MINISTRY OF EDUCATION
*Te Tāhuhu o te Mātauranga*

# Book 3
# Getting Started

**Numeracy Professional Development Projects**

## NUMERACY AND THE CURRICULUM

Numeracy arises out of effective mathematics teaching. All the strands in the mathematics and statistics learning area are important in the pathway to numeracy. Number and Algebra is central to this pathway, although the relative emphasis on this strand changes with the stages of schooling:

- in the first four years of schooling, the main emphasis should be on the Number and Algebra strand;
- in the middle and upper primary years of schooling, the emphasis is spread across all strands of the learning area;
- towards the end of compulsory schooling, number sense becomes a tool for use across the other strands.

The Venn diagrams in the curriculum statement represent this change in balance across the levels.
At all stages, students should:

- create models and predict outcomes, conjecture, justify and verify, and seek patterns and generalisations
- estimate with reasonableness and calculate with precision
- understand when results are precise and when they must be interpreted with uncertainty.

From *The New Zealand Curriculum,* page 26

Effective numeracy programmes provide students with a range of tools and skills necessary for everyday life and future endeavours.

> By studying mathematics and statistics, students develop the ability to think creatively, critically, strategically, and logically. They learn to structure and to organise, to carry out procedures flexibly and accurately, to process and communicate information, and to enjoy intellectual challenge.
>
> *The New Zealand Curriculum,* page 26

Although the groundwork is laid in mathematics, other curriculum areas also provide opportunities for numeracy learning. In addition, the home, early childhood settings, and the community assist in the development of numeracy.

2008 note: Wording in this book has been changed to align with the new curriculum. Pages 16 and 28 will be updated for 2009.

**Numeracy Professional Development Projects 2008**
Published by the Ministry of Education.
PO Box 1666, Wellington, New Zealand.



ISBN 0 478 13383 9
Dewey number 372.7
Topic Dewey number 510
Item number 13383



This book is also available on the New Zealand Maths website, at www.nzmaths.co.nz/Numeracy/2008numPDFs/pdfs.aspx

# Getting Started

## Table of Contents

# Getting Started

## Introduction

The purpose of this book is to provide some detailed research evidence supporting the main components of the New Zealand Numeracy Development Projects (NDP) and to give guidelines for organising your assessment information and developing your classroom programme. It is critical that the rationale for the directions of the NDP and the theoretical / research basis for these directions are made explicit.

The Numeracy Project Assessment (NumPA) is a diagnostic tool that provides a wealth of assessment information about students. There needs to be an appropriate link between this data and the learning experiences you provide for your students through the classroom programme. Your planning should also meet the achievement objectives of the mathematics and statistics learning area of *The New Zealand Curriculum*.

This book covers:

- The Purposes of the NDP
- Quality Teaching of Numeracy
- Teaching Tools in the Numeracy Projects
- Organising Classroom Programmes
- Developing Your Numeracy Programme
- Planning Classroom Programmes
- Student Assessment
- Classroom Resources
- Templates.

*Book 1: The Number Framework* and the NumPA data give a direct window into students' number knowledge and strategy application. This provides clear direction for teaching and allows you to choose appropriate learning experiences for your students.

## The Purposes of the NDP

The purposes of the NDP are to:

1. Improve the achievement of students in Number and Algebra and in the other strands of the mathematics and statistics learning area.
2. Develop the pedagogical and content knowledge of teachers to enable them to meet the learning needs of all their students.
3. Inform schools and their communities about the significance of numeracy to the future lives of their students.

### Quality Teaching and Learning

The essence of the NDP is to promote quality teaching. All the tools and materials provided are designed to help teachers understand the conceptual developments in students' thinking and to offer them an effective model for teaching strategic thinking in number.

For students, the projects should mean different experiences in learning number than would have been the case previously. Students in NDP classes are encouraged to solve number problems mentally and to choose critically from the numeracy strategies at their disposal. The NDP give more emphasis to the use of representations to help students make sense of mathematical ideas and to develop abstract thinking. Less emphasis is given to rote performance of written algorithms to calculate answers.

The NDP have tools that allow you to respond to the thinking of your students in flexible ways and to provide learning experiences that enhance your students' thinking. The projects are not about students learning a sequence of narrow, pre-described mental strategies in number.

## Identifying the Linkages between Number Sense and Algebraic Thinking

The emphasis on mental number strategies has two quite different mathematical purposes, both of which are important.

The historical reason for much of the teaching about number was to provide the tools of computation for a wide range of curriculum areas. These included the standard written forms (algorithms), calculators, and mental estimating to check the reasonableness of answers. This purpose remains valid even in the face of existing and emerging technology.

However, the second, and much more important, purpose for the teaching of number is to encourage mathematical thinking. In particular, algebraic thinking is assisted by flexible mental computation:

> Students use numbers only and so avoid the literal symbols of algebra and their associated difficulties. They nevertheless engage in algebraic activity that involves building generalisations of number properties and relationships.
> Irwin & Britt, 2005[1]

This is why the NDP heavily emphasises flexibility and facility with mental calculation as well as its inherent value in computation.

There is evidence, published internationally, about the positive effect on the algebraic thinking of children who have been part of the NDP (Irwin & Britt, 2005). Consequently, the 2007 curriculum merged the Number and Algebra strands into a single strand of the mathematics and statistics learning area. Looking to the future, the best measure of the long-term success of the NDP should be the improvements in algebra performance in secondary school.

# Quality Teaching of Numeracy

## Dimensions of Quality Teaching

Recent research indicates that quality teaching is critical to the improvement of student outcomes (Alton-Lee, 2003[2]; Hattie, 2002[3]). Classrooms are complex social situations. While acknowledging this complexity, it is possible to describe activities of teachers that make a positive difference to student achievement.

(i) Inclusive Classroom Climate

Successful teachers create social norms in their classrooms that give students the confidence and ability to take risks, to discuss with others, and to listen actively (Cobb, McClain, & Whitenack, 1995[4]). High expectations for student behaviour and provision of a well-organised environment that maximises students' learning time are critical. The valuing of student diversity, academically, socially, and culturally, is fundamental to the development of positive relationships between teacher and students (Bishop et al., 2003[5]).

(ii) Focused Planning

Use of a variety of assessment methods, both formal and informal, to identify the needs of students is critical to quality teaching. From this data, successful teachers target concepts and processes to be taught / learned and plan carefully sequenced lessons. They develop

---

1 Irwin, K. C, & Britt, M. S. (2005). The algebraic nature of students' numerical manipulation in the New Zealand Numeracy Project. *Educational Studies in Mathematics*, 58, 169–188.

2 Alton-Lee, A. (2003). *Quality teaching for diverse students in schooling: Best evidence synthesis.* Wellington: Ministry of Education.

3 Hattie, J. (2002). *What are the attributes of excellent teachers?* A paper presented at the New Zealand Council for Educational Research annual conference. Wellington: NZCER.

4 Cobb, P., McClain, K, & Whitenack, J. (1995). Supporting young children's development of mathematical power. In A. Richards (Ed.), *FLAIR: Forging links and integrating resources* (Proceedings of the 15th biennial conference of the Australian Association of Mathematics Teachers, Darwin). Adelaide: AAMT.

5 Bishop, R., Berryman, M., Richardson, C., & Tiakiwai, S. (2003). *Te kotahitanga: The experiences of year 9 and 10 Māori students in mainstream classes.* Wellington: Ministry of Education.

learning trajectories that map potential growth paths and can "unpack" these trajectories in detail if needed. Students are aware of (and sometimes set) the learning goals. These goals change and grow as learning occurs.

(iii) Problem-centred Activities

Cross-national comparisons show that students in high-performing countries spend a large proportion of their class time solving problems (Stigler & Hiebert, 1997[6]). The students do so individually as well as co-operatively. Fundamental to this is a shared belief, between teacher and students, that the responsibility for knowledge creation lies with the students (Clarke & Hoon, 2005[7]).

(iv) Responsive lessons

Responsiveness requires teachers to constantly monitor their students' thinking and to react by continually adjusting the tasks, questions, scaffolding, and feedback provided. To this end, quality teachers create a variety of instructional groups to address specific learning needs.

(v) Connections

Askew et al. (1997)[8] report that successful teachers of numeracy are "connectivist". Such teachers use powerful representations of concepts and transparently link mathematical vocabulary and symbols with actions on materials. The use of realistic contexts helps students to connect mathematics with their worlds.

(vi) High Expectations

Quality teachers ask questions that provoke high-order thinking skills, such as analysing, synthesising, and justifying, and they have high expectations for student achievement. They encourage students to regulate their own learning, make their own learning decisions, and be self-critical. Successful teachers provide incentives, recognition, and support for students to be independent learners.

(vii) Equity

Success for all students is a key goal, and quality teachers provide extra time for students with high learning needs. They promote respect and empathy in their students for the needs of others.

Uniformly, the research on quality teaching stresses the importance of teachers' pedagogical and content knowledge. Shulman (1987)[9] defined teacher's pedagogical content knowledge (PCK), in part as:

> an understanding of how particular topics, problems, or issues are organized, presented, and adapted to the diverse interests and abilities of learners, and presented for instruction.

It is essential that you know the mathematical content that you are teaching and understand the conceptual difficulties that your students may have in learning this mathematics. This is central to your ability to create coherent, targeted planning to assist this learning. The NDP materials provide several tools to help you to develop your pedagogical and content knowledge.

---

[6] Stigler, J. & Hiebert, J. (1997). Understanding and improving classroom mathematics instruction: An overview of the TIMSS video study. In *Raising Australian standards in mathematics and science: Insights from TIMSS.* Melbourne: ACER.

[7] Clarke, D. & Hoon, S. L. (2005). *Studying the responsibility for the generation of knowledge in mathematics classrooms in Hong Kong, Melbourne, San Diego and Shanghai.* In H. Chick & J. Vincent (Eds.), Proceedings of the 29th conference of the International Group for the Psychology of Mathematics Education, July 10–15, 2005. Melbourne: PME.

[8] Askew, M., Brown, M., Rhodes, V., William, D., & Johnson, D. (1997). *Effective teachers of numeracy.* London: King's College, University of London.

[9] Shulman, L. S. (1987). Knowledge and teaching: Foundations of the New Reform. *Harvard Educational Review, 57(1),* 1–22.

# Teaching Tools in the NDP

The main teaching tools that form the basis of the NDP are:

- a strategy teaching model;
- a framework of increasing sophistication for strategic reasoning;
- a diagnostic interview to be administered orally to every student in a teacher's class.

These teaching tools are all derived from a constructivist research method that involves the close observation of a small number of children (and sometimes of the teacher).

## The Strategy Teaching Model

The strategy teaching model is summarised in Figure 1.

The arrows on the diagram illustrate a dynamic relationship between the phases of Using Materials, Using Imaging, and Using Number Properties. Moving through these phases demonstrates greater degrees of abstraction in a student's thinking.

Progression from Using Materials to Using Imaging is usually promoted by the teacher masking materials and asking anticipatory questions about actions on those materials. Progression to Using Number Properties is promoted by increasing the complexity or size of the numbers involved, thus making reliance on the material representation difficult and inefficient.

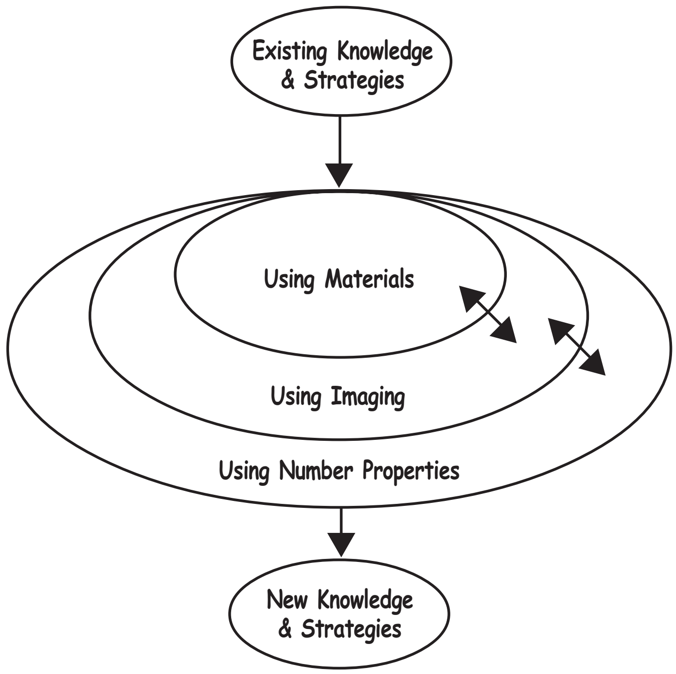



FIGURE 1

Folding back to previous phases of the model is critical as students attempt to connect the mathematical abstraction with the actions on materials. For example, a student's ability to illustrate their mental strategy with materials is evidence of strong understanding of the number properties involved.

Your students' progression through the Using Materials to the Using Imaging stages of the model is unlikely to be successful without targeted input from you. It is fundamental that you use materials and imaging as tools to encourage your students to make the abstractions and you need to observe them as to see if they do make these abstractions. From these observations, you can design opportunities for students to develop increasingly sophisticated number strategies through use of the materials, imaging, recording, and discussion with others. This cannot be done by sending the students away to work independently.

Practice is helpful for reinforcing and extending students in the Using Number Properties sections. You will need to assess when students have demonstrated the abstractions that enable them to work productively from worksheets or textbooks. Restrictions on the use of the teaching model need to occur at the simpler counting stages, as indicated in the table on the next page.

| Strategy Stages | Using Materials | Using Imaging | Using Number Properties |
|---|---|---|---|
| Emergent | ✔ | ✘ | ✘ |
| One-to-one Counting | ✔ | ✘ | ✘ |
| Counting from One on Materials | ✔ | ✘ | ✘ |
| Counting from One by Imaging | ✔ | ✔ | ✘ |
| Advanced Counting | ✔ | ✔ | ✔ |
| Early Additive Part-Whole | ✔ | ✔ | ✔ |
| Advanced Additive–Early Multiplicative Part-Whole | ✔ | ✔ | ✔ |
| Advanced Multiplicative–Early Proportional | ✔ | ✔ | ✔ |
| Advanced Proportional | ✔ | ✔ | ✔ |

Note: In some classrooms, teachers are using the teaching model in a way that was not intended by producing number problem sheets for students to use independently of the teacher. The misapplication of the teaching model comes about when students work on written sheets independently, practising on materials, then practising imaging, and then practising the abstractions without guidance and observation. *This is a serious misunderstanding of the teaching model and should never be encouraged.*

## Theoretical Background to the Strategy Teaching Model

The construction of the teaching model shown in Figure 1 (Hughes, 2002[10]), was influenced by P-K theory (Pirie & Kieren 1989[11], 1994[12], Pirie & Martin 2000[13]).

P-K theory is derived from constructivist teaching experiments. The teaching model is the basis for you to see your teaching as experimental and always in development. The critical idea in such experiments is that you do not tell students how to solve problems. Ideally, problems are given to students so that they grapple with them in order to construct meaning for themselves.

von Glasersfeld (1992)[14] indicates the problematic nature of using materials:

> Mathematics is the result of abstraction from operations on a level on which the sensory or motor material that provided the occasion for operating is disregarded. In arithmetic this begins with the abstraction of the concept of number from acts of counting. Such abstractions cannot be given to students, they have to be made by the students themselves.

---

10 Hughes, P. (2002). A model for teaching numeracy strategies. In B. Barton, K. C. Irwin, M. Pfannkuch, & M. O. J. Thomas, (Eds.), *Mathematics education in the South Pacific* (Proceedings of the 25th annual conference of the Mathematics Education Group of Australasia, Auckland, 350–357.) Sydney: MERGA.

11 Pirie, S., & Kieren, T. (1989). A recursive theory of mathematical understanding. *For the Learning of Mathematics, 9(3).*

12 Pirie, S., & Kieren, T. (1994). Beyond metaphor: Formalising in mathematical understanding within constructivist environments. *For the Learning of Mathematics, 14(1).*

13 Pirie, S., & Martin, L. (2000). The role of collecting in the growth of mathematical understanding. *Mathematics Education Research Journal, 12(2)*, 127–146.

14 von Glasersfeld, E. (1992). Aspects of radical constructivism and its educational recommendations. Paper presented at the ICME-7, Draft to working group #4, Montreal. Retrieved March 18, 2003 from www.umass.edu/srri/vonGlasersfeld/online Papers/html/195.html

In the teaching model, this abstraction is called Using Number Properties when applied to number. Using Materials is potentially very useful as an aid towards Using Number Properties, but von Glasersfeld warns it is not an end in itself:

> The teacher, of course, can help by generating situations that allow or even suggest the abstraction. This is where manipulables (materials) can play an important role, but it would be naive to believe that the move from handling or perceiving objects to a mathematical abstraction is automatic. The sensory objects, no matter how ingenious they might be, merely offer an opportunity for actions from which the desired operative concepts may be abstracted.

Importantly, von Glasersfeld also warns us that we should not underestimate the complexity and difficulty of using number properties:

> one should never forget that the desired abstractions, no matter how trivial and obvious they might seem to the teacher, are never obvious to the novice.

The gap between material and abstraction is often too great for students to bridge. The extensive use of materials without some way of promoting abstraction can lead to problems. Ross (1989)[15] gives a good example:

> even extensive experience with embodiments like base-ten blocks, and other place-value manipulatives does not appear to facilitate an understanding of place value ...

In a similar vein, Hart (1989)[16] noted that an eight-year-old boy told her "bricks is bricks and sums is sums" – he did not connect materials with the desired abstractions. Hart argued that this problem was common, which suggests, in this case, the need for a bridge between "bricks" and "sums". In the strategy teaching model, the bridge is derived from Bobis (1996)[17]. She argued for the power and usefulness of visual imagery:

> The use of concrete materials is important, but rather than moving directly from physical representations to the representations to the manipulation of abstract symbols to explain the traditional abstract procedures of algorithms, it is suggested that the emphasis be shifted to using visual imagery prior to the introduction of more formal procedures. (p 21)

Visualising is referred to as Using Imaging in the NDP.

The strategy teaching model is a subtle tool that should not be reduced to a set of mechanical rules. Indeed, history warns us of the danger of such a reduction. The ideas of the German philosopher Johann Herbart (1776–1841), who foreshadowed many of the ideas of modern-day constructivism, were used to construct a powerful, complex model for classroom teaching. In time, the model was reduced to the "five-step" method and consequently faded out of sight (Ellerton & Clements, 2005)[18]. If you apply the strategy teaching model as a rule-bound set of steps, little of significance can be expected of the students' learning.

---

15 Ross, S. (1989). Parts, wholes and place value: a developmental view. *Arithmetic Teacher, 36(60),* 41–51.

16 Hart, K. (1989). *A lecture on bridging.* A paper presented at 1st New Zealand Association of Mathematics Teachers (NZAMT) Conference. Waikato University.

17 Bobis, J. (1996). Visualisation and the development of number sense with kindergarten children. In J. Mulligan & M. Mitchelmore (Eds.), *Children's number learning: A research monograph of MERGA/AAMT.* Adelaide: AAMT.

18 Ellerton, N. & Clements, M. (2005). A mathematics education ghost story: Herbartianism and school mathematics. In P. Clarkson, A., Downton, D., Gronn, M., Horne, A. McDonough, R. Pierce, & A. Roche (Eds.), *Building connections: theory, research and practice* (Proceedings of the annual conference of the Mathematics Education Group of Australasia, Melbourne, 313–321.) Sydney: Mathematics Education Research Group of Australasia.

## The Strategy Section of the Framework

The NDP's strategy section of the Framework is derived from the work of Steffe et al (1983[19], 1988[20]), Wright (1991[21, 22, 23], 1998[24, 25]) and others. The fundamental basis of their work is the construction of theories about children's numerical reasoning from small-scale, intense observation of children's words and actions rather than large-scale, statistically based studies.

> In a teaching experiment, it is the mathematical actions and abstractions of children that are the source of understanding for the teacher-researcher. (Steffe & Kieren, 1994)[26]

Such observations led to the discovery of a hierarchy of children's counting methods:

> Children with different developmental backgrounds may well be able to get the same answers on an arithmetical task, but the ways in which they do so might differ significantly. (von Glasersfeld, 1992)

From his observations, Wright constructed a strategy framework that was used as the basis for the Mathematics Recovery (MR) programme in New South Wales, Australia. The MR framework-based diagnostic interview was adapted by Wright for use in the Count Me in Too (CMIT) project (for example NSWDET, 1999[27]). In 2000, CMIT was trialled in New Zealand, through an arrangement with the New South Wales Department of Education and Training.

CMIT was restricted to the first three years in school, so it was necessary to extend the strategy framework for use in New Zealand with older and more advanced students. The upper stages of the strategy section of the Number Framework, Advanced Additive–Early Multiplicative Part-Whole, Advanced Multiplicative–Early Proportional Part-Whole, and Advanced Proportional Part-Whole, were created for use in New Zealand from 2001.

These advanced stages were also derived from observations of students' words and actions against a background of research into the growth of place value, multiplicative and proportional reasoning, and fraction concepts (Fuson et al., 1997[28]; Jones et al., 1996[29]; Steffe, 1994[30]; Lamon,

---

19 Steffe, L., von Glasersfeld, E., Richards, J., & Cobb, P. (1983). *Children's counting types: philosophy theory and application*. New York: Paeder.

20 Steffe, L. & Cobb, P., with von Glasersfeld, E. (1988). *Construction of arithmetical meanings.*

21 Wright, R. (1991). An application of the epistemology of radical constructivism to the study of learning. *The Australian Educational Researcher, 18(1),* 75–95.

22 Wright, R. (1991). The role of counting in children's numerical development. *The Australian Journal of Early Childhood, 16(2),* 43–48.

23 Wright, R. (1991). What number knowledge is possessed by children entering the kindergarten year of school? *The Mathematics Education Research Journal, 3(1),* 1–16.

24 Wright, R. J. (1998). An overview of a research-based framework for assessing and teaching early number. In C. Kanes, M. Goos, & E. Warren (Eds.), *Proceedings of the 21st annual conference of the Mathematics Education Research Group of Australasia, 2,* 701–708. Brisbane: Griffith University.

25 Wright, R. J. (1998). Children's beginning knowledge of numerals and its relationship to their knowledge of number words: An exploratory, observational study. In A. Olivier & K. Newstead (Eds.), *Proceedings of the 22nd conference of the International Group for the Psychology of Mathematics Education, 4,* 201–208. Stellenbosh, South Africa: University of Stellenbosh.

26 Steffe, L. & Kieren, T. (1994). Radical constructivism and mathematics education. *Journal for Research in Mathematics Education, 25(6),* 711–733.

27 NSW Department of Education and Training (1999). *Count Me In Too* (professional development package). Ryde: NSWDET.

28 Fuson, K. C., Wearne, D., Hiebert, J., Murray, H., Human, P., Oliver, A., Carpenter, T., & Fennema, E. (1997). Children's conceptual structures for multidigit numbers and methods of multi-digit addition and subtraction. *Journal for Research in Mathematics Education, 28(2),* 130–162.

29 Jones, G., Thornton, C., Putt, I., Hill, K., Mogill, A., Rich, B., & Van Zoest, L. (1996). Multi-digit number sense: A framework for instruction and assessment. *Journal for Research in Mathematics Education, 27(3),* 310–336.

30 Steffe, L., (1994). Children's Multiplying Schemes. In G. Harel, & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics.* NY: State University of New York Press.

31 Lamon, S., (1994). Ratio and proportion: Cognitive foundations in unitising and norming. In G. Harel, & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics.* NY: State University of New York Press.

32 Pitkethly, A. & Hunting, R. (1996). A review of recent research in the area of initial fraction concepts. *Educational Studies in Mathematics, 30,* 5–38. Belgium: Kluwer Academic.

The advanced stages of the Framework were also supported by detailed extension of the unit structures from the early stages by Behr, Harel, Post and Lesh (1994)[33]. Evidence of student achievement from the NDP since 2001 has strongly supported the broad progressions in thinking documented in the Framework (Young-Loveridge & Wright, 2002[34]). This extended strategy section of the Framework is now used in the NDP with students from years 1 to 10.

Note that the average time taken to progress from one stage to the next is not equal. Research has shown that progress through the early counting stages is easier than through the later, more advanced part-whole stages (Young-Loveridge, 2004)[35]. In particular, note that progression between Early Part-Whole and Advanced Additive Part-Whole thinking represents a significant challenge to students (Irwin, 2003[36]). On analysis, this gap appears to be strongly related to students' understanding of place value that requires them to perform operations on numbers rather than merely identify digits in columns. This is the reason that Book 5 (Ministry of Education, 2005) is called *Teaching Addition, Subtraction, and Place Value*. This book emphasises the vital link between place value and operations by incorporating place-value thinking into most activities.

A strategy stage in the Framework refers to a structural type of reasoning in number that students use. For example, one child might solve 8 + 5 by counting from 9 to 13, whereas another might say 8 + 5 is 10 and 3, which is 13. The first child has used an advanced counting strategy and the second an early part-whole additive strategy.

In the NDP, it is important for you to recognise that part-whole thinking is seen as fundamentally more complex and useful than counting strategies. One reason is that counting methods are strictly limited, whereas part-whole methods are more powerful. For example, to work out 98 + 45 by counting is a tedious task. In part-whole reasoning, 98 + 45 is the same as 100 + 43, which is 143; this is efficient and transferable to problems of a similar type. Part-whole reasoning is essential if students are to make sense of multiplicative reasoning and fraction concepts. Counting strategies are an inadequate foundation for these ideas, and this means that for counters, many advanced number ideas are inaccessible. Therefore, your major objective is to assist students to understand and use part-whole thinking as soon as possible. While students learn to partition numbers at varying ages, it is important that you realise that this is the aim for all number teaching at all levels of schooling.

## The Diagnostic Interview

The major purpose for administering the Numeracy Project Assessment (NumPA) to individual students is to help you understand the strategic thinking that is going on in your students' minds.

> Because there is no way of transferring meaning, i.e., concepts and conceptual structures, from one head to another, teachers, who have the goal of changing something in students' heads, must have some notion of what goes on in those other heads. Hence it would seem necessary for a teacher to build up a model of the student's conceptual world. (von Glasersfeld, 1992)

Additionally, the interview has knowledge frameworks. In the NDP, *knowledge* is defined as those things that children can recall quickly with minimal mental effort. For example, when

---

[33] Behr, M., Harel, G., Post, T., & Lesh, R. (1994). Units of quantity: A conceptual basis common to additive and multiplicative structures. In G. Harel, & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics*. NY: State University of New York Press.

[34] Young-Loveridge, J. & Wright, V. (2002). *Data from the Numeracy Development Project and the New Zealand Number Framework*. Paper presented at the 2nd biennial conference of the Teacher Education Forum of Aotearoa New Zealand (TEFANZ), 28–31 August.

[35] Young-Loveridge, J. (2004). Patterns of performance and progress on the Numeracy Projects: Further analysis of the Numeracy Project data. In *Findings from the New Zealand Numeracy Development Project 2004*. Wellington: Ministry of Education.

[36] Irwin, K. C. (2003). *An evaluation of the Numeracy Project for Years 7–10, 2002:* Exploring issues in mathematics education. Wellington: Ministry of Education.

asked what number comes before 67, the child quickly says 66. *Strategy* is defined as the reasoning process by which children use their *knowledge* to find answers. The challenge for you, through the oral interview and regular classroom discussions, is to recognise your students' use of knowledge and strategy.

The distinction between knowledge and strategy has been made artificially to encourage you to consider the implications of "cognitive load" (Sweller, 1994[37]). In problem solving, access to written recording systems can help students to store information, but mental working space is the critical constraint. The knowledge section of the Framework lists the items of knowledge that must require minimal working space to retrieve if problem solving at the given strategy stages is to be successful. As the famous mathematician A. N. Whitehead (1911)[38] said:

> Civilisation advances by extending the number of important operations which we can perform without thinking of them.

The purpose of the second interview at the end of the year is to provide data on the progress of the students through the year. It also provides formative data that will be of use to their teacher the following year.

# Organising Classroom Programmes

## Assessment Information

There are three clear steps to organising your NumPA information:

1. Analysis of knowledge hot spots
2. Assigning strategy stages to students
3. Grouping for instruction.

## Analysis of Hot Spots

Examining the Knowledge section of the NumPA will give you a collective picture of your students' number knowledge as well as highlighting areas of strength and weakness for individual students. From this data, you will be able to identify "hot spots". These hot spots are problematic areas of knowledge that are common to many students in your class and can be addressed in the whole-class knowledge teaching at the warm-up phase of each lesson.

Other knowledge gaps are specific to students in certain strategy stage transitions. Choose suitable knowledge learning activities (from *Book 4: Teaching Number Knowledge*) to target the knowledge needs of your students during your warm-up or group teaching time (refer to the models for daily numeracy lessons on page 13 of this book). Pages 21–23 provide possible formats for recording knowledge hot spots on your class grouping sheet for NumPA.

It is quite common to see a mismatch between a student's strategy stage and their knowledge. This suggests that either the student has more knowledge than they are able to use or that the student has powerful strategies but lacks the knowledge to apply them to more difficult numbers. This is why the identification of knowledge hot spots is crucial for quality teaching.

## Assigning a Strategy Stage

Your initial grouping of students should be by their dominant strategy stage. Stages 0–7 describe the transition from counting to part-whole addition and subtraction strategies for fractions and decimals. Understanding fractions and decimals requires multiplicative thinking. This is why addition and subtraction of fractions and decimals is placed at stage 7 alongside multiplication and division of whole numbers. Responses to the Operational Strategy Windows of the NumPA will dictate which strategy stage to assign to your students.

---

37 Sweller, J. (1994). Cognitive Load Theory, learning difficulty, and instructional design. *Learning and Instruction, 4*, 295–312.

38 Whitehead, A. N. (1911). *Introduction to mathematics.*

Be mindful that students' strategy stages across the three domains may be out of phase. For example, a student might be at stage 5 for both the addition and subtraction and the proportions and ratios domains and at stage 6 for multiplication and division. This student understands how to derive multiplication facts but lacks the addition and subtraction strategies to do so efficiently and has insufficient knowledge to apply multiplicative thinking to fractions. Your initial focus is likely to be on addition and subtraction, so assign the student to their stage for that domain. (This is common where students have learnt times tables by rote.)

Later regrouping for multiplication and division or fractions will need to take into account formative assessment information about the students' progress and knowledge of their characteristics as well as the initial NumPA results.

Assign strategy stage 8 (Advanced Proportional) to students who have reached stage 6 for addition and subtraction and stage 7 for multiplication and division and who have also demonstrated high-level strategies for solving problems with fractions, ratios, and proportions.

## Grouping for Instruction

The students who gain most from grouping situations are those who actively engage in the discussion by asking questions or by answering the questions of others (Brophy & Good, 1986[39]). Grouping students by their strategy stages makes it easier for you to pose problems that are broadly in the students' "zone of proximal development" (Vygotsky, 1978[40]). You also need to consider the personal characteristics of your students in forming instructional groups, including their ability to work collaboratively. Friendship and heterogeneous grouping also have their place in learning classrooms (Higgins, 2005[41]).

Group your students for instruction by their assigned strategy stages for addition and subtraction. You can record this information onto a class grouping sheet for NumPA. Pages 21–22 provide a format for this purpose and a hypothetical example is provided on page 23. Choose the page that has the relevant strategy stages for your class and write each student's name in the relevant column. The progress of a student is shown by highlighting their name through to their new stage. This provides a useful record of achievement.

Most classes display a wide range of strategy stages. This can be managed in many ways, including:

(i) Putting together students from close strategy stages

(ii) Cross-grouping between classes for a few students at the extreme ends of the range

(iii) Using parent or teacher aide help to monitor group or independent work.

In some classes, a large number of students are at one strategy stage. This makes it difficult to monitor students' thinking during the group lessons. Splitting the group is an option. In regrouping, consider the key knowledge that the students have for the strategy transition involved as well as their attitudes, learning preferences, and ability to work collaboratively.

For example, you have 20 students grouped at stage 4, Advanced Counting. Eight students in the group solve problems by using counting on with their fingers. These students have limited knowledge of groupings with ten or basic addition facts. Twelve other students at Advanced Counting solve problems of counting on and back by imaging in their heads and have strong knowledge of doubles and teen numbers. There is a clear division between these students, and this enables you to divide them into two groups with confidence. The students using counting on by imaging are more ready for the transition to part-whole thinking than their counting-on-with-fingers counterparts. Finger counters are likely to need work on imaging before the part-whole transition is successful.

---

[39] Brophy, J. & Good, T. (1986). Teacher behaviour and student achievement. In M. C. Wittrock (Ed.), *Handbook of research on teaching*, 3rd ed. New York: MacMillan.

[40] Vygotsky, L.S. (1978). *Mind and society: The development of higher mental processes.* Cambridge, MA: Harvard University Press.

[41] Higgins, J. (2005). Effective teaching strategies for Māori students in an English-medium numeracy classroom. In *Findings from the New Zealand Numeracy Development Project 2004.* Wellington: Ministry of Education.

# Developing Your Numeracy Programme

## Lesson Groups

Over time, a balanced numeracy programme should contain both knowledge and strategy teaching. It should also involve the use of a variety of grouping situations, depending on the needs of students and the type of learning required. Lessons can have many structures, including:

### *(i) Whole-class Instruction*

This has the benefit of involving all students, thereby enhancing the opportunities for a diversity of responses, effective modelling of the students' strategies for others, and developing a collective learning focus. Whole-class instruction simplifies management and preparation. It poses challenges for the teacher in catering for the diversity of student ability, particularly with older students. Posing open problems that have many solution paths, providing variations of the same task, making activities creative, and using co-operative grouping are useful strategies to cater for different abilities.

Knowledge hot spots provide possible learning outcomes for whole-class lessons. Students can learn to count together, recognise numerals, and practise relevant basic facts effectively in whole-class situations.

### *(ii) Ability Groups*

Base these groups on common strategy stages. Ability groups allow students to work on problems that tightly match the next progression in their learning trajectory. The aim of a group lesson may be the learning of a new type of strategy or the learning of key knowledge that is the foundation for the development of strategies. For example, learning the meaning of fraction symbols (knowledge) is the foundation for using a fraction as an operator (strategy). Ability groups provide intense situations for dialogue and new learning, increasing students' potential for success.

Using ability groups successfully depends on the setting up of good routines and habits for independent work and therefore increases the demands on preparation time. Exclusive use of ability groups can limit students' expectations of themselves.

### *(iii) Mixed-ability Groups*

Create these groups on a social basis that allows compatible students to work together. Provide open problem solving rather than practice tasks. This type of grouping sustains student effort longer than other groupings and allows for students of differing abilities to learn from one another through questioning and explaining (Slavin, 1996[42]). It has the potential to increase students' expectations and contribute to the development of key competencies, such as relating to others, self management, and belonging.

### *(iv) Individual Work*

Individual work allows students the opportunity to reflect on their own personal confidence and their ability to recall knowledge or use strategies. It also provides practice of key skills without the assistance of other students and is a useful source of formative assessment information for you.

## Lesson Structures

This section suggests a variety of lesson models and discusses the strengths and weaknesses of each model.

---

[42] Slavin, R. E. (1996). Research on co-operative learning and achievement: What we know and what we need to know. *Contemporary Educational Psychology, 21,* 43–69.

### *Whole-class to Co-operative Group Lesson*

This model is ideally suited for beginning a unit of work and should be used to focus on knowledge, understandings, and strategies that all students need.

| Phase | Knowledge Focus | Strategy Focus |
|---|---|---|
| **Warm-up**<br>10 minutes | • Share the knowledge hot-spot focus with students as learning outcomes.<br>• Present the concept to be learned through demonstration and discussion with the whole class. | • Introduce the problem to the whole class.<br>• Discuss the conditions of the problem as well as potentially useful strategies and sources of information.<br>• Form the class into co-operative groups. |
| **Work out**<br>30 minutes | • Have practice activities organised as seat work or stations for students to visit.<br>• Students work in groups or independently. | • Students work in teams to solve the problem.<br>• You may bring the class back together to discuss progress.<br>• You may want to make available scaffolding ideas, such as hints and suggested strategies. |
| **Warm-down**<br>5 minutes | • Summarise the learning outcomes from today's lesson, making connections to previous lessons and existing knowledge. | • Students share their solutions with the whole class.<br>• Provide connections to other mathematics and contexts and have students reflect on the power of their strategies. |

### *Group Lesson*

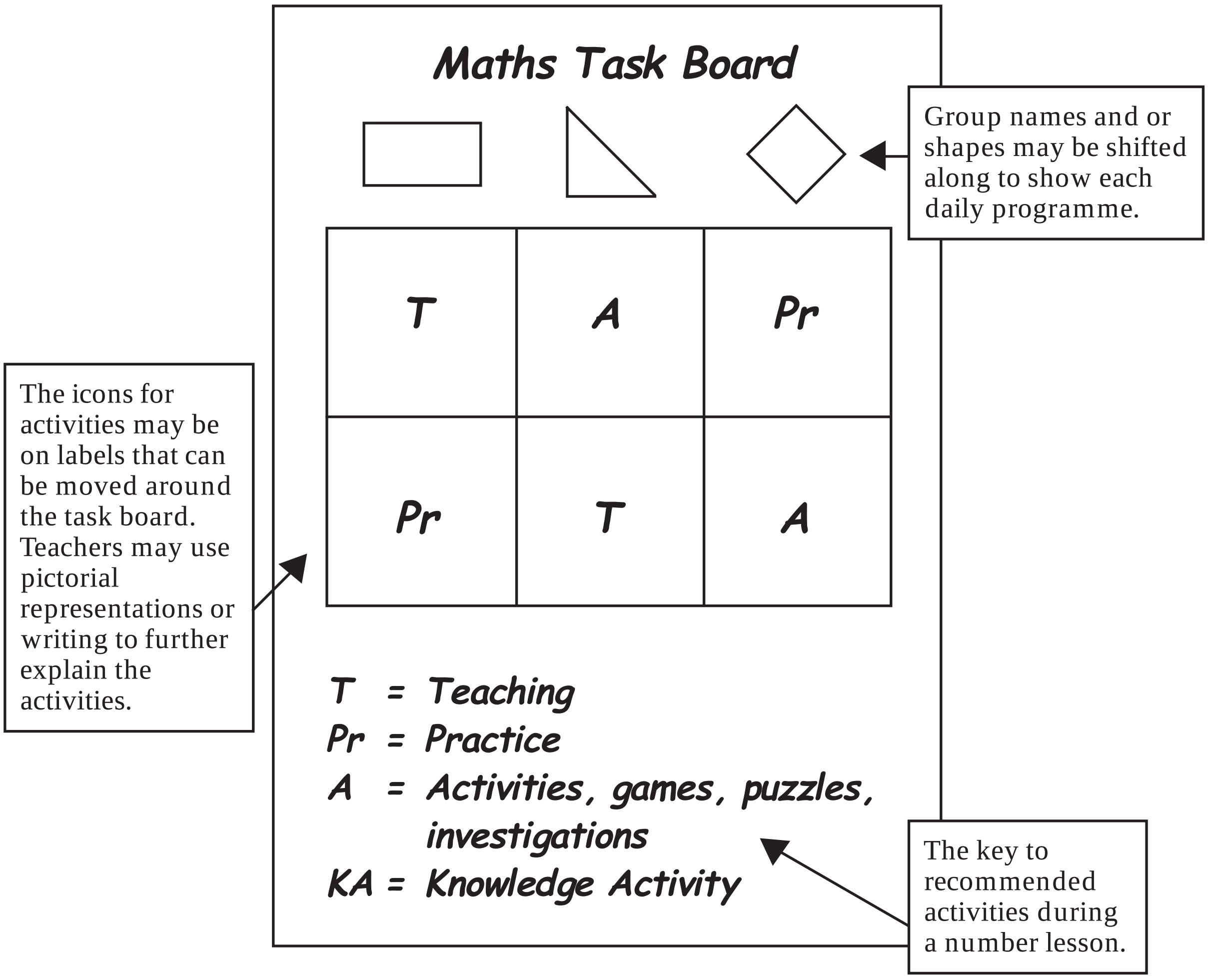
Students at different strategy stages solve operational problems at greatly differing speeds and with varying levels of sophistication. Strategy groups are a useful teaching technique for developing students' mathematical power. They allow for focused teaching of knowledge and strategies.

Below, in table form, is a model for managing a 2-day ability group rotation. It involves three groups. You may chose to use a longer or shorter rotation depending on the students' ability to work independently.

| | Monday | Tuesday |
|---|---|---|
| **Starter** | Whole class activity focused on a knowledge hot spot (optional) | |
| **Group 1** | Lesson with teacher<br>Practice of knowledge or strategy from lesson | Problem solving, games or puzzles, investigations<br>Lesson with teacher |
| **Group 2** | Problem solving, games or puzzles, investigations<br>Lesson with teacher | Practice of knowledge or strategy from lesson<br>Problem solving, games or puzzles, investigations |
| **Group 3** | Practice of knowledge or strategy from lesson<br>Problem solving, games or puzzles, investigations | Lesson with teacher<br>Practice of knowledge or strategy from lesson |

A task board is a useful tool for making students aware of the group rotation and of your expectations for their independent work.

**A Model for Informing Students About Independent Group Work**




# Planning Numeracy Programmes

## Long-term Plans

Long-term plans help with goal setting. They outline a proposed programme of work that will invariably change because of the observations of students' learning and other school events. When filling in your mathematics and statistics long-term plan, remember that in the first four years of schooling, proficiency in number is a critical outcome for your students. In the middle and upper primary years, the emphasis on number needs to be balanced with your students' needs in the other strands. For students at the advanced stages of the Number Framework, Number and Algebra integrates into the teaching of the other strands.

Pages 26–27 provide a format for flexible long-term planning and some hypothetical examples. This format allows you to develop a long-term plan through the course of a year while tracking against the key indicators of *The New Zealand Curriculum*.

## Unit Planning

Many different unit structures are possible. Number units may be generic and deal with the achievement objectives from the mathematics and statistics curriculum learning area in an integrated way, or units may target a specific operational domain, such as addition and subtraction or fractions. Theme units are useful to develop a collection of key mathematical ideas across the strands and help students to connect mathematics to their world.

You need to identify (highlight) the following aspects in planning number units:

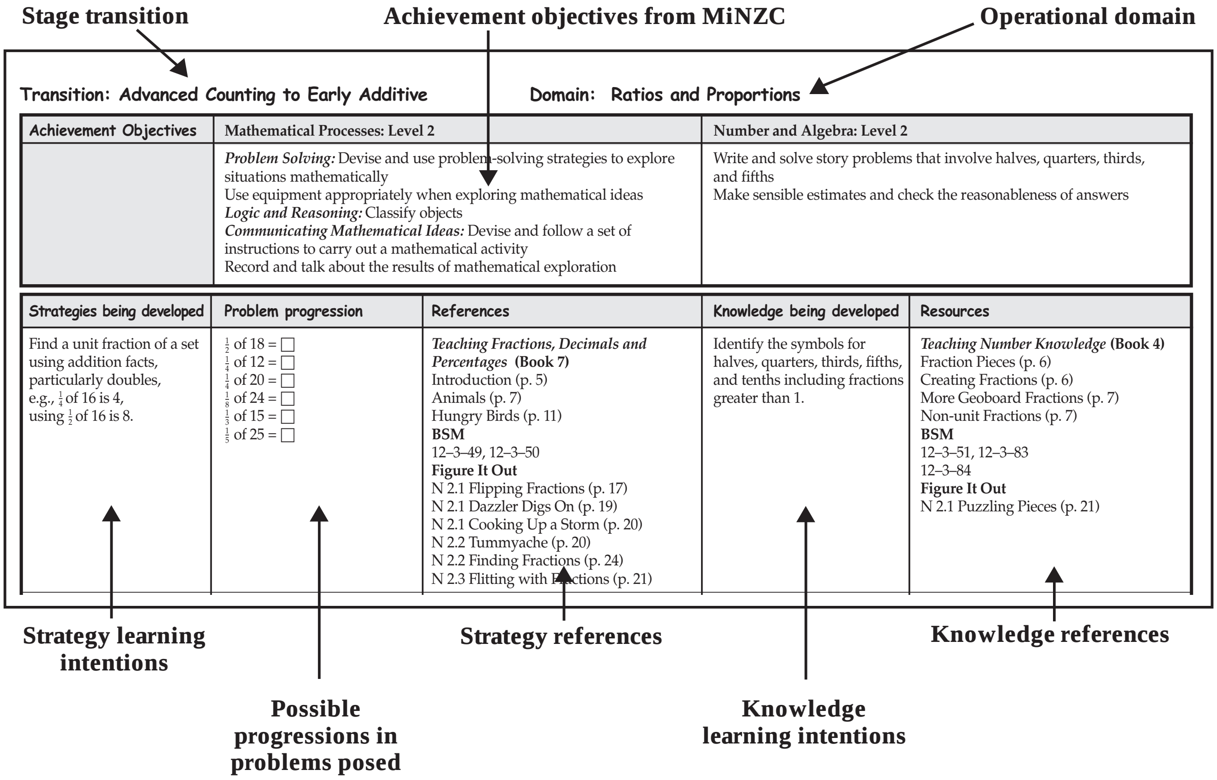
- knowledge learning intentions
- strategy learning intentions
- activities to support this learning
- materials / resources that are required
- assessment tools for monitoring the learning.

It is important to align the knowledge and strategy outcomes. Choosing a small number of learning intentions is preferable to inadequate coverage of many outcomes. To minimise preparation time, the NDP materials include unit plans that can be used and adapted. These plans are organised by stage and operational domain. They can be downloaded from www.nzmaths.co.nz / numeracy / PlanLinks / default.aspx

This link takes you to a table organised by stage transition and domain:

| Stage Transition | Addition and Subtraction | Multiplication and Division | Ratios and Proportions |
|---|---|---|---|
| **Emergent to One-to-One Counting** | E–1 AddSub | E–1 MultDiv | E–1 RatioProp |
| **One-to-One Counting to Count from One on Materials** | 1–CAM AddSub | 1–CAM MultDiv | 1–CAM RatioProp |
| **Count from One on Materials to Counting from One by Imaging** | CAM–CAI AddSub | CAM–CAI MultDiv | CAM–CAI RatioProp |
| **Counting from One by Imaging to Advanced Counting** | CAI–AC AddSub | CAI–AC MultDiv | CAI–AC RatioProp |
| **Advanced Counting to Early Additive** | AC–EA AddSub | AC–EA MultDiv | AC–EA RatioProp |
| **Early Additive to Advanced Additive–Early Multiplicative** | EA–AA AddSub | EA–AA MultDiv | EA–AA RatioProp |
| **Advanced Additive–Early Multiplicative to Advanced Multiplicative–Early Proportional** | AA–AM AddSub | AA–AM MultDiv | AA–AM RatioProp |
| **Advanced Multiplicative–Early Proportional to Advanced Proportional** | AM–AP AddSub | AM–AP MultDiv | AM–AP RatioProp |

An example of a unit plan is on page 28. The first part of a unit will look similar to this (to be updated 2009):




Transition: Advanced Counting to Early Additive Domain: Ratios and Proportions

| Achievement Objectives | Mathematical Processes: Level 2 | Number and Algebra: Level 2 |
|---|---|---|
| | *Problem Solving:* Devise and use problem-solving strategies to explore situations mathematically<br>Use equipment appropriately when exploring mathematical ideas<br>*Logic and Reasoning:* Classify objects<br>*Communicating Mathematical Ideas:* Devise and follow a set of instructions to carry out a mathematical activity<br>Record and talk about the results of mathematical exploration | Write and solve story problems that involve halves, quarters, thirds, and fifths<br>Make sensible estimates and check the reasonableness of answers |

| Strategies being developed | Problem progression | References | Knowledge being developed | Resources |
|---|---|---|---|---|
| Find a unit fraction of a set using addition facts, particularly doubles, e.g., $\frac{1}{4}$ of 16 is 4, using $\frac{1}{2}$ of 16 is 8. | $\frac{1}{2}$ of 18 = $\square$<br>$\frac{1}{4}$ of 12 = $\square$<br>$\frac{1}{4}$ of 20 = $\square$<br>$\frac{1}{8}$ of 24 = $\square$<br>$\frac{1}{3}$ of 15 = $\square$<br>$\frac{1}{5}$ of 25 = $\square$ | ***Teaching Fractions, Decimals and Percentages* (Book 7)**<br>Introduction (p. 5)<br>Animals (p. 7)<br>Hungry Birds (p. 11)<br>**BSM**<br>12–3–49, 12–3–50<br>**Figure It Out**<br>N 2.1 Flipping Fractions (p. 17)<br>N 2.1 Dazzler Digs On (p. 19)<br>N 2.1 Cooking Up a Storm (p. 20)<br>N 2.2 Tummyache (p. 20)<br>N 2.2 Finding Fractions (p. 24)<br>N 2.3 Flitting with Fractions (p. 21) | Identify the symbols for halves, quarters, thirds, fifths, and tenths including fractions greater than 1. | ***Teaching Number Knowledge* (Book 4)**<br>Fraction Pieces (p. 6)<br>Creating Fractions (p. 6)<br>More Geoboard Fractions (p. 7)<br>Non-unit Fractions (p. 7)<br>**BSM**<br>12–3–51, 12–3–83<br>12–3–84<br>**Figure It Out**<br>N 2.1 Puzzling Pieces (p. 21) |

Subsequent pages in the unit have the same headings, but there is no repetition of the achievement objectives. The "Problem progression" column in the unit plan provides problems that you might pose to students. The problems given are arithmetic, but you should frame them in contexts that are motivating to the students. How far you proceed in the problem progression depends on the responses of your students. Solving the final problems in each progression would indicate a strong grasp of the Using Number Properties phase of the teaching model.

Most units constitute a major progression for the students. These units will take more than a few weeks to work through, and you may return to them several times during a yearly programme. Highlight an existing plan using a different colour to indicate each new teaching period. After adapting the unit plans, use a weekly plan format or a planning diary to record the day-to-day details.

Extra references to Figure It Out and Beginning School Mathematics can be obtained by referring to the Numeracy Project Links resource: www.nzmaths.co.nz/numeracy/PlanLinks/default.aspx

## Weekly Planning

Pages 24–25 of this book provide a common model for weekly planning and an example of a completed plan. You may favour more or less detail than this example.

There is a risk of over-planning. It is wise to fill in weekly plans only to Wednesday or Day Three. This allows for flexibility for the rest of the week, particularly when the students have not completed activities or new learning needs are identified. The Numeracy Planning Assistant on the NZ Maths website is useful to identify relevant activities. You can access it at www.nzmaths.co.nz/numeracy/NumeracyPA/

## Student Assessment

It is important that you develop confidence in your judgments about students' thinking. Formative assessment of students that informs the next learning progression is central to the success of your numeracy programme.

"Snapshot" assessment items are helpful in providing a window into students' strategies. These snapshots can take several forms: items that you make up yourself, structured interview questions, or written items where the students communicate their strategies with pen and paper.

The NZ Maths website has four forms of Global Strategy Stage (GloSS), a shortened strategy interview. You can use these to find the strategy stage of an individual student. Use this link to access GloSS: www.nzmaths.co.nz/numeracy/Other%20material/GLoSS.aspx

The website also contains portfolio items in which students solve strategy problems. The usefulness of these tasks is dependent on the ability of your students to communicate their strategies in written form. Often you will need to interview students to discuss their responses to the items. Use this link to access the portfolio examples:
www.nzmaths.co.nz/numeracy/Other%20material/PortfolioExamples.pdf

The project materials also provide a pencil and paper knowledge test/interview. The test is very time-efficient. Access it through this link:
www.nzmaths.co.nz/numeracy/Other%20material/IKAN.pdf

You may wish to maintain individual profiles of your students. These profiles are also useful for developing students' ability to self-assess. The project profile sheets provide a collection of key learning outcomes at each stage transition. They do not state all the possible outcomes. An example of a profile sheet is on page 31 of this book.

Access the full set of profiles through this link:
www.nzmaths.co.nz/numeracy/Other%20material/Profiles.pdf

## Classroom Resources

### Material Masters

Material masters are photocopiable worksheets and resources from which you can make key pieces of cardware referred to in the NDP teaching books. Each school has been supplied with a hard copy of at least one complete set and a CD-ROM that contains all of them. Alternatively, they can be downloaded as PDF files from the NZ Maths website: www.nzmaths.co.nz/numeracy/materialmasters.aspx. Several commercial firms also produce this cardware. Purchasing is often a more cost-effective option than making the materials yourself or having ancillary support make them for you.

If a teaching activity requires a material master, it will be referenced using a code number. For example, material master 5–2 is first introduced in *Book 5: Teaching Addition, Subtraction, and Place Value*. The suffix "–2" indicates that it is the second material master referred to in that book. You will need these material masters to teach many of the strategy development and knowledge lessons effectively.

### Ministry Publications/Resources Available to Schools

#### *Figure It Out Series*

Numeracy-related books available as at March 2008:

Level 2: Number (two books)

Levels 2–3: Number* (two books), Number Sense and Algebraic Thinking (two books), Algebra (one book), Multiplicative Thinking (one book)

Level 3: Number* (four books), Number Sense and Algebraic Thinking (two books), Algebra (one book), Multiplicative Thinking (one book)

Level 3+: Proportional Reasoning (one book)

Levels 3–4: Number* (four books), Number Sense and Algebraic Thinking (two books), Algebra (one book), Multiplicative Thinking (one book)

Level 3–4+: Proportional Reasoning (two books)

Years 7–8: Number* (eight books) , Algebra (four books)

* includes Number, Number Sense, and Basic Facts books

The available books are referenced throughout the planning units on www.nzmaths.co.nz / numeracy / PlanLinks / default.aspx

### nzmaths.co.nz

This website is funded by the Ministry of Education. It contains the following material:

- units of work on all strands of the mathematics and statistics learning area of *The New Zealand Curriculum*.
- NDP materials including PDFs of all the books, material masters, and assessment resources available
- lead teacher material to guide professional learning in your school
- the Results Database for entering and processing your school data
- a Planning Assistant to help you locate resources for your students
- mathematics resources in te Reo Māori
- links to learning objects at the Ministry of Education's DigiStore (you will need your school's user name and password to enter this site) and other useful websites throughout the world
- Bright Sparks section for extending able students
- equipment animations to illustrate key concepts with materials.

The site is continually under development, and you should re-visit it regularly to keep up to date with changes.

### Connected Series

The following collection of references from the Connected series can be used in your numeracy programme. All Connected stories are included on Learning Media's Journal Search CD-ROM each quarter.

| Title | Edition Topic | Stages |
|---|---|---|
| "Two Number Ditties", Part 1, 1999 | Poems about numbers | CA–AA |
| "Patterns", Part 1, 1999 | Sequential patterns | CA–AA |
| "Number, Numbers Everywhere", Part 1, 2000 | Numbers in Daily Life | CA–AA |
| "Eggs in the Nest", Part 1, 1999 | Number Game | AC–AA |
| "The Bubblegum Machine", Part 2, 1999 | Whole Number Place Value | AC–AA |
| "The Giant Who Needed a Bed", Part 1, 2000 | Scale, Whole Numbers | AC–AM |
| "Baby Maths", Part 1, 2001 | Addition and Subtraction | AC–AA |
| "Snails", Part 1, 2001 | Addition and Subtraction | AC–AA |
| "The Monster Birthday Party", Part 1, 2003 | Fractions, Addition / Subtraction | AC–AA |
| "Multiplication Monday", Part 1, 2004 | Multiplication | AC–AA |
| "A Bird's Breakfast", Part 1, 2004 | Fractions | EA–AA |
| "Centre Stage", Part 1, 2005 | Fractions, Ratios | EA–AM |
| "The Playful Frogs", Part 2, 1999 | Number Patterns | EA–AM |
| "The Case of the 13-year-old Grandmother", Part 1, 2000 | Dates, Number Operations | EA–AM |
| "Miss Mind-reader", Part 2, 2000 | Operations | EA–AM |

| Title | Edition Topic | Stages |
|---|---|---|
| "Out of Step", Part 1, 2002 | Number Patterns | EA–AA |
| "Number Hospital", Part 2, 2000 | Classifying Numbers | AA–AP |
| "Magic Muffins", Part 2, 2001 | Operations, Decimals | AA–AM |
| "Multiplication Monday", Part 1, 2004 | Multiplication Patterns | AA–AM |
| "Split Personalities", Part 2, 2003 | Division with Remainders | AA–AP |
| "Time Zones", Part 2, 2004 | Calculations with Time | AA–AP |
| "Pond Puzzler", Part 2, 2005 | Number Patterns | AA–AP |
| "The Chain Goes On", Part 3, 2001 | Powers, Number Patterns | AM–AP |
| "Beetle", Part 3, 2002 | Probability, Fractions | AA–AP |
| "How High is That Tree? ", Part 3, 2002 | Scale, Ratio | AM–AP |
| "Who Wants to be a Billionaire? ", Part 3, 2003 | Place Value | AA–AP |
| "Down for the Count", Part 3, 2004 | Estimating populations | AA–AP |
| "The Right Beat", Part 3, 2005 | Fractions in Music | AM–AP |
| "Nailing It Down", Part 1, 2006 | Counting Strategies | AC–EA |
| "Kynan's Positive and Negative Adventures", Part 3, 2006 | Balancing Numbers | AA–AM |

### Beginning School Mathematics (BSM)

Replacement card material for specific activities is still available online from orders@learningmedia.co.nz. This material is free issue to schools. For references to BSM, refer to the Numeracy Unit Plans on www.nzmaths.co.nz / numeracy / PlanLinks / default.aspx

## Useful Hardware

The list below provides a basic set of equipment for numeracy by class levels. This hardware should be available in every NDP class.

| Item | Years 1–3 Quantity per class | Years 4–6 Quantity per class |
|---|---|---|
| Animal strips | 3 sets | 3 sets |
| Arrow cards (whole nos.) | 5 packs | 5 packs |
| Blank wooden cubes (30mm) | 50 | 50 |
| Craft (iceblock) sticks | Pkt of 1000 | – |
| Deci-pipes | | 1 set (shared) |
| Dice (1–6) | 30 | 30 |
| Film canisters (white) | 30 | 30 |
| Fly flip cards | 2 packs | 2 packs |
| Fraction circles (incl. $\frac{1}{10}$ $\frac{1}{12}$) | 3 sets | 5 sets |
| Fraction tiles (rainbow) | 3 sets | 5 sets |
| Happy hundreds | 10 boards | 10 boards |
| Icecream containers | 10 | 10 |
| Number fans | 10 | 10 |
| Number flip charts (7 digit) | 1 set | 1 set |
| Number lines (0–100) | 5 lines | 5 lines |
| Number lines (decimal) | | 5 lines |
| Number strips (1–20) | 1 set of 12 | – |
| Numeracy play money | 1 pack | 1 pack |
| Numeral flip strip | 1 | – |
| Plastic beans or kiwi eggs | 500 | 500 |
| Rubber bands | 1 bag | 1 bag |

| **Item** | **Years 1–3 Quantity per class** | **Years 4–6 Quantity per class** |
|---|---|---|
| $1 coins | 100 | 100 |
| Slavonic abacus | 1 | 1 |
| Student hundreds boards | 10 boards | 10 boards |
| Teddy bear counters | 160 bears | – |
| Tens frames (dots) | 4 packs | 1 pack |
| Tens frames (blank) | | 1 pack |
| Thousands book | 1 | 1 |
| Transparent counters | 1000 | 1000 |
| Unilink cubes (or similar) | 500 | 1000 |

| **Item** | **Years 7–8 Quantity per class** | **Years 9–10 Quantity per class** |
|---|---|---|
| Animal strips | 3 sets | 2 sets |
| Arrow cards (whole nos.) | 5 packs | 2 sets |
| Blank wooden cubes (30mm) | 50 | 10 |
| Craft (iceblock) sticks | Pkt of 1000 | – |
| Deci-pipes | 1 set (shared) | 1 set (shared) |
| Dice (1–6) | 30 | 30 |
| Film canisters (white) | 30 | 30 |
| Fly flip cards | 2 packs | 2 packs |
| Fraction circles (incl. $\frac{1}{10}$ $\frac{1}{12}$) | 4 sets | 2 sets |
| Fraction tiles (rainbow) | 5 sets | 5 sets |
| Fraction wheels (MM 7–6) | 10 wheels | 10 wheels |
| Happy hundreds | 10 boards | 10 boards |
| Icecream containers | 10 | 10 |
| Mystery stars | 1 set | 1 set |
| Number flip charts (7 digit) | 1 chart | 1 chart |
| Number lines (0–100) | 5 lines | 1 line |
| Number lines (decimal) | 10 lines | 1 line |
| Number stick | 1 | 1 |
| Numeracy play money | 1 pack | 1 pack |
| NumPA assessment kit (2006) | 1 kit | NA |
| OHP calculator | 1 calculator (shared) | 1 calculator |
| Percentage strips (MM 7–4) | 5 sets | 5 sets |
| Plastic beans or kiwi eggs | 500 | 500 |
| Playing cards (jumbo) | 3 packs | 5 packs |
| $1 coins | 100 | 100 |
| Slavonic abacus | 1 | 1 |
| Student hundreds boards | 10 boards | 5 boards |
| Tens frames (dots) | 2 packs | 2 packs |
| Tens frames (blank) | 1 pack | 1 pack |
| Thousands book | 1 | 1 |
| Transparent counters | 1000 | 1000 |
| Unilink cubes (or similar) | 1000 | 1000 |

Each set will require about $60 to be spent on plastic containers, rubber bands, plastic bags, and so on.

# Class Grouping Sheet for NumPA (Stages 0–4)

| Teacher | | School | | Operation Domain | |
|---|---|---|---|---|---|

| Knowledge Hot Spots | |
|---|---|

| Emergent | | | | | | One-to-One Counting | | | | | | Counting from One on Materials | | | | | | Counting from One by Imaging | | | | | | Advanced Counting | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | FNWS | BNWS | NI | PV | BF | | FNWS | BNWS | NI | PV | BF | | FNWS | BNWS | NI | PV | BF | | FNWS | BNWS | NI | PV | BF | | FNWS | BNWS | NI | PV | BF |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

# Class Grouping Sheet for NumPA (Stages 4–8)

| Teacher | | School | | Operation Domain | |
|---|---|---|---|---|---|
| Knowledge Hot Spots | | | | | |

| Advanced Counting | | | | | | Early Additive | | | | | | Advanced Additive | | | | | | Advanced Multiplicative | | | | | | Advanced Proportional | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | FNWS | BNWS | F | PV | BF | | FNWS | BNWS | F | PV | BF | | FNWS | BNWS | F | PV | BF | | FNWS | BNWS | F | PV | BF | | FNWS | BNWS | F | PV | BF |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

# Class Grouping Sheet for NumPA (Stages 4–8)

| Teacher | Ms Print | School | Tapworth Primary | Operation Domain | Addition/Subtraction |
|---|---|---|---|---|---|

| Knowledge Hot Spots | One more/less with whole numbers to 999 999<br>Ordering of unit fractions, improper fractions as mixed numbers<br>Tens, hundreds, thousands in whole numbers to 999 999 |
|---|---|

| Advanced Counting | | | | | | Early Additive | | | | | | Advanced Additive | | | | | | Advanced Multiplicative | | | | | | Advanced Proportional | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | F N W S | B N W S | F | P V | B F | | F N W S | B N W S | F | P V | B F | | F N W S | B N W S | F | P V | B F | | F N W S | B N W S | F | P V | B F | | F N W S | B N W S | F | P V | B F |
| Alby | 5 | 5 | 5 | 4 | 3 | | | | | | | | | | | | | | | | | | | | | | | | |
| Bic | 5 | 5 | 3 | 4 | 3 | | | | | | | | | | | | | | | | | | | | | | | | |
| Colin | 5 | 4 | 4 | 4 | 3 | | | | | | | | | | | | | | | | | | | | | | | | |
| Delia | 5 | 5 | 4 | 4 | 4 | | | | | | | | | | | | | | | | | | | | | | | | |
| Elisa | 4 | 4 | 4 | 4 | 3 | | | | | | | | | | | | | | | | | | | | | | | | |
| Frank | 4 | 4 | 5 | 4 | 3 | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | Gina | 5 | 5 | 5 | 4 | 4 | | | | | | | | | | | | | | | | | | |
| | | | | | | Herb | 5 | 4 | 4 | 4 | 4 | | | | | | | | | | | | | | | | | | |
| | | | | | | Indira | 5 | 4 | 5 | 5 | 4 | | | | | | | | | | | | | | | | | | |
| | | | | | | Joeli | 4 | 4 | 4 | 4 | 4 | | | | | | | | | | | | | | | | | | |
| | | | | | | Kayla | 6 | 5 | 4 | 5 | 4 | | | | | | | | | | | | | | | | | | |
| | | | | | | Lou | 5 | 5 | 5 | 4 | 4 | | | | | | | | | | | | | | | | | | |
| | | | | | | Manuel | 5 | 5 | 5 | 4 | 4 | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | Nadia | 5 | 5 | 5 | 5 | 5 | | | | | | | | | | | | |
| | | | | | | | | | | | | Olive | 5 | 5 | 5 | 5 | 6 | | | | | | | | | | | | |
| | | | | | | | | | | | | Pene | 5 | 5 | 5 | 5 | 5 | | | | | | | | | | | | |
| | | | | | | | | | | | | Quentin | 5 | 5 | 5 | 5 | 5 | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | Rachel | 6 | 6 | 6 | 6 | 6 | | | | | | |
| | | | | | | | | | | | | | | | | | | Somani | 6 | 6 | 6 | 5 | 6 | | | | | | |

## Weekly Numeracy Plan

Teacher:____________ Room:____________ Week:__________ Term:________

| Knowledge hot-spot focus: | | | | | |
|---|---|---|---|---|---|
| | **Monday** | **Tuesday** | **Wednesday** | **Thursday** | **Friday** |
| **Learning Intentions** | **Class warm-up:** | **Class warm-up:** | **Class warm-up:** | **Class warm-up:** | **Class warm-up:** |
| | **Teacher** | **Activity** | **Practice** | **Teacher** | **Activity** |
| | **Practice** | **Teacher** | **Activity** | **Practice** | **Flexi-time** |
| | **Practice** | **Practice** | **Teacher** | **Activity** | **Practice** |
| | **Teacher** | **Activity** | **Practice** | **Teacher** | **Flexi-time** |
| | **Practice** | **Teacher** | **Activity** | **Practice** | **Teacher** |
| | **Activity** | **Practice** | **Teacher** | **Activity** | **Flexi-time** |
| | **Class warm-down:** | **Class warm-down:** | **Class warm-down:** | **Class warm-down:** | **Class warm-down:** |

# Weekly Numeracy Plan

**Teacher:** Ms Print **Room:** 765 **Week:** 3 **Term:** 2

**Knowledge hot-spot focus: Basic multiplication and division facts**

| Learning Intentions | Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|
| | **Class warm-up:**<br>Tables Mountain self check | **Class warm-up:**<br>Creation of flash cards | **Class warm-up:**<br>2× to 4× to 8× Taranaki Sprints | **Class warm-up:**<br>10× to 9× to 11× to 21× TS's | **Class warm-up:**<br>10× to 5× to 6× to 4× to 15× TS's |
| AC-EA Solve multiplication and division problems by repeated addition. | **Teacher**<br>Animal Arrays (see Bk 5: Mult / Div, pp. 5–6) | **Activity**<br>FIO Basic Facts L 2–3, p. 24: Six Shooter, with classmate | **Practice** (Materials)<br>Draw arrays to show:<br>$6 \times 3 = \square$, $8 \times 5 = \square$,<br>$3 \times 6 = \square$, $4 \times 10 = \square$,<br>Which arrays are equal? | **Teacher**<br>Pirate Crews (see Bk 5: Mult / Div, pp. 7–8) | **Activity**<br>FIO Number 2 Bk 2, p. 18: Double Trouble |
| | **Practice** (Use Happy 100s)<br>$6 \times 5 = \square$? $8 \times 2 = \square$?<br>$8 \times 4 = \square$? $10 \times 4 = \square$?<br>If you know $6 \times 3 = 18$, what are:<br>$6 \times 4 = \square$, $7 \times 3 = \square$, $3 \times 6 = \square$, | **Teacher**<br>Animal Arrays (see Bk 5: Mult / Div, pp. 5–6) Cont. | **Activity**<br>FIO Basic Facts L 2–3, p. 9: An Apple a Day | **Practice**<br>$18 \div 3 = \square$, $24 \div 4 = \square$,<br>$36 \div 6 = \square$, $36 \div 3 = \square$,<br>If each pirate in a crew got $7 and there was $2 left over, how much did they have to share? | **Flexi-time**<br>Four in a Row Multiplication<br>Six Shooter<br>Multiplication or out |
| EA-AA Work out new multiplications from known facts. | **Practice**<br>$\square \times \square = 24$<br>What could $\square$ and $\square$ be?<br>$\square \times \square \times \square = 36$,<br>What could $\square$, $\square$, and $\square$ be? | **Practice**<br>What multiplication facts can you work out from ...<br>$6 \times 5 = 30$? $10 \times 5 = 16$?<br>$8 \times 5 = 32$? $4 \times 50 = 200$? | **Teacher**<br>A Little Bit More... (see Bk 5: Mult / Div p. 15) | **Activity**<br>FIO Basic Facts L 2–3, p. 11: Heading for Home; p. 24: Six Shooters | **Practice**<br>$19 \times 5 = \square$? $28 \times 4 = \square$?<br>$102 \times 3 = \square$? $999 \times 7 = \square$?<br>$6 \times 5 = \square \times 6$?<br>$25 \times \square = 8 \times 50$? |
| | **Teacher**<br>Fun with Fives (see Bk 5: Mult / Div, p. 12) | **Activity**<br>FIO Y7–8, Number, Bk 1, pp. 4–5: Fives and Tens | **Practice**<br>$8 \times 10 = 80$, $8 \times 10 = \square$<br>$6 \times 20 = 120$, $6 \times 19 = \square$<br>$7 \times 100 = 700$, $7 \times 102 = \square$<br>What can you work out from $12 \times 25 = 300$? | **Teacher**<br>Turnabouts (see Bk 5: Mult / Div, p. 17) | **Flexi-time**<br>Four in a Row Multiplication<br>Six Shooter<br>Remainder game |
| AA-AM Use place value and tidy numbers to solve multiplication problems. | **Practice**<br>FIO Basic Facts L 3–4, p. 5: How Many Factors? | **Teacher**<br>Multiplication Smorgasboard (see Bk 5: Mult / Div, p. 27) | **Activity**<br>FIO Number L 3 Bk 3, p. 12 What a View!<br>Number L 3–4 Bk 3, p. 14: Dogs' Dinner | **Practice**<br>Find 3 ways to solve ...<br>$16 \times 4 = \square$? $3 \times 27 = \square$?<br>$28 \times 6 = \square$? $5 \times 46 = \square$? | **Teacher**<br>Cut and Paste (see Bk 5: Mult / Div, p. 25) |
| | **Activity**<br>FIO Number L 3 Bk 2, p. 7: Singing Up a Storm; p. 8: Sweet Thoughts | **Practice**<br>Draw your strategy for:<br>$4 \times 23 = \square$? $7 \times 19 = \square$?<br>$6 \times 47 = \square$? $33 \times 5 = \square$? | **Teacher**<br>Multiplication Smorgasboard (see Bk 5: Mult / Div, p. 27) | **Activity**<br>FIO Basic Facts L 3–4, p. 3: Eleventh Heaven | **Flexi-time**<br>Six Shooter<br>Remainder game<br>A Matter of Factor |
| | **Class warm-down:** | **Class warm-down:** | **Class warm-down:** | **Class warm-down:** | **Class warm-down:** |

# Long-term Plan for Mathematics and Statistics (*The New Zealand Curriculum*)

Teacher: ____________________ Room: ____________________ Year/s: ____________________

| Week | Term One | Term Two | Term Three | Term Four |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |
| 7 | | | | |
| 8 | | | | |
| 9 | | | | |
| 10 | | | | |
| 11 | | | | |

| Number and Algebra | | | |
|---|---|---|---|
| Number strategies | Number knowledge | Equations and expressions | Patterns and relationships |
| | | | |

| Geometry and Measurement | | | |
|---|---|---|---|
| Measurement | Shape | Position and orientation | Transform-ations |
| | | | |

| Statistics | | |
|---|---|---|
| Statistical investigation | Statistical literacy | Probability |
| | | |

## Long-term Plan for Mathematics and Statistics (*The New Zealand Curriculum*)

Teacher: ______________________ Room: ______________ Year/s: ______________

| Week | Term One | Term Two | Term Three | Term Four |
|---|---|---|---|---|
| 1 | Statistical investigations “Ourselves” Using CensusAtSchool | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | GloSS/Portfolio/ IKAN | | | |
| 5 | Number strategies/knowledge Addition and subtraction | | | |
| 6 | | | | |
| 7 | | | | |
| 8 | | | | |
| 9 | Shape and space | | | |
| 10 | | | | |
| 11 | | | | |

| Number and Algebra | | | |
|---|---|---|---|
| Number strategies | Number knowledge | Equations and expressions | Patterns and relationships |
| | | | |

| Geometry and Measurement | | | |
|---|---|---|---|
| Measurement | Shape | Position and orientation | Transform-ations |
| | | | |

| Statistics | | |
|---|---|---|
| Statistical investigation | Statistical literacy | Probability |
| | | |

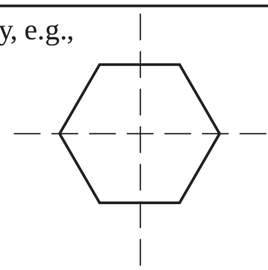
## Transition: Advanced Counting to Early Additive    Domain: Ratios and Proportions    (to be updated 2009)

| Achievement Objectives | Mathematical Processes: Level 2 | Number and Algebra: Level 2 |
|---|---|---|
| | ***Problem Solving:*** Devise and use problem-solving strategies to explore situations mathematically<br>Use equipment appropriately when exploring mathematical ideas<br>***Logic and Reasoning:*** Classify objects<br>***Communicating Mathematical Ideas:*** Devise and follow a set of instructions to carry out a mathematical activity<br>Record and talk about the results of mathematical exploration | Write and solve story problems that involve halves, quarters, thirds, and fifths<br>Make sensible estimates and check the reasonableness of answers |

| Strategies being developed | Problem progression | References | Knowledge being developed | Resources |
|---|---|---|---|---|
| Find a unit fraction of a set using addition facts, particularly doubles, e.g., $\frac{1}{4}$ of 16 is 4, using $\frac{1}{2}$ of 16 is 8. | $\frac{1}{2}$ of 18 = ☐<br>$\frac{1}{4}$ of 12 = ☐<br>$\frac{1}{4}$ of 20 = ☐<br>$\frac{1}{8}$ of 24 = ☐<br>$\frac{1}{3}$ of 15 = ☐<br>$\frac{1}{5}$ of 25 = ☐ | ***Teaching Fractions, Decimals and Percentages* (Book 7)**<br>Introduction (p. 5)<br>Animals (p. 7)<br>Hungry Birds (p. 11)<br>**BSM**<br>12–3–49, 12–3–50<br>**Figure It Out**<br>N 2.1 Flipping Fractions (p. 17)<br>N 2.1 Dazzler Digs On (p. 19)<br>N 2.1 Cooking Up a Storm (p. 20)<br>N 2.2 Tummyache (p. 20)<br>N 2.2 Finding Fractions (p. 24)<br>N 2–3 Flitting with Fractions (p. 21) | Identify the symbols for halves, quarters, thirds, fifths, and tenths including fractions greater than 1. | ***Teaching Number Knowledge* (Book 4)**<br>Fraction Pieces (p. 6)<br>Creating Fractions (p. 6)<br>More Geoboard Fractions (p. 7)<br>Non-unit Fractions (p. 7)<br>**BSM**<br>12–3–51, 12–3–83<br>12–3–84<br>**Figure It Out**<br>N 2.1 Puzzling Pieces (p. 21) |
| Find unit fractions of a continuous region, such as a length or area, using halving. | Find:<br>one-half of a length or area, e.g., circle, length, rectangle, one-quarter, one-eighth, one-third, one-fifth of a rectangle or length. | ***Teaching Fractions, Decimals and Percentages* (Book 7)**<br>Wafers (6)<br>**Figure It Out**<br>N2.1 Puzzling Shapes (p. 21)<br>N2–3 Circle Segments (p. 17)<br>N2–3 Fabulous Folding (p. 18)<br>N2–3 Getting in Shape (p. 19) | Order fractions with like denominators, e.g., $\frac{1}{4}$ and $\frac{2}{4}$. | ***Teaching Number Knowledge* (Book 4)**<br>Fraction Pieces (p. 6) |
| Order unit fractions and fractions with the same denominator and explain why they are larger or smaller. | Which is bigger? Why?<br>$\frac{1}{2}$ or $\frac{1}{3}$, $\frac{1}{8}$ or $\frac{1}{4}$, $\frac{1}{5}$ or $\frac{1}{3}$,<br>$\frac{1}{8}$ or $\frac{1}{12}$, $\frac{1}{17}$ or $\frac{1}{15}$, $\frac{1}{100}$ or $\frac{1}{99}$,<br>$\frac{2}{4}$ or $\frac{3}{4}$, $\frac{5}{8}$ or $\frac{3}{8}$, $\frac{7}{10}$ or $\frac{9}{10}$. | ***Teaching Fractions, Decimals and Percentages* (Book 7)**<br>Fraction Circles (p. 9) | | |
| Order fractions visually using materials, including improper fractions such as $\frac{5}{3}$ and $\frac{7}{4}$, and explain what the numerator and denominator mean. | Make each pair of fractions. Which is bigger?<br>$\frac{2}{3}$ or $\frac{4}{5}$, $\frac{3}{4}$ or $\frac{3}{8}$, $\frac{5}{2}$ or $\frac{9}{4}$,<br>$\frac{6}{10}$ or $\frac{3}{5}$, $\frac{1}{2}$ or $\frac{5}{12}$, $\frac{25}{5}$ or $\frac{25}{4}$. | ***Teaching Fractions, Decimals and Percentages* (Book 7)**<br>Fraction Circles (p. 9) | | |

| E | CA | AC | EA | AA | AM | AP |
|---|---|---|---|---|---|---|

## Student Profile

| | | |
|---|---|---|
| **Name:** | | |
| **Advanced Counting to Early Additive** | | **Date achieved** |
| **I am learning to ...** | | **I can ...** |
| **Knowledge** | | |
| **Read and Count** | Whole numbers up to 1 000, in ones, tens, and hundreds, e.g., 370, 380, 390, 400, 410 ... | |
| **Recall** | How many tens in a three-digit number, e.g., 456 has 45 tens. | |
| **Know** | All the addition facts to 20, e.g., $8 + 7 = 15$ | |
| **Know** | All the $2 \times$, $10 \times$, and $5 \times$ multiplication facts and the matching division facts, e.g., $35 \div 5 = 7$ | |
| **Strategy** | | |
| **Solve + and – problems by:** | Using doubles, e.g., $8 + 7 = 15$ because $7 + 7 = 14$, $16 - 8 = 8$ because $8 + 8 = 16$ | |
| | Using fives, e.g., $7 + 6 = 5 + 2 + 5 + 1 = 13$ | |
| | Making tens, e.g., $28 + 6 = 30 + 4$ | |
| | Joining and separating tens and ones, e.g., $34 + 25 = 30 + 20 + 4 + 5 = 59$ | |
| **Solve × and ÷ problems by:** | Using repeated addition, e.g., $4 \times 6$ as $6 + 6 = 12$, $12 + 12 = 24$ | |
| | Turning multiplications around, e.g., $10 \times 3 = 3 \times 10$. | |
| **Find a unit fraction of:** | A set using halving, e.g., $\frac{1}{4}$ of 20 as $\frac{1}{2}$ of $20 = 10$, and $\frac{1}{2}$ of $10 = 5$ | |
| | A shape using fold symmetry, e.g.,  | |

## ACKNOWLEDGMENTS

The Ministry of Education wishes to acknowledge the following people and organisations for their contribution towards the development of this handbook.

## THE PARTICIPANTS:

The New Zealand numeracy project personnel – facilitators and principals, teachers, and children from more than eighteen hundred New Zealand schools who contributed to this handbook through their participation in the numeracy development projects from 2000–2005.

## THE NUMERACY REFERENCE GROUP:

Professor Derek Holton, convenor (The University of Otago), Professor Megan Clark (Victoria University of Wellington), Dr Joanna Higgins (Victoria University of Wellington College of Education), Dr Gill Thomas (Maths Technology Limited), Associate Professor Jenny Young-Loveridge (The University of Waikato), Associate Professor Glenda Anthony (Massey University), Tony Trinick (The University of Auckland Faculty of Education), Garry Nathan (The University of Auckland), Paul Vincent (Education Review Office), Dr Joanna Wood (New Zealand Association of Mathematics Teachers), Peter Hughes (The University of Auckland Faculty of Education), Vince Wright (The University of Waikato School Support Services), Geoff Woolford (Parallel Services), Kevin Hannah (Christchurch College of Education), Chris Haines (School Trustees' Association), Linda Woon (NZPF), Jo Jenks (Victoria University of Wellington College of Education, Early Childhood Division), Bill Noble (New Zealand Association of Intermediate and Middle Schools), Diane Leggatt of Karori Normal School (NZEI Te Riu Roa), Sului Mamea (Pacific Island Advisory Group, Palmerston North), Dr Sally Peters (The University of Waikato School of Education), Pauline McNeill of Columba College (PPTA), Dr Ian Christensen (He Kupenga Hao i te Reo), Liz Ely (Education Review Office), Ro Parsons (Ministry of Education), Malcolm Hyland (Ministry of Education).

## THE ORIGINAL WRITERS, REVIEWERS AND PUBLISHERS:

Vince Wright (The University of Waikato), Peter Hughes (The University of Auckland Faculty of Education), Sarah Martin (Bayview School), Gaynor Terrill (The University of Waikato School of Education, Carla McNeil (The University of Waikato School of Education), Professor Derek Holton (The University of Otago), Bruce Moody (The University of Waikato School of Education), Lynne Petersen (Dominion Road School), Marilyn Holmes (Dunedin College of Education), Errolyn Taane (Dunedin College of Education), Lynn Tozer (Dunedin College of Education), Malcolm Hyland (Ministry of Education), Ro Parsons (Ministry of Education), Kathy Campbell (mathematics consultant), Tania Cotter, Jocelyn Cranefield, Kirsty Farquharson, Jan Kokason, Bronwen Wall (Learning Media Limited), Dr Gill Thomas, Joe Morrison, Andrew Tagg (Maths Technology Limited).

## REVISED EDITION 2006:

Vince Wright (The University of Waikato School Support Services), Peter Hughes (The University of Auckland Faculty of Education), Tracey Snape and Judy Callaghan (Christchurch College of Education), Judith Mills, Carla McNeil, Belinda Havill (The University of Waikato), Susan Roche (Learning Media Limited), Andrew Tagg (Maths Technology Limited).

In addition, the Ministry of Education wishes to acknowledge Professor Bob Wright (Southern Cross University, Lismore, NSW), Dr Noel Thomas (Charles Sturt University, Bathhurst, NSW), Dr Koeno Gravemeijer (Freudenthal Institute, Utrecht, Netherlands), Jim Martland (The University of Manchester, UK).

The Ministry of Education also wishes to acknowledge The New South Wales Department of Education and Training for permission to trial *Count Me In Too* in 2000 through a one-year arrangement. The findings from the use of this pilot project informed the development of the numeracy policy in New Zealand.



The cover design is by Dave Maunder (Learning Media Limited) and Base Two Design Ltd. All other illustrations are by Noel Eley and James Rae.