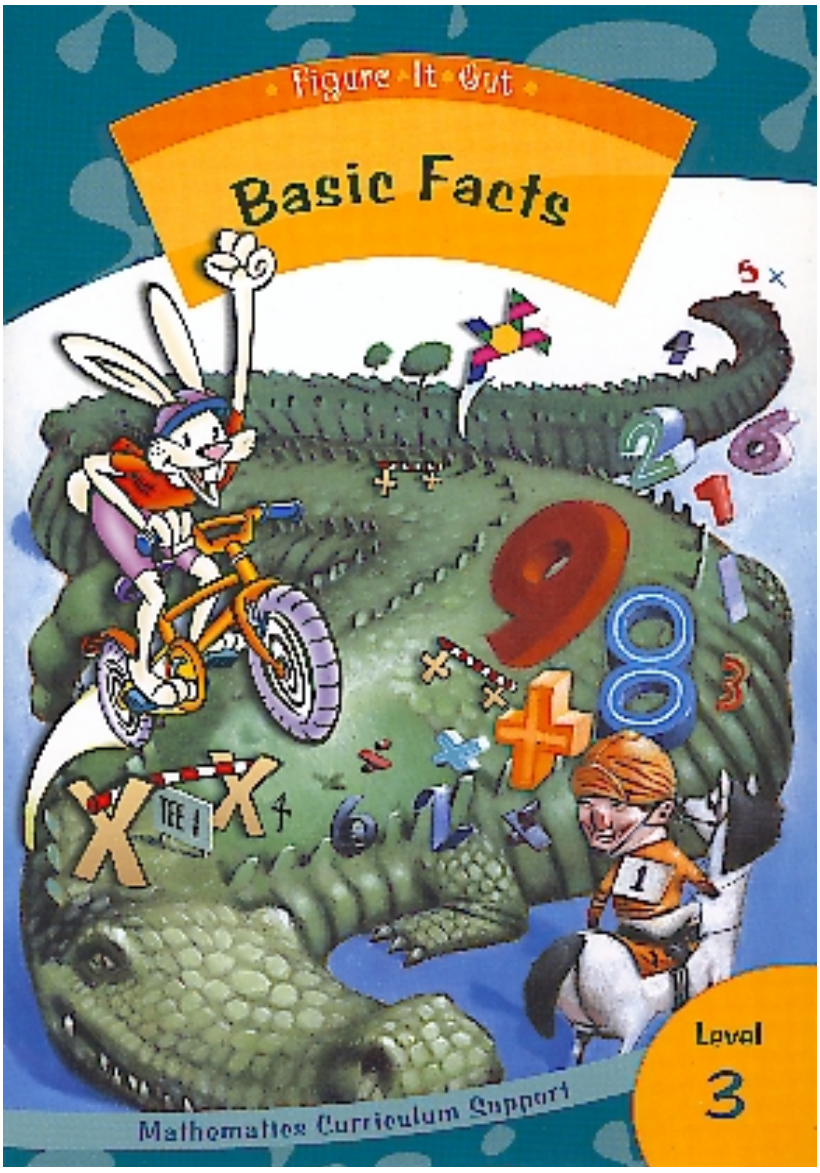
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Contents

Introduction

The Figure It Out series is designed to support *Mathematics in the New Zealand Curriculum*. The booklets have been developed and trialled by classroom teachers and mathematics educators. The series builds on the strengths of a previous series of mathematics booklets published by the Ministry of Education, the School Mathematics supplementary booklets.

Figure It Out is intended to supplement existing school mathematics programmes and can be used in various ways. It provides activities and investigations that students can work on independently or co-operatively in pairs or groups. Teachers can select particular activities that provide extension to work done in the regular classroom programme. Alternatively, teachers may wish to use all or most of the activities in combination with other activities to create a classroom programme. The booklets can be used for homework activities, and the relevant section in the teachers' notes could be copied for parents. These notes may also provide useful information that could be given as hints to students.

There are eight booklets for level 3: one booklet for each content strand, one on problem solving, one on basic facts, and a theme booklet. Each booklet has its own *Answers and Teachers' Notes*. The notes include relevant achievement objectives, suggested teaching approaches, and suggested ways to extend the activities. The booklets in this set (level 3) are suitable for most students in year 5. However, teachers can decide whether to use the booklets with older or younger students who are also working at level 3.

The booklets have been written in such a way that students should be able to work on the material independently, either alone or in groups. Where applicable, each page starts with a list of equipment that the students will need to do the activities. Students should be encouraged to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Teachers could encourage students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Where possible, suggestions have been made to encourage discussion and oral presentation of answers, and teachers may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> (*Mathematics in the New Zealand Curriculum*, page 7)

Students will have various ways of solving problems or presenting the process they have used and the solution. Successful ways of solving problems should be acknowledged, and where more effective or efficient processes can be used, students can be encouraged to consider other ways of solving the problem.

Figure It Out

Basic Facts

Answers

Page 1: Animal Antics

Activity

1. Monster **a**: 94 cents

Monster **b**: 89 cents

2. Answers will vary.

Game

A game of addition

Pages 2–3: Beat Yourself Down

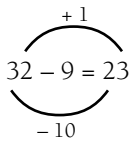
a.	b.	c.	d.	e.	f.
5	6	5	5	9	16
11	7	8	6	6	21
12	2	8	9	7	14
12	8	7	8	7	13
9	7	6	4	5	10
14	8	9	4	3	17
15	2	8	9	9	12
11	11	7	7	2	16
9	9	3	8	8	11
14	4	2	3	7	10
17	9	8	8	6	18
11	8	7	7	8	11
8	2	7	2	7	8
15	7	6	5	6	5
11	3	9	7	3	14
18	6	9	5	8	10
10	5	5	1	9	10
13	9	3	8	9	9
12	6	9	3	8	9
7	8	7	8	7	7

Page 4: Nines and Eights

Activity

1. **a.** 4 **b.** 9 **c.** 7 **d.** 1
e. 17 **f.** 23 **g.** 66

2. To find the answer to a two-digit number minus 9, take 1 off the tens digit and add 1 to the ones digit, for example,



3. Take 1 off the tens digit and add 2 to the ones digit.

4. **a.** 6 **b.** 9 **c.** 15 **d.** 92
e. 46

Page 5: Give or Take

Game

A game of addition and subtraction

Page 6: Carrot Country

Activity

1. Rufus can get:
22 (4 + 7 + 6 + 5)
41 (4 + 7 + 6 + 4 + 8 + 3 + 9)
43 (4 + 7 + 6 + 4 + 8 + 3 + 2 + 9)
24 (4 + 5 + 4 + 6 + 5)
33 (4 + 5 + 4 + 8 + 3 + 9)
35 (4 + 5 + 4 + 8 + 3 + 2 + 9)
34 (8 + 3 + 8 + 4 + 6 + 5)
50 (8 + 3 + 8 + 4 + 5 + 4 + 7 + 6 + 5)
31 (8 + 3 + 8 + 3 + 9)
22 (8 + 3 + 2 + 9)
48 (8 + 3 + 2 + 9 + 3 + 8 + 4 + 6 + 5)
64 (8 + 3 + 2 + 9 + 3 + 8 + 4 + 5 + 4 + 7 + 6 + 5)

2. Rufus should follow the 8 + 3 + 2 + 9 + 3 + 8 + 4 + 5 + 4 + 7 + 6 + 5 path, which will give him 64 carrots.

Page 7: Four in a Row

Game

A game of addition

Page 8: Array Puzzles

Activity One

1.

+	4	7	3
5	**9**	**12**	**8**
8	**12**	**15**	**11**
6	**10**	**13**	**9**

2. a.

+	3	5	8
9	**12**	**14**	17
6	**9**	11	**14**
4	7	**9**	12

b.

+	6	9	3	4
5	**11**	14	**8**	9
8	**14**	**17**	11	**12**
7	13	**16**	**10**	11
9	**15**	18	12	**13**

Activity Two

Answers will vary.

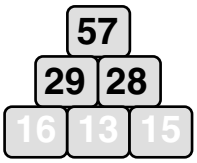
Page 9: Paths and Pairs

Activity One

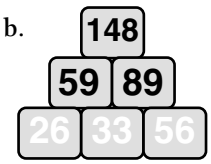
376 + 295 + 437 + 58 = 1166

Activity Two

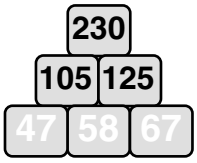
1. a.



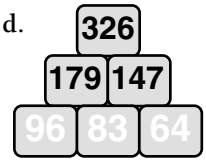
b.



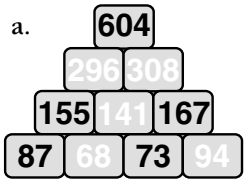
c.



d.



2. a.



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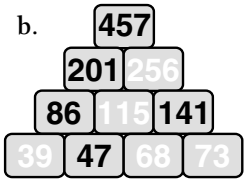
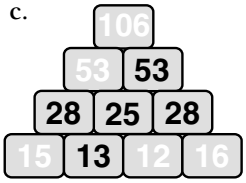


c.



Activity Three

a. 285 + 396 = 681

b. 138 + 589 = 727

c. 377 + 396 = 773

d. 246 + 174 = 420

e. 467 + 174 = 641

Page 10: Choco-blocks

Activity

a. 1 x 18; 2 x 9; 3 x 6

b. 1 x 25; 5 x 5

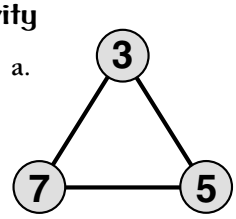
c. 1 x 36; 2 x 18; 3 x 12; 4 x 9; 6 x 6

d. 1 x 100; 2 x 50; 4 x 25; 5 x 20; 10 x 10

Page 11: Factor Puzzles

Activity

1. a.



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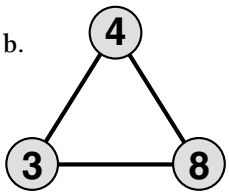
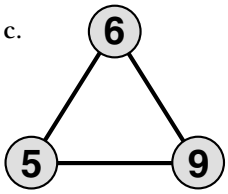


c.



2. Answers will vary.

Game

A game of multiplication

Page 12: Stars and Students

Game

A game of multiplication

Page 13: Digital Delights

Activity One

1. **a.** 9 9 9 9 9 9 9 9

b. All the numbers have the same digital sum, 9.

c. Answers will vary but could include: They are all multiples of 9.

2. 59 has a digital sum of 5 + 9 = 14 ➔ 1 + 4 = 5

Activity Two

Multiples of 8	8	16	24	32	40	48	56	64	72	80	88	96
Digital sums	8	7	6	5	4	3	2	1	9	8	7	6

A possible explanation of the pattern is: A number sequence starting with 8 and decreasing by 1. The sequence goes back to 9 after 1 is reached.

Investigation

Answers will vary but could include the following explanations with the completed multiples:

Multiples of 7	7	14	21	28	35	42	49	56	63	70	77	84
Digital sum	7	5	3	1	8	6	4	2	9	7	5	3

The pattern is either odd or even numbers, decreasing by 2.

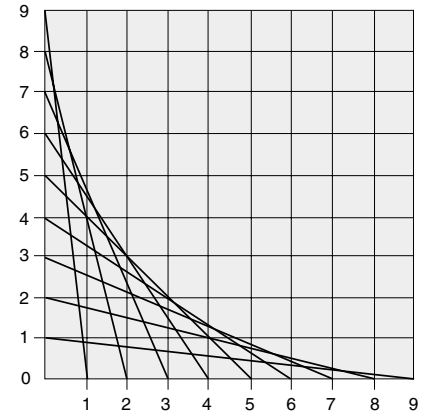
Multiples of 6	6	12	18	24	30	36	42	48	54	60	66	72
Digital sum	6	3	9	6	3	9	6	3	9	6	3	9

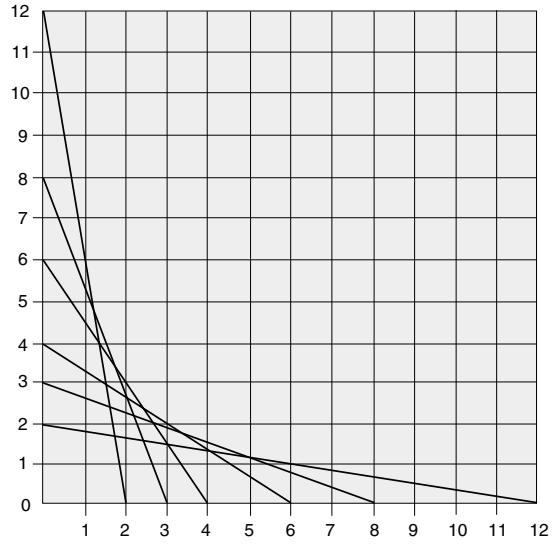
The pattern is 3 repeated multiples of 3: 9, 6, 3.

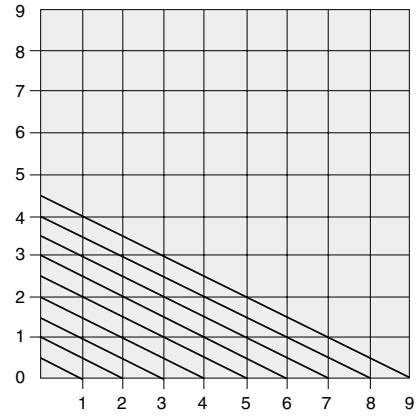
Multiples of 5	5	10	15	20	25	30	35	40	45	50	55	60
Digital sum	5	1	6	2	7	3	8	4	9	5	1	6

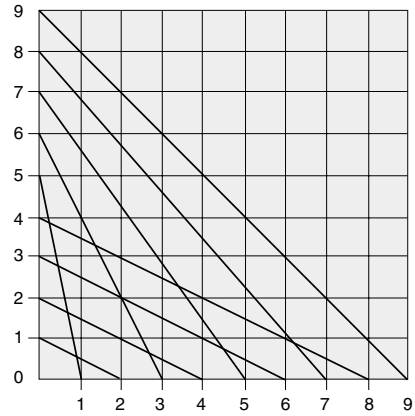
The pattern is based on – 4 in increasing pairs or on + 1 in alternate pairs.

Page 14: Joining Rules

1. 

2. 

3. 

4. 

Page 15: Almost Squares

Game

A game using addition, subtraction, and multiplication

Pages 16–17: The Field of 100 Sheep

Activity One

1. Answers will vary. Known facts can be built on to obtain 6 x 4 or 4 x 6, for example, (5 x 4) + 4 or (3 x 3) + (3 x 3) + 6.

2. **a.** 16

b. 28

c. 30

d. 35

3. **a.** Group the sheep in clusters of 6. Count the clusters (4).

b. Group the sheep in clusters of 4. Count the clusters (6).

c. Group the sheep in clusters of 3. Count the clusters (8).

d. Group the sheep in clusters of 2. Count the clusters (12).

Activity Two

1. 25

2. **a.** 42

b. 48

c. 6

d. 12

Pages 18–19: Calculator Golf

Activity

1. There are many possible answers. Two possible answers are:
First shot: 14 x 4 = 56
Second shot: 56 + 7 = 63
Total number of shots = 2

First shot: 14 x 3 = 42
Second shot: 42 x 3 = 126
Third shot: 126 ÷ 2 = 63
Total number of shots = 3

2. Answers will vary, but the most common will probably be when the number on the calculator is higher than the number on the green.

Game

A game of addition, subtraction, multiplication, and division

Page 20: Dicey Dabble

Game

A game of multiplication and division

Page 21: Multiple Mirrors

Activity

1. **a.** Each pattern has reflectional symmetry about the left-to-right or right-to-left diagonal.

b. This happens because the order of factors does not affect the product, for example, 3 x 6 = 6 x 3.

2. **a.**

x	1	2	3	4	5	6	7	8	9
1		■						■	
2	■			■		■			■
3				■		■			
4		■	■				■	■	
5									
6		■	■				■	■	
7				■		■			
8	■			■		■			■
9		■						■	

b. The pattern has 4 lines of reflectional symmetry and rotational symmetry of order 4.

3. **a. i.**

x	1	2	3	4	5	6	7	8	9
1	■								■
2									
3			■				■		
4									
5									
6									
7			■				■		
8									
9	■								■

a. ii.

x	1	2	3	4	5	6	7	8	9
1			■				■		
2									
3	■								■
4									
5									
6									
7	■								■
8									
9			■				■		

a. iii.

x	1	2	3	4	5	6	7	8	9
1				■		■			
2		■	■				■	■	
3		■						■	
4	■			■		■			■
5									
6	■			■		■			■
7		■						■	
8		■	■				■	■	
9				■		■			

a. iv.

x	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

b. Explanations of patterns will vary.

Page 22: It Remains to Be Seen

Game

A game of division

Page 23: Steeplechase

Game

A game of multiplication and rounding

Page 24: Making Numbers

Activity One

Answers will vary.

Activity Two

1. Answers will vary. Possible equations include:
1 = 3 – 2, 1 = 5 – 4, 1 = 4 – 3, 1 = 2 x 3 – 5
2 = 4 – 2, 2 = 5 – 3, 2 = 4 ÷ 2, 2 = 2 x 3 – 4,
2 = 3 + 4 – 5
3 = 5 – 2, 3 = 2 x 4 – 5, 3 = 2 x 5 – (3 + 4),
3 = 5 – (4 ÷ 2), 3 = (5 + 4) ÷ 3
4 = 3 + 5 – 4, 4 = 5 + 2 – 3, 4 = (3 + 5) ÷ 2,
4 = (5 – 3) x 2
5 = 3 + 2, 5 = 2 x 4 – 3, 5 = 3 + 4 – 2
6 = 2 x 3, 6 = (4 x 3) ÷ 2, 6 = 2 x 5 – 4,
6 = 3 + 5 – 2, 6 = (5 – 3) + 4, 6 = (5 + 4) – 3
7 = 3 + 4, 7 = 3 x 4 – 5, 7 = 2 x 3 + (5 – 4),
7 = 5 + 2
8 = 2 x 4, 8 = 32 ÷ 4, 8 = 24 ÷ 3, 8 = 5 + 3,
8 = (5 – 3) x 4
9 = 4 + 5, 9 = 54 ÷ (2 x 3), 9 = 52 – 43,
9 = (5 – 2) x 3
10 = 2 x 5, 10 = 2 x 3 + 4, 10 = (4 x 5) ÷ 2,
10 = 5 x 4 ÷ 2
11 = 3 x 5 – 4, 11 = 2 x 5 + (4 – 3),
11 = 35 – 24, 11 = 53 – 42
12 = 3 x 4, 12 = (5 – 2) x 4
13 = 3 x 5 – 2, 13 = 52 ÷ 4, 13 = 45 – 32,
13 = 2 x 5 + 3, 13 = 2 x 4 + 5
14 = 3 x 4 + 2, 14 = 42 ÷ 3, 14 = 2 x 5 + 4
15 = 3 x 5
16 = (3 + 5) x 2
17 = 4 x 5 – 3, 17 = 34 ÷ 2, 17 = 3 x 4 + 5,
17 = 3 x 5 + 2
18 = 4 x 5 – 2, 18 = 54 ÷ 3
19 = 3 x 5 + 4, 19 = 24 – 5, 19 = 23 – 4,
19 = 5 x 4 – 3 + 2
20 = 4 x 5

2. Answers will vary. It could be argued that the easiest numbers to make are the ones with the most possible equations.

Figure It Out

Basic Facts

Teachers' Notes

Overview: Basic Facts

Title	Content	Page in students' book	Page in teachers' notes
Animal Antics	Adding single-digit numbers	1	12
Beat Yourself Down	Adding and subtracting single-digit numbers	2–3	12
Nines and Eights	Subtracting single-digit numbers	4	14
Give or Take	Practising addition and subtraction	5	14
Carrot Country	Building on basic addition facts	6	14
Four in a Row	Practising basic addition facts	7	15
Array Puzzles	Using basic addition arrays	8	15
Paths and Pairs	Practising addition facts	9	15
Choco-blocks	Applying multiplication facts	10	16
Factor Puzzles	Practising basic multiplication facts	11	16
Stars and Students	Applying multiplication facts	12	17
Digital Delights	Investigating patterns in multiples	13	17
Joining Rules	Using rules involving addition, subtraction, multiplication, and division	14	17
Almost Squares	Using addition, subtraction, and multiplication	15	18
The Field of 100 Sheep	Learning basic multiplication facts	16–17	19
Calculator Golf	Applying addition, subtraction, multiplication, and division	18–19	19
Dicey Dabble	Practising multiplication and division	20	19
Multiple Mirrors	Practising basic multiplication facts	21	20
It Remains to Be Seen	Dividing by a single-digit number	22	20
Steeplechase	Practising rounding	23	20
Making Numbers	Applying addition, subtraction, multiplication, and division	24	20

About Basic Facts

The term Basic Facts refers to those equations where two single-digit numbers are combined by adding or multiplying to give a sum or product respectively. For each basic addition fact, there is a related subtraction fact, and for each basic multiplication fact (except where zero is the multiplier), there is a related division fact. *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) also includes multiplication by 10.

In the curriculum statement *Mathematics in the New Zealand Curriculum,* the overall achievement aims for Number state:

> The mathematics curriculum intended by this statement will provide opportunities for students to:
>
> - develop an understanding of numbers, the ways they are represented, and the quantities for which they stand;
> - develop accuracy, efficiency, and confidence in calculating – mentally, on paper, and with a calculator;
> - develop the ability to estimate and to make approximations, and to be alert to the reasonableness of results and measurements.
>
> *Mathematics in the New Zealand Curriculum*, pages 9 and 31

Clearly, students' mastery of basic facts is most important if they are going to achieve these aims.

The relevant achievement objectives from Number are:

- recall the basic addition and subtraction facts (level 2)
- recall the basic multiplication facts (level 3)

The students' booklet is designed to give students opportunities to practise using their basic addition facts so that they have rapid recall of these facts. It also aims to provide experiences that should help students gain an understanding of the concept of multiplication and gain confidence in knowing the basic multiplication facts. The basic facts can then be not only used in number topics but also applied to other strands, such as Algebra, Measurement, and Statistics. (See also the introduction on page 8 of *Answers and Teachers' Notes: Basic Facts*, Figure It Out, Levels 2–3.)

There are key understandings that should guide the learning of basic facts. They are:

- The commutative laws of addition and multiplication:
  $a + b = b + a$ $a \times b = b \times a$
- The role of zero in addition and the role of zero in multiplication:
  $a + 0 = a$ $a \times 0 = 0$
- The role of one in multiplication:
  $a \times 1 = a$
- Addition and subtraction are inverse operations.
  Multiplication and division are inverse operations.
- Families of facts. For example:

$3 + 4 = 7$	$5 \times 7 = 35$
$4 + 3 = 7$	$7 \times 5 = 35$
$7 - 3 = 4$	$35 \div 5 = 7$
$7 - 4 = 3$	$35 \div 7 = 5$

By level 3, students should have an understanding of the basic facts for addition and multiplication. Most students will need opportunities to practise their basic facts so that they can develop rapid recall and are able to apply this skill in a variety of situations. However, some students find it hard to learn particular facts and may need to be given strategies to help them learn facts that they have difficulty memorising.

Looking for patterns can help students develop strategies to work out unknown facts from those that they do know. One example of this is shading multiples on an array and looking for the patterns that arise, as on page **21**.

Students should encounter the basic facts in all their symbolic forms. For example:

$3 \times 9 = \square$	$\square \times 9 = 27$	$3 \times \square = 27$
$27 \div 3 = \square$	$27 \div \square = 9$	$\square \div 3 = 9$
$\frac{27}{3} = \square$	$3\overline{)27}$	

The Multiplication Facts Triangle

Students should be aware that there are only 36 basic multiplication facts. This number is based on two assumptions:

1. Any number multiplied by one is itself.
2. The commutative law of multiplication (for example, $4 \times 9 = 9 \times 4$) operates.

The 36 basic multiplication facts can be displayed on a triangular chart, or, more effectively, students could make their own triangular charts and highlight those facts that they have difficulty recalling.

				9 x 9 = 81				
				8 x 8 = 64	8 x 9 = 72			
			7 x 7 = 49	7 x 8 = 56	7 x 9 = 63			
		6 x 6 = 36	6 x 7 = 42	6 x 8 = 48	6 x 9 = 54			
	5 x 5 = 25	5 x 6 = 30	5 x 7 = 35	5 x 8 = 40	5 x 9 = 45			
	4 x 4 = 16	4 x 5 = 20	4 x 6 = 24	4 x 7 = 28	4 x 8 = 32	4 x 9 = 36		
3 x 3 = 9	3 x 4 = 12	3 x 5 = 15	3 x 6 = 18	3 x 7 = 21	3 x 8 = 24	3 x 9 = 27		
2 x 2 = 4	2 x 3 = 6	2 x 4 = 8	2 x 5 = 10	2 x 6 = 12	2 x 7 = 14	2 x 8 = 16	2 x 9 = 18	

Games and Activities

The games and activities in the students' booklet are designed to provide enjoyable and motivating opportunities to practise and use basic facts knowledge. Some of the games and activities have an element of competition, which could be competition against oneself, and other games depend on chance and skill.

Any clear connections to other strands of the curriculum statement are identified in these teachers' notes.

Page 1: Animal Antics

Activity

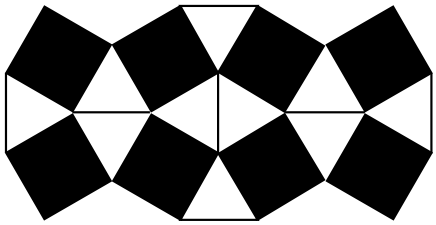
Question **1** is designed to give students practice with addition facts. Most students are likely to record the various amounts shown and then calculate the sum. Some students, though, may choose to use multiplication facts. It would be worthwhile looking at students' recordings and noting those who have used multiplication as a more efficient method.

Question **2** gives students an opportunity to be creative and could take the sum to greater than 100. Given the monetary value of the blocks, you may wish to encourage students to make connections between cents and dollars. (This links to the measurement strand.)

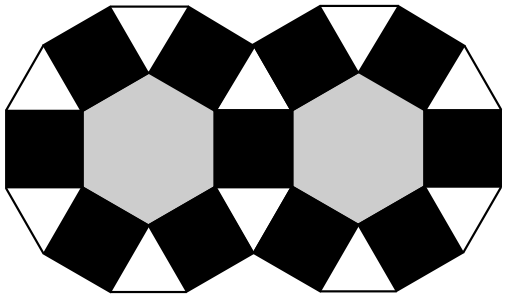
As an extension exercise, the students could be challenged to make a monster that costs exactly $1 or $1.25.

The pattern blocks are designed to fit together to create mosaic patterns. As another extension, students could create mosaics with the pattern blocks and calculate the value of their pattern. Encourage students to use multiplication facts as a more efficient way of finding the sum. (This connects to the geometry strand.)

For example:



squares: 8 x 7 cents = $0.56
triangles: 10 x 5 cents = $0.50
sum = $1.06



hexagons: 2 x 9 cents = $0.18
squares: 11 x 7 cents = $0.77
triangles: 10 x 5 cents = $0.50
sum = $1.45

Game

This game encourages students to learn addition facts for sums of 12, 13, and 14. The pairs of blocks could be sorted into those that make a sum of 12 (5 + 7, 6 + 6, 7 + 5), a sum of 13, or a sum of 14.

Discuss the strategy for finding the sum of two numbers when these two numbers are part of a sequence that has the middle number missing. For example, in 5 + 7, the missing number in the sequence is 6, which is one more than 5 and one less than 7. Double 6 is 12, so 5 + 7 = 12.

Pages 2-3: Beat Yourself Down

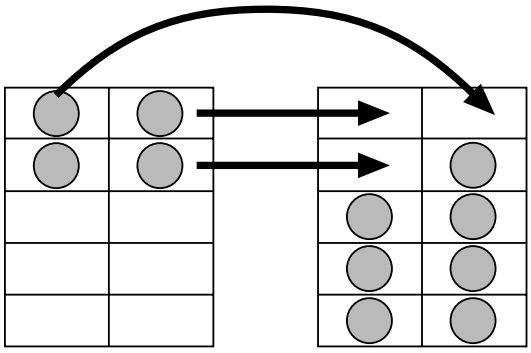
Activity

Timed self-testing activities are an excellent way for students to work out which facts they can recall with speed and which they cannot. Students should record the facts that they do not know for future learning.

The following strategies can help students learn addition facts:

- Do you know another fact close to this one (the unknown)? For example, you don't know 4 + 7, but you do know 4 + 4. How could you get from (known fact) to find the answer to (unknown fact)?

- Picture what the numbers look like in tens frames.
  Can you make a full 10, with some over?
  Draw a picture of this in your book.



4 + 7 = 10 + 1
= 11

- What other new facts will you be able to work out from (unknown fact)? For example, 4 + 7 = 11, so 5 + 7 = 12, 4 + 8 = 12, 3 + 7 = 10, 4 + 6 = 10, and 11 – 4 = 7, 11 – 7 = 4, etc.

Encourage students to keep records of the number of correct answers and the time spent on each section. This encourages self-competition and can motivate students to learn and rapidly recall the basic facts. Results can then be graphed. (This connects to the statistics strand.)

The activity needs a timed penalty for each error, otherwise there is no incentive for the student to get the correct answer.

A possible penalty system could be:

Number of errors	1	2	3	4	5	6	7	8	…
Time to be added to total (in seconds)	3	8	15	20	30	40	50	60	…

Students can highlight and record those facts they have incorrect and focus on learning these.

A class collection of sheets of calculations focusing on one particular family of facts could be developed as in the example for the seven times table below.

Get to Know Your Seven Times Table

Name: Time: Score:

4 x 7 = ❐	1 x 7 = ❐	7 x 9 = ❐	5 x 7 = ❐
9 x 7 = ❐	4 x 7 = ❐	5 x 7 = ❐	8 x 7 = ❐
7 x 3 = ❐	7 x 8 = ❐	1 x 7 = ❐	3 x 7 = ❐
2 x 7 = ❐	10 x 7 = ❐	7 x 7 = ❐	6 x 7 = ❐
7 x 0 = ❐	8 x 7 = ❐	3 x 7 = ❐	0 x 7 = ❐
❐ ÷ 7 = 3	❐ ÷ 1 = 7	❐ ÷ 8 = 7	❐ ÷ 7 = 6
❐ ÷ 7 = 7	❐ ÷ 10 = 7	❐ ÷ 6 = 7	❐ ÷ 3 = 7
❐ ÷ 9 = 7	❐ ÷ 7 = 8	❐ ÷ 7 = 10	❐ ÷ 7 = 2
❐ ÷ 2 = 7	❐ ÷ 5 = 7	❐ ÷ 7 = 9	❐ ÷ 7 = 1
❐ ÷ 7 = 0	❐ ÷ 4 = 7	❐ ÷ 1 = 7	❐ ÷ 7 = 4
❐ x 5 = 35	❐ x 7 = 49	❐ x 7 = 14	❐ x 8 = 56
8 x ❐ = 56	7 x ❐ = 0	7 x ❐ = 63	7 x ❐ = 49
7 x 9 = ❐	7 x 6 = ❐	10 x 7 = ❐	5 x 7 = ❐
❐ x 7 = 21	❐ x 7 = 7	❐ ÷ 7 = 42	❐ x 7 = 28
7 x ❐ = 70	7 x ❐ = 28	7 x ❐ = 0	7 x ❐ = 14
❐ ÷ 4 = 28	❐ ÷ 7 = 7	❐ ÷ 7 = 1	❐ ÷ 7 = 5
14 ÷ ❐ = 2	70 ÷ ❐ = 7	42 ÷ ❐ = 6	63 ÷ ❐ = 9
35 ÷ 5 = ❐	7 ÷ 7 = ❐	28 ÷ 7 = ❐	70 ÷ 10 = ❐
❐ ÷ 7 = 3	❐ ÷ 8 = 7	❐ ÷ 6 = 7	❐ ÷ 7 = 2
7 x ❐ = 7	21 ÷ ❐ = 56	49 ÷ ❐ = 7	7 ÷ ❐ = 1

The sheets could be used in the following way:

- Students (and possibly the teacher) have a timed start and sprint through the facts sheet, writing their answers in the spaces provided.
- As the students finish, their time is called out, they record it, and then they turn the sheet over and continue quietly with another task until the cut-off time. This could be 5 minutes or whatever is appropriate for the speed of students in the class.
- Students use a copy of the same sheet for 3 consecutive days and observe their improvements.
- Students can concentrate on a particular multiplication table and present the facts in various forms.

Page 4: Nines and Eights

Activity

This page provides a strategy for subtracting nine, which students often have difficulty doing. Encourage students to record their results so that they can see the patterns.

Page 5: Give or Take

Game

This game is a number version of the word game Boggle. The game could be extended to include a combination of addition and subtraction operations. For example, $15 - 3 + 4 - 9 = 7$ and $16 - 12 + 7 - 5 = 6$.

Page 6: Carrot Country

Activity

This activity asks students to record possible paths and the associated sums and gives them a chance to practise mentally adding a series of numbers. It provides an opportunity for students to explore the mathematics of networks, a topic that is developed much later in the curriculum. (This connects to the geometry strand.)

Strategies for solving the problem and recording methods can be shared. Ask them to explain and justify how they know that all possible paths have been checked.

Students might like to try question **1** as a race. Working in pairs, one student could calculate mentally while the other checks with the calculator. This approach should reinforce the fact that mental strategies are often faster than using a calculator.

Students could use the information gathered to answer this additional question: "Which path should Rufus take if he wants to collect the least number of carrots?"

As an extension, suggest that students create a new map for Rufus and change the numbers to make the addition more challenging.

Page 7: Four in a Row

Game

This is a great strategy game that requires students to use addition facts. It will help students to develop skills in logical thinking. (This connects to mathematical processes: developing logic and reasoning.) As students play the game more, they are likely to use blocking techniques and make anticipatory moves that give them more options for winning.

Page 8: Array Puzzles

Activity One

Students enjoy making and completing arrays for both addition and multiplication facts. Students are often given arrays to complete where the addends are provided and they have to find the sum, such as the array in question **1**.

Question **2** offers an interesting and challenging alternative because it has missing addends and sums. Students need to recognise that addition and subtraction are inverse operations and to be able to consider the facts in different forms. For example, to find out what number goes in the top right corner of the first array, they need to find the unknown in this equation: $9 + ❑ = 17$. Seeing that addition and subtraction are inverse operations, they can rewrite this as $17 - 9 = ❑$ and solve the problem this way.

Activity Two

As a variation to this activity, students might like to use this same method and design multiplication arrays for classmates to solve.

Page 9: Paths and Pairs

Activity One

This activity provides an opportunity for students to practise addition skills. The most efficient way to find the correct path is to add the ones digits only to find a number with a six in the ones place. There is only one path that gives an answer with six in the ones place.

Activity Two

Addition pyramids are an easy way of structuring addition calculations. Answers can be easily checked with a classmate or the calculator.

Encourage students to explain the different mental strategies they used to add the numbers. For example, to calculate 47 + 58, students could use any of these strategies:

- Front-ending, that is, $47 + 50 \rightarrow 97 + 8 \rightarrow 105$
- Adding the tens and then the ones: $40 + 50 \rightarrow 90 + (7 + 8 = 15) \rightarrow 105$
  $40 + 50 \rightarrow 90 + 7 \rightarrow 97 + 8 \rightarrow 105$
- Adding the ones first and renaming as per the standard written algorithm:

$$\begin{array}{r} {\scriptstyle 1\,\,} \\ 34 \\ +\ 27 \\ \hline 61 \end{array}$$

  Using this method, students add the ones first, rename the 10 ones as one 10, and then add the tens.

Students could design their own pyramids and make them more interesting and challenging by leaving out different numbers in the pyramid.

Students could be encouraged to compare the speed of completing a pyramid using mental calculations with using a calculator.

Activity Three

This activity encourages students to use strategies for adding mentally and to practise their estimation skills. Students could share with a classmate their strategies for finding the sums.

Page 10: Choco-blocks

Activity

This activity shows students another model for multiplication, that is, that multiplication can be shown as a rectangular array. Making chocolate bar rectangles reinforces the commutative property of multiplication (a x b = b x a). This activity also shows that multiplication can be used to find the area of a rectangle. (This connects to the measurement strand.)

This activity will help students find the factors for a given product. The set of factors for 12 (1, 2, 3, 4, 6, and 12) can be found from the three different lengths of the chocolate blocks.

As an extension, you could ask students to imagine the chocolate blocks as squares and encourage them to find all square numbers from one to 100.

Page 11: Factor Puzzles

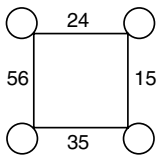
Activity

In this activity, students find the factors for given products and identify the common factors in order to solve the puzzle.

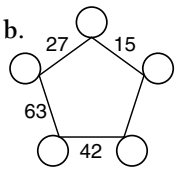
Students will need to be very familiar with basic multiplication facts to do this activity. They could identify factors for each product and then use trial and improvement as a strategy to solve the problem. Questions **1b** and **1c** include numbers with more than one common factor, so students will have to use problem-solving strategies to identify the required common factor.

Examples of polygons that students could use for question **2** are squares, pentagons, and hexagons.

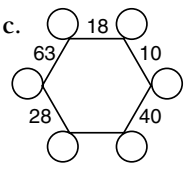
a.



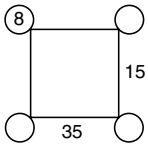
b.



c.



(**b** and **c** have one missing product.)
Students could vary their puzzles so that there are several missing factors and products, for example:



Game

This is a relatively quick game that gives students practice in finding products. Some of the products are repeated so that students can use strategies to increase their chances of winning. Playing this game several times gives students the chance to explore different strategies.

Page 12: Stars and Students

Game

This game gives students practice with basic multiplication facts. They will soon realise the benefits of throwing a one (because one is a factor of every number).

As a variation, students could use a dice that doesn't have one on it, for example, a dice marked three to eight. The numbers on the board would have to be varied accordingly.

Page 13: Digital Delights

Activity One

This activity encourages students to recognise that any multiple of nine has a digital sum of nine.

As an extension, you could ask students to work in pairs and give them large numbers to check to see whether they are multiples of nine.

Activity Two

Students can explore patterns of multiples for eight, seven, six, and five.

Multiples can be generated very quickly on the calculator if students activate the constant function. This is done on most calculators by keying in the number that students want to find multiples of and then pressing [+] [+] followed by [=] [=] [=] to generate the set of multiples, for example, [2] [+] [+] [=] [=] [=].

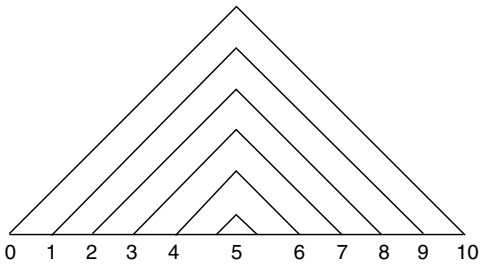
It is also interesting to include multiples of three and the corresponding digital sum. Every multiple of three has a digital sum that is a multiple of three, that is, three, six, or nine. To find out whether a number is a multiple of three, calculate its digital sum and see whether that is a multiple of three.

Page 14: Joining Rules

Activity

In this activity, students generate interesting geometric designs from number patterns. They begin with the number on the horizontal axis (x-axis) and connect it to the value on the vertical axis (y-axis). This is the conventional order for graphing algebraic equations.

In question **1**, the sums for 10 create a symmetrical curve. A visual device, the tens triangle, can also help students to learn the number combinations that add to 10.



The curve in question **2** is generated from factors of 24. This curve is also symmetrical, so drawing the curve may help students find any missing factors.

Students will find that joining the numbers in question **3** creates a series of parallel lines. Ask students why they think the lines are parallel.

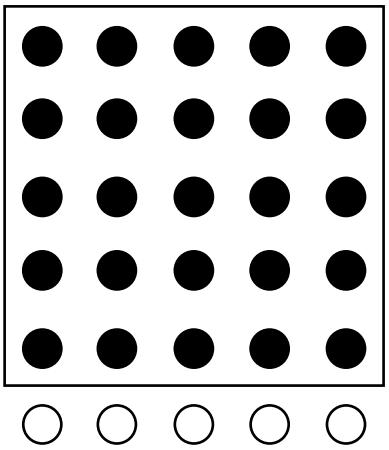
These patterns could also be made by using nails and coloured string on a board.

Page 15: Almost Squares

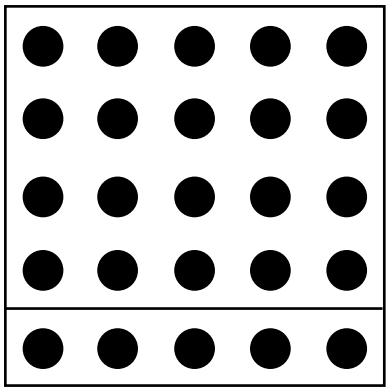
Game

Almost Squares is a game designed to encourage students to derive multiplication facts. Students usually learn the square number facts (1 x 1 = 1, 2 x 2 = 4, 3 x 3 = 9, 4 x 4 = 16, …) quite early. Square numbers also have significant application in algebra and the measurement of area.

An array model can be used to illustrate the effect of reducing or increasing a factor by one. Consider 5 x 5 = 25:

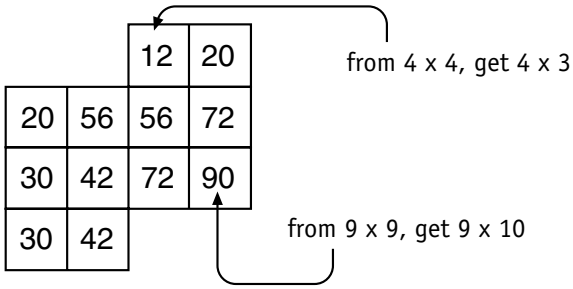


5 x 6 = 30



4 x 5 = 20

Students should realise that the lowest and highest numbers on the board are the hardest to get. All the other numbers can be obtained in two ways. For example, 56 can be obtained from 7 x 7 and 8 x 8. Therefore, it is sensible to cover 12 and 90 when the opportunity presents itself.



Students may wish to vary the game. For example, increase one factor by one or two and reduce the other by one or two. In this scenario, 6 x 6 becomes 7 x 5 or 8 x 4.

For this variation, the playing board would need these numbers:

		12	15
21	24	32	35
45	48	60	63
77	80		

Pages 16–17: The Field of 100 Sheep

Activity One

This activity provides students with an array model to help them visualise multiplication. This is a similar approach to the choco-blocks model on page **10** of the students' booklet. Students are encouraged to make links with facts that they already know in order to find the product for a new fact. For example, masking half the 6 x 4 array gives students the opportunity to view it as a 3 x 4 array doubled.

A masking card can be used to encourage skip counting. Uncovering each of the rows in turn gives 6, 12, 18, and 24, which are the multiples of six. Uncovering each of the columns in turn gives 4, 8, 12, 16, 20, and 24, which are the multiples of four.

For question **3**, students can use the array in reverse to model division. They slide the card either horizontally or vertically to find the quotient for **a** or **b**. (24 ÷ 3 and 24 ÷ 2 are not easily shown by sliding the card.) Encourage students to link their knowledge of basic multiplication facts to division.

Activity Two

This activity encourages students to learn the "twins" and then use this knowledge to learn other multiplication facts.

Pages 18–19: Calculator Golf

Activity

In this game, students use estimation skills. They keep improving on the estimates they have made until they get the number on the green.

Game

After students have played the game several times, you could ask them whether they have worked out the most efficient way to play the game. If they haven't worked out that the quickest way to move between numbers with very different values is to use multiplication or division, you could remind them that multiplication is a repeated addition and division is a repeated subtraction.

As a variation to the game, students could suggest pars for each hole and classmates could try to match the par. Another challenge could be to get a "hole in one" for a particular hole. Note: The course would have to be designed so that a "hole in one" is possible.

Page 20: Dicey Dabble

Game

This game reinforces students' recognition of factors. The points incentive encourages them to find common factors. As students replay the game, they will soon learn the best ways to arrange the dividends so that they increase their chances of finding a common factor.

To vary this game, you could change the dice to one marked three to eight or change the cards to 8, 12, 14, 15, 16, 20, 21, 28, 30, 32, 36, and 40.

Page 21: Multiple Mirrors

Activity

This activity reinforces the commutative property for multiplication. (See also question **2** on page **14** of the students' booklet.) The patterns that are created are symmetrical, so students will realise whether they are missing a particular product in their shaded pattern. (This connects to the geometry strand.)

Page 22: It Remains to Be Seen

Game

This game is valuable for practising division and addition skills because of the need to calculate the remainder. Encourage students to say the calculation out loud so that the second player can check their move. Having one as a divisor reinforces the understanding that $n \div 1 = n$.

To vary this game, you could encourage students to design another game board with larger numbers and to use a dice marked four to nine. This gives students an opportunity to practise other basic multiplication facts.

Page 23: Steeplechase

Game

This is a relatively simple game using multiplication facts and rounding and addition skills. Students will need to know that five is commonly rounded to the next 10. They will also need to count by tens to move around the board.

Page 24: Making Numbers

Activities One and Two

On this page, students are required to use various combinations of numbers and operations to make a target value. Encourage them to use more than one operation in their equations. Have them keep a record of their equations so that they can compare with others and challenge each other as to how many operations they can think of.

Students have the opportunity to use brackets. You may need to explain that this is an important symbol used by mathematicians to designate the order of operations. Students may decide to use their calculators as they try out different combinations, so this may be a good time to discuss the results that different calculators give. For example, if [5] [+] [8] [x] [3] [−] [4] [=] is keyed into a scientific calculator, it will give 25, that is, $5 + (8 \times 3) - 4$, whereas some four-function calculators may give 35, that is, $(5 + 8) \times 3 - 4$. A scientific calculator automatically follows the conventions for operations, that is, multiplication and division are calculated before addition and subtraction.

Acknowledgments

Learning Media would like to thank Brenda Bicknell, College of Education, Massey University, for developing the teachers' notes. Thanks also to Paulette Holland for reviewing the answers and notes.

The main illustrations on the cover and contents page (except for the rabbit on the bicycle, the pattern-block animal, and the jumps, which are by Lorenzo Van Der Lingen) are by Daron Parton.

Series Editor: Susan Roche
Series Designer: Esther Chua

Published 2000 for the Ministry of Education by
Learning Media Limited, Box 3293, Wellington, New Zealand.
Website: www.learningmedia.co.nz

Dewey number 510.76
ISBN 0 478 12695 6
Item number 12695
Students' book: item number 12694

105135/7/00