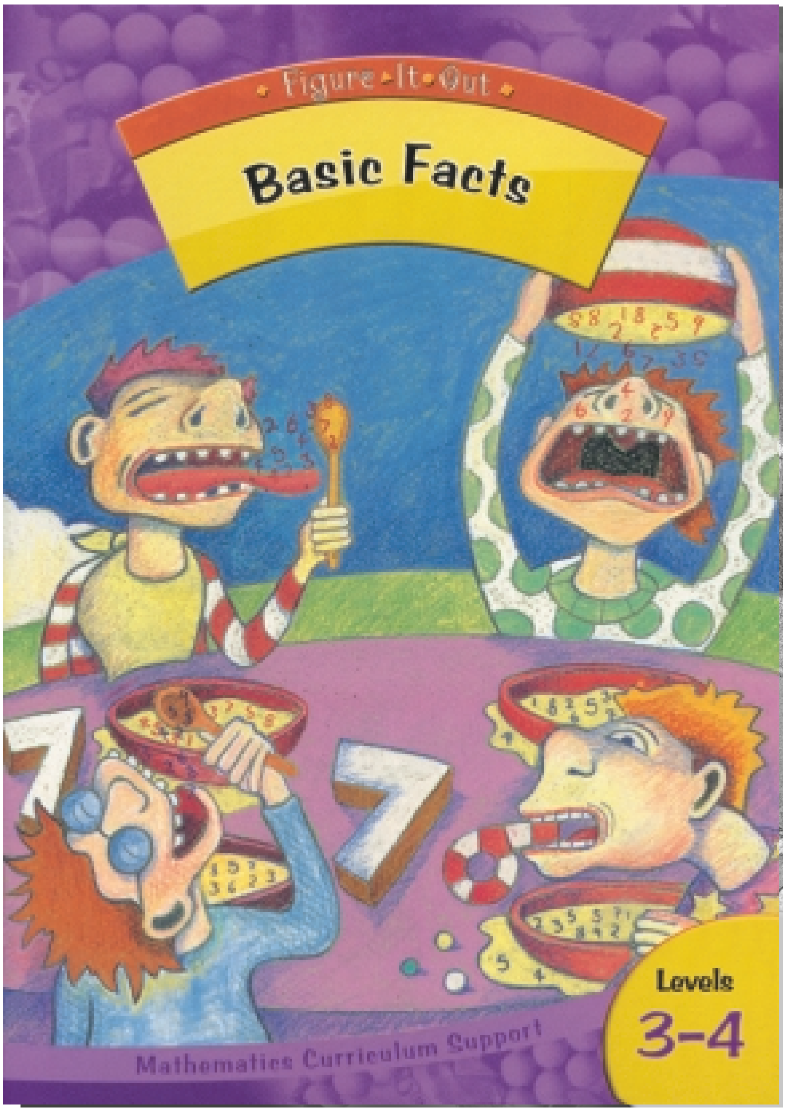
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Contents

Introduction

The Figure It Out series is designed to support *Mathematics in the New Zealand Curriculum.* The booklets have been developed and trialled by classroom teachers and mathematics educators. The series builds on the strengths of a previous series of mathematics booklets published by the Ministry of Education, the School Mathematics supplementary booklets.

Figure It Out is intended to supplement existing school mathematics programmes and can be used in various ways. It provides activities and investigations that students can work on independently or co-operatively in pairs or groups. Teachers can select particular activities that provide extension to work done in the regular classroom programme. Alternatively, teachers may wish to use all or most of the activities in combination with other activities to create a classroom programme. The booklets can be used for homework activities, and the relevant section in the teachers' notes could be copied for parents. These notes may also provide useful information that could be given as hints to students. The teachers' notes are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/community

There are eight booklets for levels 3–4: one booklet for each content strand, one on problem solving, one on basic facts, and a theme booklet. Each booklet has its own *Answers and Teachers' Notes*. The notes include relevant achievement objectives, suggested teaching approaches, and suggested ways to extend the activities. The booklets in this set (levels 3–4) are suitable for most students in year 6. However, teachers can decide whether to use the booklets with older or younger students who are also working at levels 3–4.

The booklets have been written in such a way that students should be able to work on the material independently, either alone or in groups. Where applicable, each page starts with a list of equipment that students will need to do the activities. Students should be encouraged to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Teachers could encourage students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Where possible, suggestions have been made to encourage discussion and oral presentation of answers, and teachers may wish to ask their students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. Successful ways of solving problems should be acknowledged, and where more effective or efficient processes can be used, students can be encouraged to consider other ways of solving the problem.

Figure It Out

Basic Facts

Answers

Page 1: Bunches

Game

A game of addition

Page 2: Trying Times

Activity

1. Teacher to check methods used. Answers are:

   a. 60

   b. 105

   c. 195

   d. 225

2. Answers will vary.

3. Answers will vary. You could use doubling/halving, bridging from 10 or multiples of 10, or splitting into suitable part/whole arrangements.

Page 3: Eleventh Heaven

Activity

1. a. 165, 275, 385, and 495

   b. The ones and the hundreds digits are the same as the two digits in the original factor (for example, 25 x 11 = <u>2</u>7<u>5</u>). These two digits add up to the tens digit (2 + 5 = 7).

2. a. 3_6: 3 + 6 = 9. The answer is 396.

   b. 4_3: 4 + 3 = 7. The answer is 473.

   c. 5_2: 5 + 2 = 7. The answer is 572.

3. a. The pattern works in this way:
   Think about any two-digit number, for example, 35. When the 5 is multiplied by 11, the result is 55, which is 5 tens and 5 ones. When the 30 is multiplied by 11, the result is 330, which is 3 hundreds and 3 tens.
   So, the ones digit of the answer must be 5 because this is the only way ones are created. The hundreds digit must be 3 because this is the only way hundreds are created. The tens digit in the answer will be the result of adding together 5 tens and 3 tens, that is, 8 tens.

3. b. If the two digits of the number add up to 10 or more, an extra step is necessary.
   For example, for 57, 5 + 7 = 12.
   Using Toby's method:
   57 x 11 = 5_7.
   But 5 + 7 = 12, and you can't put 12 in the tens column, so you have to add an extra 1 to the hundreds digit:
   5 / 2 / 7
   1
   Another way of showing this is

H	T	O
5	12	7

H	T	O
6	2	7

4. Answers will vary.
   For example, the 19 times tables are "one less than" the 20 times tables.
   7 x 19 = (7 x 20) – 7
   = 140 – 7
   = 133
   12 x 19 = (12 x 20) – 12
   = 240 – 12
   = 228
   You could also use the 10 times table.
   For example:
   12 x 19 = (10 x 19) + (2 x 19)
   = 190 + 38
   = 228
   Ways of remembering the 15 times table are given on page **2** of the student booklet.
   The 13 times table is a combination of the 10 times and 3 times tables. For example:
   9 x 13 = (9 x 10) + (9 x 3)
   = 90 + 27
   = 117
   16 x 13 = (16 x 10) + (16 x 3)
   = 160 + 48
   = 208

This method could be used for other tables.
For example:

2 x 14 = (2 x 10) + (2 x 4)
= 20 + 8
= 28

For "teen" times tables that are even numbers, you could use the "halve and double" method. For example, the 14 times table is double the 7 times table, the 16 times table is double the 8 times table, and so on.

Pages 4–5: Diamond Dazzle

Game

A game of addition

Pages 6–7: Memory Testing

Activity

1. **a.** 24 is in the memory. Pressing M+ three times puts in 8 + 8 + 8. 3 x 8 = 24

   b. M shows that something is in the memory.

2. 0 is in the memory. It is the same as
   9 + 9 + 9 – 9 – 9 – 9 = 0.
   (3 x 9) – (3 x 9) = 0

Game

A game using calculator memory

Pages 8–9: Napier's Bones

Activity

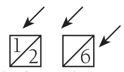
1. Answers will vary. You should see patterns if you:
   - look at the number at the top and scan down each strip or if you look across the strips
   - look at how the two-digit numbers are written
   - look for symmetrical patterns
   - look for repeating patterns.

2. Research activity. The facts that make up Napier's invention were originally carved on bones.

3. Answers will vary according to the multiplication statements used. The answers to each times table up to 9 are found vertically and horizontally. For example, look at the 5 column. Notice that each entry as you look down the column is a multiple of 5 (5, 10, 15, and so on). To find 6 x 5, the sixth entry is 30, which is 6 x 5. Likewise, to find 8 x 5, look for the eighth entry in the column.
   To find 8 x 7, look at the eighth entry in the 7 column. Each of the columns works in this way.

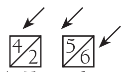
4. Each line of the working has the same numbers in it as the arrowed strips, that is, 3 tens and 5 ones and 1 hundred and 5 tens. You are also combining 7 x 5 on the seven strip (35) with 5 x 30 on the three strip (150).

5. **a.** 126: 2 strip and 1 strip, 6th row. Read as:



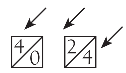
1 2 6

   b. 476: 6 strip and 8 strip, 7th row. Read as:



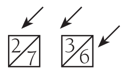
4 7 6

   c. 424: 5 strip and 3 strip, 8th row. Read as:



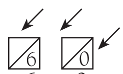
4 2 4

   d. 306: 3 strip and 4 strip, 9th row. Read as:



2 10 6
(= 306)

   e. 60: 3 strip and 0 strip, 2nd row. Read as:



6 0

Page 10: Matrix

Activity One

1. With the 6, 9, and 4 in their best boxes, the final grid should look like this:

5	6	9	
40	54	72	8
45	48	63	4
56	60	36	7

2. Her total score is: 2 + 2 + 1 + 3 + 3 + 1 = 12. (Gina gets 3 points from the 6, 3 from the 9, and 1 from the 4.)

Game

A game of factors

Activity Two

a. The highest possible score for **a** is 13. There are only four ways to get this:

i. top boxes 5 4 8 / side boxes 6 8 4

ii. top boxes 5 4 4 / side boxes 6 4 4

iii. top boxes 5 4 4 / side boxes 6 8 4

iv. top boxes 5 4 8 / side boxes 6 4 4

b. There are 72 ways to get the highest possible score of 11 points. Two of these are:

i. top boxes 9 6 7 / side boxes 9 9 7

ii. top boxes 9 9 7 / side boxes 7 5 9

Page 11: Magical Tens

Activity

1\. a. 18 + 7 = 25

●	●	●	●	●					
❍	❍	❍	❍	❍	❍	❍	❍	❍	❍
❍	❍	❍	❍	❍	❍	❍	❍	●	●

b. 28 + 7 = 35

●	●	●	●	●					
❍	❍	❍	❍	❍	❍	❍	❍	❍	❍
❍	❍	❍	❍	❍	❍	❍	❍	❍	❍
❍	❍	❍	❍	❍	❍	❍	❍	●	●

c. 38 + 7 = 45

●	●	●	●	●					
❍	❍	❍	❍	❍	❍	❍	❍	❍	❍
❍	❍	❍	❍	❍	❍	❍	❍	❍	❍
❍	❍	❍	❍	❍	❍	❍	❍	❍	❍
❍	❍	❍	❍	❍	❍	❍	❍	●	●

2\. Discussion may vary, but the statements on the boards could be written as:

a. 9 + 6 = 10 + 5
= 15

b. 7 + 6 = 10 + 3
= 13

c. 9 + 8 = 10 + 7
= 17

d. 8 + 7 = 10 + 5
= 15

3\. **a.** 20 + 5 **b.** 30 + 5 **c.** 60 + 5
d. 90 + 5 **e.** 30 + 3 **f.** 50 + 3
g. 80 + 3 **h.** 90 + 3

4\. a. 19 + 8 = 20 + 7
= 27

b. 59 + 8 = 60 + 7
= 67

c. 109 + 8 = 110 + 7
= 117

d. 1 009 + 8 = 1 010 + 7
= 1 017

e. 47 + 6 = 50 + 3
= 53

f. 67 + 6 = 70 + 3
= 73

g. 237 + 6 = 240 + 3
= 243

h. 997 + 6 = 1 000 + 3
= 1 003

Pages 12–13: A Matter of Factor

Activity

None. The only factors of 17 are 17 and 1. Stacey has just covered 17, and the 1 was covered earlier in the game.

Game

A game using factors

Page 14: Back through 10

Activity

1\. a. 7
Working: 15 – 8 = (15 – 5) – 3
= 10 – 3
= 7

b. 7
Working: 16 – 9 = (16 – 6) – 3
= 10 – 3
= 7

c. 17
Working: 24 – 7 = (24 – 4) – 3
= 20 – 3
= 17

d. 89
Working: 98 – 9 = (98 – 8) – 1
= 90 – 1
= 89

2. a. $16 - 7 = (16 - 6) - 1$
$= 10 - 1$
$= 9$

b. $12 - 5 = (12 - 2) - 3$
$= 10 - 3$
$= 7$

c. $13 - 8 = (13 - 3) - 5$
$= 10 - 5$
$= 5$

d. $26 - 8 = (26 - 6) - 2$
$= 20 - 2$
$= 18$

e. $22 - 5 = (22 - 2) - 3$
$= 20 - 3$
$= 17$

f. $27 - 9 = (27 - 7) - 2$
$= 20 - 2$
$= 18$

g. $82 - 7 = (82 - 2) - 5$
$= 80 - 5$
$= 75$

h. $143 - 8 = (143 - 3) - 5$
$= 140 - 5$
$= 135$

i. $904 - 6 = (904 - 4) - 2$
$= 900 - 2$
$= 898$

j. $1\,746 - 8 = (1\,746 - 6) - 2$
$= 1\,740 - 2$
$= 1\,738$

k. $1\,000\,014 - 7 = (1\,000\,014 - 4) - 3$
$= 1\,000\,010 - 3$
$= 1\,000\,007$

l. $5\,961 - 8 = (5\,961 - 1) - 7$
$= 5\,960 - 7$
$= 5\,953$

3. Answers will vary. You could:
- count back (not a very efficient method)
- split into 5 and a bit more, for example, for $13 - 8$: $(13 - 5) - 3$
- use relationship to addition, for example: $12 - 5 = 5 + 7 - 5$
- subtract 10 and then adjust, for example: $143 - 8 = 143 - 10 + 2$
- round, then subtract, for example: $82 - 7 = 80 - 5$.

Page 15: How Many Factors?

Activity

1: 1
2: 1, 2
3: 1, 3
4: 1, 2, 4
5: 1, 5
6: 1, 2, 3, 6
7: 1, 7
8: 1, 2, 4, 8
9: 1, 3, 9
10: 1, 2, 5, 10
11: 1, 11
12: 1, 2, 3, 4, 6, 12
13: 1, 13
14: 1, 2, 7, 14
15: 1, 3, 5, 15
16: 1, 2, 4, 8, 16
17: 1, 17
18: 1, 2, 3, 6, 9, 18
19: 1, 19
20: 1, 2, 4, 5, 10, 20
21: 1, 3, 7, 21
22: 1, 2, 11, 22
23: 1, 23
24: 1, 2, 3, 4, 6, 8, 12, 24
25: 1, 5, 25
26: 1, 2, 13, 26
27: 1, 3, 9, 27
28: 1, 2, 4, 7, 14, 28
29: 1, 29
30: 1, 2, 3, 5, 6, 10, 15, 30
31: 1, 31
32: 1, 2, 4, 8, 16, 32
33: 1, 3, 11, 33
34: 1, 2, 17, 34
35: 1, 5, 7, 35
36: 1, 2, 3, 4, 6, 9, 12, 18, 36

Investigation

Prime numbers have only two factors, themselves and 1. Prime numbers between 1 and 36 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31.

3 factors: 4, 9, 25 (prime numbers squared, plus the number itself and 1)
4 factors: 6, 8, 10, 14, 15, 21, 22, 26, 27, 33, 34, 35 (one rectangular array, plus the number itself and 1)
5 factors: 16 (fourth power of the prime number 2)
6 factors: 12, 18, 20, 28, 32 (two rectangular arrays, plus the number itself and 1)
8 factors: 24, 30 (three rectangular arrays, plus the number itself and 1)
9 factors: 36 (square number)

Page 16: To and Fro

Game

A game of multiplication and division

Page 17: Mind Boggling

Activity

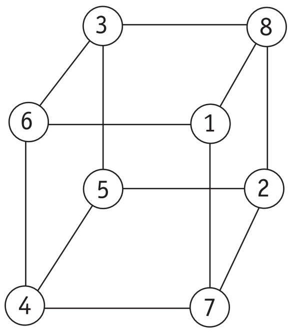
Solutions and possible equations include:

1. 9 days: 72 ÷ 8 = 9
2. 25 pūkeko: 54 – 29 = 25
3. 16 pots of soup: 64 ÷ 4 = 16
4. 7 cousins: (96 ÷ 12) – 1 = 7
5. 48 cherries: 36 ÷ 3 x 4 = 48
6. 80 goals: 56 ÷ 7 x 10 = 80
7. 24 chapters: 8 x 3 = 24
8. Answers will vary.

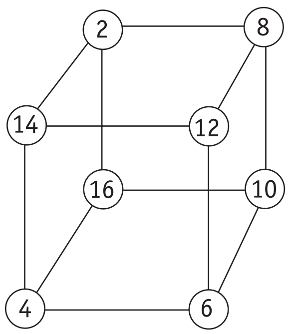
Page 18: Face Totals

Activity

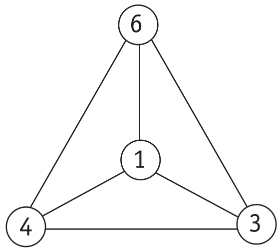
1. A possible solution is:

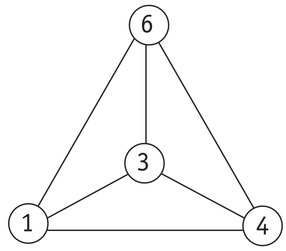


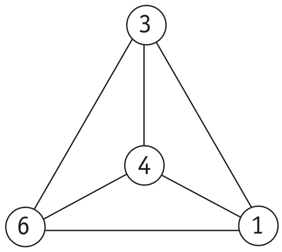
2. A possible solution is:

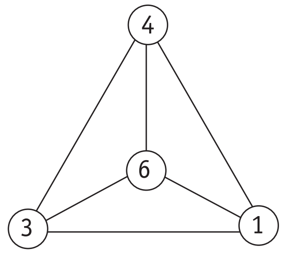


3. a. Four of the possible solutions are:







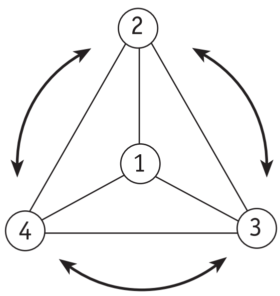


(The only digits that work are 1, 3, 4, and 6. Their positions can be varied.)

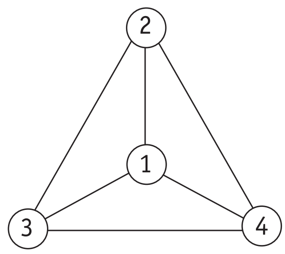
b. Solutions for the triangle puzzle with face totals of 8, 10, 11, and 13 will all have the same combinations in common, regardless of the position of digits. These are: 1–3–4 (8), 1–3–6 (10), 1–4–6 (11), 3–4–6 (13).
The combinations for the 6, 7, 8, and 9 face totals will be 1–2–3 (6), 1–2–4 (7), 1–3–4 (8), and 2–3–4 (9).

For each triangle puzzle, there are 24 solutions. Each of the four digits can be used as the centre digit with the other three digits swapping places, using rotation and reflection. For example:

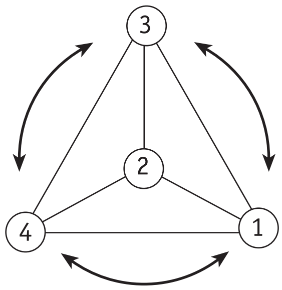
rotation:



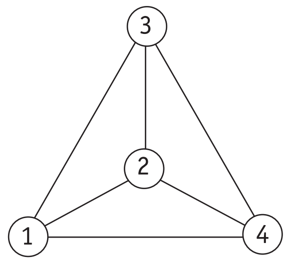
reflection:



rotation:



reflection:



Page 19: Staircase Capers

Activity

1. In each rectangle, pairs of numbers in opposite corners of the rectangle are equal.
2. In each cross, the sum of the two horizontal numbers added is 2 less than the sum of the two vertical numbers added.
3. The sum of the three corner numbers is always 2 more than the sum of the other three numbers in the staircase.

Investigation

Answers will vary and will include different shapes.
For example:

- triangles, such as

```
        1
     2  3  4
  5  6  7  8  9
```

- alphabet letters, for example, Z
- multiplying opposite corners
- comparing a 3 x 3 set of numbers on one of the staircases to 3 x 3 grids on a calendar.

Page 20: Making Equations

Activity

1. Teacher to check. (Examples of equations using four, five, and six digit cards are given on the page.)

2.–3. Teacher to check

Page 21: Mystery Decimals

Activity

1. a. $\frac{\boxed{3}}{\boxed{6}} = 0.5$

   b. $\frac{\boxed{4}}{\boxed{8}} = 0.5$

   c. $\frac{\boxed{2}}{\boxed{8}} = 0.25$

   d. $\frac{\boxed{3}}{\boxed{4}} = 0.75$

   e. $\frac{\boxed{1}}{\boxed{8}} = 0.125$

   f. $\frac{\boxed{9}}{\boxed{4}} = 2.25$

2. Answers will vary.

Pages 22–23: Primates

Activity

It is a draw. Both have a sum of 17 uncovered.

Game

A game using prime factors

Page 24: Decimal Spotting

Game

A game using decimal fractions

Figure It Out

Basic Facts

Teachers' Notes

Overview: Basic Facts

Title	Content	Page in students' book	Page in teachers' notes
Bunches	Renaming addition sums	1	12
Trying Times	Exploring two-digit multiplication	2	12
Eleventh Heaven	Exploring two-digit multiplication	3	13
Diamond Dazzle	Practising basic addition facts	4–5	14
Memory Testing	Applying operations strategies	6–7	14
Napier's Bones	Applying multiplication strategies	8–9	15
Matrix	Finding factors	10	16
Magical Tens	Applying addition strategies	11	17
A Matter of Factor	Finding factors of numbers	12–13	18
Back through 10	Applying subtraction strategies	14	18
How Many Factors?	Investigating factors of numbers	15	19
To and Fro	Applying multiplication and division facts	16	20
Mind Boggling	Using inverse operations	17	21
Face Totals	Applying basic addition facts	18	21
Staircase Capers	Investigating number patterns	19	22
Making Equations	Making equations	20	23
Mystery Decimals	Converting fractions to decimals	21	23
Primates	Finding prime factors	22–23	24
Decimal Spotting	Converting fractions to decimals	24	24

About Basic Facts

Mathematics in the New Zealand Curriculum, on page 210, defines the term Basic Facts as "the addition facts up to 9 + 9 and the multiplication facts up to 10 x 10". This also includes the related subtraction and division facts.

Students need to be able to retrieve and use basic facts so that:

- they can calculate accurately, confidently, and efficiently
- they can estimate
- they can use calculators sensibly.

Developing understanding of the basic facts, recalling them, and using them will help students to accomplish the broader objectives of Number and all the other strands.

Relevant achievement objectives from the Number strand that students need to maintain through all levels include:

- recall the basic addition and subtraction facts (level 2)
- mentally perform calculations involving addition and subtraction (level 2)
- demonstrate the ability to use the multiplication facts (level 2)
- make sensible estimates and check the reasonableness of answers (levels 1–4)

This is the third Basic Facts booklet in the Figure It Out series. By levels 3–4, most students should have a sound understanding of basic facts operations. They will have begun to commit basic facts to memory by observing patterns and taking part in other enjoyable, repetitive activities and games.

Students should continue with learning activities that improve recall, but they should not be subjected to prolonged, repetitive practice of already learnt facts that lack challenge and interest. Their learning should be based on personal errors that they record in a notebook or on a card. They can focus on manageable amounts on this list during the day or at home.

It is important that students do not just learn facts in isolation. They need to realise that if they can recall one fact, they can also retrieve many more, and that there are patterns and relationships among facts that can be useful. For example, with the families of facts:
knowing that 2 + 5 = 7
means that students can also work out that 5 + 2 = 7 and 7 – 2 = 5 and 7 – 5 = 2;
knowing that 8 x 9 = 72
means that students can also work out that 9 x 8 = 72 and 72 ÷ 8 = 9 and 72 ÷ 9 = 8.

Make a point of often asking the students to reverse the process and to explain how they can get back to where they started.

Other useful thinking strategies include:

- counting forwards or backwards from the higher number. For example:
  9 + 4: by starting at 9 and counting on in ones;
  14 – 5: by starting at 14 and counting back in ones.
- using mental images. For example:
  using a tens board or a number line.
- using the commutative rule for addition and multiplication. For example:
  I don't know 3 x 8, but I remember 8 x 3;
  I can get 4 + 7 if I think of 7 + 4.
- relating to a known fact. For example:
  9 + 5: I know 10 + 5 = 15, so it's 1 less (14);
  15 – 8: I know 15 – 10, so it's 2 more (7).
- using doubles or near doubles. For example:
  7 + 6 is 1 less than 7 + 7;
  17 is 1 more than 8 + 8, so 17 – 8 = 9;
  6 x 12 is double 6 x 6, 12 x 11 is (6 x 11) x 2

- bridging through 10. For example:
  12 x 11: first multiply by 10 and then add the extra number (120 + 12) or add the 2 times table facts to the 10 times table facts (110 + 22);
  7 + 6: 7 + 3 = 10 and 3 more is 13.
- skip counting forwards or backwards. For example:
  4 x 3: 3, 6, 9, 12
  20 ÷ 5: 20, 15, 10, 5
- splitting into useful parts. For example:
  8 x 5: 5 x 5 = 25 and 3 x 5 = 15. 25 + 15 = 40

The mental calculation activities in this booklet will also encourage students to use these types of mental strategies. Encourage them to think out and share the mental strategies they use in memory activities, games, and other applications of part-whole numbers. They will soon begin to use basic facts with greater confidence when doing mental calculations and estimations well beyond 10.

The games and activities in this booklet will also provide opportunities for students to practise and apply their basic facts. They will be able to maintain their knowledge and use of factors, multiples, prime numbers, and decimals and apply basic facts to word problems.

You will be able to extend some activities into investigations. This will encourage the students to use mathematical processes by posing questions, recording systematic lists, looking for patterns, keeping track of solutions, interpreting, and communicating results. You can therefore choose to use some of the activities either as brief, consolidating lesson starters or as more extended lessons.

Page 1: Bunches

Game

This game asks students to practise mental addition of two or more one-digit numbers. Repeated practice using games such as this will help to improve their ability to compute mentally and give them practice with digits that add up to sums from 8 to 18.

To introduce the students to the game, you could ask them what different ways they can make up a total of 11, for example, using three or more addends.

Players will be able to check their classmate's totals more easily if they use transparent counters. During the game, you could ask the students to give other possible addends to make up the thrown sum or to justify the use of a particular combination.

At the conclusion of the games, the students could consider which strategies help cover the most numbers. For example, does it help to cover the high numbers first?

As a variation, the winner could be the player with the lowest sum found by adding the uncovered digits.

You could extend the game by using multiples of 10 on the dice and game board. Another way is to enlarge the size of the triangle game board by extending the rows but keeping a similar variation of numbers as the original pattern.

Page 2: Trying Times

Activity

This activity may be quite challenging initially, especially for those students who are used to using vertical written algorithms, which they may try to visualise. The students could work in pairs to experiment with the different methods shown.

All these methods build students' skills in mental calculation and help to develop their number sense. Encourage the students to split the factors into small numbers or to use other known factors (5, 10, and 20) as a bridge. Concrete models using place value blocks (longs and little cubes) patterned in rows of 15 may help some students to understand the value of splitting the rows of 15 into groups of 10 and groups of 5 to work out the answer. You could work through an example with the class, using all the methods shown, to demonstrate that these methods are more efficient than the written algorithm.

There is no clear answer to question **2.** You need to value each student's opinion and encourage them to come up with other ways of thinking as well. For question **3**, as well as the methods shown on the page or suggested above, possible answers include:

- 18 x 15: half of 18 is 9, and double 15 is 30. 30 x 9 = 270
- 9 x 15 is 10 x 15 less 15. 150 – 15 = 135
- Counting up from a known multiple, for example:
  I know that 15 x 2 = 30, so 15 x 8 is 30, 60, 90, 120, or I know that 15 x 4 = 60, so 15 x 8 will be double that.

See also pages 15–18 in *Number*, Figure It Out, Level 3 and their accompanying teachers' notes for more examples of mental calculation strategies.

To develop fluency, students need to have opportunities to explain their methods and further chances to practise the more common strategies.

Page 3: Eleventh Heaven

Activity

If students can identify the pattern in multiplying a two-digit number by 11, they should be able to use the rule to find the product of other two-digit numbers multiplied by 11. The pattern is explained in Toby's example. The students should first consider some examples of their own, using their calculators, and then discuss findings. Finally, they should make a conjecture based on the results.

The students can use a spreadsheet to see all the two-digit numbers multiplied by 11:
In cell A1, enter **10**.
In cell A2, enter **=A1+1**.
Fill down to cell A99.

In cell B1, enter **11**.
Fill down to cell B99.

In cell C1, enter **=A1xB1**.
Fill down to cell C99.

The students will then be able to see the pattern. Question **3b** encourages the students to extend the pattern for two-digit numbers that add to 10 or more.

You could also discuss with them the strategy of multiplying a factor by 10 and then adding that factor. For example, 12 x 11 = 12 x 10 + 12. (As an extension, the students could apply the multiply-by-11 system to factors with more than two or three digits. For example, 11 x 5 218 is 10 x 5 218 + 5 218 = 52 180 + 5 218.)

After this, encourage the students to discuss and share ways of mentally calculating other two-digit and "teen times" tables. They will be able to multiply using closely related facts. Encourage them to split factors into suitable part-whole combinations and to share their results.
For example:

- 13 x 12 = (13 x 10) + (13 x 2)
  = 130 + 26
- 19 x 7= (20 x 7) – 7
  = 140 – 7
- Work out the 12 times table by adding the two times table facts to the 10 times table facts.
- For the even teen times tables, halve the teen number and multiply the product by 2.
  For example:
  14 x 4 = (7 x 4) x 2
  = 28 x 2
  = 56

Practising with a classmate, with one student checking on the calculator and creating questions, should help improve confidence over time.

As a variation, see if the students can predict the results of:

1 x 1 =

11 x 11 =

111 x 111 =

1 111 x 1 111 =

11 111 x 11 111 =

(This is also discussed in "Investigating 11" in the *Connected 2 2000* teachers' notes, page 19.)

Pages 4-5: Diamond Dazzle

Game

This game gives students practice in adding one-digit number facts to two-digit numbers.

The students record a cumulative total as they go. Encourage them to do this mentally before recording the new total and suggest that other players also confirm the score. They will need to avoid being jumped while, at the same time, trying to score the highest points.

As a variation, you could change the target number to 150 and use numbers between 13 and 19 in the circles.

Pages 6-7: Memory Testing

Activity

The calculator is an excellent tool to help students to check mental calculations and hence to improve number sense. The advantages of using a calculator in this activity include the student being able to visualise the arithmetic leading to the unseen cumulative total. A calculator also provides quick feedback to the student's responses. Many calculator users do not make good use of the memory functions on their calculator, so here is a good opportunity for the students to learn how to use this function.

The following is a practical example of the use of the memory function. The first step is to clear the memory.
Two bottles of milk at $2.95 each: 2 x 2.95 = M+
Three loaves of bread at $1.45 each: 3 x 1.45 = M+
Two cans of baked beans at $1.15 each: 2 x 1.15 = M+
Total for list: MRC

Learning to use the memory function may take time and practice. Different brands of calculators may have slightly different memory keys or functions, so the students may need to experiment with their calculators. It may be worthwhile to get the students to record the steps they followed when they used their calculators.

The students need to make sure that they clear the memory before beginning a new game.

A useful teaching strategy is to use an overhead projector calculator to play some games with the class or group and also to ask the students to play a game in pairs on the overhead projector as a demonstration. (Make sure you do not leave the calculator on the projector when you are not using it.) For example, ask the students to raise their hands when they think the total in the memory has reached 100. When over half the class thinks this has happened, press MRC.

Game

This game provides students with an opportunity to practise mental addition and subtraction. They will need to take care not to push the buttons too fast.

The students can repeat the game, in its basic or more challenging forms, several times as a warm-up activity or in spare classroom moments.

In order to play the more challenging versions of the game suggested at the bottom of the page, the students will need mental strategies to add and subtract two-digit numbers. Reflecting and discussing are good ways of sharing efficient strategies. Ask: "What did you say in your head when you added 26 each time?"

Possibilities include: adding 20 and then adding 6, or adding 30 and then subtracting 4.

As a variation, play target games using only single-digit numbers. For example:
Two players choose a target number, for example, 50. Then they take turns to enter alternate single-digit numbers (zero is not allowed), followed by M+. When a player thinks that the target will be hit in their next turn, they enter their number and M+ and then MRC to check whether they are right. If they are right, they win the game. Target numbers can be varied and the difficulty level altered quite easily.

The students can also play this game using subtraction, this time beginning from the higher number to reach the target, zero.

As an extension, challenge the students to make up a list of items for a party. For example, six bottles of coke at $2.25 each, five bags of chips at $1.75 each, seven chocolate bars at 90 cents each, and so on. The students estimate the sum mentally by rounding and then calculate using the calculator memory.

Pages 8–9: Napier's Bones

Activity

This activity uses an interesting historical context to help students to understand place value ideas as they explore multiplication.

For question **2**, the students will find that information about John Napier is readily available in reference books and on the Internet. He lived from 1550 to 1617.

Napier's invention made multiplication much easier for the people of his time. Patterns of numbers were carved on rods or sticks, but originally the facts were carved on bones and therefore became known as Napier's Bones. Napier has also been credited with the invention of the decimal point and tables of logarithms.

Ask the students to investigate what else John Napier invented.

When the students are looking for patterns to answer question **1**, encourage them to look at the number at the top and scan down each strip. What do they notice?

How are the two-digit numbers written? (They are split, with the tens digit above the diagonal and the ones digit below the line.) Encourage the students to look:

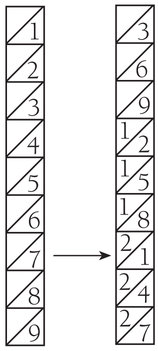
- at the vertical columns, rows, and diagonals
- at just the ones place and then just the tens place, noting their position in relation to the diagonal lines
- for a symmetrical pattern of numbers or repeating patterns of numbers, and so on.

In question **3**, the students use the strips to solve simple multiplication problems. When they investigated the patterns in the strips (question **1**), they probably noticed that the multiples of a number go down the strip with that number at the top. For example, reading vertically down the strip with 3 at the top, you have the multiples of 3, that is, 3, 6, 9, 12, 15, and so on. So to find 4 x 3, they go down to the fourth square on the 3 strip and they get the answer, 12.

Instead of counting down the strip, the students could use the 1–9 strip as a quick way of finding the other factor.

For example, to find 7 x 3, using the 3 strip:

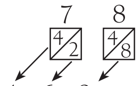
1–9 strip   3 strip



7 x 3 = 21

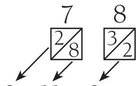
As an extension after the students have finished the problems on the page, they could use the strips to find the value of larger multiplication problems. For example, for 78 x 46:

Row 6:



78 x 6 = 4 6 8

Row 4:



78 x 40 = 2 11 2
3 1 2 0

Note: Calculate 78 x 40 as 78 x 4 on the strip and then put a zero in the ones place of your answer and move all the digits one place to the left.

78 x 46 = 468 + 3 120
= 3 588

The students could then try others, such as 48 x 93 = $\square$, 80 x 32 = $\square$, 69 x 96 = $\square$, 420 x 33 = $\square$, and 1 563 x 47 = $\square$.

Page 10: Matrix

This game gives students practice in quickly identifying common multiples of a single-digit number.

The students will need to have good recall of basic multiplication facts. They can find the common multiples, and therefore find the best position for the thrown factor, by mentally dividing each number in a column or row to see if there is a remainder.

Activity One

You could introduce the students to this page by asking what numbers on the dice are factors of 48 and 54.

This activity will help the students to understand how to play the game that follows. Review the scoring system carefully.

Discuss other possible outcomes. For example, "What might have been the outcome if Gina had put the 7 in the box at the top of the right-hand column?"

Game

Whether or not the students beat Gina's score depends on what they throw on the dice.
It is possible to beat Gina's score. The highest possible score is 15.

For example:

Top boxes 4 6 9
Side boxes 9 9 4
Points: 2 + 3 + 3 + 2 + 2 + 3 = 15

Activity Two

The students can play the game with these two grids several times. Ask them: "Where is the best place to put the first factor?"

When they have placed the first factor, they can work systematically to work out the highest possible score. For example, for the box above the left column in **a**, they need to find a number on the dice that is a factor of as many of the numbers in that column (18, 25, and 40) as possible:
6 x 3 = 18
9 x 2 = 18

5 x 5 = 25

4 x 10 = 40
5 x 8 = 40

None of the numbers on the dice is a factor of all three numbers in the column. Five is a factor of two of the numbers, 25 and 40. So the highest score that they can get for that box is two points if they put 5 in it.

As an extension, the students could try to design a matrix that will produce a result of more than 15 points or extend the size of the grid to a 4 x 4 matrix for further games.

To create a matrix that produces a score of more than 15 points means that the numbers used in the grid must have more common factors. In grid **b**, the bottom left-hand corner has 41. But 41 is a prime number, so no score can be generated from this entry in the grid. An easy way to improve the possible top score for this grid would be to replace 41 with a number that has a factor shown on the dice. A profitable replacement would be 48 because this has 4, 6, and 8 as factors. On the other hand, 84 is a bonus since it has 4, 6, 7, and 8 as factors!

The highest possible score that can be gained from a well-planned 3 x 3 grid is 18. You can achieve this by being able to score three points on every row and every column of the matrix.

Page 11: Magical Tens

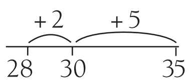
This activity will help students to bridge through 10 or a multiple of 10 as a mental strategy for addition. Initially, a Make Tens board is used as a model. Students are then encouraged to visualise the process, and finally, to work with numbers in their mind only.

Using the Make Tens board as a model for the calculation 8 + 7, it is useful to see the 7 as 5 + 2, taking the 8 up to 10, and then adding the remaining 5. To help the students split the numbers, you could ask questions like: "What do you need to add to 87 to make the next multiple of 10?" or "What two parts will you break the 6 into?" Using these models and activities, the students should see how useful part-whole thinking is.

The students can use a Make Tens board to check their answers to question **3**.

Another good model for the same strategy is a number line. Encourage the students to visualise from this model using the same process.
For example:

28 + 7



Magical Tens is complemented by the activity on page **14** of the student booklet, which uses subtraction.

Pages 12–13: A Matter of Factor

Activity

This activity will help students to distinguish between good and bad strategies for playing the games that follow, and it encourages them to look again at what composite numbers and prime numbers are.

Review with the students how to name a set of factors for a number. For example, the factors for 10 are 5, 2, 10, and 1. Discuss why Stacey thinks that 17 will not give Laird a lot of points.

Game

In this game, students need to make mental estimations, think ahead, and identify factors and primes. High prime numbers give away the fewest points.

The game board is graded at three levels of challenge. Increase the challenge gradually. If the students are faced with too many numbers early on, they can easily get confused and begin to cover numbers randomly rather than thinking strategically.

Observe carefully and encourage the students to reflect on their choices if these are not particularly sound. You may need to remind the students that all uncovered factors must be covered and that some factors may already be covered and therefore will not be available.

Page 14: Back through 10

In this activity, students will gain experience at bridging through 10 or near multiples of 10 as a mental strategy for subtraction. The activity encourages students to visualise the process and use part-whole thinking, using the same principle as in the activity on page **11** of the student booklet but this time visualising a number line.

You may wish to discuss with the students how to use brackets to separate the two steps when they are recording their answers.

Ask questions to clarify the students' thinking as they move from the visual number line model to the mental operation. For example, "What number do you take away to reach the nearest multiple of 10?"

An alternative model, the Make Tens board, may also be used in the same way as it is on page **11**.

For example:
16 – 7 = (16 – 6) – 1

1	2	3	4	5	6	7	8	9	10
•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•				

1	2	3	4	5	6	7	8	9	10
•	•	•	•	•	•	•	•	•	•

1	2	3	4	5	6	7	8	9	10
•	•	•	•	•	•	•	•	•	

Alternative mental strategies for question **3** are suggested in the Answers section.

Page 15: How Many Factors?

Activity

The students will probably have worked with sets of factors earlier in the students' book, for example, on pages **12–13**. Encourage them to work systematically to make sure that they find all the factors. They can do this by trying to divide their whole number by each number from 1 up to see whether it divides the whole number evenly and therefore is a factor. For example, to find the factors of 16:

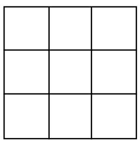
$16 \div 1 = 16$
$16 \div 2 = 8$
$16 \div 3 = 5$ remainder 1
$16 \div 4 = 4$
$16 \div 5 = 3$ remainder 1
$16 \div 6 = 2$ remainder 4
$16 \div 7 = 2$ remainder 2
$16 \div 8 = 2$

(The students should be able to do these calculations mentally.) So 1, 2, 4, and 8 are factors of 16. Students only need to try the numbers up to half the number for which they are finding the factors. Numbers greater than half will not divide evenly.

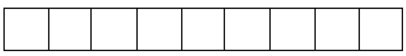
Investigation

In this investigation, the students will be working with numbers that they are familiar with. They will organise data in a systematic way, highlight the significant features, and then see what insights emerge. You can also use the investigation to reinforce the terms prime numbers, composite numbers, rectangular numbers, and square numbers.

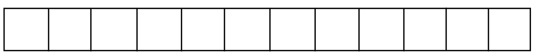
A prime number has only two factors because it is divisible only by 1 and itself. A composite number has more than two factors and can be represented by square or rectangular patterns. For example, 9 has three factors, 9, 3, and 1. It is a square number and a rectangular number. It can be represented as:

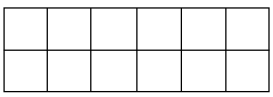


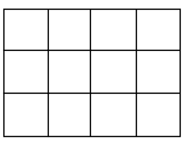
or



12 is a rectangular number. Its factors are 1, 2, 3, 4, 6, and 12. It can be represented as:







As the students begin this investigation, encourage them to explore square and rectangular patterns with concrete materials (for example, linked cubes) or diagrams (for example, squared paper).

They could put the results in a table and see what they can find out about each of the numbers with two factors, three factors, and so on.

Number	Number of factors	Factors
2	2	1 2
3	2	1 * 3
4	3	1 2 * 4
5	2	1 * * * 5
6	4	1 2 3 * * 6
7	2	1 * * * * * 7
8	4	1 2 * 4 * * * 8
⋮		

Ideas from the students will vary. Encourage them to give appropriate explanations and to record generalisations. In general, square numbers seem to have an odd number of factors, while numbers with four, six, and eight factors are rectangular. Prime numbers have only one rectangular array. Numbers with four factors have two types of rectangular array: 8 has 1 x 8 and 2 x 4. Numbers with six factors have three rectangular arrays: 20 has 20 x 1, 10 x 2, and 5 x 4. Numbers based on a multiple of 12, such as 24 and 36, will have more than six factors because 12 itself already has six factors.

Encourage the students to extend the investigation further. Further questions may be posed by you or by the students themselves. For example:
"Find a number with seven factors (64 is one)."
"How many of the first 100 numbers are prime?"
"6 = 3 + 2 + 1. This is a *perfect* number because it is equal to the sum of its other factors. Find the next perfect number." (28)
"Investigate the product of all the factors of a number. (For example, for the factors of 12:
$12 \times 6 \times 2 \times 4 \times 3 \times 1 = 1\,728$
$= 12^3$)."

Page 16: To and Fro

Game

This game gives the students practice in recognising the factors of numbers.

To help the students play strategically, you could ask questions such as: "Which is the best position to choose?"

You may want to discuss whether the students are allowed to block the path of the other player. (It is not forbidden in the rules and is part of the strategy in many games.)

As a variation, some students might enjoy the challenge of changing the numbers on the dice and then changing the numbers on the game board to multiples of the numbers that are now on the dice.

Page 17: Mind Boggling

Activity

This activity asks students to solve word problems involving numbers in real-life contexts and, in particular, to choose and use appropriate operations to solve a problem. Working in pairs will be helpful with these activities.

Read the first problem to the class, discuss it, and clarify what is happening in this situation. Discuss what numbers the students will need to use and decide which operation to use.

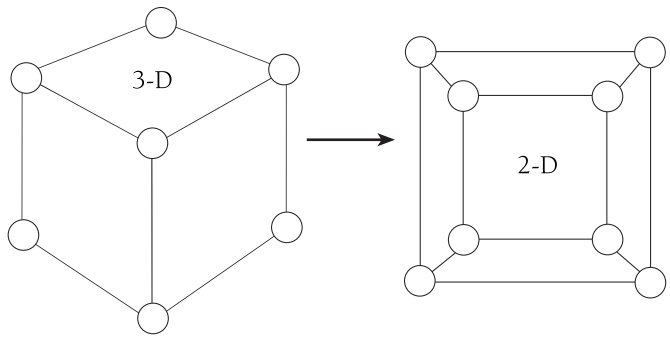
Make sure that the students check that they have got the right answer. For question **1**, they could check by using the inverse operation 8 x 9 = 72 to verify their answer.

Questions **5–7** are more challenging because they involve fractions. You could ask the students how they would calculate fractions of certain amounts, for example, $^2/_5$ of 10. Get them to demonstrate this with materials and use the same working for questions **5–7**. In question **5**, working backwards: $^3/_4$ of the amount of cherries is 36, so $^1/_4$ will be 36 ÷ 3 = 12, and $^4/_4$ can be found with 12 x 4 = 48.

Discuss question **8** initially by writing up a calculation such as 25 – 11 = 14 and asking the students to devise story problems that may be relevant to this calculation.

Page 18: Face Totals

The three-dimensional (3-D) nature of these puzzles makes them more challenging. The students will enjoy working collaboratively on them. Some students may find the puzzles easier if they write numbers on small pieces of card and experiment by sticking the numbers onto the corners of a cube. Other students may like to work with a 2-D sketch of the cubes, such as the one below, so that they can easily see the totals of all six faces.



The students can use trial and improvement to solve the puzzles, but their trials shouldn't be completely random. For example, the bottom face of the cube in question **1** already has numbers that add up to 6, so the numbers that go on the two blank circles must add up to 12. When the students have filled in these circles, they will have two numbers on each of the side faces, so they can complete the side faces next.

You could ask them to find out whether there are other solutions as well as the first one they find. Rotating and reflecting the number combinations will generate additional solutions for the cubes and tetrahedrons. See the Answers for the different number combinations.

In these puzzles, the students will explore the possible numbers that add up to a specific total and meet the conditions of the shapes provided. You will need to decide whether to allow time for the students to find alternative solutions.

With these puzzles, it should be possible for most students to successfully find a solution by using trial and improvement to locate numbers that form the appropriate relationships.

Page 19: Staircase Capers

Activity

In this activity, the students are looking for patterns in numbers. They will need to try several squares, crosses, or staircases before they can be sure that they have found the correct pattern. Students could use a calculator to speed up their operations, but discourage them from using a calculator for simple operations.

Investigation

As the students investigate different patterns, you could ask the following questions to extend their thinking:
"Will it always work?"
"Would it work on a bigger shape?"
"What other patterns can you make?"
"Can you find out anything about the numbers on diagonal lines?"
"What will happen if you multiply the numbers?"
"What happens for vertical and horizontal lines?"
"Why does it work?"

For example:

2	3	6 + 3 = 9
6	7	2 + 7 = 9

10	11	18 + 11 = 29
18	19	10 + 19 = 29

Students may also enjoy investigating patterns of numbers in other situations, for example, in calendar months or hundreds squares. Do the same patterns work?

Here is one pattern that compares 3 x 3 grids on the staircases and on a calendar.
Choose a 3 x 3 set of numbers from one of the staircases. For example:

6	7	8
12	13	14
20	21	22

Double the middle number (13 x 2 = 26). Now take any two numbers on the outside of the grid that are directly opposite each other (vertically, horizontally, or diagonally), for example, 7 and 21. Adding them gives 7 + 21 = 28, which is two more than 26. The same is true for 6 and 22, which give 28 when added. Create a new 3 x 3 grid from the staircase and check if the rule holds.

If you do this with a calendar, it gets even more interesting. For example, a grid from the calendar for May 2001 will look like this:

8	9	10
15	16	17
22	23	24

The middle number is 16, and 16 x 2 = 32. Pick a pair of opposite numbers, say 15 and 17.
15 + 17 = 32. Another pair is 10 + 22 = 32.

Use a calendar and check if this rule holds true for any 3 x 3 grid that you pick.

Page 20: Making Equations

Activity

This activity will increase the students' confidence and competence with basic facts as they create equations and think about relationships instead of completing repetitious closed exercises.

Check that the students' written expressions follow the standard order of operations (BEMA): Brackets, Exponents, Multiplication and Division (in the order they appear), Addition and Subtraction (in the order they appear).

You could consider using variations of this idea over several sessions or as part of your daily maths routine. You would need to decide how many and which digits the students can use and which operations they can use.

Showcase a few interesting examples by getting the students to write their equation on the whiteboard. For example, if a student wrote this equation: $12 \div 3 + 4 = 8$, you could point to the $12 \div 3$ and say: "Julie wanted another 4, so she did this".

Other variations include:

- Ask the students to find as many expressions as possible for a given number.
- Put out the cards in a certain order and ask the students to insert the operation signs to make a correct equation. For example, with the digit cards 3, 5, 6, 2, and 1 you could get $3 \times 5 + 6 = 21$. The students may enjoy creating these puzzles for others to solve.
- Have the students play this game in pairs. One player thinks of an equation using four to six cards. They tell the other player which cards are to the left of the equals sign and which cards are to the right. The other player has to make an equation using the same cards. A point is scored if both players agree that the equation is correct.
- Use the activity daily, using that day's date as the target number.

As an extension, suggest that the students make all the numbers 1–100, using any operations and four 4s, for example: $4 \div 4 + 4 - 4 = 1$.

Page 21: Mystery Decimals

Activity

This activity gives students practice at relating common fractions to decimals and vice versa. Students at this level should have experienced writing tenths and hundredths as a decimal and writing a decimal for any fraction expressed in tenths and hundredths. This will not necessarily mean that they can express other fractions as decimals.

Some sense of the relative size of fractions will be helpful in these situations, such as knowing that $\frac{3}{5}$ is a little more than half or that $\frac{2}{8}$ is the same as $\frac{1}{4}$. Some examples are easily changed to a decimal, for example, $\frac{4}{5} = \frac{8}{10}$, which is equal to 0.8. Alternatively, the students should think of a fraction as a division equation, for example, $\frac{4}{5}$ means $4 \div 5$ or $5\overline{)4.0}$ with quotient 0.8

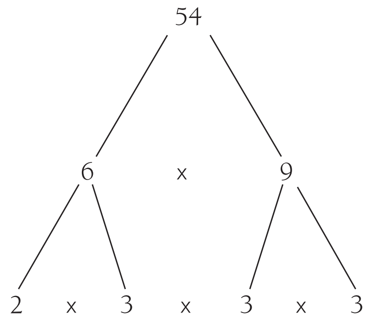
When the students are making up their own problems, you may want to limit the denominators that they use. They could use calculators to explore possibilities for creating equations.

As a variation, the students could use the inequation symbols > < in these statements, or you could give them some examples for which they have to supply the correct operator, < or >. For example, $\frac{3}{4}\ \square\ \frac{5}{8}$, $\frac{2}{3}\ \square\ \frac{3}{5}$, and so on.

Pages 22–23: Primates

Activity

You may want to talk about the concept of prime factors, which can be found by using a factor tree to split composite numbers into their factors until only prime numbers remain. The final numbers are the prime factors of the composite number that you started with.
For example:



Prime factors of 54 = 2 x 3 x 3 x 3

Game

The game gives students practice in deriving prime factors and multiplying together more than two factors to give a product.

Some students may enjoy investigating whether the game board reflects the likelihood of certain numbers occurring.

Page 24: Decimal Spotting

Game

To play this game, students need to be able to compare and order fractions according to their decimal value. The game will give them practice in converting fractions to decimals and vice versa.

Get the students to play several rounds in pairs so that they see the relationship between fractions and decimal fraction positions. If they are unsure where their fraction would go on the number line, encourage them to relate it to a known fraction. For example, $\frac{1}{9}$ is a little bit more than $\frac{1}{10}$, and $\frac{1}{10}$ = 0.1. (If they use a calculator, they will see that $\frac{1}{10}$ = 1 ÷ 10, which is equal to 0.1.) Also encourage the students to think about a position that would block off their opponent.

There are several equivalent fractions that the students could make from the numbers in the box, and this reduces the positions available. A possible follow-up activity could be to find the equivalent fractions and find out how many different ways they can be made. For example, $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, and $\frac{5}{10}$ are all equivalent to 0.5.

If the students play the game with small markers (for example, coloured sticks or toothpicks), they could re-use the same copymaster.

A possible variation could be to relate fractions to percentages, using the same rules and number matrix.

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