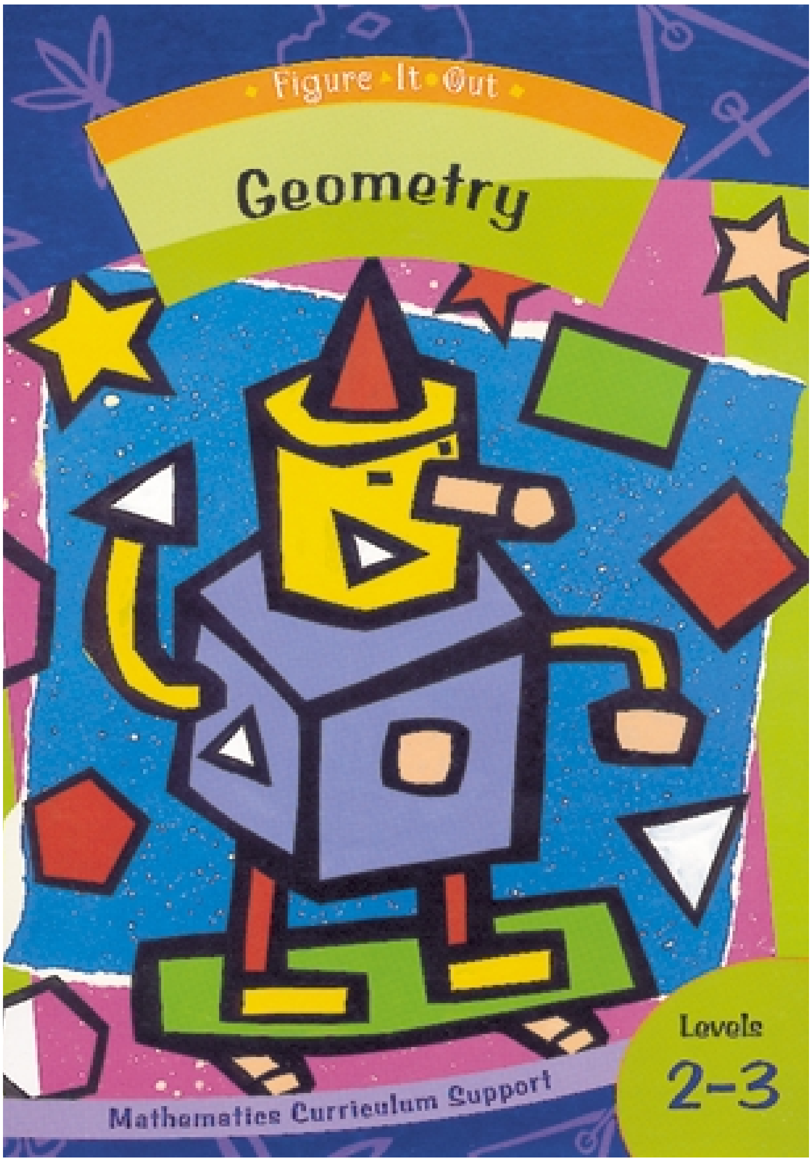
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Contents

Introduction

The Figure It Out series is designed to support *Mathematics in the New Zealand Curriculum*. The booklets have been developed and trialled by classroom teachers and mathematics educators. The series builds on the strengths of a previous series of mathematics booklets published by the Ministry of Education, the School Mathematics supplementary booklets.

Figure It Out is intended to supplement existing school mathematics programmes and can be used in various ways. It provides activities and investigations that students can work on independently or co-operatively in pairs or groups. Teachers can select particular activities that provide extension to work done in the regular classroom programme. Alternatively, teachers may wish to use all or most of the activities in combination with other activities to create a classroom programme. The booklets can be used for homework activities, and the relevant section in the teachers' notes could be copied for parents. These notes may also provide useful information that could be given as hints to students.

There are eight booklets for levels 2–3: one booklet for each content strand, one on problem solving, one on basic facts, and a theme booklet. Each booklet has its own *Answers and Teachers' Notes*. The notes include relevant achievement objectives, suggested teaching approaches, and suggested ways to extend the activities. The booklets in this set (levels 2–3) are suitable for most students in year 4. However, teachers can decide whether to use the booklets with older or younger students who are also working at levels 2–3.

The booklets have been written in such a way that students should be able to work on the material independently, either alone or in groups. Where applicable, each page starts with a list of equipment that the students will need in order to do the activities. Students should be encouraged to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Teachers could encourage students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Where possible, suggestions have been made to encourage discussion and oral presentation of answers, and teachers may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum*, page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. Successful ways of solving problems should be acknowledged, and where more effective or efficient processes can be used, students can be encouraged to consider other ways of solving the problem.

Figure It Out

Geometry

Answers

Page 1: Shapes in Life

Activity

Answers will vary.

Page 2: Post It!

Activity One

1. cylinder
2. pyramid
3. cube: the square hole and the circle (if the circle is big enough)
   cone: the triangle hole and the circle hole

Activity Two

Answers will vary.

Page 3: Pattern Play

Activity One

1. and **2.** Students will not be able to fit together the repeated shape of a circle or a hexagon without gaps, but they should be able to repeat equal-sided shapes, such as squares, triangles, and rectangles.

Activity Two

1. a. and **b.** Classmate to check

2. a.– d. Teacher to check

Activity Three

Answers will vary.

Page 4: In Shape to Race

Activity

Route 1 can be followed only by big red or blue blocks that are squares, rectangles, or hexagons.

Route 2 can be followed only by red or blue blocks that are triangles, circles, or hexagons.

Any block can get to the finish if it is not yellow and students choose the correct route for that block.

Page 5: Mind Boggle

Activity One

Amy's shape is a blue hexagon that is thin and small.

Priya's shape is a yellow square that is thick and big.

Ralph's shape is a red rectangle that is thin and small.

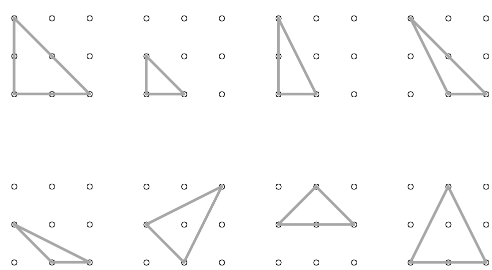
Activity Two

Space 1: yellow hexagonal blocks
Space 2: large yellow blocks
Space 3: large hexagonal blocks
Space 4: large, yellow hexagonal blocks

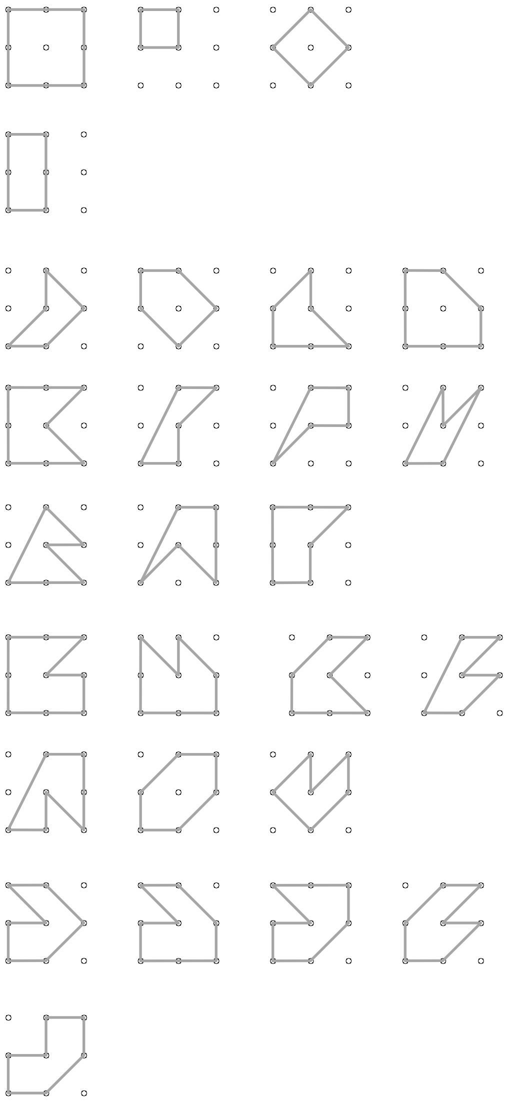
Page 6: Shaping Up

Activity One

1. Some possible answers are:

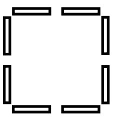


2. Some possible answers are:



Activity Two

1.



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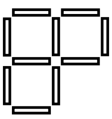

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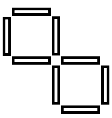

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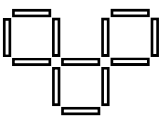

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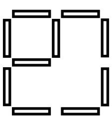
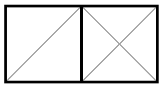


Page 7: Changing Shape

Activity One

Some possible answers:

a.



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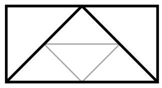

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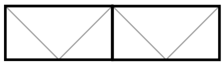
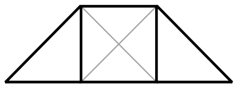


b.



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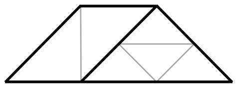

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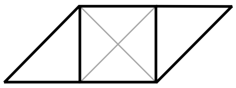

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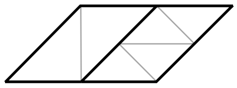

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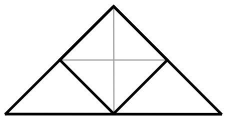

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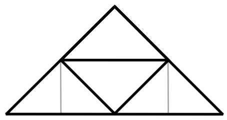


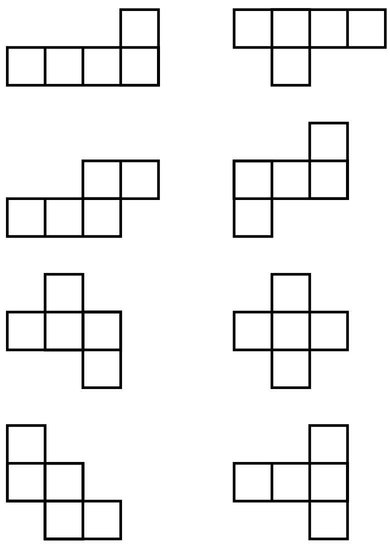
Activity Two

1. and **2.** Teacher to check

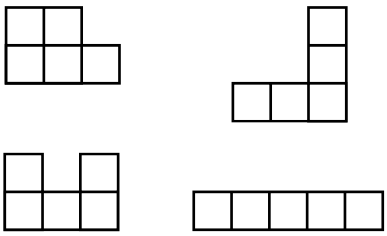
Page 8: Boxes of Tricks

Activity One

1. Pentominoes are combinations of five squares joined side to side.
2. No. An open cube is a cube box with no lid.
3. 12
4.

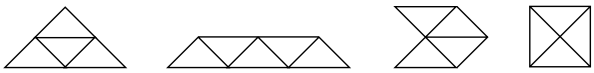


5.



Activity Two

1. 4:



2. The first two shown in the answer to question **1** will fold to form a tetrahedron.

Page 9: Roll Over

Activity One

1. Sian's method does work.
2. Teacher or classmate to check

Activity Two

The faces of the house are rectangles and pentagons.

Page 10: Little Boxes

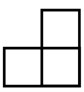
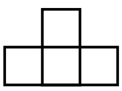
Activity

1. **a.** 4

   b. 6
2. Answers will vary.
3. Teacher to check

Page 11: Points of View

Activity One

1. Answers will vary.
2. 
3. 

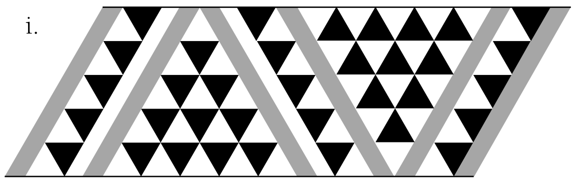
Activity Two

Answers will vary.

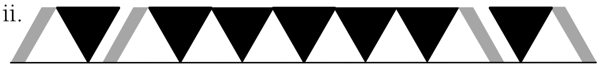
Page 12: Tricky Triangles

Activity One

1. 20
2. 25
3. 25
4. 65
5. **a.** Answers will vary. They include:
   the next section sideways:

   i. 

   or the next section downwards:

   ii. 

   b. Answers will vary, but the two examples above will give i. 35 or ii. 15 new triangles.

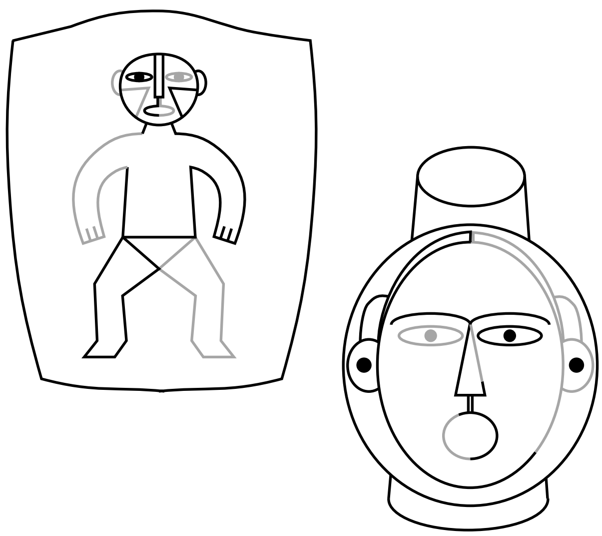
Activity Two

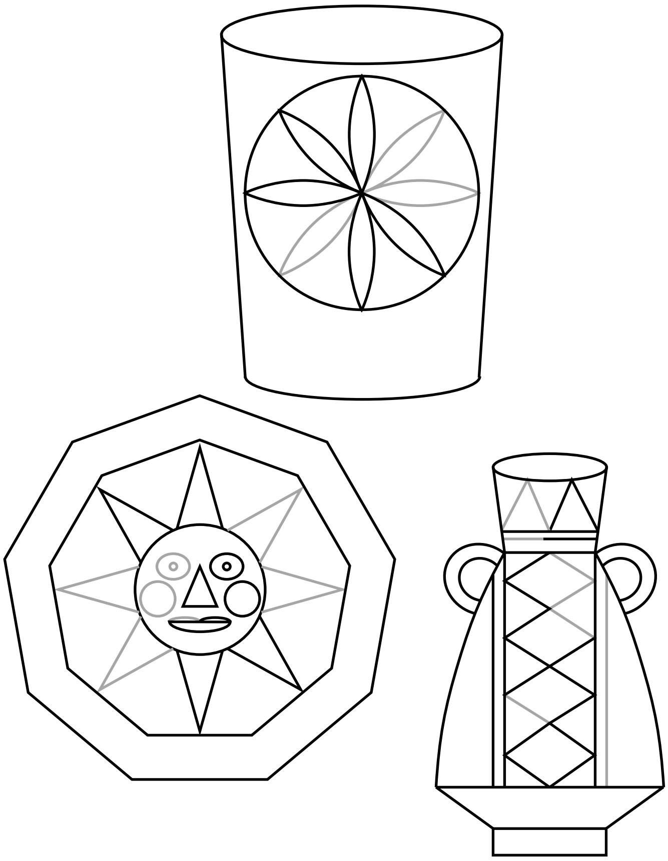
Answers will vary.

Page 13: Sketching the Etching

Activity

1.





2. Answers will vary.

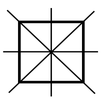
Page 14: The Mirror Game

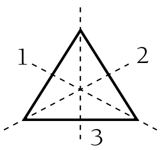
Game

A game of reflection

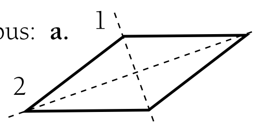
Page 15: Getting in Line

Activity

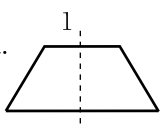
1. 

2. triangle: **a.** 

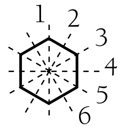
b. 3 mirror lines

rhombus: **a.** 

b. 2 mirror lines

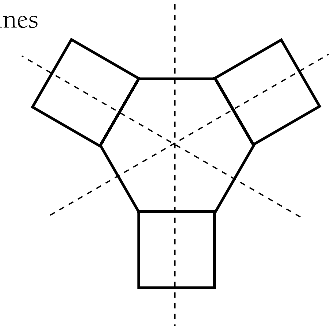
trapezium: **a.** 

b. 1 mirror line

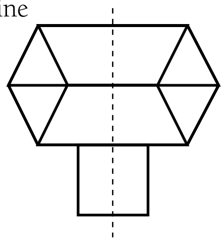
hexagon: **a.** 

b. 6 mirror lines

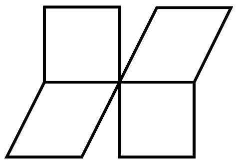
3. **a.** 3 mirror lines



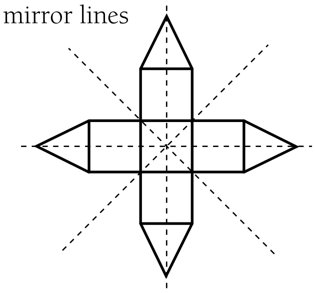
b. 1 mirror line



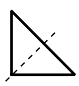
c. no mirror lines

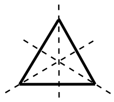


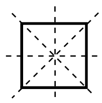
d. 4 mirror lines



4. Answers will vary. Some examples are:

a. 

b. 

c. 

Page 16: Wrap It Up

Activity

Teacher to check

Page 17: Cut It Out

Activity One

Answers will vary.

Activity Two

1. Teacher to check
2. a. They will all have at least 2 lines of symmetry.

   b. The pattern will have at least 4 lines of symmetry.

Page 18: Hidden Shapes

Activity One

Answers will vary.

Activity Two

Answers will vary.

Page 19: Follow That Thread

Activity One

Answers will vary but could include:
The space left inside the square is an ellipse.
The space left inside the angle is a parabola.
The space left inside the circle is almost another circle.

Activity Two

a. This makes approximately a $^{1}/_{4}$ circle (or a hyperbola).

b. To make a complete circle, students could make another 12 slits and create a $^{1}/_{4}$ circle in each $^{1}/_{4}$ of the square.

Pages 20 and 21: Pirate Island

Activity One

Teacher to check

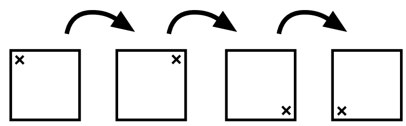
Activity Two

Answers will vary.

Page 22: Tumble Time

Activity One

a.



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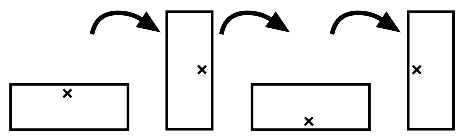

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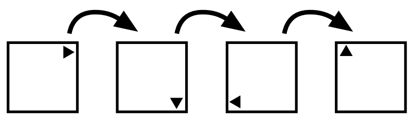

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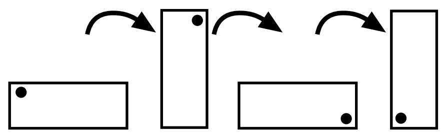

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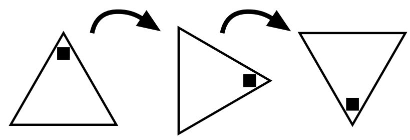

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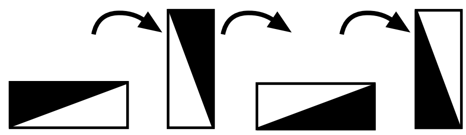
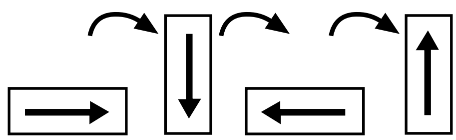


g.



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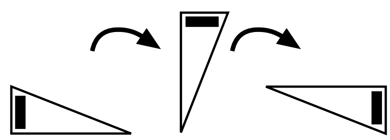
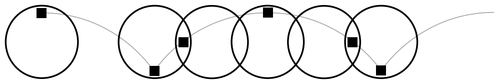


Activity Two

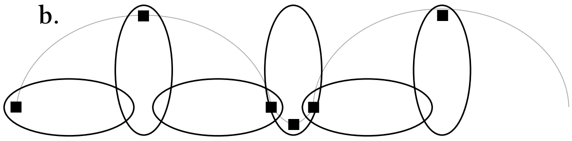
1. **a.** It moves in a circular path as the circle turns. (It stays in the same position in relation to the outside of the circle.)

   b.



2. **a.** The square in the ellipse will also move in a circular path (because the square is always the same distance from the centre of the ellipse).

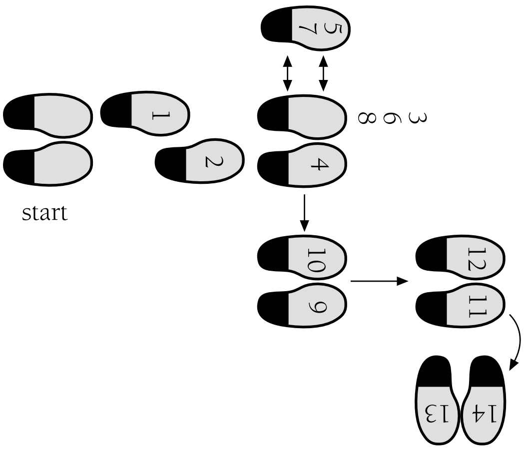
   b.



Page 23: Taking Flight

Activity One

1.



2. She does the routine four times in order to face the same way as she started.

Activity Two

Answers will vary depending on the movement of the object.

Page 24: Cutting Corners

Activity One

1. A sphere
2. The list could include balls, marbles, soaps, parts of buildings, poi, and oranges.

Activity Two

1. A cylinder
2. The list could include paper rolls, tins, jars, poles, and tubes.

Activity Three

1. A cone
2. The list could include ice-cream cones, traffic cones, and turrets.

Figure It Out

Geometry

Teachers' Notes

Overview: Geometry

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Post It!	Shapes	2	11
Pattern Play	Making patterns with shapes	3	12
In Shape to Race	Working with attributes	4	13
Mind Boggle	Problem-solving matrices	5	14
Shaping Up	Classifying shapes	6	15
Changing Shape	Shape puzzles	7	17
Boxes of Tricks	Making nets for three-dimensional solids	8	18
Roll Over	Making nets for three-dimensional solids	9	18
Little Boxes	Drawing buildings made with cubes	10	20
Points of View	Drawing buildings made with cubes	11	20
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Sketching the Etching	Making symmetrical patterns	13	22
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Getting in Line	Mirror symmetry (reflection)	15	25
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Follow That Thread	Following a path to make a shape	19	29
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Shapes in Life

Achievement Objective

- make, name, and describe, using their own language and the language of geometry, everyday shapes and objects (Geometry, level 2)

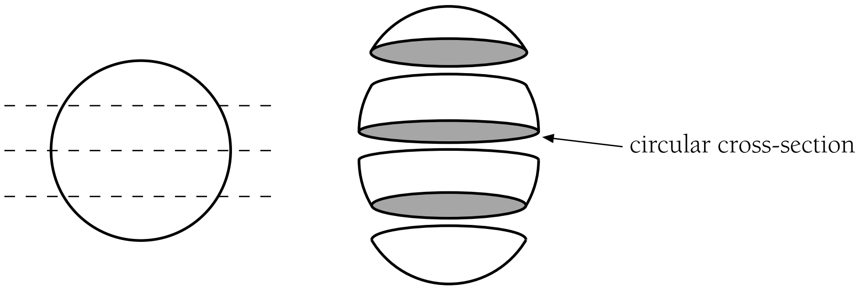
Activity

Students should not have too many difficulties finding the two-dimensional shapes (squares, circles, triangles, etc.) in the photographs. However, before they attempt to identify the solids in the pictures, it is wise to discuss the features of these solids. They can be classified into two broad categories:

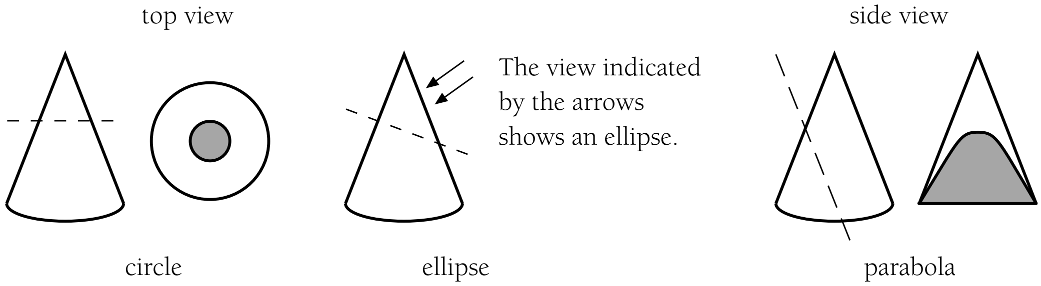
Polyhedra (poly: many; hedra: faces)	Closed surfaces
cube, rectangular prism, triangular prism, pyramid	cylinder, sphere, cone

The important features of polyhedra are the number and shape of their faces, the number of edges, and the number of vertices (corners). For example, a cube has six square faces, 12 edges, and eight vertices.

Closed surfaces are formed by the intersection of surfaces that may be flat or curved. Cross-sections of closed surfaces are interesting and can be explored by making the solids with play dough and cutting them with one straight cut. For example, no matter where you cut a sphere, its cross-section is always a circle, (though it can be argued that the cut could touch the circle at one point):



A cone has many well-known cross-sections:



Once they have found examples of these solids in the photographs, you could ask the students to create their own skyscape of buildings made from these solids. The students will need to investigate nets (flat patterns) that can be folded to form each solid (see the activities on pages 8 and 9).

Page 2: Post It!

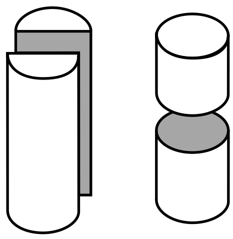
Achievement Objectives

- classify objects (Mathematical Processes, developing logic and reasoning, level 2)
- make, name, and describe, using their own language and the language of geometry, everyday shapes and objects (Geometry, level 2)

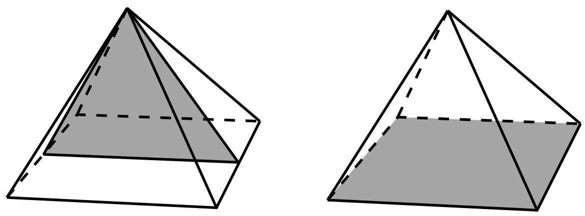
Activity One

Posting boxes encourage young children to look at the shape of the faces and cross-sections of three-dimensional solids. The dimensions of the posting holes are usually worked out so that a block may pass through only one hole. (The posting holes for this question are a circle, a rectangle, a square, and a triangle.) In this case, each shape that has a cross-section or face that is the same as a given solid will be able to post through that solid's posting hole.

For example, the cylinder has a rectanglar cross-section when sliced vertically and a circular cross-section when sliced horizontally. The cylinder will post through the circular and rectangular holes.



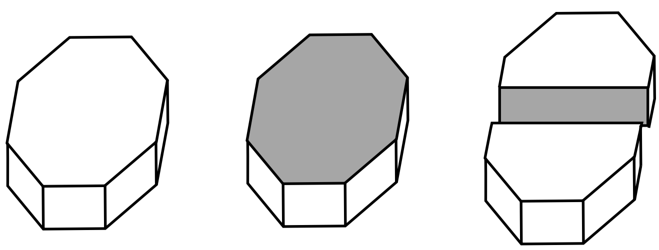
Similarly, the pyramid will fit through the triangular and square holes.



Activity Two

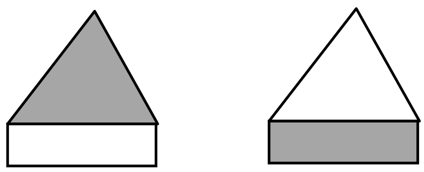
For this activity, students will need to visualise solids that have the cross-section shapes shown in the problems.

The octagonal prism will have an octagonal and a rectangular cross-section.



octagonal prism

The triangular prism has a triangular and rectangular cross-section.



triangular prism

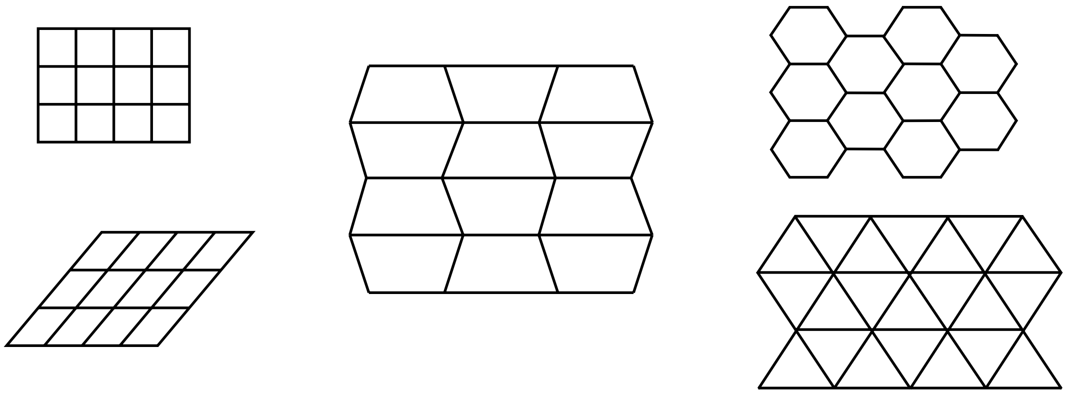
Page 3: Pattern Play

Achievement Objectives

- make, name, and describe, using their own language and the language of geometry, everyday shapes and objects (Geometry, level 2)
- create and talk about geometric patterns which repeat (show translation) or which have rotational or reflection symmetry (Geometry, level 2)

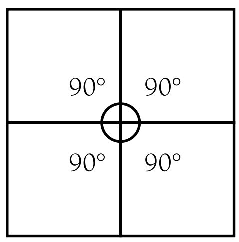
Activity One

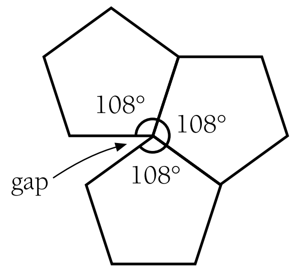
In this activity, students apply the properties of polygons to find tessellation patterns. Pattern blocks provide a large number of possibilities for tessellation. These possibilities include:



The key idea of tessellation is the size (or measurement) of the internal angles of the polygons that meet at each vertex (point). For example, consider squares and regular pentagons:

Although squares will tessellate about a point because four angles of 90° complete a full 360°, regular pentagons have internal angles of 108°. Three regular pentagons about a point give an angle sum of 324°, which leaves a gap.





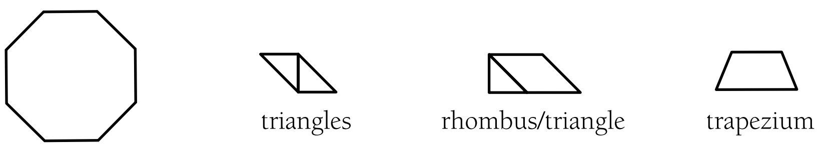
Students at this level may intuitively grasp this idea, but full understanding of tessellations is usually developed in later years. Encourage students to explain why some shapes tessellate but others do not.

Activity Two

Students will need to visualise what pattern block shapes will fit into the given silhouettes. If students look at the parts of the figure that jut out, they get the best clues about which shapes are involved in that figure.

Activity Three

This activity encourages students to investigate possible shapes that they can make with pattern blocks. Angles are very useful indicators of which blocks have been used. For example, the angles of the octagon suggest that several triangles, rhombuses, or trapezia have been used:



Page 4: In Shape to Race

Achievement Objective

- classify objects (Mathematical Processes, developing logic and reasoning, level 2)

Activity

This activity is an example of a logic track. It requires students to consider several attributes of a block simultaneously to find out whether the block can get from start to finish.

Students should justify their decision about which routes are best for the blocks they have chosen. For example, in considering the size attribute, big blocks should take Route 1 and small blocks Route 2. Selecting a quadrilateral (either a square or a rectangle) would mean taking Route 1 because the block could not pass through the tunnel on Route 2.

Students need to find ways to organise their thoughts about the characteristics of blocks that get to the finish. A table can be very helpful to record the different characteristics. The attributes can be crossed out on the table as they are eliminated:

	Route 1	Route 2
Shape	~~triangle~~ ~~circle~~ square rectangle hexagon	triangle circle ~~square~~ ~~rectangle~~ hexagon
Colour	red blue ~~yellow~~	red blue ~~yellow~~
Size	big ~~small~~	big small
Thickness	thick thin	thick thin

The table shows that any block can get to the finish provided that it is not yellow and students choose the correct route for that block. Students will enjoy creating their own logic tracks for other students.

Page 5: Mind Boggle

Achievement Objective

- classify objects (Mathematical Processes, developing logic and reasoning, level 2)

Activity One

Students who are not familiar with using tables to eliminate possible outcomes will find this problem difficult. Teachers may wish to provide an easier example, such as:

Hattie and Troy each took a different attribute block from the set. Use the clues to work out which block each person took:

1. Hattie's shape has three more sides than Troy's shape:

	Red	Yellow	Blue	⬡	□	△	○	▭	Thin	Thick	Small	Big
Hattie				✓	✗	✗	✗	✗				
Troy				✗	✗	✓	✗	✗				

2. Both Hattie and Troy did *not* take a big blue shape:

	Red	Yellow	Blue	⬡	□	△	○	▭	Thin	Thick	Small	Big
Hattie			✗	✓	✗	✗	✗	✗			✓	✗
Troy			✗	✗	✗	✓	✗	✗			✓	✗

3. Troy's shape was red and thicker than Hattie's:

	Red	Yellow	Blue	⬡	□	△	○	▭	Thin	Thick	Small	Big
Hattie	✗	✓	✗	✓	✗	✗	✗	✗	✓	✗	✓	✗
Troy	✓	✗	✗	✗	✗	✓	✗	✗	✗	✓	✓	✗

So, Hattie took a thin, yellow, small hexagon, and Troy took a thick, red, small triangle. The key idea with using a matrix as above is that a positive selection (✓) also results in some negative selections (✗).

Activity Two

The intersection is the common space shared by two or more circles. The blocks that can be placed at the intersection of these circles must have the attributes of the two or more circles involved. Note that in the second diagram, the space numbered 4 denotes the intersection of the three circles.

Page 6: Shaping Up

Achievement Objective

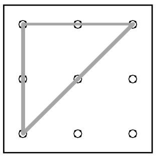
- make, name, and describe, using their own language and the language of geometry, everyday shapes and objects (Geometry, level 2)

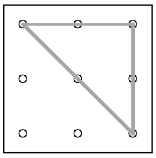
Activity One

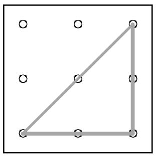
The main purpose of this activity is to get students to classify triangles and thereby broaden their knowledge about the properties of triangles. There are a limited number of different triangles that can be made on a 3 x 3 geoboard. Students may find it difficult to see a reflection or rotation of a given triangle as being the same triangle.

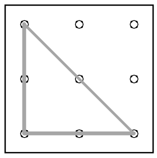
It can be very helpful to put copies of the geoboard on a transparency to show on an overhead projector. For example:

Triangles a, b, c, and d are congruent and can be mapped onto one another by rotation.

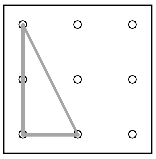

a

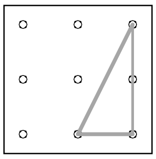

b

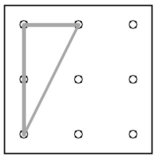

c

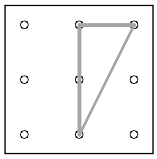

d

Triangles e, f, g, and h are all congruent and can be mapped onto each other by combinations of reflection (for example, e <–> f), rotation (for example, f <–> g), or translation (for example, g <–> h):


e

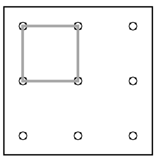

f

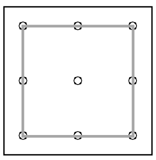

g

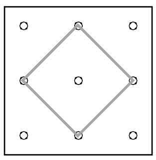

h

Asking students to come up to the projector and fit one triangle onto another by flipping, turning, or shifting is a good way to test the uniqueness of particular triangles.

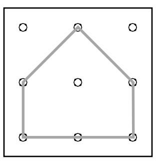
The purpose of the table in question **2** is not to make the problem more difficult but to get students to organise their results and classify the shapes they find. For example, the three different squares that are possible are also types of rectangles.

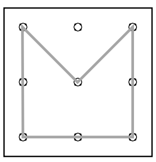


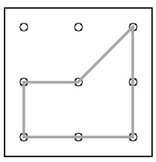


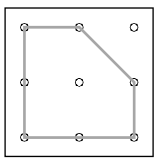


The shapes below are all examples of pentagons:



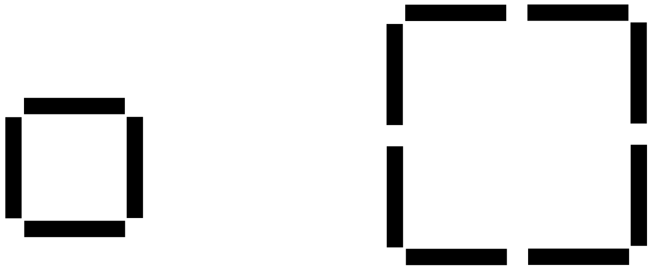




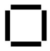
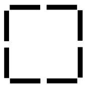


Activity Two

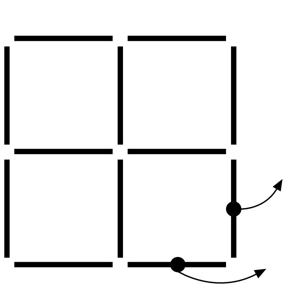
A useful strategy to begin the problems in this activity is to imagine the size that the target squares could be. Given the starting figure, they can only be of the sizes:



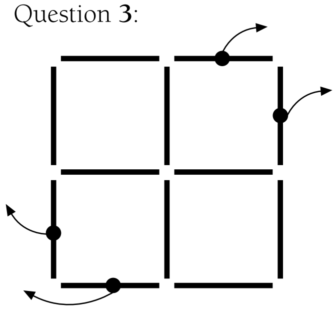
4 rods or 8 rods

Use rods or matchsticks for these exercises. Question **1** involves leaving only one square after removing four rods. To leave a smaller square of , students would have to take away more than four rods. Therefore, the target square in this question must be . To achieve this, students will take away more than four internal rods. The other problems in questions **2–5** can be solved using similar reasoning.

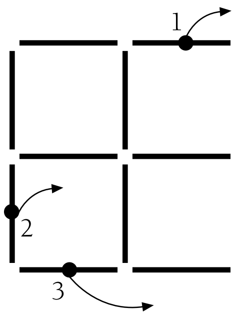
Question **2**:

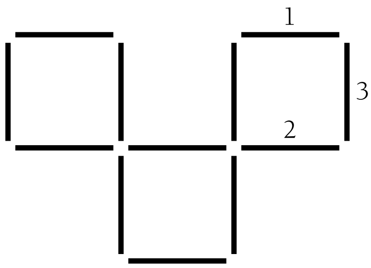


Question **3**:

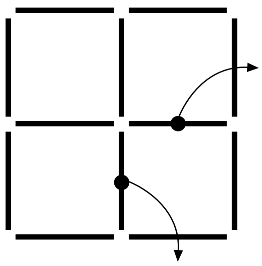


Question **4**:





Question **5**:



Page 7: Changing Shape

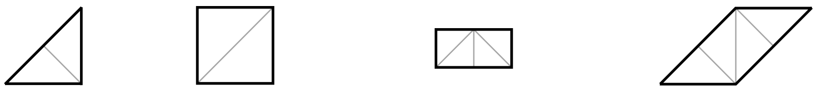
Achievement Objective

- devise and follow a set of instructions to carry out a mathematical activity (Mathematical Processes, communicating mathematical ideas, level 2)

Activity One

Students will need to make the triangle pieces for the two activities from two square pieces of card so that they can manipulate the pieces to work out the different shapes each time. As with the rod problems on page 6, it is very important to have a concept of the size and properties of the target shape. Also important are the subunits of target shapes and how they might be made using the pieces.

For example, in **Activity One**, the main subunits can be made up in the following ways:



triangle square oblong (rectangle) parallelogram

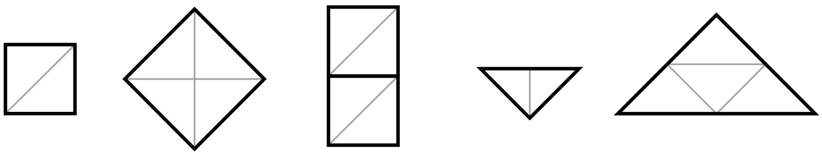
See the diagrams in the Answers section for possible solutions.

Remind students that all the pieces must be used to complete each target shape.

Note how the square, triangle, and parallelogram subunits are used in these possible answers for question **b**:

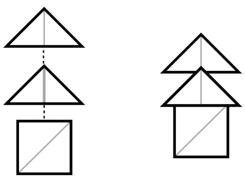
Activity Two

Similar reasoning can be used to solve these problems. It should be noted that the shapes in problems **a** and **b** overlap slightly. The subunits here are:

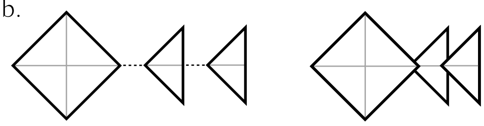


square rectangle triangle

a.



b.



Page 8: Boxes of Tricks

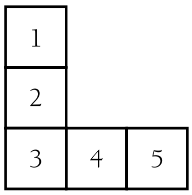
Achievement Objective

- make, name, and describe, using their own language and the language of geometry, everyday shapes and objects (Geometry, level 2)

Activity One

Pentominoes are combinations of five squares joined side to side. *Penta* is the prefix that means five.

Most students will need to cut the pentominoes out of squared paper to find out whether they will fold to make a cube. Encourage them to visualise the folding before attempting to fold each pentomino. The pentomino in question **2** will *not* fold to make an open cube (a cube box with no lid):

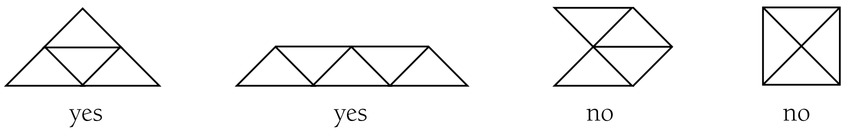


Faces 1 and 5 will overlap.

The Answers section shows the 12 possible pentominoes and whether they form open cubes.

Activity Two

Students will find the different triangular shapes by trial and improvement after drawing them on the isometric dot paper.



The two shapes labelled "yes" above will fold to form a tetrahedron; the other shapes will not.

Page 9: Roll Over

Achievement Objective

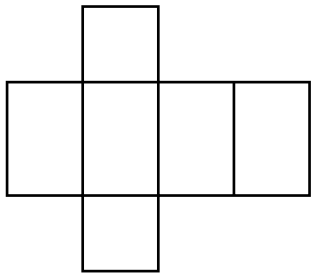
- make, name, and describe, using their own language and the language of geometry, everyday shapes and objects (Geometry, level 2)

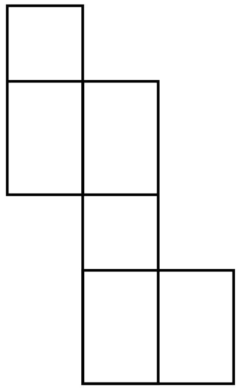
Activity One

Sian's method is a straightforward way of finding the net (flat pattern) from which a simple solid can be built. Students will need to ensure that every face is accounted for when rolling.

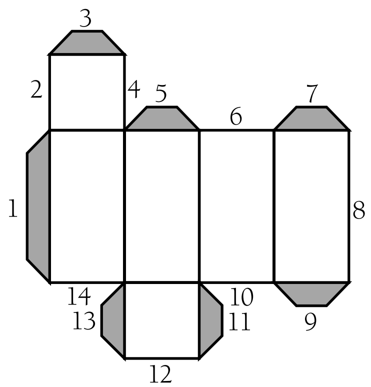
Students may be intrigued that many different nets can be generated for the same solid, depending on the order in which the faces are traced.

For example, a rectangular prism can be made with these nets:



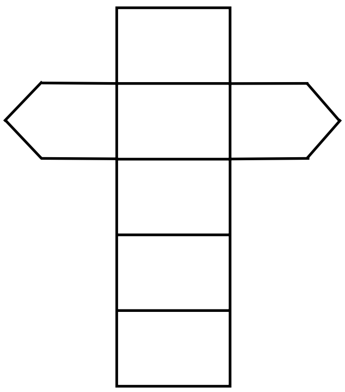


Students will notice that the box they open has tabs that are generally used to glue the net together. Packaging companies try to have the least possible number of tabs to keep their costs low. Conventionally, these tabs are located on every second side of the net, as shown in the diagram below:



Activity Two

The house in this activity has two pentagonal faces and five rectangular faces. Its net is shown below:



Page 10: Little Boxes

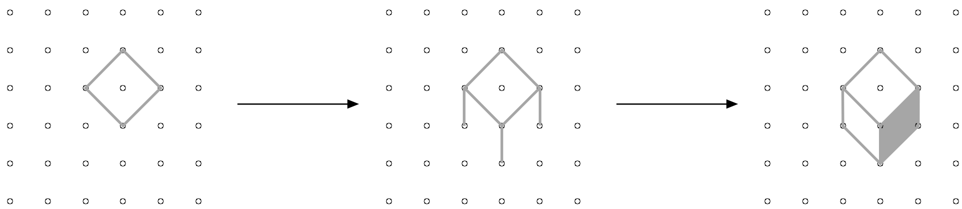
Achievement Objective

- make, name, and describe, using their own language and the language of geometry, everyday shapes and objects (Geometry, level 2)

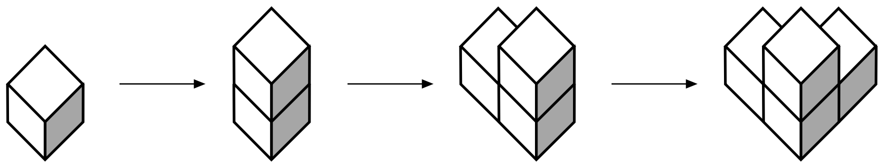
Activity

Students' spatial visualisation can be developed further by having them use isometric dot paper to draw shapes that have been made with cubes. It is easier for students to draw models in this form if they have had experience building cube models from complete drawings first.

Students who have difficulty with isometric drawing will need to start by drawing a single cube:



From this initial drawing, the students can go on to add more cubes:



The greatest difficulty students have working out how many cubes were used to make a drawn building is visualising the cubes that they cannot see. Experience in making and drawing their own cube building is vital to develop spatial awareness of hidden parts of three-dimensional solids.

Getting students to make cube buildings from other students' drawings is an important part of three-dimensional work and encourages creativity and ownership in problem solving.

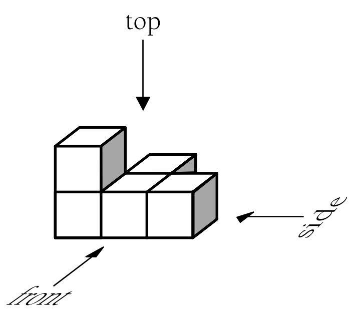
Page 11: Points of View

Achievement Objective

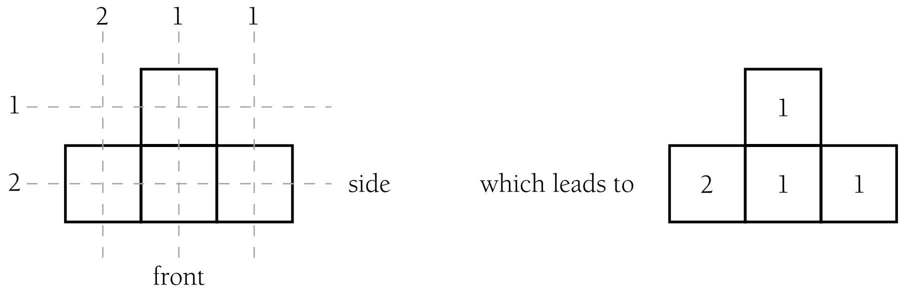
- make, name, and describe, using their own language and the language of geometry, everyday shapes and objects (Geometry, level 2)

Activity One

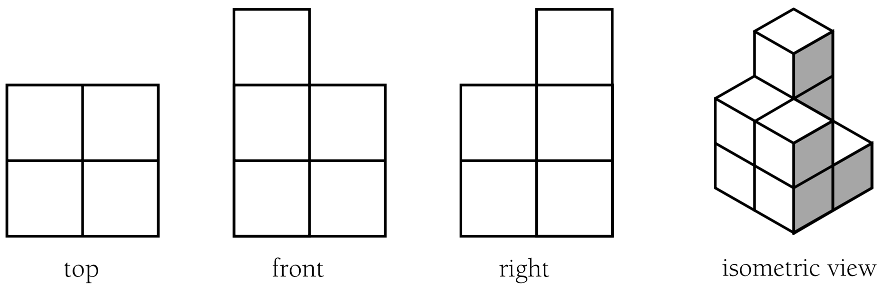
Students will need to make Zac's building through a process of trial and improvement until their model matches the top, front, and right-side views as shown:



In more difficult problems of this type, the top view can be used to organise the data from the other views. For example, with Zac's building, the maximum heights of each column and row of the top view are:



A further challenge to students is to be given three views of a cube building and to make an isometric drawing of it (as on page 10). For example:



Page 12: Tricky Triangles

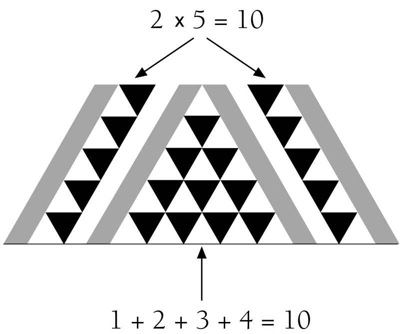
Achievement Objective

- create and talk about geometric patterns which repeat (show translation), or which have rotational or reflection symmetry (Geometry, level 2)

Activity One

This pattern is taken from a section of tapa cloth. The design is based on tessellations of triangles.

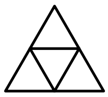
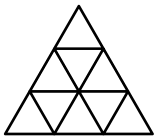
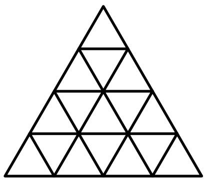
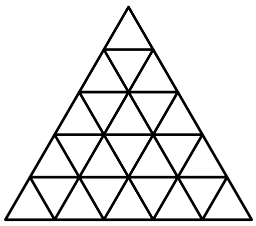
Students will need to count the triangles systematically for questions **1–4** and record their counting carefully. A knowledge of symmetry is very useful in solving these problems. For example, students can use the following method to count the number of small brown triangles:



Students can use the total number of small brown triangles to help find the number of small white triangles. In the central section of the tapa cloth design, there is one more row of small white triangles than there is of brown triangles, so the total number is: (1 + 2 + 3 + 4 + 5) + (5 + 5) = 25.

In question **3**, students may realise that each row contains two more triangles than the previous row: 1 + 3 + 5 + 7 + 9 = 25.

In question **4**, students will need to think how many triangles of different sizes there are in the pattern.

Triangle	Number
	20
	25
	10
	6
	3
	1

Question **5** asks students to generate the next section of the pattern. Students may extend the pattern down or sideways. Two possible answers are given in the Answers section.

Students can find the next pattern using translation (shifting) and half-turn rotation.

Page 13: Sketching the Etching

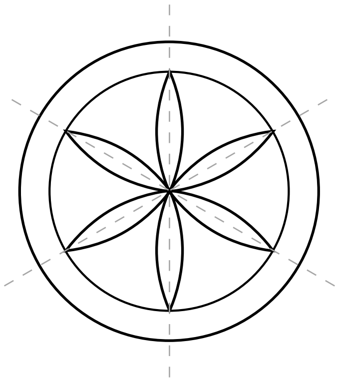
Achievement Objective

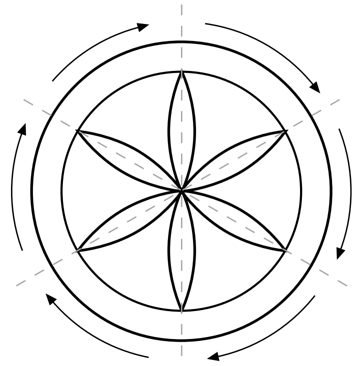
- create and talk about geometric patterns which repeat (show translation), or which have rotational or reflection symmetry (Geometry, level 2)

Activity

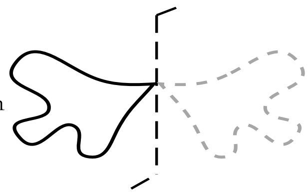
All the designs shown on this page have reflection symmetry. A mirror can be used to fill in the missing scratches provided that the lines of symmetry are established.

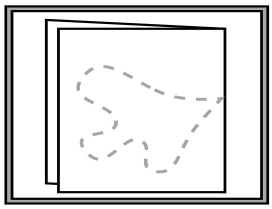
For example, the flower design has three lines of symmetry and rotational symmetry of order six. This means it turns onto itself six times in a full turn.





If students place a mirror on any given line of symmetry and look at half the design, its image will confirm what the complete design should look like. Alternatively, a tracing of the shape can be folded in half along the mirror line and the doubled-over paper held up to a window for tracing. In a similar way, students can create a design with mirror symmetry. For example:





Page 14: The Mirror Game

Achievement Objective

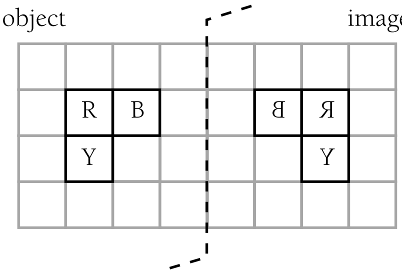
- create and talk about geometric patterns which repeat (show translation), or which have rotational or reflection symmetry (Geometry, level 2)

Game

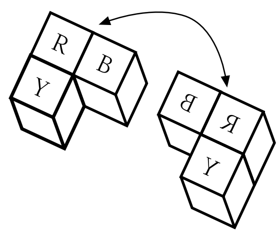
The mirror game is designed for students to apply reflection symmetry. Although Perspex mirrors are ideal because a matched cube can be seen to coincide with its reflection, backed mirrors are adequate for students to check that they have placed the matching cube correctly.

The harder version of the game, involving composite pieces, fosters discussion about the properties of reflections. For example, consider the case:

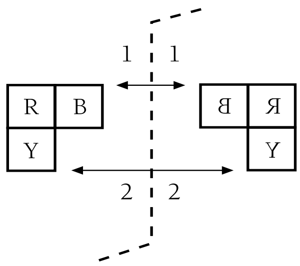
Although the size and shape of the object remains unchanged, its orientation (direction) changes. This orientation change is described as a "flip" in American mathematics books.



A flip is a useful way to describe the movement.

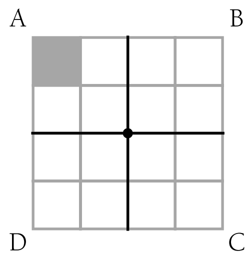


Note that the distance between any block and the mirror should be the same as the distance between the image of the block and the mirror:

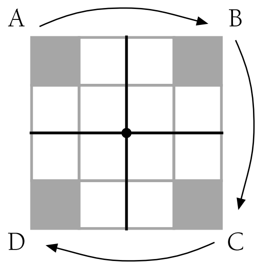


As an extension to this activity, students may like to play The Rotation Game. In this game, players match a cube that is put down with its image after a quarter turn. For this reason, people play on four grids.

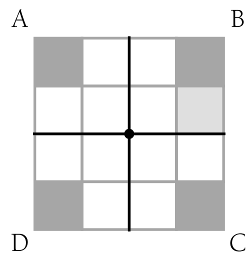
i. Player A places a cube:



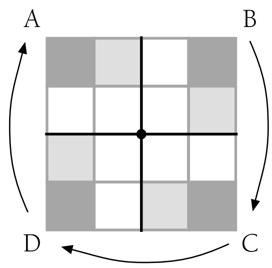
ii. Players B, C, D, in succession, match where the cube would end up in their quadrant after each quarter turn:



iii. Player B has a turn placing a different cube in their quadrant.



iv. Players C, D, and A take turns to match where the cube would end up in their quadrant after each quarter rotation:



Students can check their attempts at placement by turning the paper a quarter turn and then back to its original position. The game is much more difficult if each player has a 3 x 3 quadrant, which makes the player's total grid 12 centimetres x 12 centimetres, or a 4 x 4 quadrant grid, which is 16 centimetres x 16 centimetres in total.

As with The Mirror Game, students can take the cubes off one by one and colour the grid to record their pattern.

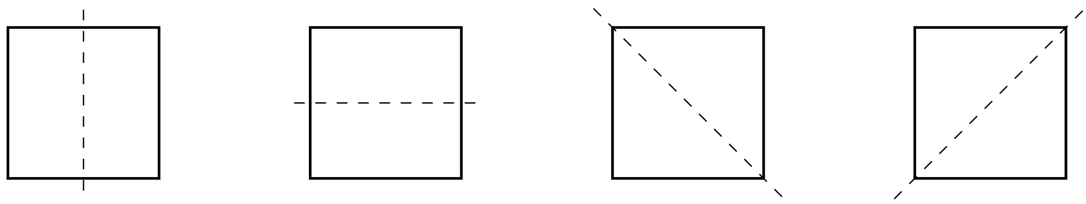
Page 15: Getting in Line

Achievement Objective

- create and talk about geometric patterns which repeat (show translation), or which have rotational or reflection symmetry (Geometry, level 2)

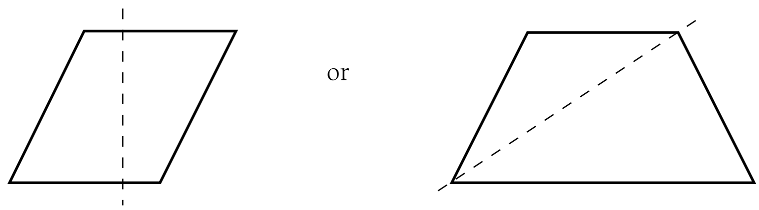
Activity

To begin the activity, students should trace around the pattern block they are considering so that they can draw in the lines of reflection (mirror lines). For a square, the four lines would be:



Some of the other pattern block shapes are more difficult to analyse (see diagrams in the Answers section).

Some students will draw lines that appear to cut the shape in half but are **not** lines of reflection symmetry. For example:



Students need to self-check these lines by looking at the whole shape and then comparing it with the image they see when they put a mirror on the mirror line.

Investigating the number of lines of symmetry of a circle is an interesting extension. Some students may colour the whole circle to show that there are so many lines that they couldn't draw them all. This is one way of describing infinity.

Finding lines of symmetry for shapes made from a number of pattern blocks is considerably more difficult than for single pattern block shapes. However, the same principles apply (see the diagrams in the Answers section).

The final challenge is to build pattern block shapes with specified numbers of lines of symmetry. In this case, students will need to consider the single pattern block shapes once more. For example, building a pattern with three lines of symmetry will need to be based around a triangle or a hexagon shape. A pattern with four lines of symmetry will need to be based around a square shape. See the Answers section for examples.

Page 16: Wrap It Up

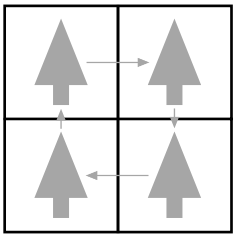
Achievement Objective

- create and talk about geometric patterns which repeat (show translation), or which have rotational or reflection symmetry (Geometry, level 2)

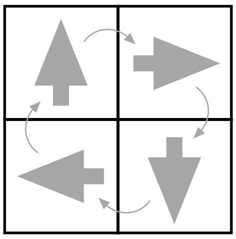
Activity

This activity gets students to apply translational and rotational symmetry to create freeze-type patterns. The method suggested uses a printing block.

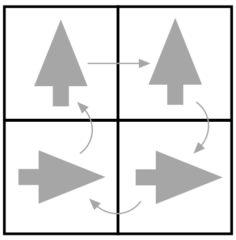
The patterns created by translation need to preserve the orientation of the figure. For example:



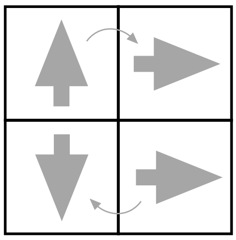
By contrast, the patterns created by rotation involve a change in orientation. For example:



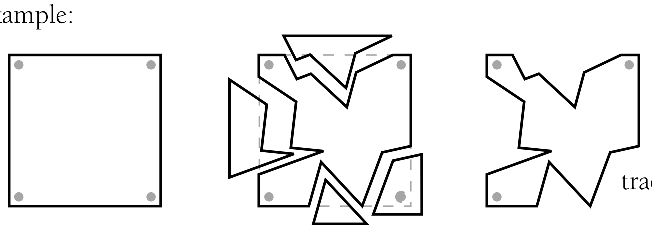
Students may find that they can apply translation and rotation in combination. For example:



or

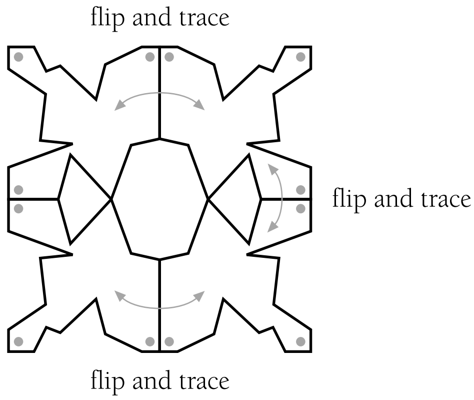


Another way of making patterns by reflection is to cut a tracing block from a square of card. Note that it is wise to mark the original corners of the square before cutting to help keep track of the original shape. For example:



tracing block

The pattern can be duplicated across the whole page by repeated reflection.



Page 17: Cut It Out

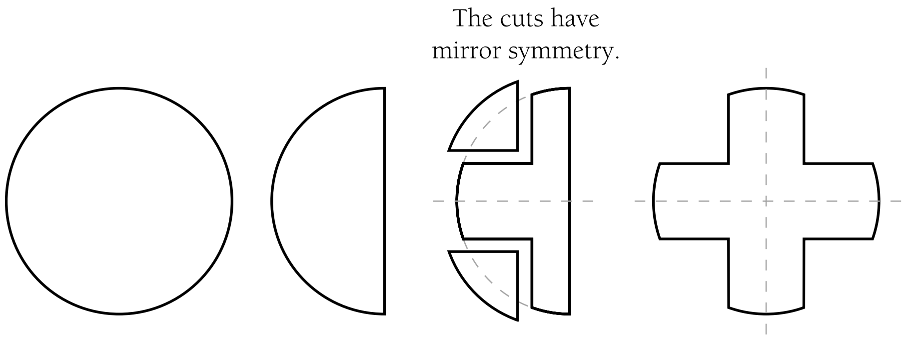
Achievement Objective

- create and talk about geometric patterns which repeat (show translation), or which have rotational or reflection symmetry (Geometry, level 2)

Activity One

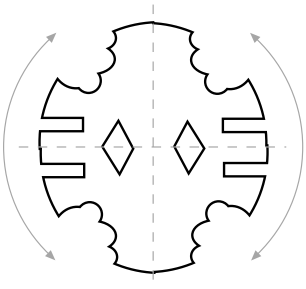
Predicting the shape before opening it up is a key part of this activity. It focuses students on the likely attributes of the shape and is useful preparation for discussing symmetry.

Circles that are folded in half and cut up will always generate shapes with at least one line of reflection symmetry along the fold line. That is why mirror symmetry and fold symmetry are sometimes used synonymously. Note that a half-folded circle cut up may have two lines of symmetry if the cuts are made symmetrically. For example:



Activity Two

In a similar way, a circle that is folded into quarters and then cut will generate a shape that has at least two lines of reflection symmetry (along the fold lines) and half-turn symmetry. The example shown in this activity will look like this when it is opened up:



The pattern continues with circles that are folded into eighths and cut up. These shapes always have at least four lines of reflection symmetry and at least quarter-turn symmetry.

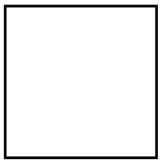
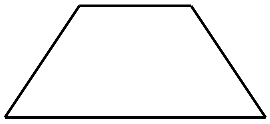
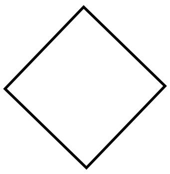
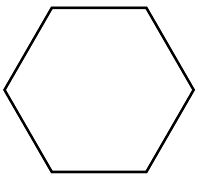
Page 18: Hidden Shapes

Achievement Objectives

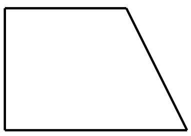
- make, name, and describe, using their own language and the language of geometry, everyday shapes and objects (Geometry, level 2)
- describe and interpret position, using the language of direction and distance (Geometry, level 2)
- make clockwise and anticlockwise turns (Geometry, level 2)

Activity One

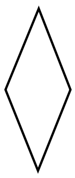
This is an excellent activity for developing visual memory and the use of geometric language. The level of difficulty of the task can be varied by altering the number of blocks used to make the figure. Discuss the appropriate names for the pattern blocks and their properties.

	Square (pl. squares)	• Four equal sides • Four right, internal angles (90°)
	Trapezium (pl. trapezia)	• Four sides • Only one pair of parallel sides
	Rhombus (pl. rhombuses)	• Sides and opposite angles equal • A square is a special rhombus.
	Hexagon (pl. hexagons)	• Six sides • A regular hexagon has six equal sides and angles.

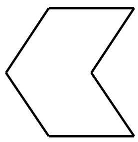
Broaden students' understanding of these shapes by asking them to identify other figures.
For example:



trapezium



rhombus



hexagon

After a few attempts at this activity, get students to list the directional words that have been commonly used. This vocabulary might include words such as above, below, next to, left, right, on top, symmetrical, turn. Focus on the most efficient use of language by showing the students a figure made from blocks and getting them to write instructions for building it. Refine the instructions collectively to create a model for future games.

Activity Two

Many students will need you to interpret the example of instructional language given on page 18. Getting students to act out a set of instructions involving quarter- and half-turns, both clockwise and anticlockwise, will be beneficial. A marching team's marching plan is a useful example of directional instructions. Relate quarter- and half-turns to the movement of the minute hand of an analogue clock and demonstrate that movement for students by moving the hand, using the knob at the back of the clock. Students can copy the clock hand movement with their own body turns.

More advanced students might be encouraged to use compass directions, especially if the activity is extended to outside the classroom. Maps of the school can be used for outside "treasure hunts". Drawing a co-ordinate system on the plan to provide an indicator of distance can also help.

Page 19: Follow That Thread

Achievement Objective

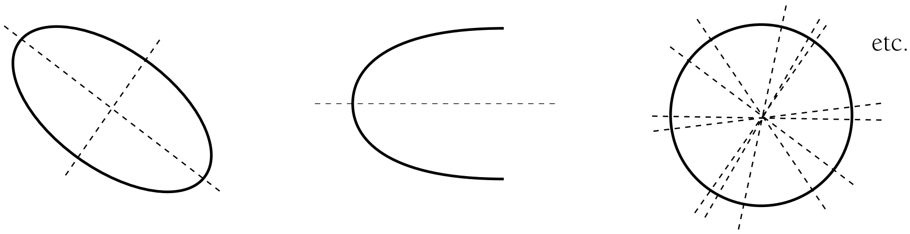
- create and talk about geometric patterns which repeat (show translation), or which have rotational or reflection symmetry (Geometry, level 2)

Activity One

The lines created by these curve-sketching activities create "envelopes" of well-known shapes (loci). A locus is a set of points often governed by a relationship or rule. For example, the lines in the square create the envelope of an ellipse. The more points that are marked on the outside of the square, the closer the envelope would come to approximating an ellipse.

The lines in the angle create the envelope of a parabola, which is the path of a ball when thrown in the air. The lines in the circle create the envelope of another circle. Again, a larger number of matching points on the circumference of the starting circle would make a shape that is closer to a circle.

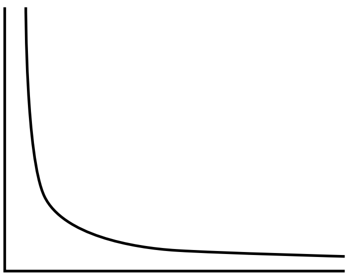
Ask the students to identify symmetry in the envelope shapes. The ellipse has two lines of reflection symmetry, the parabola has one, and the circle has an infinite number of lines.



The circle has rotational symmetry of infinite order, and the ellipse has half-turn symmetry.

Activity Two

The lines of thread will create the envelope of a hyperbola:



The curves created in this activity are known as the conic sections (see the notes for page 1) as they can be created by cutting a cone with a single cut. Students might enjoy looking up these famous loci in an encyclopedia or on the Internet.

Pages 20 and 21: Pirate Island

Achievement Objective

- describe and interpret position, using the language of direction and distance (Geometry, level 2)

Activity One

A copymaster of a map is included at the end of these notes. Students attempting this activity will need experience with compass directions, particularly the eight points north, south, east, west, north-east, north-west, south-east, and south-west. This is best learned by using a compass to identify north on a map of the school. Ask students to follow an instruction, for example, "Walk 20 metres south-west", and draw where they have travelled on their map. After some practice, they can play The Pirate Game, hiding treasure somewhere on the school grounds and writing a brief set of instructions so another group of students can find it. As a check, you can get the students to mark their journey on the map. Retain the map as a model answer.

As it stands, Pirate Island contains no reference to scale. For an extension to this activity, you could introduce a scale of 1 centimetre : 1 kilometre and tell students to calculate the distance of the journey to find the treasure. They may wish to draw a map of their own with accompanying instructions. Some discussion of land forms, such as lagoons, bays, peninsulas, and points, would be useful.

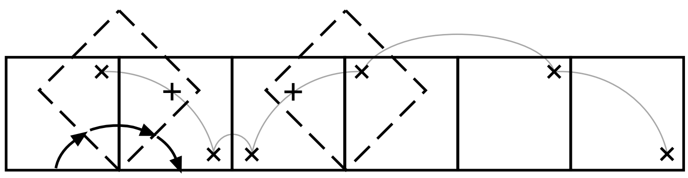
Page 22: Tumble Time

Achievement Objective

- describe and interpret position, using the language of direction and distance (Geometry, level 2)

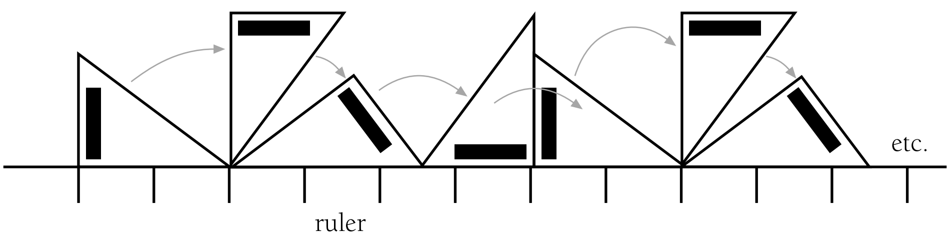
Activity One

The aim of the activity is to have students visualise the movement of well-known shapes and get them to focus particularly on the path (locus) of a point or area at a fixed position on the shape.
For example, the path of the point on the cube-shaped block in diagram **a** would be:



Encourage the students to attempt to draw the path of the marked point or area following successive quarter-turns in the case of quadrilaterals and third-turns in the case of the equilateral triangle.

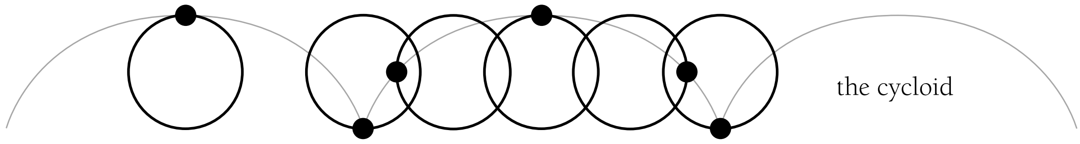
Students can confirm their answers by making a two-dimensional model of each block from card and turning it along a ruler edge (as shown below).



Another way to find the path of a point is to bore a small hole in the card and use the end of a pencil to trace the curve as the whole shape is turned.

Activity Two

The curved path traced by a point on the outer edge (circumference) of a circle as it turns is called a cycloid. Again, students can model this curve by cutting a circle from card and tracing the path of the point through a small hole. Encourage them to sketch what they think the curve will look like before experimenting:



The curve traced by a point on the circumference of an ellipse has similar characteristics, though the height of the curve depends on the location of the point. Once again, this characteristic is easily explored with a cardboard ellipse (see the diagram in the Answers section).

Page 23: Taking Flight

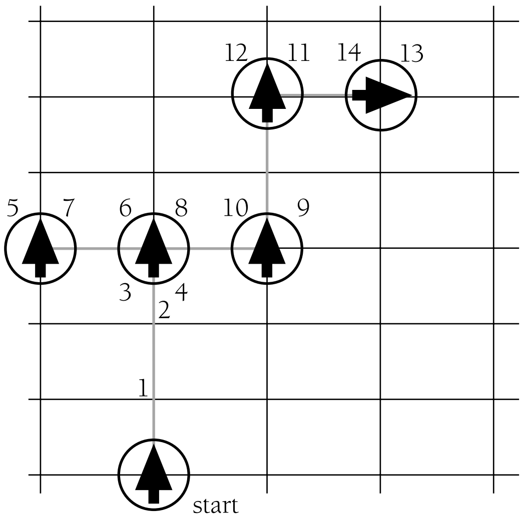
Achievement Objectives

- describe and interpret position, using the language of direction and distance (Geometry, level 2)
- make clockwise and anticlockwise turns (Geometry, level 2)

Activity One

Students need to realise that the numbers beside the footprints represent the position of each successive step as it is taken. They may want to act out Lola's dance. The dance starts with the left foot moving off and involves the left foot tapping out to the side twice for steps 5–8 before the right foot leads off to the right at step 9. This move may confuse some students.

Using a grid to mark the distance and direction of Lola from her starting position (origin) may be useful.



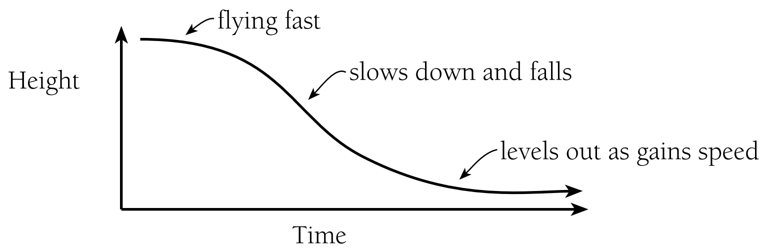
The effect of Lola's routine is to move her four steps up and four steps to her right together with a quarter-turn clockwise. Four repetitions of the routine will bring her back to the start and have her facing in the original direction.

An interesting question to ask students is: "Why might it be important to return to your starting place in dancing?" The limitations of dancing in confined spaces make this necessary, as does the need to be within earshot of the music.

Students may want to design their own dances that return them to their original starting position and direction.

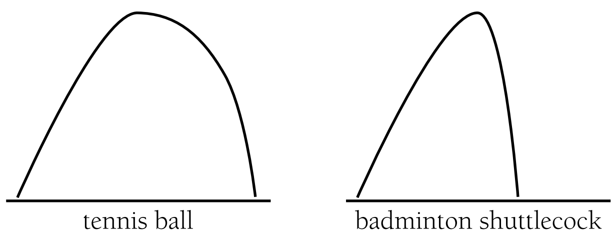
Activity Two

The flight path of a tennis ball and the path of a water jet are parabolas, as discussed in the notes for pages 1 and 19. Both paper planes and frisbees are subject to air flows and to variations in their speed and angle of flight. Ask students to try to explain patterns in the flight path they see. For example, the graph below might describe the flight of a paper dart:



Ribbons can make many different patterns. Students can see some of the different patterns in rhythmic gymnastic displays from the Olympic or Commonwealth Games.

Students could try using other objects and comparing their flight paths with the objects suggested on this page. For example:



The shuttlecock has more wind resistance than the tennis ball, and so it loses momentum faster.

Page 24: Cutting Corners

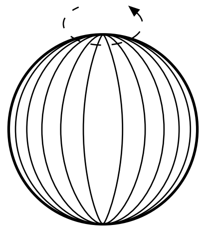
Achievement Objective

- make, name, and describe, using their own language and the language of geometry, everyday shapes and objects (Geometry, level 2)

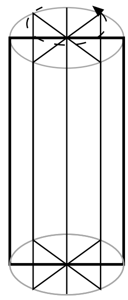
"Solids of revolution" are created when a flat (two-dimensional) shape is rotated about an axis.

Activity One

As the circle turns, its rotational image is a sphere, like the shape of a basketball or a planet.

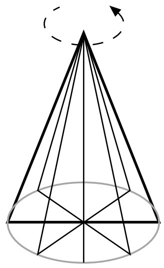


Activity Two



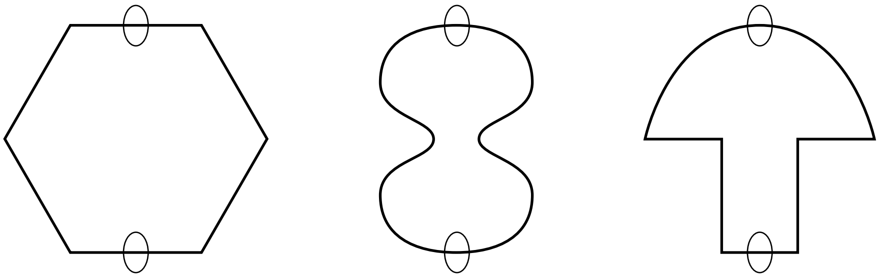
The rotational image of a rectangle is a cylinder.

Activity Three



The rotational image of the triangle is a cone.

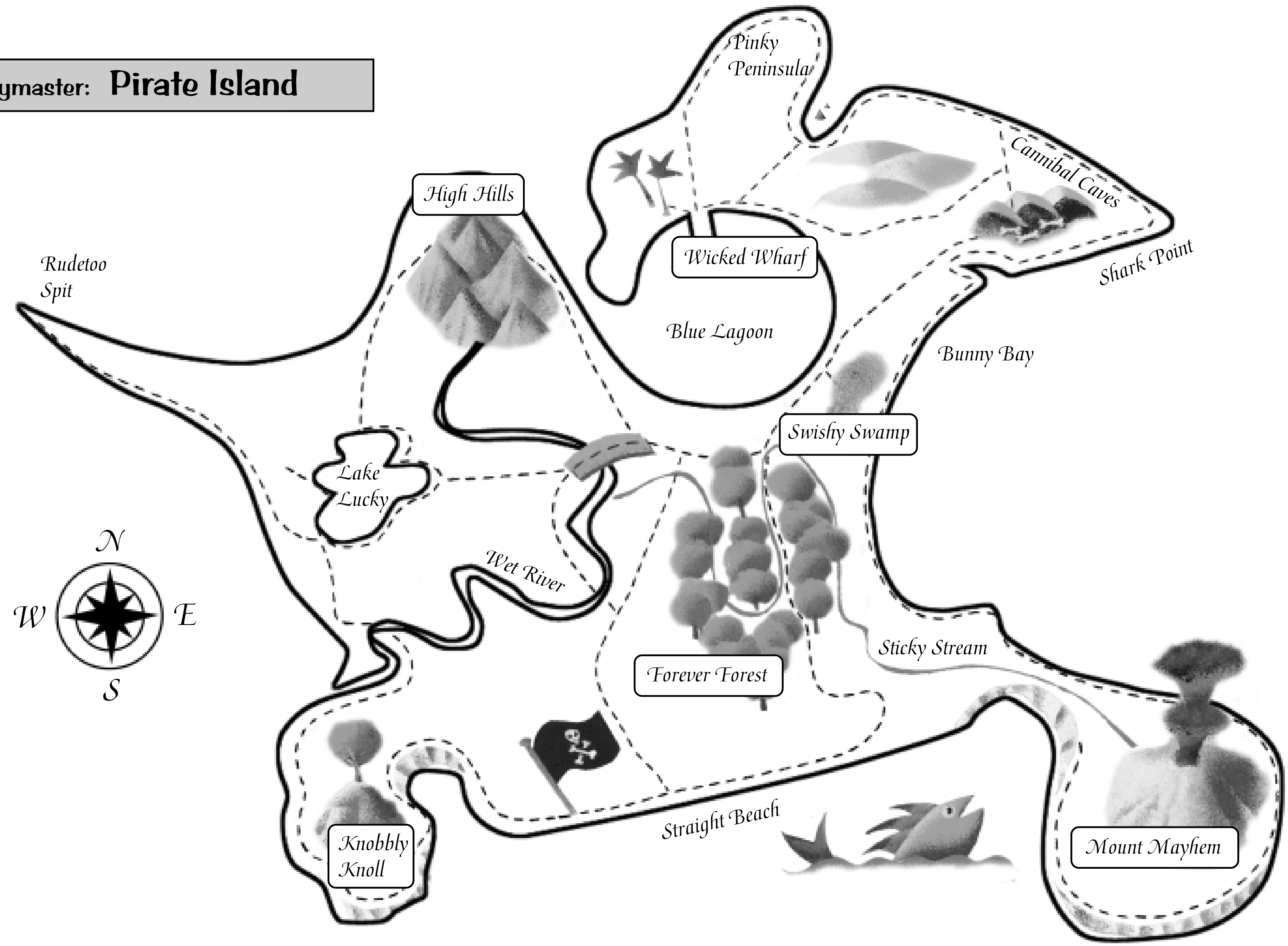
Students may wish to investigate the rotational images of other shapes, such as:



hexagon hourglass mushroom

Some examples of solids of revolution are woodturning on a lathe and figure skating.

Copymaster: **Pirate Island**



Acknowledgments

Learning Media would like to thank Vince Wright, School Support Services, School of Education, University of Waikato, for developing the teachers' notes. Thanks also to Diana Barnes and Paulette Holland for reviewing the answers and notes and to Carla Morris for her assistance to the designer.

The main illustrations on the cover and contents page and the line art on the cover, contents page, and pages 2, 3, and 9 (except for the leaves pattern, which is by Murray Grimsdale, and the circle pattern, which is by Esther Chua) are by Scott Kennedy.

Series Editor: Susan Roche
Series Designer: Esther Chua

Published 1999 for the Ministry of Education by
Learning Media Limited, Box 3293, Wellington, New Zealand.

Dewey number 510.76
ISBN 0 478 23709 X
Item number 23709
Students' book: item number 23708