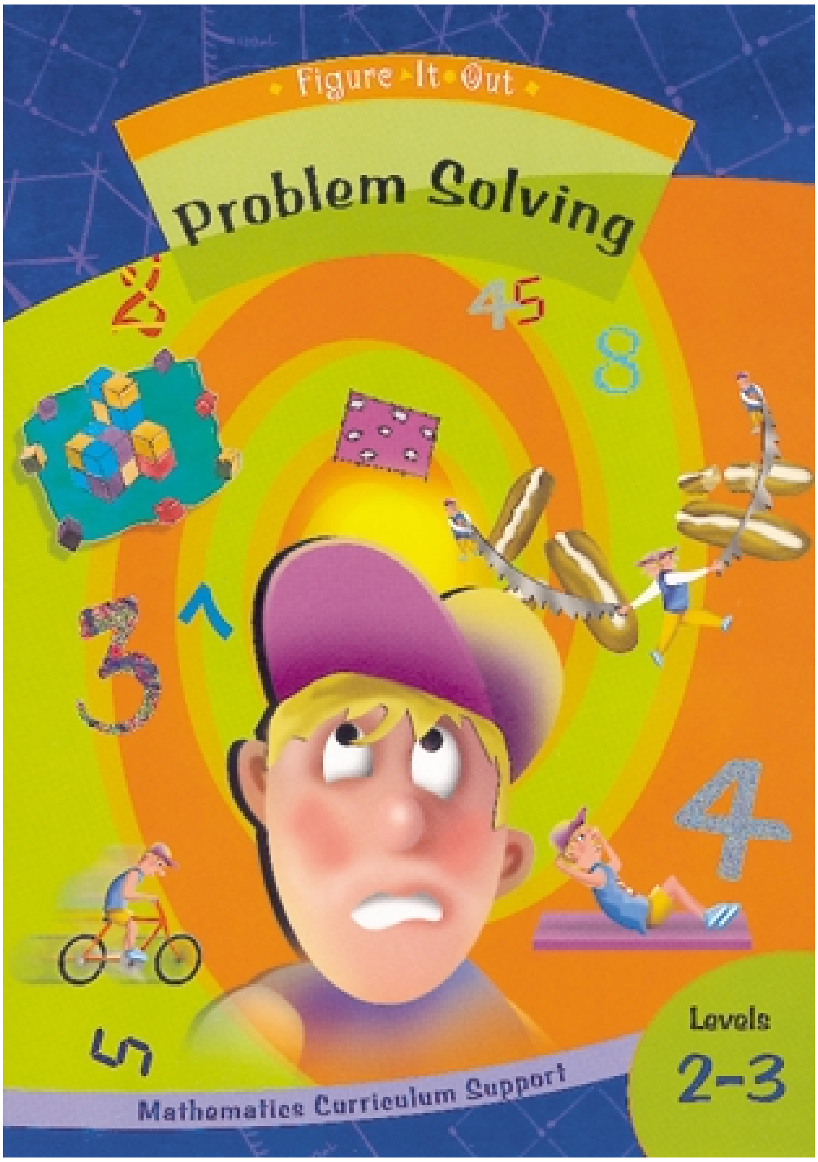
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Contents

Introduction

The Figure It Out series is designed to support *Mathematics in the New Zealand Curriculum*. The booklets have been developed and trialled by classroom teachers and mathematics educators. The series builds on the strengths of a previous series of mathematics booklets published by the Ministry of Education, the School Mathematics supplementary booklets.

Figure It Out is intended to supplement existing school mathematics programmes and can be used in various ways. It provides activities and investigations that students can work on independently or co-operatively in pairs or groups. Teachers can select particular activities that provide extension to work done in the regular classroom programme. Alternatively, teachers may wish to use all or most of the activities in combination with other activities to create a classroom programme. The booklets can be used for homework activities, and the relevant section in the teachers' notes could be copied for parents. These notes may also provide useful information that could be given as hints to students.

There are eight booklets for levels 2–3: one booklet for each content strand, one on problem solving, one on basic facts, and a theme booklet. Each booklet has its own *Answers and Teachers' Notes*. The notes include relevant achievement objectives, suggested teaching approaches, and suggested ways to extend the activities. The booklets in this set (levels 2–3) are suitable for most students in year 4. However, teachers can decide whether to use the booklets with older or younger students who are also working at levels 2–3.

The booklets have been written in such a way that students should be able to work on the material independently, either alone or in groups. Where applicable, each page starts with a list of equipment that the students will need in order to do the activities. Students should be encouraged to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Teachers could encourage students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Where possible, suggestions have been made to encourage discussion and oral presentation of answers, and teachers may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum*, page 7

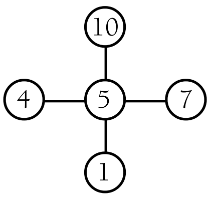
Students will have various ways of solving problems or presenting the process they have used and the solution. Successful ways of solving problems should be acknowledged, and where more effective or efficient processes can be used, students can be encouraged to consider other ways of solving the problem.

Figure It Out

Problem Solving

Answers

Page 1: Seeing Cs

1. Mere's dog is $5\frac{1}{2}$ years old.
   Mere's cat is $8\frac{1}{2}$ years old.
2. Some possible answers are:
   50c, 20c, 10c, 5c, 5c
   50c, 10c, 10c, 10c, 10c
   20c, 20c, 20c, 20c, 10c
3. 

4. 17 cubes

Page 2: Dotty Darts

1. c
2. The possible combinations need to include darts landing in the same ring:

   4 + 4 + 4 = 12
   4 + 4 + 7 = 15
   4 + 4 + 9 = 17
   4 + 7 + 7 = 18
   4 + 7 + 9 = 20
   4 + 9 + 9 = 22
   7 + 7 + 7 = 21
   7 + 7 + 9 = 23
   7 + 9 + 9 = 25
   9 + 9 + 9 = 27
3. 13
4. 5 cc

Page 3: Playing with Numbers

1. 7 x 2 + 3 – 5 or 2 x 7 + 3 – 5
2. $4
3. 120 metres
4. chocolate and boysenberry
   chocolate and apricot
   chocolate and chocolate
   boysenberry and boysenberry
   boysenberry and apricot
   apricot and apricot

Page 4: Pieces of Eight

1. Amy likes bananas.
   Barbara likes peaches.
   Pieta likes apples.
2. 47 + 39 = 86 or 37 + 49 = 86
   79 + 43 = 122 or 49 + 73 = 122
   93 + 74 = 167 or 73 + 94 = 167
3. 6 corners, 12 edges

Page 5: Losing Your Marbles

1. **a.** 8
   b. You could draw the diagram and highlight a different triangle each time, or you could label the vertices (corners) of the figure and use these labels to make a list of all the possible triangles.
2. 26 and 27
3. 3 moves
4. **a.** 8, 9, 10
   b. No. 31 is not divisible by 3.

Page 6: Join the Queue

1. **a.** 6

   b. 24

2. 10:10

3. **a.** 6

   b. 9

4. **a.** 2 + 2 + 3 =

   b. It is possible to get all integers.
   For example, [8] is [3] [+] [3] [+] [2] [=]
   and [$^{-}1$] is [2] [–] [3] [=]

Page 7: Home Sweet Home

1. **a.** 7 + 2 + 9 + 8 + 4 + 3 = 33

   b. Other totals: 26, 28, 38

2. **a.** blue and red, blue and green,
   red and red, red and green,
   green and green

   b. Most likely: red and green

3. 25

4. 20c, 10c
   10c, 10c, 10c,
   10c, 10c, 5c, 5c
   10c, 5c, 5c, 5c, 5c
   5c, 5c, 5c, 5c, 5c, 5c

Page 8: Food for Thought

1. $2.25

2. 4 big cars and 2 small cars

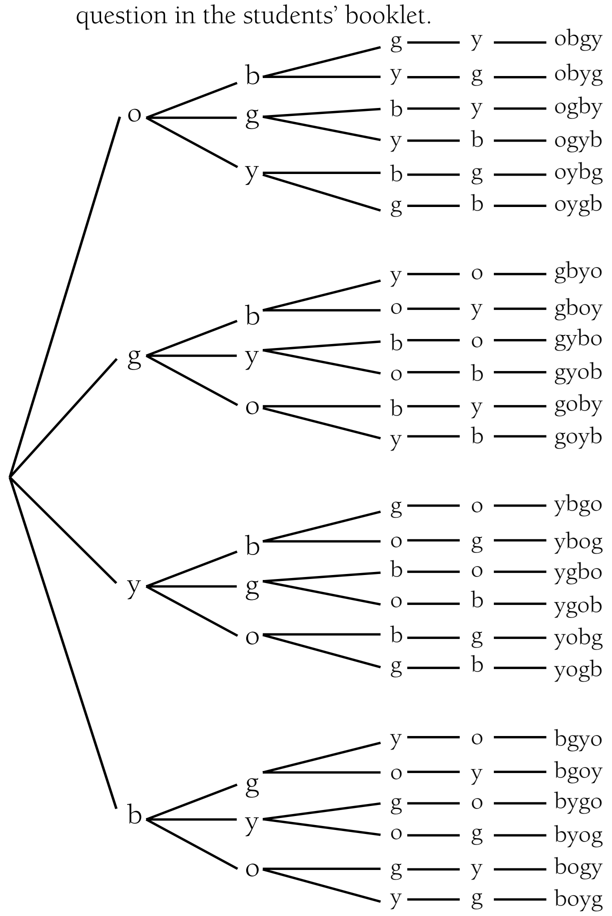
3. **a.** small yellow triangle

   b. large blue square

4. Fill the 5 L jug and use this to fill the 3 L jug, so there is 2 L in the large jug.
   Use the 2 L to water the tree. Then refill the 5 L jug and water the tree with this. The tree will receive 7 L of water.

Page 9: Teething Problems

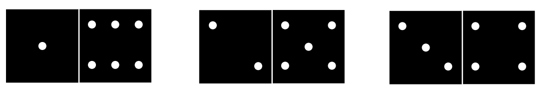
1. There are 24 different ways altogether.
   Note: ogyb is shown in the illustration for this question in the students' booklet.



o = orange dots
g = green stripes
y = yellow
b = blue squares

2. 168

3. **a.** 3 dominoes:



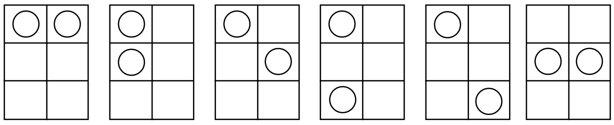
   b. 21 dots

4. **a.** hexagon: 9

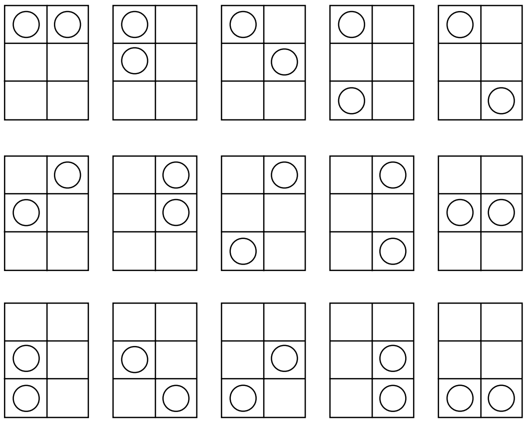
   b. octagon: 20

Page 10: Number Crunching

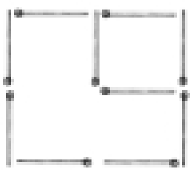
1. 60
2. If those that map onto each other by rotation or reflection are not included, these are the possible arrangements:



If rotation or reflection are included, these are the possible arrangements:



3. One possible solution:



4. **a.** 17

   b. Answers will vary.

Page 11: Reflect on This

1. 3
2. **a.** 8 is a mirror of itself, and 5 and 2 mirror each other.

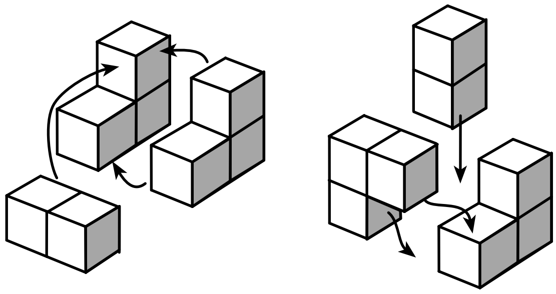
   b. 25, 11
3. 1 green and 3 yellow
   2 green and 3 pink
   3 green, 1 yellow, 1 pink
   6 green
   2 yellow and 2 pink
4. Some possibilities:

   $4, $3.50, $3.50, $1
   $4, $3.50, $3, $1.50
   $4, $3.50, $2.50, $2
   $4, $3, $3, $2
   $4, $3, $2.50, $2.50

Page 12: Making More

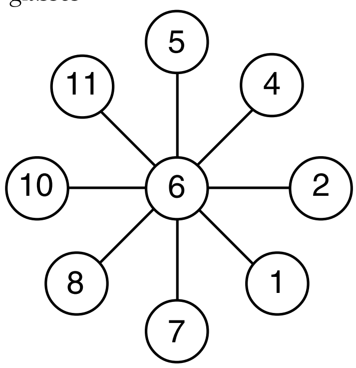
1. **a.** $127

   b. Answers will vary, but his mother probably won't agree.
2. **a.** Possible solutions:



   b. Teacher to check
3. **a.** 10 glasses

   b. 7.5 glasses
4.



Page 13: Numbers in Disarray

1. 9
2. **a.** 16 m^2

   b. There are a range of answers, depending on whether all or some of the electric fence is used.

   Possible answers include rectangles which are squares, such as 3 m x 3 m or 6 m x 6 m, or rectangles such as 6 m x 4 m or 9 m x 3 m.

   You could also have rectangles using fractions, such as 10 m x 2.5 m, 6 m x 6.5 m, or 6.25 m x 6.25 m.

c. The 6.25 m x 6.25 m rectangle has the greatest area (39.0625 m^2).

3. 47, 65, 56, 38, 83, 29, 92

4.

x	3	7	5
4	12	28	20
6	18	42	30

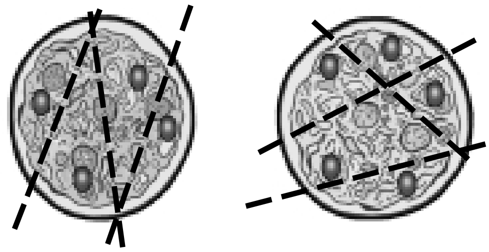
Page 14: Quacky Questions

1. 9 rectangles

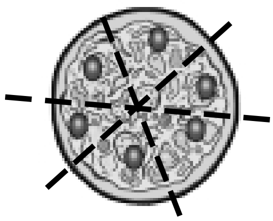
2. 4 ducks

3. 19

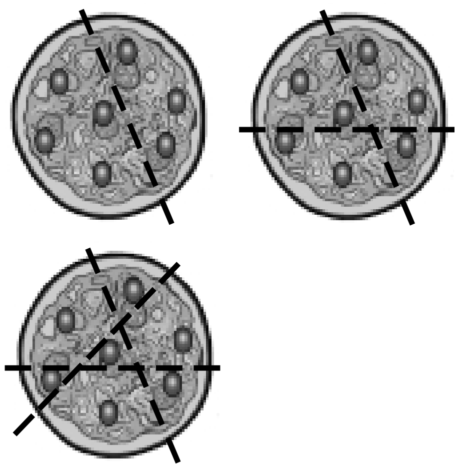
4. Pizzas **a** and **b**: various answers, for example:



Pizza **c**:

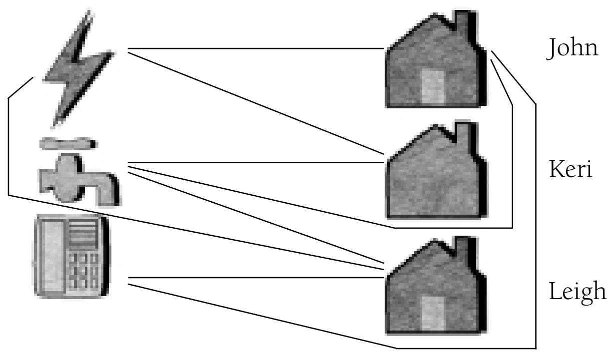


Pizza **d**: three steps:



Page 15: Power to the People

1. One solution is:



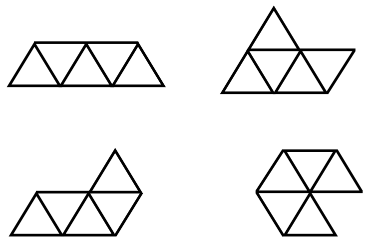
2. The digit 6

3. 24

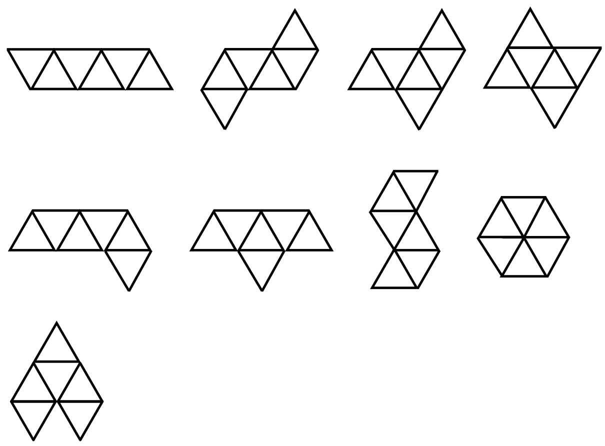
4. Any of the whole numbers from 4 to 10

Page 16: Shaping Up

1. **a.**



b. Some possible answers:



2. 33

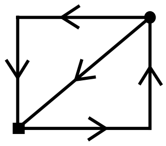
3.

43	45
x 5	x 3
215	135

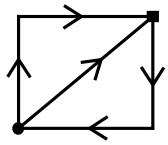
4. Answers depend on the student's description of methods. Two methods are by layers or by blocks.

Page 17: Pinning It Down

1. You can draw all the shapes.
   Two solutions for **a** are:



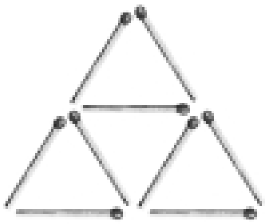
or



• = start ■ = finish

   Solutions and hints for the other shapes are outlined in the notes for this page.

2. blue: 1
   green: 4
   yellow: 3
   red: 2

3. 

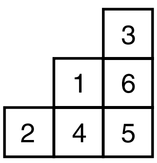
4. **a.** Answers will vary. There are 6 combinations ending in 7.

   b. She has a 50% chance of getting her money.

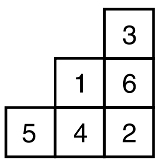
Page 18: Slippery Solutions

1. 9 minutes
2. 66
3. 16
4. 6

Page 19: Setting Up Swaps

1. 

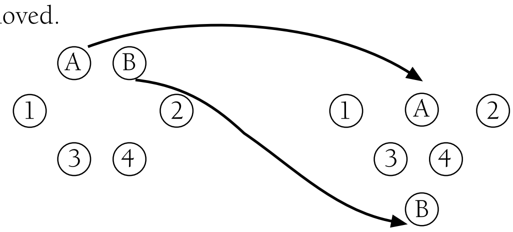
or



2. $1.90, $1.85, $1.80, $1.75, $1.70, $1.60, $1.45, $1.40, $1.30, $1, 95c, 90c, 80c, or 50c
3. August
4. 2

Page 20: Kete Counting

1. 11
2. The two counters in the top row of the figure are moved.



3. Answers will vary (see notes).
4. Answers will vary depending on street configuration. The configuration shown in the diagram would have 20 intersections.

Page 21: Problem Bits One

1. Answers will vary.
   The minimum number of moves is:
   a. 10 cents ⟷ 50 cents: 13 moves
   b. 10 cents ⟷ 20 cents: 22 moves
2. Answers will vary but may include: "There are so many sides and corners that they are too small to see."
3. Answers will vary. If you add 0, the answer isn't larger, and if you add a negative integer (for example, $^{+}3 + ^{-}2 = ^{+}1$), the answer is less than the starting addend.

Page 22: Problem Bits Two

1. Answers will vary.
2. False; a tetrahedron has the same number of corners as faces.
3. Answers will vary.

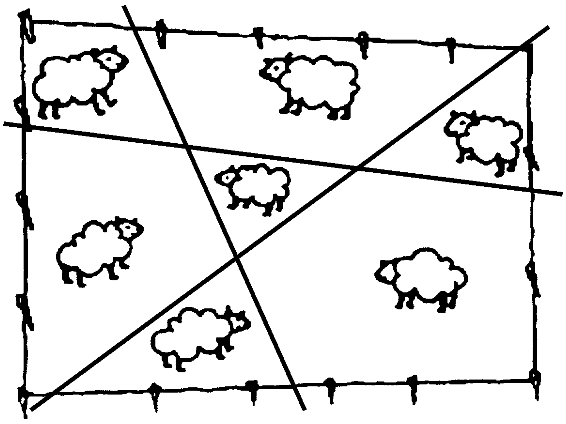
Page 23: Problem Bits Three

1. **a.** True; 13 is a prime number and has only two factors, itself and one.

   b. 3 rectangles with 12 tiles: 1 x 12, 2 x 6, 3 x 4

2. **a.** You can get an answer of $^{-}2$.

   b. Answers will vary.

3. 

Page 24: Problem Bits Four

1. 7 is only one possible solution. There are other justifiable solutions, for example, 1, 2, 4, 8 (see notes).

2. Yes. Different methods can be used to divide the doughnuts, but each person gets $1^2/_3$.

3. One way is to check whether the sum of the digits is divisible by 3.

Problem Solving

Teachers' Notes

Overview: Problem Solving

About Problem Solving

The nature of this booklet is different to the other seven in the Figure It Out series in that it focuses on students' ability to solve non-routine problems rather than focusing specifically on mathematical content. Some ways in which the problems could be used are:

- "Problem of the day" examples for the introductory part of a lesson
- Homework examples for students and parents to work on together
- Examples that students can use to write their own problems.

These teachers' notes contain suggestions about how each problem can develop effective problem-solving strategies, powerful reasoning, and communication. The notes do not list the achievement objectives for each problem because effective problem solving involves the combined application of all of the mathematical processes.

In the notes, important strategies are shown as a way of describing how a particular problem might be solved. This is not necessarily the only productive method, and you will need to be receptive to the ideas of your students.

Page 1: Seeing Cs

Problem One

Students are likely to solve the problem by trial and improvement. For example, they might try the numbers seven and four for the ages of the cat and the dog respectively. Realising that the sum of these numbers is 11, they will then make adjustments upwards until they find the correct answer. If necessary, suggest to students that they try fractions.

Check the students' ability to handle the processes of the problem by varying the conditions, for example:

"Mere's cat is twice the age of her dog. The sum of their ages is 21. How old are the animals?"

Problem Two

Allow students to come up with as many solutions as they can. Most of them will approach the problem randomly, finding possible answers by experimentation. Make sure toy money is available for them to use.

When collating the students' solutions, you could organise the answers in a table to avoid duplication. This will also help find all the possible combinations:

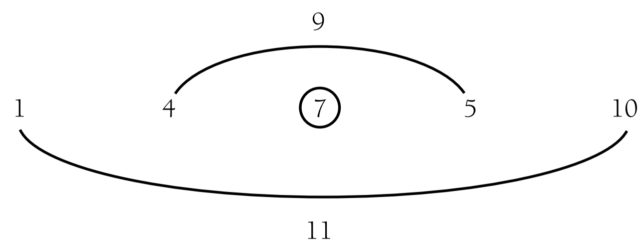
5 cent coins	10 cent coins	20 cent coins	50 cent coins
	1	4	
	4		1
2	1	1	1

Experiment with the problem by changing the conditions. For instance, if students find this example too difficult, give them an easier example:

"I have five coins that add to 50 cents. What might they be?"

Problem Three

Some students may find that making cards with the numbers 1, 4, 5, 7, and 10 and moving them around is a useful strategy. Another strategy is to systematically eliminate one number to see if the remaining numbers can be paired. For example, if 7 is used as the middle number, students will see that a solution is *not* possible because the remaining numbers cannot be organised in pairs with the same total:

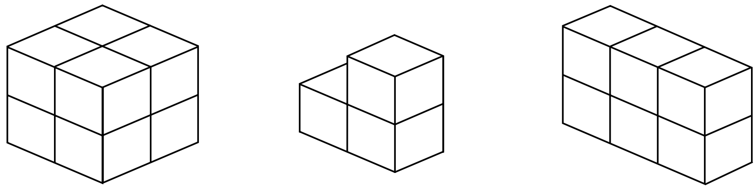


Set other problems of this type by changing the numbers involved, for example:

"What if the numbers were 2, 3, 6, 7, and 11?"

Problem Four

Students may experience difficulty visualising blocks that cannot be seen in the picture. Provide multilink cubes or similar building apparatus for these students. Encourage them to visualise how the building could be broken up into blocks to make counting easier, for example:

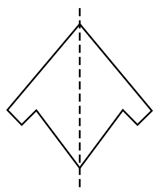


The problem can be extended by asking students to draw their own cube buildings and to give these drawings to a classmate to build.

Page 2: Dotty Darts

Problem One

Encourage the students to try visualising the problem first. This is very difficult, so ultimately they will need to confirm their solution by folding and cutting a piece of paper. Focus the discussion on how the fold line becomes a line of mirror (reflective) symmetry for the cut-out shape. You may need a mirror to show this:

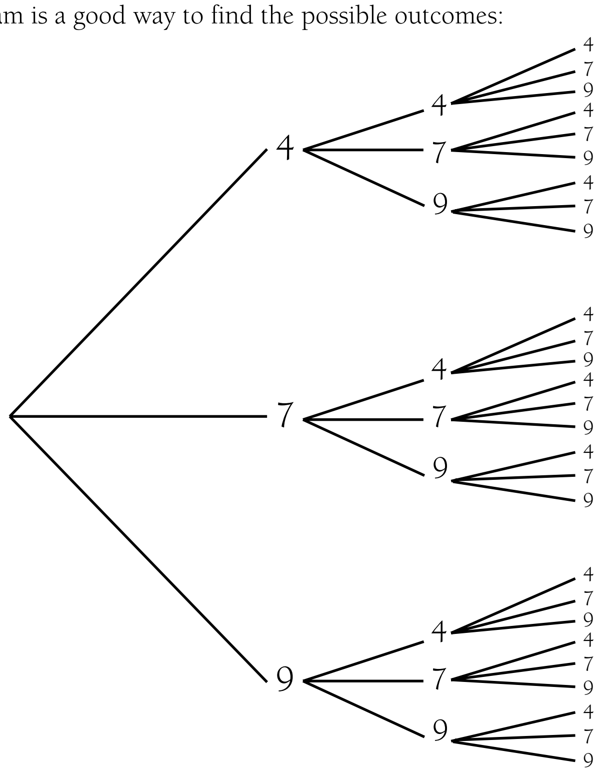


Ask students to fold other pieces of paper in half, draw in cutting lines, and predict the finished shape before cutting it out.

Problem Two

Many students will approach the problem in a random way and produce a few solutions.

A tree diagram is a good way to find the possible outcomes:



Some combinations result in the same score. For example, 4, 7, 4 has the same score as 7, 4, 4 and 4, 4, 7. One approach is to work through the tree diagram to eliminate these duplications. Another is to write an organised list:

4 + 4 + 4 = 12, 4 + 4 + 7 = 15, 4 + 4 + 9 = 17, 4 + 7 + 7 = 18, 4 + 7 + 9 = 20, 4 + 9 + 9 = 22 (This eliminates any score where a dart lands on a 4.)

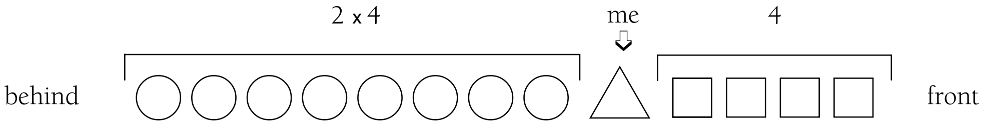
7 + 7 + 7 = 21, 7 + 7 + 9 = 23, 7 + 9 + 9 = 25 (This eliminates any further scores where a dart lands on a 7.)

9 + 9 + 9 = 27 is the only remaining score.

If students find this problem too difficult to start with, simplify it by saying that Russell threw two darts and build up possible strategies before proceeding to the three-dart problem.

Problem Three

A physical model of the problem is useful. Counters can be used, or students may act it out. Encourage a systematic approach supported by reasoning.



"I knew there were thirteen people in the line because there were four in front of me, twice that number, eight, behind me, and four plus eight plus one is 13."

The problem can be made more complex by changing the conditions, for example:

"Twelve people are in front of you, and half as many people are behind you."

"You are in a dance formation. Four people are in front of you, and four are behind you. Twice as many people are to your right, and three times as many are to your left. How many people are in the formation?"

Problem Four

Students will need to look carefully to determine that the water level rose by 25 millilitres. This means that each marble displaced 25 ÷ 5 = 5 millilitres and has a volume of 5 cubic centimetres (cm^3 or cc).

Note: 1 mL = 1 cc. Students may not know this.

Ask students how many marbles would be needed to get the water level to rise up to 100 millilitres. At that point, half the volume will be marbles and the other half water. Tell them to draw another jar where the volume is half marbles and half water.

"What about three-quarters marbles and one-quarter water?" (For this question, students will need to think about the size of the jar and the volume of water in the jar.)

Page 3: Playing with Numbers

Problem One

Most students are likely to use trial and improvement strategies to solve this problem. Some thinking about the operations and the size of the numbers involved would be very productive.

Much of the equation for the answer 12 is likely to be made up by the product of □ x □. Looking for two numbers with a product close to 12 will limit the possibilities, for example, 7 x 2.

Students may like to investigate what other answers are possible by putting the digits in different positions.

Problem Two

In this problem, students must pay attention to the important information. Drawing a flow chart of the decision-making process might help:

How much are the tickets?
⇩
What is the cost of four tickets?
⇩
What change will you get from $10.00?

Try changing conditions in the problem to check on the students' understanding of the process, for example,

"Kim buys seven tickets and hands over a $20 note. How much change will he get?"

Problem Three

This problem could be modelled with counters so that students understand that the return journey from each flag must be included in the total distance. Encourage them to write their methods mathematically, for example, 12 + 24 + 36 + 48 = □.

To check if they have generalised the process, ask: "How far would you run in total if there were two more flags to run to?" 12 + 24 + 36 + 48 + 60 + 72 = □

Encourage students to look for patterns. For example, some may express the solution using multiplication by six:

(2 x 6) + (4 x 6) + (6 x 6) + (8 x 6) + (10 x 6) + (12 x 6)

= 42 x 6

= 252

Problem Four

Making a physical model of the problem with counters or similar apparatus may help students find all the possibilities. Some discussion may focus on whether chocolate and boysenberry is a different ice cream to boysenberry and chocolate. In the explanation below, the two ice creams are considered to be the same, and duplicates are eliminated.

Students could use various strategies:

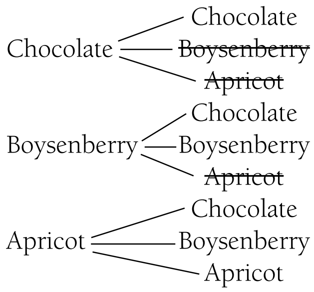
i. Writing an organised list:

C–C	B–C	A–C
~~C–B~~	B–B	A–B
~~C–A~~	~~B–A~~	A–A

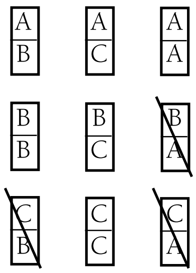
ii. Using a table:

	A	B	C
A	AA	AB	AC
B	~~BA~~	BB	BC
C	~~CA~~	~~CB~~	CC

iii. Drawing a tree diagram:



iv. Making a model:



All these strategies are valuable. Encourage the students to generalise their approach by asking, "How many different ice creams would be possible if banana ice cream were available as well?"

Page 4: Pieces of Eight

Problem One

Students may like to solve the problem by writing the names of the people and fruit on pieces of card so that they can be moved into pairs. Students can then check each pairing to see whether they satisfy all the clues.

Another method, which is more useful for more difficult combination problems, is to use a table:

	Barbara	Amy	Pieta
Peach			✗
Apple		✗	
Banana	✗		

The crosses are made from the clue "No one likes fruit that starts with the same letter as their name."

After the clue "Amy's favourite fruit has no stone", this becomes:

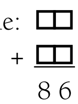
	Barbara	Amy	Pieta
Peach		✗	✗
Apple		✗	
Banana	✗	✓	✗

This leaves only one fruit for Pieta, the apple, and the peach is then left for Barbara.

Problem Two

Number sense and elimination are the most profitable strategies to use in this problem, with particular focus on the ones place of the answer.

For example:



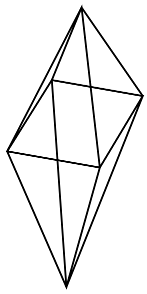
Because only 7 + 9 = 16 puts a 6 in the ones place of the answer, this indicates that the tens places of the addends must be taken by 4 and 3.

As an extension, you can ask, "Are these the only possible answers you can get by arranging 4, 7, 3, and 9 as the addends?"

Three other answers are possible: 140, 113, 131.

Problem Three

Having a model of an octahedron available would be helpful for the students, but the problem can still be solved by visualisation. For example, students might see the edges as three groups of four.



four edges meeting at the top vertex (corner)

four edges around the middle

four edges meeting at the bottom vertex (corner)

Page 5: Losing Your Marbles

Problem One

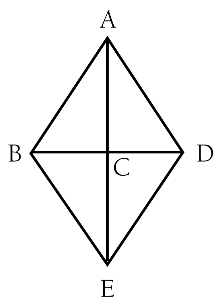
This problem requires students to think systematically. Possible strategies are:

i. Classify the triangles as those that are triangles by themselves and those that are made up of two small triangles, three triangles, or four triangles.

   Results could be organised as follows:

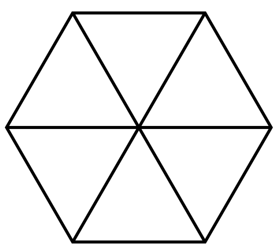
Single triangles:	4
Two triangles:	4
Three triangles:	0
Four triangles:	0

ii. Label the vertices (corners) of the figure and use these labels to make a list of all the possible triangles.



(all triangles with A as a corner)		
ABC	BCE	CDE
ABE	BDE	
ABD		
ACD		
ADE		

To extend the problem, draw more complex figures, such as:



Problem Two

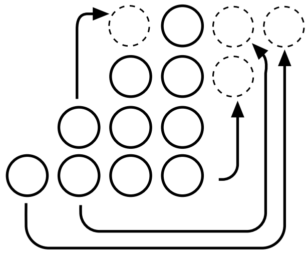
Students may solve this problem by trial and improvement. Noting that 25 is half of 50 will make this strategy more efficient. Alternatively, students might divide 53 by 2 to get the median, 26.5, which will quickly lead to the solution.

As an extension, give the students the sum of four pages in a row or the product of two pages. For example:

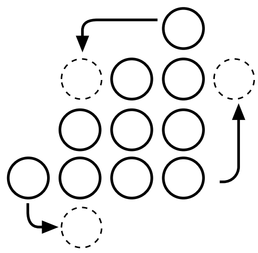
"A book is open, and the product of the two page numbers is 600. What are the page numbers?" (The solution is 24 and 25.)

Problem Three

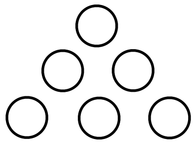
Making a model of the problem with counters will be essential for many students. Encourage the students to record their moves diagrammatically so they can recount them later. Students may initially think the smallest number of moves is four, as shown in this diagram:



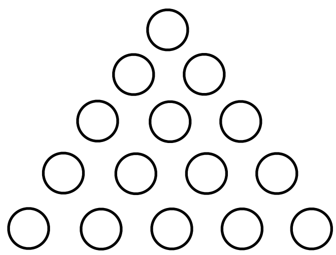
Encourage them to solve the problem in fewer than four moves, as shown here:



A related exercise might be to ask students to shift the smallest number of counters to turn these triangles upside down:



or



Problem Four

Students could make a model of the problem with counters or multilink cubes and cups. There are connections between this problem and **Problem Two** in that finding the median of the numbers leads directly to the answer. That is, 27 ÷ 3 = 9, so the middle jar holds nine marbles, the left jar one less (8), the right jar one more (10).

Thirty-one is not divisible by three, so you could not divide up the marbles between the jars in the same way. Students may recognise that these conditions can only be met when the jar totals are a multiple of three.

Page 6: Join the Queue

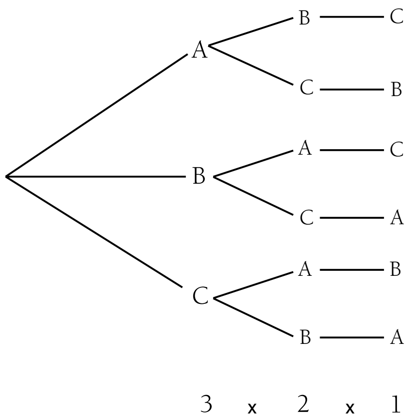
Problem One

This is an ideal problem for students to act out by taking the parts of Celia, Ally, and Bert. Making an organised list is another useful strategy:

CAB ACB BAC

CBA ABC BCA

The solution can also be found by using a tree diagram:



This tree diagram gives a clearer picture of the different orders for 3 x 2 x 1 = 6, which is also 3! (called “three factorial”).

Extending the problem by adding another person to the bus stop line increases the number of possible orders to 4 x 3 x 2 x 1 = 24, which is 4! (called “four factorial”).

Problem Two

Focusing on the way in which the digital clock shows times such as “5 minutes past 8” (that is, 8:05) will be significant for some students.

Students may discuss whether 1:01 is the next time to meet the criteria. Although it is mathematically correct, in that 01 is the same as 1, this problem requires both the hour number and the minute number to look the same.

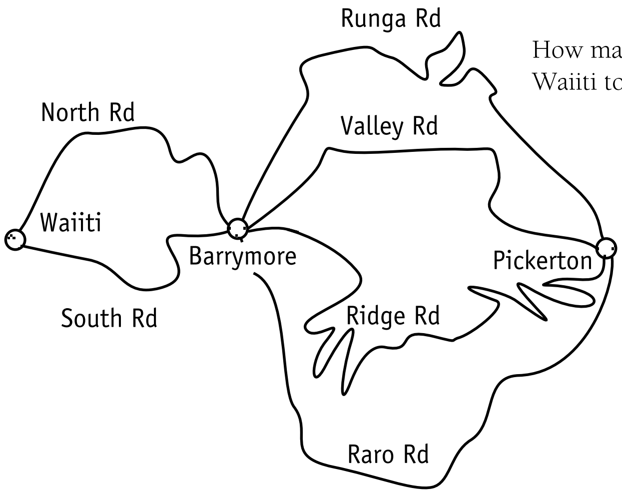
Extend the problem by suggesting that 10:01 is a “mirror” time with the colon acting as the line of symmetry. Ask students to find other mirror times, such as 12:51.

Problem Three

This problem is very similar to **Problem One** in that it involves finding all the possible combinations. Productive strategies include making an organised list, drawing a tree diagram, and making a table. For example, a table of the problem would be:

	Hill Rd	Kaimai Rd	New Rd
Swamp Rd	✓	✓	✓
Middle Rd	✓	✓	✓
East Rd	✓	✓	✓

To see whether students can generalise their strategies, give them a similar problem, such as:



How many different ways can you drive from Waiiti to Pickerton?

Problem Four

As [2] and [3] have a difference of one, it is possible to show all the integers (… $^{-}4$, $^{-}3$, $^{-}2$, $^{-}1$, 0, 1, 2, 3, 4 …). Encourage students to use the smallest number of key presses to show them.

For example:

Eight can be shown by [3] [+] [3] [+] [2]

Ten can be shown by [2] [+] [2] [+] [3] [+] [3]

Get students to experiment with other keys, such as [5] [3] [+] [–] [=], to see what numbers can be shown.

Page 7: Home Sweet Home

Problem One

A useful strategy is to add up the numbers in all the rooms, which gives a total of 44. Therefore, the rooms that are not entered must have a total of 11 because 44 – 33 = 11. Possible combinations of rooms that are not entered are 8 and 3, 9 and 2, 5 and 6, and 4 and 7. Students will be able to eliminate two of these possible combinations when they realise that rooms 7 and 3 must be entered. Rooms 9 and 2 are also eliminated as it would be impossible to travel that way without entering room 3 twice. This leaves only rooms 6 and 5 that must not be entered.

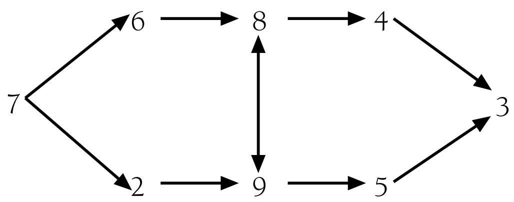
Other totals can be found by systematically working through possible routes:

7 + 6 + 8 + 4 + 3 = 28

7 + 6 + 8 + 9 + 5 + 3 = 38

7 + 2 + 9 + 5 + 3 = 26

Some students may find this diagram of the problem useful:



Encourage students to make up their own house path problems.

Problem Two

This problem can be modelled using multilink cubes or counters in a plastic container. Students may find all the possibilities by trialling, that is, taking out lots of two lollies and recording the results.

As with other combination problems, using tree diagrams or tables are useful ways to find the solution theoretically. It is helpful to use symbols like r_1 and r_2 to depict the two red lollies.

	b	r_1	r_2	g_1	g_2	g_3
b	✘					
r_1	✔	✘				
r_2	✔	✔	✘			
g_1	✔	✔	✔	✘		
g_2	✔	✔	✔	✔	✘	
g_3	✔	✔	✔	✔	✔	✘

Of the 15 possible combinations, six are a pair of one red and one green lolly, so red and green is the most likely pair.

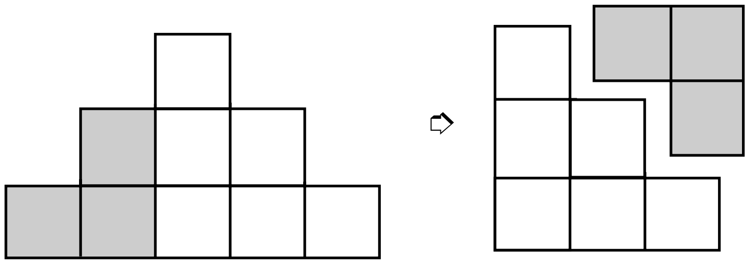
To check whether students have generalised their methods, give them other "lolly in the bag" problems. For example: two yellow, two blue, two red.

Problem Three

This problem could be solved by building the model with multilink cubes, or students could write a number sentence showing the number of cubes in each layer:

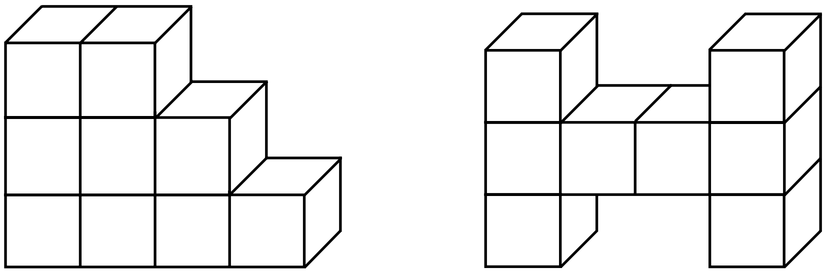
1 + 3 + 5 + 7 + 9 = 25

Some students may notice that the cubes in the illustration could be rearranged to form a square.



This could be represented as 3 x 3 = 9, so a five-high model would contain 5 x 5 = 25 cubes.

More able students could be asked to solve other five-high problems, such as:



Alternatively, they could be asked to find the number of cubes needed for a model that is 10 cubes high.

Problem Four

The different ways to make 30 cents could be organised in a table to avoid duplication:

5 cents	10 cents	20 cents
6		
4	1	
2	2	
2		1
	3	
	1	1

This moneymaking can be extended for more able students. For example:

"How many different ways can you think of to make 40 cents?"

Food for Thought

Problem One

Students are most likely to solve this problem by trial and improvement. Students know that the cost of a hamburger is greater than the cost of fries. A possible method of solving this might be:

Hamburger	Fries	Total	Difference
$2.50 (x 2)	$1.00	$6.00	$1.50
$2.20 (x 2)	$1.60	$6.00	$0.60
$2.25 (x 2)	$1.50	$6.00	$0.75

Similar types of problems could be posed as extensions, for example:

"A pie costs 50 cents more than a milkshake. Three pies and a milkshake costs $7.50. How much is a milkshake?"

Problem Two

Various solutions are possible, but not all scenarios have all the cars full. One strategy might be to start with 26 counters and experiment by dividing them into threes and fives. A systematic way of doing this is to begin with the minimum number of big cars needed:

Big cars	Small cars	All full	
6	0	No	
5	1	No	
4	2	Yes	← solution
3	4	No	
2	6	No	
1	7	Yes	← This is also a solution, but it uses two more cars.

Students could investigate how many students would be involved in scenarios where all the cars were full. For example, to take 25 children, five big cars could be used, and to take 21 children, seven small cars could be used.

Problem Three

Students need to be aware of three variables operating in the pattern:

Size	large, small, large, small …
Shape	triangle, circle, square, triangle, circle, square …
Colour	red, yellow, blue, green, red, yellow, blue, green …

Both questions can be answered by continuing these sequential patterns and relating them to the ordinal numbers. The size, shape, and colour of the fifteenth block can be found by using the repeating element for each variable. For example, the shape variable has a repeating element of three. Fifteen divides by three exactly, so the fifteenth block must be a square. Similarly, the colour variable repeats itself in every fourth object. Because fifteen divides by four with a remainder of three, the fifteenth block is coloured blue, the third colour in the sequence. Odd-numbered blocks are large, and even-numbered blocks are small, so the fifteenth block must be large.

Students can be encouraged to create their own repeating patterns involving two or more variables.

Problem Four

Students may need to record their attempts in an organised way. For example:

Tree	0	0	0	2	2	7
5 L bucket	0	5	2	0	5	0
3 L bucket	0	0	3	3	3	3
	⇧	⇧	⇧	⇧	⇧	
		Fill up 5 L bucket.	Pour from 5 L to 3 L bucket.	Empty remaining 2 L on tree.	Fill 5 L bucket.	Empty 5 L bucket on tree.

Similarly, the problem could be modelled using cut-down 2 litre drink bottles with markings on them.

Check to see whether the students have understood the process by changing the problem to "How could Filo give the tree 4 litres of water?"

A solution to the problem would be:

> Fill up the 3 L bucket and pour it into the 5 L bucket.
>
> Fill up the 3 L bucket again and pour it into the 5 L bucket until the 5 L bucket is full.
>
> This will leave 1 L in the 3 L bucket. Water the tree with this.
>
> Fill the 3 L bucket once more and pour it on the tree.

Tree	0	0	0	0	0	1	1	4
5 L bucket	0	0	3	3	5	5	5	5
3 L bucket	0	3	0	3	1	0	3	0

Page 9: Teething Problems

Problem One

This is another combination problem like the bus stop line problem on page 6. A tree diagram is one way to find all the ways of stacking the cubes (see the Answers section).

The diagram can be expressed as 4 x 3 x 2 x 1 = 24. (This is known as "four factorial" and is written as 4!.)

Problem Two

Students may need help to identify the important information in the problem.

Bridging questions include "How many days are in a week?" and "How can we find out the number of days in 8 weeks?".

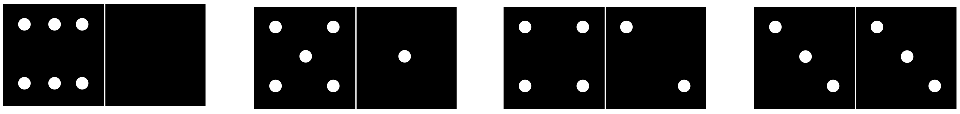
3 x 7 = 21 gives the number of brushes per week.

21 x 8 = 168 gives the number of brushes in total.

Problem Three

Some students may need to solve the problem by sorting through a set of dominoes. They will need to realise that the smallest number on one section of a domino is zero and the largest number is six.

To find the total number of dots on the three seven-dot dominoes, students should calculate either 7 + 7 + 7 = 21 or 3 x 7 = 21. Highlight the connection between repeated addition and multiplication. Extend the problem by posing similar questions, such as “What is the total number of dots showing on all the dominoes that have a total of six dots?”



Therefore, the total number of dots is 4 x 6 = 24.

A difficult problem for more able students is to work out the total number of dots on a full set of dominoes (168 dots).

The following table is one way of working this out.

	0	1	2	3	4	5	6	Total
0								0
1	1	2						3
2	2	3	4					9
3	3	4	5	6				18
4	4	5	6	7	8			30
5	5	6	7	8	9	10		45
6	6	7	8	9	10	11	12	63
								168

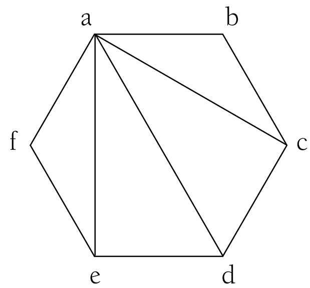
Problem Four

The definition of a diagonal will need clarification. In this problem, diagonals join a corner with every other non-adjacent corner.

For example, in this hexagon, there are three diagonals involving corner a.

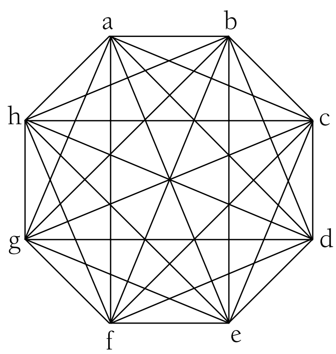
In total, there will be nine diagonals, which can be found by making an organised list:

ac ad ae bd be bf ce cf df



All the diagonals of an octagon can be found in a similar way:

ac ad ae af ag bd be bf bg bh ce cf cg ch df dg dh eg eh fh

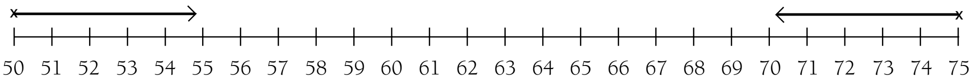


Page 10: Number Crunching

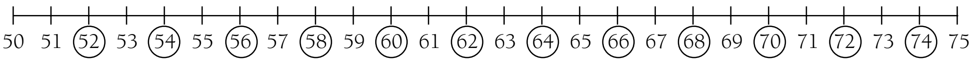
Problem One

A number line or hundreds board is a useful model for solving this problem. Numbers can be eliminated as the clues are considered.

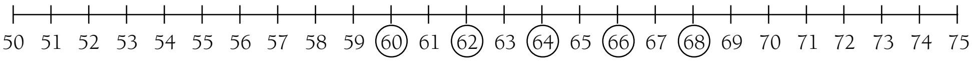
I am between 50 and 75:



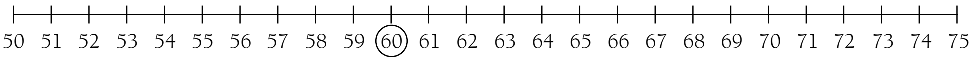
I am even:



When you add my digits, the answer is even:



I can be divided evenly by 5:

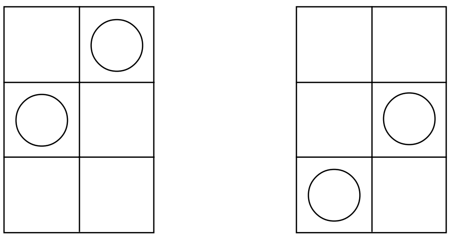


Other "What Number Am I?" problems can be solved in a similar way. For example:

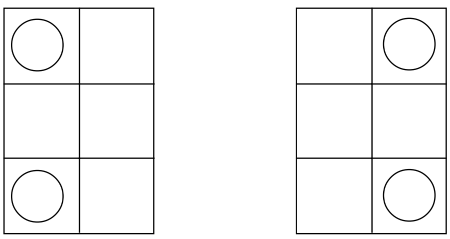
"I am between 40 and 60. I am odd. My digits add to seven. What number am I?"

Problem Two

The initial assumptions the students make play a part in the number of possible arrangements. For example, the arrangements below may be considered the same because they can map onto each other by rotation:



Similarly, these arrangements may be considered the same because they map onto each other by reflection:

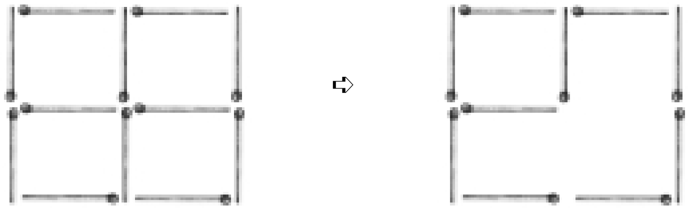


If these rotation and reflection assumptions are accepted, there are six possible arrangements (see the diagrams in the Answers section).

However, if the six positions are assumed to be individual, there are 15 possible solutions (see the diagrams in the Answers section).

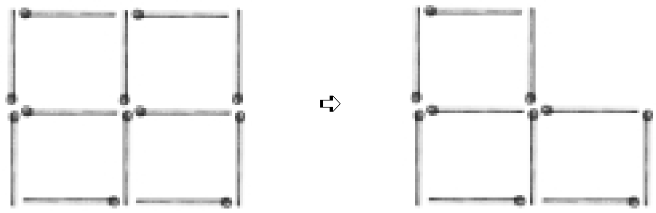
Problem Three

Students' assumptions may prevent them from solving this problem, particularly if they believe that the squares must be the same size.



Students might like to solve this related problem:

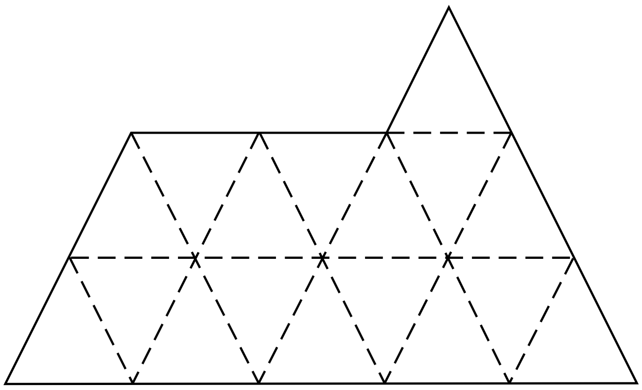
Form three squares by removing two matchsticks.



They may wish to make up matchstick removal or addition puzzles for other students to attempt.

Problem Four

Encourage the students to visualise how the triangle would fit repeatedly into the shape and what lines would be created by meeting sides:

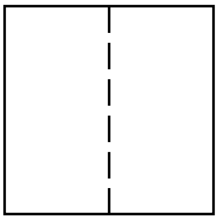


Students might use triangular pattern blocks to find other figures with an area of seventeen triangular units. Alternatively, they may want to draw their figures on isometric dot paper.

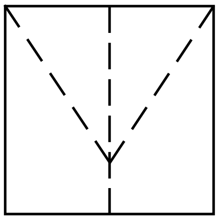
Page 11: Reflect on This

Problem One

Encourage the students to predict the number of fold lines as they follow the fold instructions. They may be able to draw where they think the fold lines will be after each step, and this can be confirmed by unfolding the square:

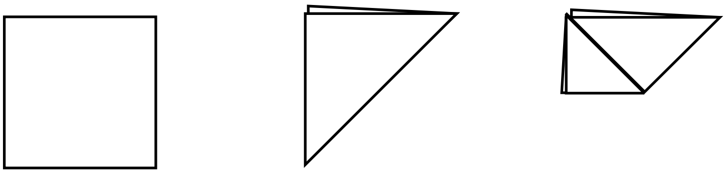


After step one



After step two

Visualisation is very difficult, so the students need experience with simpler examples, such as:



How many fold lines?

Students may also like to make up simple fold problems for someone else.

Problem Two

Students will need to identify mirror relationships between the digits.

0 and 1 and 8 are their own mirror images.

5 and 2 are mirror images of each other.

No other mirror relationships exist. This limits the number of two-digit mirror numbers to:

11 88 52 25

This knowledge can be applied to finding the different three-digit mirror numbers.

111 101 181 205 215

285 502 512 502

582 808 818 888

Problem Three

A systematic approach will be needed to find all the possibilities. This might begin with all the possibilities involving any green rods:

Six green rods	(6 x 3 = 18)
Three green rods, one yellow rod, one pink rod	(3 x 3) + 5 + 4 = 18
Two green rods, three pink rods	(2 x 3) + (3 x 4) = 18
One green rod, three yellow rods	3 + (3 x 5) = 18

Students can then find all the possibilities that can be made without using green rods:

Two yellow rods, two pink rods	(2 x 5) + (2 x 4) = 18

Students might go on to investigate how many ways a length of 24 centimetres could be made with the three different-coloured rods.

Problem Four

If Henry's pocket money is removed from the $12 total, that leaves $8 to be shared among the remaining younger children. There are several possible combinations, which can be found in an organised way (see the Answers section).

Note: If Henry is the elder by birth of twins, that opens up other possibilities, such as ages 4, 4, 3, 1 or 4, 4, 2, 2.

Students may enjoy making up their own problems about children's ages.

Page 12: Making More

Problem One

Students may express the problem as a series of calculations. For example:

$1 x 2 = $2 $2 x 2 = $4 $4 x 2 = $8 $8 x 2 = $16

$16 x 2 = $32 $32 x 2 = $64

$1 + $2 + $4 + $8 + $16 + $32 + $64 = $127

Alternatively, they may express the solution as progressive sums:

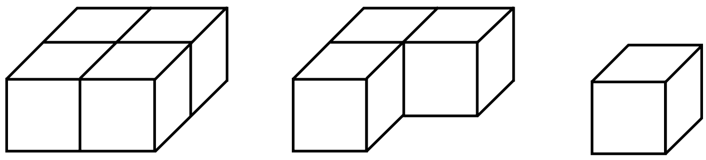
Days	1	2	3	4	5	6	7
Total	$1	$3	$7	$15	$31	$63	$127

+ $2   + $4   + $8   + $16   + $32   + $64

Extend the problem by asking "How much would Tāne get if he worked one more day (eight in total), … two more days … three more days?"

Problem Two

Students will need isometric dot paper and multilink cubes to create their own cube puzzles. For example:



Students may like to make shapes other than a cube block with the three pieces and draw the shape for a partner to build.

Problem Three

Students may need to see the glass of water poured into a 2 litre bottle. This will give them a benchmark to estimate from.

Since 2 litres is equivalent to 2000 millilitres, there will be space for ten 200 millilitre glasses. The bottle will hold a little more than this because it is not completely full of soft drink when it is bought.

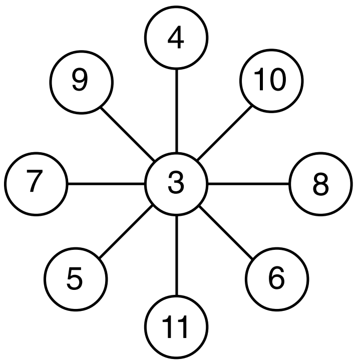
The 1.5 litre bottle will hold at least 1500 millilitres, so seven and a half glasses of water will fit in it.

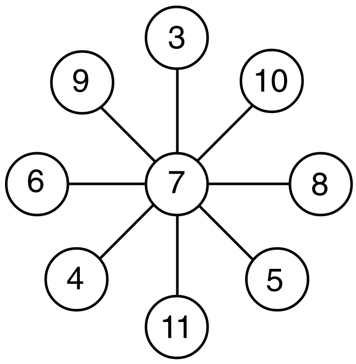
A variety of household containers could be used to find out the students' ability to estimate capacity.

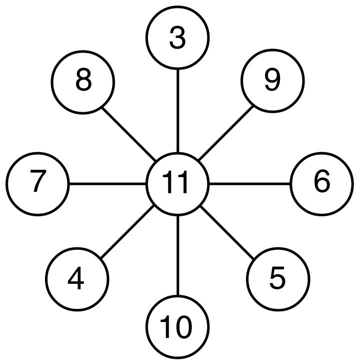
Problem Four

Students will need to look for four pairs of numbers with the same total to balance the wheel. This will mean matching a low number with a high number, for example 1 and 11, 2 and 10, etc.

In this problem, the median number, 6, is pivotal in achieving the balance needed. To extend the problem, give the students the numbers 3, 4, 5, 6, 7, 8, 9, 10, and 11 to place in the circles. This extension will have three solutions:







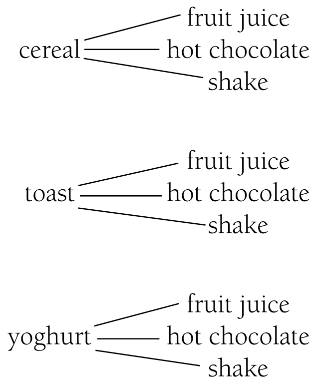
$$\begin{array}{r} 63 \\ -\ 3 \\ \hline 60 \end{array} \qquad \begin{array}{r} 63 \\ -\ 7 \\ \hline 56 \end{array} \qquad \begin{array}{r} 63 \\ -\ 11 \\ \hline 52 \end{array}$$

$$\frac{60}{4} = 15 \qquad \frac{56}{4} = 14 \qquad \frac{52}{4} = 13$$

Page 13: Numbers in Disarray

Problem One

Making a model of the problem with pictures or names of the different foods and drinks on pieces of paper will allow students to experiment with possible pairings. As with other combination problems, organised lists, making a table, and drawing a tree diagram are useful strategies.



	Fruit juice	Hot chocolate	Shake
Cereal			
Toast			
Yoghurt			

The problem can be made more difficult by adding other foods and drinks, for example, fruit salad and water, or another course, such as scrambled eggs or baked beans.

Problem Two

Note that the answers to this question will depend on whether all or some of the electric fence is used. Students may need to be reminded that area refers to the number of square units contained in the figure. This can be found by one-by-one counting, repeated addition (2 + 2 + 2 + 2 + 2 + 2 + 2 + 2), or multiplication (8 x 2), which is the most efficient strategy.

Students can draw different-sized rectangles on squared paper. This can be approached systematically by decreasing the length and increasing the width by a corresponding amount (see the diagrams in the Answers section).

Students may not realise that a square is a type of rectangle and therefore the 6.25 x 6.25 rectangle gives the greatest area. They may wish to investigate which rectangle with a perimeter of 16 metres has the greatest area. Encourage them to recognise that in both problems, the square has the greatest area.

Problem Three

This problem will be most efficiently solved in an organised way, beginning with 74 and decreasing one digit while increasing the other.

29 38 47 56 65 **74** 83 92

Note that eight two-digit numbers have a digit sum of 11. Other digit sums could be investigated.

For example, nine two-digit numbers have a digit sum of nine:

18 27 36 45 54 63 72 81 90

Problem Four

Students will need to work from what is known to what is unknown in a systematic way.

x	③		5
④		28	20
6	18		

x	3	⑦	5
4		28	20
6	18		

x	3	7	5
4	⑫	28	20
6	18	㊷	㉚

Problems of this type can be developed in the following way:

i. Put in two numbers that will tell you a third number (circled).

ii. Put in a new number that will tell you another answer (circled).

iii. Continue putting in one number at a time, circling numbers that can be found from it.

x		6	
②		12	

x		6	9
②		12	⑱

x		6	9
②		12	⑱
4		㉔	㊱

iv. When the grid is complete, erase the circled numbers.

v. This leaves a problem for another person to solve.

x	⑤	6	9
②	⑩	12	⑱
4	20	㉔	㊱

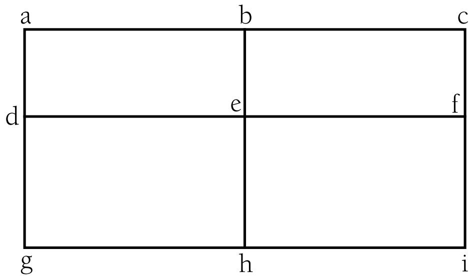
x		6	9
		12	
4	20		

Encourage students to develop their own grids.

Page 14: Quacky Questions

Problem One

This is a similar problem to **Problem One** on page 5. It can be solved using the same strategies.



Students can label the corners and make an organised list:

a b e d   b c f e   d e h g   e f i h
a b h g   b c i h   d f i g
a c f d
a c i g

Similarly, they might classify the rectangles as made up of one, two, or four smaller rectangles:

One-unit rectangles	Two-unit rectangles	Four-unit rectangles	Total
4	4	1	9

Extend the problem by adding another column or row to the figure.

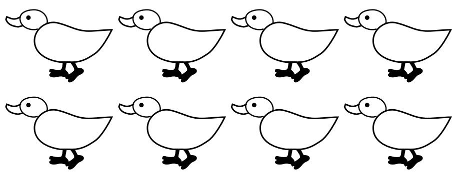
Problem Two

Knowing that, in general, ducks have two legs and sheep have four legs is important. Students may use a variety of strategies.

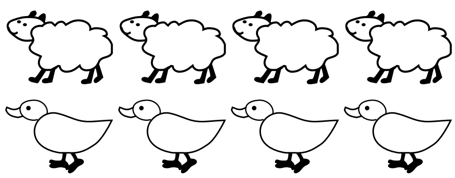
Draw a table:

Ducks	Sheep	Legs
8	0	16
7	1	18
6	2	20
5	3	22
4	4	24

Draw a diagram:



Make every animal a duck.



Then change the ducks to sheep one by one until there are 24 legs altogether.

Change the conditions of the problem to see whether students have generalised the process. For example:

"There are nine animals and 30 legs. How many ducks are there?"

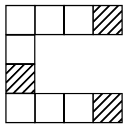
Problem Three

Making a table or writing an equation are useful strategies.

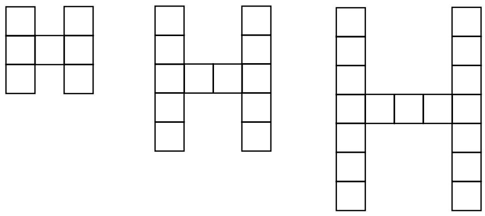
Letter	1st	2nd	3rd	4th	5th
Number of tiles	7	10	13	16	19

$7 + 3 + 3 + 3 + 3 = 19$

Students should recognise that three tiles are added each time to increase the letter size. This can be highlighted by building the letters on an overhead projector.

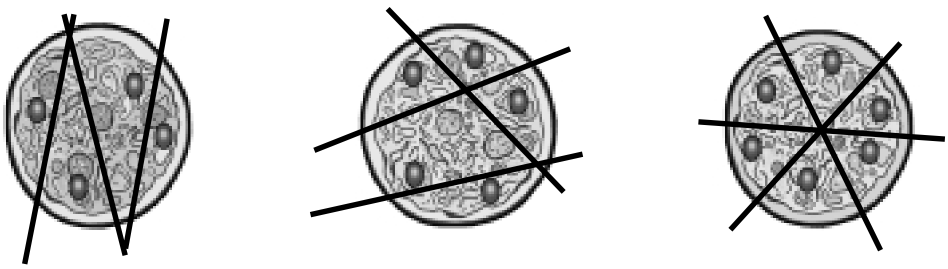


Students might solve other letter-building sequence problems, such as:



Problem Four

Students may solve the first two cutting problems randomly, but they will need to be systematic with the pizzas containing six and seven olives:

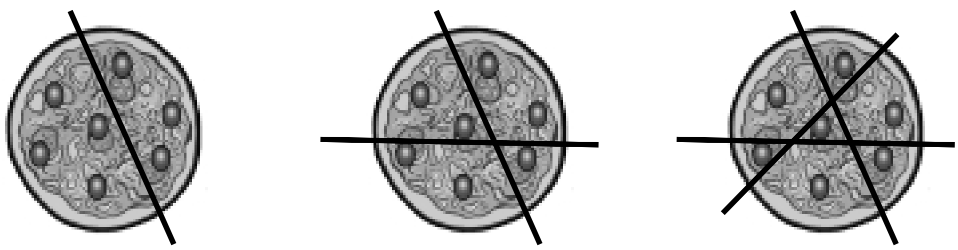


For example, with the seven-olive pizza:

Make the first cut so that four olives are on one side and three olives on the other:

Make the second cut so that the three olives are divided into one and two and the four olives are divided into two and two:

This leaves three sections with two olives in each, which need to be divided by the third cut:

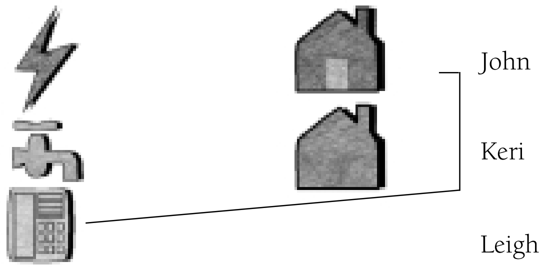


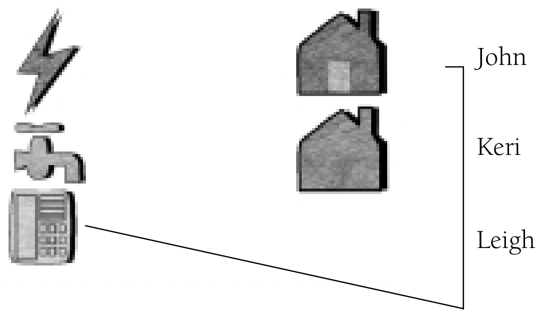
As an extension, ask the students to draw a pizza with eight olives and to make three cuts that leave each olive in a piece by itself. This is impossible and should lead students to realise that seven olives is the maximum number possible with three cuts.

Page 15: Power to the People

Problem One

Students will need to experiment with the different places that the service lines can go. For example:



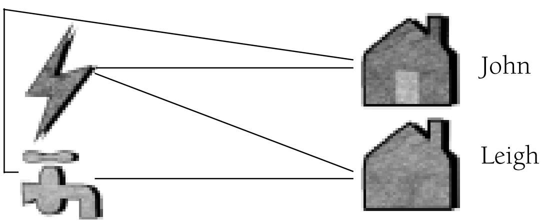


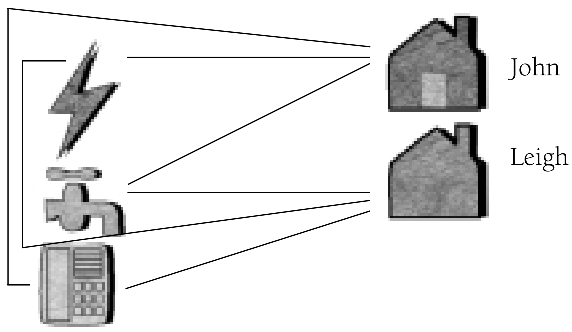
See the Answers section for a possible solution.

A table is a good way to check that all required connections have been made.

	John	Keri	Leigh
Power	✓	✓	✓
Water	✓	✓	✓
Telephone	✓	✗	✓

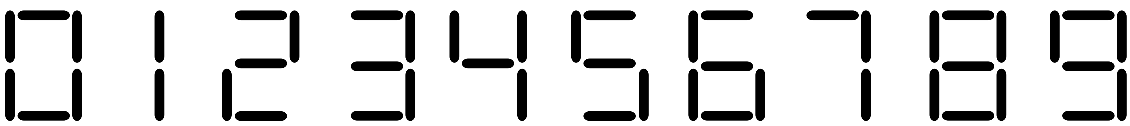
If students find the problem difficult, simplify it by having two people and two services and then two people and three services.





Problem Two

Students will need to systematically check the number of light bars needed for each digit:



Six Two Five Five Four Five Six Three Seven Six

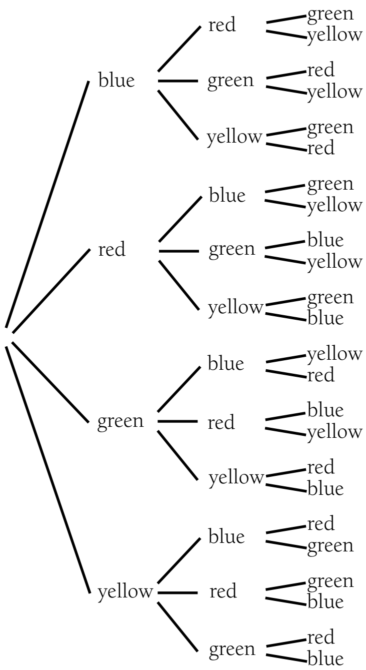
Six is the only other digit that uses as many light bars as its name.

There are other challenges that can be posed involving light bars. For example:

"How many two-digit numbers are made with 10 light bars?"

Problem Three

This is another combinations problem identical in nature to **Problem One** on page 6. The solution can be worked out with a tree diagram or organised list:



4 x 3 x 2 = 24

As an extension, increasing the number of colours to five increases the number of different possible colour combinations for the flags to 5 x 4 x 3 = 60. This assumes that each flag must contain three different colours.

Problem Four

A table is a good way to solve this combinations problem. Assuming that both the coins land, the solution can be found in this way:

+	2	3	4	5
2	4	5	6	7
3	5	6	7	8
4	6	7	8	9
5	7	8	9	10

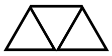
So the scores could be one of the whole numbers 4 to 10.

Students might want to make the board and trial which score is most or least likely. The table shows that students are most likely to score seven and are least likely to score four or 10.

Page 16: Shaping Up

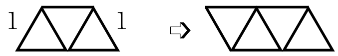
Problem One

A combination of strategies could be used to find all the possible shapes. One way is to begin with a simpler problem, that is, to find all the shapes that are possible by joining three triangles. Only one shape is possible:

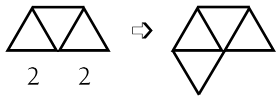


This simpler situation can then be analysed to see where the next triangle could be added to form a four-triangle shape:

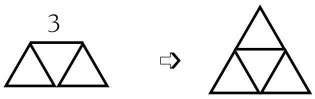
Joining to either of the sides labelled 1 gives a parallelogram:



Joining to either of the sides labelled 2 gives the following shape:

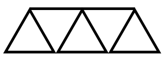


Joining to the side labelled 3 gives a larger triangle:

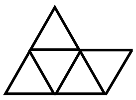


Each of the four-triangle shapes can be analysed further to find out which five-triangle shapes are possible:

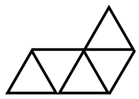
Trapezium



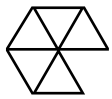
Triangle with outcrop



Parallelogram with outcrop



Letter C

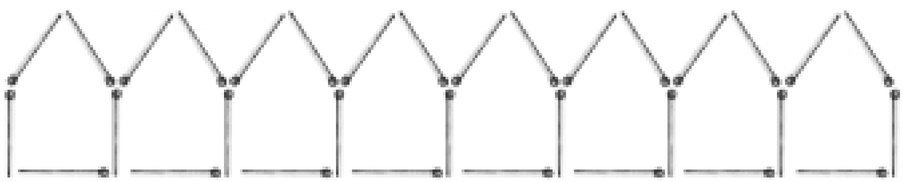


Students may need access to triangular blocks and isometric dot paper to help them find their solutions for this problem.

Problem Two

This is a typical problem that involves predicting further members of a sequential pattern. It links with the outcomes of the algebra strand. Various strategies are useful, including:

- Building the pattern and systematically counting the matches:



8	+	16	+	9	= 33
floors		roofs		walls	

- Making a table and extending the values:

Houses	1	2	3	4	5	6	7	8
Matchsticks	5	9	13	17				

+ 4 + 4 + 4

- Writing an equation: 5 + 4 + 4 + 4 + 4 + 4 + 4 + 4 = 33

Encourage students to reflect on which methods would be most powerful if the problem were extended. For example, "How many matchsticks would we need to build 15 houses?" In such cases, the table and equation strategies tend to be more efficient than the build-and-count method.

Problem Three

Students will need to apply their knowledge of place value to work out the best placement of digits. In finding the greatest product, it makes sense to have the digits with the largest value in the tens place of the top factor or as the lower factor.

This produces the arrangement with:

$$\begin{array}{r} 43 \\ \times \underline{\quad 5} \\ 215 \end{array}$$

This reasoning is reversed to give the arrangement with the smallest product:

$$\begin{array}{r} 45 \\ \times \underline{\quad 3} \\ 135 \end{array}$$

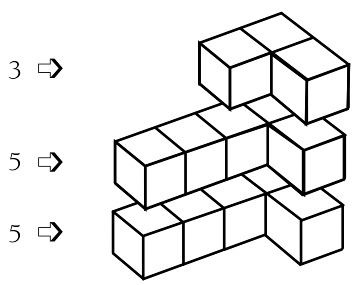
Students could investigate whether this pattern holds for different sets of digits, for example, 2, 5, 9.

Problem Four

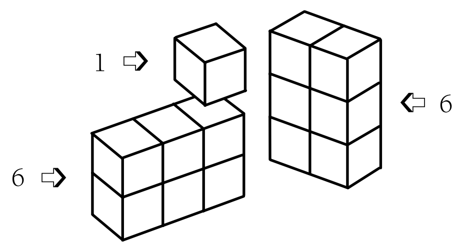
Get students to build a model of the building with multilink cubes so that they can demonstrate their methods of counting the number of cubes.

Possible methods include:

- by layers:



- by blocks:



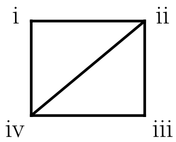
Get the students to draw up their own "how many cubes?" buildings for others to solve.

Page 17: Pinning It Down

Problem One

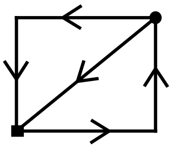
The key to solving each of these problems is to identify how many line segments meet at each point.

For example, in shape **a**:

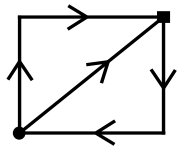


Points i and iii have two line segments joining at them. Points ii and iv have three line segments joining at them. They are called odd points. An odd point can have one, three, five, seven, etc. line segments meeting at it.

A figure can only be drawn if there are only two odd points (nodes), like ii and iv in shape **a**. If the figure has two odd points, students must start drawing the shape from one odd point and end at the other.



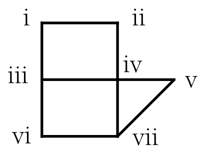
or



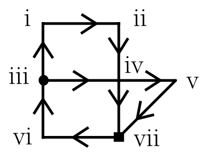
are possible pathways.

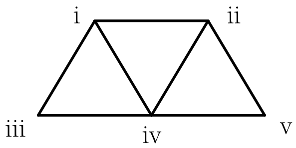
• = start ▪ = finish

Some solutions for the other shapes are:

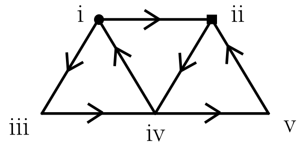


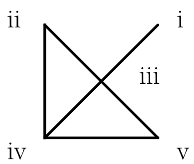
iii and vii are odd points, so one solution starting at iii would be:



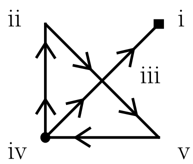


i and ii are odd points, so one solution starting at i would be:





i and iv are odd points, so one solution starting at iv would be:



As an extension, get students to draw networks like these that cannot be drawn without lifting the pencil.

Problem Two

A table (matrix) is a useful way to organise the results of the clues:

The blue and yellow cups hold an odd number of counters. This means that the green and red cups must hold an even number of counters.

	1	2	3	4
Green	✗		✗	
Yellow		✗		✗
Blue		✗		✗
Red	✗		✗	

The green cup has twice as many counters as the red cup. There is only one way this can happen, so the green cup must have four counters and the red cup must have two counters.

	1	2	3	4
Green	✗		✗	✓
Yellow		✗		✗
Blue		✗		✗
Red	✗	✓	✗	

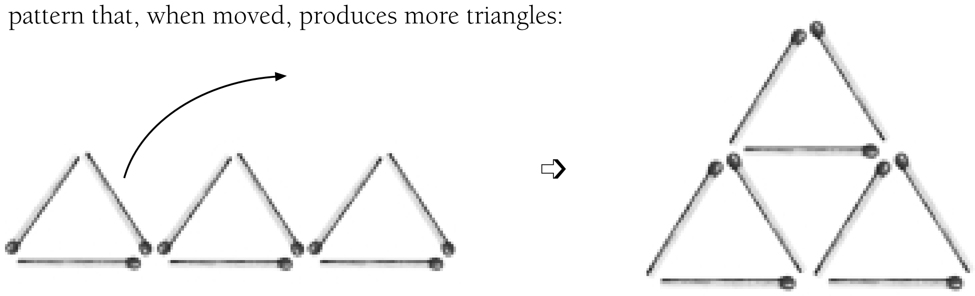
The blue cup has three fewer counters than the green cup. This must mean that the blue cup holds one counter and the yellow cup has three counters.

	1	2	3	4
Green	✗	✗	✗	✓
Yellow	✗	✗	✓	✗
Blue	✓	✗	✗	✗
Red	✗	✓	✗	✗

There is no particular best order in which to consider the clues in this problem. Some students may solve the problem by considering the clues without using tables. For example, the last clue defines how many counters are in the blue and green cups.

Problem Three

Students need to attend to the conditions of the problem, especially the fact that they do not need to create new triangle shapes that are the same size. They will need to look for an element in the pattern that, when moved, produces more triangles:



Problem Four

Since the last digit is 7, this eliminates many possibilities. An organised list of possible numbers is:

4 2 5 7	2 4 5 7	5 2 4 7
4 5 2 7	2 5 4 7	5 4 2 7

Since the machine will eat her card after three wrong tries, Trudy's mum has three out of six ($^1/_2$ or 50%) chances of getting her PIN right. Students could investigate her chances if she didn't know that seven was the final digit. In that case, there would be 24 possibilities, so she would have three out of 24 chances ($^1/_8$).

Page 18: Slippery Solutions

Problem One

Making a vertical number line and using a peg to track Snippet the Snake's position at each minute is a useful model. Organising this data in a table can also be helpful:

Minutes	1	2	3	4	5	6	7	8	9
Snippet's height	4	2	6	4	8	6	10	8	12

So Snippet reaches the top of the pole in 9 minutes. (If the pattern were to continue, he would slip back 2 metres after that ninth minute, unless he hangs on tight!) Some students will give an answer of 12 minutes based on the pattern that on every even number of minutes, Snippet is that number of metres up the pole.

Encourage the students to generalise the process by varying the conditions of the problem. For example:

"What if the pole were 20 metres high?"

"What if Snippet went up 5 metres each time and then slid back 2 metres?"

In general, for any height of pole, Snippet can climb the "up" distance on the last minute. So students need to count 1 minute for this and take the up distance off the height. Since his net gain for every two minutes is the difference between the up distance and the down distance, find out how many times this net gain will go into the remaining height. For example:

"Snippet climbs up 5 metres in the first minute and then slips back 2 metres in the second minute, and so on. The pole is 17 metres high. How long will Snippet take to reach the top?"

$17 - 5 = 12$ gives the metres to be climbed by net gains.

$5 - 2 = 3$ gives the net gain.

$12 \div 3 = 4$ gives the number of 2 minutes needed to reach 12 metres.

So Snippet takes $1 + (4 \times 2) = 9$ minutes to reach the top.

Problem Two

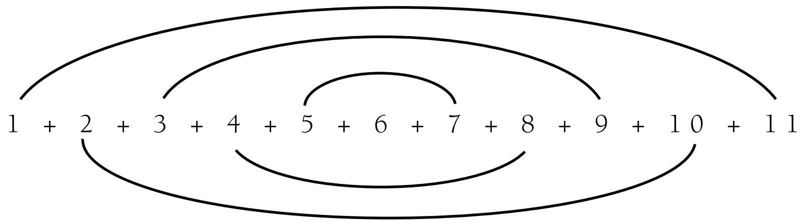
Students will need to find the sum of the first 11 counting numbers, 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11. Encourage them to look for patterns to make the calculation easier. For example:

Look for numbers that add to 10:

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + \underline{10} + 11$$

… so the answer is $(5 \times 10) + 5 + 11 = 66$

Look for pairs of numbers with the same sum, in this case, 12:



… so the answer is (5 x 12) + 6 = 66

You may want to tell the students the story about a famous mathematician called Gauss, who found the sum of the numbers 1 + 2 + 3 + … + 98 + 99 + 100 with remarkable speed at the age of nine. He used the method above and astounded his teacher.

Problem Three

Students will need to understand the dynamics of balancing scales to solve the problem. If necessary, relate this to a see-saw. A logical line of reasoning would be:

i. Three lots of B have a mass of 24, so one lot of B must have a mass of 8.

ii. Two lots of B have the same mass as A, and because B has a mass of 8, A must have a mass of 2 x 8 = 16.

These types of problems help develop algebraic thinking, particularly the ability to handle unknowns. Try varying the conditions of this problem, for example:

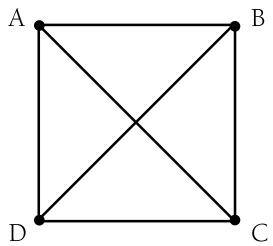
"What if four lots of B had a mass of 24?"

"What if two lots of A and two lots of B had a mass of 24?"

Problem Four

This problem is easily modelled by acting it out with the students.

Alternatively, students can represent the problem using a diagram or table:



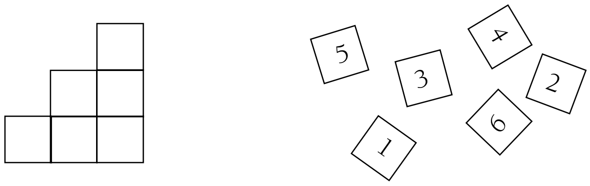
	A	B	C	D
A	✗	✗	✗	✗
B	✓	✗	✗	✗
C	✓	✓	✗	✗
D	✓	✓	✓	✗

Encourage students to generalise their methods by extending the pattern to include five people shaking hands.

Page 19: Setting up Swaps

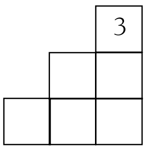
Problem One

It is easy for students to draw a model of the problem using cards with the numerals 1 to 6, which they can then use to try out different possible solutions.

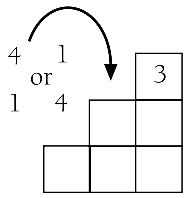


Taking account of the clues will limit the possibilities and make the problem quicker to solve.

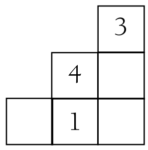
The top block is 3:



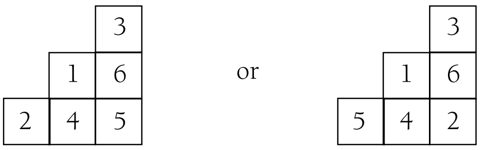
The middle tower numbers add to 5:



Students cannot make 10 with the remaining numbers if they arrange the middle tower like this:



The bottom blocks add to 11:

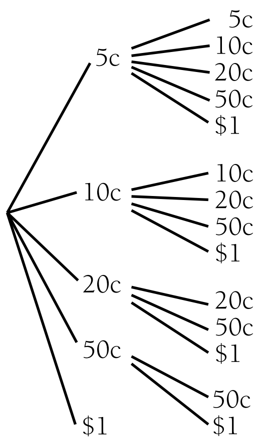


Students may enjoy making up their own problems with these digits.

Problem Two

The first step is to work out which amounts of change are possible with two coins.

A table or tree diagram showing the change would be useful:



	5c	10c	20c	50c	$1
5c	✓				
10c	✓	✓			
20c	✓	✓	✓		
50c	✓	✓	✓	✓	
$1	✓	✓	✓	✓	

So the following amounts could be given as change: 10c, 15c, 20c, 25c, 30c, 40c, 55c, 60c, 70c, $1, $1.05, $1.10, $1.20, or $1.50, in which case the article could cost $1.90, $1.85, $1.80, $1.75, $1.70, $1.60, $1.45, $1.40, $1.30, $1, 95c, 90c, 80c, or 50c respectively.

You could discuss with students which of these amounts the chippies are more likely to cost.

Problem Three

A table is a good way to keep track of the amount that each person had saved:

	Start	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Kuini	$12	$14	$16	$18	$20	$22	$24	$26	$28	$30	$32	$34	$36
Romi	$4	$7	$10	$13	$16	$19	$22	$25	$28	$31	$34	$37	$40

Students can quickly calculate each person's saving by using the constant function on a calculator.

Kuini 12 + 2 = = = … (12 times)

Romi 4 + 3 = = = … (12 times)

(On some calculators, you have to press [+] twice to make the constant function work.)

As an extension, ask students "What would have happened if Romi had saved only $2 per month? Would the amounts in their piggy banks ever have been the same?"

Problem Four

Students can act out the problem by making up three sets of A, B, C, D cards. A good place to start is with the cards that are not wanted in each player's hand. These cards are Simon (A), Cathy (C and D), and Beth (B). The players need to make two swaps to get the cards they need:

Simon (A) ⟷ (D) Cathy, and Beth (B) ⟷ (C) Cathy.

Students might investigate more complex swap situations, such as:

AAAB	CCCB	DDDB
Beth	Cathy	Simon

or where there are five cards in a set:

AAAEE	CCCBB	DDDEE
Beth	Cathy	Simon

Page 20: Kete Counting

Problem One

Students might solve this problem by trial and improvement. They could estimate how many kete Mere made in the first hour. For example:

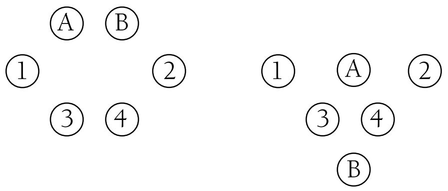
2	+	4	+	6	+	8	+	10	= 30	So the estimate of two kete in the first hour is too small.
first hour		second hour		third hour		fourth hour		fifth hour		

If they increased this amount by one more kete every hour, the overall total would increase by five, which leads to the right total.

An alternative strategy would be to find the middle number in the sequence. This would be 35 ÷ 5 = 7 (the average). Students could then work out the number of kete Mere made in the last hour by adding two each time until the total reaches 35.

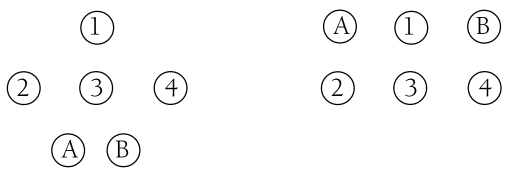
Problem Two

This problem requires students to visualise the smallest number of counters needed to transform the figure on the left to that on the right. One way to solve the problem is to draw both figures in different colours and place them on top of one another against a window. When the maximum number of counters match, it is easy to see which counters need to be moved:



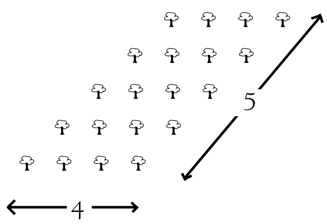
Counters 1, 2, 3, and 4 will overlap, leaving only counters A and B to be moved.

The overlapping method is also easily demonstrated on an overhead projector. Students can use it to create new counter moving puzzles, such as:

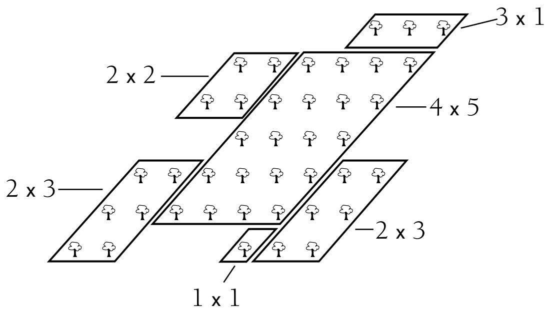


Problem Three

There are many ways to count the number of trees. The most efficient method involves the multiplication array. For example, this section of trees can be counted using 4 x 5.

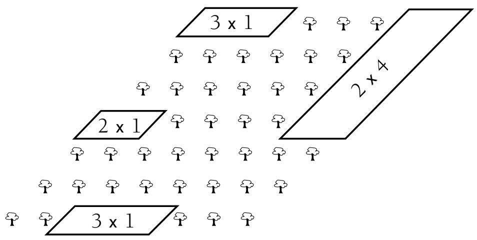


Students can either break the orchard into blocks, find the total of each block, and add these totals together:



(4 x 5) + (2 x 3) + (1 x 1) + (2 x 3) + (2 x 2) + (3 x 1)
= 20 + 6 + 1 + 6 + 4 + 3
= 40

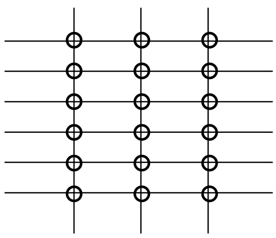
or they can work out the number of trees that would be in the complete parallelogram and subtract those that are missing:



(7 x 8) – (2 x 4) – (3 x 1) – (2 x 1) – (3 x 1)
= 56 – 8 – 3 – 2 – 3
= 40

Problem Four

There are many ways of arranging the nine streets. Students may not recognise this problem as an application of multiplication. Putting counters on a diagram of the intersections helps to illustrate this point, in this case, by showing multiples of three:



They could also use a table to find all the possible numbers of intersections systematically:

Streets down	Streets across	Intersections
0	9	0 x 9 = 0
1	8	1 x 8 = 8
2	7	2 x 7 = 14
3	6	3 x 6 = 18
4	5	4 x 5 = 20

To extend the problem, ask the students: "How many intersections might there be with 10 streets?"

Page 21: Problem Bits One

"Bits" are designed to be focus statements for group discussion. As such, they could be used as short starting points during a mathematics lesson. It is important that the results of the group discussion are shared with the whole class.

Bit One

Although the aim is to swap the 10c ⟷ 50c or 10c ⟷ 20c, other coins might also end up in a different position.
Students' answers will vary, but the minimum number of moves is:

a. 10 cents ⟷ 50 cents (13 moves)

b. 10 cents ⟷ 20 cents (22 moves)

It is important for students to have a way to record the moves they make. This may involve drawing the whole frame each time or just recording the coins that move in each situation.

For example, the 10 cent and 20 cent coin movements could be recorded as:

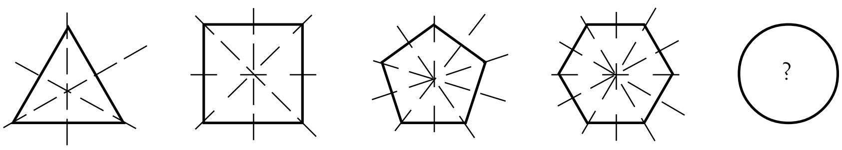
5, 10, 20, 5, $1, 50, 10, $1, 5, 20, $1, 5, 50, 10, 5, 50, 10, 5, 50, $1, 20, 10 (the 5c + $1 have swapped as well)

The first move, "5", indicates that the 5 cent coin moves to the only vacant space, followed by the 10 cent coin moving to the space created by moving the 5 cent coin.

Bit Two

The circle is the limiting case of a polygon with an infinite number of sides and corners. Students are likely to answer along these lines, with responses such as "There are so many sides/corners that they are too small to see."

Students may like to look at the number of lines of reflective symmetry that each polygon has:



The circle has an infinite number of lines of reflective symmetry.

Bit Three

The "addition makes bigger" belief is one that students frequently adopt as an informal rule. This becomes an obstacle when they add negative integers. For example, $^{+}3 + ^{-}2 = ^{+}1$, so the answer is less than the starting addend.

Students should note that adding zero is an exception to the rule because the answer and first addend are the same, for example, $4 + 0 = 4$.

Page 22: Problem Bits Two

Bit One

Students may not be aware of the convention of rounding up if the measure is exactly halfway between two measurement points. For example, a pencil that measures 14.5 centimetres would have that measure rounded up to 15 centimetres if a whole number was required. The discussion should embrace some interesting issues, such as "If it were the price of something, like $1.27, I would want it rounded down"; "If someone gave me half a cup of milk and told me it was rounded up to a whole cup, I wouldn't be happy"; "If I were trying for the high jump record, I'd like the height rounded up".

Students need to realise that rounding up halfway linear measures is a convention, a widely accepted practice, and often there is no other explanation as to why it occurs. However, measuring someone for fitting clothes is a good reason why you should round up – it's better to cut something too big than too small!

Bit Two

Students may need access to models of simple polyhedra, such as tetrahedra (4 faces), various prisms, octahedra (8 faces), and dodecahedra (12 faces). Encourage them to present their data in an organised format to support their belief.

For example:

	Faces	Corners
Tetrahedron	4	4
Cube	6	8
Triangular prism	5	6
Octahedron	8	6
Dodecahedron	20	12

The table reveals that three-dimensional solids do not always have more corners than faces.

Bit Three

There is no one correct answer in this discussion. Students may focus on the way the two coinage systems could meet certain prices. For example:

Price	20 cent coin scenario	25 cent coin scenario
25c	(20) (5)	(25)
30c	(20) (10)	(25) (5)
45c	(20) (20) (5)	(25) (10) (10)
50c	(20) (20) (10)	(25) (25)
70c	(50) (20)	(50) (10) (10)
75c	(50) (20) (5)	(50) (25)
85c	(50) (20) (10) (5)	(50) (25) (10)

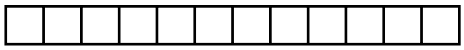
The comparison indicates that for some prices, the 25 cent coin scenario would be advantageous because fewer coins are needed to make those amounts.

Page 23: Problem Bits Three

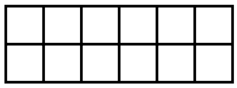
Bit One

Students will find it impossible to build any other rectangle using all 13 square tiles. This is because 13 is a prime number and has only two factors, itself and one.

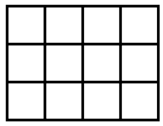
The rectangles that are possible with 12 tiles can be found by listing all the factors of 12:



12 x 1



6 x 2

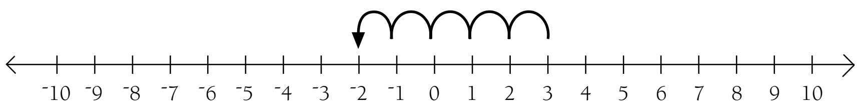


4 x 3

Bit Two

Students will need calculators to try this problem bit. They will find that when they take a greater whole number from a smaller whole number, the result is a negative integer. For example, $3 - 5 = {}^{-}2$.

Have students draw a number line descending from 10 as they key [10] [–] [1] [=] into their calculators and press the equals button repeatedly. Students will notice the introduction of the set of integers below zero as one is subtracted each time:

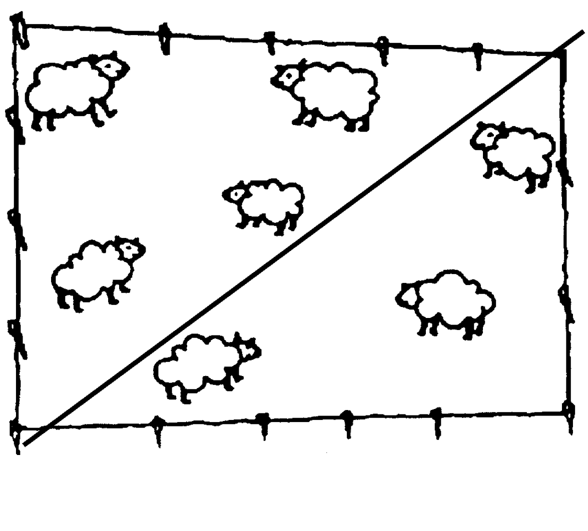


The result of $3 - 5 = {}^{-}2$ can be shown by using "jumps". Students should attempt other similar calculations, for example, 4 – 8 = □.

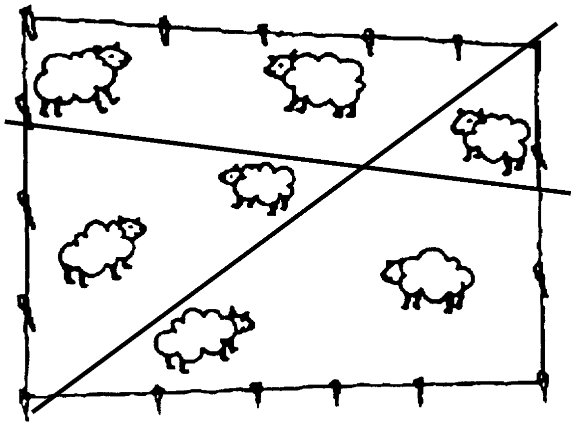
Bit Three

Students will need to organise their lines carefully if they are to enclose each sheep individually:

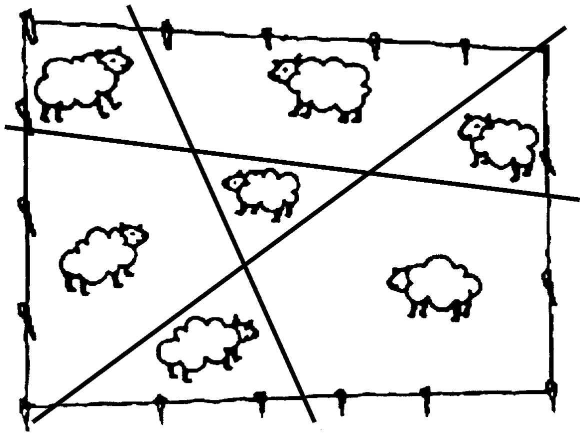
i. Divide the sheep as evenly as possible into two subsets, that is, 3 and 4:



ii. Divide each subset as evenly as possible into halves, that is, 1 and 2, 2 and 2:



iii. Position the third fence so each sheep is enclosed individually:



As an extension, ask the students if they think eight sheep could be enclosed in this way. This is impossible because seven is the maximum number of sections an area can be divided into with three straight lines (see notes for page 14, **Problem Four**).

Page 24: Problem Bits Four

Bit One

Seven is one possible number to follow in the pattern. It is not the only solution.

For example:

1 2 4 7 — The difference increases by one.
+ 1 + 2 + 3

1 2 4 8 — The next number is double the one before.
x 2 x 2 x 2

1 2 4 1 — The pattern repeats.

1 2 4 6 — The pattern has "settled" to increasing by 2 each step.
+ 1 + 2 + 2

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The main illustrations on the cover and the contents page and the illustrations on pages 26 (the matches, which are also used on page 5 of the answers), 33 (the pizzas, which are also used on page 6 of the answers), 34 (the houses and services, which are also used on page 6 of the answers), 36 (the matches), 39 (the matches, which are also used on page 7 of the answers), and 48–49 (the sheep diagrams, which are also used on page 8 of the answers) are by Watermark.

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