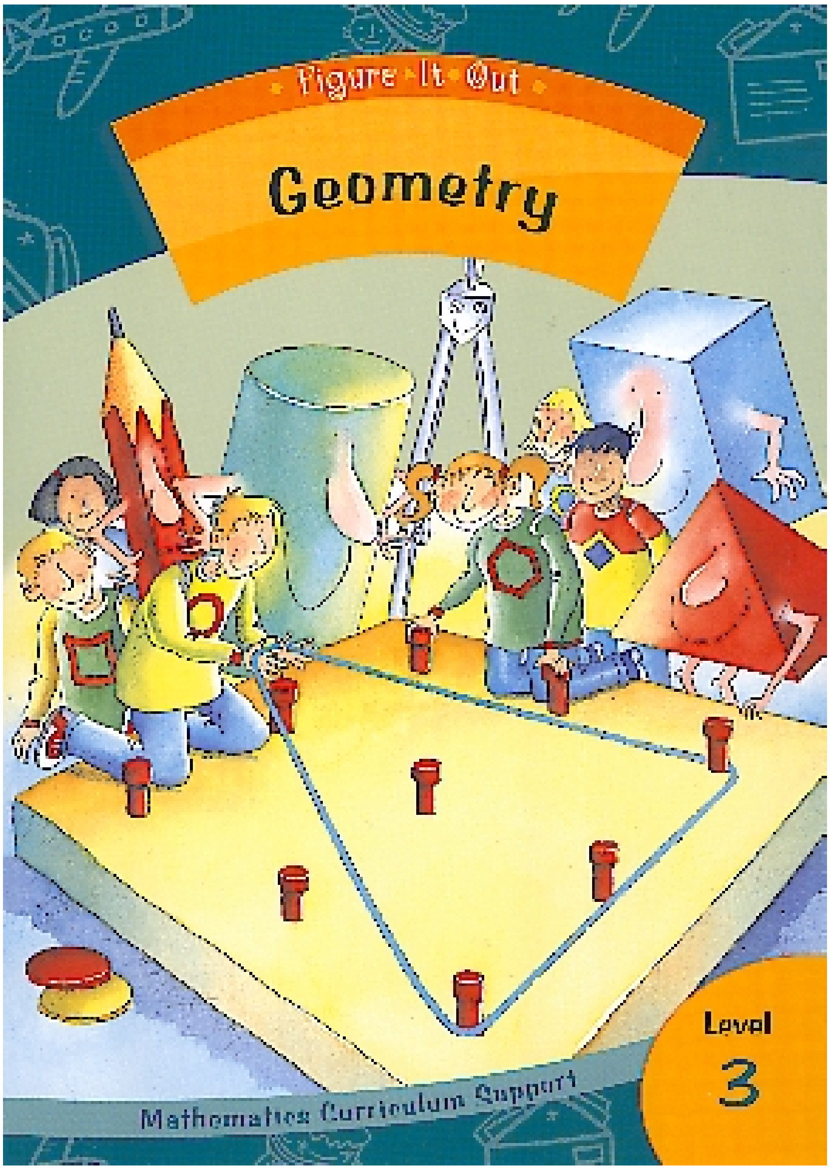
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Contents

Introduction

The Figure It Out series is designed to support *Mathematics in the New Zealand Curriculum*. The booklets have been developed and trialled by classroom teachers and mathematics educators. The series builds on the strengths of a previous series of mathematics booklets published by the Ministry of Education, the School Mathematics supplementary booklets.

Figure It Out is intended to supplement existing school mathematics programmes and can be used in various ways. It provides activities and investigations that students can work on independently or co-operatively in pairs or groups. Teachers can select particular activities that provide extension to work done in the regular classroom programme. Alternatively, teachers may wish to use all or most of the activities in combination with other activities to create a classroom programme. The booklets can be used for homework activities, and the relevant section in the Teachers' Notes could be copied for parents. These notes may also provide useful information that could be given as hints to students.

There are eight booklets for level 3: one booklet for each content strand, one on problem solving, one on basic facts, and a theme booklet. Each booklet has its own *Answers and Teachers' Notes*. The notes include relevant achievement objectives, suggested teaching approaches, and suggested ways to extend the activities. The booklets in this set (level 3) are suitable for most students in year 5. However, teachers can decide whether to use the booklets with older or younger students who are also working at level 3.

The booklets have been written in such a way that students should be able to work on the material independently, either alone or in groups. Where applicable, each page starts with a list of equipment that the students will need to do the activities. Students should be encouraged to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Teachers could encourage students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Where possible, suggestions have been made to encourage discussion and oral presentation of answers, and teachers may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> (*Mathematics in the New Zealand Curriculum*, page 7)

Students will have various ways of solving problems or presenting the process they have used and the solution. Successful ways of solving problems should be acknowledged, and where more effective or efficient processes can be used, students can be encouraged to consider other ways of solving the problem.

Figure It Out

Geometry

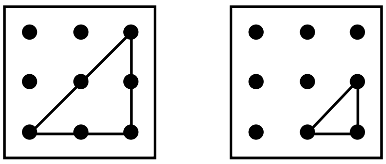
Answers

Page 1: Good Points about Triangles

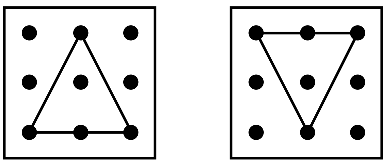
Activity

1. Answers will vary. Possible groupings are:
   Group 1: Isosceles triangles (2 sides are the same length)
   Group 2: Right-angled triangles (triangles with 1 right angle)
   Group 3: Scalene triangles (all the sides are a different length)
   Group 4: Acute-angled triangles (triangles with 3 acute angles)

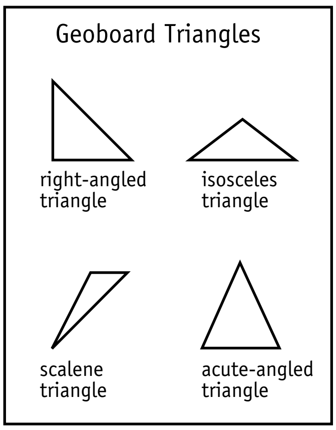
2. Triangles are similar if they have the same shape. Triangles are congruent if they have the same shape and size.
   An example of 2 similar triangles is:



   An example of 2 congruent triangles is:



3. Answers will vary. A possible chart is:

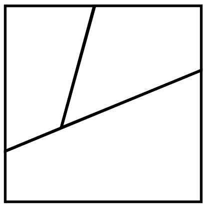


4. Answers will vary. An equilateral triangle cannot be made on a geoboard because 2 sides will always be longer than the third side. An equilateral triangle has all sides and angles congruent.

Page 2: Shaping Up

Activity One

1.



2. Answers will vary. A useful start is identifying the right angles on each piece to make the corners. Students might also look at the length of the sides or the size of the angles.

Activity Two

Practical activity

Page 3: Getting Round to It

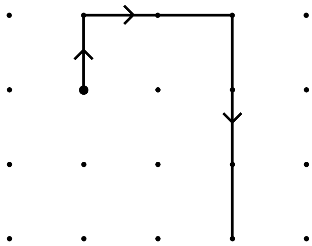
Activity

1. a. Practical activity
   b. The rope works in the same way as the compass by ensuring that there is a fixed distance from the centre of the circle. (This distance is called the radius.)
2. Practical activity
3. a. The ellipse becomes closer in shape to a circle.
   b. As the 2 people move closer together, the 2 lines begin to behave more like the radius of a circle. Eventually the 2 points would become 1 point, and this would be the centre of the circle.

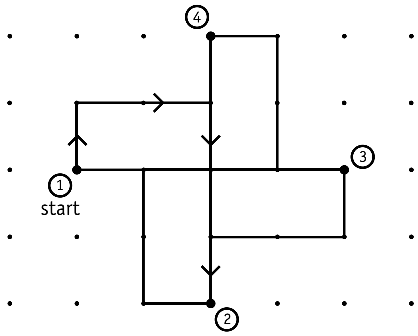
Page 4: Snail Trails

Activity

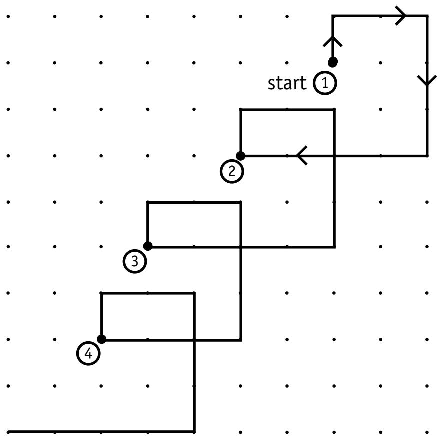
1.



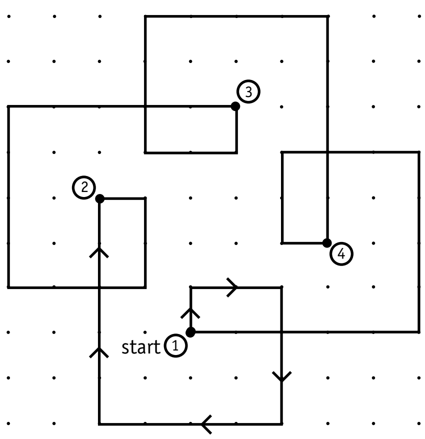
2. 4 sets



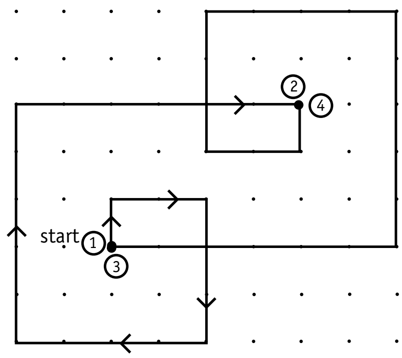
3. a.



b.



c.



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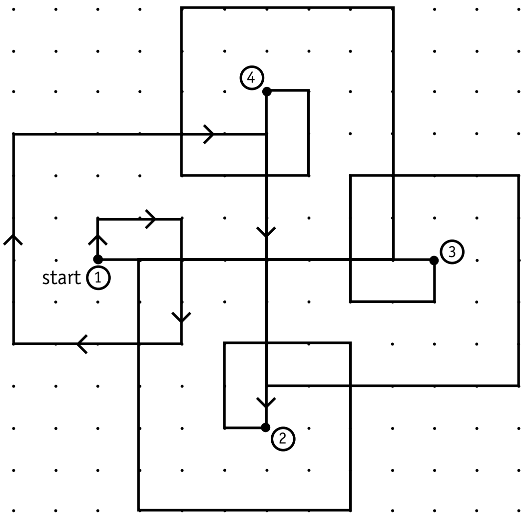


4. Discussion points will vary.

Page 5: Let's Face It

Activity

1. Practical activity

2.

Shape	Edges	Faces	Vertices
Tetrahedron	6	4	4
Cube	12	6	8
Octahedron	12	8	6
Dodecahedron	30	12	20
Icosahedron	30	20	12

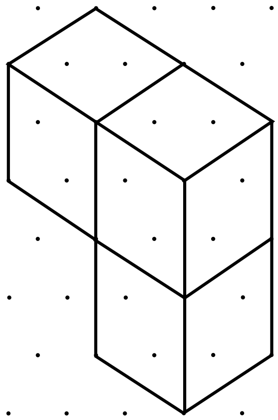
Investigation

Research on Plato

Pages 6–7: Starting Blocks

Activity

a.



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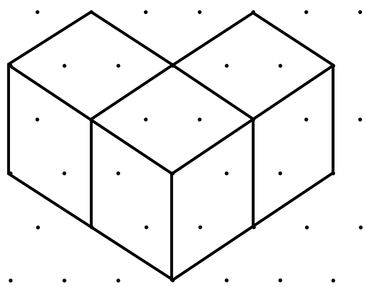
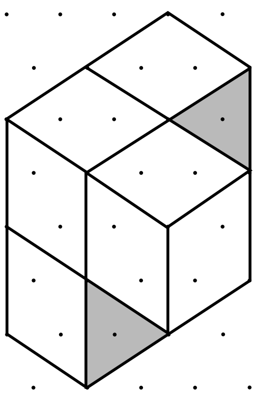
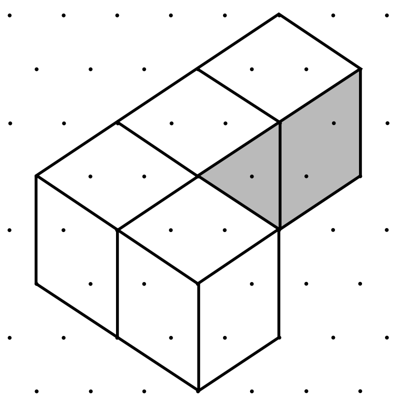


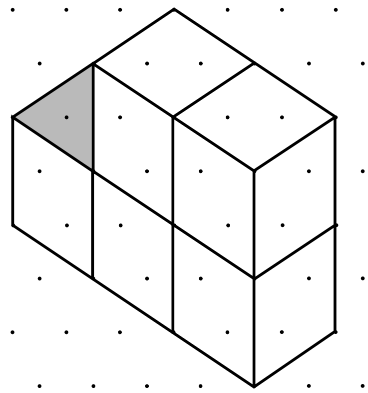
c.



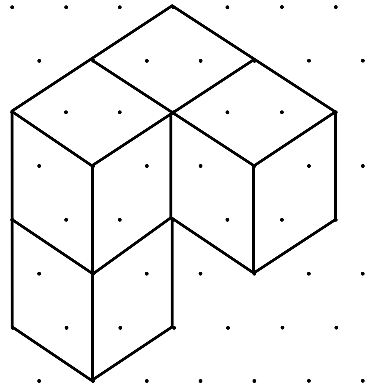
d.



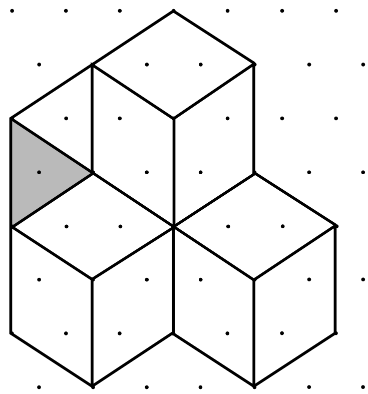
e.



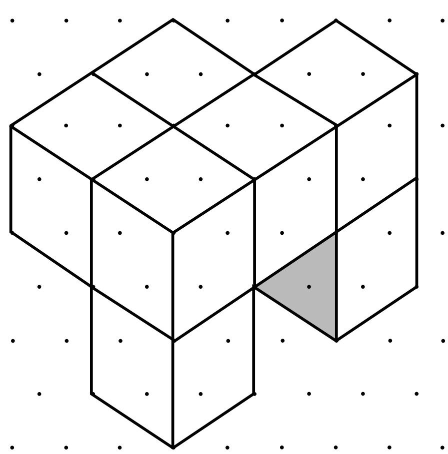
f.



g.



h.

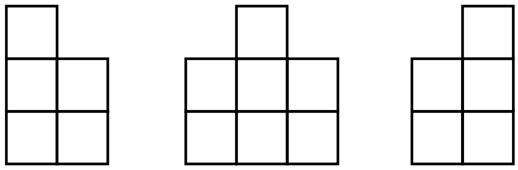


Pages 8–9: Different Viewpoints

Activity

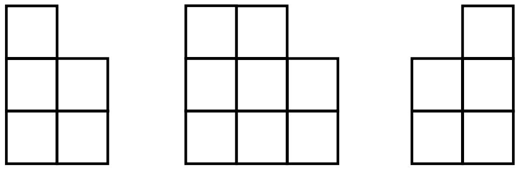
1. The views are identical.

2. a.



left-hand side front right-hand side

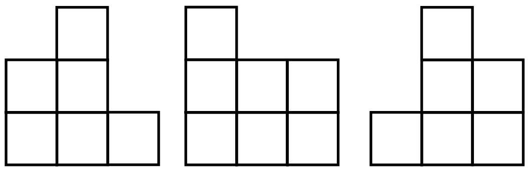
b.



left-hand side front right-hand side

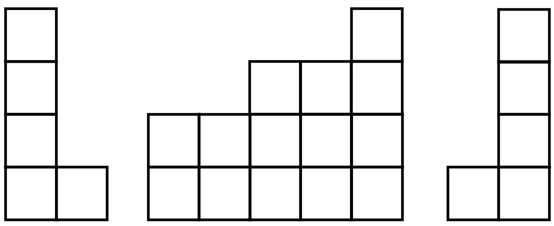
3. a. Practical activity

b. i.



left-hand side front right-hand side

ii.



left-hand side front right-hand side

Page 10: A Chip off the Old Block

Activity One

Answers will vary. The number of blocks used will depend on whether "hidden" blocks are included.

Activity Two

Practical activity

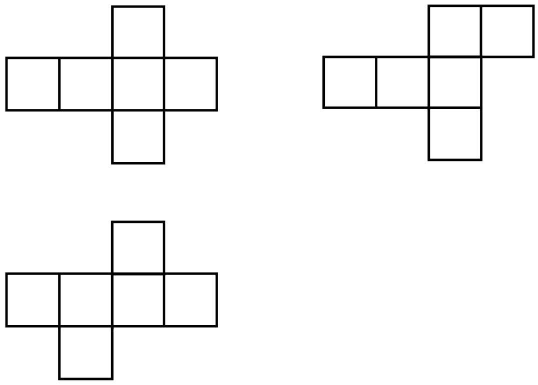
Page 11: Nutting out Nets

Activity

1. a. The correct prediction is that the net will make a cube.

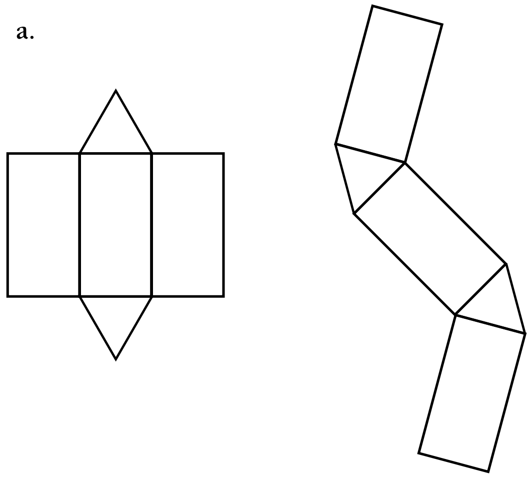
b. Practical activity

c. Answers will vary. Some examples are:

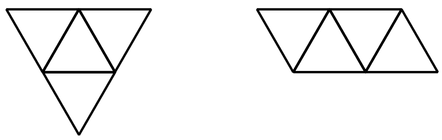


2. Answers will vary. Some examples are:

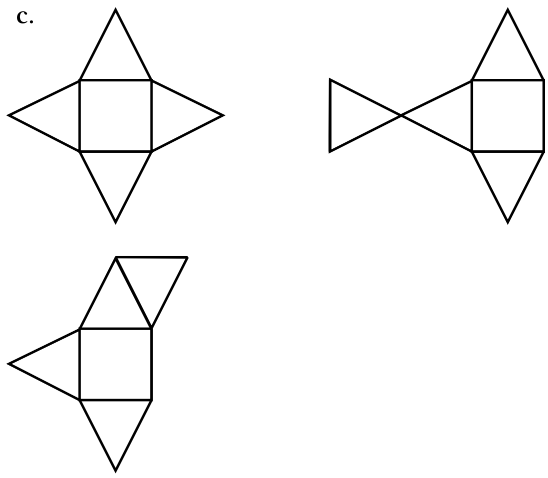
a.



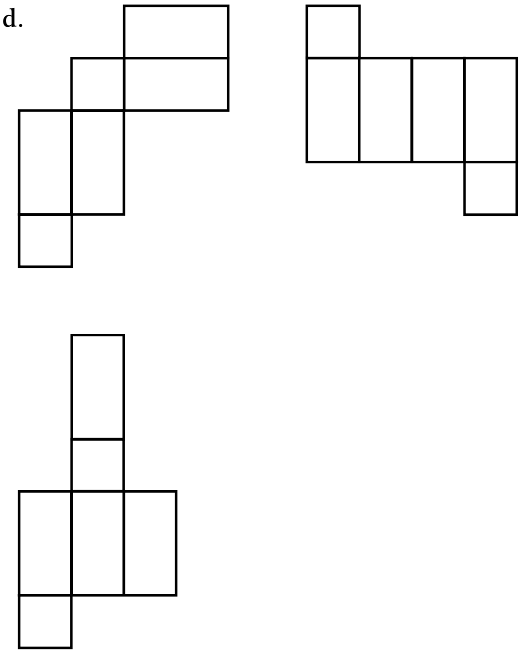
b.



c.



d.

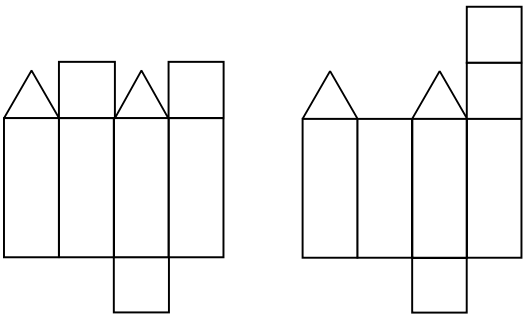


3. Practical activity. The net of **a** makes a cone. The net of **b** makes a cylinder.

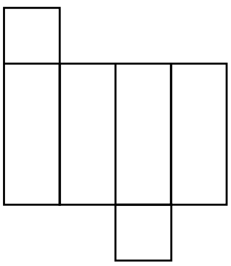
Page 12: City Skyline

Activity

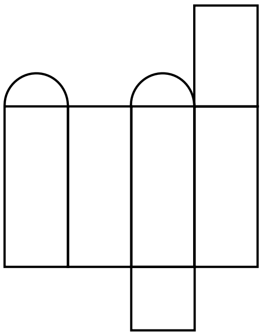
1. Yes. Two other nets that will work for Pyramid Plaza are:



2. Some possible nets are:

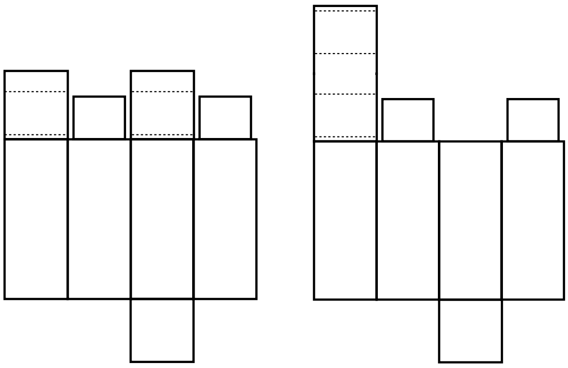


Capital Chambers

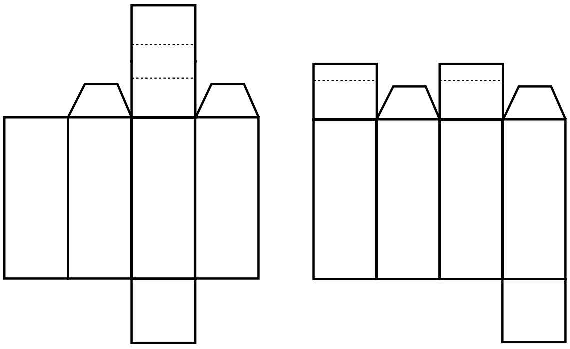


Civic Centre

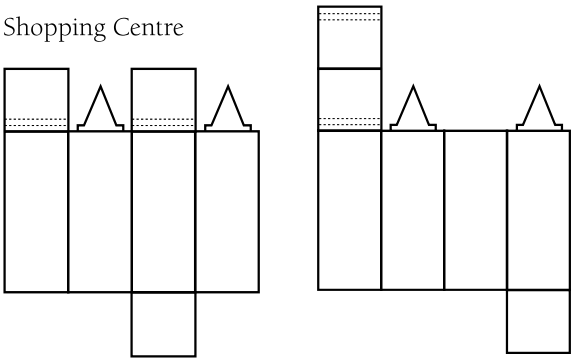
Colonial House



Archie's Arcade



Shopping Centre



3. Practical activity

Page 13: Room with a View

Activity

Practical activity

Pages 14–15: Fun Run

Activity

1. Answers will vary.
2. **a.** On the map it measures approximately 180 mm, which is 3.6 km.

   b. It measures approximately 145 mm, which is approximately 2.9 km. (Note that you have to go round the roundabout.)

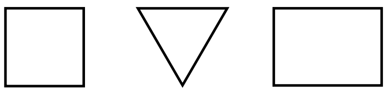
   c. It passes over the playground and the bakery.

Pages 16–17: Slice of Life

Activity

1.–3. Practical activities

4. The sections should look like this:



5. Practical activity

Page 18: Walking Triangles

Activity

1. **a.** 4, 5, 4, 2

   b. 2, 4, 2, 4

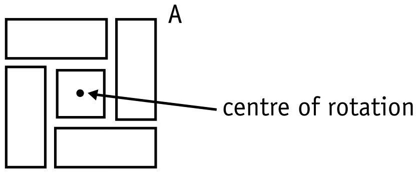
   c. 5, 3, 1, 2 or 4, 6, 2, 2

   d. 1, 1, 4, 5 or 1, 5, 2, 1
2. Answers will vary.

Page 19: Pattern Pieces

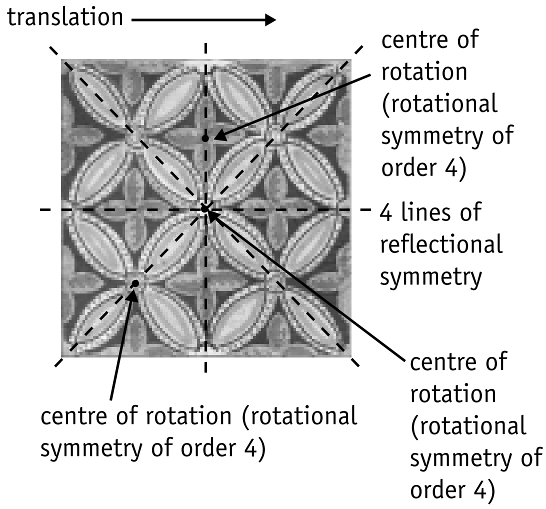
Activity

1. **a.** The symmetry of this design involves rotation, for example:

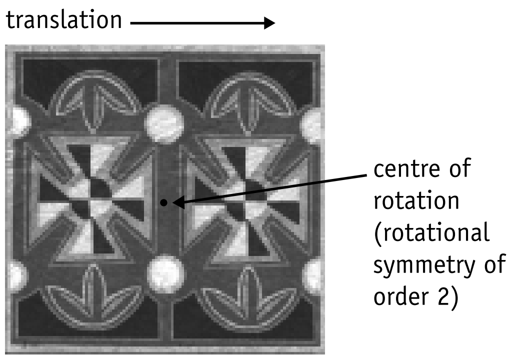


(This part of the design has rotational symmetry of order 4. This means that it will fit onto itself 4 times before returning to the start position A.)

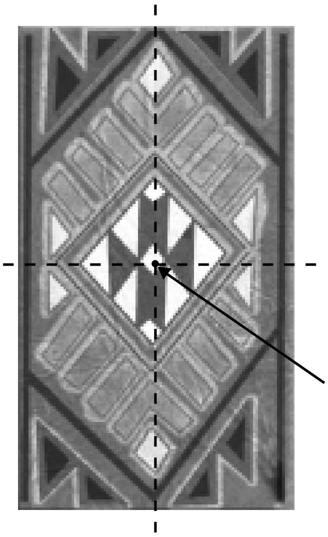
b. This design has reflectional and rotational symmetry and involves translation.



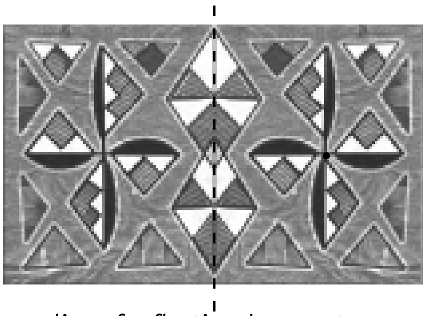
c. This design has rotational symmetry and involves translation.



d. This design has reflectional and rotational symmetry.



e. This design has reflectional symmetry. (Reflection also occurs within the pattern, for example, in individual triangles.)

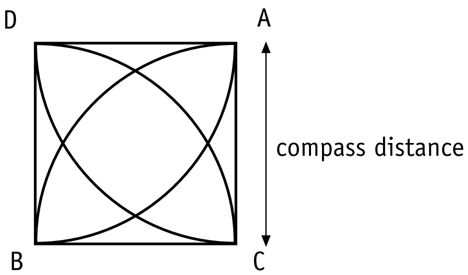


2. Practical activity

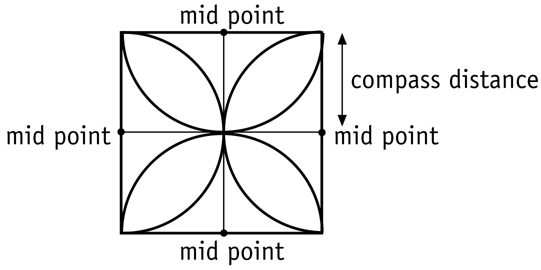
Page 20: Circle Power

Activity

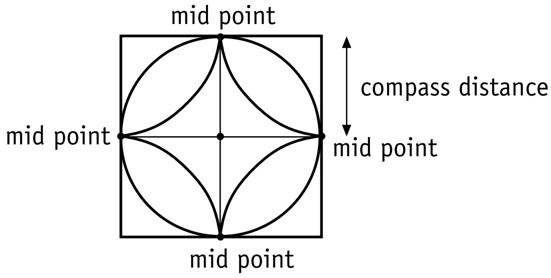
1. Teresa drew a square. She made the compass distance (radius) the same as that of the side of the square. Then she put the compass on point A and drew an arc. She put the compass on point B and drew an arc. She did the same with points C and D.



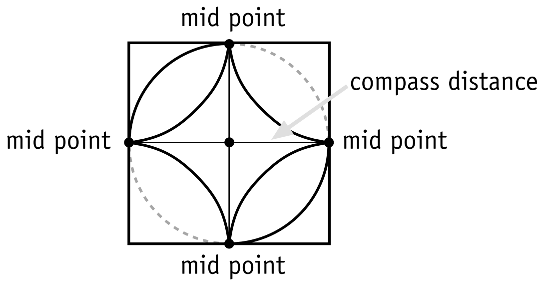
Harry drew a square. Then he marked in mid points on each side. He set the compass distance as shown and then put the compass on each mid point and drew a half circle.



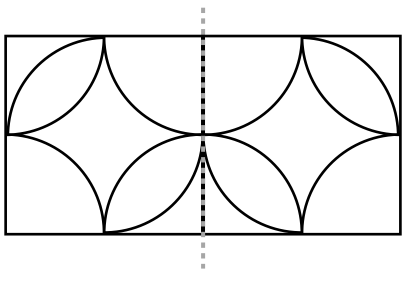
Antony drew a square and marked all the mid points. He used these to work out where the centre of the circle was. Then he drew a circle within the square, using the compass distance as shown. Finally, he put the compass on each corner of the square and drew an arc.



Lorena used the same method as Antony. As a final step, she rubbed out the circle lines shown as dotted lines in the diagram below.



2. Lorena's border pattern has been made by reflection. It is the same basic shape as Antony's pattern.



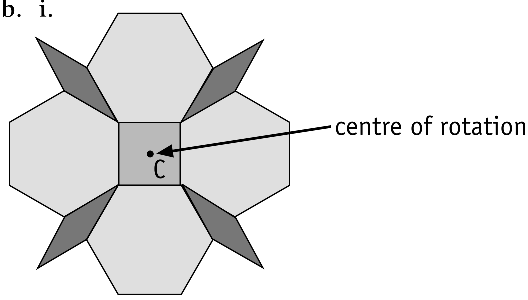
3. Practical activity

Page 21: One Good Turn

Activity

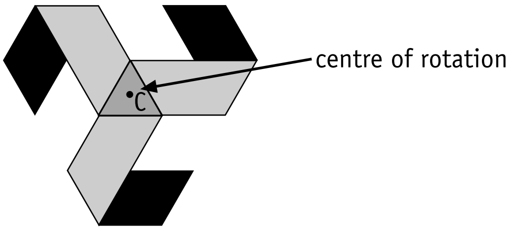
1. a. Practical activity

b. i.



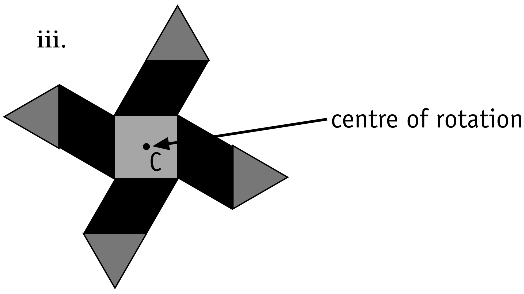
The figure fits onto itself every $^1/_4$ turn about the centre (90°). It has rotational symmetry of order 4.

ii.



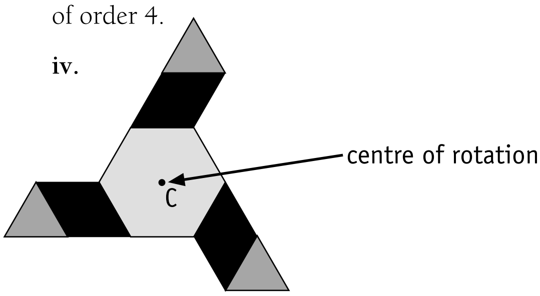
The figure fits onto itself every $^1/_3$ turn about the centre (120°). It has rotational symmetry of order 3.

iii.



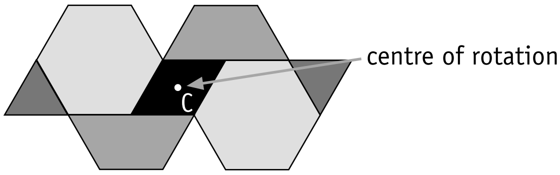
The figure fits onto itself every $^1/_4$ turn about the centre (90°). It has rotational symmetry of order 4.

iv.



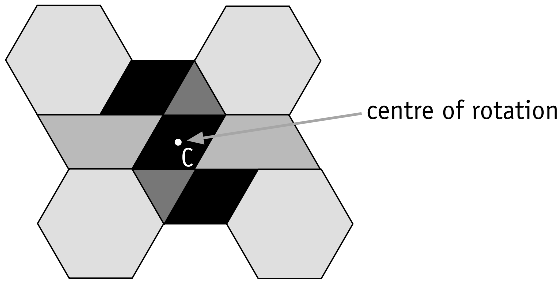
The figure fits onto itself every $^1/_3$ turn about the centre (120°). It has rotational symmetry of order 3.

v.



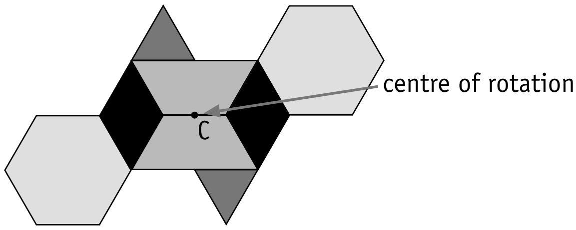
The figure fits onto itself every $^1/_2$ turn about the centre (180°). It has rotational symmetry of order 2.

vi.



The figure fits onto itself every $^1/_2$ turn about the centre (180°). It has rotational symmetry of order 2.

vii.



The figure fits onto itself every $^1/_2$ turn about the centre (180°). It has rotational symmetry of order 2.

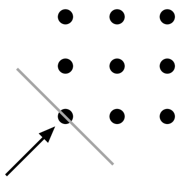
2. Practical activity

Page 22: Reflecting on Counters

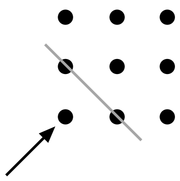
Activity

These answers assume that you have the reflective side of the mirror facing you. The mirror must be one without a frame.

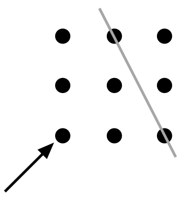
1. **a.** Position the mirror halfway over 1 counter as shown below:



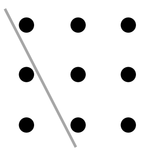
b. To see 3 counters, position the mirror as shown below:



c. To see 12 counters, position the mirror as shown below:



2. All numbers up to 18 are possible if your grid allows you enough room to position the mirror across rows as shown above. Some of the more difficult numbers include:



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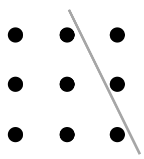

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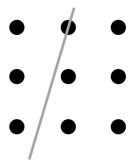

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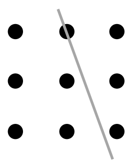

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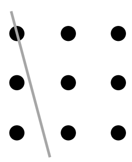

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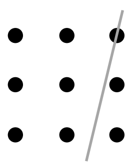

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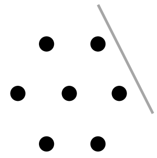
3. 14, for example,



Page 23: The Big Picture

Activity

1.–2. Practical activities

Page 24: Growing Changes

Activity One

a. The shape stayed the same, and the vertices remained right angles.

b. The length of the sides and the area

Activity Two

a. Features that have stayed the same are the shape, the vertices, and the ratio of side lengths.
Features that have changed are the length of the sides and the area.

b. Features that have stayed the same are the shape and the centre point of the circle.
Features that have changed are the circumference, the radius, the diameter, and the area (all increased).

Figure It Out

Geometry

Teachers' Notes

Overview: Geometry

Title	Content	Page in students' book	Page in teachers' book
Good Points about Triangles	Classifying two-dimensional shapes and describing their features	1	13
Shaping Up	Applying the features of two-dimensional shapes	2	15
Getting Round to It	Describing two-dimensional shapes (circles and ellipses)	3	16
Snail Trails	Describing two-dimensional shapes and paths	4	17
Let's Face It	Naming and describing polyhedra	5	19
Starting Blocks	Interpreting pictures of three-dimensional buildings	6–7	20
Different Viewpoints	Drawing buildings made from cubes	8–9	21
A Chip off the Old Block	Building and drawing three-dimensional solids	10	22
Nutting out Nets	Drawing nets	11	22
City Skyline	Drawing nets	12	24
Room with a View	Drawing to scale	13	25
Fun Run	Interpreting simple scale maps	14–15	26
Slice of Life	Three-dimensional cross sections	16–17	27
Walking Triangles	Drawing and interpreting directions and movement	18	28
Pattern Pieces	Describing patterns	19	28
Circle Power	Designing and making a pattern	20	29
One Good Turn	Investigating rotational symmetry	21	30
Reflecting on Counters	Creating and analysing symmetry	22	31
The Big Picture	Enlarging a figure to a given scale	23	31
Growing Changes	Recognising changes in properties with enlargement	24	33

Page 1: Good Points about Triangles

Achievement Objective

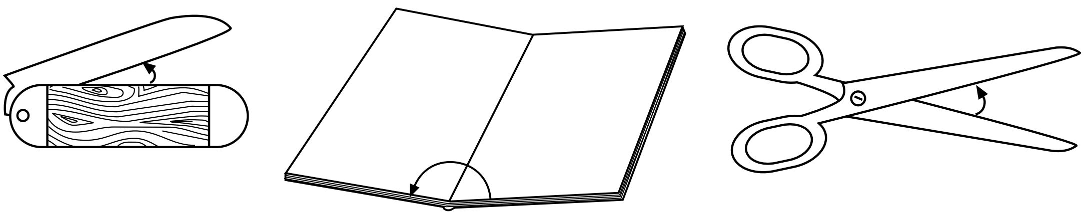
- describe the features of 2-dimensional and 3-dimensional objects, using the language of geometry (Geometry, level 3)

Activity

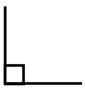
The main purpose of this activity is for students to discover the number of different triangles that can be made on a nine-pin geoboard and to be able to classify triangles. Give students the opportunity to both make and draw these triangles because this will reinforce their understanding. (A geoboard copymaster is provided at the back of this book.) Students can easily manipulate and compare triangles when they use a geoboard.

Students should be able to identify isosceles, scalene, right-angled, and acute-angled triangles for question **1**.

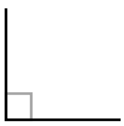
There are several ideas that you could discuss with the class to increase their understanding of triangles and angles. After students have identified the different groups of triangles, you could discuss what angles are. This builds on quarter and half turns that were covered in level 1 geometry. Use real-life situations to illustrate that an angle is the amount of turn from one line to another, for example:



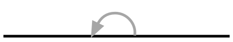
Students need a good understanding of the amount of turn of a right angle. This will help them to decide whether an angle is greater or less than a right angle. You could also show the students the notation for a right angle:



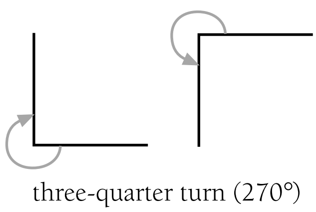
Next, you could explain degrees: "The Babylonians divided the circle into 360 small parts or sectors. Each part or sector is called a degree. The symbol for a degree is °." Find some practical examples, such as a plate (360 degrees), an open birthday card (180 degrees), or a cake with a quarter taken out (90 degrees and 270 degrees), to use with the students to demonstrate the following:



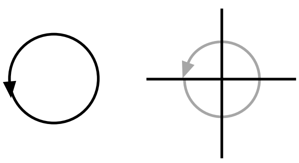
quarter turn (90°)



half turn (180°)



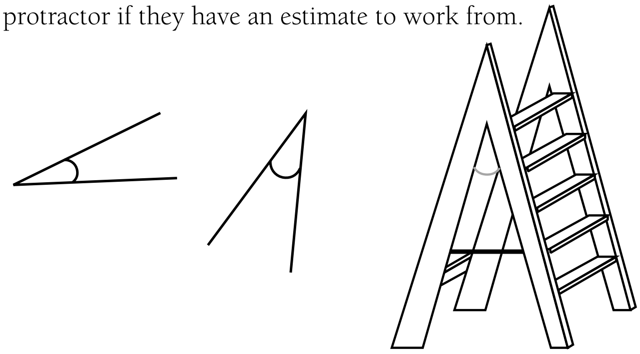
three-quarter turn (270°)



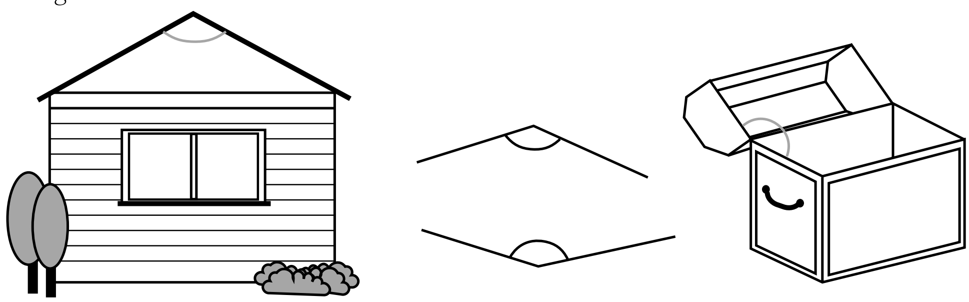
full circle (360°)

Students can make angles using two cardboard strips and a split pin. They can also relate the angles they create to the quarter turn, half turn, three-quarter turn, and full turn.

An acute angle is a sharply pointed angle that is less than 90 degrees. You could show students different angles and ask them "Is this angle an acute angle? How big might this angle be?" The ability to estimate becomes important here. Students who can do this will have very little trouble using a protractor to measure angles accurately because they are less likely to be confused by the markings on the protractor if they have an estimate to work from.



As an extension, you could introduce the concept of obtuse angles, that is, angles between 90 and 180 degrees.



You can now ask students if there are any ways that they might classify the triangles further, for example, right-angled isosceles or right-angled scalene triangles.

As an extension, you could tell students that the total of a triangle's three angles is 180 degrees and then ask them questions such as: "In an isosceles right-angled triangle, we know that one of the angles is 90 degrees and the other two angles must be the same size. What would the size of these angles be? What is the total of the three angles?"

Note that for question **3**, there are a limited number of triangles that can be made on a nine-pin geoboard. See *Answers and Teachers' Notes: Geometry*, Figure It Out, Levels 2–3, page 15 for more comments about rotation, reflection, and translation.

For the investigation given in question **4**, students need to realise that the equilateral triangle can't be made because the two angled lines on the geoboard are slightly longer than the base line. They could then go on to look at extending the triangle into a quadrilateral and looking at the diagonal. The diagonal of any quadrilateral is always longer than any one of its sides. Any triangle made on a geoboard can be extended into a quadrilateral, with one side of the triangle being the diagonal.

To do further work with triangles, you could:

- Identify triangles in the environment, such as road signs or a girder in a bridge. (You could ask students "Why is the triangle used in construction?")
- Make links to technology, science, and languages, such as the way the balls are set up in a triangle in the game of pool and the Māori name for a triangle (tahatoru). Taha means sides and the shape is named by the number of sides it has, in this case, three (toru). The word triangle means three angles, with the tri derived from Latin tres and Greek treis (three).

Page 2: Shaping Up

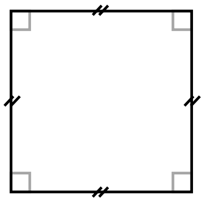
Achievement Objectives

- describe the features of 2-dimensional and 3-dimensional objects, using the language of geometry (Geometry, level 3)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 3)

For both activities, students need to visualise the shape they are trying to make. You could help students by relating the shapes to their use in the environment, for example, a stop sign (octagon), a hazard sign (rhombus), and a road patrol sign (circle).

Activity One

For this activity, students need to consider these features of a square: four equal sides and four right angles. (The comments for page **1** provide further details on the right angle.)



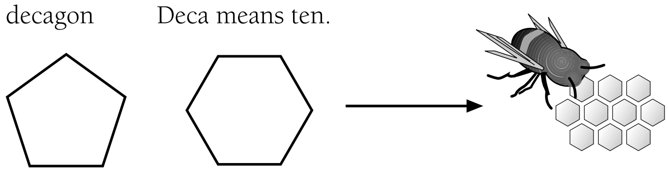
Ask students, "Where are the right angles in a square?"

Activity Two

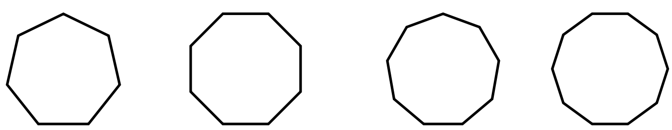
This activity is useful for introducing polygons. Poly means many and gon means angles. If a polygon is regular, all its angles are equal and all sides are equal.

Polygons that students might use include:

- pentagon — Penta is the Greek word for five. (A pentathlon is an athletic competition that has five events.)
- hexagon — Hex means six.
- heptagon — Hepta means seven.
- octagon — Octa means eight.
- nonagon — Nona means nine.
- decagon — Deca means ten.



pentagon hexagon hexagons in the environment



heptagon octagon nonagon decagon

Polygons with sides and angles that are not equal are called irregular polygons.

irregular pentagon irregular hexagon

Page 3: Getting Round to It

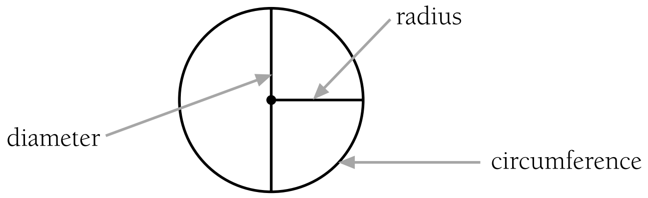
Achievement Objectives

- describe the features of 2-dimensional and 3-dimensional objects, using the language of geometry (Geometry, level 3)
- use equipment appropriately when exploring mathematical ideas (Mathematical Processes, problem solving, level 3)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 3)

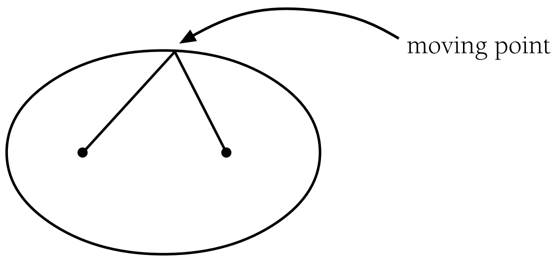
Activity

In this activity, students discover the differences between a circle and an ellipse and how these two shapes are related. Ask students what they know about circles and ellipses and how they might go about drawing them. (For example, an egg is an elliptical shape. Planets follow an elliptical path.)

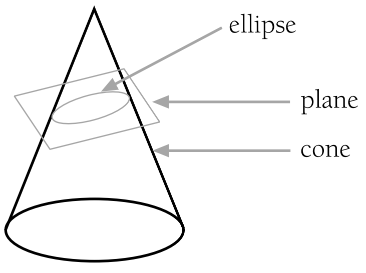
The activity provides opportunities to use mathematical language. Students need to understand that to make their circle as round as possible, the person standing in the centre needs to make sure that their end of the rope remains in the same place so that every point on the circle (the circumference) is the same distance from the centre. The distance from the centre of the circle to the circumference is called the radius. The line segment that passes through the centre and goes from one side of the circle to the other is called the diameter.



An ellipse is the path formed by a moving point so that the sum of the distances from two fixed points to the moving point is always the same.



An ellipse can be described as a regular oval. An ellipse is also produced when a cone is cut obliquely by a plane.



Discuss with students which parts of the ellipse are regular or equal, for example, the two halves of an ellipse, divided either vertically or horizontally.

Page 4: Snail Trails

Achievement Objectives

- describe and interpret position, using the language of direction and distance (Geometry, level 2)
- describe patterns in terms of reflection and rotational symmetry, and translations (Geometry, level 3)

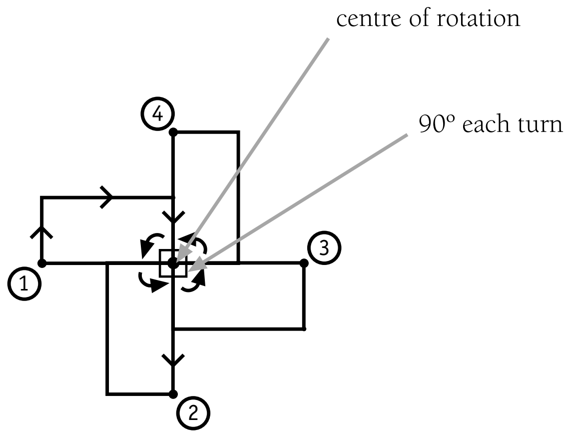
Activity

The aim of this activity is for students to begin to recognise that each sequence is either a rotation, a slide (translation), or a combination of both.

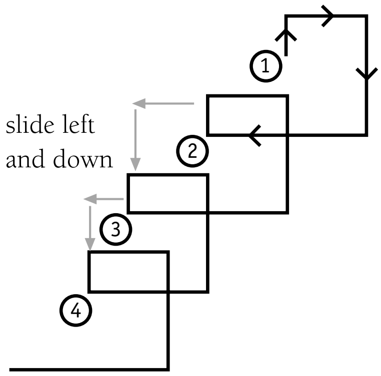
To reinforce the idea, students could use a two-dimensional model of a rectangle and complete the rotations and slides. This will reinforce their ability to visualise and make predictions.

A rotation occurs when an object is turned about a fixed point, called the centre of rotation. (See the notes for page **19**.) A translation occurs when a shape slides horizontally or vertically without turning. The shape does not change its size or orientation.

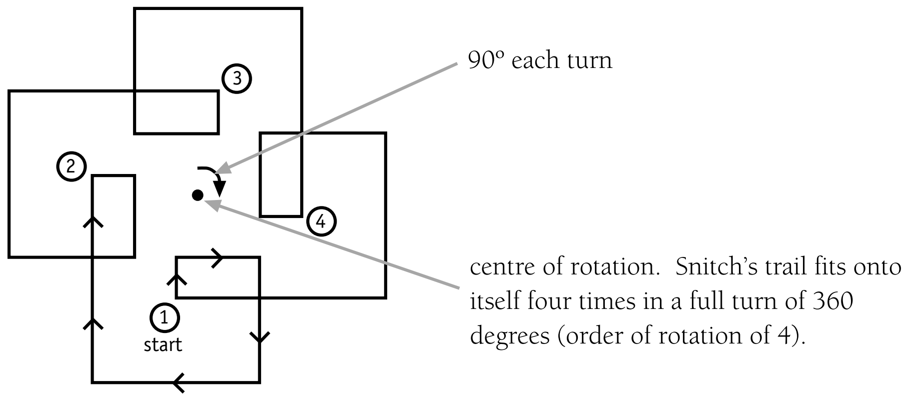
In question **2**, although Snitch the Snail travels clockwise, the rotation of his trail is 90 degrees in an anticlockwise direction. The trail he leaves has an angle of rotation of 90 degrees.



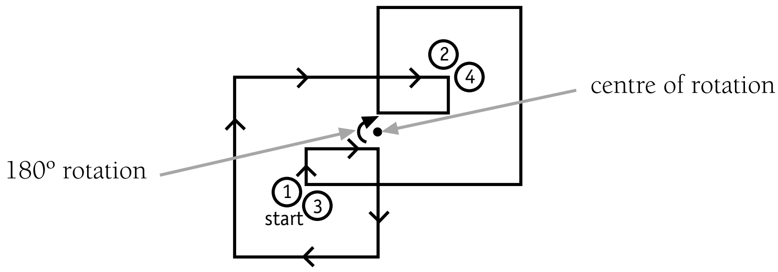
Question **3a** shows a slide or translation left and down:



Question **3b** shows a rotation of 90 degrees in a clockwise direction.

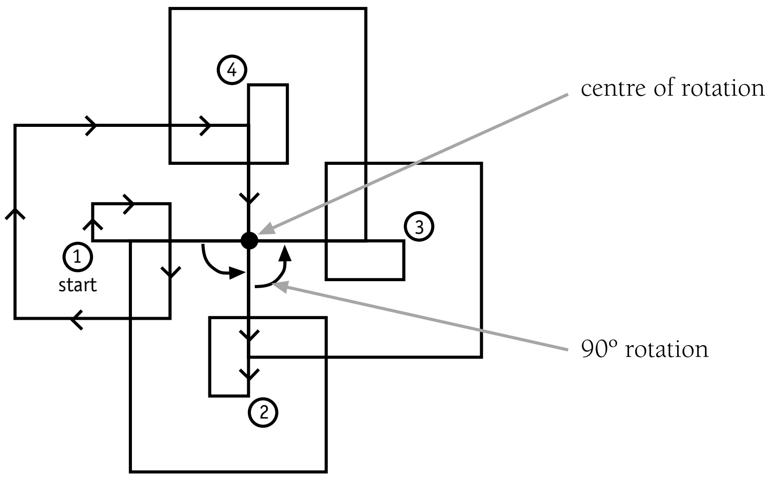


Question **3c** shows a rotation of 180 degrees in a clockwise direction.



Snitch's trail fits onto itself twice in a full turn of 360 degrees (order of rotation of 2).

In question **3d**, although Snitch travels clockwise, the rotation of his trail is 90 degrees in an anticlockwise direction. Note the centre of rotation:



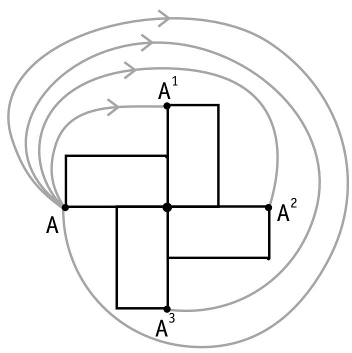
The shapes in questions **2** and **3b**, **3c**, and **3d** have rotational symmetry, that is, they fit onto themselves exactly when rotated about a centre point. For example:

$A \rightarrow A^1 = 90°$

$A \rightarrow A^2 = 180°$

$A \rightarrow A^3 = 270°$

$A \rightarrow A = 360°$



The order of rotation is 4. The shape fits onto itself four times in a full turn of 360 degrees.

Page 5: Let's Face It

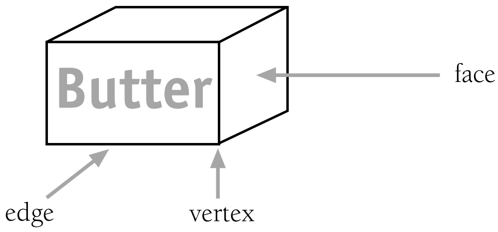
Achievement Objectives

- describe the features of 2-dimensional and 3-dimensional objects, using the language of geometry (Geometry, level 3)
- classify objects, numbers, and ideas (Mathematical Processes, developing logic and reasoning, level 3)
- use equipment appropriately when exploring mathematical ideas (Mathematical Processes, problem solving, level 3)

Activity

Students will need to understand the following:

- A face is a flat surface.
- An edge is where two faces meet.
- A vertex or corner is where edges meet.



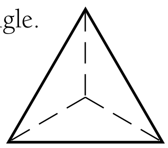
- A polyhedron is a solid shape with flat sides. A solid shape has three dimensions – length, width, and depth. The faces of a polyhedron are all polygons.

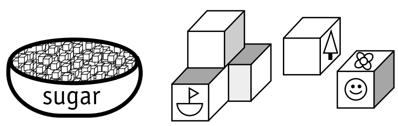
You could begin by making the cube and discussing with students the properties of the cube, especially the polygons that make up the faces and the number of faces, edges, and vertices.

Encourage students to look for polyhedra in everyday shapes found in the environment.

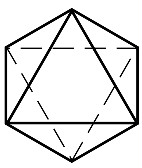
You may find the following definitions useful:

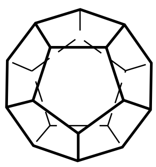
- A tetrahedron or triangular pyramid is a solid with four triangular faces. It is the same as a triangular-based pyramid.
- A cube is a regular solid with all its faces square and all its edges equal in length. Most dice are cubes.
- An octahedron is a solid shape with eight faces. In a regular octahedron, each face is an equilateral triangle.
- A dodecahedron is a solid shape with 12 faces. All the faces of a regular dodecahedron are regular pentagons.
- An icosahedron is a solid with 20 faces. In a regular icosahedron, each face is an equilateral triangle.

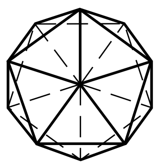

tetrahedron



cube


octahedron


dodecahedron


icosahedron

An interesting extension and discussion point is Euler's formula: faces plus vertices minus two will equal the number of edges (F + V – 2 = E). (Euler [1707–1783] was a Swiss mathematician who produced a number of important mathematical works and hundreds of mathematical and scientific memoirs.) If we use an icosahedron as an example of Euler's formula, 20 + 12 – 2 = 30. An icosahedron has 30 edges.

A useful question could be: "If I added the number of faces and vertices together, what do I need to do to get the same number of edges as shown in the table? Does Euler's formula always work?" This formula introduces some algebra and links to the achievement objective: describe in words, rules for continuing number and spatial sequential patterns, Algebra, level 3.

Investigation

The cube, tetrahedron, octahedron, dodecahedron, and icosahedron are known as Platonic solids. They are named after Plato (428–347 B.C.), who studied each solid intensively. Plato attributed the four main elements in the universe to the platonic solids: fire – tetrahedron, earth – cube, air – octahedron, and water – icosahedron. He also attributed the world to the dodecahedron.

Pages 6–7: Starting Blocks

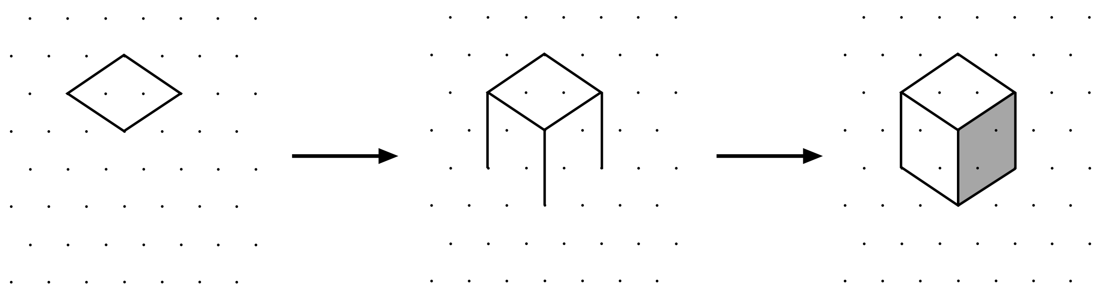
Achievement Objective

- model and describe 3-dimensional objects illustrated by diagrams or pictures (Geometry, level 3)

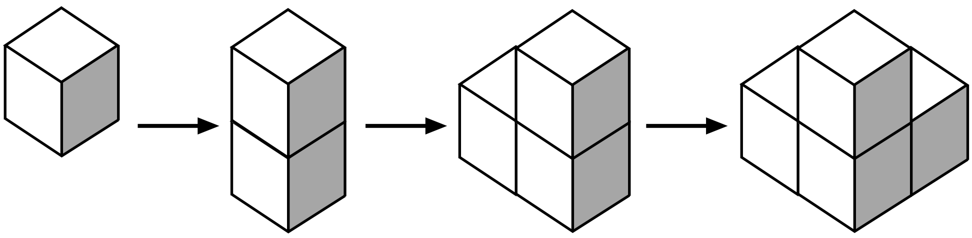
Activity

The aim of this task is to develop students' spatial visualisation.

The greatest difficulty students have is visualising the cubes that they cannot see. To help with this, have students make the completed building, including the missing piece, before attempting to draw it. Throughout the process, encourage students to communicate their understandings and ideas. To further reinforce understanding, the drawings can be done on isometric dot paper.



From this initial drawing, students can go on to design their own cube buildings:



Encourage students to make models from other students' designs. This helps students to consolidate their understanding of three-dimensional shapes and encourages ownership of ideas.

Pages 8–9: Different Viewpoints

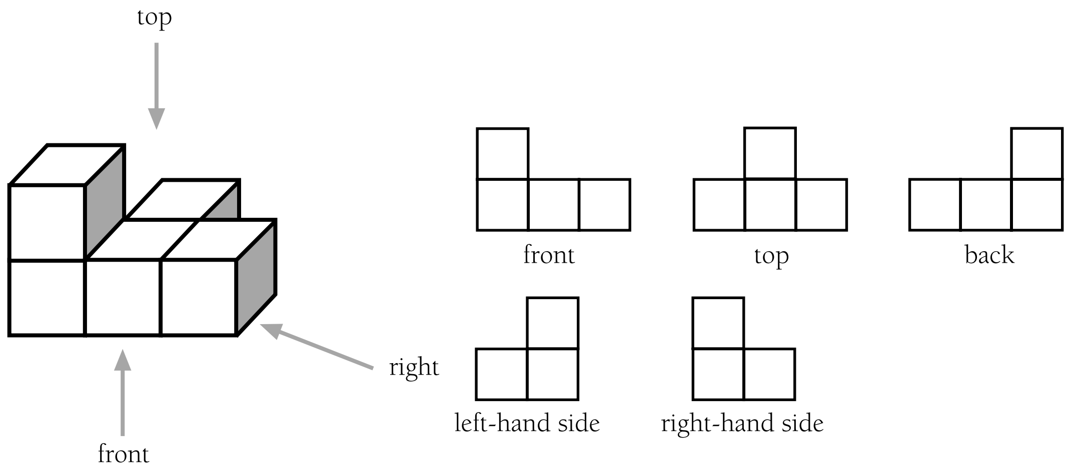
Achievement Objectives

- describe the features of 2-dimensional and 3-dimensional objects, using the language of geometry (Geometry, level 3)
- draw pictures of simple 3-dimensional objects (Geometry, level 3)

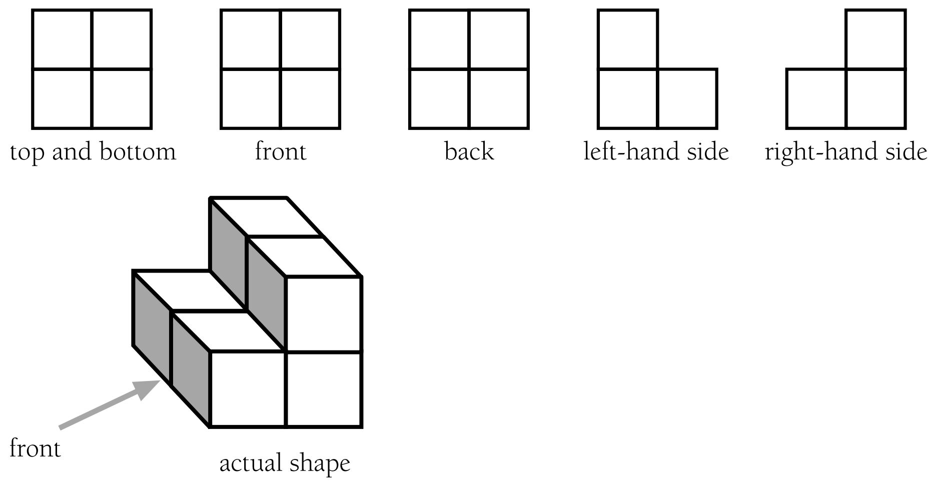
Activity

This activity builds on the activity in *Geometry*, Figure It Out, Levels 2–3, page 11.

Students must have the opportunity to make the buildings illustrated in this activity and then discuss and draw the different perspectives of the buildings. Some students will find visualisation of this kind very difficult. Talk with them about the perspectives that they are looking for. Remind them that, in each case, they are looking for the number of flat surfaces that can be seen.



A further challenge is to give students the different views and ask them to build the shape and draw isometric drawings.



Page 10: A Chip off the Old Block

Achievement Objectives

- model and describe 3-dimensional objects illustrated by diagrams or pictures (Geometry, level 3)
- draw pictures of simple 3-dimensional objects (Geometry, level 3)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 3)

The greatest difficulty students have in working out how many cubes were used to make a drawn building is visualising the cubes they cannot see.

Activity One

You could ask students to predict the number of cubes they think is in each shape before they begin building. Students can then build their shapes and compare the physical shape with their answers. This should lead to some useful discussion about the cubes that they cannot see and should help students to understand why they need to include these cubes.

Activity Two

Students need to have had experience drawing simple three-dimensional shapes onto isometric dot paper before attempting this more difficult example.

Page 11: Nutting out Nets

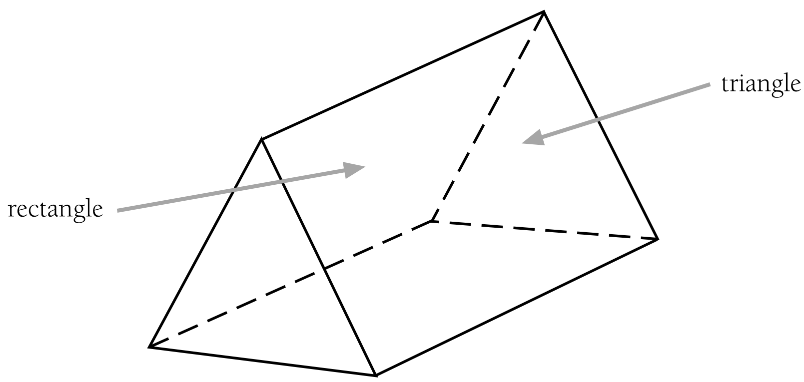
Achievement Objectives

- describe the features of 2-dimensional and 3-dimensional objects, using the language of geometry (Geometry, level 3)
- design and make containers to specified requirements (Geometry, level 3)

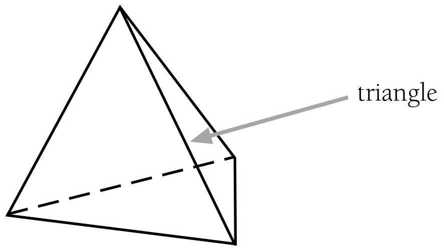
Activity

Give students the opportunity to predict, visualise, and explain before they make up the nets shown on this page. The following definitions may be helpful:

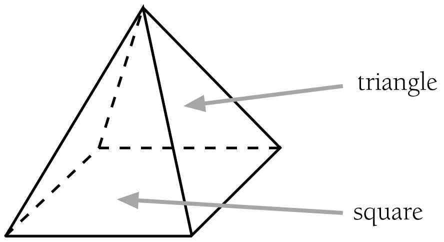
- Triangular prism: A prism has a pair of congruent and parallel faces, in this case, triangles. All other faces are rectangles.



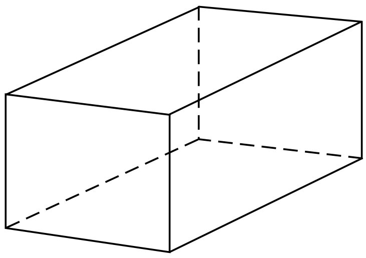
- Tetrahedron: All faces are triangles. Another name for this solid is a triangular-based pyramid.



- Square-based pyramid: A solid shape that has triangles for faces and a square for the base.



- Rectangular prism: All six faces are rectangles.

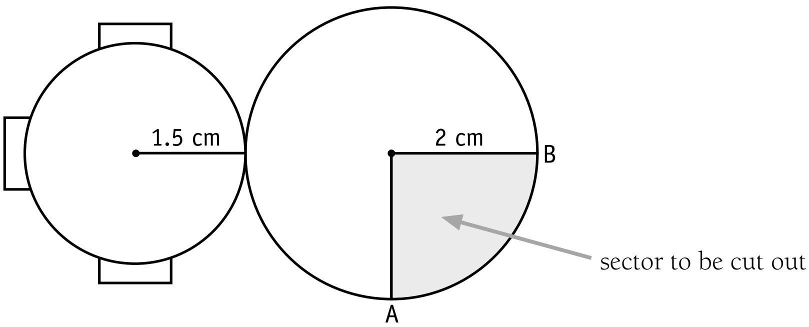


For question **2**, students will need to know the shape of each face in each of the three-dimensional shapes. Students could look at the net of one of these shapes and discuss why it is an accurate net. For example, they could carefully pull apart the glued edges of a cereal box, which is a rectangular prism. They could look at where the tabs are on the cereal box and discuss why they might be there. See also the nets on page 19 of *Answers and Teachers' Notes*: *Geometry*, Figure It Out, Levels 2–3.

A good introduction to both the cone and cylinder, in question **3**, is to consider both shapes in a real-life situation. A cone-shaped ice-cream wrapper is a good example of a cone net. Remember to include the lid of the wrapper, which is the base of the cone. A can of baked beans is a good starting point when thinking about the net of a cylinder.

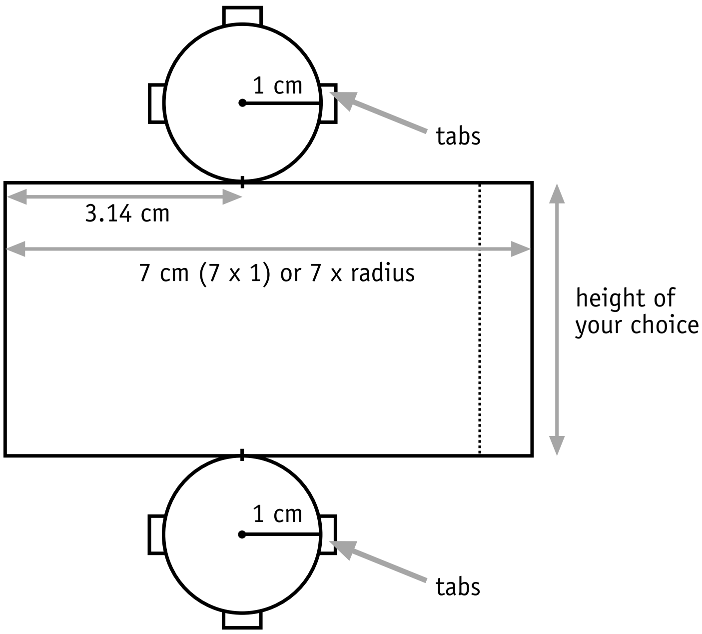
To draw a net for a cone:

- Choose a radius length for a circle, for example, 2 centimetres.
- Mark a quarter of the circle, as shown. (This is cut out later and A is joined to B.)
- Draw a base with a radius that is three-quarters of the first circle's radius. Tabs are optional.



To draw a net for a cylinder (a sheet of A4 paper is ideal for this):

- Choose a radius length for a circle. A radius of 1 centimetre is ideal. (The instructions below are based on this radius.)
- Draw the circle with tabs (optional), as shown.
- To make the body of your cylinder, draw a rectangle that is seven times as long as the length of the radius. Make sure each circle is 3.14 centimetres (π) from one end of the rectangle, as shown. The height of the cylinder can be your choice.
- Draw another circle with tabs at the other side of the rectangle.



Page 12: City Skyline

Achievement Objective

- model and describe 3-dimensional objects illustrated by diagrams or pictures (Geometry, level 3)

Activity

In this activity, students need to draw nets of the buildings and then make them into models. Although the Pyramid Plaza net is the one shown on the page, the more simple net of the Capital Chambers building could be a starting point for a discussion about how to draw the nets for the other buildings.

Key questions to prompt students with include:

- What shapes will you need to add to the Capital Chambers net to make nets for the other buildings?
- Look at the Capital Chambers net. Where do you think the shapes to make the other nets will need to go?

There is a definite pattern that emerges with the nets, and there are several possible places where students might place the shapes for the roofs.

Patterns

- The floor of each building can be attached to any side.
- The different shapes are attached alternately at the top of the net. (See the diagrams in the answers.)
- The top of the building can be completed in a variety of ways, for example:
  - Pyramid Plaza – two small rectangles. These could be placed separately or on top of one another. Two small triangles are also needed.
  - Civic Centre – one large rectangle and two rounded shapes.
  - Colonial House – two large squares, which can fit on top of each other. Two small rectangles are also needed.

Let students experiment with different nets. The important point to get across in this activity is that different nets can achieve the same result.

Page 13: Room with a View

Achievement Objectives

- draw and interpret simple scale maps (Geometry, level 3)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 3)

Activity

For this activity, students will need to take part in a teleconference. Before the teleconference, you will need to discuss with students the type of information they will need from the students from another school and who will ask the questions during the teleconference.

The information that they might need includes:

- the size of their classroom – length and width – and the distance between objects. For greater accuracy, students should decide in advance what scale they will use. They will need to have some square dot paper in front of them so they can easily rule up the basic shape of the room when they get this information.
- items in the classroom, for example, whiteboard, computer, class library, and mat area. They will need to find out how big in metres these items are, for example, the computer area may be approximately 2 metres by 1 metre.
- what is at the back of the room, front of room, etc. They will need to use terms such as right front, middle back, and halfway along.
- where the entrance to the classroom is.
- how many desks or tables there are and how these are arranged.

As the information is gathered, students can record it on the square dot paper. A sample checklist is shown below:

	Size	Position
Whiteboard	6 m long	middle front
Computer area	2 m x 1 m	middle back
Class library	2 m x 1 m	back right
Technology corner	2 m x 2 m	left middle
Mat area	5 m x 5m	middle front
Door	1.25 m x 0.75 m	back left

Pages 14–15: Fun Run

Achievement Objectives

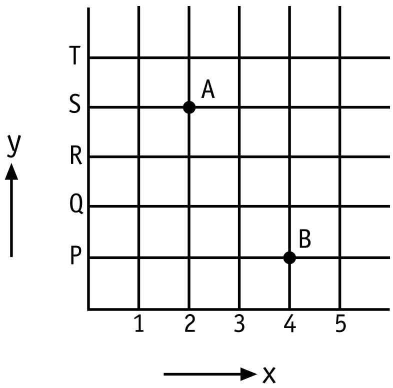
- draw and interpret simple scale maps (Geometry, level 3)
- specify location, using bearings or grid references (Geometry, level 4)

Activity

To answer questions **1**, **2a**, and **2b**, students need to work out the scale used on the map. From the key at the bottom of the map, they can see that 2.5 centimetres equals 0.5 kilometres, so the scale is 5 centimetres : 1 kilometre.

Students could look out for interesting places the run might go past. They should realise that there are a number of routes the fun run might take and that certain routes might be better than others.

Co-ordinates have been used in question **2**. Students need to understand that to find co-ordinates on most maps, they need to first move up and then out. Note that graphs normally have co-ordinates read as (x, y). Maps reverse the order.

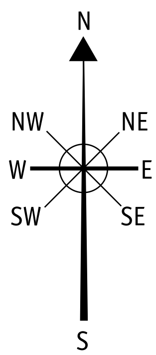


A is at (S, 2).

B is at (P, 4).

These co-ordinates are called ordered pairs. Up is known as the y axis. Out is known as the x axis.

Check that students know the points of the compass.



Here are some extension ideas:

- Increase the distance of the fun run.
- Students who are participating in the fun run need to train. Plan some interesting jogging routes of 2, 3, or 4 kilometres.
- Measure a 1 kilometre distance. How long would it take you to run this distance? (This connects to Measurement.)
- Discuss and use the scale found in an atlas. Plan various routes around New Zealand for a range of long-distance activities.

Pages 16–17: Slice of Life

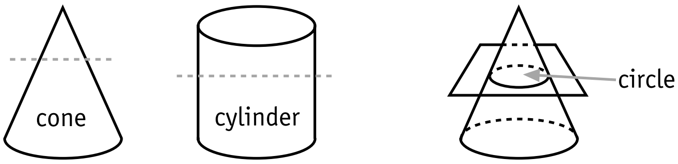
Achievement Objectives

- describe the features of 2-dimensional and 3-dimensional objects, using the language of geometry (Geometry, level 3)
- model and describe 3-dimensional objects illustrated by diagrams or pictures (Geometry, level 3)

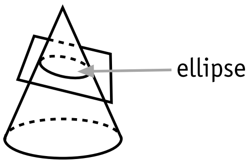
Activity

Predicting results is an important part of this activity.

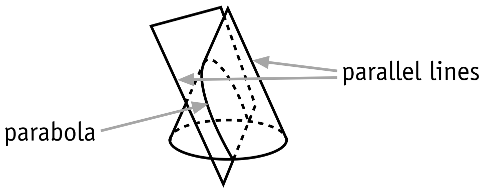
The angle that the cut is made on will affect the shape of the section. For example, a cut that is at right angles to the axis will give a circle as the cross section of a cone and a cylinder.



These right-angled cuts are much easier to predict than those that are slanted. When a slanted cut is used, the section is very much influenced by how and where the cut is made. A cut through a circle-based cone at a slant will give an ellipse as a section.

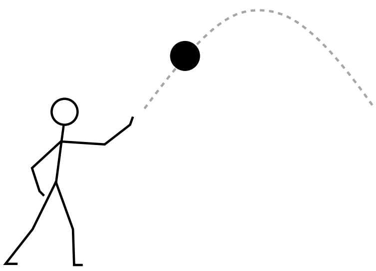


A parabola is formed by cutting through a circle-based cone parallel to a sloping side.

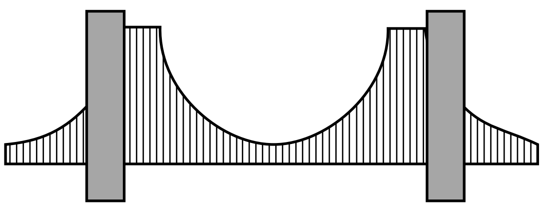


Examples of a parabola shape are:

- the path followed by a ball in flight



- the shape of wires supporting a suspension bridge.



Page 18: Walking Triangles

Achievement Objectives

- describe and interpret position, using the language of direction and distance (Geometry, level 2)
- pose questions for mathematical exploration (Mathematical Processes, problem solving, level 3)

Activity

Discuss with the class and interpret the directions indicated in the diagram that Trevor Triangle is holding. The walks already illustrated should be a good basis for this discussion.

To further extend thinking, use questions such as:

- What would Trevor Triangle have to throw to end up back where he started?
- What would Trevor's path be if he threw a 1, 2, 3, or 4 repeatedly?
- What are all the possible paths Trevor could take if he threw a mixture of 1, 2, 3, or 4?

Students could physically walk out the paths taken by Trevor if they are having difficulties visualising the possible options.

Page 19: Pattern Pieces

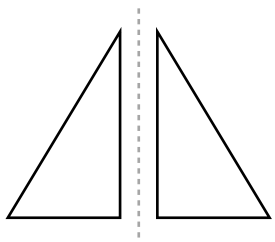
Achievement Objective

- describe patterns in terms of reflection and rotational symmetry, and translations (Geometry, level 3)

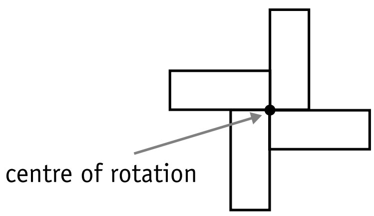
Activity

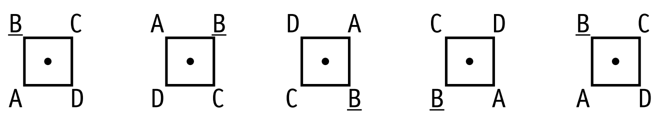
Students need to understand what reflection, rotation, and translation are. Students may need to reflect/flip, rotate, and translate simple shapes in order to visualise what is happening. Foam shapes, cut-out shapes, and/or mirrors will help students to do this.

A **reflection** is a flip:



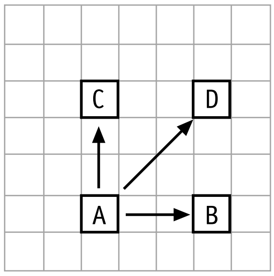
A **rotation** is when an object is turned about a fixed point called the centre of rotation. The order of rotational symmetry is the number of times a shape matches itself exactly during one complete turn. (See notes for page 4.)





The square will match its own outline four times as it rotates through 360 degrees. It has rotational symmetry of order 4.

A **translation** is when a shape slides without turning. This can be to the right, to the left, up, or down. The shape does not change its size and still faces in the same direction.



- A translates to B three grids to the right.
- A translates to C up three grids.
- A translates to D up three grids on the diagonal.

The activity on this page focuses on the symmetry of each design. As an extension, you could encourage students to also look for symmetry **within** each design.

Page 20: Circle Power

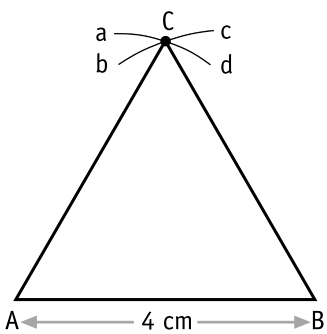
Achievement Objective

- design and make a pattern which involves translation, reflection, or rotation (Geometry, level 3)

Activity

Students will need experience with reflection, translation, and rotation. (See also pages **4** and **19** of the students' booklet.) They could practise drawing simple circles and patterns so that they learn how to be very exact when drawing with a compass, for example, drawing a circle with a set radius or drawing an equilateral triangle, using the following method:

- Measure 4 centimetres on the compass. Place the point of the compass on A and draw an arc a–d.
- Place the compass point on B and draw an arc b–c.
- Join AC and CB where the arcs cross.



Page 21: One Good Turn

Achievement Objectives

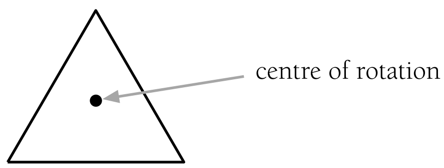
- describe patterns in terms of reflection and rotational symmetry, and translations (Geometry, level 3)
- use equipment appropriately when exploring mathematical ideas (Mathematical Processes, problem solving, level 3)

Activity

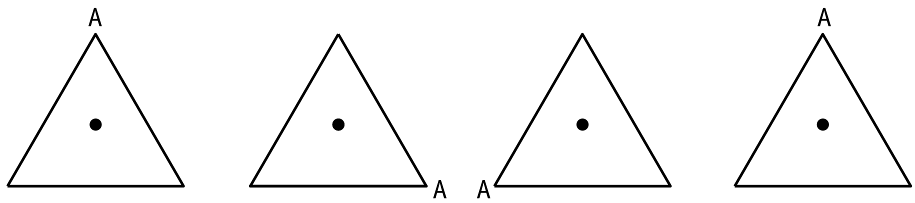
Students need to recognise that each of the shapes in question **1** is made to a pattern. When students have made the various shapes, give them the opportunity to discuss the patterns they see. Encourage them to look for mirror lines in each shape and to recognise that some parts of the shape are mirror images or rotations of another part of the shape. This will help with visualisation.

Once students have made their shapes, have them either trace or draw these onto graph paper so that they can physically rotate the shapes and see how many times each shape maps onto itself. (The use of graph paper will help students be accurate in their drawing.)

The centre of rotation is the point that does not move when the shape is rotated. It is normally shown with a dot.



Rotational symmetry is the number of times a shape matches itself exactly during one complete turn.



equilateral triangle

This has rotational symmetry of order 3. The triangle matches itself exactly three times during one complete turn.

See also pages **4**, **19**, and **20** of the students' booklet.

Page 22: Reflecting on Counters

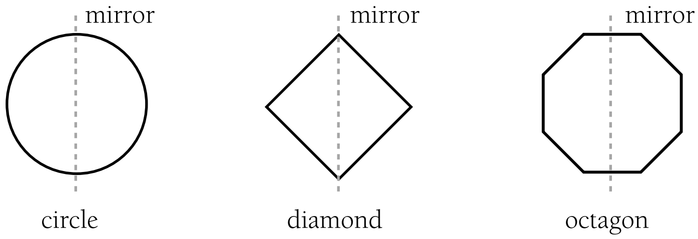
Achievement Objectives

- design and make a pattern which involves translation, reflection, or rotation (Geometry, level 3)
- use equipment appropriately when exploring mathematical ideas (Mathematical Processes, problem solving, level 3)

Activity

In this activity, students are developing the concept of symmetrical reflection using mirrors.

If you place a mirror across half a shape, the full shape is reflected in the mirror. This activity allows students to experiment with this concept in a fun problem-solving way.



Page 23: The Big Picture

Achievement Objective

- enlarge, on grid paper, simple shapes to a specified scale (Geometry, level 3)

Activity

An enlargement maps a shape onto a similar shape, using a scale factor. For example, if the enlargement is to be double the size of the original picture, the scale factor is two. The enlarged picture has the same proportions as the original.

This activity uses the grid method of enlargement. To produce a design that is twice the size, students will need to use 2-centimetre-square grid paper. (The grid paper in the illustration is 1 centimetre square). Students then need to copy the lines inside each 1 centimetre square onto the 2-centimetre-square grid. If students have drawn a grid in pencil instead of using square grid paper, they can rub it out to leave just the enlarged design. You could also refer to "How Do You Measure a Dinosaur?" in *Connected 1 1998*.

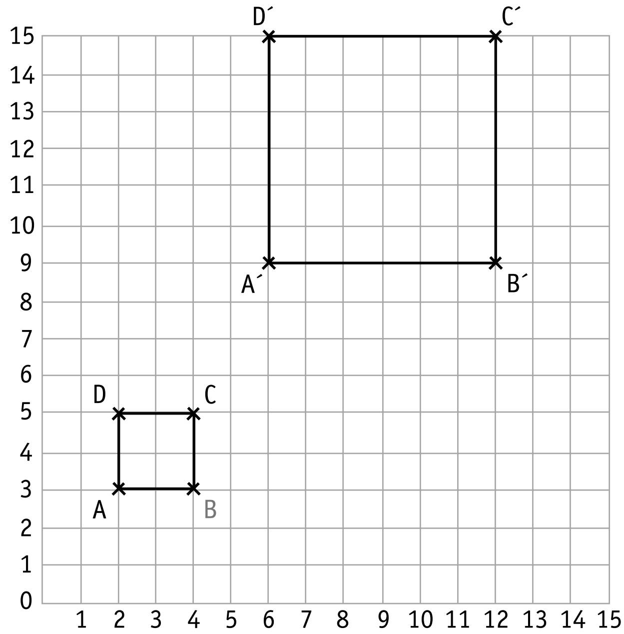
Another way of enlarging a basic shape is to use ordered pairs and a matrix. For example:

- Map these ordered pairs and label them A, B, C, and D: A = (2, 3), B = (4, 3), C = (4, 5), and D = (2, 5).
- Join A–B, B–C, C–D, and D–A.
- To enlarge this square by three, you need to multiply the ordered pairs by three: A´ = (6, 9), B´ = (12, 9), C´ = (12, 15), D´ = (6, 15).

Matrix

$$3 \times \begin{pmatrix} 2 & 3 \\ 4 & 3 \\ 4 & 5 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 6 & 9 \\ 12 & 9 \\ 12 & 15 \\ 6 & 15 \end{pmatrix}$$

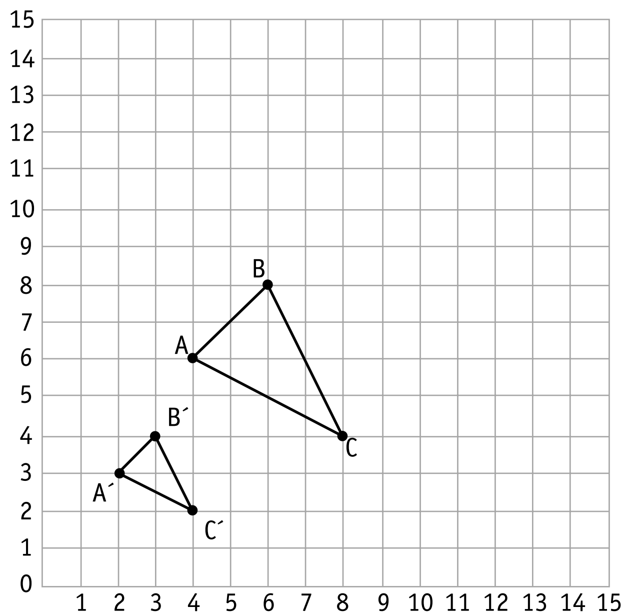
original ordered pairs — Map these onto the grid for a square three times as large.



- To reduce a shape, you divide instead of multiplying. To reduce the shape by half, divide the ordered pairs by two. For example, to reduce a triangle using ordered pairs and a matrix:

$$\begin{pmatrix} 4 & 6 \\ 6 & 8 \\ 8 & 4 \end{pmatrix} \div 2 = \begin{pmatrix} 2 & 3 \\ 3 & 4 \\ 4 & 2 \end{pmatrix}$$

original ordered pairs — Map these onto the grid for a triangle reduced by half.



Page 24: Growing Changes

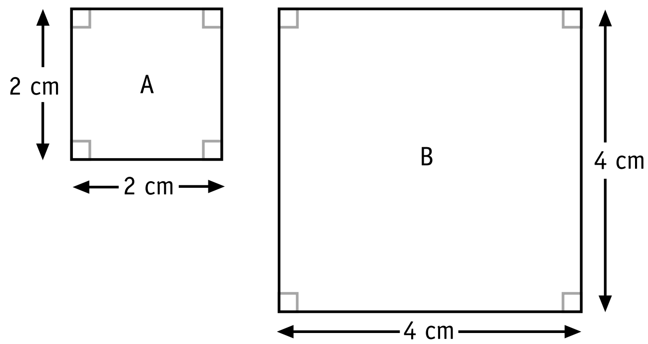
Achievement Objective

- enlarge, on grid paper, simple shapes to a specified scale (Geometry, level 3)

Activity Two

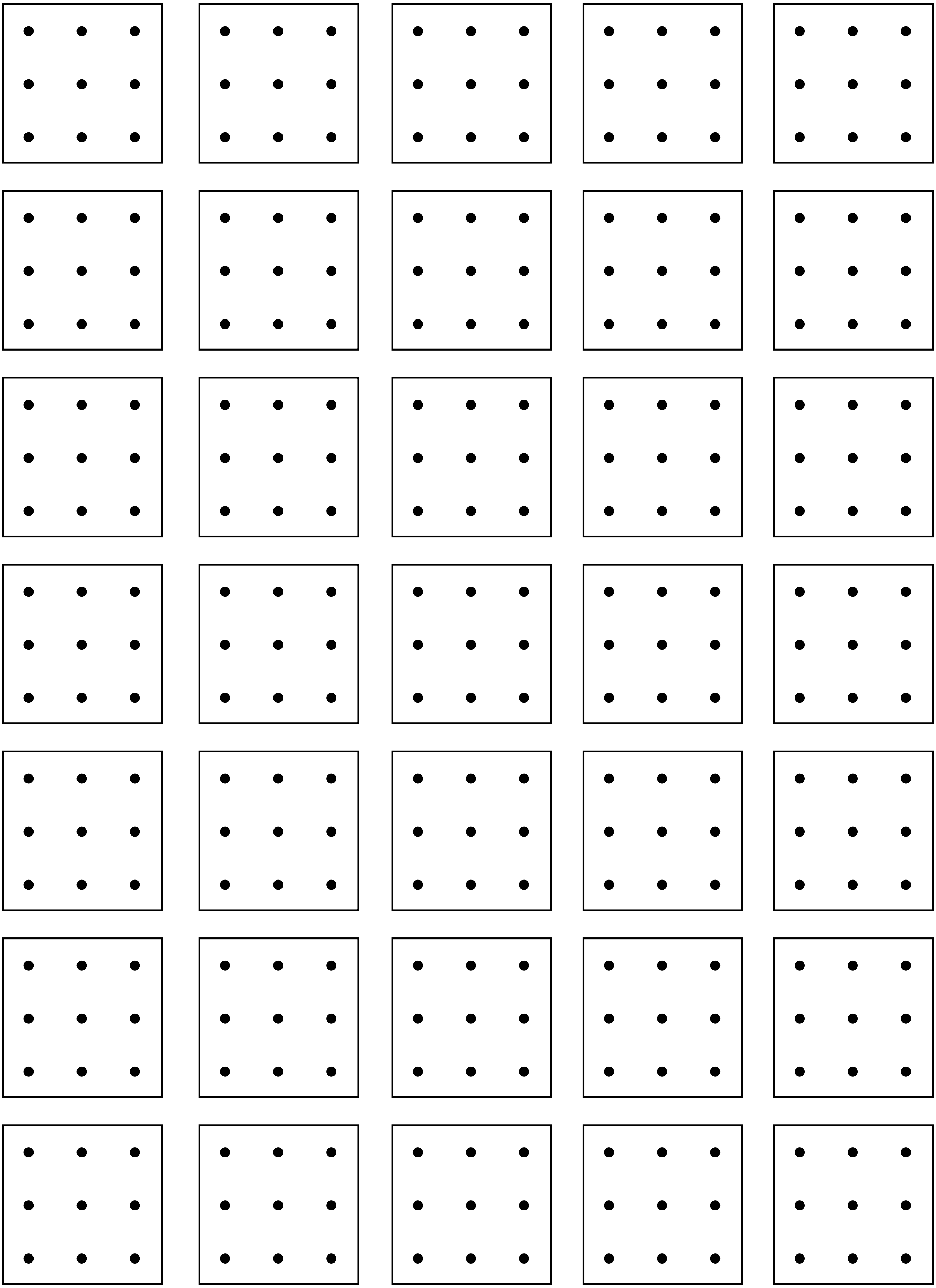
Warn students not to move the overhead projector while it is switched on, or the bulb may blow.

If students are having difficulty with the triangle and the circle, you could start with a simple square, as shown in **Activity One**:

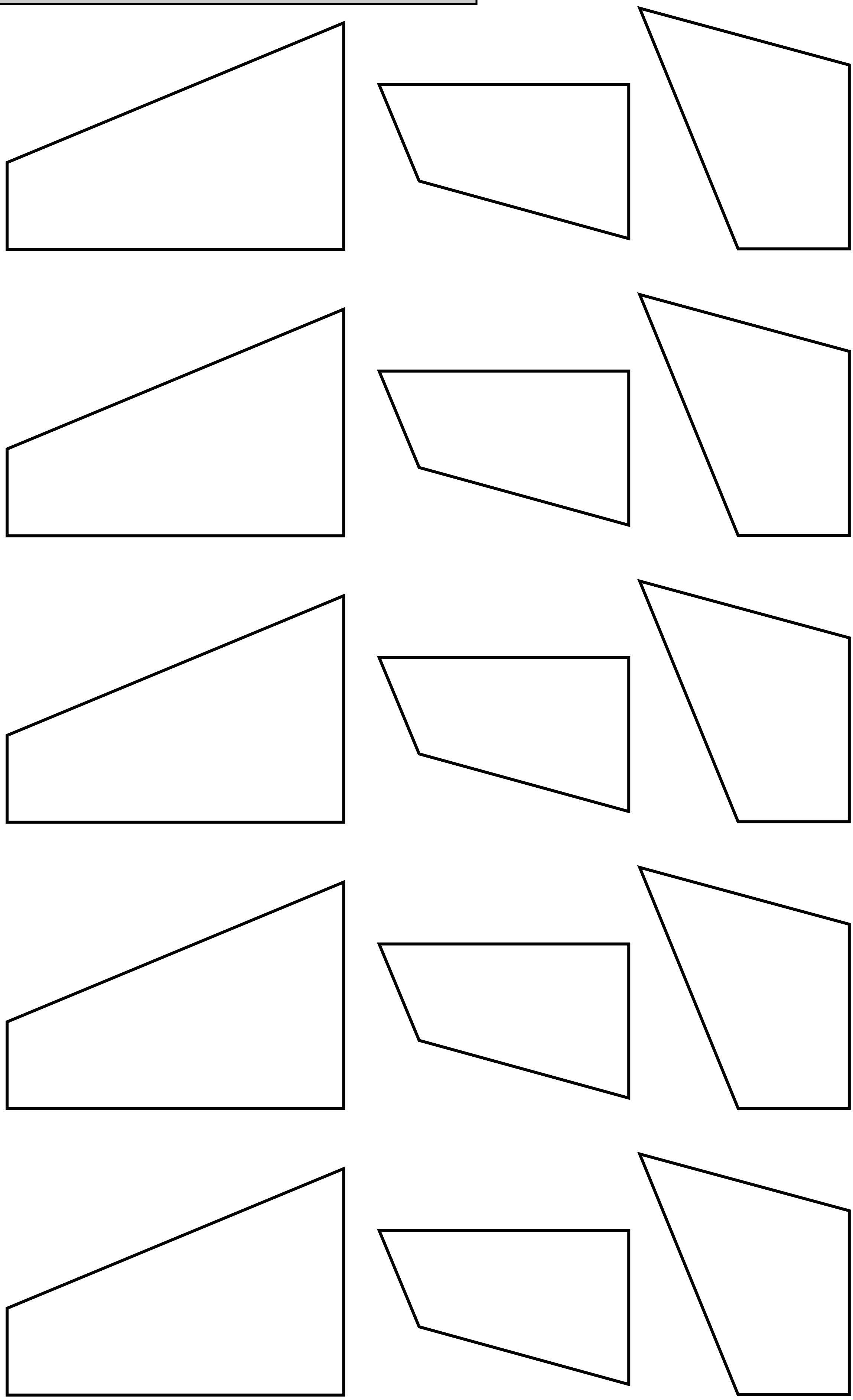


B is an enlargement of A. The scale factor is two.

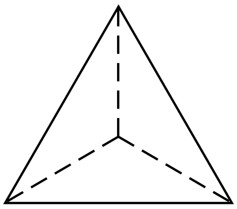
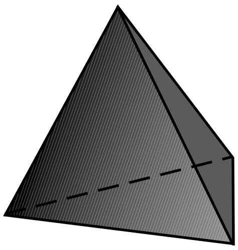
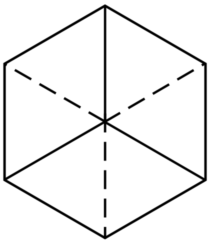
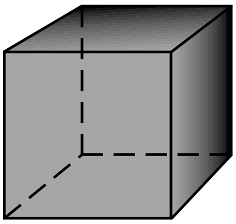
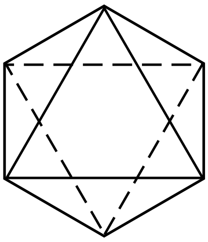
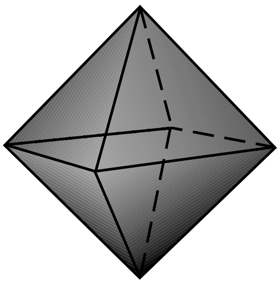
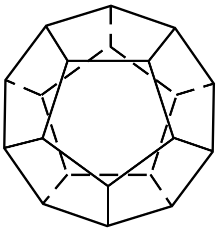
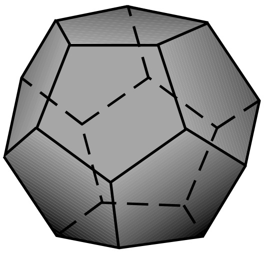
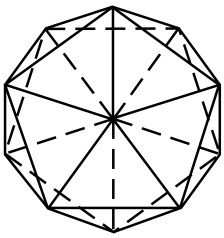
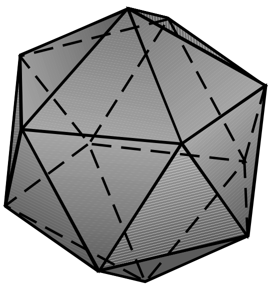
Copymaster: Good Points about Triangles



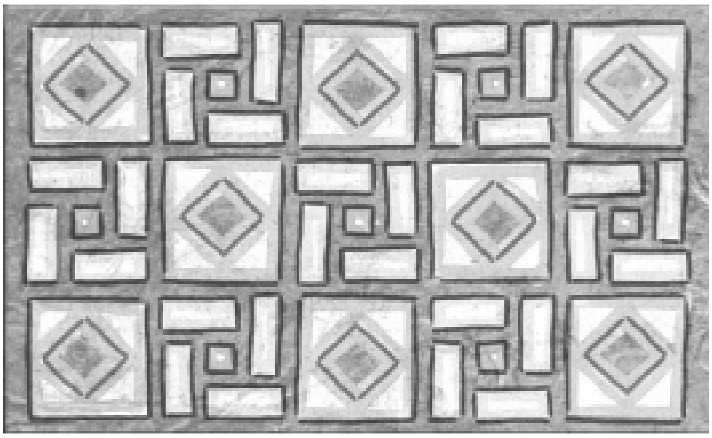
Copymaster: Shaping Up

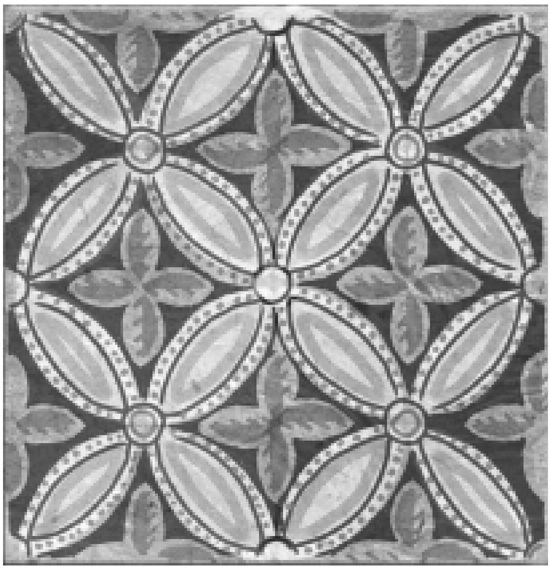


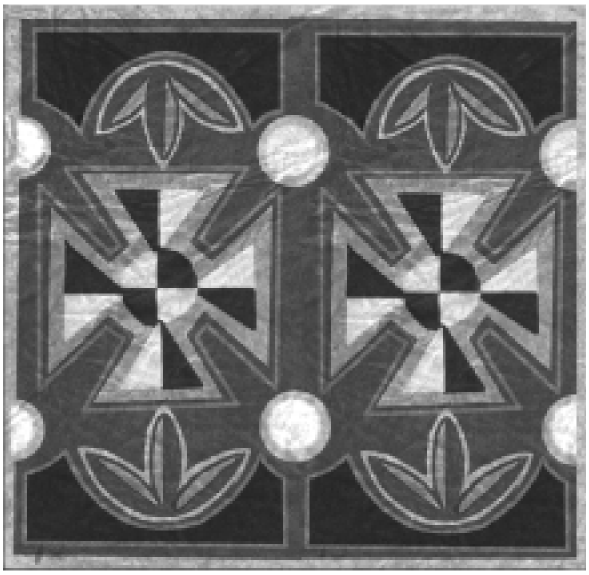
Copymaster: Let's Face It

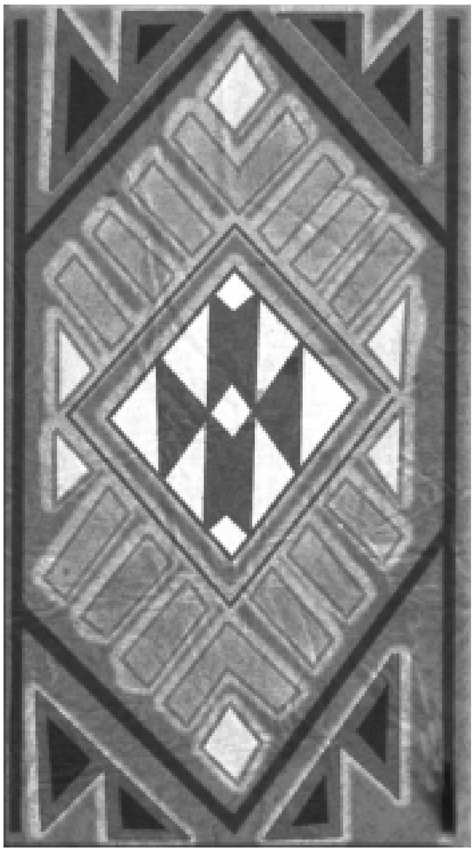
Plan view	Three-dimensional view	Edges	Faces	Vertices
 Tetrahedron				
 Cube				
 Octahedron				
 Dodecahedron				
 Icosahedron				

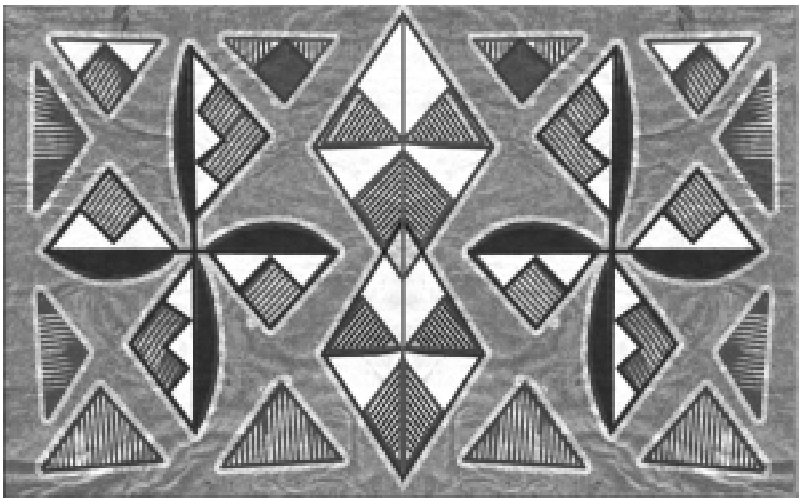
Copymaster: Pattern Pieces











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