

Session 1

Cricket Scores

Batsman	Mean score	Range
A	50	106
B	50	25

Both batsmen have the same mean score so for that reason either could be chosen to represent his country. They do differ in their reliability though. Batsman A's range is over four times that of B. Discussions of the differences between the two batsmen use words like reliability and consistency. Do the selectors choose a batsman who on the day can obtain a higher score or do they choose the more consistent?

Netball Scores

Player	No. games	Mean points	Range
A	6	12	4
B	6	13	10
C	4	14	6

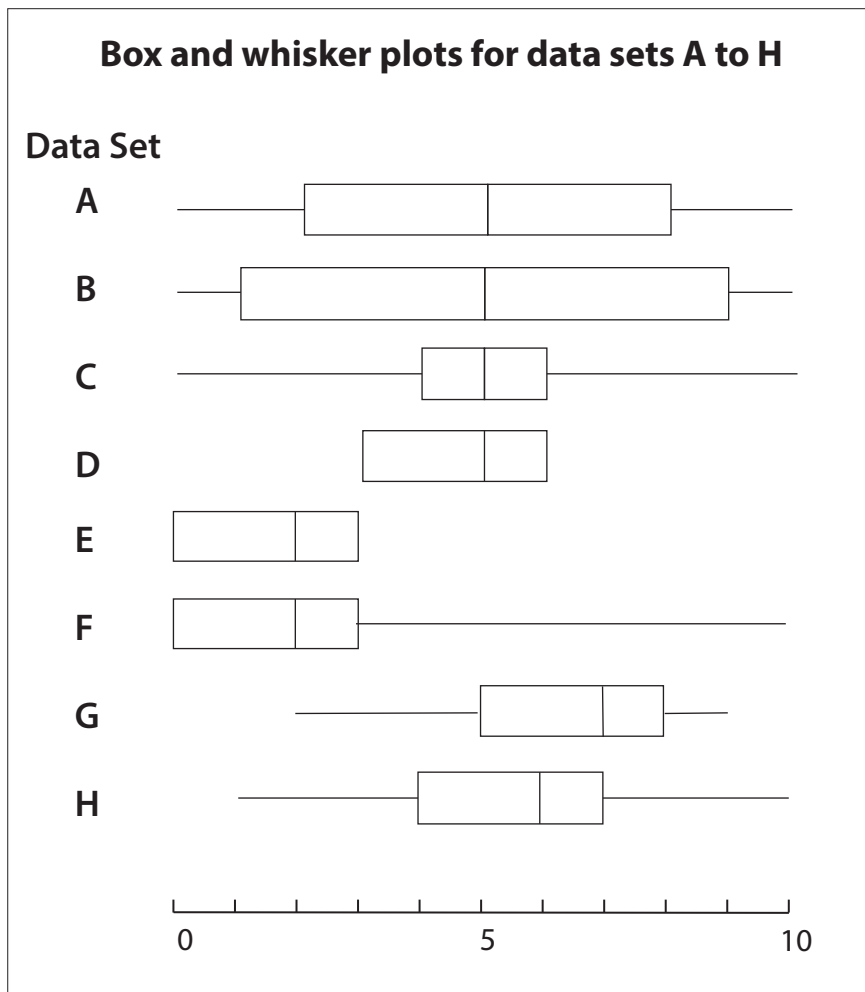
Here we have to balance consistency and scoring ability against the fact that one player has only had four games compared with six for the others. The problem is compounded by the fact that we do not know players shooting success as a percentage of shots attempted.

Rugby Scores

It may help for comparisons to express relative successes as percentages.

	Conversions (%)	Penalties (%)	Tackles (%)
Player A	80	75	85
Player B	75	75	90
Player C	70	90	80

Here we begin to realise that being a selector is not such an easy task. It is not always possible to choose a player based on statistics alone. For example, how do we decide the relative merits of penalty kicks compared with conversions? Should points scored be the sole criterion? How do we rate the importance of successful tackles? Although few conclusions may be drawn here, the amount of discussion is likely to be intense resulting in some understanding of the issues involved and the relative importance of calculated statistics.



The idea here is to have an informal look at what is called Expectation Algebra. Students will discover, for example, that if the mean number of peoples' feet is eight then the mean number of toes is $8 \times 5 = 40$, that if the mean pay packet for a small company is \$825 and everyone gets a bonus of \$100, then the mean pay packet becomes $825 + 100 = \$925$.

Discussions of applications might include the fact that calculations do not have to be made again if units are changed. For example, if an average speed in miles per hour is known it is a simple matter to obtain the average in kilometres per hour without going back to the original data, converting all the data points from m.p.h. to k.p.h and recalculating the mean.

The effect of an outlier being added to data skews both the range and mean. Medians and interquartile ranges are unaffected by the addition of an outlier since they do not rely on extreme values in their calculation.

Sessions 2 and 3

Data set A

The data might be described as widely spread, evenly spread and symmetrical.

Mean = 5, median = 5, range = 10, lower quartile = 2, upper quartile = 8, interquartile range = 6.

At this stage, when students are determining the interquartile range, they might discern why 11 data points were chosen. The lower quartile of n items is given by the value of the $\frac{1}{4}(n+1)$ th item when the items are ordered, while the upper quartile is the value of the $\frac{3}{4}(n+1)$ th item. In these data sets they are therefore the values of the third and ninth items. The interquartile range is the distance between the upper and lower quartile.

Data set B

The data might be described as widely spread and symmetrical, with some clustering at the extremities.

Mean = 5, median = 5, range = 10, lower quartile = 1, upper quartile = 9, interquartile range = 8.

Note that only the value of the interquartile range has changed from Data Set A.

Data set C

The data might be described as widely spread and symmetrical, with some clustering towards the middle.

Mean = 5, median = 5, range = 10, lower quartile = 4, upper quartile = 6, interquartile range = 2.

Note again that only the value of the interquartile range has changed from the two previous data sets. The last two examples show how useful the interquartile range is as a measure of spread since it does not rely on just the extreme values of a data set like the range.

Because data sets A, B and C are all symmetrical the mean and the median are the same.

Data set D

The data might be described as narrowly spread or clustered towards the centre and bimodal. Note that bimodal here means the data has two peaks. They are not necessarily the same height.

Mean = 4.82, median = 5, mode = 6, range = 3, lower quartile = 3, upper quartile = 6, interquartile range = 3.

Note that the mode has no significance as it occurs at the limit of the range and in no way represents an average.

Note that the range and interquartile range are the same.

Data set E

The data might be described as narrowly spread or clustered, skewed to the lower end and bimodal.

Mean = 1.82, median = 2, mode = 3, range = 3, lower quartile = 0, upper quartile = 3, interquartile range = 3.

Data set E is set D moved three to the left. The calculated statistics reflect this. Again the mode has no significance as a measure of average.

Note that moving the whole distribution left or right only affects the mean, mode and median, the range and interquartile ranges are unchanged.

Data set F

The data might be described as narrowly spread with an outlier or widely spread and skewed to the lower end, bimodal.

Mean = 2.45, median = 2, mode = 3, range = 10, lower quartile = 0, upper quartile = 3, interquartile range = 3.

By comparing data set F with that of E, it can be seen how the outlier affects the range and mean.

Data set G

The data might be described as skewed to the right.

Mean = 6.64, median = 7, mode = 8, range = 7, lower quartile = 5, upper quartile = 8, interquartile range = 3.

Data set H

The data might be described as random.

Mean = 5.73, median = 6, mode = 7, range = 9, lower quartile = 4, upper quartile = 7, interquartile range = 3.

Comparisons between the data sets can be made by studying the box and whisker plots.

Session 4

As an alternative, teachers may present the following scenario to students. The data, in fact, were the estimates given by a class to card F. The scenario was wrapped round the data, as follows:

Police gathered information concerning the events surrounding a fracas after a social event. Among the questions asked by police were how many people witnesses thought were involved in the riotous behaviour. They received the following answers:

14, 14, 18, 12, 16, 17, 15, 15, 21, 16, 17, 25, 13, 20, 15, 13, 16, 14, 16, 15, 20, 15, 19, 18, 16, 15, 17, 18, 17, 15, 16

Can the police conclude how many people were involved in the fracas?

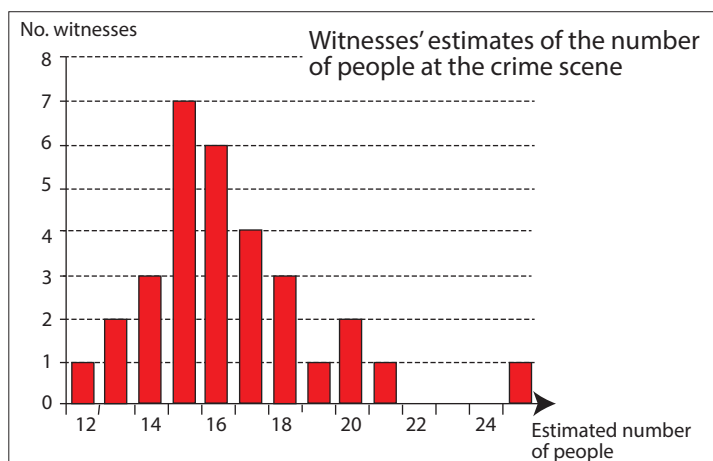
Students discuss how the problem may be investigated, what steps should be taken in analysing the data so that a conclusion can be drawn. These include ordering and tabulating the data, displaying the data, calculating measures such as range, mode, median, mean, interquartile range. Looking at the effect of any outliers and other features that are noticed.

The first thing to do with the raw data is tabulate it.

Estimate	12	13	14	15	16	17	18	19	20	21	22	23	24	25
No. witnesses	1	2	3	7	6	4	3	1	2	1	0	0	0	1

The most appropriate display here is a bar chart. We can now quickly determine that the mode is 15, that there are 31 pieces of data and the median is the value of the middle item in the ordered data, i.e. 16.

The lower quartile is the value of the eighth estimate from the left, i.e. 15. The upper quartile is the value of the eighth estimate from the right, i.e. 18.



The graph shows various features that need discussion:

1. People are not good at making estimates!
2. The mode is 15. Two witnesses said that there were 20 people involved in the fracas and one suggested 25. When people estimate numbers rounding often takes place. For example, estimating a number between 100 and 200 often results in 150 and that may be what is happening here. In this case people are rounding to the nearest five. It should certainly be taken into account when any conclusions are drawn.
3. The mean of the data is 16.4. However, there is some argument that 25 is clearly a spurious result and should be ignored. If 25 is ignored the mean is 16.1.
4. From the discussion, the graph and calculated statistics the police could be fairly confident that about 16 people were involved in the fracas.

Session 5

The results for card G make it clear that people tend to underestimate large numbers. This data needs to be grouped before a bar chart can be drawn. Here the 31 estimates have already been ordered.

25, 28, 30, 34, 35, 35, 40, 40, 40, 40, 45, 45, 48, 50, 50, 50, 50, 50, 52, 54, 55, 58, 60, 60, 60, 62, 65, 65, 70, 75, 88.

The first thing to notice is the rounding-off that occurred. Students obviously felt that with the likelihood of error occurring estimating to the nearest five was accurate enough.

A bar chart is the most appropriate way to display the data but the question is how many groupings are appropriate?

Obviously, grouping in twos; 25-26, 27-28, 29-30 and so on, would give too many bars and too many gaps. On the other hand grouping in 25s; 25-49, 50-74, 75-94 gives too few bars.

Some compromise which brings out the 'shape' of the data is required. Students should explore various alternatives. This concept is not quite as straightforward as at first appears. If groupings of 15 are decided upon, from what value should the first grouping occur? 25 perhaps, or 20, or 15 even? Different choices give different results. Taking the first grouping to begin with the lowest data point gives:

Estimate	25-39	40-54	55-69	70-84	85-99
No. students	6	14	8	2	1

Centralising the data somewhat gives:

Estimate	>20-34	35-49	50-64	65-79	80-94
No. students	4	9	13	4	1

resulting in two quite different bar charts.

