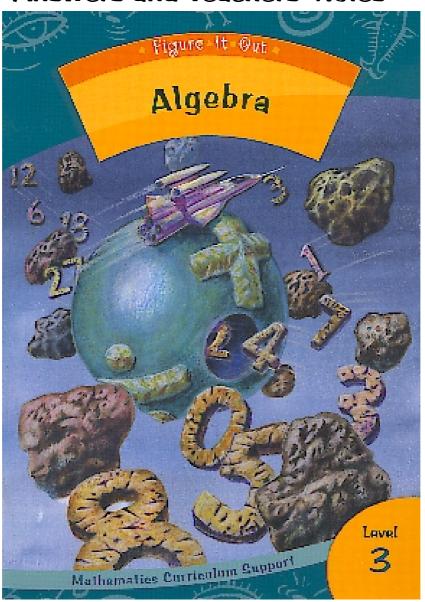
Answers and Teachers' Notes





Contents

Introduction	2
Answers	3
Teachers' Notes	14

Introduction

The Figure It Out series is designed to support *Mathematics in the New Zealand Curriculum*. The booklets have been developed and trialled by classroom teachers and mathematics educators. The series builds on the strengths of a previous series of mathematics booklets published by the Ministry of Education, the School Mathematics supplementary booklets.

Figure It Out is intended to supplement existing school mathematics programmes and can be used in various ways. It provides activities and investigations that students can work on independently or co-operatively in pairs or groups. Teachers can select particular activities that provide extension to work done in the regular classroom programme. Alternatively, teachers may wish to use all or most of the activities in combination with other activities to create a classroom programme. The booklets can be used for homework activities, and the relevant section in the teachers' notes could be copied for parents. These notes may also provide useful information that could be given as hints to students.

There are eight booklets for level 3: one booklet for each content strand, one on problem solving, one on basic facts, and a theme booklet. Each booklet has its own *Answers and Teachers' Notes*. The notes include relevant achievement objectives, suggested teaching approaches, and suggested ways to extend the activities. The booklets in this set (level 3) are suitable for most students in year 5. However, teachers can decide whether to use the booklets with older or younger students who are also working at level 3.

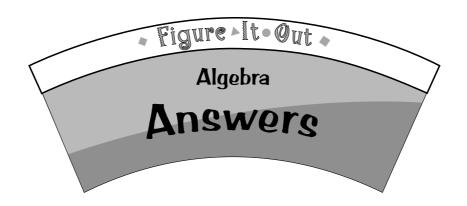
The booklets have been written in such a way that students should be able to work on the material independently, either alone or in groups. Where applicable, each page starts with a list of equipment that the students will need to do the activities. Students should be encouraged to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Teachers could encourage students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Where possible, suggestions have been made to encourage discussion and oral presentation of answers, and teachers may wish to ask the students to do this even where the suggested instruction is to write down the answer.

The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.

Mathematics in the New Zealand Curriculum, page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. Successful ways of solving problems should be acknowledged, and where more effective or efficient processes can be used, students can be encouraged to consider other ways of solving the problem.



Page 1: Terrific Tiles

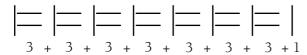
Activity

- 1. 52 tiles
- 2. 21 rhombuses
- 3. 31 triangles
- 4. 42 trapezia

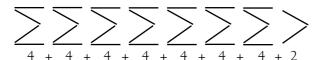
Page 2: Sticking Around

Activity One

- 1. Answers will vary. Possible counting methods and rules for each pattern are:



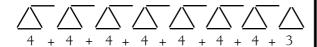
b. Count the arrows in groups of 4 sticks $(4 + 4 + 4 + 4 + 4 + 4 + 4 + 2 = 7 \times 4 + 2 = 30)$:



c. Count the houses in groups of 4 sticks $(4 + 4 + 4 + 4 + 4 + 4 + 4 + 1 = 7 \times 4 + 1 = 29)$:



d. Count the triangles in groups of 2 sticks $(15 \times 2 + 1 = 31)$ or in groups of 4 sticks $(4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 3 = 7 \times 4 + 3 = 31)$:



- **2**. **a**. 46
 - **b**. 62
 - **c.** 81
 - **d**. 41

Activity Two

Answers will vary.

Page 3: Tukutuku Patterns

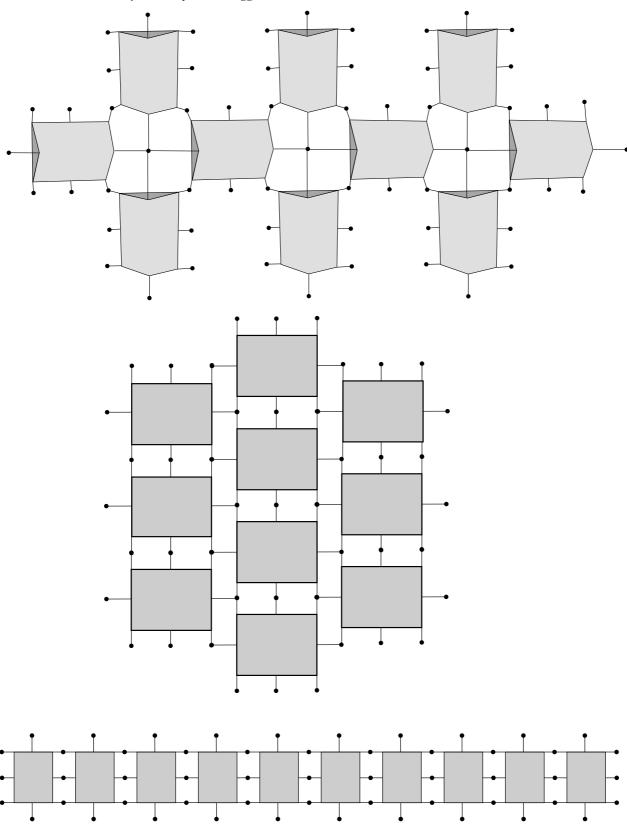
Activity

- a. Answers will vary. The quickest way is to see the first step as 6 crosses and to add 4 crosses for each new step. These 4 crosses could be inserted above the bottom line (the 2 single crosses) each time.
 - b.
 - **c.** 42
- **2**. **a**. 30
 - **b**. 56
 - **c**. 41

Page 4: Pegging Problems

Activity

a. Answers will vary. Three possible suggestions are:

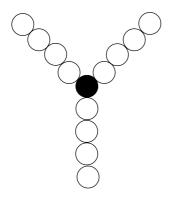


- **b.** Answers will vary.
- **c.** Answers will vary.

Page 5: Pattern Parade

Activity One

a. i.



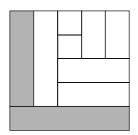
ii.



iii.



iv. Answers will vary. One answer, based on using the next size rod, would be:



(The 2 new rods are shaded.)

v.



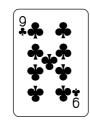
b.



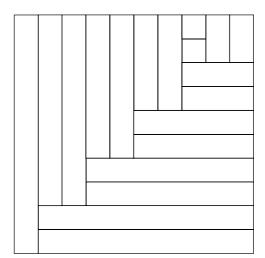
ii.



iii.



iv. Answers will vary. One answer, based on using the next size rod, would be:



V.



c. Answers will vary. The patterns for each are:

i. Each arm increases by 1.

ii. +3-4+3-4 etc.

- iii. Cards follow a suit sequence of spades, clubs, diamonds, and hearts. Each time a suit is repeated, the number is 3 higher than the previous time. For example, the next spade card after the ace of spades (1) is the 4 of spades and the next club card after the 3 of clubs is the 6 of clubs.
- **iv.** Add 1 rod of the last new colour and 1 of the next size up.
- **v.** The triangle passes through a 90° clockwise rotation in each step.

Activity Two

Answers will vary.

Pages 6-7: Possum Poles

Activity

1. a., b., c.

In the graphs in the following columns, x marks the first 5 minutes (for question a),

- shows the progress at 6 minutes (question **b**), and \blacksquare shows the progress at 7 minutes (**c**).
- **d.** Peta climbs 6 m in 1 minute and rests for 2 minutes.

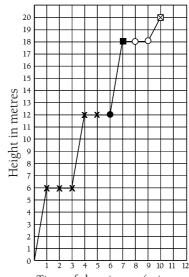
Prue climbs 1 m per minute for the first 3 minutes, 2 m per minute for the next 3 minutes, and 3 m per minute for the next 3 minutes.

Patrick climbs 2 m every 2 minutes.

Priya climbs 7 m in 1 minute then slips 3 m in the next minute.

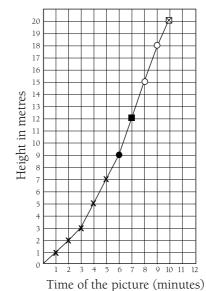
- **2. a.** ⊠ on each graph shows that the possum has reached the 20 m mark, and **O** shows where they are at 8 and 9 minutes.
 - **b.** Priya reaches the 20 m mark before the other 3 possums.
 - **c.** Between 8 and 9 minutes. (Priya reached the 20 m mark in less than a minute after she had climbed for 8 minutes.)

Peta

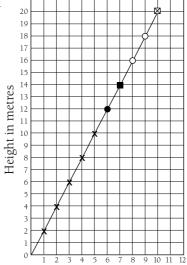


Time of the picture (minutes)

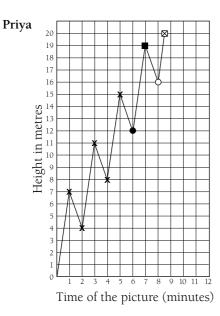
Prue



Patrick



Time of the picture (minutes)



Page 8: Preparing for the Hangi

Activity One

Everybody gets an equal amount of chicken and pork. Each person would eat only half as much chicken as they would normally eat if they were served only chicken. The same is true for the pork. So, the new rules are:

 $1\ \mbox{chicken}$ for every 4 people and $1\ \mbox{kg}$ of pork for every 8 people.

For 30 people, Hine and Rāwiri need $30 \div 4 = 7.5$ chickens and $30 \div 8 = 3.75$ kg pork. (They may have to buy 8 chickens.)

Activity Two

Shopping list:

40 kūmara

50 cobs of corn

36 carrots

15 potatoes

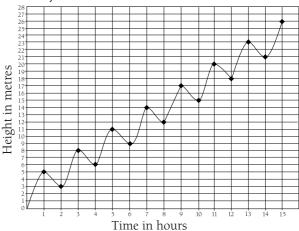
30 handfuls of pūhā

6 kamokamo

Page 9: Ups and Downs

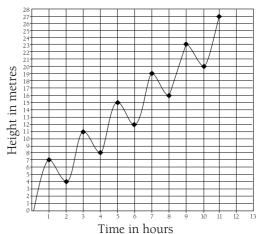
Activity

1. Slimey's Climb



It takes Slimey 15 hours to get to the top of the well.

2. Sluggish's Climb



It takes Sluggish 11 hours to get to the top of the well.

Pages 10-11: Operation Time

Activity One

(The number closest to the arrow goes through first.)

a. 4, 6, 8

b. 14, 18, 22

c. 12, 24, 28

d. 0, 6, 11

Activity Two

1.

Number in	Number out
1	4
2	5
3	6
4	7

2.

Number	Number
in	out
2	22
4	24
5	25
6	26

b.

Number in	Number out
6	2
12	4
15	5
24	8

Activity Three

1. a.

Number	Number
in	out
5	18
6	20
7	22
8	24

b.

Number	Number
in	out
3	5
5	11
7	17
9	23

c.	Number in	Number out
	3	14
	5	24
	7	34
	10	49
	10	50

d.	Number in	Number out
	6	7
	10	9
	14	11
	20	14
	22	15

2. Answers will vary.

Pages 12-13: Spreadsheet Challenge

Activity

Answers will vary.

Pages 14-15: Biscuit Binge

Activity One

1.

a.	Days	1	2	3	4	5	6	7	8	9	10
	Biscuits	2	4	6	8	10	12	14	16	18	20

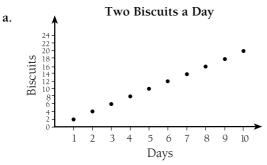
b.	Days	1	2	3	4	5	6	7	8	9	10
	Biscuits	5	10	15	20	25	30	35	40	45	50

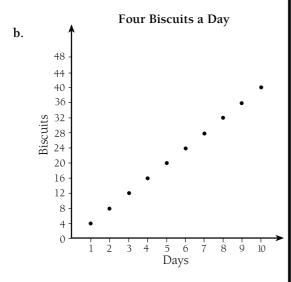
c.	Days	1	2	3	4	5	6	7	8	9	10
	Biscuits	21/2	5	71/2	10	121/2	15	171/2	20	221/2	25

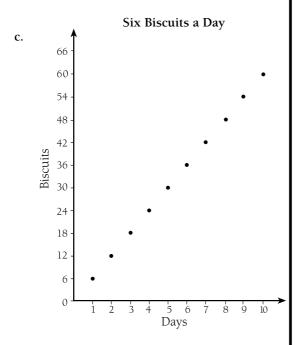
2. Answers will vary. A possible answer is: The total of the biscuits increases evenly in each chart by the number of biscuits eaten that day. To find the total number of biscuits eaten, multiply the number of days by the number of biscuits eaten each day.

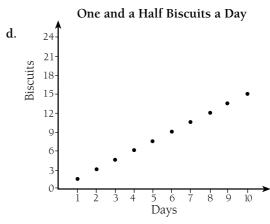
Activity Two

- 1. 3 biscuits each day
- 2. Numbers along the vertical axis will vary. Some possible graphs are:









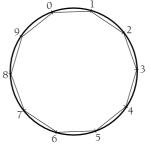
3. Answers will vary. A possible answer is: The points on the graphs are evenly spaced according to the number of biscuits eaten that day. The points lie on a straight line. This is because the difference between points is the same (for example, in **b**, 4 up for every 1 across).

Activity Three

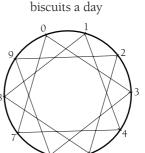
1.

The 3-biscuits-a-day pattern creates a star polygon. You may see other patterns.

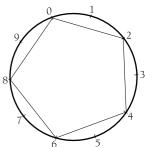
2. Answers will vary. Some examples (using the ones digit of the biscuit numbers):



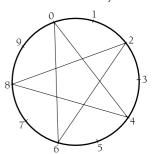
Pattern for 1 and 9 biscuits a day



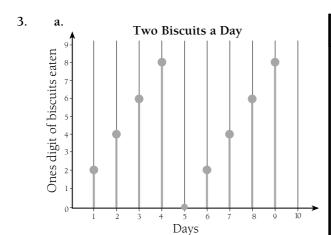
Pattern for 3 and 7 biscuits a day

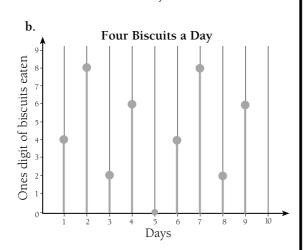


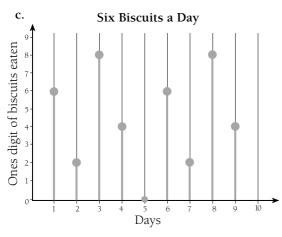
Pattern for 2 and 8 biscuits a day

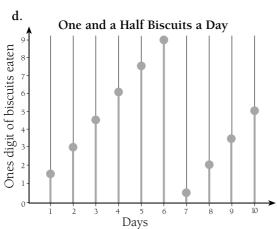


Pattern for 4 and 6 biscuits a day





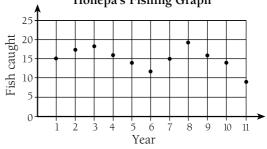




Page 16: Kai Moana

Activity

Hōhepa's Fishing Graph 1. 25 20



- 2. Answers will vary. Daylight saving and the type of fishing rods used are unlikely to affect the number of fish caught. The graph does not show that the number of fishing boats around in the last 5 years has had a major impact on the number of fish caught. So it may just be that this year is low for no particular reason. ("The fishing comes and goes around here.")
- 3. Answers will vary. Some students may think that it will get worse. On the other hand, the graph may be part of a pattern and the fishing could get better again. (Some students may see a pattern of an increase for 3 years and then a decrease for 3 years.)

Page 17: Duncan's Day

Activity

These answers include Duncan's final walk back to the bore.

8 bottles: 8 one-way walks

9 bottles: 8 one-way walks

7 bottles: 6 one-way walks

- 2. Answers will vary. Duncan delivers 2 bottles for every 2 one-way walks except on his last trip of the day, when he delivers up to 3 bottles. A rule could be: For odd numbers of bottles, take 1 off the number of bottles to find the number of oneway walks. For even numbers of bottles, the number of bottles gives the number of one-way
- 3. Answers will vary. Duncan delivers 3 bottles for every 2 one-way walks except on his last trip of the day, when he delivers up to 4 bottles. A rule could be: The number of one-way walks goes up in steps of 2, for example, 2, 3, and 4 bottles: 2 one-way walks; 5, 6, or 7 bottles: 4 one-way walks. Each group of 3 has a multiple of 3 in the

centre. Finding ${}^{2}/_{3}$ of the number of bottles for these multiples of 3 gives the number of one-way walks, for example, ${}^{2}/_{3}$ of 6 is 4 one-way walks.

Page 18: Puzzling Patterns

Activity One

- 1. 100 Working: 10 x 10 = 100
- **2**. 100
- **3.** The answer is the same for the tenth object in each pattern. The number of squares is the same for both patterns, even though they are arranged differently. Each part of both patterns is a square number.

Activity Two

- 1. Ten identical objects can be arranged in a triangular shape.
- **2. a.** Answers will vary. The first picture could be used to find 1, 3, 6, 10, 15, and 21. Adding on layers would give 28, 36, 45, etc.
 - **b.** Answers will vary, but one way of looking at the pattern is:

$$1 = 1$$

$$3 = 1 + 2$$

$$6 = 1 + 2 + 3$$

$$10 = 1 + 2 + 3 + 4$$

$$15 = 1 + 2 + 3 + 4 + 5$$

This can be explained as:

First triangular number: 1

Second triangular number: first triangular number + 2 = 3

Third triangular number: second triangular number + 3 = 6

Fourth triangular number: third triangular number + 4 = 10, and so on.

- 3. Answers will vary. One explanation is: We know that we can arrange numbers such as 3, 6, 10, and 15 so that they are a triangular shape. If you take the bottom layer off each triangular shape, you get the triangular number above it. For example, 15 5 = 10; 10 4 = 6; 6 3 = 3. Following this pattern, if you take the bottom layer (2) off 3, you get 1. So 1 is a triangular number.
- 4. The tenth triangular number is 55 (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10).

Activity Three

a.	Square numbers	1	4	9	16	25	36
	Triangular numbers	1	3	6	10	15	21

b. Answers will vary.

In each set of numbers, there is a pattern in the differences between the numbers.

Square numbers:

$$1 \underbrace{4}_{+3} \underbrace{9}_{+5} \underbrace{16}_{+7} \underbrace{25}_{+9} \underbrace{36}_{+11}$$

Triangular numbers:

Two adjacent triangular numbers added together equal the square number directly above the larger triangular number. For example, 3 + 6 = 9 and 15 + 21 = 36.

Square numbers:

The differences between square numbers and the triangular numbers below them in the table form the set of triangular numbers (except for the initial zero).

Square numbers	1	4	9	16	25	36
Triangular numbers	1	3	6	10	15	21
Difference	0	1	3	6	10	15

Page 19: **See-saw Numbers**

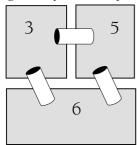
Activity

- 1. **a.** 7 + 3 > 3 + 6
 - **b.** $9 + 7 = 2 \times 8$ [or 9 + 7 = 8 + 8]
 - c. $2 \times 8 > 2 \times 6$ [or 8 + 8 > 6 + 6]
 - **d.** $7 + 5 + (2 \times 3) < 3 + 5 + (2 \times 7)$ [or 7 + 5 + 3 + 3 < 3 + 5 + 7 + 7]
 - e. $4 + (2 \times 2) = 2 + 6$ [or 4 + 2 + 2 = 2 + 6]
 - f. $(2 \times 8) + 3 + 1 > 1 + (2 \times 9)$ [or 8 + 8 + 3 + 1 > 1 + 9 + 9]
- **2.** Answers will vary.

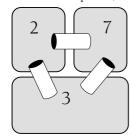
Page 20: Putting Pens to Paper

Activity One

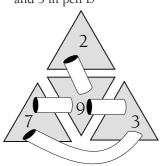
1. 3 goats in pen A, 5 in pen B, and 6 in pen C



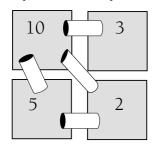
2. a. 2 otters in pen A, 7 in pen B, and 3 in pen C



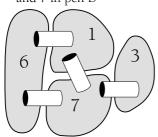
b. 2 monkeys in pen A, 9 in pen B, 7 in pen C, and 3 in pen D



c. 10 meerkats in pen A, 3 in pen B, 5 in pen C, and 2 in pen D



d. 6 birds in pen A, 1 in pen B, 3 in pen C, and 7 in pen D



Activity Two

Answers will vary.

Page 21: Losing Your Marbles

Activity

- 1. 10 cat's-eyes
- 2. 10 milkies
- 3. 3 queenies (10 galaxies = 30 cat's-eyes = 6 boulders = 3 queenies)
- 4. **a.** 4 boulders (1 kingie = 2 queenies = 4 boulders)
 - b. 20 cat's eyes (1 kingie = 2 queenies= 4 boulders = 20 cat's eyes)
 - c. 10 milkies (1 kingie = 2 queenies= 10 milkies)

Investigation

Answers will vary.

Pages 22-23: Pene's Puzzles

Activity One

- 1. **a.** $5 \div 5 = 1$
 - **b.** $5 \times 5 = 25$
 - **c.** $6 \times 1 + 1 = 7$
- 2. a. $10 \div 10 = 1$
 - **b.** $10 \times 10 = 100$
 - c. $11 \times 1 + 1 = 12$
- **3.** Answers will vary.

Activity Two

- **1**. **a**. Yes
 - **b.** One explanation is: Mark knew that x = 2 and $\div 2$ cancelled each other out, $6 \div 2 = 3$, and $\Box \Box = 0$. The only number left is 3, which is the answer.
- 2. a. Yes
 - **b.** This can be explained in 2 steps. Step 1: The puzzle can be written as $(\square \times 2 - 7 + 21) \div 2 = \boxed{?}$ This can be simplified to $(\square \times 2 + 14) \div 2 = \boxed{?}$ and then to $\square + 7 = \boxed{?}$

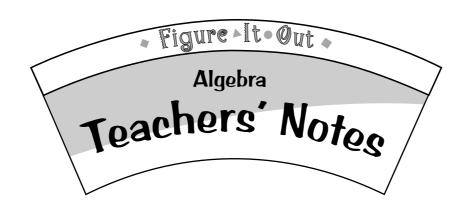
Step 2: ? - = 7. (? will always be your number plus 7. When you subtract your number, you will always be left with 7 as an answer.)

3. Puzzles and solutions will vary.

Page 24: The Potluck Paint Company

Activity

Answers will vary.



Overview: Algeb	ora		
Title	Content	Page in students' book	Page in teachers book
Terrific Tiles	Finding and applying rules for sequential patterns	1	15
Sticking Around	Finding and applying rules for sequential patterns	2	16
Tukutuku Patterns	Finding and applying rules for sequential patterns	3	17
Pegging Problems	Finding and applying rules for sequential patterns	4	19
Pattern Parade	Continuing sequential and repeating patterns	5	21
Possum Poles	Using rules and graphs to model practical situations	6–7	22
Preparing for the Hāngi	Using rules to find amounts	8	23
Ups and Downs	Graphing relationships	9	24
Operation Time	Applying rules to make tables of a value	10–11	24
Spreadsheet Challenge	Finding rules to describe relationships	12–13	26
Biscuit Binge	Representing relationships in tables and graphs	14–15	27
Kai Moana	Graphing relationships	16	28
Duncan's Day	Finding rules to describe relationships	17	29
Puzzling Patterns	Predicting further members of number patterns	18	30
See-saw Numbers	Using greater than, less than, equal to	19	32
Putting Pens to Paper	Solving sets of equations	20	32
Losing Your Marbles	Expressing relationships	21	33
Pene's Puzzles	Solving sets of equations	22–23	33
The Potluck Paint Company	Using algebraic symbols	24	34

Algebra is a very powerful problem-solving tool. The activities in this booklet encourage students to use algebraic problem-solving techniques.

The main techniques include ways of:

- identifying patterns in numbers and shapes
- expressing patterns in words and symbols
- predicting future results by identifying patterns and general rules
- making tables
- plotting points on graphs.

Page 1: Terrific Tiles

Achievement Objectives

- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- use words and symbols to describe and continue patterns (Mathematical processes, developing logic and reasoning, level 3)

Activity

If students are unsure how to begin these problems, ask them first how many tiles there are in a one-person pattern. Then ask how many extra tiles they will need for each extra person. Suggest that they record the number of tiles needed in a table.

Number of people	Number of tiles
1	7
2	12
3	17

Students should see that each time another person is added, they need five extra tiles. To find out how many tiles they need to make the pattern with 10 people, students can extend their table until they get to 10 people.

Number of people	Number of tiles
1	7
2	12
3	17
4	22
5	27
:	:
10	52

Although students are not asked to find a general rule or formula for the pattern, as an extension exercise, you could work with students to find the general rule for the pattern.

You could use this to show them that a shorter way of writing this is $10 \times 5 + 2 = 52$. So if n stands for the number of people, the general rule is: number of tiles needed for n people = $n \times 5 + 2$.

This can also be developed further in a table:

Number of people	1	2	3	4	 10	n
Number of tiles	7	7 + 5 = 12	12 + 5 = 17	17 + 5 = 22		
Using rule	$1 \times 5 + 2 = 7$	2 x 5 + 2 = 12	$3 \times 5 + 2 = 17$	4 x 5 + 2 = 22	 $10 \times 5 + 2 = 52$	n x 5 + 2

Students can follow the same procedure to answer the other questions on this page.

Page 2: Sticking Around

Achievement Objectives

- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- use words and symbols to describe and continue patterns (Mathematical processes, developing logic and reasoning, level 3)

Activities One and Two

These activities encourage students to work in pairs to:

- look for patterns
- talk about their results
- develop a method of recording.

These activities are similar to the ones on page 1 of the students' booklet. You may need to explain to students that the shaded part is one section of fence. Ask students how many extra sticks are needed for each new section of fence. Once again, encourage them to use tables to record the number of sticks needed:

Number of sections of fence	Number of sticks needed
1	4
2	7
3	10
4	13
5	16
6	19
7	22

From this, students should recognise that each time another section is added to the fence, the number of sticks increases by three.

A quick way of counting in **Activity One**, **1a**, is to count the sticks in threes and have one left over, for example, 3 + 3 + 3 + 3 + 3 + 3 + 3 + 1 = 22, or students might say "Seven groups of three plus one left over makes 22". Ask students if they can think of a shorter way to write this answer, for example, $7 \times 3 + 1 = 22$.

Students could use this equation as a rule to find out how many sticks there are in a larger (or any) number of sections, such as in question 2a. In 2a there are three extra sticks added on for each new

section and there are 15 sections. The one added on at the end of the equation is the number of sticks needed to close off the end of the pattern. So the new equation would be $15 \times 3 + 1 = 46$.

As a general rule, this could be written as:

the number of sticks in n sections of fence = $n \times 3 + 1$.

Follow the same procedure for the arrows, houses, and triangles. The general rule for the arrows will be $n \times 4 + 2$. For the houses, it will be $n \times 4 + 1$, and for the triangles, it will be $n \times 2 + 1$.

Page 3: Tukutuku Panels

Achievement Objectives

- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- state the general rule for a set of similar practical problems (Algebra, level 3)

Activity

To answer question 1, encourage students to draw the steps of the patterns for themselves on square paper and count the number of crosses that they draw for each step of the pattern. Some students may need help to see the pattern. Others may be able to visualise how the pattern develops without needing to redraw it. They may see that two lots of two are added at each step or that four crosses are added at each step. There are a variety of ways in which students may visualise this. It would be useful to go around the group or class and discuss the different techniques that students are using to visualise the pattern development.

Students could record the number of crosses needed for each step in a table:

Number of steps	Number of crosses needed
1	6
2	10
3	14
4	18

This will help them to work out quickly how many crosses are needed. They may also be able to predict the general rule, using the same procedure as that used to find the general rules for the patterns on pages 1 and 2 of the students' booklet. For this activity, the general rule is:

the number of crosses needed for n steps = $n \times 4 + 2$.

Encourage students to make a table or work out the general rule to answer question 2 rather than drawing all 10 steps. In 2a and 2c, less able students will probably find the tenth pattern by adding on a constant (+ 3 for 2a and + 4 for 2c) while more capable students should be able to find general rules. Question 2b has a pattern that does not involve a constant, so using a table may be the best strategy for all students working on this pattern. Part of the challenge for students may be realising that a simple rule is not applicable in this pattern.

2.	a.	Number of steps	Number of crosses needed
		1	3
		2	6
		3	9
		4	12

Three extra crosses are added for each extra step. The number of crosses needed for n steps is $n \times 3$.

d
J
_

After the initial two steps, each step has one more cross added onto it than was added to the previous step.

2.	c.	Number of steps	Number of crosses needed
		1	5
		2	9
		3	13
		4	17

Four extra crosses are added at each step, so the number of crosses needed for n steps is $n \times 4 + 1$.

Page 4: Pegging Problems

Achievement Objectives

- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- state the general rule for a set of similar practical problems (Algebra, level 3)

Activity

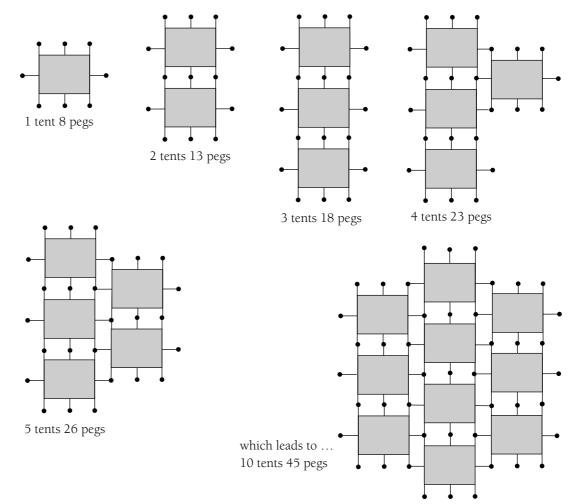
Remind students before they begin that they need a way of recording their tent arrangements and the number of pegs used.

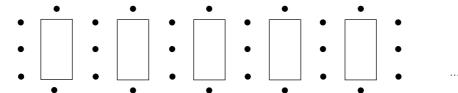
Students could use paper and counters to model the tents and pegs and use trial and improvement to work out the most efficient way to peg out the tents. Encourage them to begin with a small number of tents and for each arrangement, record in a table how many pegs they need for each arrangement of tents. When they have filled in each table up to six tents, see whether they can predict how many pegs they will need for 10 tents. Encourage them to see if there is a general rule.

Number of tents	1	2	3	4	5	6	
Number of pegs							

Be aware that some students will make a ratio error, believing that the number of pegs needed to make 10 tents will be twice that needed for five tents. The error ignores the effect of joining, but you can correct it by modelling with equipment.

Students will use the least number of pegs if they attach each new tent to pegs that have already been used for other tents. For example:

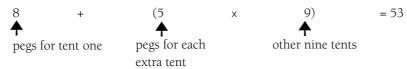




The number of pegs used for some ways of putting up the tents can be expressed as general rules. These types of general rules are quite difficult and it is unwise to teach them. Be aware that some more advanced students may discover them. An example is:

Tents	1	2	3	4	5	•••		
Pegs	8	13	18	23	28			
+5 +5 +5 +5								

The table shows that five more pegs are needed for each extra tent. The first tent took eight pegs, so the number of pegs needed for 10 tents could be found by:

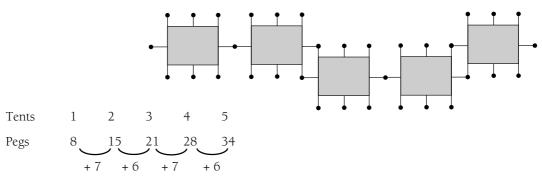


Another expression of the rule is $3 + (5 \times 10) = 53$.

These rules for the number of pegs could be written as algebraic expressions:

$$8 + 5(n - 1)$$
 or $3 + 5n$, where n is the number of tents.

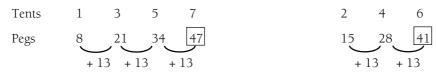
Patterns where there is not a constant difference between the peg numbers are not so easy to find general rules for. Consider this pattern:



The pattern of differences is not constant. One way to resolve this is to find a rule for odd numbers of tents and a rule for even numbers of tents.

Consider the odd numbers:

Consider the even numbers:



The average increase per tent is $6^{1}/_{2}$, so we could try $6^{1}/_{2}$ as a multiplier:

Odd number of tents:

Tents	1	3	5	7
Tents $\times 6^{1}/_{2}$	61/2	19¹/ ₂	321/2	45 ¹ / ₂
Pegs	8	21	34	47

A rule of $(6^{1}/_{2} \times n) + 1^{1}/_{2}$ will work.

Even number of tents:

Tents	2	4	6
Tents $\times 6^{1}/_{2}$	13	26	39
Pegs	15	28	41

A rule of $(6^{1}/_{2} \times n) + 2$ will work.

Page 5: Pattern Parade

Achievement Objective

 describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)

Activities One and Two

Students need to work systematically with a classmate to identify and continue patterns. A table is an effective way to do this, especially if there is more than one attribute in the pattern. For example:

c. i.

Place in pattern	1	2	3	4	 10	n
Number of circles in each arm	1	2	3	4	 10	n
Total of arm circles	3	6	9	12		

Students are not asked to calculate the total number of arms or related pattern, but finding the totals makes an interesting extension.

c. ii.

Place in pattern	1	2	3	4	5
Number displayed	10	13	9	12	10
	+ 3	- 4	+ 3	- 4	+ 3

c. iii.

Place in pattern	1	2	3	4	5	6	7
Suit	spades	clubs	diamonds	hearts	spades	clubs	diamonds
Number	ace or 1	3	2	5	4	6	5



Question \mathbf{c} . \mathbf{iii} is the most difficult pattern on this page. If students are struggling, suggest that they take one suit and look at the relationship between the numbers on the cards for that suit. Then look at another suit, and so on.

Students can continue the tables to work out the next and tenth object in each pattern.

Similar work with these types of patterns is given in *Algebra*, Figure It Out, Levels 2–3, pages 1–2 and *Answers and Teachers' Notes: Algebra*, Figure It Out, Levels 2–3, pages 11–12.

In **Activity Two**, students are asked to make up their own patterns. If necessary, you may like to stipulate that each pattern has to repeat fairly often (at the most, only four objects in the pattern) otherwise the pattern will be too hard to identify.

Pages 6-7: Possum Poles

Achievement Objective

• use graphs to represent number, or informal, relations (Algebra, level 3)

Activity

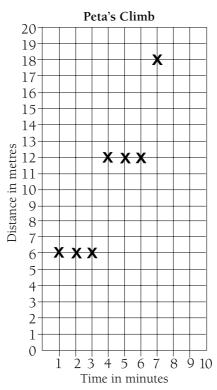
This activity will probably need teacher guidance. You could draw a set of axes and work with the class or group to record the progress of one possum on the graph.

If students have trouble converting the pictures to graphs, they could use a table as an intermediate step. This will help them to identify each possum's climbing pattern.

Pet	a
Time in minutes	Distance in metres
1	6
2	6
3	6
4	12
5	12
6	
7	18

You may need to point out that the last picture shows the possums' positions at 7 minutes, not 6 minutes as students may expect. The sixth minute has been missed out so that students can predict the position of the possums at that time (question 1b).

When students are plotting the points on a graph, make sure that they move both up the y axis and across the x axis by 1 minute for every new minute that they are plotting. Some students may forget to move across the x axis and move only up the y axis.



Question **1d** asks students to interpret the graphs. You may want to help students with this. For example, the higher up the y axis, the further the possum has climbed. If the possum moves closer to the x axis, it has slipped down.

Students can use their descriptions of the possums' climbing patterns to continue the graphs and answer question 2.

Page 8: Preparing for the Hangi

Achievement Objective

make up and use a rule to create a sequential pattern (Algebra, level 3)

Remind students to read and consider all the information, including that in the speech bubbles, before trying to solve the problems on this page.

Activity One

Before students begin working on this activity, you may want to discuss how to vary Aunty Wikitoria's rules so that people get both chicken and pork. The most likely scenario is that Hine and Rāwiri need to halve the amount of chicken and pork, but you could discuss any other options that students have. When students have decided what the revised rules will be, encourage them to use division as the most efficient way to answer the question.

There are good discussion points here. Hine and Rāwiri need 7.5 chickens. Can you buy half a chicken? They need 3.75 kilograms of pork. Is it possible to buy part of a kilogram of pork?

Activity Two

Students can use algebraic symbols and equations to answer these questions. For example, to calculate how much kūmara the children need:

```
number of kūmara = one kūmara \times number of people or number of kūmara for n people = 1 \times n.
```

The calculation for the number of carrots needed is slightly more complex. You might want to talk through this with students: "There are six carrots for every five people. How do we find out how many groups of five people there are?" Students can do this by dividing by five. To find out how many carrots are needed, they multiply the number of groups of five people by six:

```
number of carrots = (number of people \div 5) x 6.
```

If students have difficulty with this method, they could use a table or model the problem with counters or multilink cubes.

Page 9: Ups and Downs

Achievement Objective

use graphs to represent a number, or informal, relations (Algebra, level 3)

Activity

This activity requires students to construct two graphs of the movements of two different snails. The graph at the bottom right-hand side of the page models the correct procedures for labelling and setting out graphs.

Students could calculate the number of hours, but generally they will draw the pattern on the graph. Make sure that they start plotting their graph on the left-hand side of their page as the snails may take more hours than they would expect.

As a discussion point, if students calculate the number of hours, they may see that over a period of 2 hours, Slimey climbs a total of 3 metres. Dividing 26 metres by 3 metres and multiplying it by 2 hours gives a total of 17.3 hours. The catch is that Slimey reaches the top during one of his 5 metre ascents rather than after a 2 metre slide down. Students' graphs should show that Slimey reaches the top in 15 hours (presuming that he remains resting at the top of the well rather than sliding down again!).

Sluggish the Snail's climb is along similar lines, but the rate is different. As an extension exercise, students could compare the slope of each graph. If the slopes are different, the snails must have been moving at a different speed. Ask students "Would a steeper slope mean that that snail is faster or slower? Is Slimey faster or slower than Sluggish?"

Pages 10-11: Operation Time

Achievement Objective

 describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)

Activity One

Arranging the answers in tables will help students to order the results and identify any patterns. This will be useful practice for the more complex operations machines in the later activities provided on these two pages. For example:

a.	Number	Number
	in	out
	20	4
	30	6
	40	8

There is a difference of 10 between the consecutive "in" numbers and a difference of two between the consecutive "out" numbers. The out numbers are five times less than the in numbers (or the in numbers are five times more than the out numbers).

b.	Number in	Number out
	4	14
	8	18
	12	22

There is a difference of four between consecutive in numbers and consecutive out numbers. Because the differences are consistent, addition or subtraction must have been used rather than multiplication or division. The out number is 10 more than the in number.

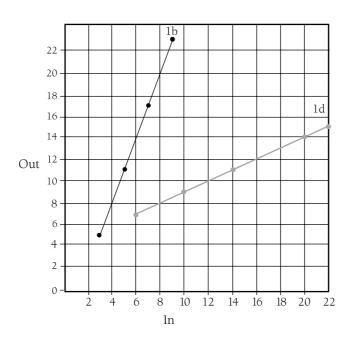
A useful exercise would be to have students graph the results. This would help them to develop the important concept of relation: how the value of one variable (the number on the left) maps onto the value of another variable (the number on the right).

See Answers and Teachers' Notes: Algebra, Figure It Out, Levels 2–3, page 14.

Activities Two and Three

To answer these questions, students will need to use their basic facts knowledge that addition and subtraction are inverse operations and that multiplication and division are inverse operations. For example, when they are given the in number for **Activity Two**, question **2b**, the operations machine tells them to divide it by three to get the out number. But when they are given the out number, they need to use the reverse operation, that is, multiplication, to get the in number. This becomes more complex in **Activity Three**, when students need to reverse two or three operations. For example, when they are given the out number in question **1d**, they need to add three, subtract seven, and then multiply by two to get the in number. Some students may notice that they can combine the +7-3 part of the machine to get +4 (or -4 when doing the reverse). In other words, inverse operations cancel each other out to a certain extent.

Encourage students to use the skills practised in **Activity One** to check their answers and look for patterns. For example, if the in numbers increase by a consistent amount, the out numbers should also increase by a consistent amount. They could also check their answers by graphing their results. If their results are correct, the plotted graph should form a straight line. This is because the relation between the in and the out numbers is always the same. Alternatively, students could use graphs to find the missing values in the In/Out tables. For example, the following graph shows $\bf 1b$ (Number in \times 3 – 4) and $\bf 1d$ (Number out \div 2 + 7 – 3):



Pages 12-13: Spreadsheet Challenge

Achievement Objective

make up and use a rule to create a sequential pattern (Algebra, level 3)

Activity

A spreadsheet is a useful teaching tool in mathematics. It will repeatedly perform calculations, following formulae that students have entered.

Important: Before students attempt this activity, do the challenge yourself on the computer that students will be using, and make sure that you know how to write formulae and what the menu commands are. The spreadsheet you are using may have different terms from the ones described in the students' book.

You may need to show students how to use spreadsheets before they do this activity. Make sure they know:

- what a spreadsheet is
- what a cell is
- how cells are labelled
- how to enter data or formulae into cells.

The following is included for teachers who are unfamiliar with spreadsheeting.

	A5 x	/		
	Α	В	С	
	→			
2				
3				
4			←	
		[

In any spreadsheet, the small boxes are called cells.

The cell at the top left is in column A and row 1 so it is called A1.

For the same reason, the cell at the bottom right is called C4.

You can type straight into these cells. When you put the cursor in the cell and click there, a border appears around that cell and when you begin to type, your typing appears at the top of your document rather than in the box. When you press return or enter, the cursor moves to the next cell and the typing appears in the cell where you want it to appear.

You can arrange the position of your typing by going to the alignment part of the Format menu.

Row 1 is usually where the headings for the columns in a table are put, for example, "In", "Try", and "Out", as given on page 12 of the students' booklet.

To enter a formula:

Type a formula such as =(A2*3)+1 in a cell (a different cell from A2). The = tells the computer that you are entering a formula. The answer appears in the cell that contains the formula. (The formula works on whatever is already in the A2 cell. If, for example, 4 were already typed into the cell, the answer would be 13.)

To copy this formula down the column, put the cursor in the cell containing the formula, hold the mouse button down, and drag the cursor down the column as far as you wish.

Release the mouse, go to the *Calculate* menu, and choose the *Fill down* command. This will then fill in the missing values.

To practise using formulae on the spreadsheet, students could try entering some of the rules or formulae that they identified in the pattern exercises on pages 1 and 2 of the students' booklet.

You could point out to students that the spreadsheet is just like Simon's operations machines on pages 10–11 of the students' booklet. But in this activity, they have to work out the rule rather than working out the in and out numbers. This activity reinforces the skills used on pages 10–11, and students should look for patterns in the In/Out tables in the same way that they did on the earlier pages. For example, are the in numbers bigger or smaller than the out numbers? If they are bigger, addition or multiplication has probably been used. If they are smaller, subtraction or division has probably been used. Is the difference between each consecutive in number the same as the difference between each consecutive out number? If so, only addition or subtraction will have been used. If not, multiplication or division will have been used.

As an extension, you could ask students to investigate what kind of graphs their computer can make from the data in the In/Out tables.

Pages 14-15: Biscuit Binge

Achievement Objectives

- use graphs to represent number, or informal, relations (Algebra, level 3)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 3)

Activity One

Encourage students to use a spreadsheet to develop their charts. Some students may compare the charts and notice that if one person ate two biscuits a day and another ate three a day, then the answers are the same as if one person ate five biscuits a day. Or they may notice that the totals for two and a half biscuits a day are half the totals for five biscuits a day.

Activity Two

You may need to remind students that usually the constant variable (in this case, the days) is on the x axis (the horizontal axis) and the variable that changes is shown on the y axis (the vertical axis).

Students will need to vary the numbers on the y axis so the graphs are a manageable size.

Activity Three

This activity shows different ways of recording patterns. Because they are on a base of 10, the patterns can be explained as multiples of the rate per day, for example, 0, 3, 6, 9, 12, 15, 18, 21, 24, and 30. The pattern has gone around the digit wheel three times before each digit is visited. The number visited is the ones digit of the total number of biscuits eaten.

Make sure that students understand how the digit wheel works. You could work through question ${\bf 1}$ with them. First, complete Sigmund's three-biscuit-a-day chart.

Days	0	1	2	3	4	5	6	7	8	9	10
Biscuits	0	3	6	9	12	15	18	21	24	27	30
					A			A	A		A

Ask students to identify the ones digits in the biscuit totals. The ones digit pattern is: 0, 3, 6, 9, 2, 5, 8, 1, 4, 7, 0.

You could suggest that students use this method to complete the digit wheel: "Start at the 0 on the digit wheel. Draw a line to the next digit in the pattern (3). Draw another line to the next digit (6).

Continue until you reach the end of the digit pattern."

Encourage students to explain the process in their own words and compare the patterns they have developed on the digit wheel for question **2**. Useful questions include:

- "How many times do you have to go around each digit wheel before you have touched each digit?"
- "What do you notice about the wheels for even numbers of biscuits eaten each day? Can you explain why this happens?"
- "Which biscuit-a-day patterns eventually touch all the numbers?"
- "Can you explain the five-biscuit-a-day pattern?"

Page 16: Kai Moana

Achievement Objectives

- use graphs to represent number, or informal, relations (Algebra, level 3)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 3)

Activity

Check that students can tell which person is Hōhepa. (He's the person on the left.) They need to know this to answer question ${\bf 1}$.

Various explanations of the graph are given in the answers. If students graphed the data in the In/Out tables on pages 10-11 and 12-13 of the students' booklet, you could ask them to compare these graphs with their fishing graph for this page. They should notice that the points on their graphs of the In/Out tables form straight lines. This is because the relationship between the x axis values and the y axis values is constant. But the points on $H\bar{o}hepa$'s fishing graph are not in a straight line, and this is because the relationship between the year and the number of fish caught is not constant. If it were constant, it would have been easier for $H\bar{o}hepa$'s whānau and the students working on this activity to explain the pattern in the number of fish caught.

Another discussion point is the way the slope of a line that joins the points shows how quickly or slowly the number of fish caught is changing. If the line slopes down sharply, the number of fish caught is decreasing rapidly. If the line slopes up gently, the number of fish caught is increasing gradually. Graphs are often interpreted in this way in the news when organisations or politicians are trying to prove how quickly or slowly inflation or employment is changing.

From the table of data used for question 1, students should be able to add realistic data to their chart. The catch for years 11, 12, and 13 should increase if the pattern continues.

Page 17: Duncan's Day

Achievement Objectives

- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- use words and symbols to describe and continue patterns (Mathematical Processes, developing logic and reasoning, level 3)

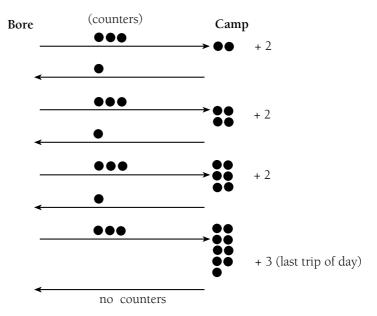
Activity

Before trying this activity, make sure that students read the questions carefully and note down important facts or restrictions.

- three bottles at a time
- always has one bottle with him except on last trip back
- no bottle on last trip
- ends back at the bore
- count one-way walks.

Students could model Duncan's walks, using counters, pen, and paper.

For example:



Using a table will help students identify patterns and suggest a rule:

Number of bottles	Number of one-way walks
1	2
2	2
3	2
4	4
5	4
6	6
7	6
8	8
9	8

Students could work in pairs or groups to suggest rules and then test them by modelling. Some examples could be:

- Answers must be even.
- Even numbers of bottles have the same value as the number of one-way walks taken to deliver them to the mining camp.
- Odd numbers of bottles would have a value one less than the number of trips taken.

The rule in question 3 is a complicated one for students to recognise. Duncan now takes four bottles per trip, three of which can be left at the mining camp unless it is the last trip of the day.

Ways in which students write their answers will vary, but they should notice that apart from the first group of four, the numbers of trips required for the numbers of bottles are in groups of three (compared to an initial three and then groups of two for the previous rule). That is, the same number of trips is required for one, two, three, or four bottles, and likewise for five, six, or seven bottles, and for eight, nine, or 10 bottles. This occurs because three bottles are left after each trip to the mining camp except for the last trip, when four can be left. Students might suggest this set of rules:

- Take one off the number of bottles needed (Duncan can leave the extra one on his last trip).
- Round this number up to the next multiple of three (because three can be left each trip).
- Divide the number by three (because three bottles are left each time).
- Multiply by two (because Duncan has to go there and back).

Another set of rules is given in the answers.

Encourage students to explain their line of reasoning logically, either orally or in written form.

Page 18: Puzzling Patterns

Achievement Objectives

- continue a sequential pattern and describe a rule for this (Algebra, level 2)
- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)

Activity One

Students can build on their geometry knowledge to find a quick way to count the number of small squares and predict the tenth square number. When calculating the area of a square, students will know that they multiply the height by the width. They can use the same method to calculate the number of small squares in each larger square. They will notice here that the height and the width are the same, that is, a square number is the result of one number being multiplied by itself. They should also notice that:

- the first square number is $1 \times 1 = 1$
- the second square number is $2 \times 2 = 4$
- the third square number is $3 \times 3 = 9$.

If they extend this pattern, they can predict that the tenth square number will be $10 \times 10 = 100$.

If students have difficulty grasping these calculations, they could model the shapes with counters or multilink cubes or use a table to record the square numbers:

Square number	Number of squares
first	1
second	4
third	9
fourth	16
fifth	25

From the table, they can use the pattern of differences to predict the tenth square number or they may notice that (as explained above) the first square number is 1×1 , etc. However, encourage students to use calculation because it is a far more efficient method.

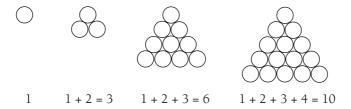
The building pattern of the squares in question 2 is not as easy to recognise or calculate immediately. However, students could notice that:

- If they start counting the squares in each building, the totals are the same as the square numbers above.
- The squares to the right of the tallest column can be moved to the left side of the building to complete a square shape.

They can then use the same methods as they used in question 1 to calculate the number of small squares in the tenth building.

Activity Two

If students have trouble seeing why 10 is a triangular number, you could draw smaller triangular numbers, such as three and six, in the same configuration.



The answers explain the patterns of triangular numbers.

Activity Three

The patterns in the differences between the terms in each set of numbers are illustrated in the answers.

Page 19: See-saw Numbers

Achievement Objective

 use the mathematical symbols =, <, > for relationships "is equal to", "is less than", and "is greater than" (Algebra, level 2)

Activity

Check that students know how to convert a number balance to an equation, for example, $7 \times 1 + 3 \times 1 = 5 \times 2$.

Questions 1c to 1f are an opportunity to introduce the use of brackets, for example, $7 + 5 + (2 \times 3) < 3 + 5 + (2 \times 7)$ rather than 7 + 5 + 3 + 3 < 3 + 5 + 7 + 7.

See also Answers and Teachers' Notes: Algebra, Figure It Out, Levels 2–3, page 26.

Page 20: Putting Pens to Paper

Achievement Objective

• solve problems of the type \square + 15 = 39 (Algebra, level 3)

Activity One

This activity develops students' logic and reasoning. It will involve students looking at combinations of equations in order to select the one that is correct. Again, the use of a table may help. The first example of the goat enclosure could be developed as follows.

If pen A has:

A	1	2	3	4	5	6	7	8
В								
С								

and A + B = 8, therefore pen B must have:

A	1	2	3	4	5	6	7	8
В	7	6	5	4	3	2	1	0
С								

and A + C = 9, therefore pen C must have:

A	1	2	3	4	5	6	7	8
В	7	6	5	4	3	2	1	0
С	8	7	6	5	4	3	2	1

For B + C to equal 11, B has to have five and C has to have six, which is when A has three.

Similar tables can be developed for the other questions.

It is possible to solve these problems by using quite complex equations, but students will not yet be using the level of algebra required to do such equations.

Page 21: Losing Your Marbles

Achievement Objective

• solve problems of the type \square + 15 = 39 (Algebra, level 3)

Activity

The names of the marbles in this exercise might be different from the names used at your school. The investigation asks students to look at the names used for marbles in your school.

Students could begin the activity by using the information in speech bubbles to work out marble exchange rates. Some students may need to organise this information into a table. However, when you are sure that they have understood the exchange rate information, you could encourage them to use algebraic notation to prepare them for later work in algebra. For example, "b" could represent boulders, "q" could represent queenies, and so on.

The exchange rates are:

```
2b = 1q

2g = 6c

2q = 10m (This can be simplified to 1q = 5m.)

3b = 15c (This can be simplified to 1b = 5c.)
```

Students have the exchange rate for cat's-eyes and boulders (15 cat's-eyes equal three boulders), so question 1 is quite straightforward: 1b = 5c so 2b = 10c. (Both sides of the exchange rate have been multiplied by two, so the rate remains the same.)

Students don't have the exchange information for the swaps in the remaining questions, so they have to go through several steps to calculate the swaps. For example, for question 2:

```
4b = \coprod m. They know that 2b = 1q and 2q = 10m, so by doubling both sides of the first exchange rate, they have 4b = 2q = 10m.
```

See also Answers and Teachers' Notes: Algebra, Figure It Out, Levels 2–3, page 27.

Pages 22-23: Pene's Puzzles

Achievement Objectives

- state the general rule for a set of similar practical problems (Algebra, level 3)
- solve problems of the type \square + 15 = 39 (Algebra, level 3)

Activity One

The sequence in these patterns leads towards students developing rules. Students may wish to complete the pattern through to the tenth element in order to answer question **2**, but do encourage them to also use more efficient methods. See *Answers and Teachers' Notes: Algebra*, Figure It Out, Levels 2–3, page 25.

Activity Two

If students aren't convinced that Mark's answer for question **1** is always three, have them try the puzzle several times with different numbers. You may like to work through the puzzle with students to help them identify what is happening. Using the notation in Mark's speech bubble is especially useful because it makes students more familiar with algebraic notation, and writing the puzzle down is much easier than doing it mentally, as Mark has done.

In order to always get an answer of three, the steps in the equation must, to a certain extent, cancel each other out. So students need to look for inverse operations that do cancel each other out:

The operations \times 2 and \div 2 cancel each other out. Six has been added on, but this is also divided by two so is now three. The original number is cancelled out by subtraction, leaving an answer of three

Remind students to keep these points in mind when they make up their own puzzles. Most of the operations that they include in their puzzles will have to cancel each other out.

You may find the following algebraic proofs interesting (although students will not yet be using this level of algebra).

1.

Let n be the number thought of.

Multiply n by 2: 2n.

Add 6: 2n + 6 = 2(n + 3).

Divide by 2: $\frac{2(n+3)}{2} = n + 3$.

Take away the first number thought of: n + 3 - n = 3.

2.

Let n be the number thought of.

Double it: 2n.

Take away 7: 2n - 7.

Add 21: 2n - 7 + 21 = 2n + 14.

Divide by 2: n + 7.

Subtract n: n + 7 - n = 7

Page 24: The Potluck Paint Company

Achievement Objective

 devise and follow a set of instructions to carry out a mathematical activity (Mathematical Processes, communicating mathematical ideas, level 3)

Activity

This activity is a fun way to develop students' confidence and familiarity with algebraic notation. Students should recognise that y, r, and b stand for the three primary colours yellow, red, and blue respectively.

Check that students understand how to use the notation correctly and realise that "2r" means two squirts of red and "y" means one squirt of yellow.

Acknowledgments

Learning Media would like to thank Ray Wilson and Jan Wallace, both of the Education Advisory Service, Auckland College of Education, for developing the teachers' notes. Thanks also to Paulette Holland for reviewing the answers and notes.

The main illustrations on the cover and contents page are by Keith Olsen; the background line art (bird, fish, camera, splash, eye, spectacles, and spiral) on the cover, contents page, and pages 2, 3, and 14 is by Fraser Williamson; and the background illustration of the snail and snail trail on the contents page and pages 2 and 14 is by Jenny Cooper.

All illustrations are copyright © Crown 2000.

Series Editor: Susan Roche Series Designer: Esther Chua

Designer: Dawn Barry

Published 2000 for the Ministry of Education by Learning Media Limited, Box 3293, Wellington, New Zealand.

Website: www.learningmedia.co.nz

Copyright © Crown 2000

All rights reserved. Enquiries should be made to the publisher.

Dewey number 510.76 ISBN 0 478 12685 9 Item number 12685

Students' book: item number 12684

105475/00