when each side of an inequality contains an operation, for example, 4 x 3 ❑ 5 x 2.





4 x 3 > 5 x 2

Students need to have the ability to process operations as mental ideas and understand the equals sign in order to cope with inequalities. For example, the student who has 4 + 7 = 11 as an idea in their mind without having to calculate it with equipment or mentally is much better prepared to solve the inequality 13 ❑ 4 + 7. The ability to process inequalities mentally is the aim of these activities; manipulation of equipment is a means towards that end.

Activity Two

This activity involves the students being able to interpret a flow chart for grouping children for swimming lessons. Students could be "walked through" how the flow chart works, using the example of actual students from their class. As an extension, students may like to create a flow chart about sorting a set of attribute blocks into triangles, squares, hexagons, and circles.

Page 21: Coin Control

Achievement Objectives

- mentally perform calculations involving addition and subtraction (Number, level 2)
- demonstrate the ability to use the multiplication facts (Number, level 2)

Activities One, Two, and Three

The aim of these activities is to get students to connect the operations of repeated addition with multiplication. This idea is critical at later levels for the development of direct rules that describe relationships. Encourage students to try to process the problems mentally at first. They can confirm their predictions later by using equipment models, such as bags of counters or multilink cubes.

It is important that you demonstrate the symbolic representations of these models. Some students may use equations without help, and their methods can be shared with others. Alternatively, you may need to demonstrate this recording.

For example, Carlos' number of gold pieces can be written as:

$2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 16$ or $8 \times 2 = 16$

Donny's number of gold pieces can be written as:

$3 + 5 + 3 + 5 + 3 + 3 + 5 + 5 + 3 + 5 = 40$ or $3 + 3 + 3 + 3 + 3 + 5 + 5 + 5 + 5 + 5 = 40$

or $(5 \times 3) + (5 \times 5) = 15 + 25 = 40$

It is important that students understand that the equals sign is a statement of equality and not a statement of "works out to be".

You can help reinforce this idea of the meaning of the equals sign by reversing the way equations are conventionally written. For example, you could write $12 = 7 + 5$ instead of $7 + 5 = 12$, which suggests an "answer on the right hand side" view. Missing-number problems also reinforce the purpose of the equals sign, for example, $3 + ❑ = 8$ and $❑ - 6 = 4$.

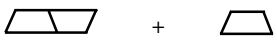
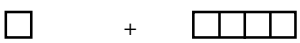
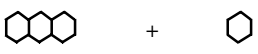
Page 22: Something Fishy

Activities One and Two

Although there is no achievement objective at this level that can be applied to these activities, there is evidence to suggest that students at levels 2–3 can gain some proficiency in manipulating symbolic expressions in context. The activities are limited by the fact that letters are represented as fixed unknowns rather than as generalised numbers or variables. Students' views about letters can be broadened through the patterning activities used at later levels.

Students who experience difficulties with these problems may require a physical model to assist or confirm their thinking. These models may be pictures of the fish, hot dog, and chips or may be equipment such as pattern blocks or multilink cubes with a key, for example,  for a fish,  for a hot dog, and for chips.

Develop the recording of algebraic symbols during this activity. For example, the Jones' order could be represented as:

Equations	Models
$2f + f = 3f$	
$h + 4h = 5h$	
$3c + c = 4c$	

Page 23: Train Talk

Achievement Objectives

- mentally perform calculations involving addition and subtraction (Number, level 2)
- demonstrate the ability to use the multiplication facts (Number, level 2)

Activities One and Two

The main focus of this activity is the relationship between repeated addition and multiplication. It provides another context for the activities given on page 21. Although many students will need Cuisenaire rods and rulers in order to confirm predictions, the primary purpose of the activity is to develop mental addition and multiplication strategies.

To this end, it is important that students predict which rods might make up a train of a given length and justify their reasoning before they build the train to confirm their predictions. Initially, students may record their solutions in rod pictures, which can be used to show the power of recording equations. For example, three students might record the information about their 18 centimetre train as:

$6 + 6 + 6 = 18$ or $3 \times 6 = 18$

By comparing these three methods during a discussion time, students will make the connection between the representations and appropriate oral description of the equation. For example, "Three multiplied by six tells you how much three sets/lengths of six come to."

Note that the requirement to use only one type of rod in each train means that 17, 13, and 23 centimetre trains can only be built with unit cubes. This is because 17, 13, and 23 are prime numbers and therefore can only be divided evenly by themselves and one.

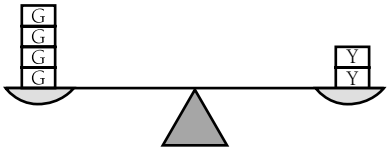
Page 24: Balancing Act

Achievement Objectives

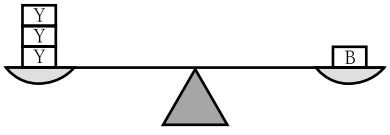
- mentally perform calculations involving addition and subtraction (Number, level 2)
- demonstrate the ability to use the multiplication facts (Number, level 2)

Activity One

The main idea in this activity is that balance on a set of scales demonstrates an equality relationship. For example:



could be written as $4g = 2y$ and suggests that yellow cubes are twice as heavy as green cubes.



could be written as $3y = b$ and suggests that a blue cube is three times heavier than a yellow cube.

To solve the question "How many G s balance two B s?" students will need to work out how to convert the blues into an equivalent mass of yellows so they can use the first relation shown above.

If one blue balances three yellows, then two blues balance six yellows. From the first set of scales, we know that each yellow is balanced by two greens. So 6 x 2 = 12 greens will be needed to balance two blues.

Often the difficulty for students in these problems is keeping track of the information, so it is a good idea to encourage students to keep a written record.

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