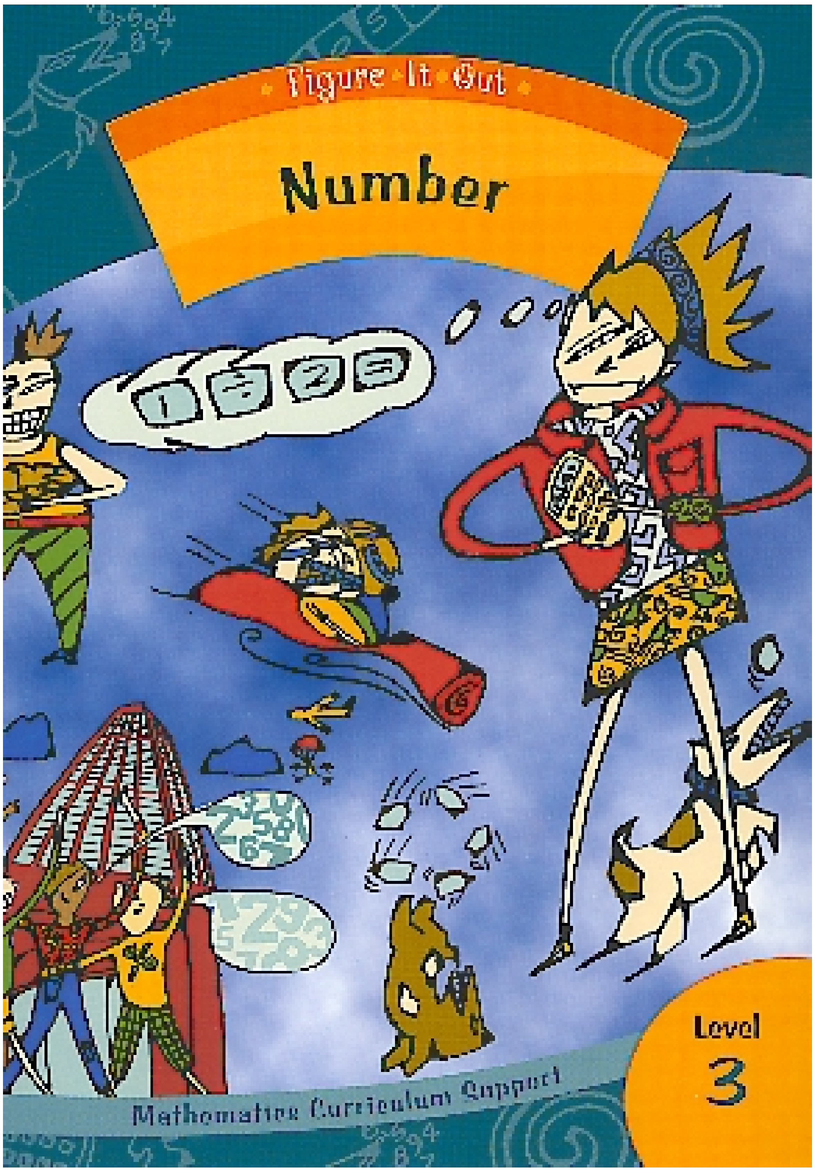
Answers and Teachers' Notes



Contents

Introduction

The Figure It Out series is designed to support *Mathematics in the New Zealand Curriculum.* The booklets have been developed and trialled by classroom teachers and mathematics educators. The series builds on the strengths of a previous series of mathematics booklets published by the Ministry of Education, the School Mathematics supplementary booklets.

Figure It Out is intended to supplement existing school mathematics programmes and can be used in various ways. It provides activities and investigations that students can work on independently or co-operatively in pairs or groups. Teachers can select particular activities that provide extension to work done in the regular classroom programme. Alternatively, teachers may wish to use all or most of the activities in combination with other activities to create a classroom programme. The booklets can be used for homework activities, and the relevant section in the teachers' notes could be copied for parents. These notes may also provide useful information that could be given as hints to students.

There are eight booklets for level 3: one booklet for each content strand, one on problem solving, one on basic facts, and a theme booklet. Each booklet has its own *Answers and Teachers' Notes*. The notes include relevant achievement objectives, suggested teaching approaches, and suggested ways to extend the activities. The booklets in this set (level 3) are suitable for most students in year 5. However, teachers can decide whether to use the booklets with older or younger students who are also working at level 3.

The booklets have been written in such a way that students should be able to work on the material independently, either alone or in groups. Where applicable, each page starts with a list of equipment that the students will need to do the activities. Students should be encouraged to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Teachers could encourage students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Where possible, suggestions have been made to encourage discussion and oral presentation of answers, and teachers may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> (*Mathematics in the New Zealand Curriculum,* page 7)

Students will have various ways of solving problems or presenting the process they have used and the solution. Successful ways of solving problems should be acknowledged, and where more effective or efficient processes can be used, students can be encouraged to consider other ways of solving the problem.

Figure It Out

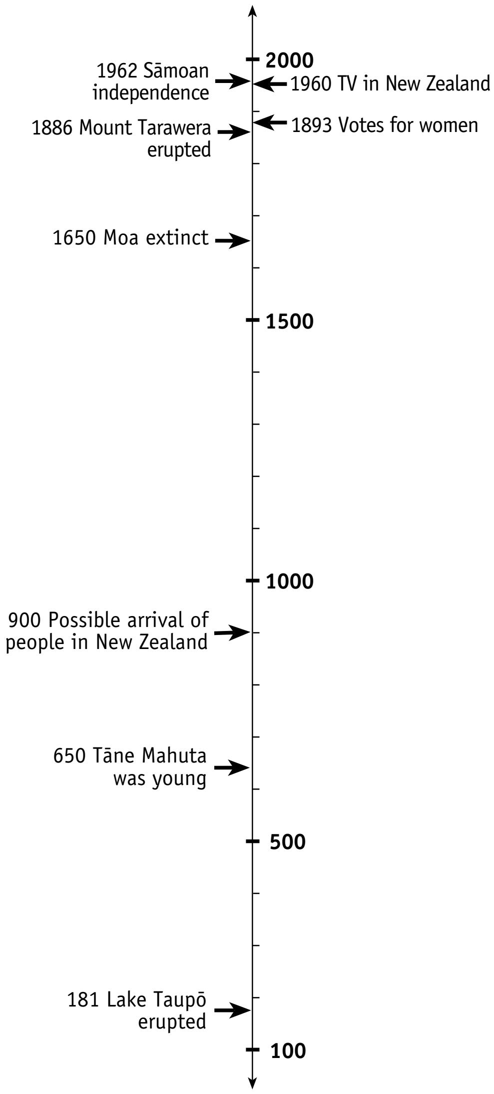
Number

Answers

Page 1: When?

Activity

1. a. Your timeline could look like this:



b. The Treaty of Waitangi was signed in 1840, so it would come before 1886 on the timeline.

2. Answers will vary. They could include comments on aspects such as the life span of plants or fewer people-related events early in the timeline.

3. Answers will vary.

Pages 2–3: Number Stretches

Activity One

1. a. Excluding the number 8 567, there are 53 possible 4-digit numbers. These include 8 967, 8 961, 8 927, 8 921, 8 561, 8 527, 8 521, 4 967, 4 961, 4 927, 4 921, 4 567, 4 561, 4 527, 4 521, and all the many ways of using 0, for example, based on 8 967: 8 000, 8 900, 8 960, 8 907, 8 060, 8 067, and 8 007.

b. 4 967

c. 4 000

2. a. 7000 200 5

b. 1000 900 90 3

c. 6000 80 2

d. 4000 500 80 1

e. 2000 200 70 9

f. 5000 90

Activity Two

1. Key pushes are:

a. 7 285 + 410 =

b. 4 163 + 4 031 =

c. 5 834 – 120 =

d. 6 945 – 2 422 =

2. Answers will vary. Amelia could use compact numeral cards to work out what needs to be added to or subtracted from each number. For example, for **b**, Amelia needs to add 4 000 + 30 + 1 (4 031) to change 4 163 to 8 194.

Activity Three

a. 200 is 20 tens and 90 is 9 tens ➔ 29 $10 notes

b. 700 is 70 tens and 50 is 5 tens ➔ 75 $10 notes

c. 1 000 is 100 tens, 800 is 80 tens, and 90 is 9 tens ➔ 189 $10 notes

d. 8 000 is 800 tens, 900 is 90 tens, and 60 is 6 tens → 896 $10 notes

Page 4: Standing Room Only

Activity

1. 28 800
2. Mereana's estimate is the closest.

Page 5: Expanding Horizons

Activity

1. 29 291 m
2. 17 573 km
3. These answers are based on measuring from the centre of each circle marking a port (to the nearest centimetre).
   a. 1 225 km
   b. Approximately 5 250 km

Page 6: Up in the Air

Activity

1. a. Only two itineraries use up all the air points in one trip: either Auckland – Nadi – Honolulu – Rarotonga – Auckland or the reverse: Auckland – Rarotonga – Honolulu – Nadi – Auckland.
   b. Itineraries can include only a return trip to either Brisbane, Nadi, or Nuku'alofa.
   c. Answers will vary. The itinerary in **a** could be extended to include a return trip from Rarotonga to Papeete (2 284 km) or a return trip from Auckland to Nuku'alofa. Another itinerary could be a round trip taking in Brisbane, Osaka, and Nadi.
2. 35 897 air kilometres. The route that uses the least number of air kilometres is Auckland – Brisbane – Osaka – Nadi – Honolulu – Rarotonga – Papeete – Rarotonga – Auckland – Nuku'alofa – Auckland.

Page 7: Fun Rides

Activity

1. Room 1: Log Flume, Minibungee, and the Roller Coaster

   Room 2: Room 2 can go on all rides.

   Room 3: Pterodactyl Flight and Minibungee

   Room 4: Pterodactyl Flight and Minibungee
2. Room 1: $90

   Room 2: $90

   Room 3: $63

   Room 4: $72

Page 8: Tens Time

Activity One

1. He can think of the 5 as being 50 tenths and then divide the 50 tenths by 10. The answer is 5 tenths, which is 0.5 ($\frac{1}{2}$).
2. a. 43.5
   b. Answers will vary but could include: I used Mike's method and then added 40 + 3 + 0.5 to get 43.5.
3. a. 76
   b. 27.5
   c. 6.4
   d. 70.3
   e. 5.94
   f. 3.5
   g. 56
   h. 0.8

Activity Two

$86.40

Discussion will vary.

Page 9: Fun with Fractions

Activity

1. Practical activity
2. Answers will vary. They could include $^2/_8 = {}^1/_4$, $^2/_6 = {}^1/_3$, and $^4/_8 = {}^1/_2$.
3. Answers will vary. Patterns will probably be based on multiples.
4. a. $^1/_3 = {}^2/_6$
   b. $^2/_8 = {}^1/_4$
   c. $^2/_3 = {}^4/_6$
   d. $^3/_4 = {}^6/_8$
   e. $^2/_3 = {}^8/_{12}$
   f. $^3/_6 = {}^1/_2$
   g. $^1/_2 = {}^5/_{10}$
   h. $^1/_4 = {}^{25}/_{100}$

Page 10: More Fractions

Activity

1. He can cut the pie from one mark across to the opposite mark.
2. a. $^1/_2 = {}^5/_{10}$
   b. $^1/_2 = {}^{50}/_{100}$
3. a. He cuts the pie in half using two opposite marks. On each piece, he finds the mark that is $^{25}/_{100}$ from the cut and makes another cut to its opposite mark. For example, a pie marked 0–100 could be cut from 0 to 50 and then from 25 to 75.
   b. $^1/_4 = {}^{25}/_{100}$
4. a. 100 ÷ 5 = 20. So cut from mark 0 to the centre, then from mark 20 to the centre. Do the same from marks 40, 60, and 80.
   b. 100 ÷ 10 = 10. Make five diameter cuts across the pie, the first from 0 to 50, then from 10 to 60, etc. Another way is to cut from 10 to the centre, 20 to the centre, etc.
   c. 100 ÷ 8 = 12.5. Make four diameter cuts: the first from 0 to 50, the second from 12.5 across to 62.5 (12.5 + 50), the third from 25 across to 75 (25 + 50), and the fourth from 37.5 across to 87.5 (37.5 + 50). Another way is to cut from 12.5 to the centre, 25 to the centre, etc.
5. a. $^2/_2 = {}^{10}/_{10}$
   b. $^3/_4 = {}^{75}/_{100}$
   c. $^2/_5 = {}^4/_{10}$
   d. $^7/_{10} = {}^{70}/_{100}$

Page 11: To Market, to Market …

Activity

1. a. $^{20}/_{100}$ ($^1/_5$)
   b. $^1/_{10}$
   c. $^{50}/_{100}$ ($^1/_2$)
   d. $^8/_{10}$ ($^4/_5$)
   e. $^5/_{10}$ ($^1/_2$)
   f. $^{25}/_{100}$ ($^1/_4$)
2. Answers will vary. The $^1/_{10}$ ripe in one of the tens trays must be balanced against the $^5/_{10}$ and $^8/_{10}$ ripe in the other 2 tens trays. $^{14}/_{30}$ (47%) of the tomatoes in the 3 tens trays are ripe, and $^{95}/_{300}$ (32%) in the hundreds trays are ripe. This could mean that, on average, fewer tomatoes in the hundreds trays will ripen before they get to market. Alternatively, you could say that there is not enough evidence to decide either way.
3. 75
4. No, because neither 10 nor 100 is evenly divisible by 3

Page 12: Sweet As

Activity

1. a. 8
   b. 6
   c. The fractions add up to more than 1. $^1/_3 = {}^4/_{12}$, $^1/_2 = {}^6/_{12}$, and $^1/_4 = {}^3/_{12}$. This gives a total of $^{13}/_{12}$, which is more than one whole.
2. a. $^1/_{10}$ ($^1/_2 = {}^5/_{10}$, $^2/_5 = {}^4/_{10}$)
   b. 10
3. She should choose jar **d**, with 16 lollies in it. $^1/_4$ of 36 = 9, $^1/_2$ of 20 = 10, $^2/_3$ of 15 = 10, and $^3/_4$ of 16 = 12. Therefore Jennifer would get more lollies from the **d** jar.

Page 13: Friendly Fractions

Activity

1. 0.5
2. 0.75
3. a. 1 ÷ 10 =

   b. 1 ÷ 4 = or 2 ÷ 8 =

   c. 8 ÷ 10 = or 4 ÷ 5 =

   d. 1 ÷ 3 = or 2 ÷ 6 =

   e. 3 ÷ 2 = or 6 ÷ 4 =

   f. 5 ÷ 8 =

   g. 2 ÷ 3 = or 4 ÷ 6 =

   h. 7 ÷ 4 =

   i. 3 ÷ 8 =

   j. 1 ÷ 6 =
4. a. All the fractions equal 1.

   b. The top number is the same as the bottom number. A number divided by itself gives 1.

Page 14: Racing to New Heights

Activity

1. Start – Before step 1

   $^{1}/_{4}$ way up – step 250

   $^{1}/_{2}$ way up – step 500

   $^{3}/_{4}$ way up – step 750

   $^{1}/_{3}$ way up – between step 333 and step 334

   $^{2}/_{3}$ way up – between step 666 and 667
2. 1 000 is not evenly divisible by 3.
3. $^{3}/_{8}$ way up
4. She has run up $^{400}/_{1\,000}$ of the steps, so she still has $^{600}/_{1\,000}$ ($^{6}/_{10}$ or $^{3}/_{5}$) of the steps to climb.
5. $^{375}/_{1\,000}$ ($^{3}/_{8}$)

Page 15: Decimal Day

Activity

1. Centre: Minnie (tallest)

   Guards: Joe and Stretch (second and third tallest)

   Forwards: Ruth and Kevin (shortest)
2. a. Joe and Stretch

   b. Joe is a guard, and Stretch is a forward.

   c. Ruth and Kevin

Game

A game using decimals

Page 16: Dealing with Decimals

Activity One

a. 8.37 8.57 8.54 1.34 1.37 1.54 1.57

b.

8.	0.3	0.07
8.	0.5	0.07
8.	0.5	0.04
1.	0.3	0.04
1.	0.3	0.07
1.	0.5	0.04
1.	0.5	0.07

Activity Two

1. 6.09 **+ 0.2** =
2. a. 4.71 **– 0.7** =

   b. 8.88 **– 0.08** =

   c. 3.25 **– 3** =

   d. 2.743 **– 0.04** =

   e. 3.55 **– 0.5** =

   f. 4.52 **– 0.02** =

   g. 7.39 **+ 0.3** =

   h. 9.027 **– 0.007** =

Page 17: Rounding Up and Down

Activity

1. a. 30
   b. 70
   c. 90
   d. 110
   e. 280
   f. 1 000
2. a. 2
   b. 3
   c. 4
   d. 8
   e. 18
   f. 277
3. Answers will vary. Making clothes is one example. Rounding down when cutting material could make the clothes too small. Another example would be buying timber for a house or renovation project.

Investigation

The cheapest solution is for Hirini to buy two 3 m lengths and six 5 m lengths. This is based on the following:

1.8 + 3.2 = 5 m	1.8 + 1.2 = 3 m
1.8 + 3.2 = 5 m	1.8 + 1.2 = 3 m
0.8 + 0.8 + 1 + 2.4 = 5 m	2.4 + 2.4 < 5 m
0.8 + 0.8 + 1 + 2.4 = 5 m	2.4 + 2.4 < 5 m

Page 18: Quick Thinking

Activity One

1. a. Adjust his rounding like this:

   20 (+ 4) + 40 (– 3) + 20 (– 1) + 40 (+ 2)
   = 120 (+ 4 – 3 – 1 + 2)
   = 120 + 2
   = 122

   b. Both 24 + 37 and 19 + 42 are equal to 61. 61 + 61 = 122.

   c. Adjust each number from 30. 24 is 30 – 6, 37 is 30 + 7, 19 is 30 – 11, 42 is 30 + 12. 120 – 6 + 7 – 11 + 12 = 122.

   d. Calculate the exact total of the ones digit and then add the tens digit totals:
   4 + 7 + 9 + 2 = 22. 22 + 100 = 122

2. Methods will vary. Likely estimates are:
   a. 160 **b.** 160 or 170
   c. 160 **d.** 170 or 180

   (Students may allow for the fact that most of the numbers in **b** and **d** round up.)

Activity Two

Answers will vary. A likely estimate is $4.60.

Page 19: Dead Calculators

Activity One

1. Answers will vary. They could include:

   a. (10 + 10 + 6 + 1) + (10 + 30 + 3 + 1 + 1) = 72, or 30 – 3 + 63 – 18 = 72, or (3 x 9) + (6 x 9 – 9) = 72

   b. 349 + **1** = 350

   350 + **250** = 600

   600 + **14** = 614

   So, 614 – 349 = 1 + 250 + 14
   = 265

   c. 47 + 47 + 47 + 47 + 47 + 47 + 47 + 47 = 376

   d. 312 – 13 – 13 – 13 … until the answer is 0, and count the number of times you have subtracted 13 (24) or use the constant function (312 – 13 = = = =)

2. Answers will vary.

Activity Two

1. Yes
2. Some ways of making each number are:

   1 = 5 – 2 – 2

   2

   3 = 5 – 2

   4 = 2 + 2

   5

   6 = 2 + 2 + 2

   7 = 5 + 2

   8 = 5 + 5 – 2

   9 = 5 + 2 + 2

   10 = 5 + 5

11 = 5 + 2 + 2 + 2

12 = 5 + 5 + 2

13 = 5 + 5 + 5 – 2

14 = 5 + 5 + 2 + 2

15 = 5 + 5 + 5

16 = 5 + 5 + 2 + 2 + 2

17 = 5 + 5 + 5 + 2

18 = 5 + 5 + 5 + 5 – 2

19 = 5 + 5 + 5 + 2 + 2

20 = 5 + 5 + 5 + 5

Page 20: Time for Change

Activity

1. 70c each

2. One way is for Leigh to pay Tio 80c, and Tio would give her 10c change. Greg can then pay Tio $1, and Tio would give him 30c change.

3. One way is: Leigh pays the total fare of $2.25. Greg pays Leigh $1.05. Tio pays Leigh 80c and gets 5c change. Leigh then pays Greg the 30c change she owes him.
Another way is: Leigh pays the total fare of $2.25. Tio pays Greg 80c and gets 5c change. Greg then pays Leigh $1.50.

Page 21: Speedy Types

Activity One

1. Practical activity. Answers will vary.

2. Answers will vary. (Estimates of the number of words in books should be discussed.)

Activity Two

1. **a.** Odd

b. Odd

c. One of the page numbers will always be odd and the other even. Only two even numbers added together or two odd numbers added together can give an even result.

2. 1 482

3. Answers will vary.

Page 22: High-powered Thinking

Activity

1. **a.** Answers will vary but could include:

i. The office block is 2 offices deep.
If you look at each column of windows in the front, there are 2 x 26 offices behind each column. Since there are 5 columns, there are 5 x 52 = 260.

ii. Each floor has 2 x 5 = 10 offices.
There are 26 floors, so there are 10 x 26 = 260 offices in the whole building.

iii. There are 5 x 26 offices with windows on the front of the building and 5 x 26 offices with windows on the back of the building. 5 x 26 = 130, and 130 x 2 = 260.

b. Answers will vary.

2. Strategies will vary. Actual number of offices are:

a. 290

b. 392. (All offices have windows, so there are no internal offices in the middle row.)

Page 23: Human Pyramids

Activity One

1. 14.5 kg

2. Todd 23.25 kg, Sue 45.5 kg, Vince 22.25 kg

3. Jo none, Sue and Tina 14.5 kg, Rua 27.25 kg, Kim 52.5 kg, Vince 25.25 kg, Henry 35.625 kg, Andrew 75.875 kg, Matt 70.375 kg, Barbara 30.125 kg

Activity Two

a. Answers will vary.

b. Answers will vary.

c. The middle person supports the most mass (just over the mass of 3 people), the next 2 support about 2.5 times the mass of 1 person, and the end people support less than the weight of 1 person.

Page 24: Budgie Buying

Activity

1. **a.** $47.95

   b. $22.05

2. **a.** $5.39, rounded to $5.40

   b. $6.23, rounded to $6.25

   c. $10.40

3. $8.15

4. Yes. The bell, ladder, and cuttlefish would cost $5.21 (rounded to $5.20) at sale price. Alison had $8.55 left after she bought the extra perch ($10.40 – $1.85). The packet of seeds would cost $1.70. $5.20 + $1.70 = $6.90, so Alison does have enough money for extra bird seed.

Figure It Out

Number

Teachers' Notes

Overview: Number

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Sweet As	Finding fractions of whole numbers	12	22
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Page 1: When?

Achievement Objective

- explain the meaning of the digits in any whole number (Number, level 3)

Activity

This activity uses a timeline showing some aspects of New Zealand history as a context for recognising and ordering numbers up to four digits in size. Students need to understand that the range of dates chosen for this timeline represents only a small fraction of time. They should also understand that time did not start at 100 AD.

Question 1 may be introduced by brainstorming the approximate dates of events in world history known to students and ordering these in sequence. The following dates may help keep the timeline in perspective:

- Dinosaurs vanished approximately 65 million years ago.
- The Stone Age lasted from about 3 000 000 BC to 8000 BC.
- The Bronze Age ranged from 1900 to 500 BC.
- The Giza pyramids and the Sphinx were built about 2500 BC.
- Alexander the Great was born in 356 BC and died in 323 BC.
- Julius Caesar was born in 100 BC and was assassinated in 44 BC.

(These last two facts can be used to show how we count back down to 1 when using BC dates before we count forward again with AD dates.)

- The ancient Roman city of Pompeii was destroyed by the volcanic explosion of Vesuvius in 79 AD.

Timelines can be different from number lines because dates do not have to be marked on the timeline in regular intervals (although the timeline in this exercise does lend itself to regular spacing of dates).

There is no date between 1 BC and 1 AD, but a number line would have to have a 0 between $^{-}1$ and $^{+}1$. (This historical anomaly is the reason that mathematicians say that the second millennium finishes only at the end of 2000 AD. The first millennium didn't start until 1 AD as there was no year 0.)

Question **2** is open-ended. Students who need prompting may be helped by questions like:
"Which fraction of the timeline shows events about people?"
"Why can't we find many people events in the first quarter of the timeline?"
"Can you prove that some plants live longer than people?"
"What other volcanoes or earthquakes could we put on the timeline?"

(The students could look up dates for the Rangitoto eruption, the Napier earthquake, the Murchison earthquake, and some of the Ruapehu eruptions.)

Students could add these dates to the timeline as well as other dates that are important for them, such as when they started school or when brothers and sisters were born.

Pages 2–3: Number Stretches

Achievement Objectives

- explain the meaning of the digits in any whole number (Number, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)

Activity One

This activity helps develop understanding of the meaning of digits in four-digit whole numbers. There is a copymaster of Amelia's set of compact numeral cards at the back of this booklet.

The arrows on the right of each card help students align the cards correctly because they are arranged with the ones numeral card on top through to the thousands numeral card on the bottom.

Question **1a** is a challenging investigation if students are going to find all the possibilities. You may wish to let students work by themselves or in pairs to see how many four-digit numbers they can find in 5 minutes and then bring all the students together to discuss their findings and the strategies they used. Highlight the need for a strategic approach. If they need help, you could suggest:
"If you work out all those numbers that use the 8 000 card, how can you use this list of numbers to easily find the numbers for the 4 000 card?"
"Have you tried recording all the possible numbers from the largest to the smallest?"
"If you know the four-digit numbers that use the 8 000 and the 500 cards, how can you use this list of numbers to make all the numbers with the 8 000 and the 900 cards?"
These sorts of prompts may help students find the 54 possible four-digit numbers that can be made with the set of cards if you include all the ways of using zero.

The question asks "What other four-digit numbers ...", so they will have to exclude the number 8 567, which will leave 53 different four-digit numbers.

Questions **1b** and **1c** are ideal for exploring the students' mental thinking strategies if, once they have decided on an answer in their heads, you ask them: "How do you know?" or "How many different ways did we use to find the solution?"

In question **1b**, the solution will clearly have to be the biggest possible number that can be made using the 4 000 card. This will be 4 967.

Question **1c** compares the difference between 8 967 and 6 500 with the difference between 4 000 and 6 500.

Question **2** could be used as a good assessment task to find out whether the students have grasped the objective of the activity (understanding the meaning of digits in four-digit whole numbers).

Activity Two

You may have to tell students that they cannot use the cancel button on the calculator.

For students having difficulty with question **1a**, use the compact numeral cards as suggested in question **2**. The students could make 7 285 with compact numeral cards and then work out which cards would need to change and by how much. They should find that the 200 card will need to become 600 and the 80 card will need to become 90, which is a total increase of 410. Ask students which buttons they should press on the calculator to increase the total by 410. They could then continue with the rest of question **1**.

Activity Three

If students come up with a quick method, such as "Amelia needs to subtract only a zero from the $430 and this will tell her she has 43 $10 notes", validate the idea but correct their incorrect use of the term "subtract": "If Amelia subtracts zero from $430, she should still have $430."

It may be better to say “If Amelia covers or removes the ones digit, she can see how many $10 notes she will get.”

As an extension question, ask them “How many $20 notes could Amelia get for each of the amounts?”

Page 4: Standing Room Only

Achievement Objectives

- explain the meaning of the digits in any whole number (Number, level 3)
- make sensible estimates and check the reasonableness of answers (Number, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)

Activity

This activity lends itself to students working co-operatively. Organise the students into mixed-ability teams of three. Give each member one of the statements in the speech bubbles. They may read it to the others, but they cannot pass it over to them. They work together to solve question **1**. When the group thinks it is solved, each member reads their statement and checks that it was used in the solution.

Groups having difficulty could try a simpler example, such as a stadium that has four sections, five rows in each section, and 10 seats in each row. Highlight this use of a simpler approach as a good problem-solving strategy for them to use in the future.

Ways of solving the questions may then be shared between the groups. Guide the discussion by asking “Which of the methods is the most efficient?”

Discuss the equation $12 \times 30 \times 80 = \square$ as a model of an efficient way of solving this type of problem. Students could be asked to make up other problems that would be solved by multiplying each part of the given information.

Page 5: Expanding Horizons

Achievement Objective

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

Activity

This activity uses interesting real-life large numbers to provide a context for making and solving addition and subtraction problems.

Have the students estimate before calculating.

Questions **1** and **2** may be good challenges as mental calculation for some students. Discuss the “in the head” strategies that some students may have used. Some students may add the thousands first. This is acceptable, for example, “In the thousands, there are three lots of 8 and another 3, that’s 27 thousands. In the hundreds, there is a 6, a 7, and an 8, that’s 21 hundreds. We now have 29 thousand, 1 hundred. Seventeen tens makes another 170. Now we have 29 270. Twenty-one in the ones place will make a total of 29 291 metres.”

You may wish to generate more calculations with additional questions, for example, "What is the difference in height between Mount Everest and Mount Cook?" "Approximately how many times longer than the Clutha is the Nile?"

This activity is a good model to help students generate their own interesting geographical questions. Use online or book sources, such as atlases or encyclopaedias that feature large numbers.

Page 6: Up in the Air

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)

Activity

This activity provides the opportunity for addition and subtraction of large whole numbers in a problem-solving context that will encourage logical thinking.

Make it clear that each of the Chua family has 15 000 air kilometres, so they will all be travelling together. Students will also have to realise that the Chua family will need to start and finish at Auckland because it is their home town. The Chua family will not want to pay for any extra air travel to get home again.

Students may understand the problem better if they make a chart showing combined totals of the different routes out of Auckland:

Route	Distance (km)	Route number
Auckland–Brisbane–Osaka	9 451	1
Auckland–Nadi–Osaka	9 470	2
Auckland–Nadi–Honolulu	7 260	3
Auckland–Nuku‘alofa–Honolulu	7 090	4
Auckland–Rarotonga–Honolulu	7 740	5
Auckland–Nuku‘alofa	2 003	6
Auckland–Rarotonga	3 010	7
Auckland–Brisbane	2 295	8
Auckland–Nadi	2 157	9
Rarotonga–Papeete–Rarotonga	2 284	10

Route 10 is included because Papeete can only be accessed through Rarotonga.

Because Osaka is over 9 000 kilometres away from Auckland, there is no way that the Chuas can go there in question **1a**. Route numbers 3 and 5 add up to 15 000 and provide a workable itinerary.

Questions **1b** and **1c** ask where the Chua family could travel. Decide whether you want your students to find some or all of the possible routes. Remind the students that it is all right for the Chuas to do less than the limit of their air kilometres but they cannot do more.

Question **2** can be done by trial and error, but some hints may help the students approach the problem logically, for example, "Only two places need to be visited more than once. Which are they? Discuss your reasons." (These are Auckland and Rarotonga: Rarotonga because you must come back from Papeete, and Auckland because it is home and also, in the least air-kilometres itinerary, because it must be visited to get to Nuku'alofa.)

Page 7: Fun Rides

Achievement Objective

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

Activity

In this activity, students use the factors of a number to solve some problems. It also provides an opportunity to discuss some of the divisibility rules.

Ask students to discuss the implications of Mr Mo's rules. They may need to use a simpler example to grasp the full meaning of his instructions, for example, "Using Mr Mo's rules, which rides could a class of seven pupils take? How about a class of eight pupils? How about a class of nine pupils?"

Multiplication table charts could be used as well. Ask students to find all the tables that contain the multiple 30. Do the same with 24, 28, and 32. Students would then be able to use these to decide which rides each class can take.

Students could also use calculators to see whether the class size has a remainder when divided by the number of riders per carriage for each ride. If it does, then that class cannot go on that ride.

If the students understand the divisibility rules for 2, 3, 4, and 6, then they could apply these to the problem. Some divisibility rules that may be useful include:

- A number is divisible by 2 if it is an even number.
- A number is divisible by 3 if the sum of its digits is divisible by 3.
- A number is divisible by 4 if half the number is an even number.
- A number is divisible by 6 if it is divisible by 2 and by 3.

Page 8: Tens Time

Achievement Objectives

- explain the meaning of the digits in decimal numbers with up to 3 decimal places (Number, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

Activity One

This activity explores what happens to whole numbers when they are divided by 10 and 100. Students will need some understanding of the meaning of the digits in one- and two-place decimals.

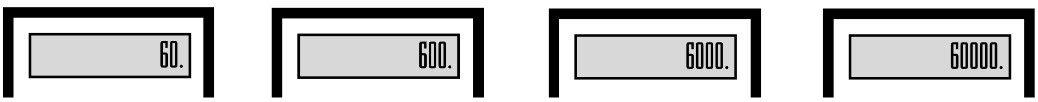
This is a major concept for students in their understanding of our number system and how they operate it. It will probably take a number of lessons before most students are confident with the workings of the decimal system. (The *Number* booklet also deals with decimals on pages **13**, **16**, and **17**.)

The following concepts may have to be revisited before this activity is attempted:

- What happens to numbers when they are multiplied by 10 and 100.
- Each column in our number system grows 10 times larger as we move to the left.
- Each column in our number system grows 10 times smaller as we move to the right.
- The meaning of fractional notation, for example, $^{3}/_{10}$ means three parts out of the 10 that make up the unit.
- The values of digits in the first and second decimal places.
- The role of the decimal point in establishing the ones or units column immediately to its left.

Patterning can be a valuable way of helping students grasp the ideas involved in this activity. Use the constant function on a simple calculator to show the patterns of multiplying and dividing by 10. This can be very effective in helping students understand the concept of decimal places. To set up multiplying by 10 as a constant function, they will have to press the [10] [x] keys first. (Some calculators may require [10] [x] [x] keys to be pressed.)

After they have entered [10] [x], ask students to enter [6] on their calculator. Ask them "What will the digits be if we multiply this by 10? (60 or six zero) We can answer this by simply pressing the [=] key. This will multiply 6 by 10, and the display will now show 60. Watch what happens to the 6 as we continually multiply by 10. (60 becomes 600, 600 becomes 6 000, 6 000 becomes …)"

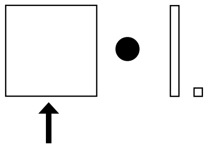


The students should be able to see the pattern. You may want the class to verbalise this: "As the 6 moves from tens to hundreds to thousands and so on, it moves one place to the left."

To show dividing by 10 on the calculator using the constant function, the students could enter [600 000] [÷] [10] [=] [=] [=] [=]. Each time they press [=], the calculator will divide by 10. (On some calculators, students may need to enter [10] [÷] [÷] [600 000] [=] [=] [=] [=].)

Have the students press [=] until the 6 has moved down to the ones place. Discuss what will happen if they press [=] again. Discuss the meaning of the display 0.6 as six-tenths and how the 6 in this place is 10 times less than the 6 was in the ones place. Have them press [=] again to show 0.06. Discuss the meaning of this as well.

You may wish to model this with place value blocks. Emphasise the use of the decimal point to create the ones place as shown in the following diagram:

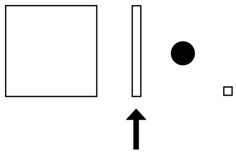


The decimal point makes this block the ones block.

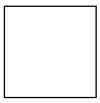
The ▯ block is $^{1}/_{10}$ of the ones square and shows the tenths place.

The ▫ block is $^{1}/_{100}$ of the ones square and shows the hundredths place.

Vary the place of the decimal point in the place value block model to emphasise its role in defining the ones place.



The decimal point makes this block the ones block.

The  block is 10 of the ones rod and now shows the tens place.

The ▫ block is $^{1}/_{10}$ of the ones rod and now shows the tenths place.

Activity Two

This activity is a good context for applying the students' understanding of the place value when a number is divided by 10.

After students have calculated the share value as $86.40, take the opportunity to compare the language of "eighty-six dollars and forty cents" with the decimal equivalent, "eighty-six point four dollars." Students may not realise that these are equivalent statements. Make sure students can explain why 40 cents is the same as 0.4 of a dollar.

Page 9: Fun with Fractions

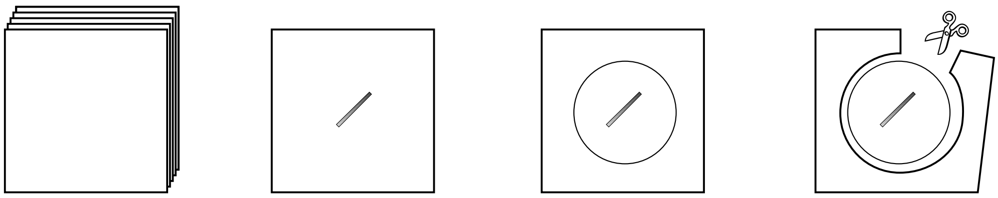
Achievement Objective

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)

Activity

This activity will help students find simple equivalent fractions.

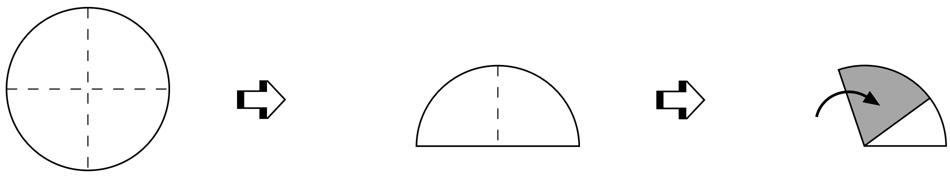
An efficient way to cut out paper circles is to:



Put several pieces of paper in a pile. Staple through the pile. Draw around the staple, using a circular plate. Cut out the circles and then undo the staple.

Note: You will still need to hold the edges of all your pieces of paper as you cut around your circle shape to stop the bottom pieces slipping out of line.

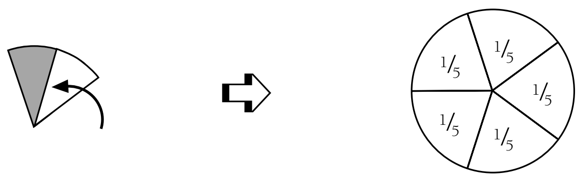
Folding the circles into quarters is straightforward, but students will have to estimate their folds for thirds and also for fifths, if you wish to extend the activity to include fifths. For example, for fifths:



Find the centre by folding quarters.

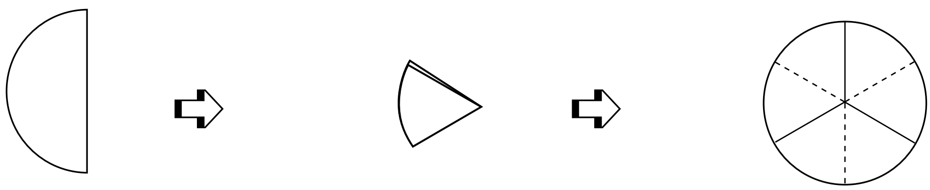
Fold in half.

Starting at the centre, fold so that the section being folded over is twice the size of the remaining section and crease firmly.



Fold the remaining section halfway across the already-folded section and crease firmly.

Thirds can be folded as follows:



Fold the circle in half.

Fold the half in thirds by bringing the outside edges into the centre to form a cone shape. Flatten the cone and crease.

To understand fraction notation, students need to keep in mind the unit of which the fraction is a part. In question **1**, the unit is the whole circle. Use language that shows the meaning of the fraction notation. Half of a circle can be described as “one of the two pieces that make the whole circle” and three-quarters as “three out of the four quarters that make the whole circle”. This focus will help students see that the numerator tells them how many pieces they have, while the denominator tells them how many pieces the whole unit is divided into.

An interesting, open-ended starter question is “Can a quarter be bigger than a half?” Encourage students to justify either a positive or a negative response, for example, “A quarter of an elephant is always bigger than half a flea.” “A quarter of this apple is always smaller than a half of this same apple.”

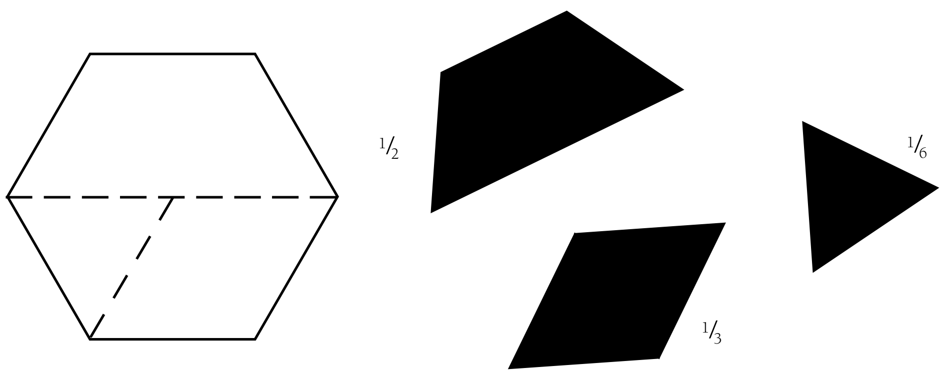
You could then ask the students “So how do you know how big a fraction is?” They should realise that this depends on how big the whole unit is. This shows the importance of both the numerator and the denominator working together to describe the fraction.

When students are looking at the pattern of their digits in question **3**, check to see whether they have mistaken their equivalent fraction pattern for a sequential pattern.

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$$

If students focus on the numerator and denominator as separate entities, they may think of the numbers as increasing in size. They need to see that the fraction has not changed, but the unit has broken into more and more parts.

Use other shapes besides a circle. A rectangle is a good shape to use to show halves, quarters, and eighths because it has four sides. Hexagons are excellent for showing halves, thirds, and sixths because they have six sides. Pattern blocks are very good hands-on equipment for showing equivalent fractions based on the hexagon.



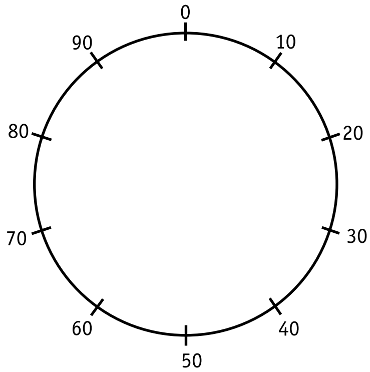
Page 10: More Fractions

Achievement Objective

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)

Activity

This activity uses a circle with 100 divisions to help students express simple fractions as tenths and hundredths. A copymaster is provided at the back of this booklet.



Although the students could use any two opposite points to answer question **1**, it would be useful if they labelled the large marks in tens starting with a zero and moving either clockwise or anticlockwise.

They would then be able to count the number of tenths that makes a half of the circle. They would need to realise that there are 100 small divisions to answer question **2b**.

To do question **4a**, students will have to find the centre of the pie and make their cuts from 0, 20, 40, 60, and 80 to the centre. The centre can be found at the intersection of any two diameters.

Question **4c** will present an interesting challenge because students will not be able to divide 100 marks exactly by eight. Once students have found out that 12.5 divisions are needed for each of the eighths, they could make a table of eighths translated to their respective hundredths amount.

Eighths		Hundredths
$^1/_8$	=	$^{12.5}/_{100}$
$^2/_8$	=	$^{25}/_{100}$
$^3/_8$	=	$^{37.5}/_{100}$
$^4/_8$	=	$^{50}/_{100}$
$^5/_8$	=	$^{62.5}/_{100}$
$^6/_8$	=	$^{75}/_{100}$
$^7/_8$	=	$^{87.5}/_{100}$
$^8/_8$	=	$^{100}/_{100}$

Page 11: To Market, to Market ...

Achievement Objectives

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- effectively plan mathematical exploration (Mathematical Processes, problem solving, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 3)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 3)

Activity

Questions **2** and **4** provide opportunities for the mathematical processes achievement objectives listed above. You may decide to select one or two of them for particular attention. Interpreting information and results in context would be the objective to focus on when you ask students to think logically in question **2**.

This activity provides a context for converting simple fractions to tenths and hundredths.

Question **2** has the potential for some rich involvement in mathematical processes because it is open-ended and presents a number of variations. Students may choose tray **b** as the best kind because only $^{1}/_{10}$ of this tray is ripe and that is less than the others. Challenge this response with the question "Is this type of tray always the best?" Students may then look more carefully at the two types of tray being compared.

Students could draw a chart that converts fractions to percentages:

Hundreds tray	**Tens tray**
a = $^{20}/_{100}$ = 20%	**b** = $^{1}/_{10}$ = $^{10}/_{100}$ = 10%
c = $^{50}/_{100}$ = 50%	**d** = $^{8}/_{10}$ = $^{80}/_{100}$ = 80%
e = $^{25}/_{100}$ = 25%	**f** = $^{5}/_{10}$ = $^{50}/_{100}$ = 50%

Overall, the hundreds tray is better. Students could explain this in a number of ways. They may see that tray **c** matches tray **e** and that tray **a** is 10 percent worse than tray **b**. But tray **f** is 55 percent better than tray **d**, so the hundreds tray must be better. Students could average the ripeness. The hundreds trays have an average ripeness of approximately 32 percent, and the tens trays have an average ripeness of approximately 47 percent.

In question **3**, work as a class to make a chart showing some of the different arrangements for three-quarters ripeness.

For further investigation, see "One Bad Banana" in *Connected 2 2000*.

Page 12: Sweet As

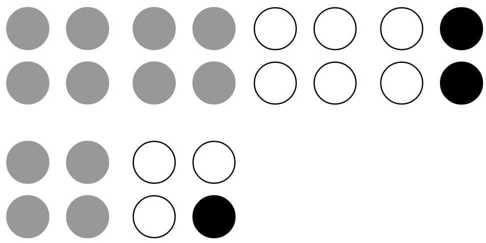
Achievement Objective

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)

Activity

This activity provides a context for finding fractions of whole number amounts, and it also highlights the use of the common denominator when combining and comparing fractions.

Have students use counters or multilink cubes to model the arrangements in each mix of sweets. If they find different ways of making each mix, have them compare equivalent fractions; for example, Kane's Winegum Mix may be arranged using 16 sweets or eight sweets:



$^1/_2$ Blue	$^3/_8$ White	$^1/_8$ Green
$^8/_{16}$	$^6/_{16}$	$^2/_{16}$
$^4/_8$	$^3/_8$	$^1/_8$

Ask students "What fraction of Kane's mix is white?"

Point out that the lowest common denominator is eight, so the smallest number of sweets in the Winegum Mix is eight. Also point out that the fractions have to add up to a whole because the factory has to make a whole packet of sweets.

Students will need to add $^1/_3$ and $^1/_2$ and $^1/_4$ for question **1c**. To do this, they will need to translate these fractions to get a common denominator, such as 12.

As explained in the answers, $^4/_{12} + ^6/_{12} + ^3/_{12} = ^{13}/_{12}$, which is more than one whole, and so the factory cannot make a whole packet of sweets with that combination of fractions.

In question **3**, students could make an attractive chart to present to the class, showing the results of their exploration.

Page 13: Friendly Fractions

Achievement Objective

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)

Activity

In this activity, students will learn to use a calculator to convert fractions to decimals. They will also explore fractions equivalent to 1.

Make sure that students understand that $0.5 = ^5/_{10} = ^1/_2$ and that $0.75 = ^{75}/_{100} = ^3/_4$. Although question **3** is based on the use of whole numbers from 1 to 10, students could explore the basic structure of tenths, hundredths, and thousandths to give them a correct sequence. For example, $1.75 = ^{175}/_{100} = ^7/_4 = 1^3/_4$.

Have students prove their answers on a calculator.

Question **4** highlights how important it is that students understand equivalent fractions of one whole. An example would be a pie chart divided into thirds. Help students to understand that $\frac{2}{3}$ means two of the parts that make one whole and $\frac{3}{3}$ means three of the three parts that make one whole, so $\frac{3}{3}$ is the same as one.

Page 14: Racing to New Heights

Achievement Objective

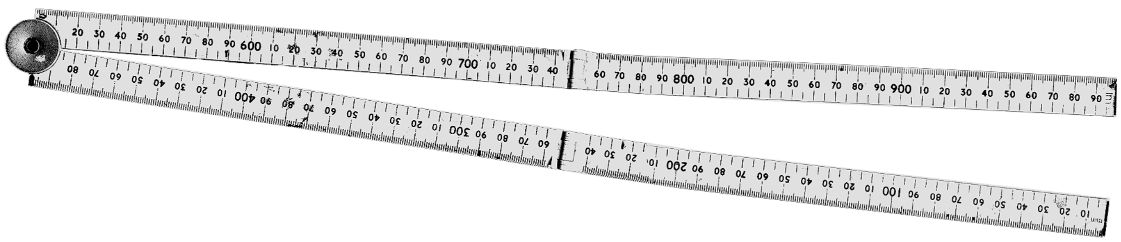
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)

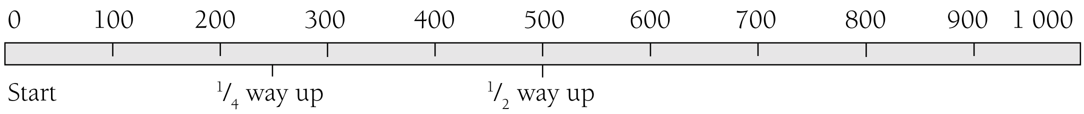
Activity

This activity looks at finding fractions of one thousand.

Check where students would put the start label. It should go before the first step, not on it. When the students put the $\frac{1}{2}$ way up sign at the 500th step, they should explain their choice as "Halfway is 500 out of 1 000, so we place the sign at the 500th step."

A number line or a metre rule would be a useful aid for students.





After students have found the solutions, ask them to investigate the equivalences with 1 000. A chart similar to the one below would be useful here.

Fraction	Out of 1 000	As a thousandth	Simplest form
$\frac{1}{2}$	500	$\frac{500}{1\,000}$	$\frac{1}{2}$
$\frac{1}{4}$	250	$\frac{250}{1\,000}$	$\frac{1}{4}$
?	600	$\frac{600}{1\,000}$	$\frac{3}{5}$
?	375	$\frac{375}{1\,000}$	$\frac{3}{8}$

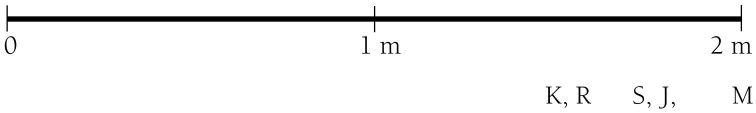
Page 15: Decimal Day

Achievement Objectives

- order decimals with up to 3 decimal places (Number, level 3)
- explain the meaning of the digits in decimal numbers with up to 3 decimal places (Number, level 3)

Activity

This activity focuses on ordering decimals to two places. Students could model the height of each player using two metre rulers on the floor.



K, R, S, J, and M are the initial letters of the players.

Introduce question **2** carefully. Students may have difficulty fully understanding the question. They need to be clear who is in the original team and who the new players are. Read it through with the students. Highlight the team changes by asking: "Who are the two new players? Who must have been left out if Aroha is now the shortest?"

Game

This game is short and simple but very effective for challenging students' understanding of the digits involved in decimal numbers.

Have the winning students justify their result by talking about the place value of their digits. Vary the rules to create more learning opportunities. For example, you could ask students to "Make the largest decimal with a two-digit whole number in it. Make the smallest decimal with a one-digit whole number in it. Make the biggest number you can that is smaller than 7 but larger than 3.9."

Page 16: Dealing with Decimals

Achievement Objective

- explain the meaning of the digits in decimal numbers with up to 3 decimal places (Number, level 3)

Activity One

This activity extends students' place-value understanding to three-digit decimals.

Note that these decimal compact numeral cards differ in shape from those used for page **2** in that they have the alignment arrow on the left so that the decimal numeral can be expanded to the right. A copymaster of compact numeral cards is provided at the back of this booklet.

Activity Two

If students have difficulty with question **2**, they could use a set of compact numeral cards. They could then explore what to add or subtract to make the required change in one operation.

Page 17: Rounding Up and Down

Achievement Objective

- make sensible estimates and check the reasonableness of answers (Number, level 3)

Activity

This activity highlights the rules used to round numbers up or down. The most difficult rounding rule occurs when the last digit is a 5. The rule here is to round up.

You may wish to compare the mathematics rules for rounding with the way a local shop rounds prices when people pay for an item with cash. This is a necessary application of rounding brought about by the absence of 1 cent and 2 cent coins. Some shops round $4.98 up to $5, and others round $4.98 down to $4.95.

“Shopping Around” page 17, *Measurement*, Figure It Out, Levels 2–3 and its accompanying teachers’ notes deal with rounding.

Question **3** shows that in contextual situations, common sense sometimes dictates the way people round. For example, if 94 people were expected to attend a parents’ evening at a school, it might be more sensible to put out 100 chairs rather than 90.

Investigation

This investigation builds on the rounding practice in the activity on this page. Students will need to look for ways to minimise wastage as well as cost.

Page 18: Quick Thinking

Achievement Objective

- make sensible estimates and check the reasonableness of answers (Number, level 3)

Activity One

Encourage students to develop mental strategies to solve number problems such as those in this activity. This is one of the most important things that teachers can do on a daily basis in the classroom to develop real mathematical thinking in students. Note that it is the developing and sharing of strategies for solving a question that is the important focus, not the number of questions solved mentally.

Think about ways of encouraging students to try some new strategies and to share successful ones with the class. A display showing “Strategy of the Week” may be encouraging for students. “Question of the Week” could be another display, with the question being the one that was solved using the greatest variety of strategies over the week. Show the different strategies on the chart.

Challenging variations will include three- or four-digit addition, where students may group thousands or hundreds before tens and ones, for example, 4 268 + 862 + 4 349 + 721. “Two lots of 4 000 is 8 000. 200 + 800 is 1 000. 300 + 700 is another 1 000. So far, we have 10 000. Two sixties is 120, and four tens and two tens is another 60. That’s three sixties, which is 180. 8 + 2 makes 10, and so does 9 + 1, so the ones add up to 20. Now we have 10 000 and 200, that’s 10 200.”

More challenging contexts would involve operations other than addition as well as combinations of operations.

Activity Two

This activity applies rounding skills to estimation in a practical context.

Estimation is more realistic if there are three or more addends involved.

Other situations in which students could estimate total prices of items are:

- buying items that are in dollars only (two-digit, three-digit, or four-digit amounts)
- buying items that are in combinations of dollars and cents (three-digit or four-digit amounts).

You can use other contexts besides money, such as:

- sports: the total points scored by your favourite netball or rugby team over a whole season
- distances: the total distance travelled when you know the distance between each stop on the way
- time: the total number of minutes on a music CD when you know the length of each track.

Page 19: Dead Calculators

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)

Activity One

This activity encourages students to use problem-solving strategies with the four operations and whole numbers.

Question **1a** is more open-ended than the others because it is the digits that cannot be used rather than the operations. Encourage students to find as many ways as possible of solving this particular question. Suggest that they may wish to combine operations, including multiplication and division, for example, 30 – 3 + 63 – 18 or (3 x 9) + (5 x 9) – 9. To do this on a basic calculator, students may need to use the memory keys because the calculator will not have brackets. Students could see whether their calculator can do this: [3] [x] [9] [M+] [on/c] [5] [x] [9] [M+] [MR] [–] [9] [=].

Students who use mental strategies may come up with 8 x 9 = because they recognise that 27 is 3 nines and 45 is 5 nines, so 27 + 45 must be 8 nines.

Activity Two

This is a good problem-solving activity. You may wish to ease students into it by suggesting that they start by only making the numbers from 1 to 5. Check that students are being systematic. Recording their results in sequence from 1 to 5 is one systematic approach.

Another approach is to find the easy ones first. This way, you will soon see which students have not got the idea of combining a number of twos or a number of fives as well as making combinations of twos with fives.

Extend this activity with questions such as:
"Try two other numbers besides 2 and 5 with the addition and subtraction keys."
"Could we make all the totals from 1 to 20 using two even numbers?"
"Try to solve the problem using the multiplication key as well."

Page 20: Time for Change

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)

Activity

These are challenging problem-solving questions involving addition and subtraction with money.

In problem solving, ensure that students understand the question before they attempt the problem. Have some ways of helping the students if they are finding the problem too difficult. Hints are a good way of keeping the students going. Use them sparingly, not too soon but before frustration sets in.

A hint for question **2** could be "Sort out Leigh's debt to Tio before you do Greg's."

Question **3** is more challenging. The students may need several hints. Let them try out some of their own solutions before giving each hint.

First possible hint:
"Find out who could pay the exact fare for all of them." (Leigh)

Second hint:
"What coins, adding up to more than 75 cents, would Tio and Greg pay Leigh to get things started?"
(Tio would give Leigh 50c + 10c + 10c + 10c, and Greg would give her $1.)

Third hint:
"What coin does Leigh now need to give Tio, and where will it come from?" (5 cents, and it must come from Greg.)

Fourth hint:
"If Greg gives Leigh another 5 cents, how much does Leigh have to give back to Greg?"

Page 21: Speedy Types

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)

Activity One

This activity provides some challenging problems using addition, multiplication, and division.

Students may need to use a calculator to multiply and divide.

You may want to decide which type of book to give to different groups of students. A book with large type may make the activity a little easier.

If minute timers with alarms are not available, use partners to time students' typing. Students should identify the final word of the 10 lines before they start to type.

To answer question **1b**, students will have to divide the number of words by the time. The time will probably be in minutes and seconds. They may need help here. They could change the time to seconds. Divide the words by the seconds and then multiply by 60 to find the words per minute. Have students round their answers to the nearest whole number. For example, "I did 80 words in 3 minutes 25 seconds. That's 80 words in 205 seconds (3 x 60 = 180. 180 + 25 = 205 secs). That means I did 80 ÷ 205 words each second. I times that by 60 to find words in 1 minute. That's about 23 words a minute."

For question **2a**, students will need a sensible strategy for working out the approximate number of words in the book. For example, each student in a group could randomly select a page, count the words on that page, and then regroup to work out the average for the group. Another strategy would be to find the average number of words in a line by sampling 10 lines. Then find the average number of lines on a page over five pages. Multiply the average number of words in a line by the average number of lines on a page to find the average number of words per page.

Have the students check whether there are any illustrations. If there are, students will need to subtract the amount of space or pages that the illustrations take up.

Remember that this is an estimation activity, so students should not worry if the answer is not particularly exact.

Activity Two

Once the students realise that a book opening will always produce pages with two consecutive numbers, they should be able to work out the problems.

In question **3**, students could use the previous question as a model for making up their own problem. If they choose an even number for the sum of the two pages, refer them to the answer to question **1**.

Page 22: High-powered Thinking

Achievement Objective

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

Activity

This activity encourages students to solve multiplication problems mentally.

After students have attempted question **1**, make sure you discuss the feature of this problem that would make multiplication the most efficient way of solving it. This feature is the arrangement of the windows in equal groups so that counting and adding are unnecessary if you know how to multiply.

Have students produce other problems that could be solved by multiplying mentally. Ideas may include problems such as: "What is the number of baked bean tins in a stack of cartons if one carton holds 12 tins and there are 20 cartons?" "If the average CD has 20 tracks, how many tracks might there be in a stack of 25 CDs?"

Try to have students include some problems that use three factors, such as: "A park has 15 pūriri trees. Each tree has 20 branches, and each branch has six kererū. How many kererū are there altogether?"

As a daily routine, ask students to solve a maths question "in their heads". It is also important that at least three of the mental strategies used to answer one of the daily questions be shared by the class every day. It is more worthwhile to spend part of the lesson time doing this than having students complete the traditional 10 mental questions. If time is at a premium, consider doing fewer questions so that students can focus on sharing strategies.

Page 23: Human Pyramids

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- effectively plan mathematical exploration (Mathematical Processes, problem solving, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 3)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 3)
- record, in an organised way, and talk about the results of mathematical exploration (Mathematical Processes, communicating mathematical ideas, level 3)

Activity One

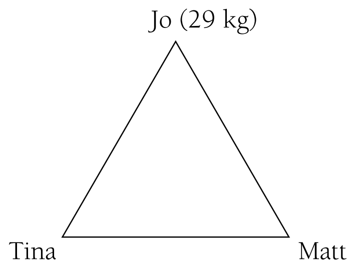
As with those for page **11**, this activity provides opportunities for the mathematical processes achievement objectives listed above. You may decide to select one or two of them for particular attention.

These activities use an unusual context to encourage problem solving that involves the four operations. Warn students not to attempt to build a pyramid with their classmates because of the dangers involved.

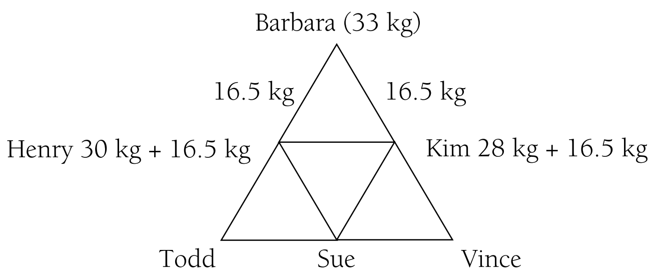
The pyramid structure implies that the mass of each person is distributed equally onto the two people below. As some people have two people above them, they have half of each of those people's mass as well a share of any mass that has been placed on those above them.

If necessary, suggest that students use a diagram to help solve this problem.

Question **1**.



Question **2**.



Todd gets half of 46.5 kilograms. Vince gets half of 44.5 kilograms. Sue gets the same as Todd and Vince combined.

Activity Two

Suggest to students that they use the same mass for each person in their pyramid model and they keep it a simple number. This way, they have a better chance of seeing how much the halving effect reduces the mass that each person on the bottom row has to support. They can also see that the end people have less mass to support than the middle people.

For example, what would happen if every person weighed 40 kilograms and you used 15 people?

Row	**Number of people in each row**	**Mass (kg) on each person**
1 (top)	1	0
2	2	20 20
3	3	30 60 30
4	4	35 85 85 35
5	5	37.5 100 125 100 37.5

Look at the bottom row. The end people each still have less than one person's mass to support. The second person in has 2.5 times the mass of one person. The middle person has the most to support, and it is just over the mass of three people, even though there are 10 people above that person.

Page 24: Budgie Buying

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 3)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 3)
- record, in an organised way, and talk about the results of mathematical exploration (Mathematical Processes, communicating mathematical ideas, level 3)

Activity

As with those for pages **11** and **23**, this activity provides opportunities for the mathematical processes achievement objectives listed above. You may decide to select one or two of them for particular attention.

This activity uses a series of transactions as a problem-solving context. The problems involve a variety of computation skills.

Encourage students to be systematic and to list each of the individual transactions in a debit or expenses column on a chart. They could also keep a running total of what has been spent to date and another of what amount of money is left. At any one time, the sum of the rows across the columns should be $70.

This is an ideal activity to work out on a computer spreadsheet. The spreadsheet may look something like this example:

Item	Debit ($)	Debit total (rounded)	Credit remaining ($)
Nil	0.00	0.00	70.00
Budgie	10.00		
Cage	30.25		
Seeds	6.80	47.05	22.95
Mirror	1.55		
Water dispenser	2.09		
Seed dispenser	1.75	5.40	17.55

The debit total column will have a formula that adds all the previous debits.
The credit remaining column will have a formula that subtracts the debit total from $70.

Copymaster: Compact Numeral Cards

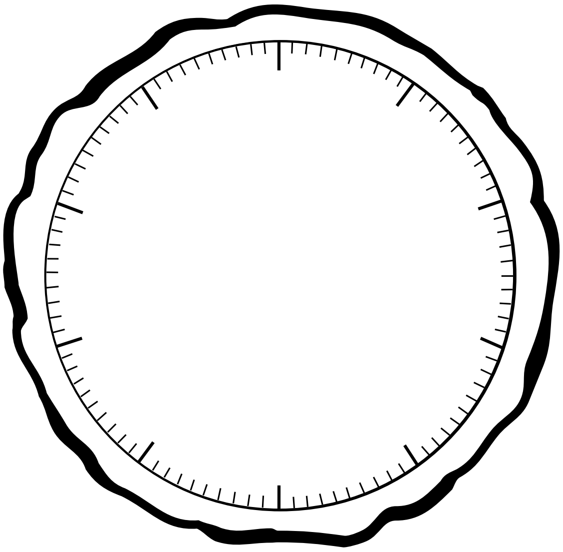
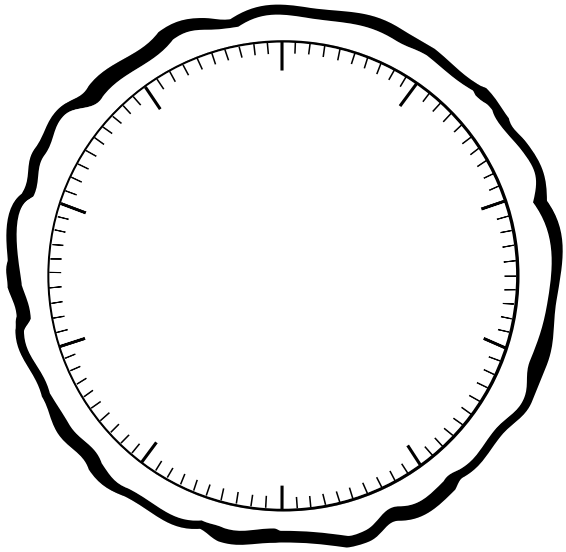
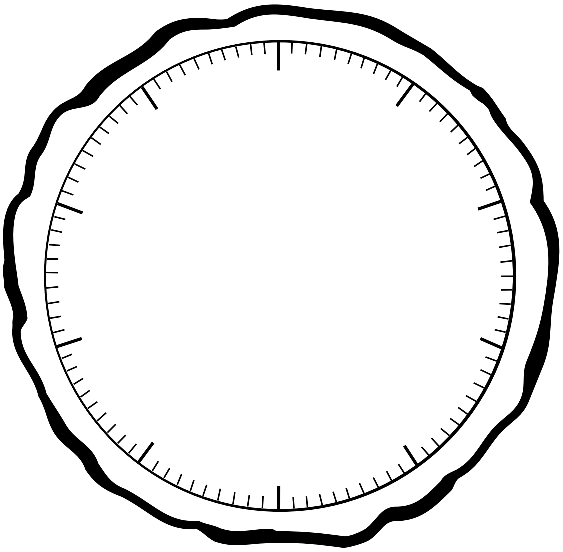
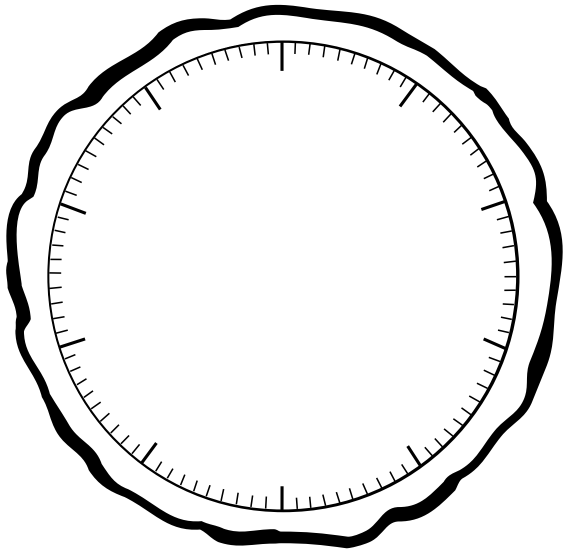
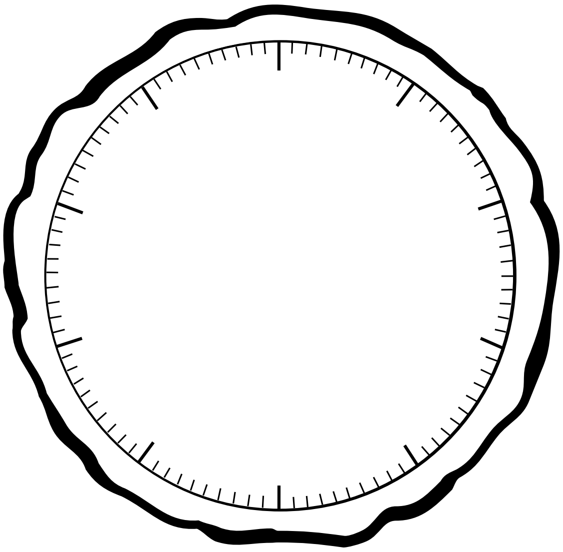
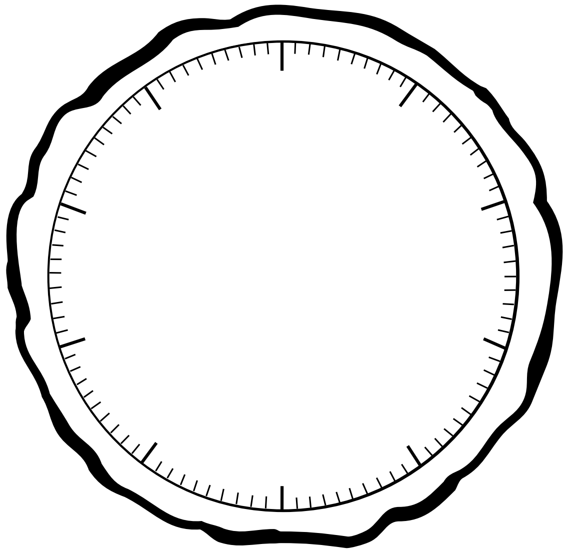
1 2 3 4 5 6 7 8 9

10 20 30 40 50 60 70 80 90

100 200 300 400 500 600 700 800 900

1000 2000 3000 4000 5000 6000 7000 8000 9000

Copymaster: Pies

Copymaster: Decimal Compact Numeral Cards

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0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1.	2.	3.	4.	5.	6.	7.	8.	9.

Acknowledgments

Learning Media would like to thank Geoff Woolford, Education Advisory Service, Auckland College of Education, for developing the teachers' notes. Thanks also to Paulette Holland for reviewing the answers and notes.

The main illustrations on the cover and contents page and the background line art on the cover, contents page, and pages 2, 3 and 10 are by Wol Jobson.

Series Editor: Susan Roche
Series Designer: Esther Chua
Designer: Dawn Barry

Published 2000 for the Ministry of Education by
Learning Media Limited, Box 3293, Wellington, New Zealand.
Website: www.learningmedia.co.nz

Dewey number 510.76
ISBN 0 478 12691 3
Item number 12691
Students' book: item number 12690

105261/00