Notes for parents (1).

The purpose of the activity is to help your student to:

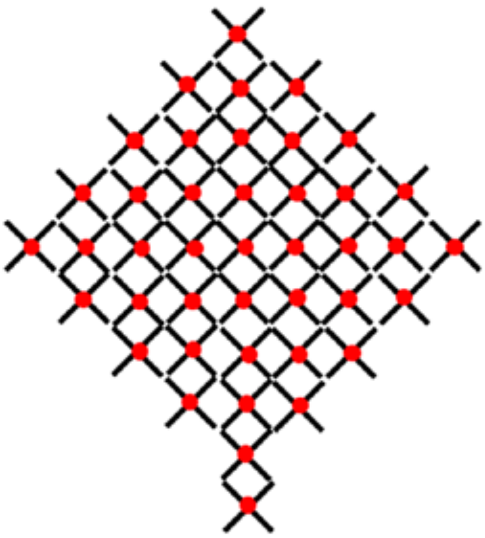
- Predict further members of a quadratic pattern.
- Create a rule for the pattern and express the rule using algebra.
- Graph a quadratic pattern and connect the slope of the graph to the difference between terms.

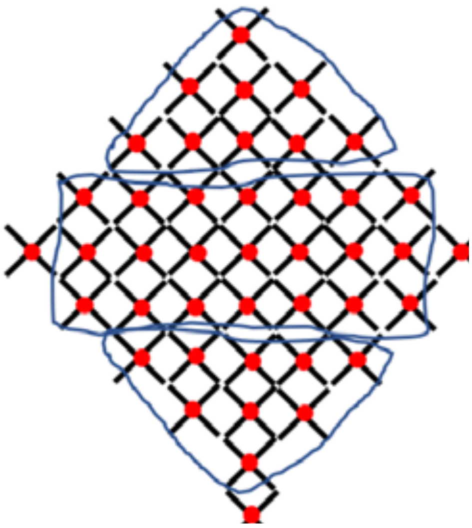
Here is what to do:

Look at the growing pattern for the pātiki visually. Discuss features such as:

The main body of the flounder is a tilted square.
The tail is always an extra cross or counter.
The middle row of circles or crosses is always an odd number.
There are triangles top and bottom.

Try to draw the pātiki that is 5 years old. It should look like this:



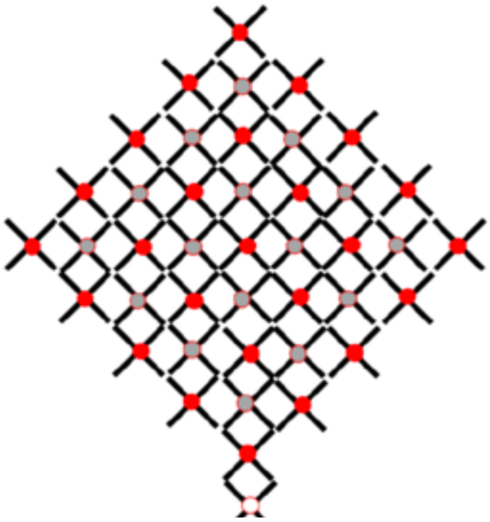


Counting the number of crosses one by one is laborious and prone to error. Look for efficient ways to count. This will involve structuring the pātiki into chunks.

Methods might include:

Making a rectangle (3 x 7), two triangles and three other crosses (sides and tail). Both triangles are made of 5 + 3 + 1 = 9 crosses.

Notes for parents (2).

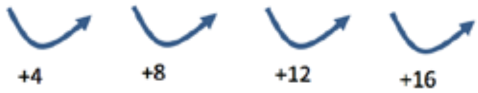


Seeing a small square of circles inside a big square of circles, plus the tail. The red circles number 5 x 5 and the grey circles number 4 x 4. So, the total number of circles is 25 + 16 + 1 = 42.

There are many other ways to chunk the pattern.

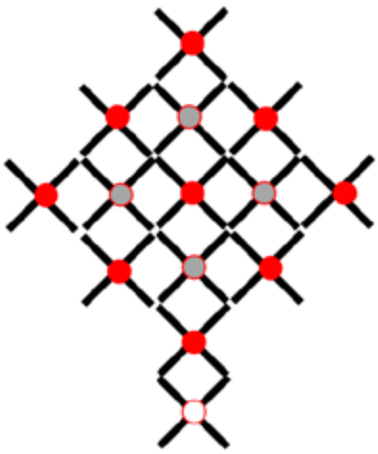
With the pātiki at age 5 years you have enough data to create a table like this:

Age in years	1	2	3	4	5	6
Number of crosses or circles	2	6	14	26	42	



The pattern in the table shows that the differences are growing by four each time so the pātiki at age six should have 42 + 20 = 62 crosses or circles. Continuing the table leads to the pātiki at year ten having 182 crosses or circles. That certainly beats counting.

However, the best rules are direct or function rules. That means the number of circles or crosses can be worked out from the age of the pātiki. Since the pattern involves tilted squares experimenting with multiplying the age number by itself is a good way to start. Looking at the pattern visually again also helps.



The pātiki at age three can be looked at like this:

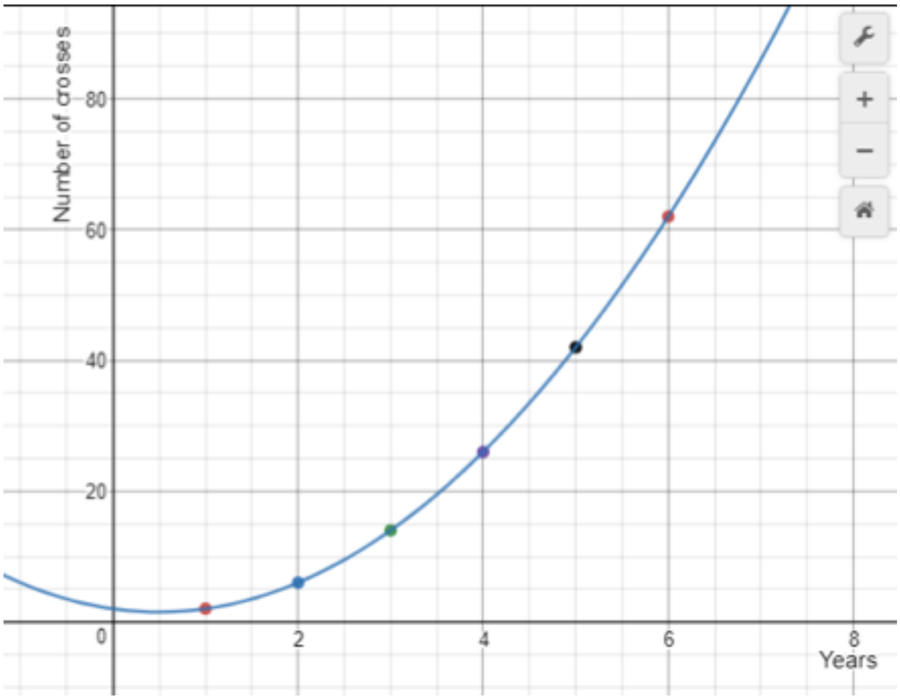
There are 3 x 3 red circles, 2 x 2 grey circles and one white circle. That is a total of 9 + 4 + 1 = 14 circles. But how does that relate to age?

3 x 3 is age squared and 2 x 2 is age less one squared. So, using y for age in years and c for total number of circles or crosses we can write:

$c = y^2 + (y - 1)^2 + 1$ noting that y^2 means y times itself.

Notes for parents (3).

A graph of the table and the rule looks like this:

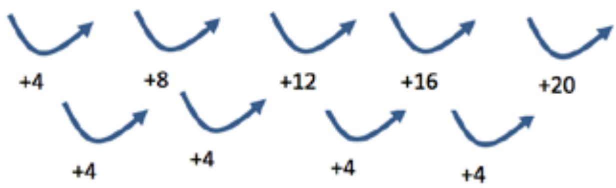


Notice that the slope of the graph increases as the number of years increases. That is due to the differences between term increasing by four each time.

Points to note:

Quadratic functions are common in everyday life especially in the world of physics. For example, the relationship between breaking distance and speed of a car is a quadratic relationship. Quadratic relationships can be recognised by constant second order differences between terms. Look at the table for the pātiki pattern.

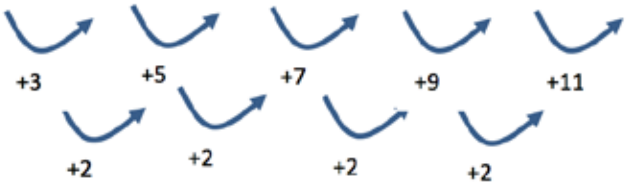
Age in years	1	2	3	4	5	6
Number of crosses or circles	2	6	14	26	42	62



The second order differences are constant, four. The simplest quadratic is the set of square numbers. Look at the second order differences of that relationship.

Notes for parents (4). Activity next page.

Number	1	2	3	4	5	6
Number squared	1	4	9	16	25	36



Notice how the pattern of squares has a constant second order difference of two. The pātiki pattern has a second order difference of four so it is sensible to assume it involves twice the age squared. Let's see how that works out.

Age in years	1	2	3	4	5	6
Number of crosses or circles	2	6	14	26	42	62
Age squared times two	2	8	18	32	50	72

Two times age squared gets close but needs some adjustment. That adjustment is subtracting twice the age and adding two. For example, for age four years the number of circles will be $2 \times 4^2 – 2 \times 4 + 2 = 32 – 8 + 2 = 26$. It works.

In algebra there are usually many ways to express the same relationship. Some are more efficient than others. The pātiki relationship can be written in these ways as well:

$c = y(2y-2) + 2$ and $c = (y - 0.5)^2+1.5$

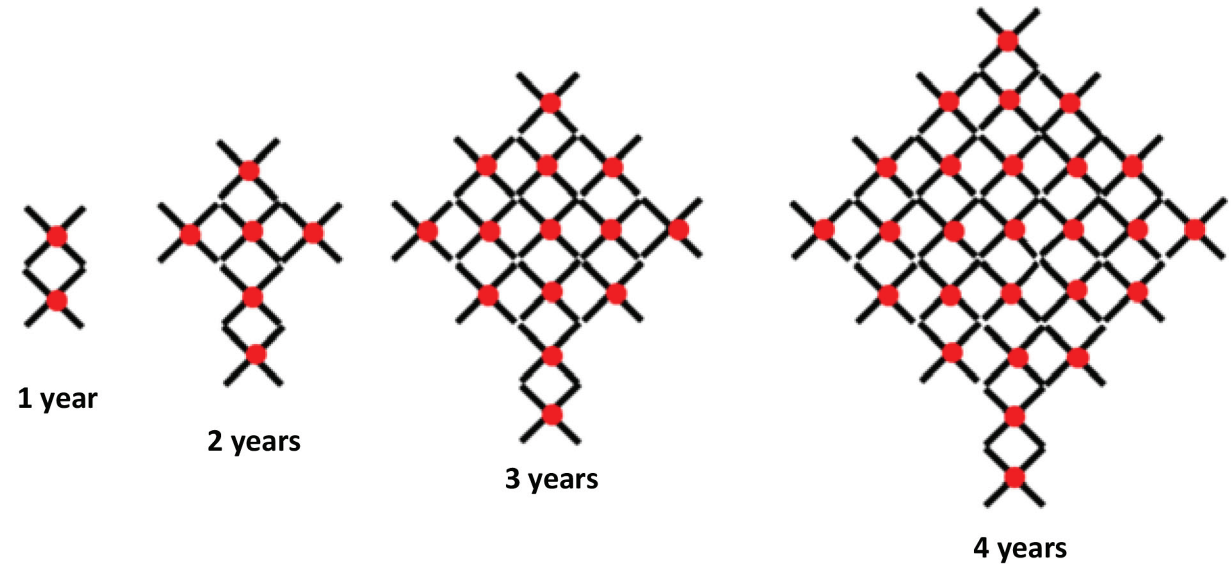
The graphs of quadratic relationships are always parabolas. That shape is approximately that of a ball thrown into the air, if wind resistance didn't play a part.

Activity | **Pātiki pattern**

Y9–10

This is a modern take on a traditional tukutuku design.

The pattern is made from black crosses with a red circle in the centre.



How many crosses or circles would be in a pātiki that is ten years old?

Write a rule for finding the number of circles or crosses that make up a pātiki of any age (number of years). You might express your rule as an equation using y for the number of years and c for the number of crosses or circles.

Draw a graph to show the relationship between age in years and number of circles or crosses.