Squaring Off

ACTIVITY

Rose and Kate are exploring areas of successive rectangles made with multilink cubes.
They start with a 6 by 6 square:

6 x 6   **5 x 7**   **4 x 8**

1. **a.** What is happening to the shape and the area of each successive rectangle?
    b. The girls discover an interesting pattern in the differences between the areas of each successive rectangle. They show this in a table:

Size	Area	Difference from previous area
6 x 6	36	–
5 x 7	35	1
4 x 8		

    i. Copy and complete the table up to a 1 by 11 rectangle.
    ii. What is the pattern?
    c. Investigate to see if there is a pattern when you start with other squares.
    For example, if you start with an 8 by 8 rectangle, your next rectangles will be 7 by 9, 6 by 10, and so on.

2. Rose and Kate know that 100 x 100 = 10 000.
    They use a pattern like the one above to work out these multiplication expressions:
    i. 98 x 102   **ii.** 94 x 106   **iii.** 88 x 112
    a. Draw up a table that shows the pattern.

Size	Area	Difference from previous area
100 x 100	10 000	–

    b. What products did Rose and Kate get for the three expressions above?

3. **a.** Rose and Kate suddenly realise that the areas in their 6 by 6 pattern are reducing from the original area by square numbers.

For example:

Size	Area	Difference from previous area	Difference in area from the original
6 x 6	36	–	–
5 x 7	35	1	1
4 x 8	32	3	4
3 x 9	27	5	9
2 x 10	20	7	16

Does this work with the other squares you started with in question **1c**?

b. Find another way to record this pattern. Write a general formula if possible.
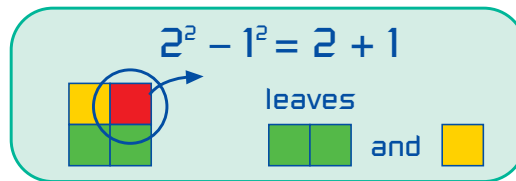
4. Rose and Kate find another pattern using squares:
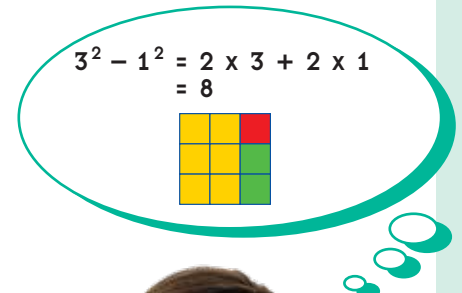
$2^2 - 1^2 = 2 + 1$

$3^2 - 2^2 = 3 + 2$

$4^2 - 3^2 = 4 + 3$

$5^2 - 4^2 = 5 + 4$

$6^2 - 5^2 = 6 + 5$

$$2^2 - 1^2 = 2 + 1$$

leaves

and

a. Investigate this pattern. Then show the pattern using multilink cubes or square grid paper.

b. What would the tenth equation in the pattern be?

c. Use the pattern to answer these:

$3^2 - 1^2 = 2 \times 3 + 2 \times 1 = 8$

i. $87^2 - 86^2 =$ **ii.** $104^2 - 103^2 =$

iii. $562^2 - 561^2 =$ **iv.** $2\,088^2 - 2\,087^2 =$

d. Discuss with a classmate why the pattern always works.

5. **a.** Kate wonders if she will find a pattern if there is a gap of two in the squares.

Investigate this pattern and explain it using diagrams, sentences, or formulae.

b. What will $10^2 - 8^2$ equal? Try to use the pattern you thought of in question **a.**